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Optimum control of pump operations

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Abstract

The report presents the outcome of a study of the operational control of land drainage pumping stations. After amplifying the multi-objective nature of pumping station operation, a methodology is presented for the optimum control of pump operations (OCOPO). The theory is put into practice through a detailed description of an application of OCOPO to the Newborough pumping station, situated near Peterborough and operated by the North Level Internal Drainage Board. Further developments and wider applications of the methodology are discussed, and conclusions drawn from the research initiative.

Acknowledgements

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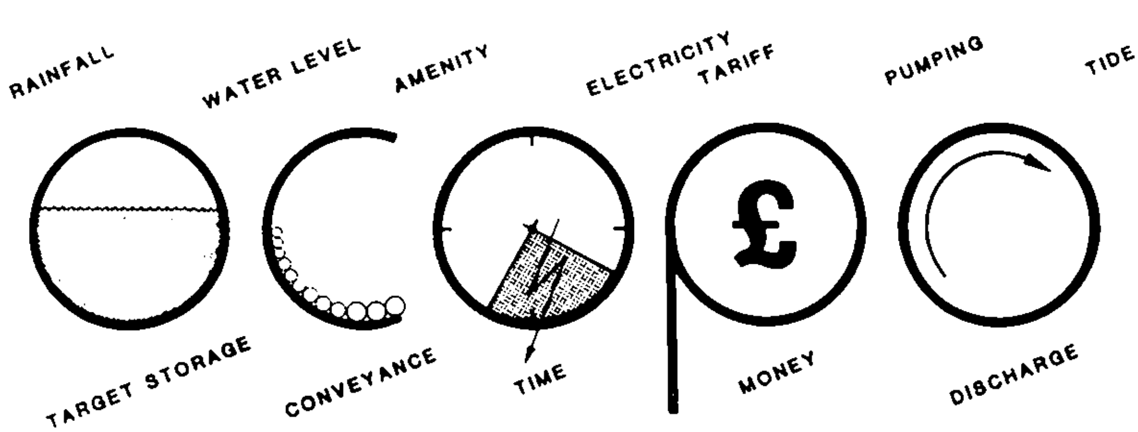
Implementation of OCOPO within the Operational Computer Control system at North Level IDB was carried out by John McMillan of McMillan Computing Services; without his commitment, OCOPO would have remained an interesting but unproven idea.

The contribution of colleagues at the Institute of Hydrology, most notably David Marshall, Adrian Bayliss and Max Beran, is gratefully acknowledged. Paul Samuels of Hydraulics Research is also thanked for sustaining interest in this research.

The views expressed in the report are those of the author alone.



OCOPO: Optimum Control Of Pump Operations

Contents

1 Land drainage pump operations

1.1 Existing practice

The primary objective of pumping station operation is to avoid flooding, usually of high-grade agricultural land. Public safety and security of food production remain watchwords for the operation of land drainage pumping stations. However, to these must be added the need for economy of operation and it is in this context that the Institute of Hydrology has investigated the operational control of pumping stations. The study relates principally to the control of electrically driven pumps.

At present, most pumping stations operate in a semi-automatic fashion, whereby pump control is achieved by local water level sensors and time-switches. A pump-run is triggered when water rises to a pre-set level and continues until it falls below a lower threshold. The time-switch overrides the water level sensors to "disable" pumping within tariff periods where high unit energy costs or penalties apply. During a flood, the water may continue to rise and, at a higher pre-set level, operation of a further pump is triggered. In order to ensure even wear of pumps, and to allow routine maintenance, the sequence in which pumps are introduced is varied from time to time; thus it is usual to speak of the "first duty pump", the "second duty pump", etc..

Electricity supply companies encourage organizations with large but sporadic power requirements to be flexible in their use of energy, particularly on winter weekdays. Many low-lying catchments, reliant on pumped drainage, have drainage channels and basins of substantial dimension; these provide significant short-term *storage* for flood runoff, over and above their primary function of conveying runoff to the pumping station. The relatively slow response of such catchments (Beran, 1982; IWEM, 1987), in comparison to the time-scale of a few hours over which electricity supply companies seek to limit peak demands, gives Internal Drainage Boards (and other drainage authorities) the scope to use this storage to manipulate pumping schedules to minimize energy costs.

In the event of a major flood it may be necessary to "enable" reserve pumps and to suppress the time-switch override. Generally these adjustments have to be made manually. As manning levels are cut to economize on staff costs, the possibility arises of failing to respond adequately to a major flood. Thus there is a requirement to monitor conditions remotely, to provide some form of alert or warning and, in certain cases, to provide fully automatic control of pumping stations.

1.2 The need for control rules

During extreme floods, the storage available in the main drain is required to absorb inflows during periods when the runoff rate from the catchment exceeds the capacity of the pumping station, with conditions in the receiving watercourse sometimes imposing an additional restraint. Thus there is a potential conflict between using storage in the main drain to economize on pumping costs in discharging minor floods, and using it to minimize inundation in the event of a major flood. A further constraint can be a requirement to maintain a relatively high water level (in the main drain) at certain times of year. Institute of Hydrology research suggests that the conflict can be resolved by adopting specific control rules for pump operation.

The need for systematic control rules is being increasingly recognized by many drainage authorities. For example, a programmable logic controller has been implemented at Winestead booster pumping station (see Section 5.4) to represent complex operating rules developed in flowchart form (Moore *et al.*, 1988).

2 Principles

2.1 Designing to meet multiple objectives

There are three main objectives to pumping station operation: flood mitigation, cost minimization and amenity preservation. The last term is used here rather loosely, to refer to a seasonal requirement to maintain the depth of water in the drain within a preferred range, for example to meet irrigation demands.

The principal role of land drainage pumping is to avoid flooding where possible. However, the risk of flooding can never be wholly eliminated. Flood defence works are designed to cope with flood peaks as large as the T-year event, where T defines the return period of flooding and 1/T is the corresponding annual exceedance probability. For rural and suburban areas, use of the 50-year flood is not unusual.

Although there may be some short-term dependence in flood risk - for example, for some time after a major flood it is likely that the ground conditions will be such as to make further flooding likely should heavy rainfall recur - there is no significant long-term memory in weather systems responsible for major flooding in the UK. Thus the risk of experiencing a major flood is independent of the period since the last major flood, if this is more than a few weeks. In particular, the occurrence of an extreme event in one year has no bearing on the likelihood of experiencing a major flood in the following year. It is perhaps for this reason that the practice in Australia is to quote the annual exceedance probability (1/T) rather than the return period.

Correct interpretations of risk require clarity of thought. A useful relationship is the risk formula:

$$r = 1 - (1 - 1/T)^M \qquad [2.1]$$

This gives the risk, r, of experiencing the T-year event in a period of M years. Substituting, for example, values of r=0.5 and M=50, and rearranging, reveals that the flood with an even chance of being experienced in a 50-year design life is the 73-year event.

Energy costs can be reduced in two ways. Most often, this is achieved by encouraging pumping within off-peak periods (when the lowest possible unit energy cost is incurred) and discouraging pumping during the highest-rate periods (when, under some electricity tariffs, the unit energy cost is punitively high). The other possibility is that water can be stored in the main drain until such time as the water level in the receiving watercourse subsides enough to allow drainage by gravity. The effect of water level in the receiving watercourse on pumping station operation is discussed in Chapter 5 (principally in Section 5.6).

Electricity tariffs to industrial users are varied and can be relatively complex; an introduction is given by Price (1986), although such texts soon date. In addition to differential unit energy costs according to time of day, day of week and calendar month, some tariffs prescribe an additional fixed charge according to the maximum demand that the installation actually draws in a given month, quarter or year. Once incurred, the "maximum demand" charge may generate penalty payments through an entire twelve-month period. Because such charges are neither completely fixed, nor wholly related to the quantity of energy used, a general treatment is likely to be very complicated. In practice, it is reasonable to assume that either the operating policy is chosen to conform to the tariff known to be applicable, or a tariff is chosen (or negotiated) such that it accommodates the known operational requirements of the pumping station. Presumably it is not in a drainage authority's interest to advertise too widely any skill gained in adapting to a particular electricity tariff.

While flood mitigation and cost minimization are essential objectives of pumping station operation, there is often a requirement to seek to maintain drain water levels within a specific range. A moderate depth of water may be required for irrigation, "wet fencing" (to discourage animals from crossing the drain), encouraging the feeding and breeding of birds, navigation, and other "amenity" objectives.

Nearly always there is a seasonal element to the request, and typically there is a higher "target" water level during the summer months. This does not imply that the flood risk is necessarily higher in summer. In well-drained permeable soils it is not unusual for appreciable moisture deficits to develop in the summer and early autumn. Thus, severe summer storms - such as those of 26 August 1912 and 26 August 1986 - may arrive when the runoff potential of catchments is intitially low.

2.2 Design and operation

Within UK river engineering practice, it is customary to treat flood design and flood warning as somewhat separate activities. This is convenient and largely justifiable for levees and other fixed structures. However, in the design of flood gates, and other structures which incorporate a major element of control, it is important to consider how this will be exerted in real time, and the extent to which operations can be refined by flood forecasting. The special character of flood control is less well recognized in the UK than in countries such as the USA where there are many reservoirs with a major flood control function.

Land drainage pumping stations exert a high degree of flood control. The pumps determine the discharge of water directly, and exert a marked influence on water levels in the main drain; in some instances, their effect may be felt throughout the drainage system.

Living with an invalid assumption
In practice, it appears that the problem is circumvented by basing the design of one pumped drainage scheme on that of another that has been found to perform satisfactorily. Because of the artificial nature of many pumped catchments, there may be sufficient local homogeneity to make this "design based on experience" approach work well. However, it relies on there being open communication within the profession so that exceptional conditions or difficulties experienced in the operation of a particular pumping station are placed in a wider context, and lessons learned where appropriate.

A further consequence of neglecting the link between design and operation is that the actual standard of flood protection provided by the pumping station will only be known in very loose terms or in retrospect.

The "design based on experience" approach would appear to be particularly vulnerable when applied to pumping stations whose operation can be affected by downstream conditions (see Section 5.6), since these may be site-specific.

Designing for multi-objective operation
The key factor in achieving multi-objective operation is that there should be *flexibility* (i.e. a usable reserve) in the main elements of the drainage system. There are three chief elements: the drain conveyance, the drain storage and the installed pumping capacity.

The operational effectiveness of the pumped drainage system will be degraded if the conveyance characteristics of the main drain are insufficient to sustain a prolonged pump-run. The resultant cyclic operation - with frequent pump starts and stops - is termed "hunting"; the system is constantly looking for, but failing to find, a stable state. The use of mixed-size pumps or variable-speed pumps (see Section 5.2) may avoid this difficulty, in addition to enhancing overall flexibility.

Secondly, if the installed pumping capacity is too small, much of the storage in the main drain will need to be reserved for flood mitigation. There will then be little scope to meet other objectives safely.

Finally, if the provision of excess storage capacity - in effect "freeboard" - in the main drain is neglected, it will not be possible to avoid pumping during periods of peak energy cost or high tide.

Should design practice change?
Ideally, the three main elements of the pumped drainage system would be designed collectively, in the light of clearly stated (multiple) objectives and an intended strategy for pump operation.

The additional costs of constructing a system that allows flexible operation - both in terms of capital cost and any loss of agricultural production consequent upon a larger main drain - has to be balanced against the likely benefits that will accrue, whether these be through reduced energy costs, increased amenity, or greater confidence that the intended degree of flood mitigation will be achieved.

While it is incorrect, in principle, to separate design and operation, it would be unrealistic to expect this to be honoured in practice. A comprehensive analysis linking design and operation would be highly complex. It would require hydrological modelling of the catchment, the construction and testing of operating rules, and a hydraulic model of the main drain; the last item is needed to demonstrate that the conveyance characteristics will suffice.

The practical requirement is more often to re-assess the design and performance of *existing* pumped drainage systems. There may be reasons to favour a particular change. For example, changing pumping rules may be easier than installing an additional pump, while increasing the storage or conveyance characteristics of the main drain may be especially difficult.

2.3 Proving the benefit of new operating rules

It may be difficult to confirm that savings have resulted from the introduction of modified pump operating rules. Unless a fully dynamic hydraulic model of the drain is derived (see Samuels, 1993), it may not be possible to say what would have happened had the new regime not been implemented. A further difficulty is that of having to evaluate performance on what may be a very limited sample. The time at which a flood occurs in relation to the electricity tariff is largely a matter of chance, and the incurrence/avoidance of premium unit energy costs or maximum demand charges in a particular flood may misrepresent the true calibre of the operating rules being followed.

3 A methodology for optimum control of pump operations

In conjunction with field experimentation at Newborough Fen, within the North Level IDB area, the Institute of Hydrology has developed a methodology for the optimum control of pump operations (OCOPO). The basic methodology is now described. Implementation details follow in Chapter 4, while Chapter 5 discusses further developments of OCOPO.

3.1 The concept of target storage

The concept of a preferred range for the water level in a main drain is widely recognized, and is implicit in the pre-set water levels at which pump-runs are triggered or terminated. Reasons why a higher water level may be required at certain times of year have been given in Section 2.1.

In present practice, it is generally the water level at the pumping station (either inside, or immediately outside, the weedscreen) that determines when pump-runs start or stop. Because of the drawdown effect of pumping, the water level at the pumping station is often a poor guide to the real hydrological demand for pumping. A more relevant measure is the *runoff rate*, defined as the flow rate from the catchment to the main drain.

For a natural river draining by gravity, it is generally possible to measure the catchment runoff rate directly, at a flow gauging station upstream of the flood control reservoir; one such instance is the River Wyre flood control scheme (Porter, 1989). However, for a flat catchment controlled by a pumping station, it is only possible to estimate the runoff indirectly, by monitoring the change of storage in the main drain and allowing for the quantity discharged by the pumps. Thus:

$$RO = STO_2 - STO_1 + PUMPED \qquad [3.1]$$

where RO denotes runoff into the main drain, and PUMPED the quantity pumped, in some aggregation period over which the storage in the main drain changes from STO_1 to STO_2.

Implicit in this approach is that the main drain is defined to be that part of the drainage system that is directly influenced by the pumping station. Clearly, there is an element of subjectivity in defining this, although Section 5.1 provides some guidance.

OCOPO exchanges the concept of a preferred water level for that of a target storage (or stock), STO_{tar}, within the main drain. Thus it uses an estimate of the quantity of water stored in the main drain as a key variable.

The storage is estimated in real time by imposing water levels at known points on to a simple 3-D geometrical representation of the main drain. Calibration of the drain geometry model follows Reed (1985); the model for the main drain to Newborough pumping station is illustrated in Section 4.3. In practice, it has been found that two water level sensors can provide a useful estimate of the storage if one is sited (as is usual) at the pumping station, and the other positioned toward the far end of the main drain. Water levels at sites intermediate to the recorders are estimated by linear interpolation, while the water level at the site farthest from the pumping station is taken to be indicative of levels in the main drain system beyond that point.

Calculation of an approximate pump "backwater" length may assist in siting the more distant recorder so that it is more representative of water levels in the catchment than of the recent pattern of pumping. However, the backwater calculation is sensitive to the assumed water level at which a pump-run is triggered. Moreover, empirical friction coefficients are poorly defined for the wide shallow channels, and low gradients, typical of fenland drains (Slade, 1985; Samuels, 1993).

Detailed water level observation at Newborough Fen has shown that the curvature of the longitudinal water surface profile is less marked for pump-runs during major runoff events than for isolated pump-runs occurring during periods of relatively low catchment runoff. A more refined estimate of storage could, however, be contemplated. Possibilities include the deployment of additional water level recorders and/or the adoption of a more sophisticated interpolation method, e.g. one that portrays the longitudinal water surface profile according to the recent pattern of pumping. Section 5.1 discusses the siting of water level recorders.

There is a mean water level associated with a particular storage value. In quiescent periods the longitudinal water surface profile in the main drain becomes horizontal and there is a direct correspondence between water level and storage. That, in the absence of pumping and high wind, the water level in the main drain rises uniformly (i.e. like water in a bath), was demonstrated unequivocally in the research at Newborough Fen (Beran, 1982; Reed, 1985); an example is given later in Figure 4.2.

Having estimated the current storage in the main drain, it is a relatively simple matter to evaluate the excess quantity relative to the target storage:

$$STOEXC = STO_{now} - STO_{tar} \qquad [3.2]$$

STOEXC denotes the stock excess. (The term stock is preferred to storage because the latter can sometimes be confused with storage capacity rather than content.) STOEXC can of course be negative, representing a deficit. The stock excess represents the extent to which current conditions in the main drain depart from the target. Any large discrepancy is referred to as a stock imbalance.

3.2 Determining the rate of inflow to the main drain

A second feature of OCOPO is the estimation of inflow rates to the main drain using Equation 3.1. The runoff volume (from the catchment to the drain), RO_{obs}, during the previous operating period (Section 3.4) is estimated by:

$$RO_{obs} = STO_{now} - STO_{prev} + PUMPED \qquad [3.3]$$

where PUMPED is the quantity of water pumped during the period. If the installed pumps are of similar rating - and the pump characteristic curve is relatively flat (i.e. insensitive to the applied head) - it is feasible to express the terms in Equation 3.3 in units of pump-hours. This makes it much easier to understand the practical significance of any stock imbalance.

3.3 Forecasting the inflow rate

The decision on the number of pumps to use in the next operating period (Section 3.4) is aided by forecasts of the expected inflow to the main drain during that period. A coarse estimate would be to assume that inflow continued at the rate most recently observed. A slightly more reliable estimate would be to take an exponentially weighted average of inflow rates observed over recent periods; the weighting reduces the effect of a poor estimate of storage that might arise for the reasons given in Section 3.1. However, if telemetered rainfall data are available, it is natural to consider use of a rainfall-runoff model to make a more informed forecast of runoff.

Given the relatively slow response of flat, low-lying catchments, a relatively simple model will suffice to forecast runoff over the next operating period. That derived for use in fenland catchments is a nonlinear storage model (Reed, 1984), and is illustrated in Section 4.3.

Rainfall-runoff modelling is prone to error and it is advisable to correct forecasts by reference to recent estimates of actual runoff derived by Equation 3.3. Details of the correction procedure are given on page 21. The outcome is a runoff forecast, RO_{fct}, of the aggregate inflow to the main drain expected in the next operating period. These operating periods are now explained.

3.4 The pump decision algorithm

Operating periods
With a view to the automation of pump decisions, it is helpful to define fixed operating periods according to time of day. The periods are chosen in sympathy with the schedule of times appearing in the electricity tariff. However, in order to ensure that pump operations are reviewed frequently, longer periods are subdivided into two or more operating periods. Given the relatively slow response of fenland catchments, there is no hydrological need to review pump operation more frequently than about every three or four hours. Thus, if the tariff specifies that the cheap-rate period is 00.30-07.30, it is convenient to represent this as two operating periods: 00.30-04.00 and 04.00-07.30. If the schedule of times appearing in the electricity tariff requires particular periods to be shorter than three hours (e.g. 19.00-20.00 on winter weekdays), these are of course adopted. Shorter operating periods would also be required to ensure timely review of pump operations in quickly responding (e.g. heavily urbanized) catchments.

Thresholds for running pumps
The methodology is more easily explained in the case where the pumps are of equal size. For convenience, all volumes are expressed in units of pump-hours, all runoff rates in units of pump capacity, and all times in hours.

Basic runoff thresholds, $ROTB_i$, are set for running i pumps in the next operating period. In order to choose the pumping rate that most closely matches the runoff rate from the catchment, it is appropriate to select i pumps when the forecast runoff lies between i-0.5 and i+0.5.

Thus the basic thresholds are: $ROTB_1 = 0.5$, $ROTB_2 = 1.5$, $ROTB_3 = 2.5$, etc.. These basic thresholds are then modified in the light of the current stock excess, and in respect of the electricity tariff.

The basic thresholds for running i pumps are first modified according to the current stock excess (Equation 3.2), to yield modified runoff thresholds:

$$ROTM_i = ROTB_i - STOEXC/DUR_{rec}, \quad [3.4]$$

where DUR_{rec} denotes the duration over which rectification of the current stock imbalance is sought, excepting adjustments for the electricity tariff.

The runoff thresholds for using i pumps in the next operating period are further adjusted in respect of the electricity tariff, to yield the adjusted runoff thresholds:

$$ROTA_i = ROTM_i + ROADJR + ROADJM_i + ROADJP + ROADJD \quad [3.5]$$

These adjustments are explained in Section 3.5.

The pump decision
The decision is made to run i pumps in the next operating period if the forecast runoff rate, RO_{fct}, exceeds $ROTA_i$. If $RO_{fct} < ROTA_1$, no pumps are to be run. In order to protect the integrity of the pump, it is axiomatic that detection of a "low water" condition within the pumping well over-rules any instruction to continue pumping.

3.5 Adapting to the electricity tariff

The ROADJ terms in Equation 3.5 are differential adjustments to the runoff thresholds which encourage or inhibit pumping according to the electricity tariff. Their values depend on the clock time, day of week, calendar month, and - in the case of ROADJM - on the recent history of pumping.

ROADJR relates to the energy cost rate (or unit energy cost) in the next period. A negative adjustment to the runoff threshold has the effect of encouraging pumping; for example, ROADJR could be set to -0.5 for a cheap rate period.

$ROADJM_i$ relates to the maximum demand charge for starting the ith pump. If this has already been incurred for the accounting period, or if a maximum demand charge is not applicable to pumping in the next operating period, $ROADJM_i$ is set to zero and will not affect the pump decision. However, if starting the ith pump in the next operating period would incur an additional maximum demand charge, $ROADJM_i$ will be positive to discourage pumping. The value of $ROADJM_i$ could also be varied through the calendar month, so that incurring the maximum demand for the ith pump is discouraged more strongly toward the month end than in the early part of the month.

ROADJP is a subtle adjustment which seeks to regulate the profile of pumping through the day, according to the sequence of unit energy costs applying to the various operating periods. The subdivision of electricity tariff periods into two or more shorter periods has been mentioned in Section 3.4. The idea of ROADJP is to distinguish operating periods that share the same unit energy cost, so that pumping is encouraged slightly more in one than another. For example, in the case of the two cheap-rate operating periods (e.g. 00.30-04.00 and 04.00-07.30), ROADJP is given a positive value (e.g. ROADJP=0.2) for the first period, and a negative value (e.g. ROADJP=-0.2) for the second period, to encourage a pump-run more in the second period than the first. This is desirable because, should the runoff rate be higher than expected, this will have less consequence in the first period (when there will be a further cheap-rate period to follow) than in the second period (when a higher unit energy cost will be faced in the subsequent period). Similar adjustments can be used to assign minor preferences for pumping in one standard-rate period than another, on days when a higher unit energy cost period intervenes (e.g. 16.00-19.00 on a winter weekday).

A further refinement is to encourage pumping rather more on a Monday (when the next weekend is distant) than on Friday (when it is imminent), in situations where the tariff provides more scope for economic pumping at weekends. This day-of-week adjustment, ROADJD, might take a value of -0.2 on Monday, -0.1 on Tuesday, 0.1 on Thursday, 0.2 on Friday, and 0.0 on other days. The version of OCOPO implemented at North Level IDB excludes the ROADJD term from Equation 3.5 but achieves a similar effect by adjusting the target stock, STO_{tar}, on a daily basis. However, it is conceptually neater to make the adjustment through the ROADJD term, so that STO_{tar} is reserved exclusively to define the amenity objective (see page 22).

3.6 Summary

The decision as to the number of pumps to be run in the next period depends on whether the forecast runoff rate, RO_{fct}, is greater than a given threshold. Uncertainties in RO_{fct} are reduced by correcting the forecast according to recent water level and pump-run observations. The basic thresholds are adjusted according to the current stock imbalance and, in various ways, according to the electricity tariff. Unfavourable pump-runs (e.g. that would incur high unit energy costs or maximum demand penalties) are discouraged, while favourable ones are encouraged.

The OCOPO methodology ensures that pump-runs incurring higher unit energy costs or maximum demand charges *will* be recommended (or initiated) if runoff rates are sufficiently high. The approach has the merit of following operating criteria that are fully defined and quantified.

Because it continually seeks out forecasts of the runoff expected in the next operating period, OCOPO responds to changing hydrological conditions far more quickly than is possible using only water level data at the pumping station intake. The correction of runoff forecasts - by reference to recent observations of storage variations and pumped quantities - means that OCOPO cannot be seriously misled by incorrect or unrepresentative rainfall measurements.

A further merit of the approach is that, because the pump decision algorithm is fully coded, it is possible to incorporate this within an automatic telemetry and pump control system, such as AFCOPS. This is now discussed.

4 Experimental application

4.1 The AFCOPS system for telemetry and pump control

North Level IDB has pioneered a system for the Automatic Flood Control Of Pumping Stations (AFCOPS). The system provides telemetered information on pumping station operation, drain water levels, and rainfall. In addition, the system allows automatic control of pumping stations, either locally (by water level) or remotely (by computer and telemetry); in the latter case, the pumps can be operated with, or without, intervention by IDB personnel.

AFCOPS is based on radio telemetry (Charnley & Ewen-Smith, 1985); this contrasts with the use of telephone communication in the Welland & Deepings IDB telecontrol scheme (Roberts, 1986). The outstations monitored by AFCOPS include ten pumping stations, 18 water level recorders and two raingauges. In addition to centralized computer control, it is possible for IDB engineers to interrogate outstations, and control pump operations, from home or vehicle, using a mobile data terminal (Charnley & Ewen-Smith, 1985). The system includes audible alarms to alert staff to exceptional conditions, such as failure of the power supply to a pumping station. Visual display of water level variation is of particular assistance in confirming that pumping stations and drains are behaving as anticipated. The "hands on" nature of AFCOPS means that IDB engineers can use their knowledge of particular drainage systems and electricity tariffs to ensure that cost-effective operation is achieved.

AFCOPS also provides a comprehensive log of system transactions, including data archiving.

Beyond the contribution of IDB engineers, AFCOPS owes much to its principal executors: Essex Communications and Spectronics MicroSystems (initially) and McMillan Computing Services (from 1985).

4.2 The OCOPO software for optimum control of pump operations

Where AFCOPS ends, and OCOPO begins, is blurred by the integrated nature of the control software, sometimes referred to as the Operational Computer Control (OCC) system. This now comprises code written by McMillan Computing Services in the programming language C, running on a personal computer. In addition to managing the telemetry and pump control interfaces, and other tasks, the OCC system implements the Institute of Hydrology's OCOPO method.

OCOPO takes water level, rainfall and pump-run data supplied by the OCC system, evaluates the current storage in the main drain, forecasts runoff in the next operating period, and decides on the number of pumps to be run. In certain phases of system development, OCOPO has controlled pumps directly. At other times, OCOPO has fulfilled a "decision support" role, with the pump control being managed by IDB engineers.

Application of OCOPO to a particular pumped catchment requires a drain geometry model and calibration of a rainfall-runoff model. Because of the broadly similar nature of fenland catchments, and the fact that the rainfall-runoff model has only a limited influence on the pump control, the model developed for Newborough Fen has been adjusted for catchment area and used on neighbouring catchments.

The Institute of Hydrology's research and development of OCOPO was commissioned by MAFF. Because of the strategic nature of the commission, it was inappropriate that this should underwrite further application to North Level IDB catchments. This report provides sufficient detail about the method to allow drainage board engineers to apply OCOPO to additional pumped catchments, if desired.

Implementation of OCOPO within AFCOPS was carried out in conjunction with McMillan Computing Services. The choice of programming language (i.e. C rather than FORTRAN) was made by McMillan Computing Services. Despite their use of "modular" programming techniques, the code for the OCC system remains complex and highly specialized. At least in part, this is inevitable, given that the system carries out many functions (e.g. outstation monitoring, pump control, data archiving and analysis reporting) in addition to executing the OCOPO software. A comprehensive manual was not available to the Institute of Hydrology at the time of writing this report, but is believed to be in preparation. The OCC system is the responsibility of McMillan Computing Services, except insofar as it relies on theory and parameter values supplied by the Institute of Hydrology.

The Institute of Hydrology holds FORTRAN code for the original version of OCOPO, as tested against experimental data recorded at Newborough Fen. However, because pump decisions interact directly with the hydraulic behaviour of the main drain, it is not possible to test the real-time correction features of OCOPO - which are known to be important - without coupling to a comprehensive hydraulic model of drain behaviour. In any event, detailed experimental data are available for one further catchment only: Boy Grift pumping station operated by the Alford Drainage Board (Marshall, 1993).

It is recommended that any further development or testing of OCOPO should be by implementation in particular telemetry and pump control schemes.

4.3 Example catchment: Newborough Fen

The Newborough Fen catchment is described by Marshall (1989), who reports the instrumentation deployed in the Institute of Hydrology's "Drainage of low-lying land" project commissioned by MAFF. The drainage area assessed for use in the operational study was 32.5 km^2, slightly smaller than the value given by Marshall. The pumping station houses three units, each with an estimated discharge capacity of 1.548 m^3s^{-1}.

The fen has a dominant main drain, stretching some 6 km back from the pumping station. As part of the field experiment, water level recorders were placed along it at intervals of about 2 km (see Figure 4.1). Early work (Beran, 1982) found that, in the absence of pumping, water levels rise synchronously throughout the main drain, as illustrated in Figure 4.2.

Drain geometry model

Dimensions of the main drain were taken initially from design drawings of the Newborough scheme. The main drain is of trapezoidal section with 1:2 side slopes; the bed width tapers from 5.9m close to the pumping station to 1.1m at a chainage of 6 km.

With the above definition of "main drain" it was found that stock changes inferred from the water level data were substantially smaller than the quantities pumped in isolated pump-runs. This suggests that the storage on which the pumping station "pulls" extends beyond the 6 km of main drain. In developing an empirical model of the extended drain system, the dimensions of the main drain were first checked by survey.

Detailed surveys at the four water level recording sites (Figure 4.1) were complemented by rough surveys at a further 19 sections. To aid comparison of data from different sources, the drain geometry was summarized in terms of the channel width at selected reference heights (0.0 mAOD and -2.0 mAOD) and plotted against chainage (i.e. distance measured along the drain) from the pumping station. The outcome is Figure 4.3.

Generally reasonable agreement was found between the detailed and rough surveys, but the surveyed channel widths were found to be somewhat narrower than values taken from the design drawings of the scheme.

An empirical model of drain geometry was derived by regression analysis of the surveyed channel widths, giving greater weight to the data derived from the detailed survey. The model is:

$$W(h,x) = 16.4 + 3.85\,h - 1.18\,x \qquad [4.1]$$

where W is the water surface width (m) at water level h (m AOD) and chainage x (km) from the pumping station. The model can be interpreted as a tapering triangular drain with side slopes of 1:1.925 (i.e. 2/3.85 m/m), a longitudinal bottom slope of 1:3260 (i.e. 1.18/3.85 m/km), and a width of 16.4 m at the pumping station for a water level of 0.0 mAOD. A visualization of the model is given as Figure 4.4. That in reality the cross-section of the drain is trapezoidal rather than triangular is of no consequence since the essential use of the model is in the calculation of stock *changes*.

A feature of lowland pumped catchments is that, at times of very low inflow, water levels along the drain respond to an isolated pump-run in a characteristic manner (see Figure 4.2). Knowing the quantity of water pumped, it is possible to adjust the parameters of the drain geometry model to achieve a closer fit to the actual storage variations observed. The adjusted drain geometry model for Newborough Fen is:

$$W(h,x) = 18.6 + 4.37\,h - 1.41\,x\,. \qquad [4.2]$$

This re-calibrated model is identical to Equation 4.1, except that all widths are 13.5% greater. It is thought that this represents the effect of side drains and, perhaps, some soil water storage in the banks of the main drain.

More generally, the drain is represented by a longitudinally tapering channel of triangular cross-section defined by:

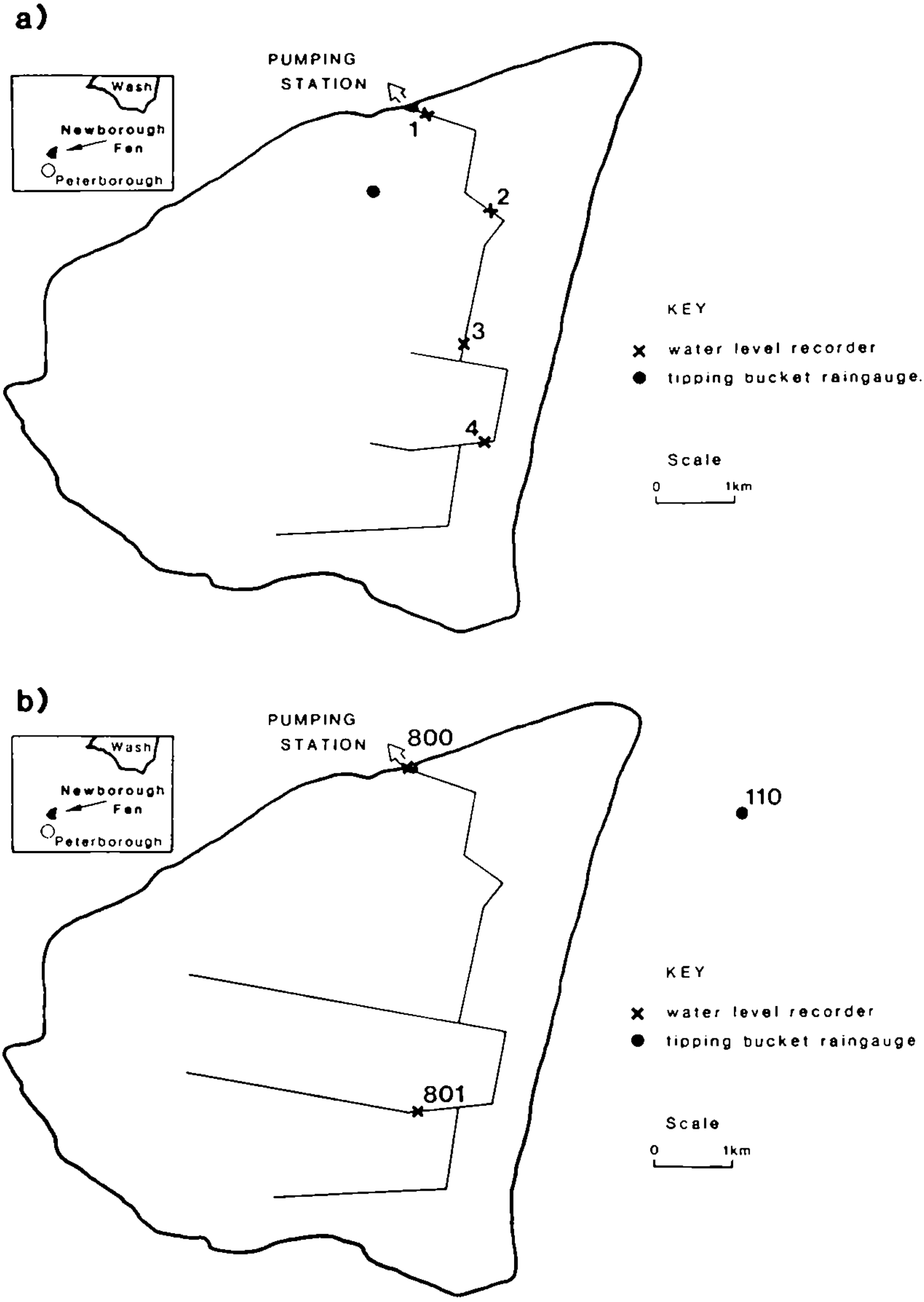


Figure 4.1 *Catchment plan, Newborough Fen a) Sites used in research b) Sites used in OCOPO implementation*

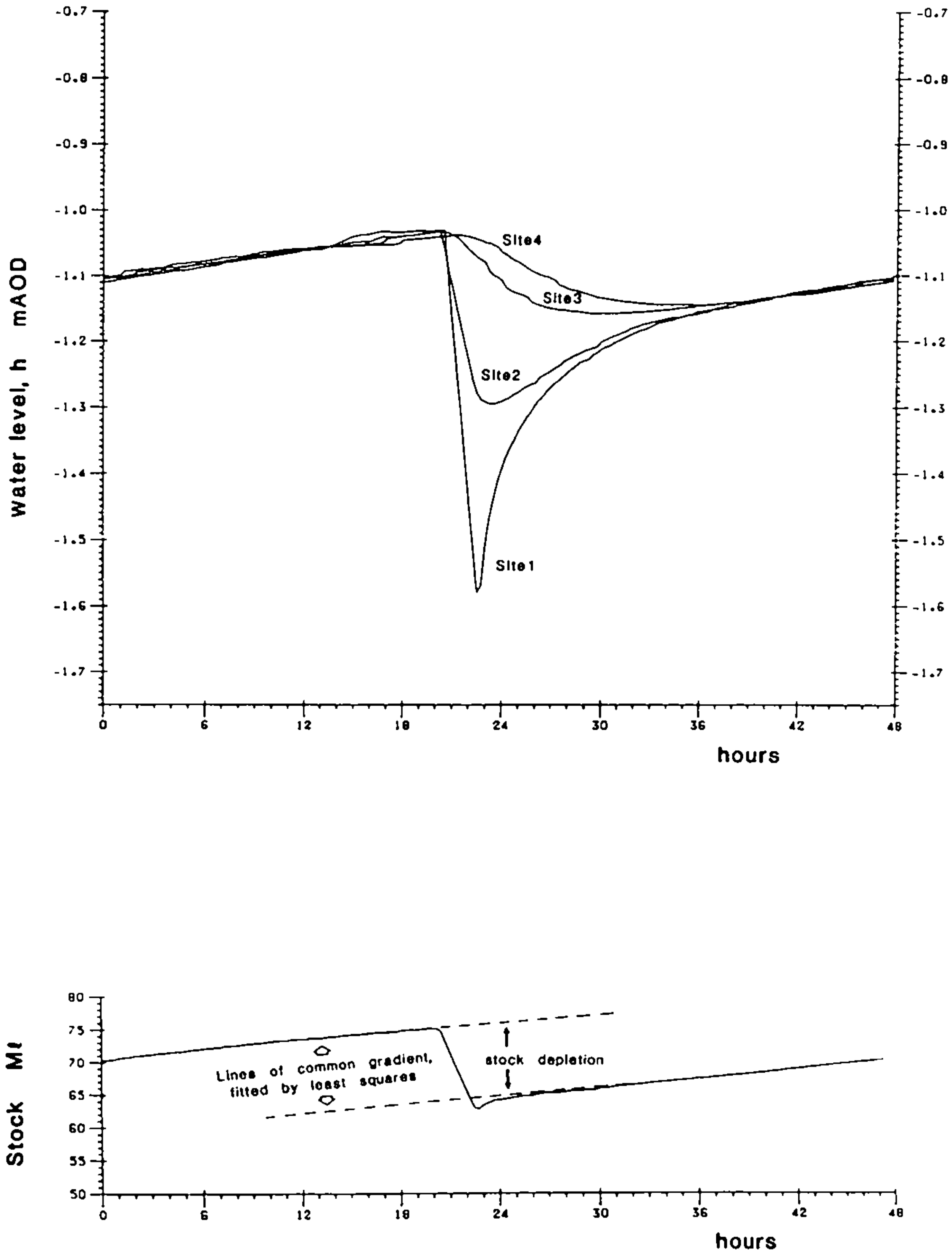


Figure 4.2 *Water level and stock variation for an isolated pump-run at Newborough Fen, 20/21 May 1981*

$$W(h,x) = a_0 + a_1 h - a_2 x \quad [4.3]$$

The side slope of the drain is $2/a_1$ m/m and the longitudinal bottom slope is a_2/a_1 m/km.

For static (i.e. horizontal) water levels throughout the main drain, the following relations give the length, L, the surface area, A, and the volume (or stock), V, of water in the main drain:

$$L(h) = (a_0 + a_1 h)/a_2 \text{ km} \quad [4.4]$$

$$A(h) = (a_0 + a_1 h)^2/(2a_2) \text{ m km} \quad [4.5]$$

and

$$V(h) = (a_0 + a_1 h)^3/(6a_1 a_2) \text{ m}^2 \text{ km (i.e. Ml)} \quad [4.6]$$

As referred to earlier, the drain geometry model is only required to represent stock differences. Inserting the parameter values for Newborough from Equation 4.2 into Equation 4.6 shows, for example, that there is a stock difference of 96.2 Ml between h=-1.0 mAOD (a typical retention level) and h=0.0 mAOD (a typical flood level). For a pump rating of 5.574 Ml/h (i.e. 1.548 m^3s^{-1}), the intervening storage is seen to be equivalent to 17.25 pump-hours. This suggests that, at Newborough, there is considerable scope to exploit storage in the main drain to avoid pumping during unfavourable tariff periods.

There is a requirement to be able to estimate the stock at times when the longitudinal water surface profile is slanted. In the real-time implementation to control Newborough pumping station, water level data are telemetered from recorders at the pumping station and at a site on the main drain 5.6 km away. Level readings from this remote site are taken to represent water levels beyond that section; i.e. it is assumed that the water surface profile is horizontal. Between the remote site and the pumping station a uniform gradient is taken to apply. Relevant equations for calculating the stock from water level readings are given in Appendix A.

It should be noted that a drain geometry model derived from design drawings can be checked without extensive instrumentation. The procedure is to monitor the water level at the pumping station for a few days before, and a few days

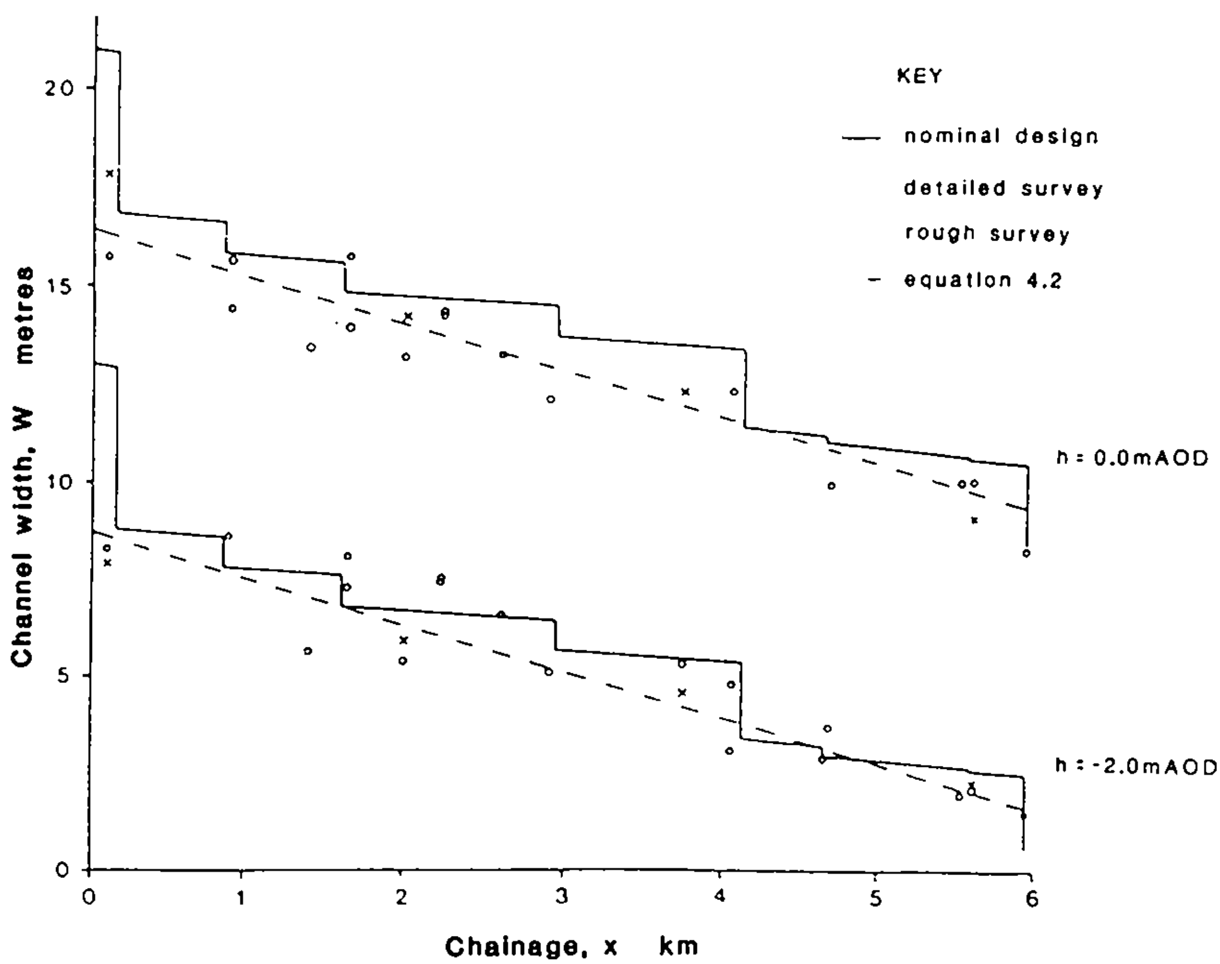


Figure 4.3 *Main drain geometry in terms of channel width*

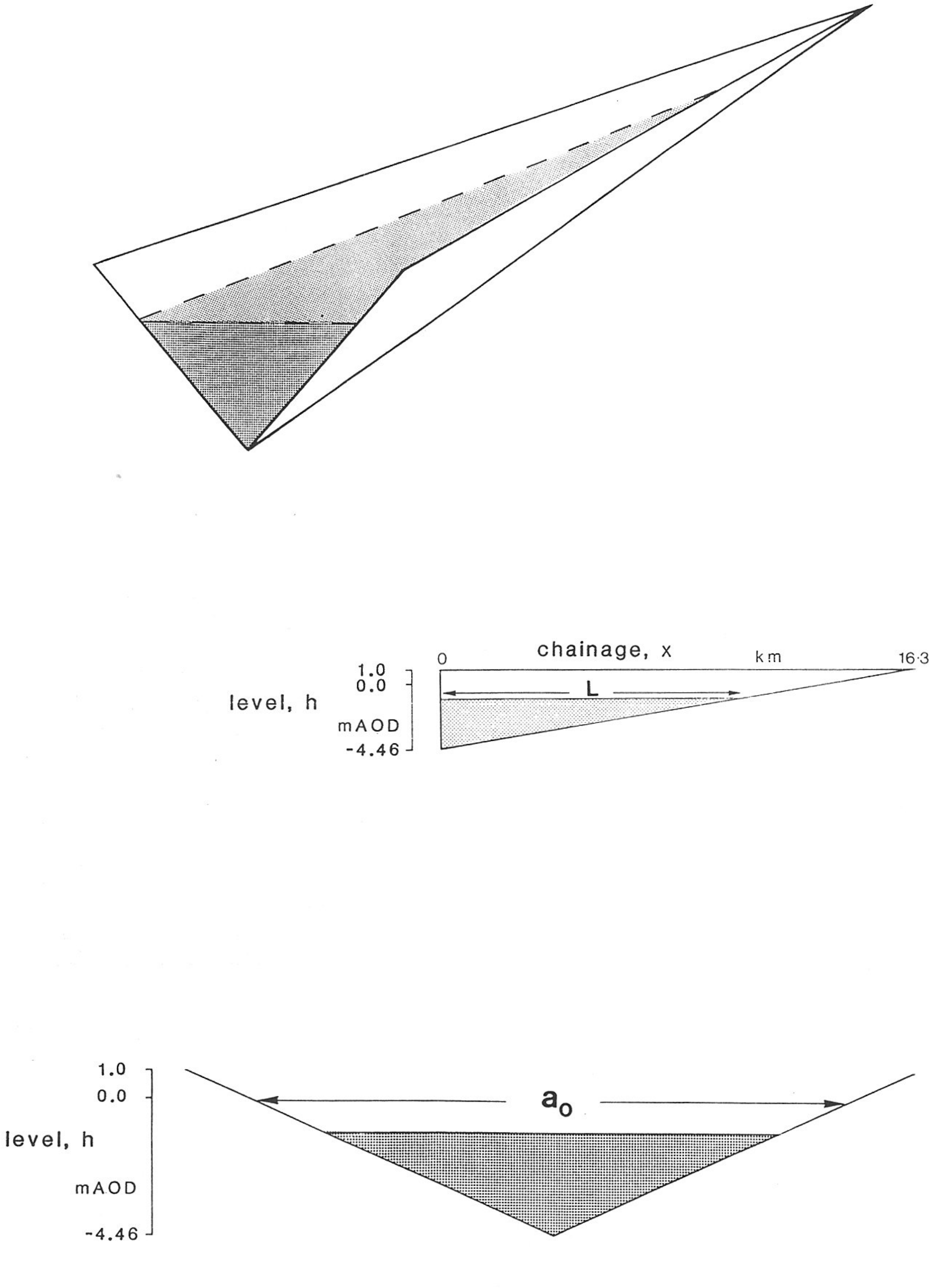


Figure 4.4 *Visualization of the tapering triangular model of drain geometry*

after, an isolated pump-run of known length occurring during a prolonged dry spell. Because wind stress can distort water levels (Marshall & Beran, 1985), it is advisable to avoid use of a period in which gales occurred. A graph of stock similar to Figure 4.2b is constructed using Equation 4.6, and the stock depletion arising from the pump-run compared to the known quantity pumped. As can be seen from Figure 4.2a, the water level at the pumping station is indicative of water levels throughout the main drain except during, and immediately following, the pump-run. If the modelled stock depletions are consistently smaller or larger than the quantities pumped, a correction factor - such as the 1.135 at Newborough - can be applied to the drain geometry model.

While it is desirable that the pump rating should be known accurately, a further advantage of re-calibrating the drain geometry model as above is that this can compensate for an error in the pump rating. In essence, it is the link between drain storage in pump-hours and pumping rate in pumps that matters; to know absolute values of either is of secondary importance.

Rainfall-runoff model

The hydrological response of the Newborough catchment is represented by a relatively simple rainfall-runoff model that links rain falling on the 32.5 km² catchment to the resultant runoff rate. For the purpose of rainfall-runoff modelling, the main drain is considered as a concentrated storage occupying no area. In theory, it would be correct to distinguish rain falling directly on the main drain from that falling on the remainder of the catchment, since the former suffers no "loss" (i.e. all of the rain enters the drain) whereas the latter suffers losses due to surface detention and infiltration (i.e. only a proportion of the rain enters the drain). However, from Equation 4.5 it can be shown that the surface area of water stored in the main drain forms only a very small fraction of the drainage area (about 0.4% at a flood level of 0.0 mAOD), making the refinement unnecessary.

The catchment rainfall is estimated by a tipping bucket raingauge, sited just outside the catchment (see Figure 4.1); this registers each 0.5 mm of rainfall that accumulates. For the type of rainfall-runoff model used here, it is convenient to express the rainfall rate and runoff rate in common units of mm h^{-1}.

The rainfall-runoff model used at Newborough Fen is a nonlinear storage model, illustrated in Figure 4.5. Although the version of the model implemented is relatively simple, its derivation is reasonably intricate, as is now described.

Reed (1984) defines a nonlinear storage model by the equations:

$$n_t = \text{ROP}\ p_t \qquad [4.7]$$

and

$$dq/dt = (n_{t-L} - q_t)\ dq/dS\ , \qquad [4.8]$$

where ROP denotes the runoff proportion and n_{t-L} is the net rainfall lagged by time L. L is termed the pure time delay, since all rainfall entering the storage is first delayed by time L. The term "net rainfall" means net of any losses. The term "losses" includes any process by which rainfall is prevented from running off. For fenland catchments, the most obvious losses are infiltration and surface detention (e.g. on vegetation or in puddles).

The function dq/dS determines the "routing" behaviour of the nonlinear storage. If dq/dS=1, the discharge is directly proportional to the quantity of water in the store; this corresponds to a linear storage, in which the discharge decays exponentially if the input, n_t, is zero. Because the rainfall-runoff model uses a single storage to represent many physical processes, it has no simple conceptual interpretation. The function represents the routing effect as the net rainfall is temporarily detained by vegetation, reaches the ground, passes through the upper layers of the soil, then to a field drain or minor watercourse, before finally arriving at the main drain.

In the version used here, the runoff proportion is assumed constant:

$$\text{ROP} = c \qquad [4.9]$$

and the routing function corresponds to a type I ISO-function (Lambert, 1972; Reed, 1984; Lambert & Reed, 1986):

$$dq/dS = q/k\ . \qquad [4.10]$$

The parameter c is a runoff coefficient (dimensionless), while k denotes the characteristic depth of the soil moisture store (mm). Thus the model reduces to the differential equation:

$$dq/dt = q\ (n - q)/k \qquad [4.11]$$

The solution is considered in two cases, according to whether the lagged net rainfall is zero. For n=0:

$$dq/dt = -\ q^2/k\ , \qquad [4.12]$$

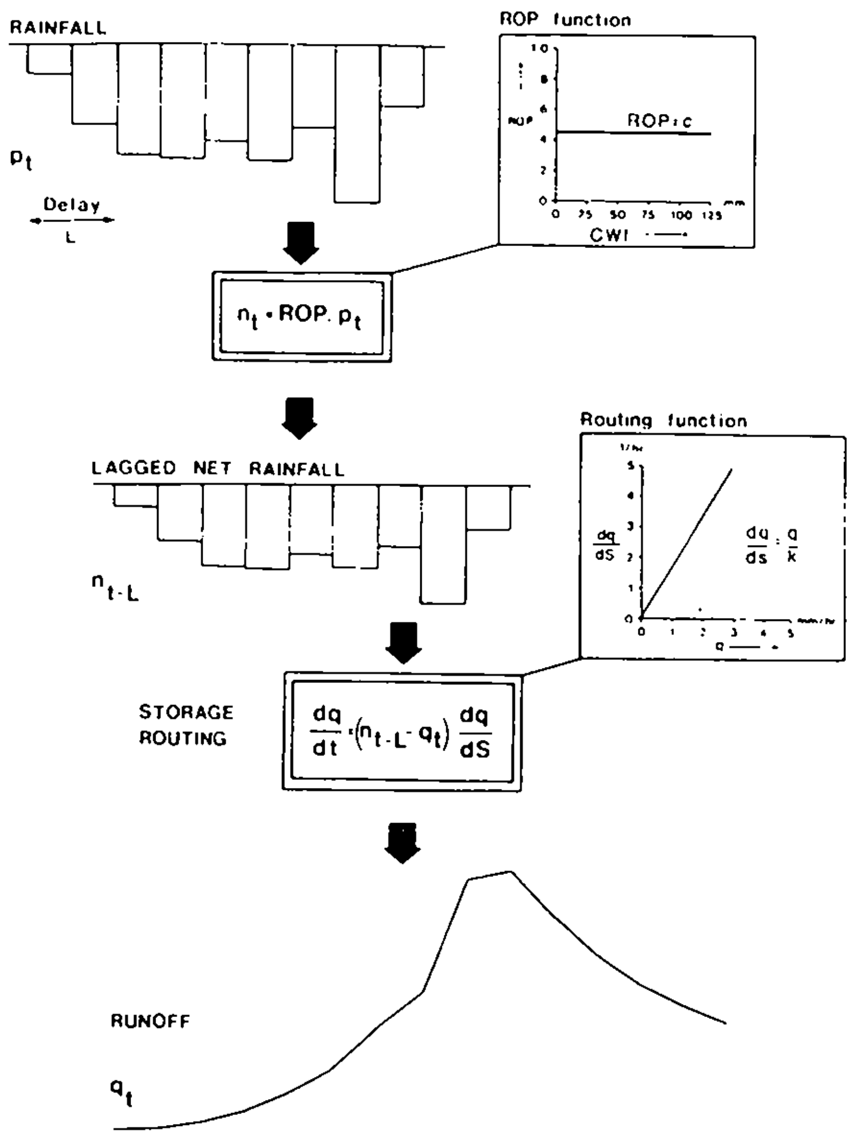


Figure 4.5 *Nonlinear storage model (after Reed, 1984)*

which, on integration, yields the solution:

$$1/q = 1/q_0 + t/k\,, \quad [4.13]$$

where $q=q_0$ at $t=0$. For $n>0$ we have:

$$dq/dt = q\,(n - q)/k\,, \quad [4.14]$$

which yields the solution:

$$1/q = e^{-nt/k}/q_0 + (1-e^{-nt/k})/n\,. \quad [4.15]$$

Calibration of the model on historical rainfall and runoff data gathered for Newborough Fen (see Reed, 1985, for a description of how the inflow series was calculated) led to adoption of the parameter values: c = 0.45, k = 5.41 mm, and L = 3.5 hours. The value of c means that the model derives runoff from 45% of the rainfall. The OCOPO code presently uses the reciprocal of k, i.e. a = 0.185 mm^{-1}, as the parameter defining the routing function.

Real-time version of rainfall-runoff model
The low gradient and predominantly rural nature of fenland catchments give rise to a relatively slow response to rainfall. Thus there is no hydrological requirement to consider rainfall data at a very fine time interval; research at Newborough Fen suggests that a data interval of 3 hours would suffice. However, implementation of the rainfall-runoff model to simulate and forecast runoff in real time is complicated by other factors, notably the electricity tariff and the

need to consider operational periods of uneven length. Use of a half hour data interval allows a match to be obtained with the operational periods applicable at Newborough, and is therefore adopted as the basic data interval in the real-time implementation.

The telemetry system maintains, and updates, a record of half-hourly rainfall depths over the last 24 hours. Because of the need to allow time for telemetered data to be gathered and processed, and the pump decision software executed, the periods used in assessment are a quarter of an hour in advance of the pump operating periods. Thus, for example, the decision on the number of pumps to be used in the operating period beginning at 07.30 is based on telemetered observations up to 07.15. The various periods relevant to operation of Newborough pumping station are listed in Table 4.1.

Table 4.1 *Schedule of assessment and operating periods*

Index	Operating period	Assessment period
1	00.30 to 04.00	19.45 to 00.15
2	04.00 to 07.30	00.15 to 03.45
3	07.30 to 12.00	03.45 to 07.15
4	12.00 to 16.00	07.15 to 11.45
5	16.00 to 19.00	11.45 to 15.45
6	19.00 to 20.00	15.45 to 18.45
7	20.00 to 00.30	18.45 to 19.45

At each assessment, the modelling is carried out in two steps. First, Equations 4.13 and 4.15 are used to simulate runoff in the period since the last assessment (e.g. 03.45 to 07.15). One or other equation is applied each half hour, according to whether the lagged net rainfall is zero. The initial runoff rate (i.e. that at 03.45 in the above example) is taken as the runoff rate at the corresponding time estimated as part of the previous assessment, whereas the final runoff rate (i.e. at 07.15) is saved for use in initializing the runoff simulation next time round. Then the average runoff rate, $ROMOD_1$, over this first period (i.e. 03.45 to 07.15), is evaluated and converted from mm h^{-1} to units of pump capacity (see Section 3.4).

The second step is to extend the modelling over the next assessment period, i.e. 07.15 to 11.45 in the example. In this instance, the second period is 4.5 hours long, one hour longer than the 3.5 hour pure delay time in the rainfall-runoff model. As a consequence, the runoff forecast is partly dependent on yet-to-be-observed rainfall. Rather than seeking a rainfall forecast, the model simply assumes that rainfall beyond time "now" is zero. The rainfall-runoff modelling task ends by evaluating the average runoff rate, $ROMOD_2$, over the second period (i.e. 07.15 to 11.45), again expressing this in units of pumps. An alternative approach would be to extend the modelling over the pump operating period (i.e. 07.30 to 12.00); however, this would fit in less easily with the use of half-hourly rainfall data. It is adequate, certainly for these slowly responding fenland catchments, to use an estimate of runoff rate that is 15 minutes in arrears.

Refinement of runoff forecasts

Modelling, or forecasting, runoff from rainfall data alone is subject to large uncertainty. There are many reasons for this. Firstly, the observed rainfall may not be representative of catchment rainfall as a whole. Secondly, rainfall-runoff models oversimplify catchment response processes. A weak point in the rainfall-runoff model used here is the assumption that the proportion of rainfall that contributes to runoff in the short term (i.e. the runoff proportion, ROP) is constant. In reality, the runoff coefficient will depend on the current wetness of the catchment, being generally much smaller (than 0.45) when the soils are dry, and possibly larger when they are already saturated. A complication for fenland catchments is that the runoff rate into the main drain is influenced by the water level, which is itself influenced by the recent history of pumping. Thus it is highly desirable that modelled, and forecast, quantities of runoff are corrected by reference to observed runoff rates.

"Observed" runoff over the previous period is evaluated using the water balance procedure described in Section 3.2. In applying Equation 3.3 it is necessary, of course, to evaluate the quantity over the relevant assessment period (e.g. 03.45 to 07.15).

In correcting the modelled runoff rate, it is assumed that the relative error (i.e. the factorial error) exhibited by the rainfall-runoff model over the last few hours is likely to persist into the next operating period. A correction factor is calculated as the ratio of observed to modelled runoff rates, i.e:

$$CF = ROOBS_r / ROMOD_r ,$$

where the suffix r denotes that both quantities are weighted averages calculated recursively. Thus:

$$ROOBS_r := (ROOBS_r)^{1-w} (ROOBS_1)^w$$

and

$$ROMOD_r := (ROMOD_r)^{1-w} (ROMOD_1)^w$$

where the weight, w, is defined as the ratio of the duration of the assessment period to a reference duration, i.e.

$$w = DUR_{ap}/DUR_{ref}$$

DUR_{ref} represents a typical catchment response time; a nominal value of 12 hours is appropriate for the fenland catchments studied. Clearly it is preferable if the real-time correction is based on runoff observations over several assessment periods. This is achieved by tailoring operational periods to the response characteristics of the catchment (as well as the structure of the electricty tariff), e.g. ensuring that no assessment period is longer than 0.5 DUR_{ref}.

The correction factor, CF, is then applied to correct the modelled runoff, $ROMOD_2$, over the forthcoming assessment period:

$$RO_{kt} = CF\ ROMOD_2$$

This yields the corrected runoff forecast required by the pump decision algorithm (see Section 3.4).

Target stock
The concept of a target retention level, RET_{tar}, in the main drain is fundamental to OCOPO. At Newborough Fen, the target water level adopted is -0.75 m AOD in summer and -1.00 m AOD in winter. The summer target applies in April to September, while the winter value applies in November to February. March and October are transition months in which RET_{tar} is adjusted day by day.

The target retention level in m AOD is transformed to a target stock in Ml using Equation 4.6. The summer and winter values are 97.5 and 78.0 Ml respectively. Finally, the target stock is converted to pump-hours by dividing by the pump rating of 5.574 Ml/hour (i.e. 1.548 m^3s^{-1}). The resultant summer and winter values of STO_{tar} at Newborough are 17.5 and 14.0 pump-hours respectively.

4.4 Example electricity tariff

The DMA mod3 tariff operated by East Midlands Electricity is characterized by high unit energy costs between 07.30 and 20.00 on winter weekdays (November to February inclusive), and punitively high unit energy costs between 16.00 and 19.00 on weekdays in December and January. The tariff provides a cheap-rate period from 00.30 to 07.30. Relative to the standard unit energy cost, the cheap rate is approximately half as expensive, while the high and punitively high rates are about 1.5 and 8.5 times more expensive, respectively. The operating periods (Table 4.1) are chosen to fit in with the key times in the tariff.

4.5 Application of OCOPO to other catchments

OCOPO has been applied to two further pumping stations operated by North Level IDB. Postland is a somewhat similar set-up to Newborough pumping station. While the installed pumping capacity is broadly equivalent, the available storage in the main drain is notably smaller than at Newborough (see Table 4.2). This suggests that there is less flexibility to meet one objective (e.g. minimizing flood risk) without violating the others (i.e. cost minimization and amenity preservation).

Some further developments were required to accommodate Dog-in-a-Doublet, the other pumping station to which OCOPO has been applied. These are discussed in Sections 5.1 and 5.2.

4.6 Performance monitoring

The Operational Computer Control (OCC) system, within which the OCOPO software sits, provides both a hard-copy log of key system variables (if the printer is switched on) and summary information in tabular and graphical form (on demand). Visual inspection of water level variation over time is helpful in confirming that the drain, pumping station, telemetry system and OCOPO are behaving as expected. The hard-copy log was designed as an aid to system debugging and is not particularly user-friendly.

The key variables for monitoring performance are water levels (m AOD), the stock excess (in pump-hours), the number of pumps recommended to be operated, the number of pumps actually operated, and the number of pump-hours subsequently achieved. Clearly it is of interest to evaluate the number of hours of pumping achieved at the various unit energy cost rates.

Table 4.2 *Catchment-specific parameters*

	Units	Newborough	Postland	Dog-in-a-Doublet
Catchment area	km^2	32.5	25.7	24.8
Pump capacity (and number)	m^3s^{-1}	1.548 (#3)	1.204 (#3)	1.105 (#2) 0.708 (#2)
Max. pumped runoff rate	$mm\ h^{-1}$	0.515	0.506	0.526
Summer retention level	m AOD	-0.75	-0.75	-1.45
Winter retention level	m AOD	-1.00	-1.00	-1.45
Drain geom. model parameters				
a_0	m	18.6	12.1	29.0
a_1	$m\ m^{-1}$	4.37	4.00	6.50
a_2	$m\ km^{-1}$	1.41	0.93	2.67
Corresponding target stock				
Summer	pump-h	17.5	7.8	18.1*
Winter	pump-h	14.0	5.5	18.1*
Chainage to remote water level recorder	km from PS	5.6	6.0	2.45 (GD) 3.7 (MD)
Time over which rectification of stock imbalance sought, DUR_{rec}	h	4.0	2.5	4.5

* Expressed in terms of larger pump-size

It has not been possible to evaluate the performance of OCOPO in isolation since the OCC system has remained under intermittent development. North Level IDB engineers have used the system to control pumps directly only for short periods, preferring to let OCOPO advise on pump operation rather than to control the pumps directly. This rather undermines the aim of OCOPO, since it was specifically designed as a comprehensive pump control system. Consequently, the potential for Optimum Control of Pump Operations has not been fully demonstrated. Some of the difficulties of proving a benefit were outlined in Section 2.3.

It is particularly difficult to monitor the performance of the rainfall-runoff model and to evaluate the extent to which its use enhances operation. There is the suspicion that the important element is the real-time refinement (see page 21) rather than the rainfall-runoff model itself (page 19). It may be sufficient to use the simple forecast:

$$RO_{fct} = ROOBS_{r}$$

A rainfall-runoff model would not be needed and there would then be no requirement to telemeter rainfall measurements.

5 Further developments

5.1 The siting of water level recorders

The siting of recorders to monitor the quantity of water stored in the main drain is guided mainly by drain geometry. The minimum requirement is one water level recorder at the pumping station and one toward the remote end of the main drain. It is helpful if the latter water level recorder is sufficiently far from the pumping station to be only moderately affected by pump starts and stops. However, if it is placed too far from the pumps, the water level readings will not be relevant to the storage that is actually controlled and manipulated by the pumping station.

Calculation of a backwater length may be one way of defining the extent of the main drain system, but such a calculation is sensitive to the assumed water level at which a pump-run is triggered. Moreover, empirical friction coefficients are poorly defined for the wide shallow channels, and low gradients, typical of fenland drains (Slade, 1985). A further difficulty is that the hydraulic performance of the main drain can be affected by seasonal weed growth and resultant weed-cuts.

Reference has already been made (Section 3.1) to the possible error induced by assuming a uniform longitudinal water surface profile between the remote site and the pumping station. Practical experience suggests that the pump decision algorithm is insulated from occasional poor estimates of storage (e.g. due to the drawdown effect in the early phase of a pump-run) by exponential weighting of the inferred runoff rates (see Sections 3.3 and 4.3). The deployment of additional water level recorders along the drain would, if required, produce better real-time estimates of the amount of water stored in the drain.

North Level IDB's Dog-in-a-Doublet pumping station is served by a dual main drain system. In such cases it is natural to use two remote water level recorders, one in each branch. The modelling of drain geometry is inevitably a little harder

The approach taken was to derive the water width (W), at given chainage (x) and water level (h), separately for each branch. The two values of W were then summed and the usual approach followed of representing the drain storage as a tapering triangular trapezoid. This resulted in the composite drain geometry model:

$$W(h,x) = 29.0 + 6.5\,h - 2.67\,x \qquad [5.1]$$

Estimation of the storage in real-time is undertaken as follows. Firstly, the remote water level readings at the pumping station and on Gore Drain are taken to be representative, and the storage calculated from the composite drain geometry model. Then, the process is repeated using water level readings at the pumping station and on Middle Drain. This yields an alternative estimate of the volume of water in the dual main drain system. The two estimates are then averaged. Weights of 0.45 and 0.55 are applied, to reflect the slightly larger storage capacity of Middle Drain.

5.2 Mixed-size pumps

A further feature of the Dog-in-a-Doublet pumped catchment is that there are mixed-size pumps: two discharging 0.708 m^3s^{-1} and two discharging 1.105 m^3s^{-1}. This complicates the construction and implementation of pump control rules.

The approach taken was to continue to define runoff (and pumping) rates in units of pumps, and storage in units of pump-hours - arbitrarily choosing to define the unit to be equal to the discharge capacity of the larger pumps. The alternative would be to revert to working in terms of m^3s^{-1} and Ml. This has the disadvantage of providing much less "feel" for the significance of a given runoff rate or stock excess.

A facet of mixed-size pumps is that a small unit can be operated for much longer periods (e.g. Hobson & Carne, 1986) and the tendency towards "hunting" (see Section 2.2) largely suppressed. Birks (1986) refers to the use of small pumps designed to cope with the 95-percentile flow.

Some drainage authorities have begun to use variable-speed pumps, for example by adding one to an existing array of fixed-speed pumps. While this allows a much more flexible pumping regime, it has the potential to complicate the pump decision algorithm. Krutzch (1984) presents information about the efficiency of variable-speed centrifugal pumps under various operating heads.

5.3 Tariffs with maximum demand penalties

Electricity tariffs with maximum demand penalties are less widespread than formerly. Eastern Electricity (who supply power to the Dog-in-a-Doublet pumping station) introduced a new tariff system in April 1988, under which there are no maximum demand charges. The Newborough and Postland pumping stations operate under the tariff described in Section 4.4. It was therefore unnecessary to implement the runoff threshold adjustments for maximum demand charges (see Section 3.5). However, a brief discussion is warranted, in case maximum demand charges continue to apply elsewhere or are re-introduced.

The practice of using time-switches to avoid pump-runs which might incur a maximum demand charge (or a punitive unit energy cost), irrespective of the hydrological demand, is risky. In some systems using less sophisticated time-switches, it is believed that pumping is disabled at certain times (e.g. between 14.00 and 17.00 in the winter period) on *all* days, even though penalties can only be incurred on weekdays.

For pumping stations operating under tariffs with maximum demand (MD) penalties, the OCOPO methodology includes specific adjustments to the runoff thresholds required to start the ith pump (see Section 3.5). Prescription of how these adjustments, $ROADJM_i$, are to be calculated will be a matter of judgement. If the maximum demand has already been incurred for running i pumps, $ROADJM_i$ will be zero. Otherwise it will take a suitably discouraging value, perhaps increasing from 0.5 at the beginning of the month to 1.5 at the end of the month. Clearly the formulae for $ROADJM_i$ must reflect the detailed structure of the MD element of the tariff. If the seasonal period in which MD penalties can be incurred is almost over, a particularly strong disincentive to acquiring one may be warranted.

In setting values of $ROADJM_i$, it is important to appreciate the "one-off" nature of MD charges. For example, it may be preferable to incur the MD charge for one pump early in the month than to run too great a risk of incurring the MD charge for several pumps near the month end.

5.4 Booster stations

Booster stations are pumping stations sited *within* the drain system to raise water from one level to another. Thus the drain into which the booster station discharges is itself controlled by a pumping station. Consequently, the decision on the number of pumps to be run in the next operating period should take account of storage conditions on both sides of the booster station.

The "target storage" concept (Section 3.1) can be readily modified to deal with booster stations. One approach is to replace the stock excess term in Equation 3.4 by the excess of the stock excess on the suction side over that on the discharge side, i.e.

$$STOEXC = STOEXC_{ss} - STOEXC_{ds} \qquad [5.2]$$

In effect, the objective shifts to ensuring that any excess of storage is evenly balanced through the main drain system. Other aspects of the pump decision algorithm are unchanged.

If this approach is adopted, it is important that operation of the main outfall station is adjusted to reflect the hydrological demand throughout its catchment, i.e. on both sides of the booster station.

Operation of Yorkshire Water's Winestead booster pumping station is relatively sophisticated, both in terms of its treatment of the electricity tariff and the manner of implementation (viz. a programmable logic controller based on operating rules developed in flowchart form). However, the role played by the water level in the receiving drain is relatively simple; should this rise above a pre-set level all pumping is suspended, irrespective of conditions on the suction side of the booster station.

Another example of a booster station is the French Drove pumping station, sited on the North Level main drain.

5.5 Main outfalls

Operation of a main outfall pumping station should ideally reflect the hydrological demand throughout the drainage area. This can be evaluated by aggregating the stock excesses from all drains, including the main drain to the outfall station and each tributary main drain. Forecast runoff in the next operating period can be similarly aggregated. Of course, in such an application, it would be necessary to convert units to a common base, e.g. expressing stock imbalances and runoff rates in terms of outfall station pump-hours and pumps, respectively.

An opportunity to apply OCOPO to a main

outfall pumping station - such as North Level's Tydd pumping station (see Sturgess, 1987) - has not yet emerged.

5.6 Stations affected by conditions in the receiving watercourse

In the majority of fenland catchments, almost all the water has to be pumped at least once before it can discharge to the sea by gravity. For most pumping stations in the North Level IDB area there is no scope for discharge by gravity. Consequently the water level in the receiving watercourse is largely irrelevant to pump operation, an exception being the case of a booster station. However, in other areas there is scope for significant discharge through "gravity doors" or "tide gates" during favourable phases of the tide.

Tide-influenced pumping stations
Marshall (1993) gives an account of pumping station operation at a site subject to marked tidal influence: the Boy Grift pumping station operated by Alford Drainage Board. He defines unit energy costs in adverse electricity tariff periods *relative* to the cheap-rate unit energy cost. Similarly, he evaluates the unit cost of pumping in adverse tidal conditions *relative* to those applicable when the pumping station is discharging freely over a fixed sill.

Clearly these balances are affected by the particular characteristics of the pumps, the stilling basin and the tidal regime. For Boy Grift, it was found that fitting in with the electricity tariff was more important than fitting in with the tidal cycle, if energy costs were to be contained; "the electrical tariff structure is so dominant that, during the winter, trying to avoid pumping against high tides should be regarded as very much a secondary consideration" (Marshall, 1993). However, the analysis neglected the possibility of some discharge by gravity, since the gravity doors were kept closed during the field experiment.

Tide level variation does not, of course, conform to a neat 24-hour cycle to which pump operating periods might be moulded. However, a prediction of the astronomical tide level can be obtained, most conveniently as the harmonic series formula underlying the "tide table" for the nearest standard reference site. Precise transformation of projected tide levels from this site to the pumping station is not required, since it is only the *relative* severity of the tide that is of interest in setting differentials. Correction for storm surge (or other forecast variations in the expected "total" tide) could be an important refinement but would be difficult to integrate into OCOPO.

In the spirit of OCOPO it may be sufficient to introduce a simple tidal adjustment into the runoff threshold formula (Equation 3.5) that is central to the pump decision. ROADJT might be set to a number between (say) -0.5 and 0.5 according to the favourableness (or otherwise) of the water level expected in the receiving watercourse in the operating period. For example:

$$\text{ROADJT} = \text{TMEAN}/\text{TRANGE} \qquad [5.3]$$

where TMEAN is the mean tide level expected over the coming operational period, while TRANGE is the long-term average tidal range (i.e. twice the amplitude).

Allowing for drainage by gravity
In cases where drainage by gravity is possible at low tide, it is important that checks are made for exceptional conditions. In particular, it is advisable to ensure that any return flow through imperfectly sealed or damaged gates is insignificant in comparison to the rate at which water is being pumped. Such a check may be difficult to automate.

It would be quite difficult to generalize OCOPO to deal with cases where gravity discharge can contribute significantly to economic drainage. For the refinement of runoff forecasts (see page 21) to work, it would be necessary to know the volume of runoff draining by gravity. This in turn would require a discharge rating for the gravity door, together with telemetered water level measurements inside and out.

Other cases
Local drainage of flood water can sometimes be impeded by high water levels in a receiving watercourse, even where this is non-tidal. This can be a particular problem where a relatively quickly responding catchment drains directly to a much larger river which exhibits a different flood regime. A notable example is the river Foss which joins the river Ouse in York. When water levels in the Ouse are high it is necessary for the discharge of the Foss to be pumped if backing-up, and consequent flooding, are to be avoided in the lower reaches of the Foss.

A barrier and pumping station (discharge capacity 30 m^3s^{-1}) were constructed and a 2MW electrical power supply (with standby diesel generators) installed (Bramley, 1987; Moore & Grace, 1989). The Foss is a navigation: preservation of water levels, coupled with limited freeboard, leaves little scope for flexible operation. Thus, to prevent flooding, it may be necessary to pump soon after a forced barrier closure. Presumably, operation using the standby generators might be considered if a Foss flood is forecast to clash both with high levels in the Ouse and a particularly expensive electricity tariff period.

6 Review of other applications

Many urban areas are partially dependent on pumped drainage to alleviate local flooding. Some of these cases bear similarity to those of Section 5.6, in that pump operation is only required at times when local drainage is impeded by high water levels in the receiving watercourse. A not uncommon situation is that construction of a river flood improvement scheme on the receiving river calls for installation or upgrading of pumped drainage schemes on these minor, often urbanized, catchments. Examples are the Tutt pumping station to the Ure at Boroughbridge, North Yorkshire, and local pumping stations on the Rhondda Fawr at Gelli, South Wales.

Other urban drainage schemes are of a different character, and are more intimately linked to the storm sewer system. Examples include the Isle of Dogs pumping station in London (Bennett *et al.*, 1988), systems in Kingston upon Hull, and others serving Hastings and Bexhill (Armstrong *et al.*, 1989). Detailed hydraulic modelling of sewer systems may be required to understand the consequences if free discharge at the outfall is impeded. However, in terms of multi-objective control of pump operations, the storage approach used in OCOPO may have merit. Indeed, perhaps the nearest precursor to OCOPO was concerned with optimization of sewage pumping (Evans, 1981). Subsequent WRc research has shown that energy savings in storm pumping can be achieved if sets of equal-sized fixed-speed pumps are replaced by systems which allow more flexible rates of pumping (Hobson & Carne, 1986).

A somewhat similar problem is the real-time control of "in-sewer balancing" storage, through electrically-actuated penstocks. While there may be little or no scope to manipulate electricity tariffs, sophisticated operating rules are required to balance the twin objectives of regulating flows to treatment (to improve the efficiency of treatment costs) and minimizing adverse overflows in storm conditions. The system described by Robinson (1990) uses "head" and rate of rise of head as key variables. Use of the latter "state variable" differs from OCOPO, since a rise in water level may be attributable to system outlet conditions, as illustrated in Figure 6.3 of the Water Practice Manual (IWEM, 1987). In contrast, OCOPO is driven by the actual "demand" for drainage, by estimating the inflow rate to the main drain system in real time.

Computer-aided control of flows to treatment are considered also by Naghdy and Helliwell (1987). They incorporate short-term forecasts of flow, and of ammonia concentration, using the Box-Jenkins method of time series analysis (e.g. Chatfield, 1980). These stochastic models provide a natural alternative to simple deterministic models (such as the nonlinear storage model used in OCOPO), and are likely to have particular merit where the inflows have a pronounced cyclical component.

Pump scheduling for water supply is a further activity where cost minimization has to be balanced against supply security (Coulbeck & Orr, 1983; Brockton, 1987; Lumbers & Cook, 1993). There is only a very minor hydrological element to this: namely, climatic effects on short-term demand for water.

The principles of OCOPO could nevertheless be adapted to encompass such applications. In essence, the problem is an inversion of the pumped drainage one. The objective is to maintain the water level in the service reservoir at or above that needed to provide supplies at the required pressure, while meeting fluctuating demands, and avoiding pumping in adverse periods of the electricity tariff. The runoff thresholds are replaced by demand thresholds, while it is a short-term forecast of demand, rather than runoff, that is required. Of course a key element is again the refinement of forecasts by reference to recent observations of stock changes in the reservoir and quantities pumped.

In fact, the original concept of OCOPO was born out of recognition that what was already being achieved in the management of water resource systems (see Walsh *et al.*, 1988, for a historical review) could be applied to an inverse problem in land drainage pump control!

It is as yet unclear whether OCOPO could be adapted to assist in the control of structures, as opposed to pumping stations, e.g. discharge gates at flood retention reservoirs (Porter, 1986), inlet gates to washland storage areas, or tidal barriers. This is an area for further research.

7 Conclusions

- The drainage of flat low-lying catchments gives rise to an exceptional class of river engineering problems and solutions. The design and maintenance of drainage systems in such catchments are highly specialized tasks. But it is possibly the *operation* of land drainage pumping stations that most distinguishes fenland river engineering, calling for the particular skills and experience of Internal Drainage Boards and sister drainage organizations.

- The design of pumped drainage systems should take into account how they will be operated, either explicitly or by analogy with experience gained with similar drainage systems. Land drainage pumping stations generally have to meet the multiple objectives of flood mitigation, cost minimization and amenity preservation. The profession should perhaps recognize this more explicitly, and accept that giving precedence to meeting one objective may degrade the performance achieved in one or both of the other objectives.

- Rainfall, which is the primary input to non-tidal flood formation, is a complex stochastic process, characterized by high irregularity and intermittency. Irrespective of what has passed before, there is an inherent risk of an overwhelming flood event occurring. For a well designed station, the risk will be acceptably small. However, the procedures for operation of the station should give some thought to possible consequences should an overwhelming event occur within the design life of the works.

- In comparison to naturally draining catchments, there is less diversity in fenland catchments. Both drainage networks and constituent catchment soils tend to be rather regular. However, it is the lack of pronounced topographic-forcing that most strongly characterizes the flood response of fenland catchments. It has been demonstrated at Newborough Fen that, in response to heavy rainfall, and in the absence of pumping, water levels rise synchronously throughout the drainage system. Thus it is principally the pumping station operation and the characteristics of the flow through the soil that determine the temporal pattern of catchment response.

- A rainfall-runoff model is presented for use in forecasting inflows to fenland main drains, and it is suggested that the parameter values calibrated for Newborough Fen might be transferred for use in broadly similar catchments.

- Not all flat low-lying catchments fall into this mould. For example, some receive drainage from adjacent "highland" areas, while others have an unusual drain or catchment configuration, or receive flood runoff from urban areas.

- If there is to be scope to operate a pumping station to meet multiple objectives, it is essential that the system design allows flexibility; in essence, the suit must be generously cut. The factors seen to be most influential are: the discharge capacity of the pumping station, the conveyance characteristics of the main drain, and the capacity of the main drain to store water temporarily. Existing pumped drainage systems differ from station to station according to which of these factors binds most closely on pump operation. A fourth factor impinging on pumping station operation is the electricity tariff. In some cases, the number and differential sizing of pumps may also be important.

- The report presents a methodology for the optimum control of pump operations (OCOPO), with particular regard to fenland catchments. The approach has been shown to be entirely feasible through implementation of OCOPO within the automated flood control of pumping stations (AFCOPS) system operated by North Level Internal Drainage Board.

- An important element of the approach is the representation of the drain storage commanded by the pumping station. The model of drain storage is derived from design drawings and augmented by site survey of the main drain. Depending on the geometry of the drain system, its conveyance characteristics, and the pump operating regime, the storage commanded by the pumping station may be larger or smaller than that surveyed. A procedure is described by which the drain geometry model is re-calibrated to match actual behaviour of the prototype.

- In real-time application, the quantity of water stored in the main drain is estimated from the drain geometry model using water level measurements at the pumping station and at one or more remote sites.

- Hydraulic modelling of the main drain is not essential to optimizing the operation of existing pumping stations. Such modelling might be required on main drains which include a constriction, such as a culvert, and is always to be recommended if major new works are planned.

- Hydrological modelling of the rainfall-runoff process is helpful in refining forecasts of the hydrological demand for pumping, which is represented by the inflow rate from the catchment to the main drain. However, such modelling is by no means essential. An adequate estimate of inflow rate can be deduced from recent water level variations in the main drain and the record of quantities pumped, using a simple water balance. Thus, although rainfall measurement and rainfall-runoff modelling can refine runoff estimates, it may be sufficient to monitor and interpret drain water levels.

- OCOPO provides a set of operating rules which determine the number of pumps to be used in the next operating period. It is helpful to choose operating periods that reflect the response characteristics of the catchment and drain, and mesh with the electricity tariff. For many fenland catchments and electricity tariffs, a daily cycle of six or seven operating periods is likely to be satisfactory.

- It is helpful to express runoff rates and storages in units of pumps and pump-hours respectively. The significance of a particular inflow rate or stock imbalance is then immediately apparent.

- Because water level measurements are central to OCOPO, and crucial to satisfactory pump operation and avoidance of flooding, it is important that facilities are provided to alert staff to exceptional (or missing) values, so that instrument or telemetry malfunctions can be investigated promptly.

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Appendix A Equations to estimate stock from water level observations

A.1 The drain geometry model

The mathematical formulation of the drain geometry model of Section 4.3 page 14 is now amplified. This represents the main drain as a tapering triangular channel (see Figure 4.4). The water width is given by:

$$W(h,x) = a_0 + a_1 h - a_2 x \qquad [A1]$$

where h is the water level in mAOD, and x is the chainage (i.e. distance along the drain) in km, measured from the pumping station.

A.2 Interpretation of parameters

The parameter a_0 is the water width at the pumping station (i.e. at x=0) for the reference water level of 0.0 mAOD (i.e. h=0).

Partial differentiation with respect to h shows that a_1 is the rate at which the water width increases with water level. Given the symmetry of the cross-section, the side slope of the main drain therefore corresponds to $2/a_1$ m/m.

The drain bed is the level, h_0, at which the water width is zero. From Eqn. A1, this is:

$$h_0 = (a_2 x - a_0)/a_1 \qquad [A2]$$

Partial differentiation of h_0 with respect to x indicates that the longitudinal bottom slope is a_2/a_1 m/km.

A.3 Static case: water level horizontal throughout the main drain

The length, L, of the wetted drain is found as the chainage at which the water level, h, equals the bed level, h_0. Thus, from Eqn. A2:

$$h = (a_2 L - a_0)/a_1 ,$$

so that:

$$L = (a_0 + a_1 h)/a_2 . \qquad [A3]$$

This is the wetted length of the main drain in km for a given water level of h mAOD.

At any chainage, x, the cross-section of the wetted drain is an inverted isosceles triangle of "base" W and "height" $h-h_0$. Thus the cross-sectional area, A_s, is given by:

$$A_s = (h - h_0)\, W/2 \qquad [A4]$$

Substituting for W (from Eqn. A1) and h_0 (from Eqn. A2) yields:

$$A_s = (a_0 + a_1 h - a_2 x)^2/(2a_1) ,$$

or:

$$A_s = W^2/(2a_1) \qquad [A5]$$

Like W, A_s is a function of both h and x.

Evaluating the volume, V, of water in the main drain is obtained by integrating A_s with respect to x along the length of the wetted drain. Thus:

$$V = \int_0^L W^2/(2a_1)\, dx .$$

Substituting W for x in the integration, and noting that:

$$dW = -a_2\, dx ,$$

gives:

$$V = \int_{a_0+a_1h}^{0} -W^2/(2a_1a_2)\, dW = \int_0^{a_0+a_1h} W^2/(2a_1a_2)\, dW .$$

Thus:

$$V = \Big[W^3/(6a_1a_2) \Big]_{W=0}^{W=a_0+a_1h}$$

so that:

$$V = (a_0 + a_1 h)^3/(6a_1a_2) \qquad [A6]$$

This is the volume of water in stock for a static water level of h throughout the main drain.

A.4 Dynamic case: water level varies along the main drain

OCOPO represents the water in the main drain by reference to real-time water level measurements at one or more cross-sections. For generality we consider the case where there are n water level recorders positioned at chainages $x_1, x_2, \ldots, x_n$; the suffix 1 indicates the site closest to the pumping station, while n denotes the most remote site.

The method assumes - for the specific purpose of stock assessment for pump control - that the water surface profile between gauges is adequately represented by linear interpolation. If the first recorder is sited more than a few metres from the pumping station, it is necessary to estimate the intervening storage. OCOPO does this by extrapolating the water surface gradient from the subsequent reach, i.e. that between recorders 1 and 2.

It is similarly necessary to represent the storage beyond the farthest station but, in this case, extrapolation is not appropriate. The water level at x_n is taken to indicate the level throughout the remainder of the main drain; indeed, this is the *raison d'être* of the siting of the remote water level recorder.

Thus, the water surface profile is assumed to be horizontal beyond the farthest station, and a modified Eqn. A6 is used to calculate the volume contribution beyond the nth recorder. [The required modification is to replace a_0 by the water width at chainage x_n at the reference level of h=0.0 mAOD, i.e. by $a_0-a_2x_n$.]

The volume element, v_r, contributed by the rth reach ($2 \leq r \leq n$) is calculated as follows. The water level h is given by the linear interpolation:

$$(h - h_{r-1})/(h_r - h_{r-1}) = (x - x_{r-1})/(x_r - x_{r-1}) \quad [A7]$$

Because h is taken to vary linearly between x_{r-1} and x_r, so too does W. The derivation of a formula for v_r mirrors that of Eqn. A6, the difference being that:

$$dW = (a_1 s_r - a_2)\, dx\ ,$$

where s_r is the gradient of the water surface profile from Eqn. A7, i.e.:

$$s_r = (h_r - h_{r-1})/(x_r - x_{r-1})\ .$$

Thus:

$$v_r = \int_{x_{r-1}}^{x_r} W^2/(2a_1)\, dx$$

and:

$$v_r = \int_{x=x_{r-1}}^{x=x_r} W^2/\,[2a_1(a_1s_r-a_2)]\, dW$$

so that:

$$v_r = \Big[\, W^3/\,[6a_1(a_1s_r-a_2)]\, \Big]_{x=x_{r-1}}^{x=x_r}$$

Hence:

$$v_r = [\,(a_0 + a_1 h_r - a_2 x_r)^3 - (a_0 + a_1 h_{r-1} - a_2 x_{r-1})^3\,]\,/\,[6a_1(a_1s_r-a_2)\,] \quad [A8]$$

This is the contribution of the rth reach to the total volume of water in stock in the main drain. The equation applies for $2 \leq r \leq n$.

For the first reach we have the special relationship:

$$v_1 = [(a_0 + a_1 h_1 - a_2 x_1)^3 - (a_0 + a_1 h_1)^3\,]\,/\,[6a_1(a_1s_2-a_2)] \quad [A9]$$

where the slope of the second reach has been applied to the first reach.

For the (n+1)th reach we have:

$$v_{n+1} = (a_0 + a_1 h_n - a_2 x_n)^3/(6a_1a_2) \quad [A10]$$

for a static water level of h throughout the remainder of the main drain (i.e. beyond x_n).

Combining Eqns. A8, A9 and A10 we arrive at the overall volume, or stock, of water in the main drain:

$$V = v_1 + \sum_2^n v_r + v_{n+1} \quad [A11]$$