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A simplified model for sewered catchments

by

G A Price, J C Packman
and C H R Kidd

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I N S T I T U T E
O F
H Y D R O L O G Y

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ABSTRACT

This report describes the development of a rainfall-runoff model for urban sewered subcatchments up to 100 ha in area. The model uses the Wallingford Urban Subcatchment Model previously developed for the conversion of a rainfall hyetograph into an inlet hydrograph to the sewer system. Thereafter an equivalent pipe model is used in which the pipe network is conceived as a single tapered pipe run of length and slope equal to that of the prototype system. The performance of the model is shown to be in good agreement with that of the full Wallingford Model in which the entire sewer system is simulated. The model as developed is suitable for use in situations where detailed catchment information and sewer system layout are not available.

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1. INTRODUCTION

1.1 Background

The Hydraulics Research Station and the Institute of Hydrology are collaborating in the development of improved methods for the hydraulic design of storm sewers. To this end, a suite of rainfall-runoff models of varying degrees of complexity has been developed. These models have, as their basic unit of areal discrimination, a single pipe length (from manhole to manhole) of the sewer system. As such, catchment data are required at every manhole in the system, which is a level of data requirement that is often not practicable.

There are two main situations under which this degree of detail may not be warranted: firstly, where part of a catchment is already developed but catchment details are either not available or considered not necessary for the required simulation; and, secondly, where part of the catchment is designated for future development, but exact details are not yet available. Under both these circumstances, some simplified model is required, and this report describes just such a model for incorporation in the new design methods. A further advantage of this development is that it allows the models to be used more easily as planning tools in addition to their main function as design tools.

1.2 The Wallingford Models

There are four models available in the Wallingford design package:

(a) a peak flow model (c.f. the Rational Formula),

(b) a peak flow model incorporating pipe-slope optimisation,

(c) a hydrograph model for design and simulation (including allowance for only single pipe surcharging); and

(d) a hydrograph simulation model including full solution for surcharged flow (i.e. when pipes are flowing full and discharge is governed by pressure flow hydraulics instead of open channel hydraulics).

Model (d) is the most detailed in its simulation of the rainfall-runoff process and model (c) is effectively a special case of model (d). Model (d) was chosen for use in this study.

The model may be divided into two:

(a) an above-ground hydrological model (The Wallingford Subcatchment Model), described in detail in a companion report (Kidd & Lowing, 1979) and incorporating (i) a contributing area runoff volume submodel, and (ii) separate nonlinear reservoirs for paved/pervious areas and for roofed areas.

(b) a below-ground hydraulic model, incorporating (i) a Muskingum-Cunge pipe routing model for part-full flow (Price & Kidd, 1978) and a full solution of the simultaneous differential equations for surcharged pipe flow (Bettess, Pitfield & Price, 1978).

Catchment data requirements for each manhole are: (i) the area of paved, roof, and pervious surface; (ii) the average surface slope and area-per-gully, each entered as one of three categories (steep, medium or flat for slope; small, medium or large for area-per-gully); (iii) pipe length, slope, diameter and roughness; and (iv) manhole depth and area, together with a "floodable area" (for surcharge solution). Global values for some of these inputs may be specified (eg surface slope, area-per-gully, pipe roughness). Values for the surface and pipe routing coefficients are calculated by the model from these inputs.

The hydrological input to the model consists of a rainfall event (or, in design, a rainfall depth, duration, and profile) and a value for the catchment wetness index UCWI, defined as:

$$UCWI = 125 + 8\ API5 - SMD \tag{1.1}$$

where API5 is the 5-day antecedent precipitation index (mm)
and SMD is the soil moisture deficit (mm)

Overall catchment percentage runoff (PRO) is estimated from UCWI, and from the catchment characteristics PIMP (the overall percentage imperviousness) and SOIL (the soil type) using the following regression equation (Kidd and Lowing, 1979):

$$PRO = -20.7 + .829\ PIMP + 25.\ SOIL + .078\ UCWI \tag{1.2}$$

Where some observed data are available, the model may be constrained to adopt values of the routing parameters and of PRO other than those predicted by the equations.

1.3 The approach to the problem

As described earlier, the problem is to develop a model to predict the hydrograph for a subcatchment larger than the single pipe unit; - this subcatchment would include both overland and sewer flow (ie a "sewered subcatchment"). One approach to this problem would be to develop an alternative to the existing "above ground" subcatchment model (see Kidd and Lowing, 1979). This new model would then allow for the attenuation associated with pipe flow as well as overland flow. Sarginson & Nussey (1975) have developed a model of this type consisting of two linear reservoirs in series, one to simulate surface routing and the other pipe routing. The major problem with this type of model is to ascribe appropriate degrees of attenuation to each of the two reservoirs.

An alternative approach, and the one adopted in this study, is to retain the existing "above ground" model in its present form, and develop a new model associated only with the pipe-flow part of the problem. This pipe-flow model would then be directly analogous to the second of

Sarginson & Nussey's (1975) linear reservoirs. The parameters of the above ground model would be associated with average conditions of all the subcatchments within the "sewered subcatchment" and the parameters of the pipe-flow model would be associated only with characteristics of the sewer system. This approach puts a new perspective on the problem. Observed rainfall-runoff data contain elements of surface routing as well as pipe-flow and thus are not suitable for developing a model for pipe-flow alone. However, suitable data can be generated using the full Wallingford Model described earlier. In this case, a new pipe-flow model is required to simulate the pipe-flow behaviour of the full model; what is required is a model of a model.

At first sight, development of the sewered-subcatchment model on synthetic data might seem unwise, but two points should be borne in mind. Firstly, the surface routing and pipe-flow models used in the Wallingford Model were developed independently, and each has been shown separately to yield true representations of the processes involved. Secondly, the main use for the sewered-subcatchment is as an approximation to the full model for areas in which the detailed pipe layout is not relevant to the current task. As such, it is arguably more important to yield a good fit to the full model than to observed data

2. THE MODEL

2.1 Introduction

As discussed in Section 1.3, the model developed for sewered subcatchments consists of an above-ground model and a pipe flow model. The above-ground model is the same as that used in the full Wallingford Model, and is described in detail by Kidd & Lowing (1979). For application to the larger, sewered subcatchments, the only differences are that (i) the areas of paved, roofed, and pervious surfaces are now the totals over all the individual subcatchments within the sewered subcatchment, and (ii) the surface slope and area-per-gully are the average over all the individual subcatchments in the sewered subcatchment. The outflow from the above-ground model is therefore, the lumped sum of all the individual above-ground hydrographs feeding to the pipe system of the sewered subcatchment. This summed hydrograph forms the input to the pipe flow model.

The pipe flow model (called the equivalent pipe model) is based on a simplification of the pipe network used in the full model. There are two main processes to be simulated in routing a flood wave through a pipe system

(a) translation of the flood downstream, with lateral distribution of inputs

(b) attenuation of the flood peak due to storage within the pipe network

In the full Wallingford Model these processes are adequately represented by the large number of small discrete inputs and short routing lengths involved. For the sewered subcatchment model, it was necessary to simulate this process but with a much reduced data input, and ideally in a form that could be easily incorporated in the full model.

The equivalent pipe model is a conceptual model based on hydraulic principles, utilising a simplified "equivalent" pipe system to represent the complex branched network of the actual (prototype) catchment. It comprises a single branch of pipes arranged in series. For a given catchment, all pipes are of the same length and slope, and each pipe is of constant diameter throughout its length. The diameter of successive pipes are reduced in an upstream direction to give a realistically tapered system. The inflow hydrograph derived for the catchment as a whole is distributed equally to the upstream ends of each pipe. The model is represented schematically in Figure 2.1. Flow is routed through the system using the same Muskingum-Cunge procedure as is used in the full Wallingford Model.

Since the sewered-subcatchment model uses the same above-ground model and the same pipe flow method of routing (once the equivalent system has been defined) as the full model, it is easily incorporated as an option in the Wallingford Model.

2.2 Method of Routing

The Muskingum-Cunge routing method is defined by the continuity equation:

$$\frac{d\Sigma}{dt} = I - O \qquad (2.1)$$

and a storage equation of the form:

$$\Sigma = K\{\varepsilon I + (1-\varepsilon)O\} \qquad (2.2)$$

where Σ = storage

I = Inflow

O = Outflow

K = routing time constant

ε = routing attenuation constant

Rewriting these equations in finite difference form over the interval t to $t+\Delta t$ and rearranging gives the following well known recurrence relationship for $O_{t+\Delta t}$

$$O_{t+\Delta t} = C_1 I_t + C_2 I_{t+\Delta t} + C_3 O_t \qquad (2.3)$$

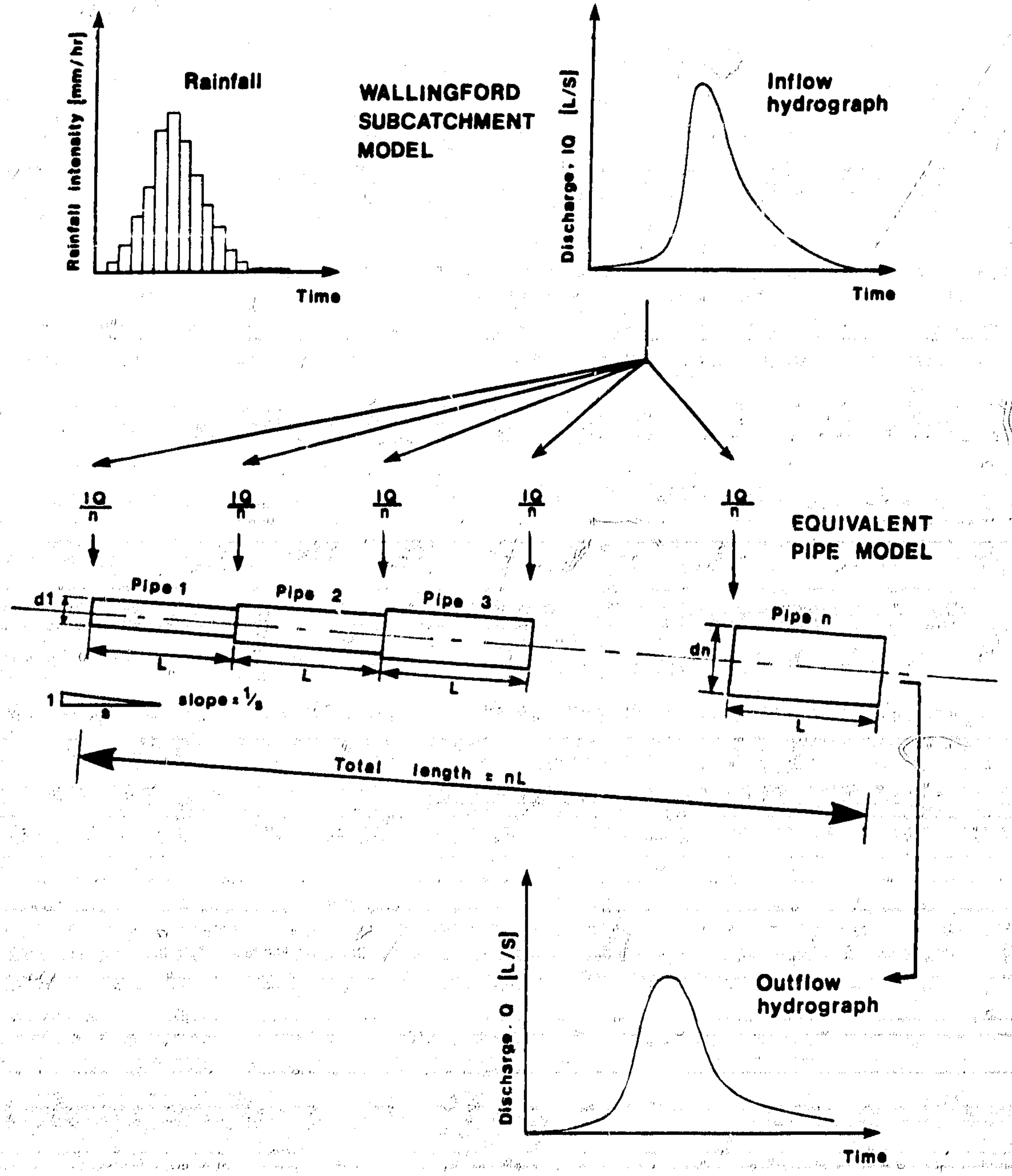


FIGURE 2.1 Schematic representation of equivalent pipe model

where $C_1 = \{2K\varepsilon + \Delta t\}/\{2K(1-\varepsilon) + \Delta t\}$ (2.4)

$C_2 = -\{2K\varepsilon - \Delta t\}/\{2K(1-\varepsilon) + \Delta t\}$ (2.5)

$C_3 = \{2K(1-\varepsilon) - \Delta t\}/\{2K(1-\varepsilon) + \Delta t\}$ (2.6)

Equations (2.3) to (2.6) form a very quick method of flood routing once values for K and ε have been determined. Cunge (1969) showed that these equations gave, in fact, a first order solution to the convective-diffusion routing equation, and thus were an approximation to the full Saint-Venant equations for flood routing. Hence he determined the following expressions for K and ε for a prismatic channel:

$K = L/\bar{W}$ (2.7)

$\varepsilon = \frac{1}{2}\{1 - \bar{Q}/(\bar{B}\ SL\bar{W})\}$ (2.8)

where L is channel length

S is channel slope

$\bar{W}$ is average wave speed along the channel

$\bar{Q}$ is average discharge along the channel

B is average channel breadth

Use of equations (2.7) and (2.8) with equations (2.3) to (2.6) gives an accurate solution to the flood routing problem provided attenuation along the reach is not too large (ε > .25) and channel length is well conditioned in relation to the wave speed and the timestep, ie

$\bar{W}\Delta t < L < 2\bar{W}\Delta t$ (2.9)

This condition may be interpreted as: channel length should be between one and two times the distance the wave travels in one time step. In the Wallingford Model, short pipe lengths are dealt with by routing the wave along a pipe of length $\bar{W}\Delta t$ and linearly interpolating at length L; long pipes are dealt with by subdividing into two or more shorter pipes. In practice, this is enough to ensure accuracy of the method.

2.3 Equivalent pipe system

The equivalent pipe system, described in Section 2.1 and shown in Figure 2.1, is defined by four parameters: the diameter of the most downstream pipe (DIAM); the overall length of the pipe system (LENGTH); slope (SLOPE); and a constant (c) determining the degree of tapering. The system consists of n individual pipes each of length equal to LENGTH/n, where n is determined from:

integer value of $\{\text{LENGTH}/(1.5\ \bar{W}\Delta t) + 1\}$ (2.10)

and Δt the solution timestep

$\bar{W}$ is the calculated mean wave speed for the most downstream pipe (diameter = DIAM, slope = SLOPE)

Equation (2.10) ensures that for all practical cases equation (2.9) holds for all pipes in the system.

The diameter of successive pipes in the equivalent system is reduced in an upstream direction according to the following equation.

$$\text{DIAM}_i = \text{DIAM}_n \left(\frac{i}{n}\right)^c \quad (2.11)$$

where DIAM_i is the diameter of the i^{th} pipe in the system

(i = 1 for most upstream pipe, i = n for most downstream pipe)

n is the number of pipes

c is the tapering constant

2.4 Distribution of inflows

The inflow to the equivalent pipe system consists of the lumped above-ground hydrograph, distributed as equal lateral inputs to the upstream end of each pipe. Thus, the inflow to each pipe is given by the sum of the above-ground contribution to that pipe and the outflow from the previous pipe (see equation 2.12).

$$QI_i = QO_{i-1} + IQ/n \quad (2.12)$$

where

QI_i = discharge entering equivalent pipe i

QO_{i-1} = discharge from the upstream equivalent pipe i - 1

IQ = lumped above-ground input for whole catchment.

This equation is resolved for every time increment over the whole hydrograph, taking each pipe in turn. At each time step, the inflow discharge to the pipe under consideration (QI_i) is compared with the pipefull discharge (QP_i). If QI_i exceeds QP_i, the pipe is surcharged. The model as described above makes no allowance for surcharging as the Muskingum-Cunge routing coefficients remain unchanged regardless of whether or not the pipe is surcharged. A second form of the model was also considered incorporating the surcharging solution used in the full Wallingford Model (see Bettess, Pitfield & Price, 1978). The two forms of the model are discussed more fully and compared in Section 4.

A time increment of 30 secs was adopted both in the Wallingford Model and the equivalent pipe model. Tests using the Wallingford Model on catchments with extensive surcharging indicated that this was the maximum time increment possible to obtain a stable solution. Further development of the pipe routing procedures used in the Wallingford model since this work was started permit the use of 60 secs as the time increment. Tests of the sensitivity of the equivalent pipe model to time increment (see Section 4.5) showed that the resulting outflow hydrographs were largely independent of the time increment adopted. This may have been because the equivalent pipe model handled surcharging differently.

The equivalent pipe model was incorporated into an existing package of computer programs interfaced with a custom built data base system (Kidd, 1978). The package operates in three modes - simulation, optimisation and error surface mapping and allows for model runs to be performed on single or multiple events. The package also allows plotting of inflow, outflow and modelled hydrographs.

3. DATA GENERATION

3.1 Catchment data

In order to test and develop the equivalent pipe model, it was first necessary to generate a data base of inflow and outflow hydrographs, using the full Wallingford Model, for a large range of catchment types. In this context, the inflow hydrograph is defined as the sum of all the individual above ground hydrographs feeding into the pipe system, and the outflow hydrograph is derived at the outfall of the sewered subcatchment after routing and combining the individual above-ground hydrographs through the full pipe system. To generate suitable data, two approaches are possible:

(a) To design a sewer network for each of a series of completely hypothetical catchments and pipe layouts

(b) To base the analyses on existing sewer networks.

The latter approach was adopted in this study since it was considered more likely to cover the range of catchment conditions encountered in urban areas, and also was more likely to yield truly realistic catchment types. Another advantage was that detailed pipe layouts and information on paved, roof, and pervious areas contributing to each pipe junction (necessary for running the full model) were already available.

The analyses were based on the Shephall catchment in Stevenage, a 1940's residential development comprising houses, shops, schools and open grassed areas. Overall 24% of the area is impervious. The catchment is roughly oval in shape, 142 hectares in area and of moderate slope. It is drained by a separate system. Several quite large pervious areas are present at the bottom of the catchment, including playing fields and a running track. Consequently, in order to exclude such local effects from the data base only the top two thirds of the catchment was considered for this study. For the same reason, several large pervious areas from the top of the catchment were also excluded. Using the remaining area data, and the existing pipe layout, pipe sizes were redesigned using the Wallingford Model with a 2 year return period storm of 30 minute duration.

From the resulting pipe network, twelve subcatchments were chosen to cover a range of catchment areas from 1 to 68 hectares and a range of slopes and catchment shapes (see Figure 3.1). Each subcatchment was in itself fairly homogeneous in terms of impermeability and in terms of the slope of the main sewer. Longitudinal sections of the main sewers are plotted in Figure 3.2. Subcatchment characteristics, including area, number of pipes and the overall length and slope of the main sewer run (the main drag) are shown in Table 3.1.

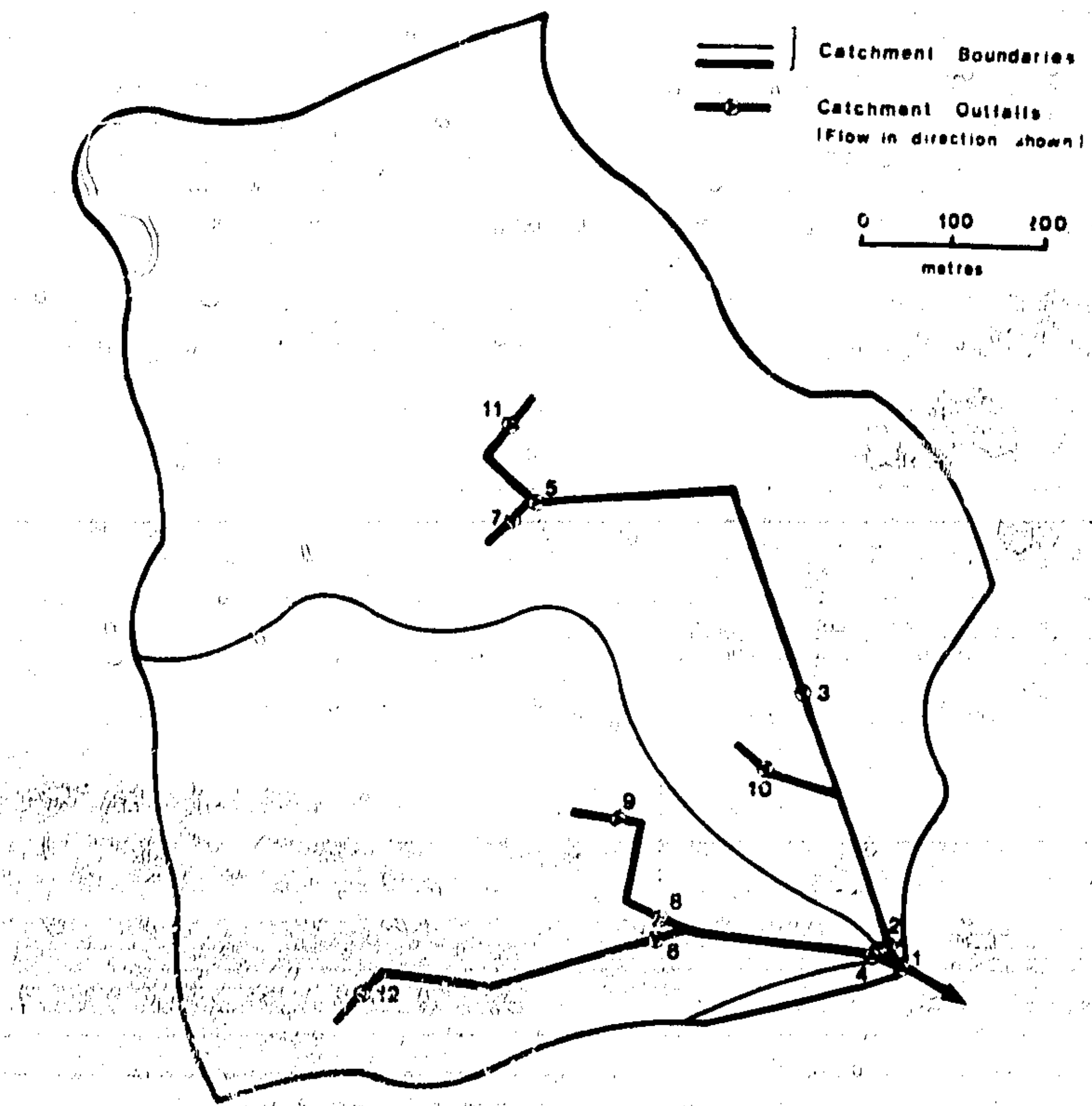


FIGURE 3.1 Shephall, Stevenage: location of catchments

Two measures of slope are included in Table 3.1. The first, called main drag slope, is simply the difference in level between the upper and lower end of the main drag. It is therefore a measure of overall topographic slope. The second measure, called the Taylor-Schwartz slope, is an "equal travel time" slope. It is based on the premise that travel time along a reach of length L and slope S is proportional to $L/\sqrt{S}$; hence the travel time along n reaches is proportional to $\sum_{i=1}^{n} (L_i/\sqrt{S_i})$. Thus, for a channel of constant slope S' and length

$\sum_{i=1}^{n} L_i$ to have the same travel time, S' is given by:

$$S' = \{ \sum_{i=1}^{n} L_i / (\sum_{i=1}^{n} L_i/\sqrt{S_i}) \}^{\frac{1}{2}} \quad (3.1)$$

S' is the Taylor-Schwartz slope, and unlike the main drag slope, it is sensitive not only to overall catchment slope, but also to the range of individual slopes between reaches. Obviously it ignores the effect of hydraulic radius on travel time. It has been included in Table 3.1 because it is ideally suited to the present study where the pipe system is replaced by an equivalent system. The values quoted were found by solving equation (3.1) over all the individual pipes comprising the main drag of the catchment.

TABLE 3.1 SUBCATCHMENT CHARACTERISTICS - FIRST SERIES[1]

CATCHMENT NO.	OUTFALL PIPE NO.	TOTAL AREA (ha)	% PAVED	% ROOF	NO. OF PIPES	MAIN DRAG LENGTH (m)	MAIN DRAG SLOPE (%)	TAYLOR SCHWARTZ SLOPE (%)
1A	1.34	67.7	20.6	17.1	587	1498	1.79	1.50
2A	1.33	41.4	19.3	17.6	354	1475	1.79	1.50
3A	1.26	30.1	20.3	17.4	260	1195	1.85	1.54
4A	113.30	26.3	22.6	[illegible]	222	1034	[illegible]	1.80
[illegible]	1.17	23.9	20.1	16.6	189	789	[illegible]	1.62
6A	140.18	13.0	20.5	15.9	90	853	2.23	1.76
7A	31.15	14.6	17.2	13.4	94	738	1.91	1.43
8A	113.22	11.4	23.0	15.8	100	782	2.75	2.06
9A	113.18	8.6	21.0	15.8	71	658	2.71	1.94
10A	97.06	3.2	20.2	26.8	31	285	2.44	1.67
11A	19.06	2.8	20.6	23.5	26	390	2.59	1.46
12A	144.07	1.3	12.5	20.3	9	237	1.59	1.17

[1] Second series - catchment nos. 1B to 12B - identical to first series, but all slopes halved.

It is evident from Table 3.1 that all catchments are in a fairly narrow band of main drag slope (1.6 to 2.75%). Consequently, to extend the data set to encompass a larger range of slopes, a second series of catchments was formed in which the slope of each pipe was half that of the first series. Pipe sizes were redesigned using the

same 2-year, 30 minute storm. In every other respect, this second series of catchments (numbered 1B to 12B) was identical to the first series, with catchment characteristics as given in Table 3.1.

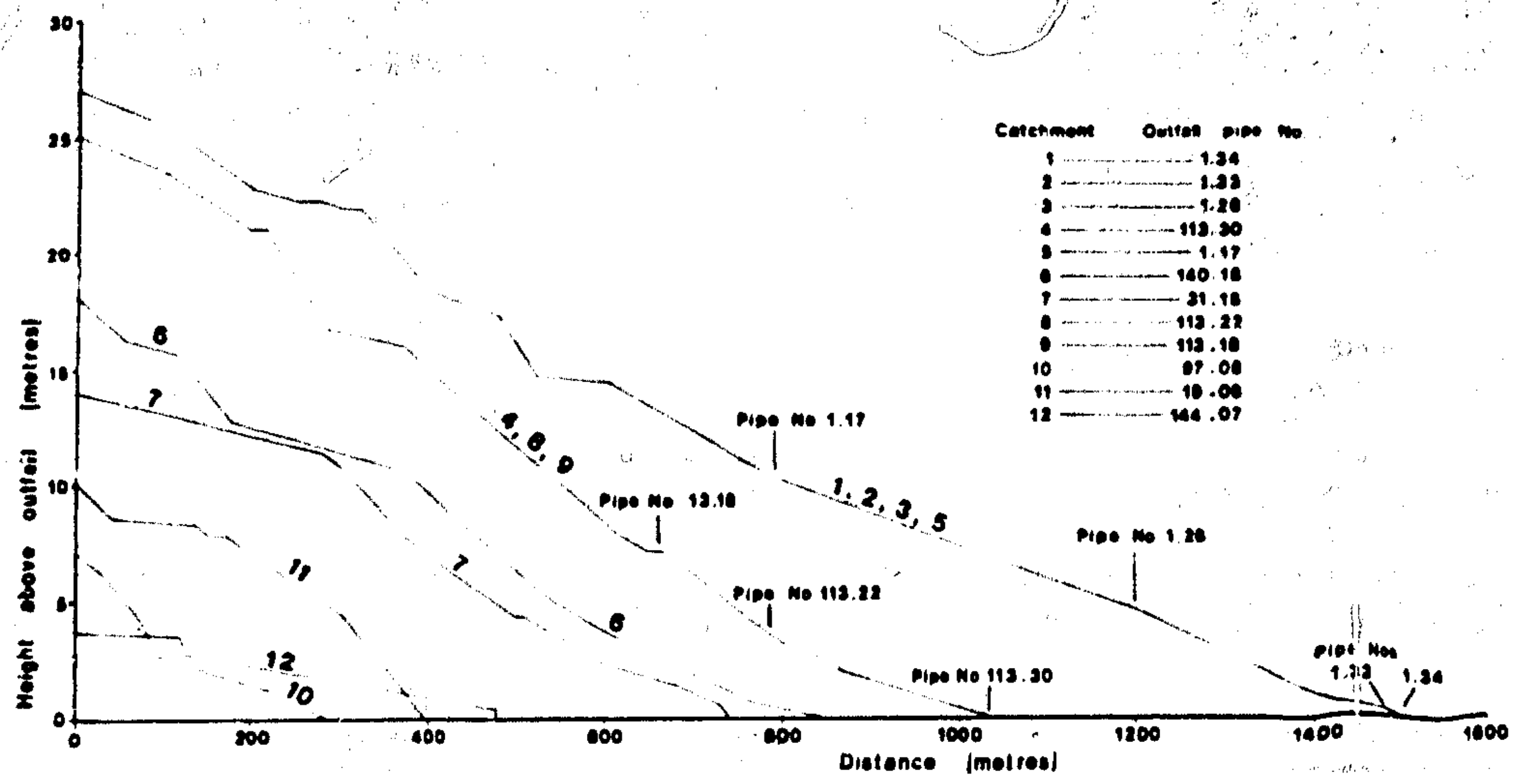


FIGURE 3.2 Shephall, Stevenage: longitudinal sections of main drag pipes for 12 catchments

3.2 Generation of hydrographs

In order to test the model on a wide variety of conditions, ten rainfall events were used as input to the full Wallingford Model. These comprised five design storms and five observed events. The design storm hyetographs, at 1 minute increments were generated for 1, 2, 5, 10 and 25 year return periods using a 50% summer profile and a 30 minute duration in all cases. These rainfall profiles are symmetrical about the peak 1 minute intensity and are compiled according to the method outlined in the Flood Studies Report Vol. II (NERC, 1975). The observed events were extracted from the autographic record for a raingauge sited within the Shephall catchment. The events were all of a less than five year return period and included several multi-peaked rainfall profiles. The wide range of storm types was chosen to reproduce the different degrees of surcharging and cover the spectrum of conditions encountered by a design engineer.

Inflow and outflow hydrographs were generated for each storm on each of the 24 catchments using the Wallingford model with full solution for surcharged flow. The inflow hydrograph is effectively a lumped input to the sewer system for the whole catchment, based on the relative contributions from paved, roof and pervious areas. The outflow hydrograph incorporates both the above and below ground phases, and involves calculation of the inflow hydrograph for each individual pipe, and routing this through the sewer system (see Bettess, Pitfield and

Price, 1978). It therefore approximates the integration process of simultaneous inputs and routing observed in reality. Table 3.2 gives brief details of the input storms and whether they gave rise to surcharging in the pipe system.

TABLE 3.2 STORM CLASSIFICATION

STORM NO.	TYPE OF STORM	SURCHARGING
1	5 year design storm	Yes
2	1 year design storm	No
3	2 year design storm	No
4	Observed storm	No
5	Observed storm	No
6	Observed storm	Yes
7	Observed storm	No
8	Observed storm	No
9	10 year design storm	Yes
10	25 year design storm	Yes

The resulting 240 pairs of inflow and outflow hydrographs were stored on the data base system (referred to in Section 2.4) together with all the necessary catchment data.

4. PARAMETER OPTIMISATION AND CHOICE OF FORM OF THE MODEL

4.1 Introduction

The equivalent pipe model has been outlined in Section 2. It is a four parameter model based on pipe length (LENGTH), pipe slope (SLOPE), pipe diameter (DIAM) and a tapering constant (c).

This section describes the optimisation of parameter values and outlines tests to determine the most suitable form of the model. Analyses are made of the sensitivity of the model to changes in parameter values, and estimation of parameter values from catchment characteristics is considered.

Having established the final form of the model, two further points, in relation to the applicability of the model were investigated.

(i) The effect of model performance on points downstream - it was important to know how any errors in hydrograph simulation would be translated downstream, since the sewered subcatchment model is intended as an integral part of the full Wallingford model.

(ii) The effect of catchment nonhomogeneity - the parameters of the model are essentially catchment average values, and it was important to know how sensitive the model was to non-homogeneities, particularly slope.

These analyses are described in detail in Appendix A and B respectively, since they are peripheral to the main development and testing of the equivalent pipe model.

4.2 Optimisation technique

A Rosenbrock optimisation routine was used to determine optimum parameter values, which could be related to catchment characteristics. This routine was included in the modelling package described by Kidd, 1978.

In order to optimise parameter values, two objective measures of fit were used:

(i) Integral square error (ISE) - this is a version of the common least squares function, but scaled to a dimensionless measure of fit. Although biassed towards higher discharges, it provides an estimate of the overall fit of the hydrographs.

$$ISE = \{ | \Sigma(Qm - Qmp)^2 |^{\frac{1}{2}} / \Sigma Q_m \} \times 100$$

where

Qm = modelled outflow discharge using full Wallingford Model

Qmp = modelled outflow discharge using the equivalent pipe model

The summation is over the hydrograph at intervals equal to 1 time step (30 secs in this case). ISE may be interpreted as the standard error of the simplified model about the full model, expressed as a percentage of the average discharge. ISE = 0 denotes a perfect fit, so minimisation of ISE is desirable.

(ii) Error in peak estimation (PEAK)

$$PEAK = \{(Pm - Pmp)/Pm\} \times 100\%$$

where

Pm = modelled outflow peak discharge using full Wallingford Model

Pmp = modelled outflow peak discharge using equivalent pipe model.

Note that PEAK is insensitive to timing errors, while ISE is very sensitive to timing errors.

In addition to these two objective measures of fit a visual assessment of the fit of the two modelled hydrographs was also made.

4.3 Parameter Optimisation

Although the four parameters (LENGTH, SLOPE, DIAM, c) of the equivalent pipe model all have separate physical meaning, there is a marked dependence between three of them (LENGTH, SLOPE, DIAM) in terms of model performance. In other words, a similar model response can be achieved with a range of combinations of these three. This dependence arises because, although the Muskingum-Cunge solution allows for both translation and attenuation, attenuation in the pipe system is small, and translation is the dominant effect. Consequently, the Muskingum-Cunge solution tends to a time offset method, with the offset, T, given by:

$$T = LENGTH/(n\bar{W}) \qquad (4.1)$$

where n is the number of pipes in the system

and $\bar{W}$ is the mean wave speed.

Wave speed can be shown to depend on slope to the power ½ and hydraulic radius to the power 2/3. Thus:

$$T \propto LENGTH/(n\ SLOPE^{1/2} DIAM^{2/3}) \qquad (4.2)$$

As the Muskingum-Cunge model tends to a time offset model, so the equivalent pipe model tends to a moving average, of order n and interval T. In fact, the equivalent pipe model is not that simple because (i) the effect of the tapering constant, c, is to reduce the diameter and hence increase the time interval for the upstream pipes, and (ii) some attenuation *is* present in the sewer system. Consequently, the dependence between the model parameters is not quite as given by equation (4.2). However, the dependence still exists, and thus to obtain a sensible relationship for any one parameter, values for the other two must be fixed *a priori*.

Since equation (4.2) suggests that the model would be most sensitive to changes in LENGTH, it was decided to fix values for SLOPE and DIAM and optimise on LENGTH. Besides these parameters, a value was required for c. Initial experience with the model showed that while LENGTH affected both peak flow and time to peak, time to peak was fairly insensitive to c. Consequently it was decided to fix c also and optimise only in terms of LENGTH. The value of c could be adjusted later, if necessary, to improve the fit to peak.

One final decision remained before optimisation could begin and this concerned the method of routing flows greater than pipe full. It was

decided that initially the model fit to non-surcharge events was more important, and that a good fit to surcharge events was a "bonus". The model fit was therefore optimised using only the non-surcharge events, and the fit to surcharge events was examined at a later stage.

Values for the three parameters SLOPE, DIAM and c were chosen as follows:

SLOPE: This was set equal to the Taylor-Schwartz slope of the real (prototype) catchment. The Taylor-Schwartz slope was chosen in preference to the simpler "overall slope of the main drag" because it was expected to be more closely related to travel time. The rationale behind this measure of slope is given in Section 3.1.

DIAM: This was chosen such that the equivalent pipe system should be representative of the real (prototype) system. The diameter of the last pipe in the equivalent system was defined such that its pipe full discharge should be the same as the last pipe in the prototype system (its slope will, in general, be different). This definition assumes the last prototype pipe has a design standard consistent with the rest of the system.

Tapering constant, c: Three values of tapering constant were considered in the analysis; c = 0.0 (ie a non-tapered system), c = 0.2 and c = 0.3. These last two values were chosen in order to try and simulate the average development of surcharging that might be expected in the prototype catchment. Clearly, with c = 0.0, upstream pipes are larger than necessary - surcharging would occur first at the bottom of the system and move upstream. For large values of c (approaching unity) the opposite would occur. A compromise between these two extremes was necessary. The spread of surcharging through the equivalent pipe system was studied for a range of values of c. It was found that for values of c = 0.4 and larger, surcharging moved in a downstream direction, while for c = 0.2 and below it generally commenced at a downstream pipe and spread upstream. For values of c between 0.2 and 0.3 there was uniform development of surcharging and it was decided that this best represented the rather localised surcharging (upstream of any constrictions) in the full Wallingford Model.

For the purposes of optimisation, LENGTH was redefined as

LENGTH = K x (length of main drag of prototype catchment).

Optimisation was then performed on K, the ratio of the lengths of the main drags in the model and prototype systems. Optimum values for K were determined for each catchment using the six non-surcharge events combined, and optimising in terms of both ISE and PEAK. Table 4.1 gives the values of k obtained for a tapering constant (c) of 0.2. It can be seen that in terms of ISE the optimum K was 1.084, while in terms of PEAK the value was 0.914.

TABLE 4.1 **OPTIMAL K VALUES FOR c = 0.2**

CATCHMENT NO.	OBJECTIVE FUNCTION PEAK	OBJECTIVE FUNCTION ISE	CATCHMENT NO.	OBJECTIVE FUNCTION PEAK	OBJECTIVE FUNCTION ISE
1A	0.892	1.186	1B	0.891	1.192
2A	0.868	1.108	2B	0.900	1.156
3A	0.858	1.172	3B	0.832	1.162
4A	0.917	1.132	4B	0.835	1.256
5A	0.975	1.126	5B	1.002	1.172
6A	0.831	0.927	6B	0.705	0.924
7A	1.019	1.149	7B	1.141	1.218
8A	0.921	1.026	8B	0.869	1.025
9A	0.967	1.062	9B	0.902	1.048
10A	1.074	1.221	10B	0.630	0.752
11A	1.001	1.049	11B	0.896	0.961
12A	0.993	0.899	12B	1.012	1.037
AVERAGE (STEEP)	0.943	1.093	AVERAGE (FLAT)	0.885	1.074
OVERALL AVERAGE	0.914	1.084			

A visual assessment of the fits achieved showed that the discrepancy between the optimum K values was largely due to the difficulty in fitting both peak and time to peak. In order to achieve a good overall fit, an average of the two optimum K values has been taken, yielding the intuitively attractive result:

$$K = 1.00$$

or: the model main drag length = prototype main drag length.

The analysis was repeated for c = 0.0 and 0.3 yielding average K values of 1.07 and 0.97 respectively. Neither value is as attractive as the value 1.00 achieved for c = 0.2.

Table 4.2 shows the ISE and PEAK error obtained when K = 1, and also the ISE and PEAK errors obtained using the catchments optimum K (in terms of ISE). The trade off in terms of ISE and PEAK achieved for K = 1 is quite favourable. It is interesting but not surprising to note the general increase in PEAK and ISE for the larger catchments, reflecting the increased importance of sewer flow.

The number of pipes in the equivalent system depends on LENGTH (see equation 7.10) and thus also on K. The 24 catchments studied were represented in the equivalent pipe system by between 4 and 15 pipes, compared with between 9 and 587 pipes in the prototype system. In all cases, equivalent pipe slopes were in the range 0.6 to 1.5%, individual equivalent pipe lengths were between 60 and 150 metres, and outfall pipe diameters between 0.313 and 0.981 metres.

TABLE 4.2 TRADE OFF BETWEEN PEAK ERROR AND ISE FOR CHANGE IN K (c = 0.2)

CATCHMENT NO.	K=OPTIMUM		K = 1		CATCHMENT NO.	K=OPTIMUM		K = 1	
	ISE	PEAK	ISE	PEAK		ISE	PEAK	ISE	PEAK
1A	1.98	13.0	2.70	5.2	1B	2.27	13.7	3.06	8.0
2A	1.95	11.1	2.43	6.2	2B	2.17	12.4	2.74	5.3
3A	2.20	14.2	2.85	7.4	3B	2.48	15.0	3.17	8.2
4A	1.97	12.0	3.06	4.6	4B	2.38	16.4	3.37	7.6
5A	0.80	6.0	1.47	1.1	5B	0.94	6.5	1.75	- 0.1
6A	0.90	5.5	1.23	9.0	6B	1.50	11.3	1.74	15.5
7A	0.77	4.2	1.43	- 2.4	7B	0.43	2.8	1.77	- 5.8
8A	0.70	4.7	0.74	5.7	8B	0.99	7.0	1.02	6.9
9A	0.62	4.3	0.74	1.0	9B	0.85	6.4	0.97	4.5
10A	0.52	3.5	1.96	- 2.0	10B	0.79	6.8	2.75	17.9
11A	0.72	1.5	0.82	0.1	11B	0.66	3.6	0.76	5.6
12A	0.54	2.7	0.96	1.1	12B	0.51	1.4	0.57	- 2.6

4.4 Choice of final form of the model

Having established suitable definitions for the parameters LENGTH, SLOPE and DIAM in terms of catchment characteristics, it now remained to determine whether to adopt a tapered or non tapered version of the equivalent pipe model. Another consideration was whether a complete solution for surcharged pipe flow, similar to that in the full Wallingford model, should be adopted.

The criterion for comparing the different forms of the model was the goodness of fit between the modelled hydrographs and those generated from the full Wallingford model. Fit was assessed both visually and using the more objective measures, ISE and PEAK.

Initially the effect of varying the tapering constant c was examined more closely. Hydrographs were generated for a 1 year design storm with c in the range 0.0 to 0.3 as suggested by surcharging considerations (Section 4.3). Table 4.3 summarizes the results for 12 catchments and a typical set of hydrographs are plotted in Figure 4.1. It is clear that a non-tapered system (c = 0.0) yields a closer fit to the full Wallingford model hydrograph peak, and increasing c progressively flattens and broadens the peak. PEAK errors are therefore

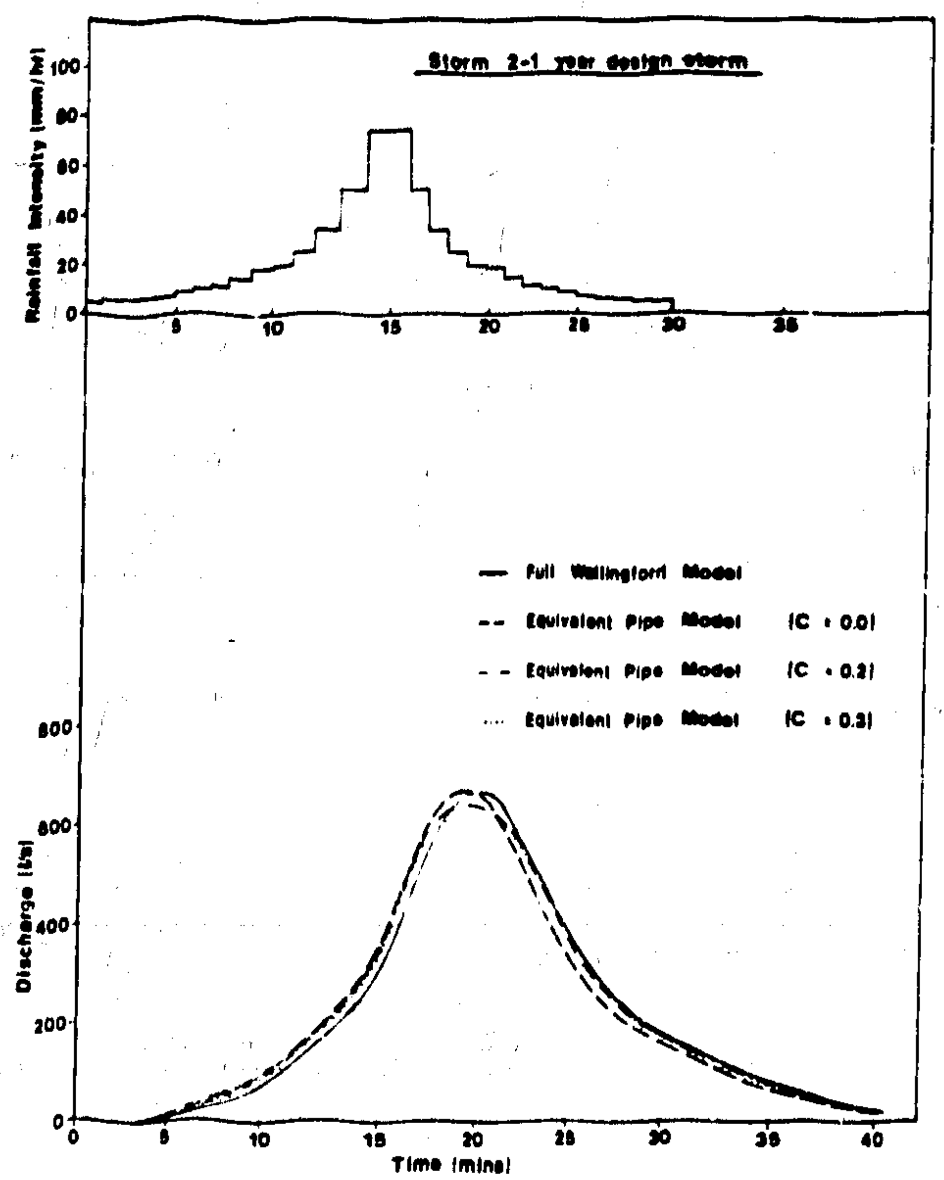


FIGURE 4.1

Comparison of different values of tapering constant

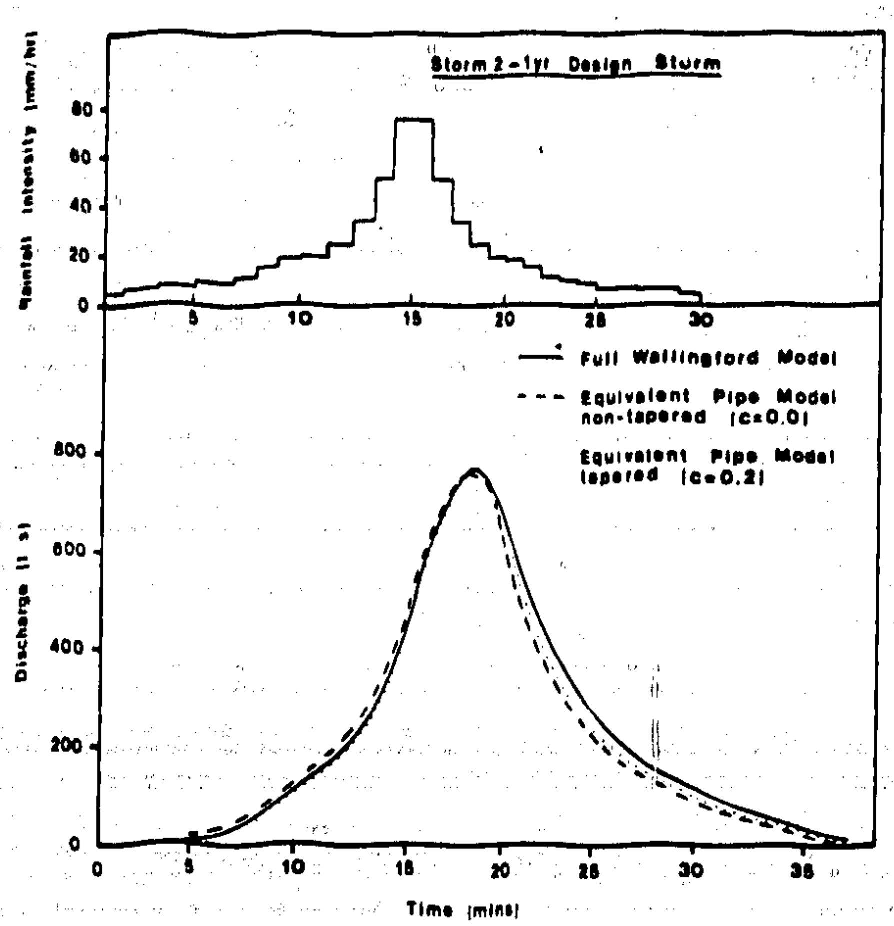


FIGURE 4.2

Catchment 5A - comparison of tapered and non tapered equivalent pipe models

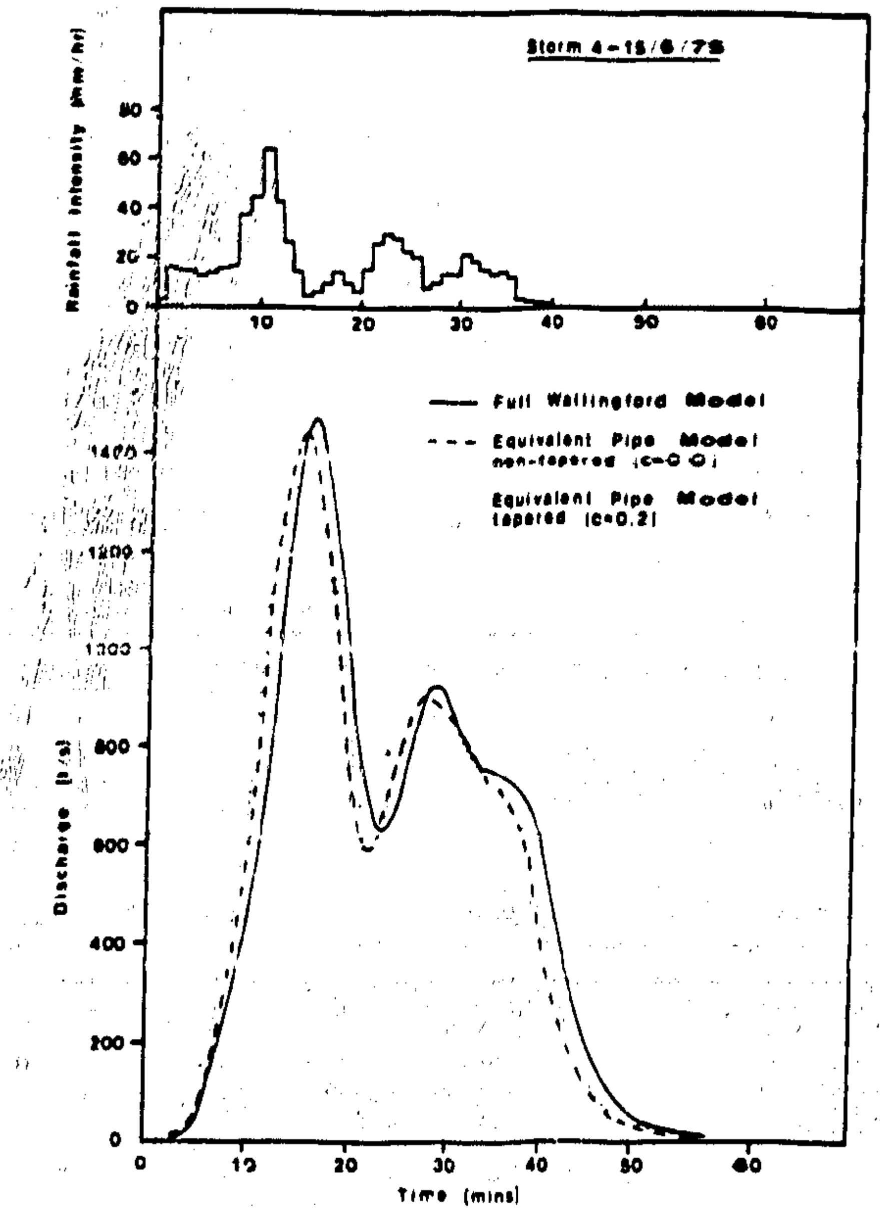


FIGURE 4.3

Catchment 1A - comparison of tapered and non tapered equivalent pipe models

FIGURE 4.4

Catchment 12B - comparison of tapered and non tapered equivalent pipe models

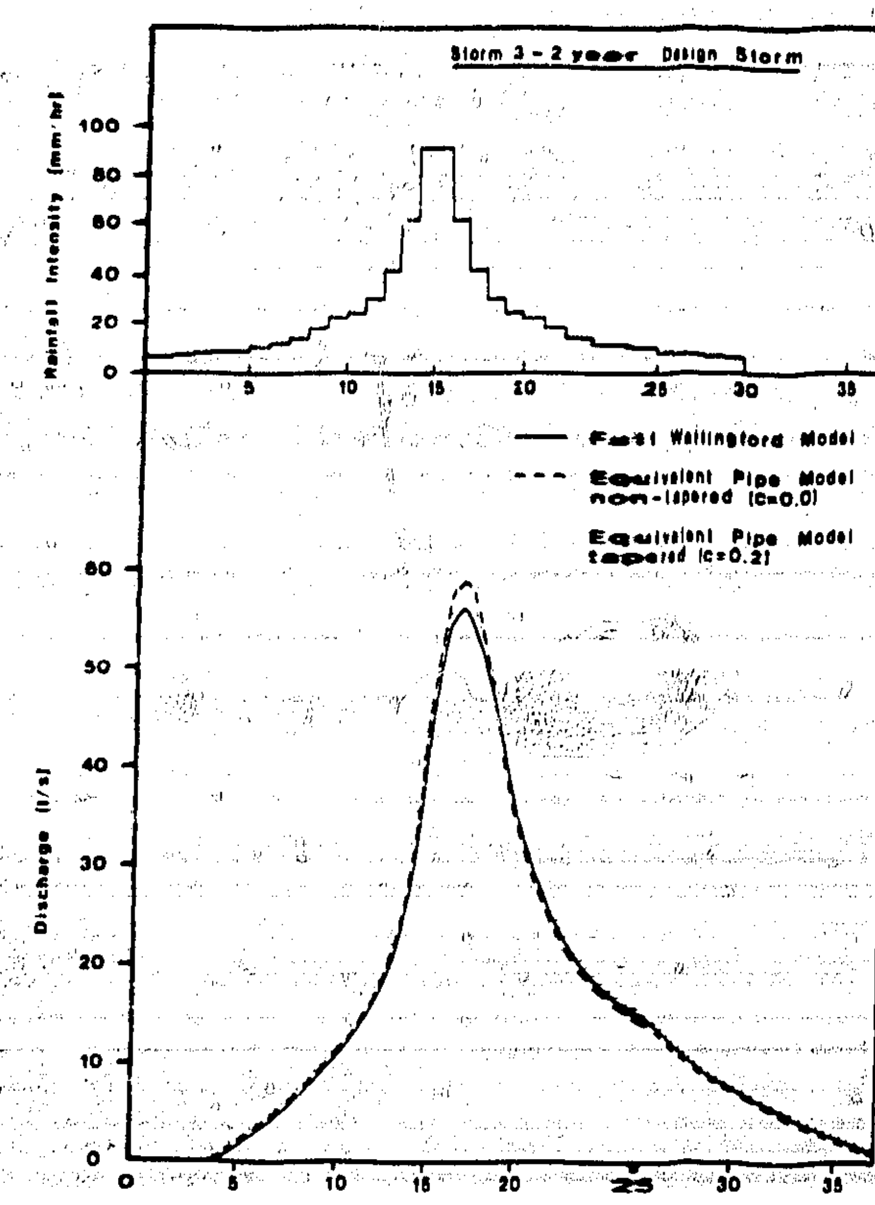


TABLE 4.3 EFFECT ON OUTFLOW HYDROGRAPH OF VARYING TAPERING CONSTANT IN EQUIVALENT PIPE MODEL

STORM 2. (1 YEAR DESIGN STORM)

CATCHMENT	PEAK ERROR(%) c			PEAK TIMING ERROR(mins) c			ISE OBJECTIVE FUNCTION c		
	0.3	0.2	0.0	0.3	0.2	0.0	0.3	0.2	0.0
1B	10.2	7.6	3.4	+ 1.0	+ 1.0	+ 1.0	2.55	2.69	3.15
2B	10.4	7.7	2.1	+ 1.0	+ 1.0	+ 1.0	2.28	2.38	2.83
3B	13.5	11.1	6.2	+ 1.0	0.5	1.0	2.53	2.63	3.02
4B	8.2	5.9	1.2	+ 1.5	1.5	1.5	3.22	3.46	4.00
5B	5.7	3.6	0.4	0.0	0	0	0.89	0.99	1.40
6B	15.8	13.7	9.4	0.0	0	0	1.71	1.43	1.02
7B	- 4.6	- 6.7	-10.4	+ 0.5	0.5	0.5	1.70	1.97	2.48
8B	9.2	6.9	2.6	- 0.5	- 0.5	0	0.90	0.74	0.83
9B	7.3	5.2	- 1.2	- 0.5	- 0.5	0	0.73	0.63	0.83
10B	1.2	- 0.1	- 2.3	0.0	0	0	1.37	1.54	1.89
11B	7.1	5.5	2.2	- 1.0	0	0	1.00	0.79	0.49
12B	- 2.6	- 3.6	- 5.1	0.0	0	0	0.35	0.46	0.72
AVERAGE	6.8	4.7	0.7	.3	.3	.4	1.60	1.64	1.89

Negative error - modelled peak exceeds/occurs after full model peak

smaller for a non-tapered system, while ISE values, which consider the fit of the hydrograph as a whole, are larger for a non-tapered than a tapered system. These results indicate that c = 0.2 is preferable to c = 0.3 and thus c = 0.2 was adopted in future analyses.

The comparison of tapered (c = 0.2) and non-tapered models (c = 0.0) was then extended to a greater range of catchments and storm types. The same trends were exhibited by the two groups of catchments, and the results from the flatter set are summarized in Table 4.4. Typical hydrographs have been plotted in Figures 4.2 to 4.4. As noted in the pilot study the non-tapered model gave the closest fit to the peak for non-surcharge events, but a marginally higher overall ISE value. In both cases timing discrepancies of up to one minute were found with the simplified model hydrograph tending to occur before the full model hydrograph.

It is apparent from Table 4.4, that for events with surcharging (storm 1 - minor surcharging, storm 9 - major surcharging) this trend is reversed and c = 0.2 generally gave the better fit to the outflow hydrograph, reflected in the lower ISE and PEAK error values. The equivalent pipe model in virtually all cases overestimated the full model peak discharge. This is to be expected as the equivalent pipe model, in the form described, makes no allowance for the attenuating effects of surcharging. On balance, the improvement in performance of the tapered system under surcharging conditions, more than compensates for the fact that the non-tapered system was slightly better for non-surcharged events. For this reason, a tapered system with c = 0.2 is recommended.

TABLE 4.4 COMPARISON OF DIFFERENT FORMS OF EQUIVALENT PIPE MODEL

(1) TAPERED/NON TAPERED (WITHOUT SURCHARGING SOLUTION)

Storm No.	Type	Average values for 12 catchments (series B): Average peak error % c = 0.2	Average peak error % c = 0.0	Average peak timing error (mins) c = 0.2	Average peak timing error (mins) c = 0.0	ISE objective Function c = 0.2	ISE objective Function c = 0.0
7	Non surcharge	4.8	0.8	0.3	0.4	1.65	1.89
4	Non surcharge	5.8	1.6	0.5	0.4	1.12	1.22
9	Surcharge	2.9	- 2.5	- 1.1	- 1.1	1.53	2.05
10	Surcharge	- 4.4	- 9.5	- 1.9	- 2.1	2.62	3.01

(2) WITH/WITHOUT FULL SURCHARGING SOLUTION

Storm No.	No. of catchments with surcharging	Average values for catchments with surcharging: Average peak error % without	Average peak error % with	Average peak timing error (mins) without	Average peak timing error (mins) with	ISE objective function without	ISE objective function with
1	6	4.1	1.9	- 1.8	0.2	1.66	1.76
9	11	- 4.5	- 14.1	- 2.1	0.7	2.03	2.63

Negative error - modelled peak exceeds/occurs after full model error

The introduction of a surcharging solution of the same form as used in the full Wallingford model (see Section 1.2) can now be considered. There has previously been no allowance for surcharging in the equivalent pipe model, with flows always routed by the Muskingum-Cunge method. Surcharging and non-surcharging forms of the model were compared for two major storms on all catchments which had pipes surcharged in the Wallingford model (see Table 4.4 and Figures 4.5 to 4.7). In general a full surcharging solution tended to overestimate peak discharge, especially for large catchments. A typical outflow hydrograph (Figure 4.7) shows a rapid rise to peak as the flood wave is translated rapidly downstream followed by a flattening of the recession. This can result in errors of peak estimation of up to 33%. Timing discrepancies between the full and simplified models however tend to be smaller when a surcharging solution is adopted.

At first sight it might be expected that the full surcharging solution should provide a better simulation than the simpler non-surcharging Muskingum-Cunge solution. There are two probable reasons why this is not so.

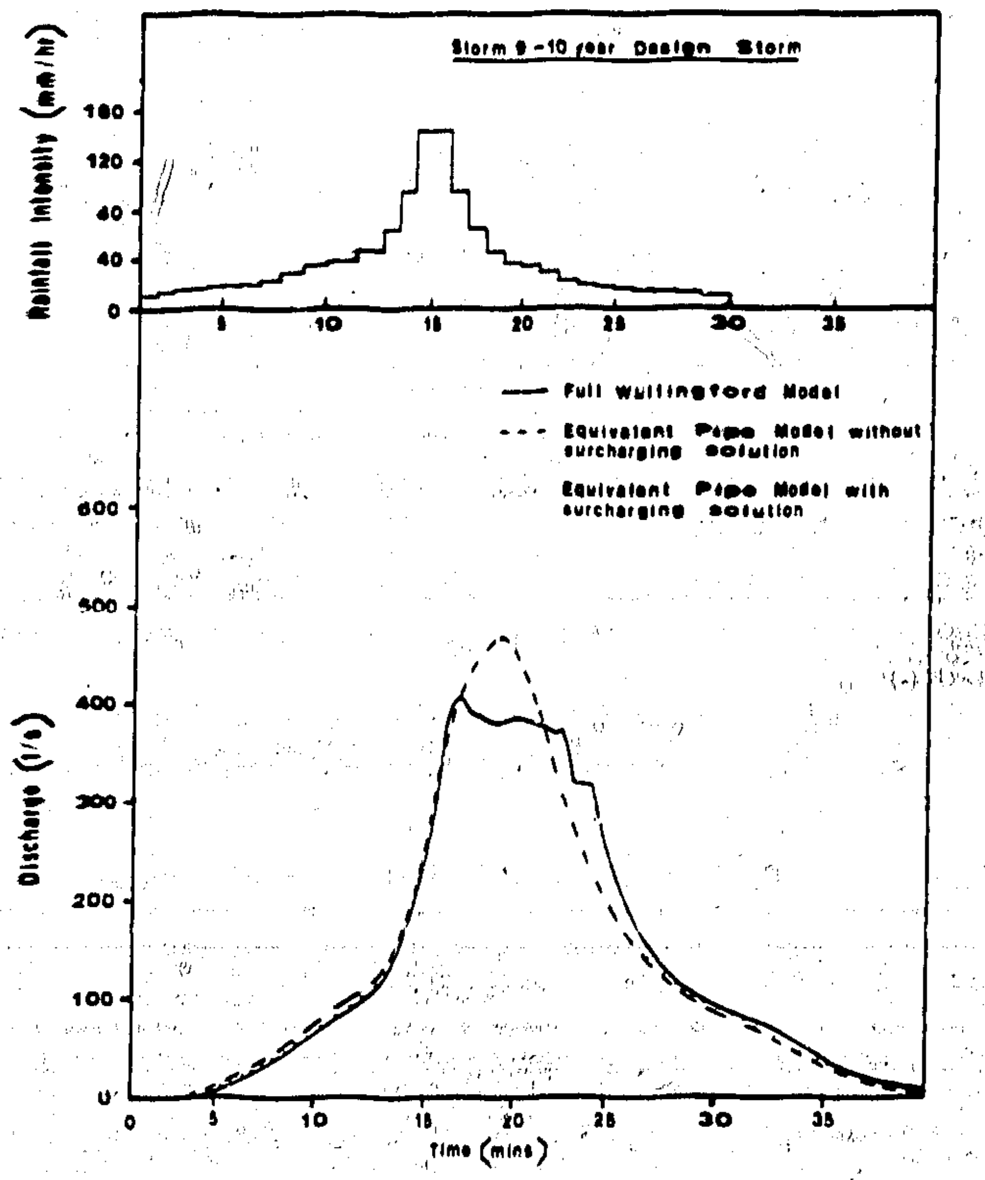


FIGURE 4.5

Catchment 9B - comparison of hydrographs from equivalent pipe model with/without a full surcharging solution

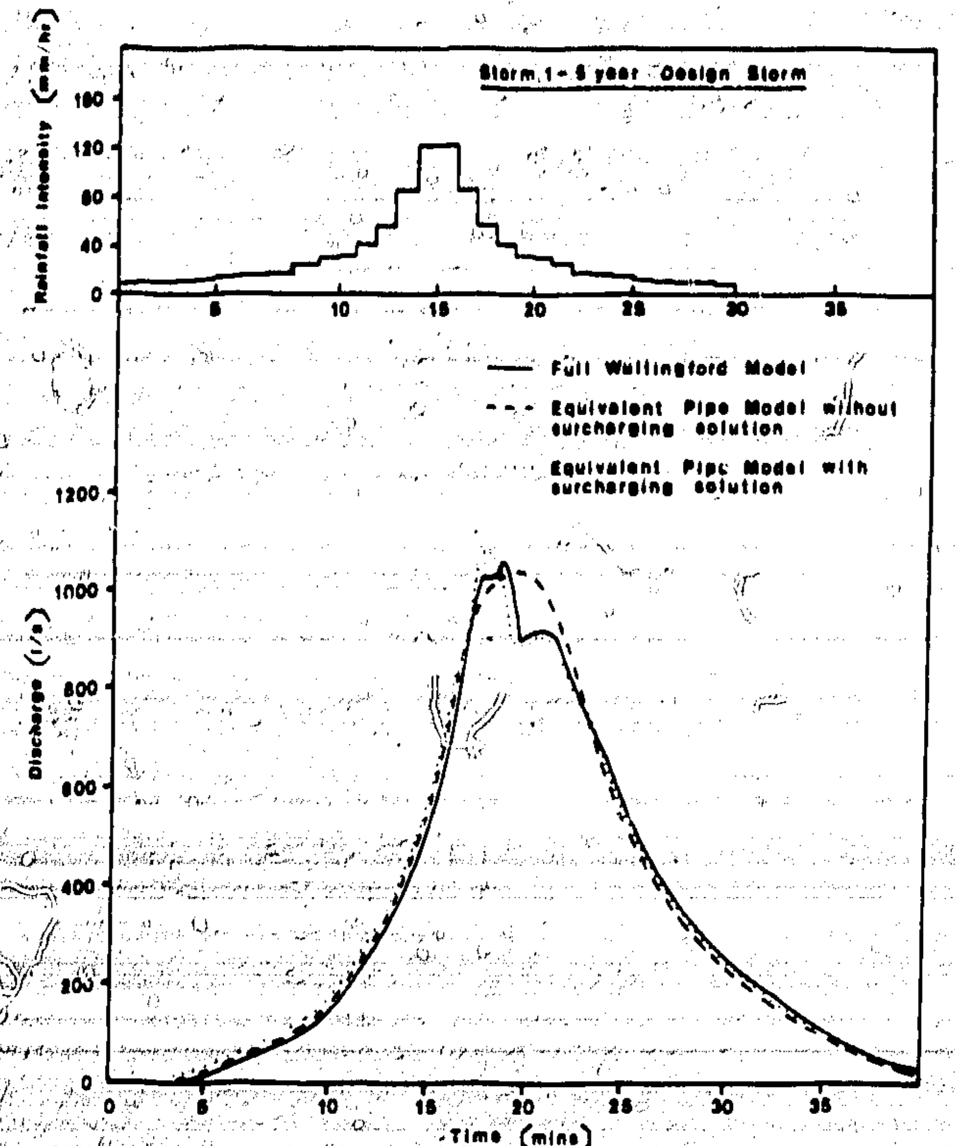


FIGURE 4.6

Catchment 5B - comparison of hydrographs from equivalent pipe model with/without a full surcharging solution

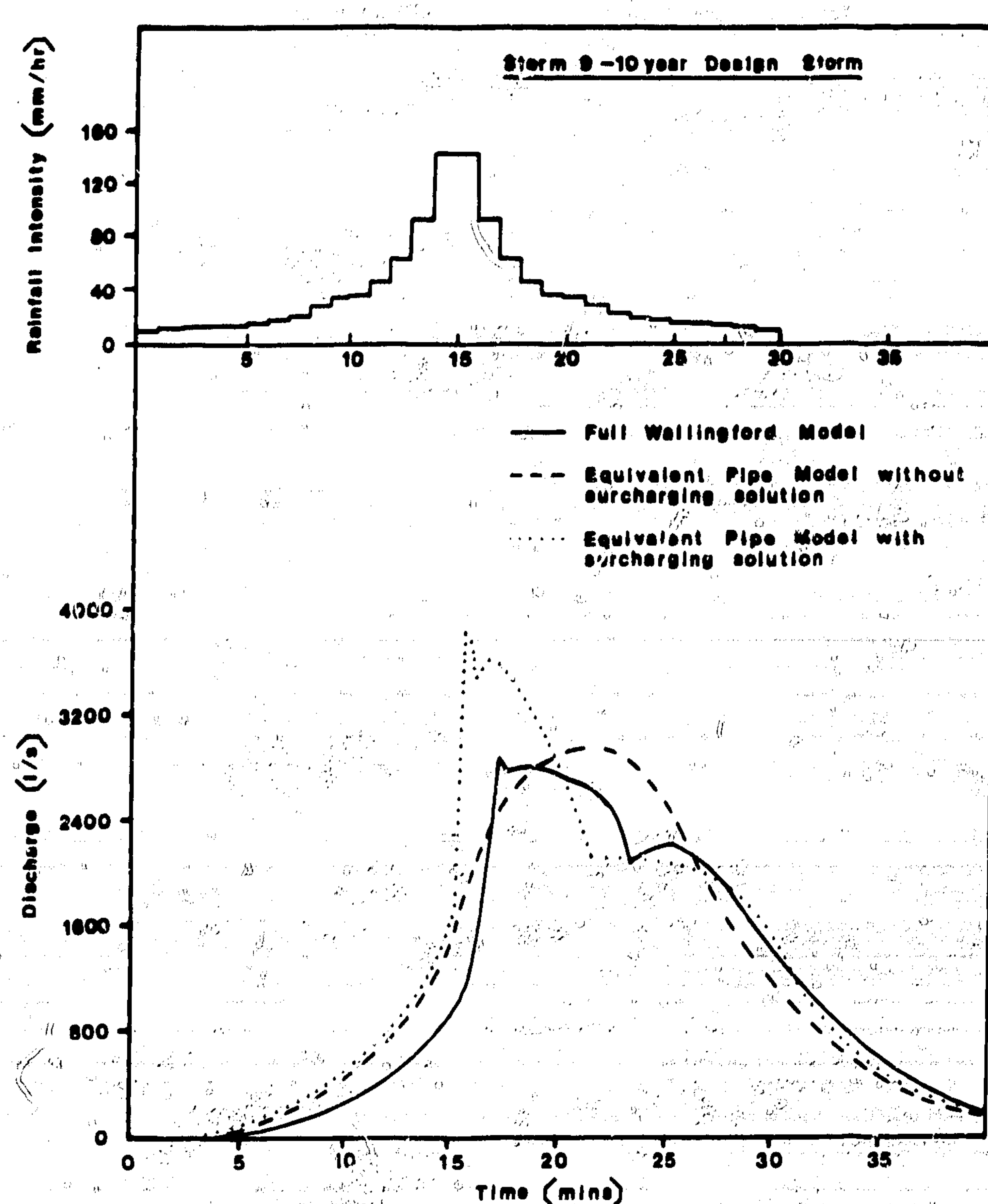


FIGURE 4.7 Catchment 1B - comparison of hydrographs from equivalent pipe model with/without a full surcharging solution

Firstly the equivalent system does not exactly mirror the prototype in that surcharging occurs along the whole length of the equivalent system and not in small localised "pockets". It might be possible to overcome these difficulties by the use of commercial pipe sizes (ie in increments of 75 mm). Secondly, the equivalent system has far fewer manholes than the prototype, and manhole storage plays an important role in reducing wave speed for surcharged events. The use of larger diameter manholes might overcome this difficulty. These options were not however investigated in the present study.

Based on the above discussion, it was decided to adopt the tapered form of the model (c = 0.2) using Muskingum-Cunge routing with no explicit allowance for surcharging.

4.5 Sensitivity of model parameters

The sensitivity of the model, as reflected by the peak discharge, to changes in the values of the four model parameters (LENGTH, SLOPE, DIAM and tapering constant c) was examined for a selection of catchments using storm 2, a non-surcharge event. The results are presented in Figures 4.8 to 4.11. These figures demonstrate that model performance is fairly insensitive to changes in parameter values. A 10% increase in length gave approximately 4% decrease in peak discharge, while a 10% increase in slope or diameter gave increases in peak discharge of approximately 2% and 1% respectively. The relationship between the parameters is not quite the same as given by equation (4.2) for the "time offset", indicating the effect of attenuation in the system. Figure 4.11 shows model sensitivity to tapering constant, c. Peak discharge decreases with the degree of tapering, a change from c = 0.2 to 0.0 resulting in an increase in peak discharge of approximately 4%. Not surprisingly, model sensitivity increases with catchment size reflecting the importance of the below ground phase of the response.

Besides the four parameters discussed above, there is another model parameter which has been embedded in the analysis, the time increment. A time increment of 30 seconds was used for both the full model and the sewered subcatchment model in this study. However, the full Wallingford Model has now been developed to run at a time increment of 60 seconds. With a 60 second time increment, the number of pipes used in the equivalent pipe model is halved and individual pipe lengths are doubled. However, tests showed that the effect on model performance is slight.

4.6 Estimation of model parameters in design

As noted in the introduction, there are basically two situations in which the equivalent pipe model is likely to be applied.

1. An existing development where either the detailed information required by the full Wallingford Model is not available, or the use of the full model is not appropriate to the particular design problem.

2. A proposed development for which exact details of the sewer layout are not yet available.

In the first case it should be possible to obtain the length (L_i) and slope (S_i) for each pipe along the main drag of the catchment and hence determine LENGTH (the total length of the main drag) and SLOPE (the Taylor-Schwartz slope - see equation 3.1).

Given the diameter ($DIAM_p$) and slope (S_p) of the last pipe in the prototype system the equivalent pipe diameter (DIAM) may be found iteratively from the Colebrook-White equation using the equivalent pipe SLOPE. However this is a lengthy process and the following equation derived from Manning's formula will give results of sufficient accuracy.

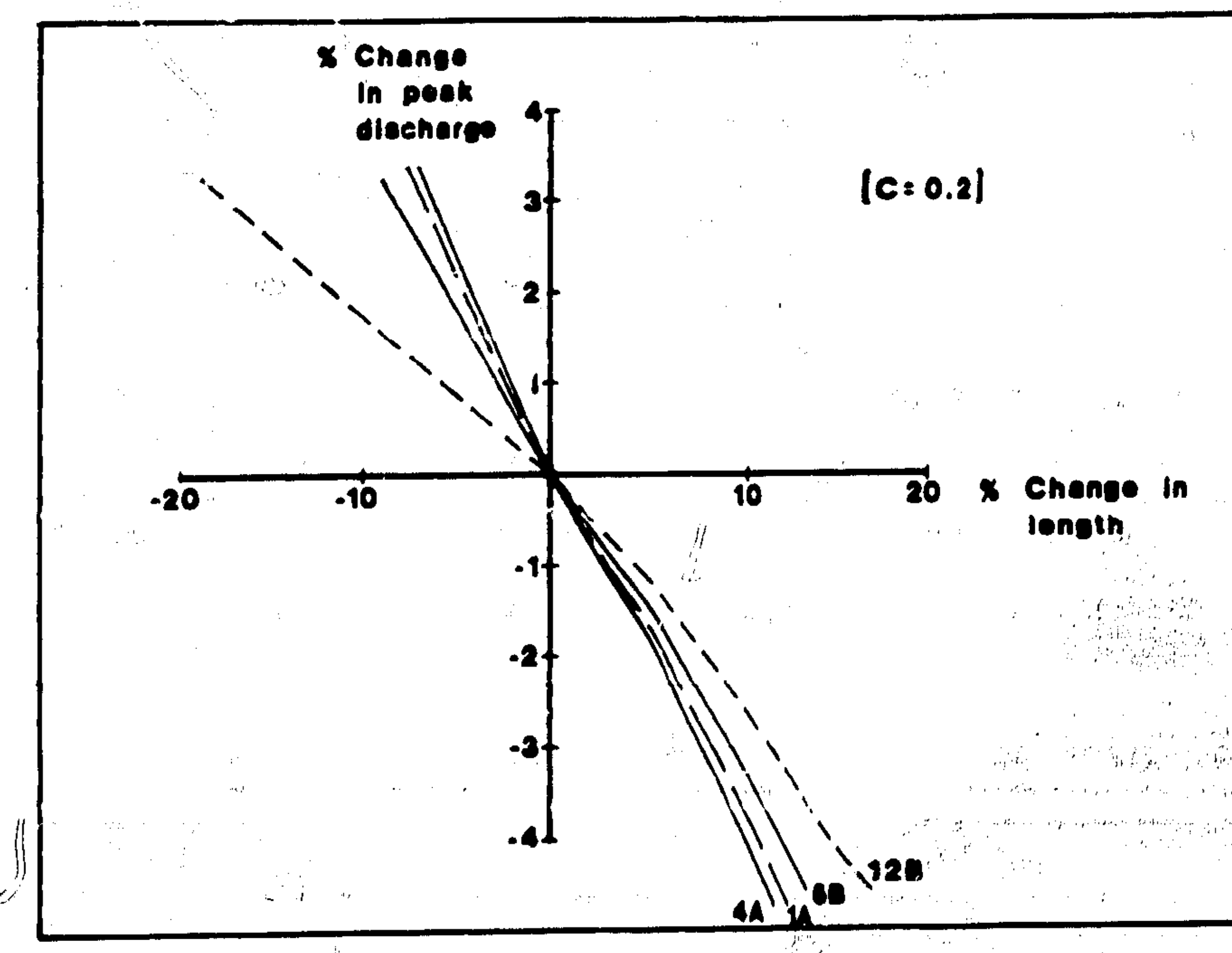


FIGURE 4.8 Sensitivity of equivalent pipe model to length

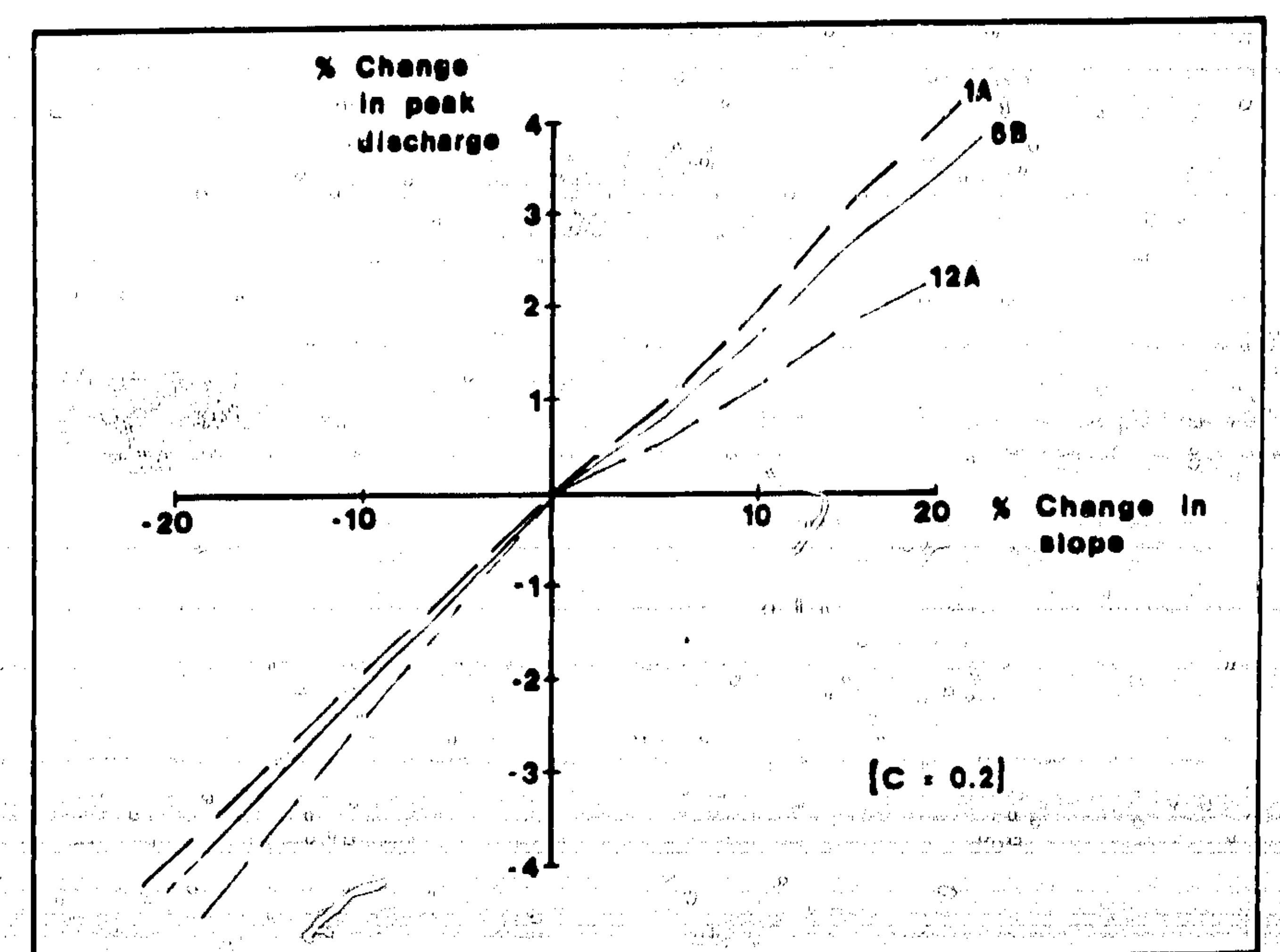


FIGURE 4.9 Sensitivity of equivalent pipe model to slope

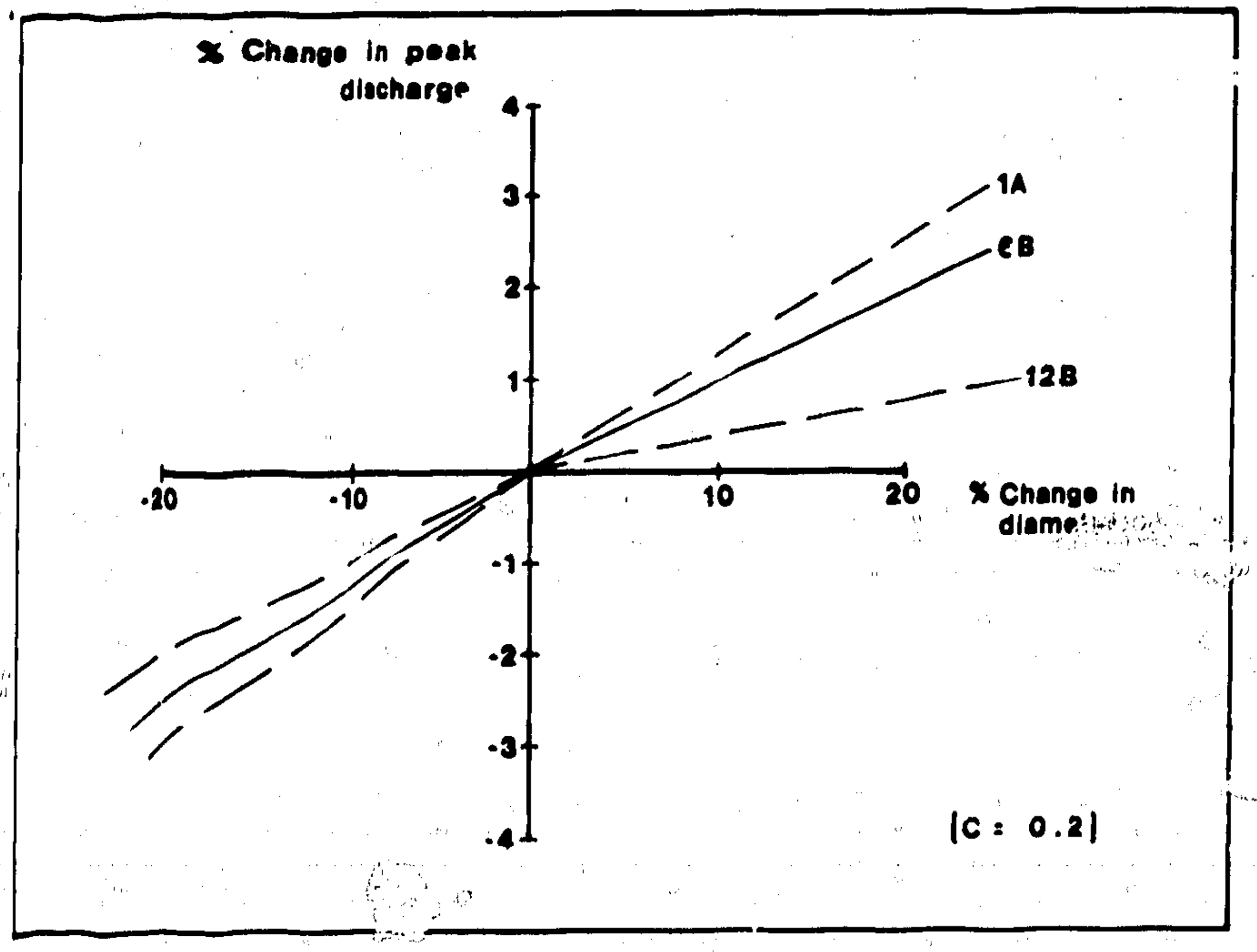


FIGURE 4.10 Sensitivity of equivalent pipe model to diameter

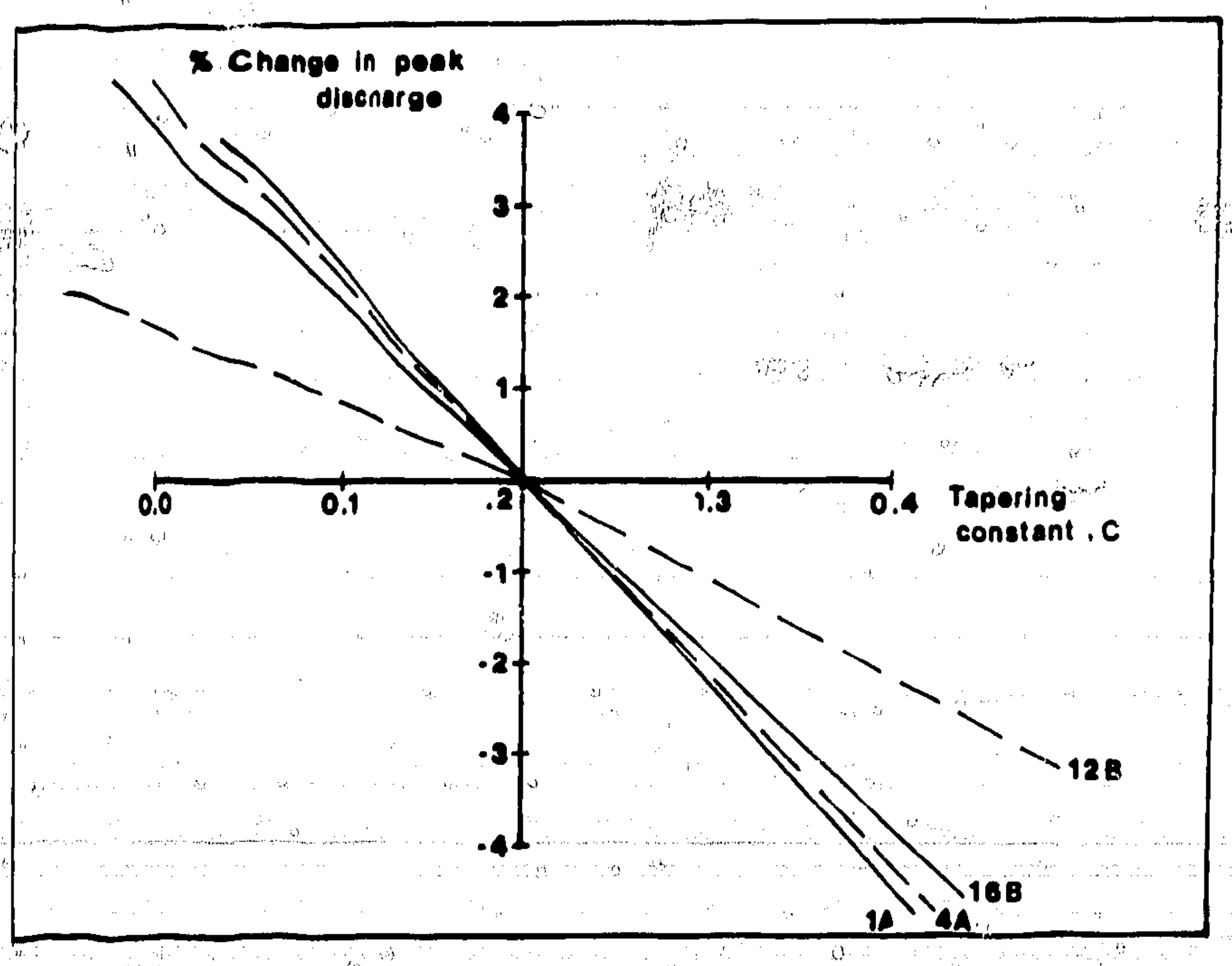


FIGURE 4.11 Sensitivity of equivalent pipe model to tapering constant c

$$DIAM = DIAM_P \left(\frac{S_P}{SLOPE}\right)^{0.2} \quad (4.3)$$

In this context it should be realised that sizing DIAM on the last pipe in the prototype system may not give a truly representative diameter. For example, if the last pipe is at a steeper slope than pipes upstream, but (as is usual) pipe diameters are never reduced downstream, the capacity of the last pipe could be much greater than is necessary. In this situation, a more representative diameter may be achieved using the next pipe up the system. However, it should be noted that the model is less sensitive to DIAM than its other parameters, and accurate estimation is not of vital importance.

In the second situation (where development is proposed) estimation of model parameters may be more difficult. However, the engineer should be able to decide on an approximate main drag pipe layout and from this determine the length and slope of the main drag pipes. To help in this respect, a pilot study was undertaken to relate Taylor-Schwartz slope to the overall slope of the main drag. Based on pipe data from several catchments in the UK the following relationship was obtained.

$$SLOPE = S' = 0.75 \times (\text{topographic slope of main drag}) \quad (4.4)$$

Having decided on a main drag, and determined LENGTH and SLOPE, an estimate of DIAM may be made using the Rational Formula

$$Q = CIA \quad (4.5)$$

where Q is pipe full discharge

C is runoff coefficient

I is rainfall intensity in time of concentration

A is catchment area

To determine time of concentration, it can be shown that for a tapering constant of 0.2, the flow time through the sewer system is given by

$$T_f \approx 0.05 \frac{n \, LENGTH}{DIAM^{2/3} SLOPE^{1/2}} \text{ minutes} \quad (4.6)$$

where n is Manning's n

Thus, if an estimate is made of DIAM, equation (4.8) may be used to yield an estimate of T_f. Adding a time of entry 9 minutes gives a time of concentration. The Rational Method may then be used to estimate Q and determine an updated value for DIAM. This may be used for a second iteration of equations (4.8) and (4.7) if necessary.

In estimating model parameters, it is encouraging to note from Figures 4.8 to 4.11 that a ± 10% error in each parameter in the worst sense will result in a maximum error of peak estimation of only about 7½%. This figure of course only relates to the choice of parameter values and not to model error. In Section 5 of this report the fit of both the full model and the sewered subcatchment model (ie above ground model + equivalent pipe model) to observed hydrographs is investigated.

5. TESTING MODEL PERFORMANCE ON DIFFERENT CATCHMENTS

Once the final form of the model had been established (see Section 4.4), it now remained to test the performance of the model on catchments different from those used in its calibration.

Two widely different catchments at Derby and Blackpool, both with readily available pipe and rainfall/runoff data were selected for the analysis. Since the sewered subcatchment model is essentially a model of a model, it was the fit of the outflow hydrograph generated by this model to the full Wallingford Model hydrograph which was of prime importance. Comparisons were performed for five observed rainfall events, including both single and multipeaked profiles and events of a high rainfall intensity. As observed flow data was available for these events, it has also been included and model runs were performed using observed rather than calculated percentage runoff values.

5.1 Catchment characteristics for test catchments

Grange Park, Blackpool is a small (5.04 ha) steeply sloping catchment, approximately square in shape. It is a post-war residential development, with steeply pitched roofs, and as indicated on Figure 5.1,

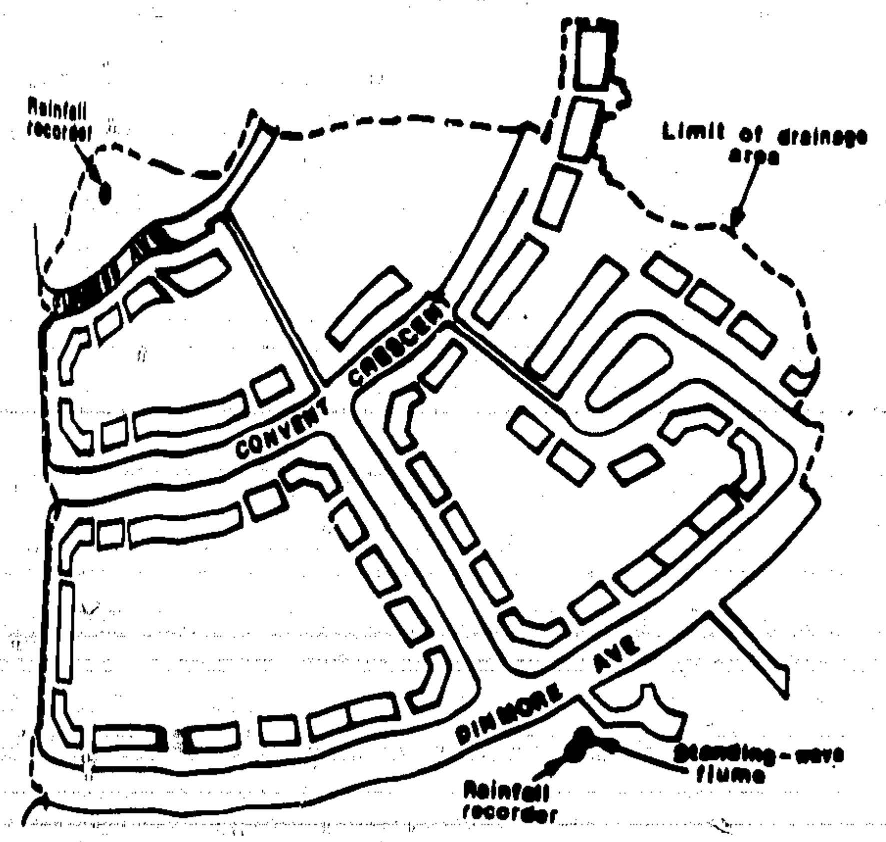


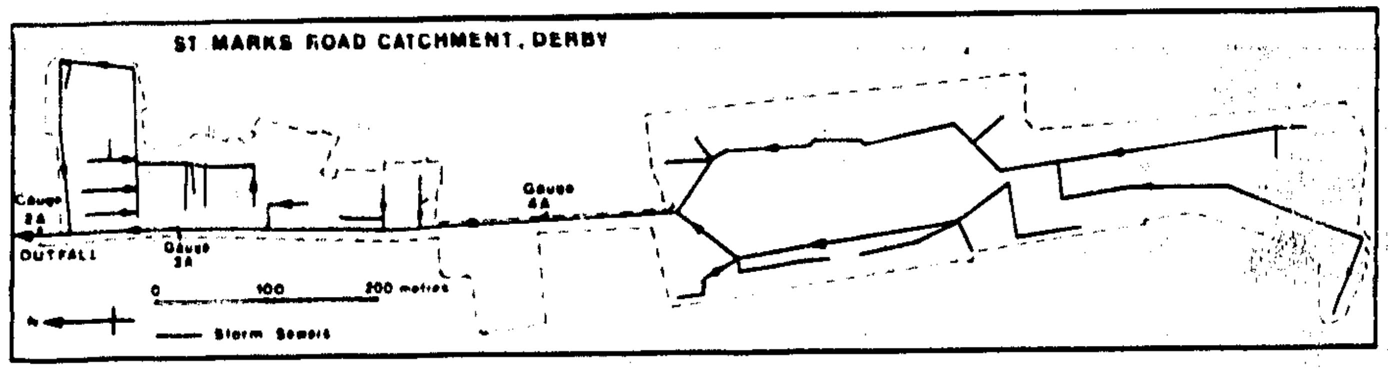
FIGURE 5.1

Catchment plan of Blackpool

comprises four large grassland areas surrounded by housing. 42.6% of the catchment is impervious. Rainfall and runoff were monitored from 1953 to 1958 as part of the Road Research Laboratory study (Watkins, 1962). Two autographic raingauges were sited in the north of the catchment, and flow was monitored in the 18 inch outfall culvert of the combined system, using a standing wave flume.

St Marks Road, Derby is a small (10.3 ha) flat catchment situated on the flood plain of the River Trent. As Figure 5.2 indicates, it is an elongated catchment comprising three nested catchments having areas 7.2, 8.5 and 10.3 ha respectively. It comprises a mixed residential development, which is uniformly distributed over the three catchments. 53% of the area is impervious. Runoff has been monitored at the outfall of each of the nested catchments since 1973, using an Arkon air purge system. Rainfall records are obtained from an autographic gauge just off the catchment.

FIGURE 5.2 CATCHMENT PLAN OF DERBY



Details of the catchment configurations of both Derby and Blackpool are summarised in Table 5.1. It can be seen that each provides sharp contrast to the Stevenage catchments.

TABLE 5.1 CATCHMENT DATA FOR DERBY AND BLACKPOOL

	OUTFALL PIPE NO.	TOTAL AREA(ha)	% PAVED	% ROOF	LENGTH OF MAIN DRAG(m)	TAYLOR-SCHWARTZ SLOPE (%)	NO.OF ACTUAL PIPES	NO.OF EQUIV. PIPES
Blackpool	1.25	5.0	27.8	14.8	337	1.4	90	4
Derby 1	1.26	10.3	36.8	16.4	1387	0.4	87	29
Derby 2	1.23	8.5	34.4	16.8	1257	0.4	60	28
Derby 3	1.17	7.2	31.2	17.7	948	0.4	41	26

5.2 Model Performance at Blackpool

The Blackpool catchment was represented in the equivalent pipe network by just four pipes, each 84 metres in length with a slope of 1 in 71. Despite the high rainfall intensities, the equivalent pipe system did not surcharge, and the full Wallingford model showed minor surcharging (four pipes) for only one storm.

TABLE 5.2 **BLACKPOOL: SUMMARY TABLE OF PEAK FLOWS**

STORM NO.	DATE	PEAK FLOWS (L/S) OBSERVED	FULL WALLINGFORD MODEL	SEWERED SUBCATCHMENT MODEL
1	17/9/53	100.9	102.7	102.9
12	12/9/54	229.7	314.5	318.2
15	22/8/56	263.0	206.3	209.3
21	27/9/56	92.1	89.9	86.1
24	15/5/57	132.7	125.9	124.4

Table 5.2 gives details of the goodness of fit obtained, and typical hydrographs are presented in Figures 5.3 to 5.5. In general, the sewered-subcatchment model gave a close simulation of both the full Wallingford model and observed data. For non-surcharge events peaks were with 1.5% of the full model peak, and within 6% of the observed peak. There was a tendency for both models to underestimate observed peak flows. For the one surcharge event (storm 12), both models overestimated the peak by about 37%, but overall fit to the hydrograph was good. The timing discrepancy of up to 2 minutes evident in the hydrograph plots can largely be attributed to non-synchronisation of the rainfall and runoff recorders and is relatively unimportant in the context of design.

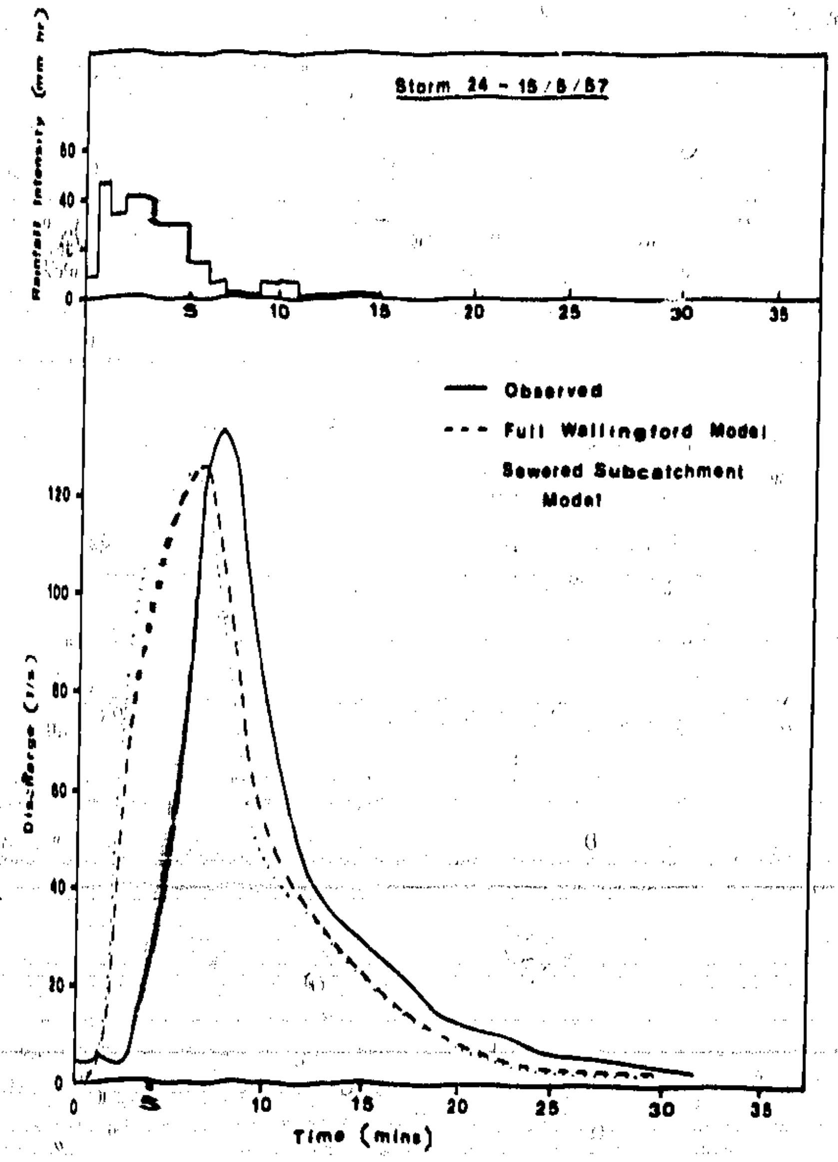


FIGURE 5.3

Blackpool - comparison of observed and modelled outflow hydrographs for a non surcharge event

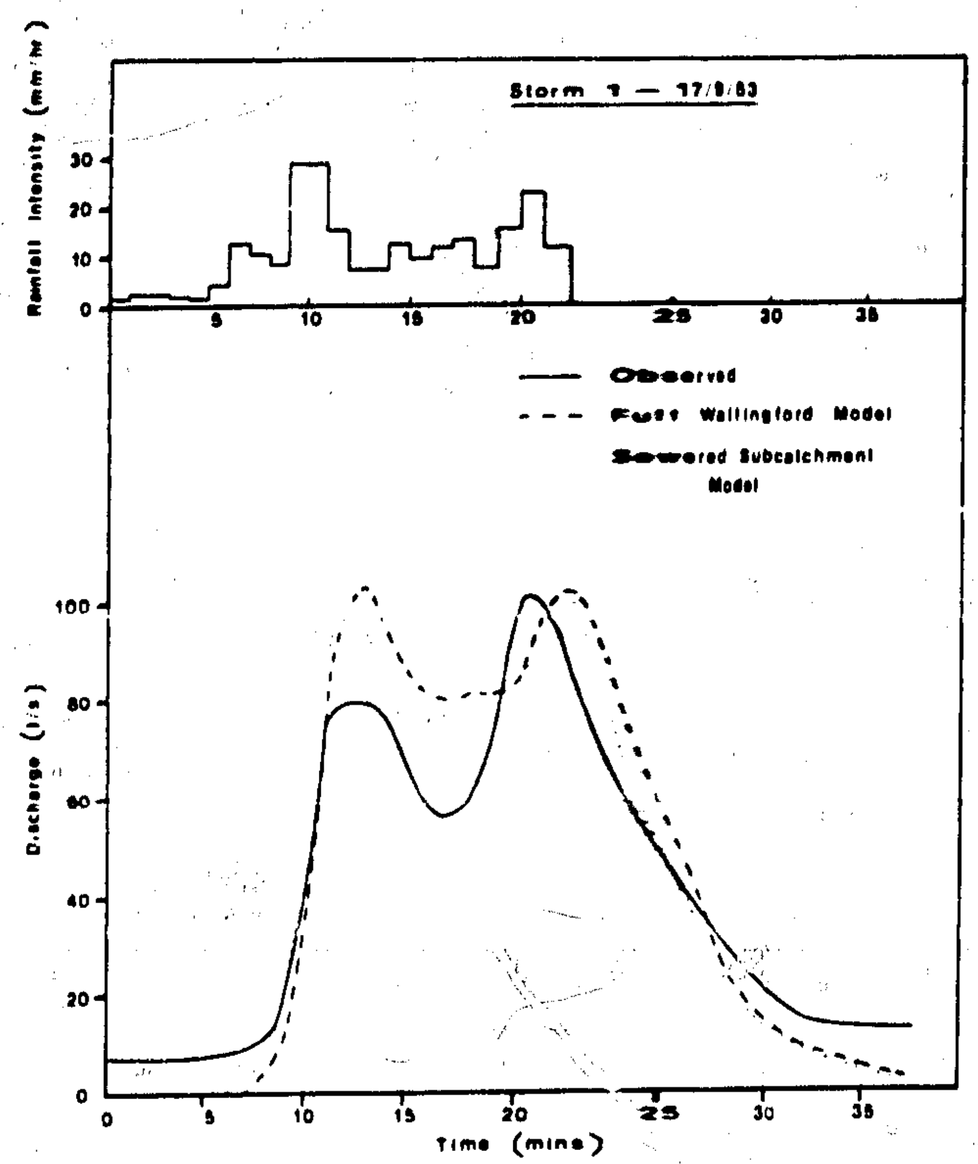
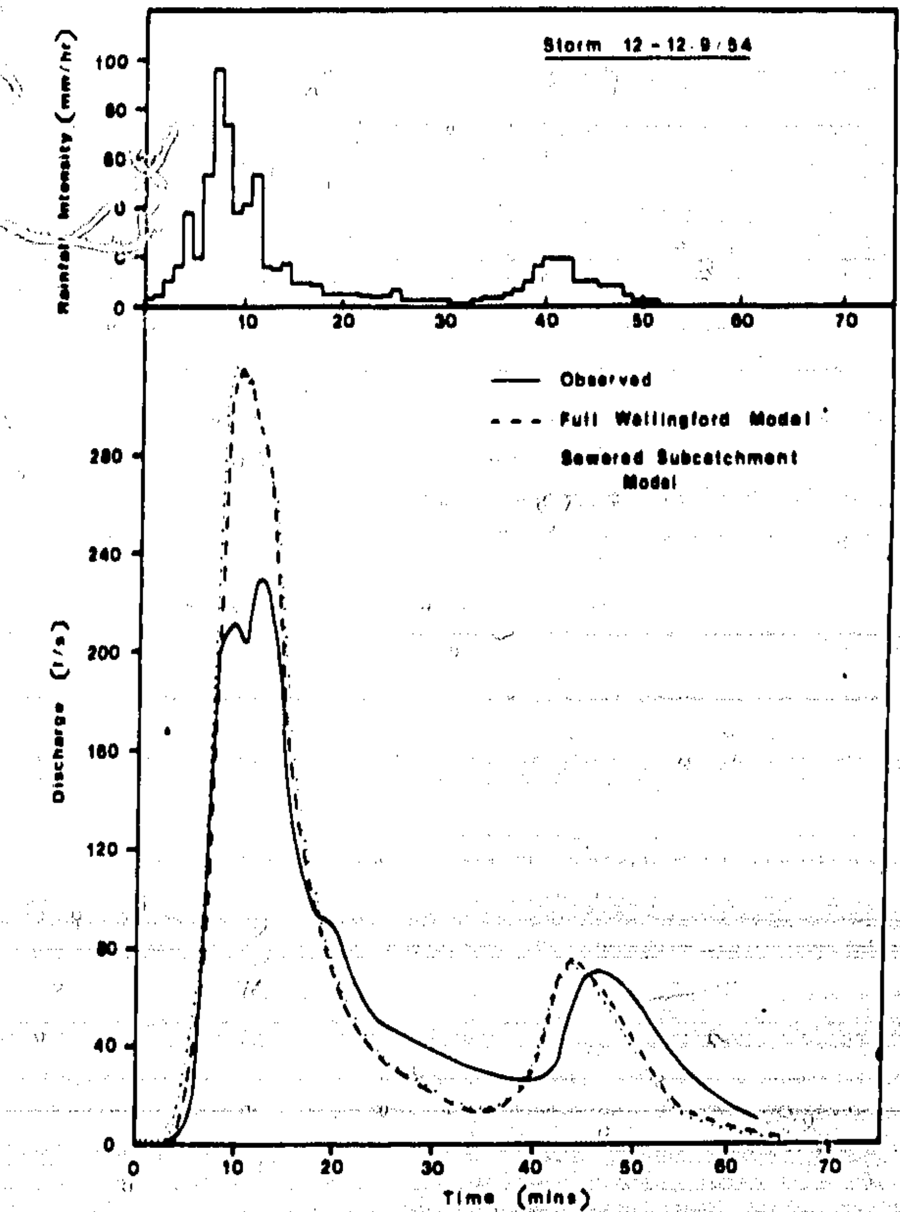


FIGURE 5.4

Blackpool - comparison of observed and modelled outflow hydrographs for a non surcharge event

FIGURE 5.5

Blackpool - comparison of observed and modelled outflow hydrographs for a non surcharge event



5.3 Model performance at Derby

For each of the three nested catchments at Derby (upstream of pipes 1.17, 1.23 and 1.26 respectively) hydrographs were generated using both the full and simplified models. Each catchment was considered separately except that the percentage runoffs from paved, pervious and roof area for each catchment were derived from the observed hydrograph at 1.26 (1.23 for event 10). Thus observed and predicted volumes of runoff were only exactly equal at 1.26 (1.23 for event 10).

The largest of the catchments (1.26) was represented by 29 equivalent pipes each of length 48 metres, decreasing to 26 pipes each of length 36 metres for the smallest catchment (1.17). This large number of equivalent pipes is a direct result of the low wave speeds, created by the flat pipe gradients.

The results of the model simulations are summarised in Table 5.3, and typical hydrographs are presented in Figures 5.6 to 5.10. For the full system all five rainfall events resulted in surcharging, ranging from a minimum of six pipes in storm 14 to 19 pipes in storm 10. Storm 10 is the largest storm on record for which reliable gauging information is available. In all cases the fit of the sewered subcatchment model to the observed and full model hydrographs is worse than at Blackpool. Averaging over all events there was a 12% difference between full and lumped model peaks at 1.26 and 1.23, and a 20% difference at 1.17. The average error between the lumped model and the observed peaks was 17% at 1.26 and 1.23, while at 1.17 it exceeded 30%.

TABLE 5.3

DERBY - SUMMARY OF PEAK FLOWS

STORM NO.	DATE	OBSERVED	FULL WALLINGFORD MODEL	SEWERED SUBCATCHMENT MODEL
			Peak Flows (L/S) at Pipe No.1.26	
10	31/7/72	-	234	195
13	8/9/72	122	145	134
14	10/10/72	170	152	133
19	12/2/73	132	176	157
24	8/8/74	141	129	111
			Peak Flows (L/S) at Pipe No.1.23	
10	31/7/72	179	191	161
13	8/9/72	103	121	109
14	10/10/72	125	127	109
19	12/2/73	112	145	129
24	8/8/74	-	114	91
			Peak Flows (L/S) at Pipe No.1.17	
10	31/7/72	122	173	138
13	8/9/72	70	108	92
14	10/10/72	99	115	130
19	12/2/73	75	127	144
24	8/8/74	-	109	153

FIGURE 5.6

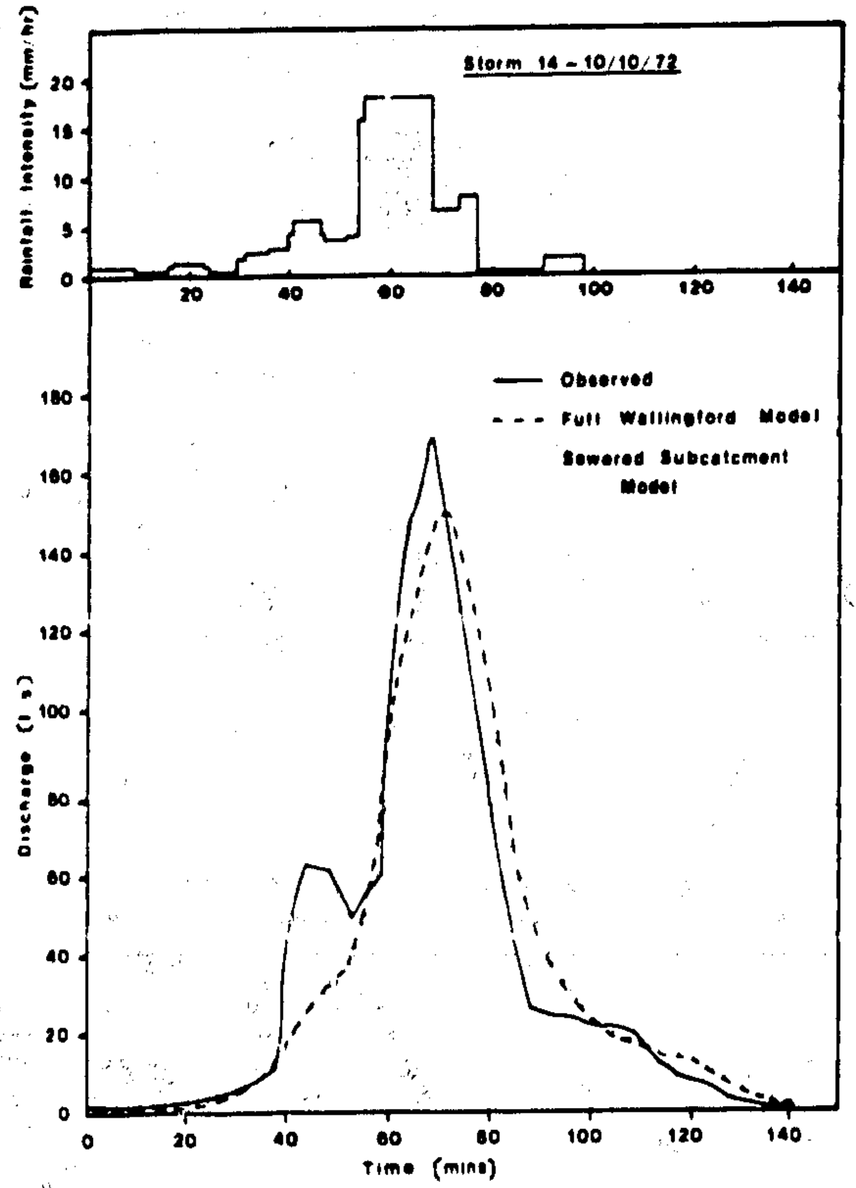


FIGURE 5.7

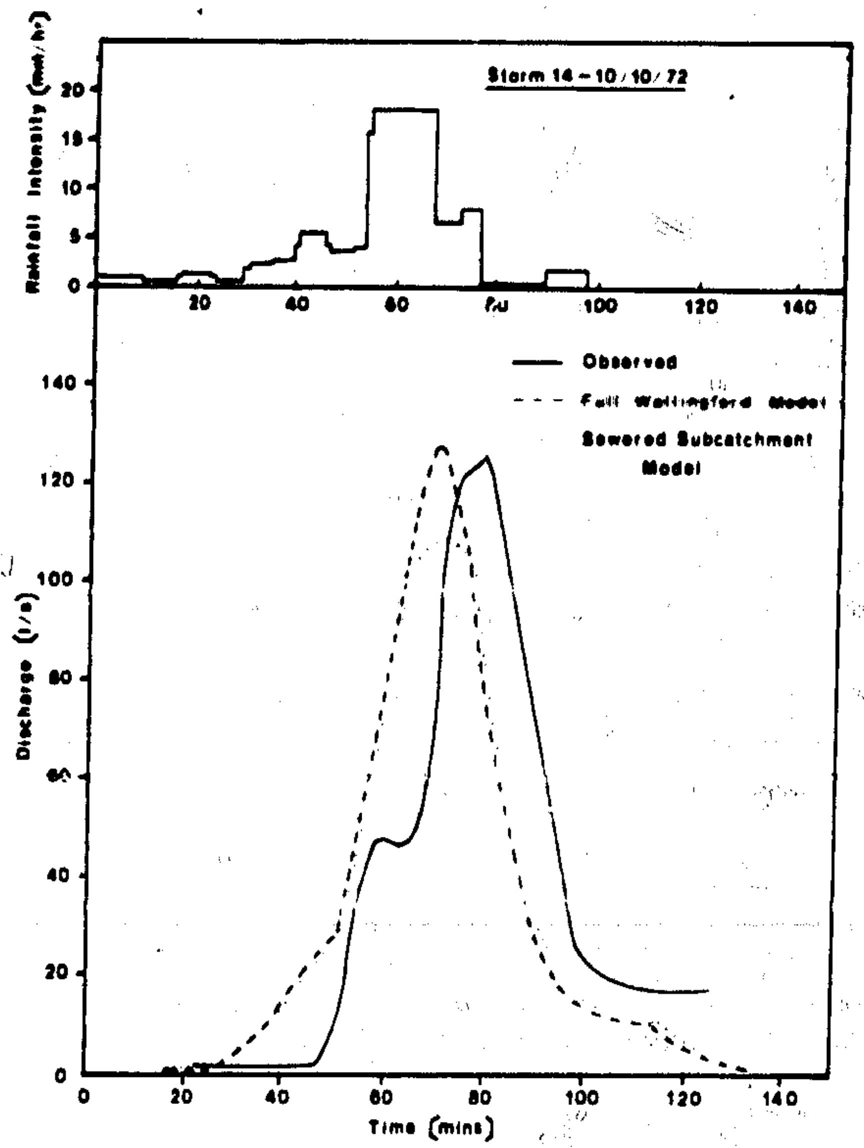
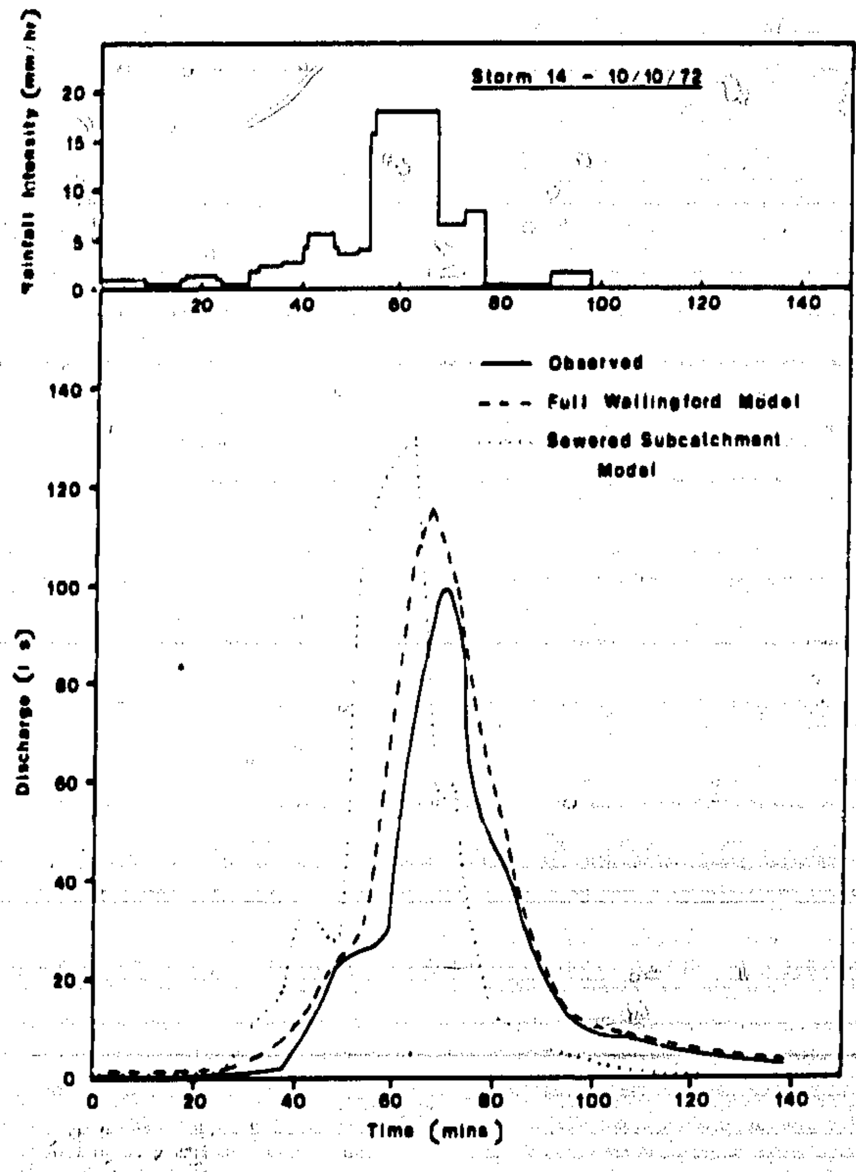


FIGURE 5.8



Figures 5.6 to 5.8 Derby - comparisons of observed and modelled hydrographs at pipes 1.26, 1.23 and 1.17 respectively

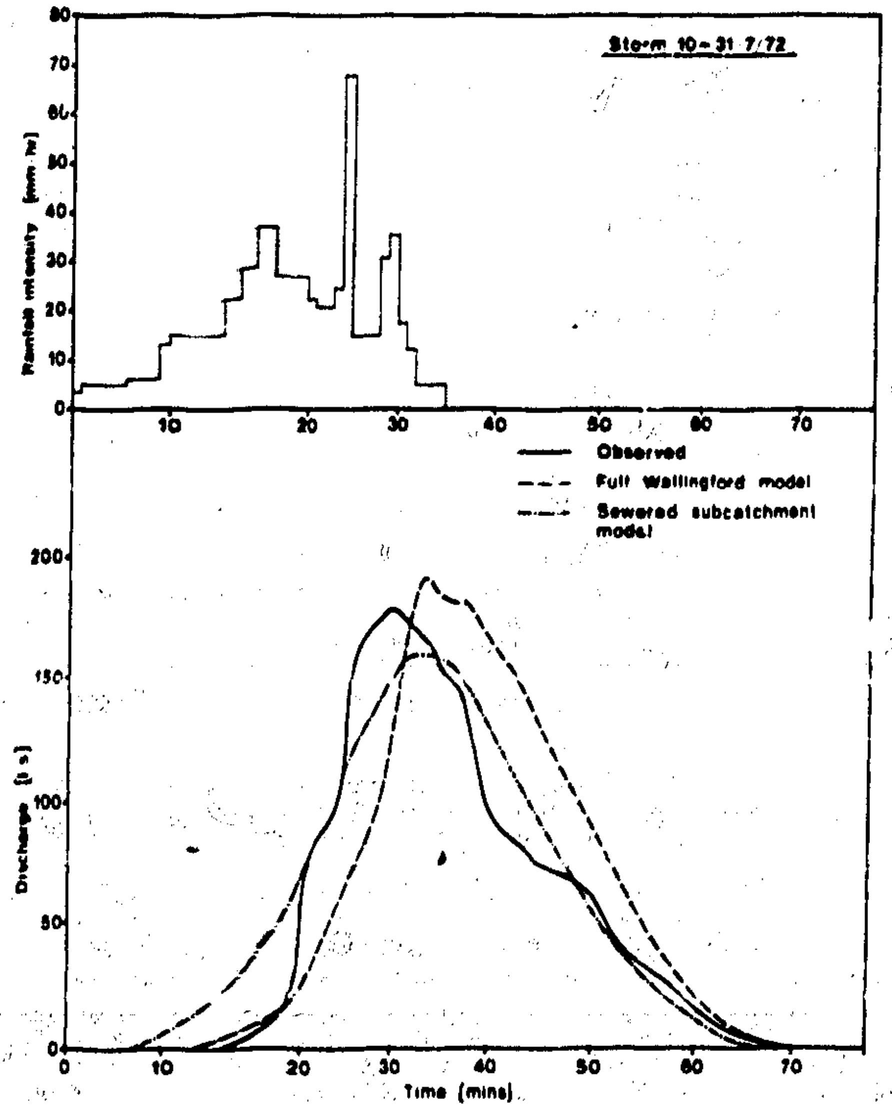


FIGURE 5.9

Derby - comparison of observed and modelled hydrographs hydrographs at pipe no. 1.23

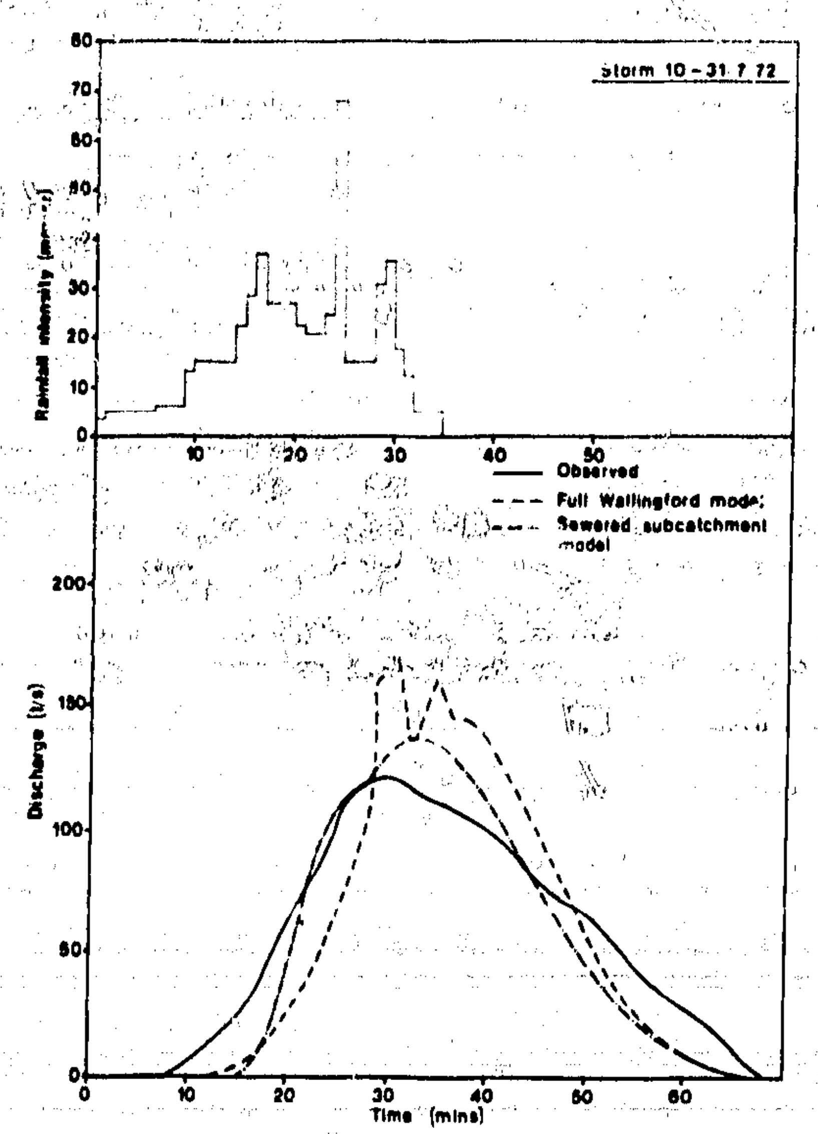


FIGURE 5.10

Derby - comparison of observed and modelled hydrographs at pipe no. 1.17

There are several possible explanations for the poorer model fit at Derby. One might be the increased surcharging of the Derby system, which as noted in Section 4 causes increased errors. Another explanation might be the use of average catchment conditions in the equivalent pipe model, and how well these reflect the full system. It can be seen from Figure 5.11 that the longitudinal profile of the main drag pipes at Blackpool is reasonably uniform. At Derby (see Figure 5.12) a sharp discontinuity in slope at pipe 1.05 is evident. This distinct break in slope cannot be adequately represented by a single pipe length at a constant slope. The effect of the break in slope is most significant in the smallest catchment (1.17) which is also the catchment which yields the largest errors.

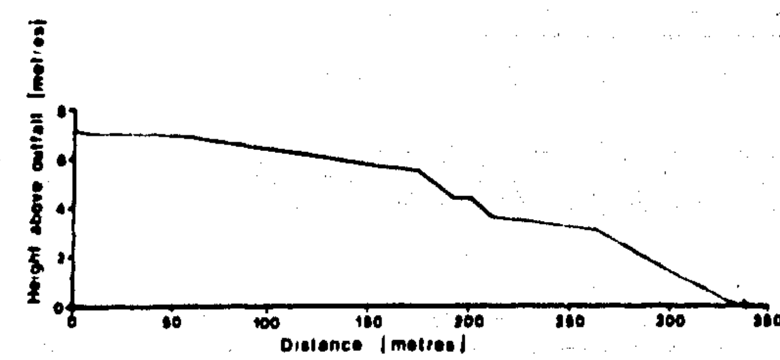


FIGURE 5.11

Blackpool - longitudinal section of main drag pipes

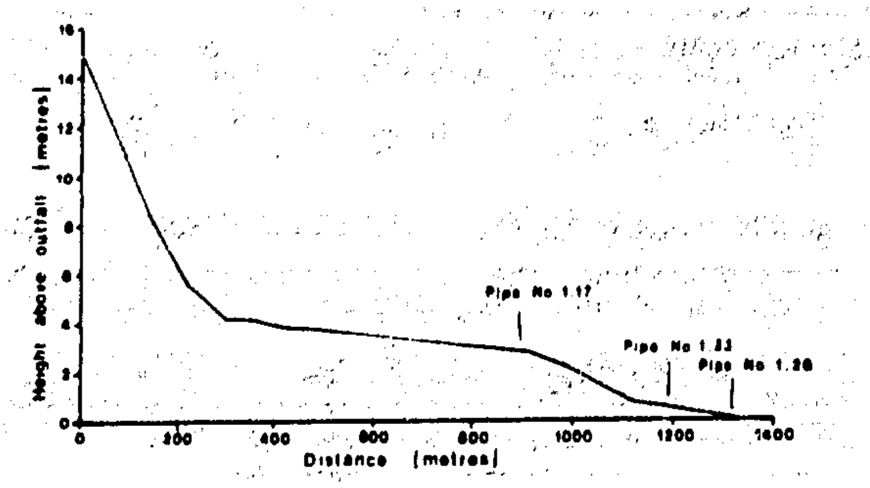


FIGURE 5.12

Derby - longitudinal section of main drag pipes

A third possible explanation for the poorer model fit at Derby might be the use of the same percentage runoffs from paved, pervious and roof surfaces for each catchment. The events used in this analysis were chosen as far as possible for consistency of response between the three catchments. However, this consistency was not always exact and additional modelling errors may have been introduced.

To summarise, these results serve to highlight two situations where the sewered subcatchment model may be expected to give less accurate results:

(i) in catchments with such low pipe gradients that surcharge development is extensive;

(ii) in catchments with distinct non-homogeneities of slope or non-uniform distribution of impervious cover.

Problems in simulating extensive surcharging are still unresolved, but the effect of non-homogeneity is investigated more fully in Appendix B.

6. CONCLUSIONS AND POSSIBILITIES FOR FURTHER DEVELOPMENT

The model described in this report has been developed for particular use with the new Wallingford Storm Sewer Design Models. It can be applied in situations where detailed catchment information is not available either because the area is not yet fully developed or because catchment details are not readily accessible. It uses the same surface routing model and the same pipeflow model as the full Wallingford Model, but replaces the detailed pipe network with an "equivalent" series of pipes forming a single branch. Each pipe has the same length and slope, but diameters of successive pipes are increased in a downstream direction. It is easily included as an option within the full Wallingford Model, the only new code required being that to set up the equivalent pipe system. The model has been developed to a stage where it gives good agreement with the full model (using the detailed area and sewer layout). However, several possibilities are for future development are discussed below.

Firstly, the model fit to non-surcharge events has been shown to be good, but some improvement could be made for surcharge events. In this respect it is thought that the treatment of manhole storage in the equivalent system is a possible source of error. When the full network of say 100+ pipes is replaced by say 10, then obviously approximately 100 manholes are also replaced by 10. This is a considerable loss of storage capacity to balance heads for surcharged flow. One way to compensate for this would be to increase the size of manholes for the equivalent system. Another way modelling of surcharge events could be improved might be to restrict pipe sizes in the equivalent system to commercial pipe diameters. This, combined with increased manhole storage might better reproduce the "patchy" occurrence of surcharging in the full system.

One other useful way in which the equivalent pipe model could be improved would be to develop the ability to lump intermediate areas along a pipe. At present, the whole area above a specified point is lumped, and all the area below that point must be considered in detail. Although this is not unduly restrictive in most cases (for example, the design of a trunk sewer with lumped inputs at various points along its length is quite possible), the ability to have a lumped catchment feed into another lumped catchment might be a useful feature. It may, for example, be one way of improving the fit to non-homogeneous catchments, allowing for the effects of both slope and area distribution.

The model in its present stage of development has been considered mainly as an approximation to the full Wallingford Model. However, as shown by Section 5, it has merit as a catchment model in its own right. Consequently, the model might be used on its own, without the full Wallingford model. In this way it would form a link between sewer design models and models to investigate the effect of urbanisation on the flood response in natural catchments. It would also be suitable for use as a planning model to investigate the advantages of alternative urban development strategies. As identified by Packman (1980), development of such a "distributed" model for application to urbanisation problems would be highly desirable.

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APPENDIX A

DEMONSTRATION OF MODEL PERFORMANCE DOWNSTREAM

In Sections 4 and 5 the sewered subcatchment model has been tested for its ability to predict the peak and hydrograph shape at the outfall to a particular catchment. Although a good fit has been achieved, a timing discrepancy has arisen in some cases. The question arises, therefore, what effect does use of the sewered subcatchment model have on discharges downstream? Model performance in this respect has been tested for two situations: (i) where one single subcatchment has been treated as a lumped area, and (ii) where all inputs to a main drag have been considered as lumped. The first case might arise where sewer information for a certain area is not available, the second case might arise in, for instance, the design of a trunk sewer.

For this analysis, the sewer system designed for the first, steeper series of catchments was used, and catchments to be treated as lumped inputs were selected. The catchments selected were not, in general, the same as used in development of the sewered subcatchment model (Sections 3 and 4). Figure A.1 gives a schematic representation of the catchment with the inputs to the main drag between pipes 1.16 and 1.34 shown as lumped inputs.

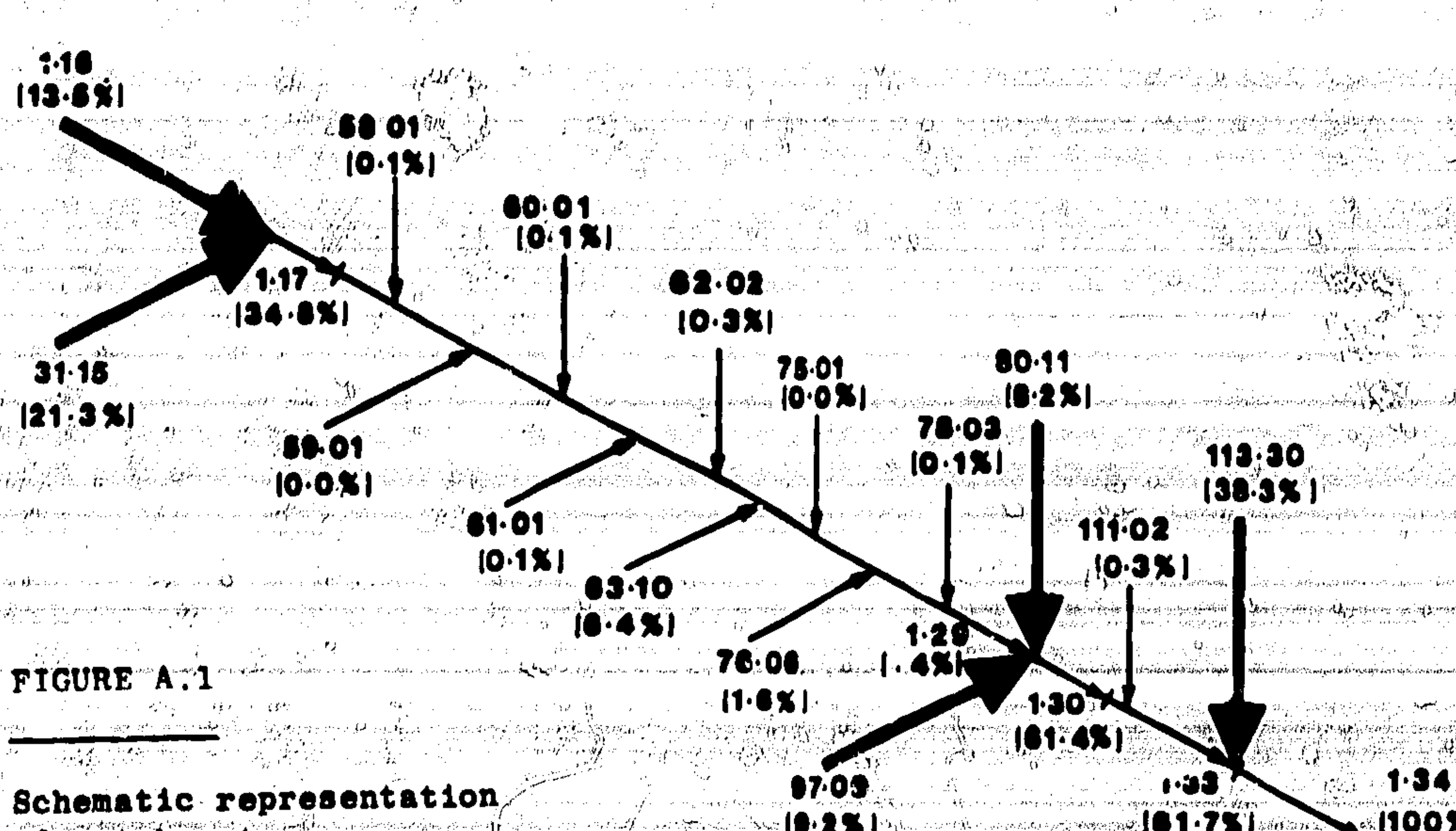


FIGURE A.1

Schematic representation of catchment

Two storms were chosen for the analysis, storm 2 (the one year design storm - a non surcharge event) and storm 9 (the ten year design storm - a surcharge event). In all, eight model runs were performed for each storm; the degree of lumping and the position at which flow hydrographs were obtained are given in Table A.1.

TABLE A.1 MODEL RUNS

RUN NO	CATCHMENTS TREATED AS LUMPED	OUTPUT HYDROGRAPHS OBTAINED AT						
1	None	1.17	1.23	1.24	1.29	1.30	1.33	1.34
2	1.16, 31.15	1.17				1.30		1.34
3	1.17	1.17				1.30		1.34
4	1.29, 80.11, 97.09					1.30		1.34
5	1.30					1.30		1.34
6	113.30, 1.33							1.34
7	1.34							1.34
8	1.16 to 113.30 exluding main drag	1.17	1.23	1.24	1.29	1.30	1.33	1.34

With these model runs it was possible to compare not only the effect of lumping an increased proportion of the catchment but also the effect of lumping two adjacent catchments separately, or together; for example, comparing the fit with catchments 1.16 and 31.15 lumped separately, or lumped together at 1.17. In this way, the effect of errors in phasing between the two lumped areas would be maximised.

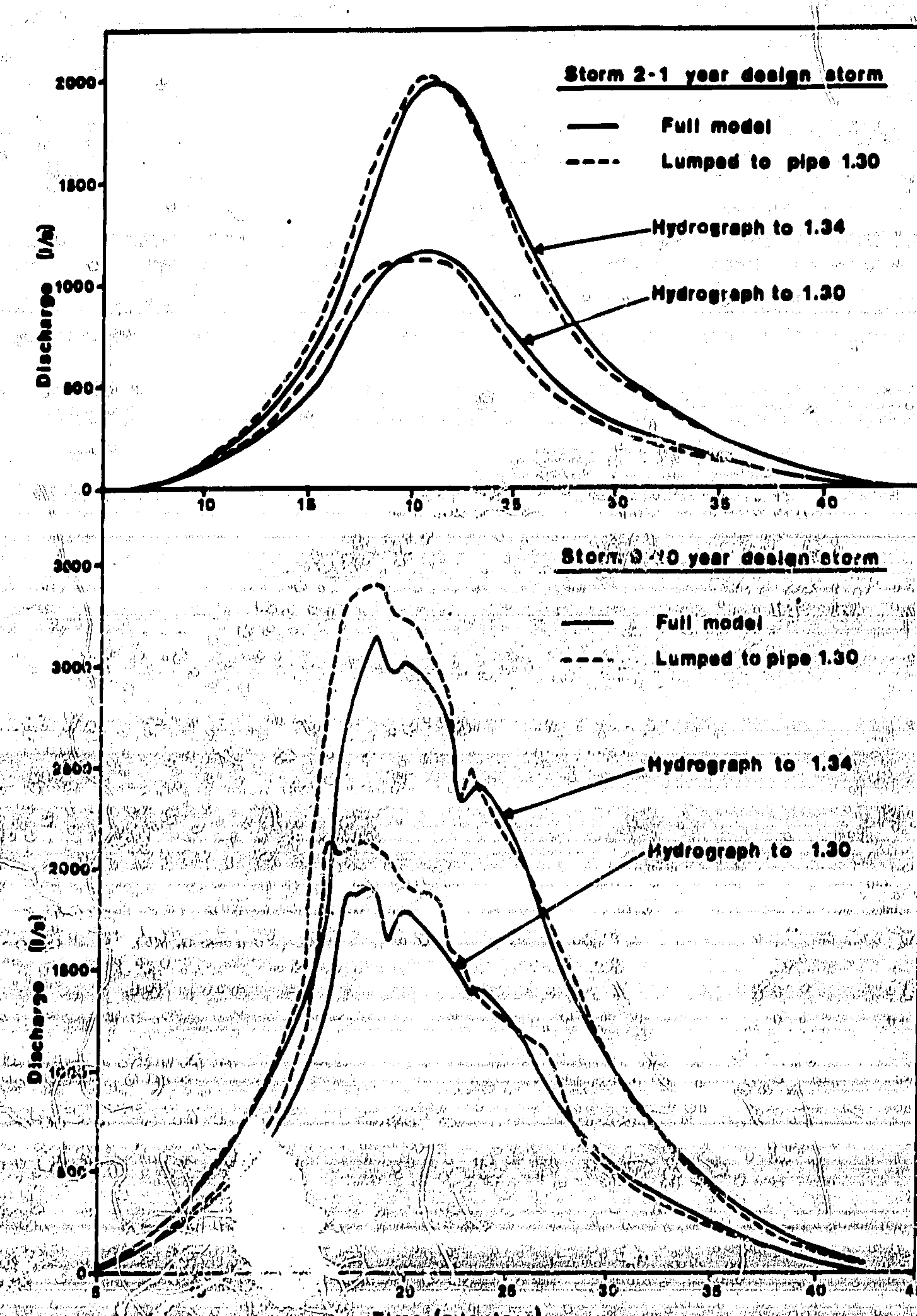
Table A.2 gives the percentage error in peak estimation at pipes 1.17, 1.30, and 1.34 for each storm for runs 2 to 7. It can be seen that with 35% and approximately 60% of the catchment considered as lumped a good fit to the full model is achieved. Figure A.2 illustrates the fit at two locations.

TABLE A.2 THE EFFECT OF LUMPING DOWNSTREAM

RUN NO	CATCHMENTS LUMPED	AT 1.17			AT 1.30			AT 1.34		
		% OF AREA LUMPED	PEAK ERROR (%)		% OF AREA LUMPED	PEAK ERROR (%)		% OF AREA LUMPED	PEAK ERROR (%)	
			STORM 2	STORM 9		STORM 2	STORM 9		STORM 2	STORM 9
2	1.16 & 31.15	100%	0.0%	- 7.7%	57%	-1.2%	-2.0%	35%	- 1.0%	0.0%
3	1.17	100%	-1.5%	-12.0%	57%	-3.5%	- 5.6%	35%	-2.3%	-1.5%
4	1.29, 80.11, 97.09				100%	2.0%	-14.2%	61%	0.1%	-9.4%
5	1.30				100%	1.4%	-21.7%	61%	0.0%	-13.0%
6	113.30, 1.33							100%	5.8%	-17.0%
7	1.34							100%	6.8%	-25.7%

These results show that the errors in estimating the outflow hydrograph from a lumped subcatchment do not cause bigger errors downstream; in fact the errors are dissipated quite quickly. For example, in run 2

FIGURE A.2 Comparison of hydrographs at pipes 1.30 and 1.34 for catchment lumped to 1.30 (61% of catchment to 1.34)



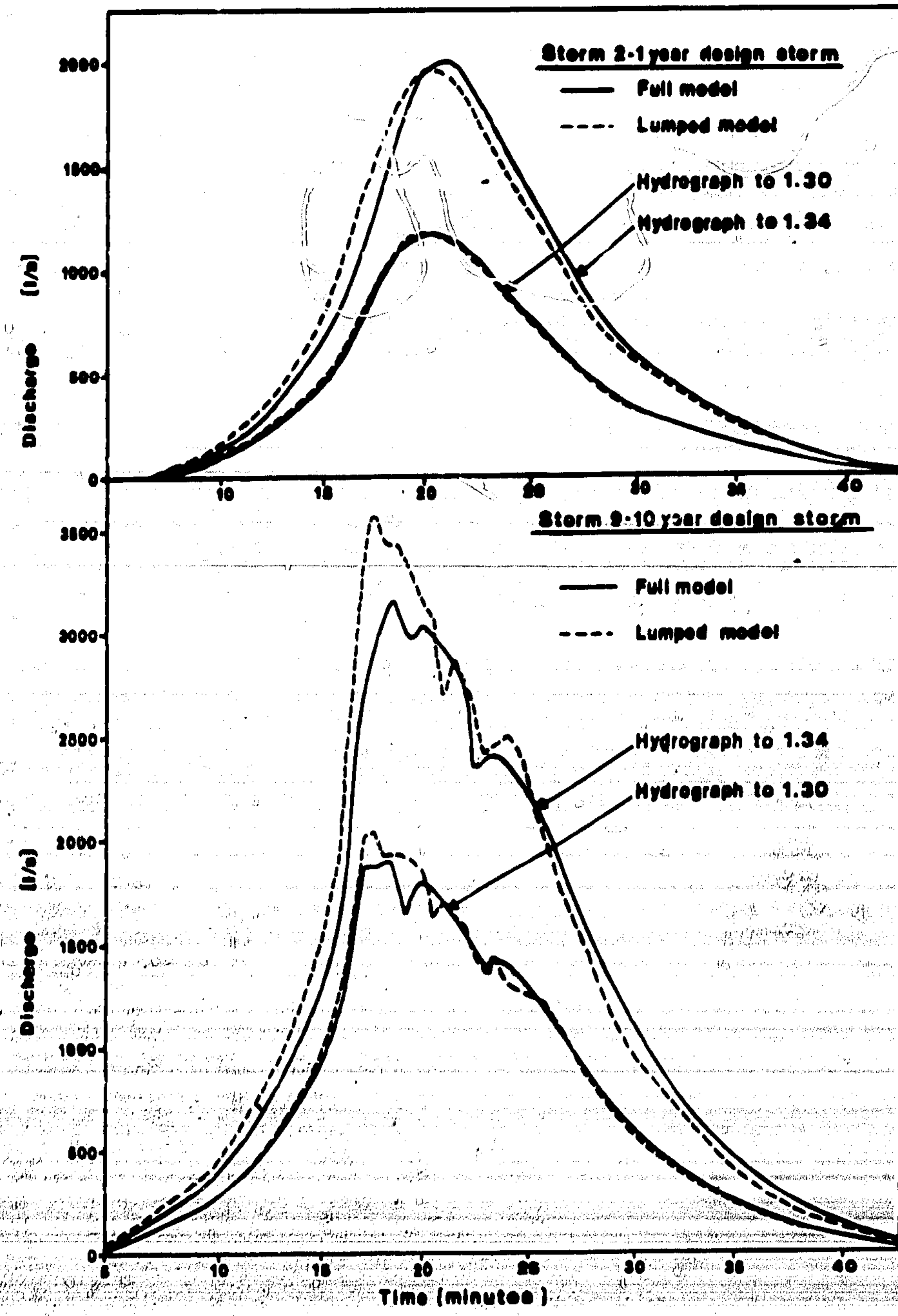
a 7.7% error in estimating peak for storm 9 has dissipated to a 0.0% error by the time pipe 1.34 has been reached. Similarly, in run 4 a 14.2% error has dissipated to 9.4% after only 4 pipes. These results show once again that the model provides a good fit to non surcharge events but not such a good fit to surcharge events. Storm 9 results in severe surcharging - in the full model it yields surcharging in 105 of the 587 pipes. (In spite of this, no surface flooding is predicted and the maximum head in any manhole was .79 metres). Figure A.2 shows a discrepancy in volume as predicted by the two models for storm 9. This discrepancy is due to an under-prediction of volume by the version of the full model used in this study under conditions of extensive surcharging. Normally this error was less than 5%, but was confined to the peak of the hydrograph. Later versions of the surcharge solution have overcome this difficulty. The dissipation of errors downstream as shown by Table A.2 suggests that, even for surcharge events, if the lumped input is sufficiently far upstream of the point of interest, any errors induced will be negligible.

Table A.3 gives the errors at several points along the main drag for run 8 and Figure A.3 shows typical hydrographs. In this run, the full pipe system of 587 pipes has been replaced by 15 lumped inputs and 18 main drag pipes. As can be seen, the model has given an extremely close fit to the non-surcharge event, and considering the few number of pipes used, a good fit to the surcharge event. Once again the dissipation of errors downstream between large lumped inputs can be seen. These results show the model does not lead to further errors downstream.

TABLE A.3 A "TRUNK SEWER" SIMULATION

STORM NO	% ERROR IN PEAK AT PIPE NO. 1.17	1.23	1.24	1.29	1.30	1.33	1.34
2	0.6%	0.0%	1.4%	1.1%	1.1%	0.9%	2.5%
9	- 6.7%	- 3.5%	- 1.5%	- 4.3%	- 8.4%	- 8.1%	- 14.6%

FIGURE A.3 Comparison of hydrographs at pipes 1.30 and 1.34 for all catchment lumped except main drag between 1.16 and 1.34



APPENDIX B

EFFECT OF CATCHMENT NON-HOMOGENEITIES

The equivalent pipe model, with runoff inputs distributed at equally spaced intervals along its length, assumes in effect a rectangular catchment of constant slope. While the assumption of a rectangular catchment may be justified for most catchments, the use of a constant slope may give rise to errors. Clearly, if the pipe slope is constant for all main drag pipes in the prototype, the equivalent pipe model will be an accurate representation of the real system. In this section the effect of departures from a constant slope are investigated in order to define limits on application of the equivalent pipe model, if necessary.

B.1 Data generation

Two imaginary catchments with widely differing slope profiles, but represented by the same equivalent pipe model, were created. These were based on the largest of the Shephall catchments upstream of pipe 1.34. As indicated by Figure 3.2, the longitudinal profile of the main drag pipes for this catchment is reasonably uniform. Discontinuities in slope were introduced at pipe 1.17, since the drainage boundary at this point divides the catchment transversely into two approximately equal parts. The existing pipe slopes were altered, in the ratio of approximately 4:1 upstream and downstream of pipe 1.17 respectively; this resulted in two catchments, one with a concave and the other a convex slope profile, but each with the same average slope of the maindrag pipes. Longitudinal sections of the main drag for these two slope configurations are plotted in Figure B.1. Pipe diameters for each catchment were designed using the Wallingford Model with storms of 15 and 30 minute duration and 2-year return period.

Using the same ten storms as in previous analyses, outflow hydrographs from the two catchments were derived by the full Wallingford Model.

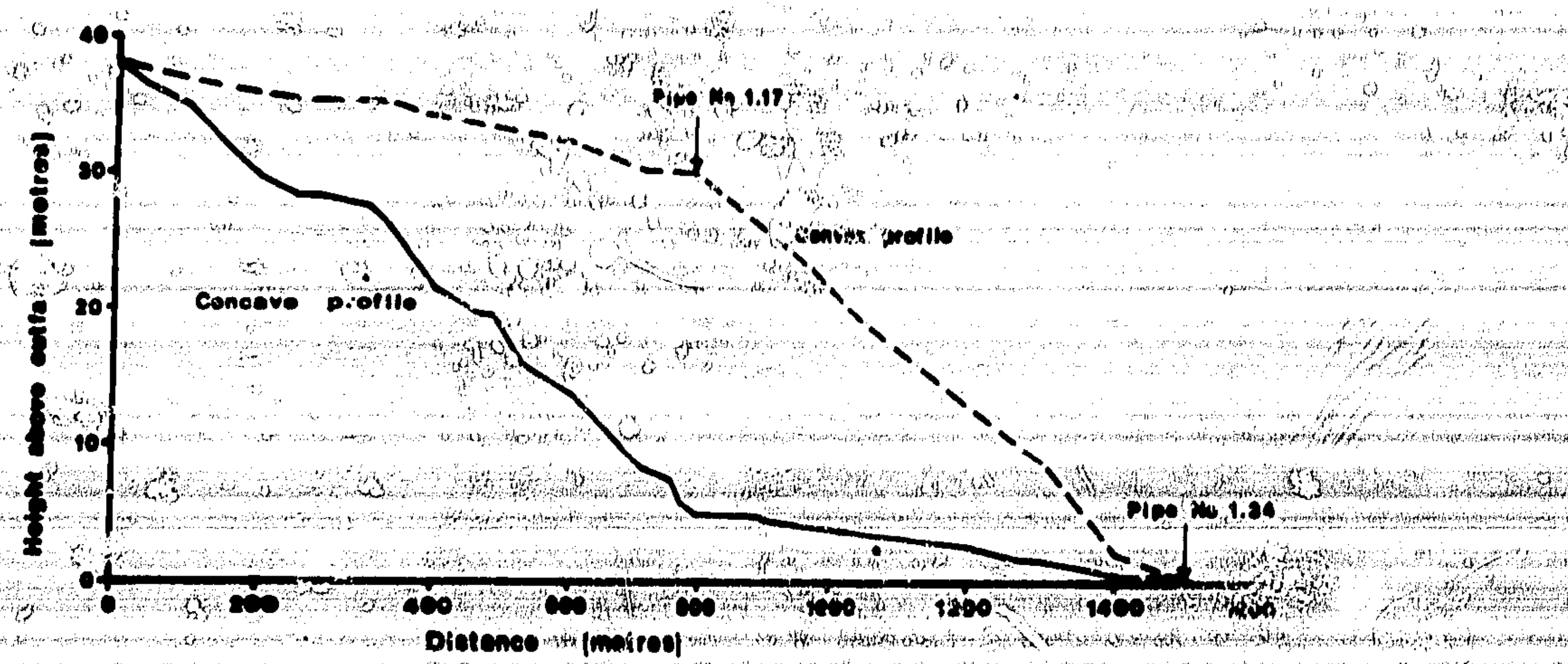
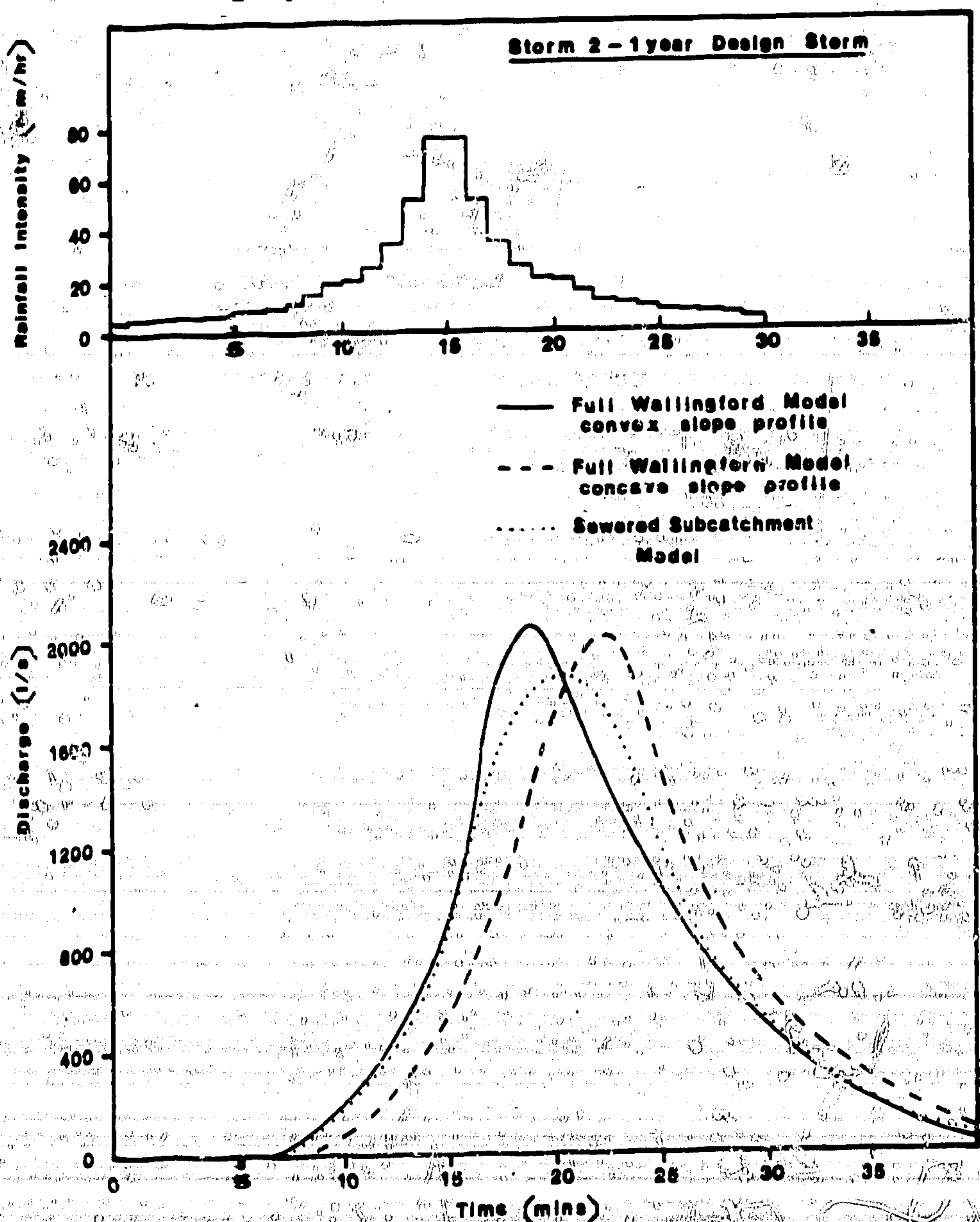


FIGURE B.1 Longitudinal section of main drag pipes for 2 different slope profiles

FIGURE B.2 Comparison of modelled hydrographs for different slope profiles (non surcharge event)



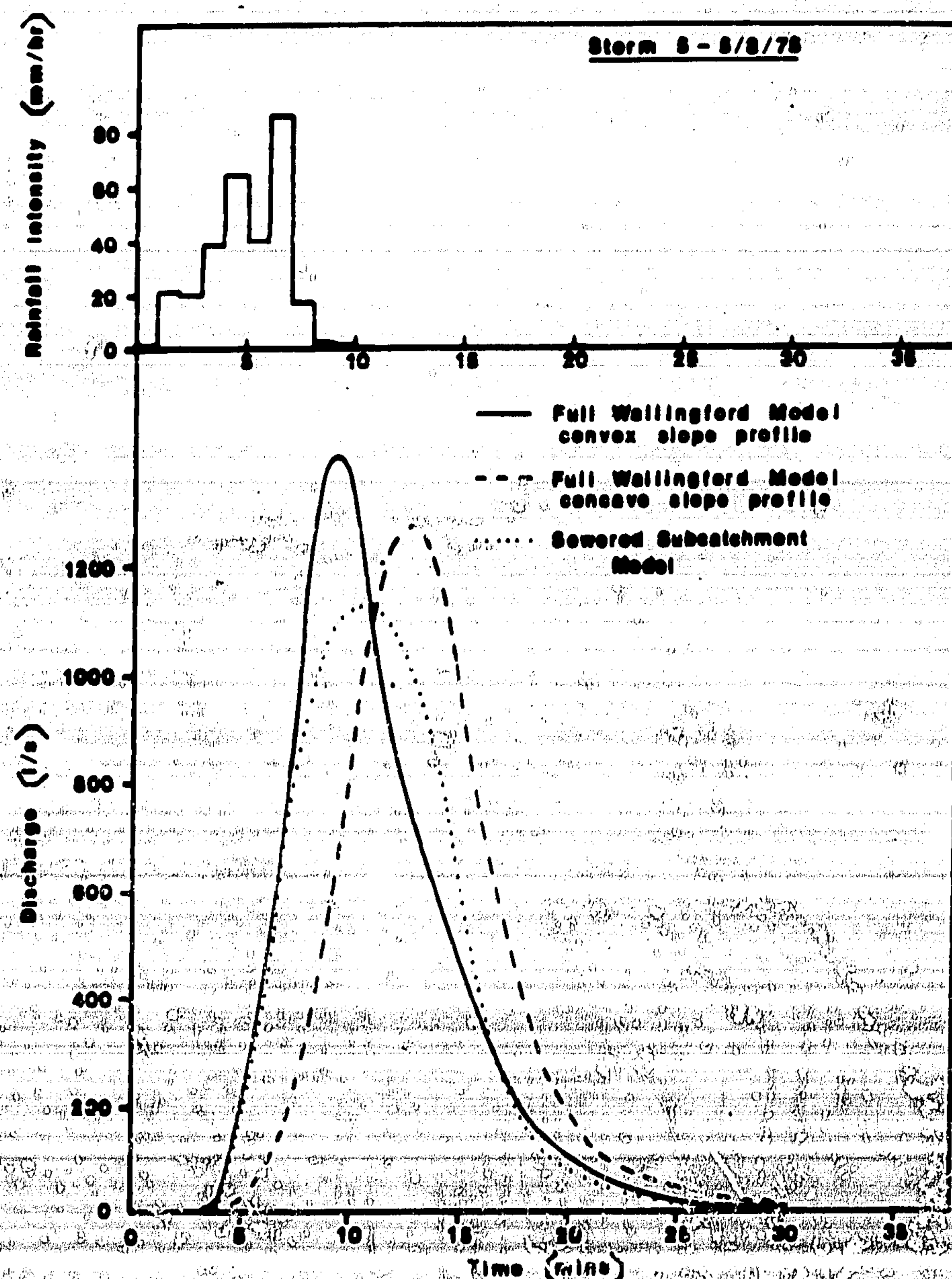
B.2 Model Simulations

Outflow hydrographs for each of the ten storms were simulated using the sewered-subcatchment model, treating the whole catchment as a single lumped input. To enable an exact comparison between hydrographs from the two slope profiles, the same value of DIAM (the largest of the two) was applied to both catchments. Thus the two equivalent pipe models were identical, consisting of 11 pies, each of length 136 metres and at a slope of 1 in 90.

B.3 Non-surcharge events

Two sets of outflow hydrographs, typifying the results from events with no surcharge development, are plotted in Figures B.2 and B.3. It is clear that despite the significantly different slope profiles, the sewered subcatchment model provides a close simulation of the full model peak and particularly the shape of the full model hydrograph. As anticipated, errors in peak estimation are slightly higher (at about 10%) than for the more uniform slope profile, adopted in Section 4, which gave about 7%.

FIGURE B.3 Comparison of modelled hydrographs for different slope profiles (non surcharge event)



The error in the timing of the peak of the hydrograph is more significant. In all cases, the hydrograph from the sewered subcatchment model occurred up to two minutes after the full model hydrograph for a concave slope profile, and up to two minutes before the full model hydrograph for the convex profile. This indicates that it is the pipe slopes in the vicinity of the catchment outfall that have the greatest influence on the outflow hydrograph - steep slopes near the outfall yield a more rapid and slightly peakier hydrograph than shallower slopes.

B.4 Surcharge events

Three of the ten storm events analysed, using the full model, resulted in surcharging of the pipe network. The outflow hydrographs plotted in Figure B.4 provide a typical example. The results indicate that non-homogeneities in slope are far less important for events with surcharging than those without. When the pipe system surcharges, the rate of flow is in response to a pressure gradient, rather than to the pipe slope; hence the flood hydrographs will peak at approximately the same time (see Figure B.4) regardless of slope profile. The errors in peak estimation between the sewered-subcatchment and full model hydrographs are comparable to those for non-surcharge events.

It is therefore evident that despite significant departures from a uniform main drag slope profile, the sewered-subcatchment model can still adequately represent the full model hydrograph in all but the most extreme of non-uniform conditions. However, errors in the phasing of response from the sewered-subcatchment and the rest of the catchment will be greater when non-homogeneity is present.

FIGURE B.4 Comparison of modelled hydrographs for different slope profiles (surcharge event)