INSTITUTE
OF
hydrology

AN IMPROVED SUBCATCHMENT MODEL FOR THE RIVER DEE

by

C S GREEN

ABSTRACT

This report describes improvements to the original (Lambert) model used for real-time flow forecasting on the River Dee (North Wales) subcatchments. The model produces a flow prediction from a knowledge of rainfall and the present telemetered flow using two parameters; the catchment lag, L, and the storage parameter k. By allowing the parameter k to vary with discharge it is shown that a much improved flow forecast is possible. Modification of the lag parameter is also considered. A technique for deriving the relationship between k and discharge from historic data is also described. The model is applied to data from five gauged subcatchments of the Dee and tested on eight flood events from each catchment. The improved model is now used in real time operation for flow forecasting on the River Dee.

THE NATURAL ENVIRONMENT RESEARCH COUNCIL

REPORT NO 58

March 1979

CONTENTS

ILLUSTRATIONS

FIGURES

TABLES

LIST OF SYMBOLS

k = storage parameter (used generally)

k_1 = storage parameter (used specifically for I.S.O. Function Type I)

k_2 = storage parameter (used specifically for I.S.O. Function Type II)

k_{1a} = as k_1, but the first part of a two-part k versus q relationship

k_{1b} = as k_1, but the second part of a two-part k versus q relationship

L = catchment lag (hours)

q = flow

qo = present flow value

qn = flow at time T hours from now

r = rainfall

S = catchment storage (mm)

T = time period or data interval (hours)

t = time

x = central rainfall fraction

1. INTRODUCTION

1.1 A brief history of the I.S.O. function model

The *I.S.O.* (*Inflow-Storage-*Outflow) rainfall-runoff model was first proposed by Lambert in 1969 and applied to the Ceiriog, a tributary of the River Dee (N. Wales). Subsequently the model was defined in two forms (linear and non-linear) and applied to the Afon Dyfrdwy (a tributary to Llyn Tegid on the River Dee) as a subcatchment model and as a routing model to Llyn Tegid itself (Lambert, 1972).

The Lambert model was chosen as the basic rainfall-runoff model for five gauged subcatchments of the River Dee and employed in the real-time flow forecasting system based at Bala (Harvey and Lowing, 1976). The five subcatchments, shown in Figure 1, are:

1. Hirnant 2. Ceiriog 3. Dee at New Inn (Afon Dyfrdwy)
4. Alwen (below Alwen reservoir) 5. Gelyn

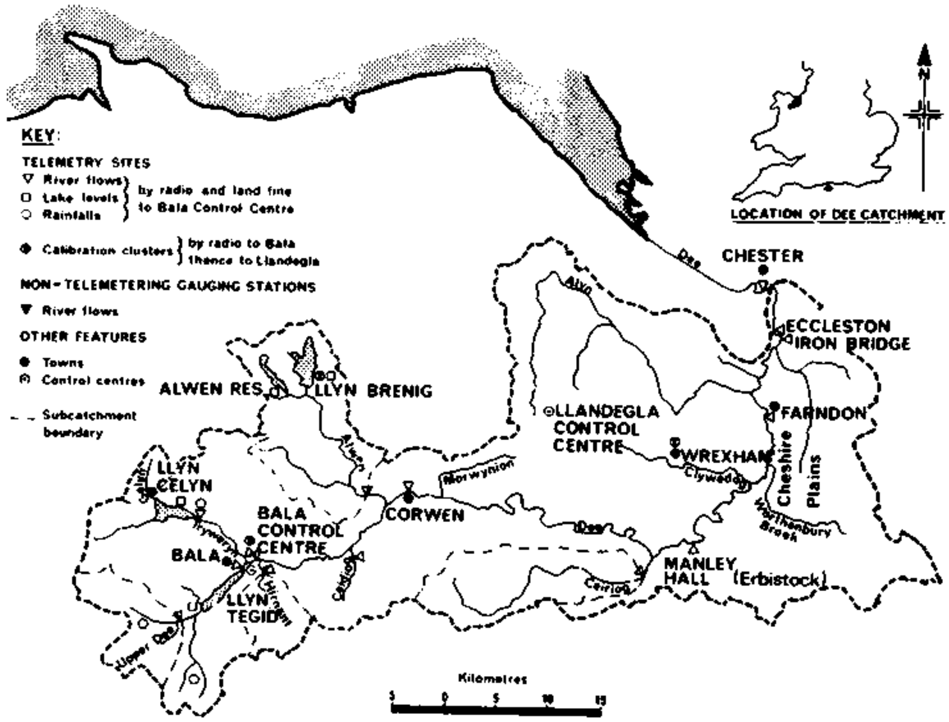


FIGURE 1 The River Dee catchment

Application of the Lambert model to these subcatchments has been described by McKerchar, 1975, and Lowing, Price & Harvey (1975).

1.2 Recommended developments

Following implementation and initial evaluation of the real-time forecasting system at Bala, recommendations for further development of the rainfall-runoff model were made by the project's Steering Committee in 1978. Two areas of further study were proposed, both involving the form of the model's two parameters; the catchment lag, L, and the storage parameter, k:

(1) The replacement of the existing single 'lag' measure by a simple triangular time area diagram.

At present the rainfall input to the model is considered to be that occurring during one basic time interval (½hhour on the Dee), L hours in the past. However, it was suggested that variation of travel time along the length of the catchment could be represented by a symmetrical distribution of contributing area around the central lag estimate of L hours. This effect is achieved by smoothing the rainfall thus:

$$r_T = yr_{(L-T)} + xr_{(L)} + yr_{(L+T)}$$

where

r_T = Total rainfall input to the model

r = Rainfall recorded over basic time period (T)

L = Catchment lag (hours)

y = $(1 - x)/2$

x = central rainfall fraction $1 > x > 0$

At present x = 1, but one suitable model might be defined by a value of x = 0.6 giving a rainfall profile 0.2 - 0.6 - 0.2 as shown in Figure 2.

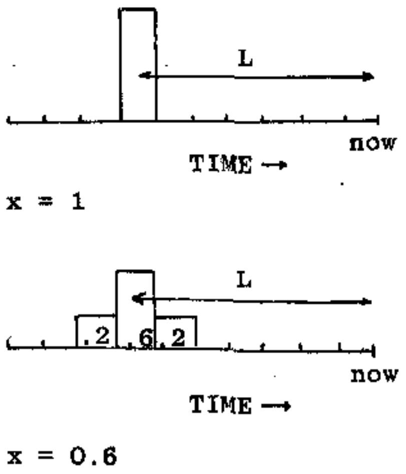


FIGURE 2

Rainfall models

(2) The replacement of the existing storage parameter k, by a more flexible relationship between k and q (discharge).

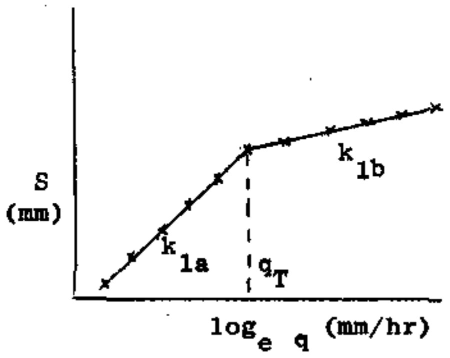
Originally McKerchar derived a two part storage-outflow relationship for some subcatchments, incorporating a threshold above which a second storage parameter was employed (Figure 3). In addition, two seasonal parameter sets were derived separately for summer (May-October) and winter (November-April) months. However, it was considered that a fully flexible k vs q relationship would improve the model performance.

Storage parameter

k_1 is slope of

S vs $\log_e q$

relationship



Threshold discharge = q_T

(1) Storage parameter k_{1a} below flow of q_T

(2) Storage parameter k_{1b} above flow of q_T

FIGURE 3 **Two-part S vs q relationship**

1.3 Improvement objectives

The Lambert model is currently in operational use in the Dee basin to produce subcatchment flow predictions. It is stored as a subroutine on the Bala control centre computer as part of the total hydrological/hydraulic model. Any improvements made to the subcatchment model should therefore result in more reliable flow forecasts at all points on the system, thereby raising confidence in their use by the engineers responsible for flood warning and reservoir control.

The Lambert model in its original form of fixed parameters was ideally suited to real-time application because of its ability to continually self correct and produce a revised forecast in the light of new telemetered flow and rainfall information. Although its ability to reproduce long lengths of historic record is not therefore its main attribute, there is no doubt that an improved performance in this (the simulation mode as distinct from the forecasting mode) must increase the reliability of the forecasts made 24 hours ahead. Lack of accuracy at this lead time is not a serious constraint at present because such a forecast on a subcatchment requires almost as long a forecast of rainfall. But as a quantitative precipitation forecasting (QPF) improves, so must the model performance in simulation.

2. THE LAMBERT MODEL

2.1 Assumptions

In the derivation of the basic equations, Lambert (1972) assumes:

" (1) That run-off is principally controlled by water stored naturally within the surface layers of the catchment area.

(2) That the rate of run-off (q) from a catchment area at any time is uniquely related to the amount of water stored naturally at that time within the catchment area (in surface depressions, soil strata, aquifers, etc), collectively called 'catchment storage' (S).

(3) That, as a first approximation, it is unnecessary to subjectively divide river flow into components (surface run-off, interflow, base-flow, etc).

(4) That the water-balance for the catchment shall be satisfied at all times."

2.2 Model formulation

The water balance for a catchment may be written as:-

$$\frac{dS}{dt} = r - e - q \qquad \ldots(1)$$

where,

S = catchment storage

r = rainfall input
e = losses due to evaporation etc.
q = outflow from the catchment
} Expressed as instantaneous rates

The losses term, e, in Equation (1) has been ignored not only because evaporation tends to be low in North Wales (particularly in the winter time) but also because optimisation of the model's parameters can, to a certain extent, compensate for it.

Equation (1) then becomes:-

$$\frac{dS}{dt} = r - q \qquad \ldots(2)$$

To solve Equation (2) a second equation relating storage to outflow is required, two forms of which were proposed by Lambert.

(1) <u>ISO-function Type I</u> (log-linear)

$$S = k_1 \log_e q \qquad \ldots(3)$$

Differentiation gives:-

$$\frac{dS}{dq} = \frac{k_1}{q} \qquad \ldots(4)$$

(2) <u>ISO-function Type II</u> (linear)

$$S = k_2 q \qquad \ldots(5)$$

Differentiation gives

$$\frac{dS}{dq} = k_2 \qquad \ldots(6)$$

It should be noted that the units of k_1 are storage units (mm) for the Type I model and time units (hours) for k_2 of the Type II version. The symbol k is used to represent either k_1 or k_2 as appropriate.

Since,

$$\frac{dq}{dt} = \frac{dq}{dS} \cdot \frac{dS}{dt} \qquad \ldots(7)$$

Equation (2) may be combined with either Equation (4) or Equation (6)

For model Type I, therefore,

$$\frac{dq}{dt} = \frac{q}{k_1} \cdot (r-q) \qquad \ldots(8)$$

and Type II,

$$\frac{dq}{dt} = \frac{(r - q)}{k_2} \qquad \ldots(9)$$

Integration of Equations (8) and (9) give the final equations of the Lambert model:-

Type I model

(a) When $r \neq 0$

$$q_n = q_o \cdot \frac{1}{X + (1 - X) \cdot \frac{1}{Y}} \qquad \ldots(10)$$

where

q_n = predicted flow at time T hours from now

q_o = present telemetred flow

$X = e^{-\frac{rT}{k_1}}$

r = total rainfall input to model

T = basic time interval (0.5 hours for Dee)

k_1 = storage parameter

$Y = \frac{r}{q_o}$

(b) When $r = 0$

$$q_n = \frac{q_o}{(1 + q_o \cdot \frac{T}{k_1})} \qquad \ldots(11)$$

Type II model

$$q_n = q_o \cdot (W - WY + Y) \qquad \ldots(12)$$

where,

$$W = e^{-T/k_2}$$

$$Y = \frac{r}{q_o}$$

Equations (10), (11) and (12), therefore, form the basic operating equations. The second parameter, the catchment lag, L, is introduced into the model by delaying the effect of rainfall by L hours. The total rainfall input to the model, r, is that which occurred during the time period T, L hours previously.

Although it is the log-linear form of the model (I.S.O. function type I) currently in use on the Dee subcatchments, both model forms are considered in this report for two reasons:

(1) Other catchments where the Lambert model could be applied in future may not require a log-linear type model but would benefit from a variable k.

(2) To determine, in the light of increased parameter flexibility, whether the more complex Type I log-linear I.S.O. function was really necessary.

3. DATA

The data used in this report comes from the five subcatchments shown in Figure 1:

Dee Subcatchment	Area
Dee at New Inn (Afon Dyfrdwy)	53.9 km^2
Hirnant	33.9 km^2
Ceiriog	113.7 km^2
Gelyn	13.1 km^2
Alwen	137.2 km^2

The main body of this report uses data from the first catchment studied, the Dee at New Inn; they were also used in the development work on the model, the results of which are reported in Section 8.

The Dee at New Inn, in common with the other subcatchments, is a mountainous catchment with steep valley sides covered with only a thin layer of soil on impermeable rock; there is a rapid response of runoff to rainfall. Rainfall is greatest in the winter months, but the occurrence of thunderstorms in summer means flooding is possible any time of year.

The data come from a time when the area was covered by a dense network of recording raingauges for the Dee Weather Project (Steering Committee Report, 1978) and are therefore of a relatively high quality. Rainfall and flow data are both at half-hour intervals. In determining the best parameter values for the Dee at New Inn, an 11 month period from November 1972 to September 1973 was used.

Lambert model parameters, in use before completion of the work reported here, are given in Table 1.

TABLE 1. Lambert model parameter values
(In use until August 1978)

	WINTER (NOV-APR)	SUMMER (MAY-OCT)			ALL YEAR
	k_1 (mm)	k_1 or k_{1a} (mm)	Threshold*q (mm/h)	k_{1b} (mm)	L (h)
Ceiriog	17.0	60.4	0.08	17.9	2.5
Alwen	8.6	23.6	0.10	9.1	2.5
Celyn	6.9	24.2	0.10	6.8	0.5
Hirnant	11.7	15.9	-		1.0
Dee at New Inn	4.9	7.9	-		1.0

* Denotes q at which change from k_{1a} to k_{1b} occurs

4. RAINFALL MODEL

This section considers the replacement of the existing single lag measure by a simple time-area diagram concept. Formulation of the rainfall smoothing process was given previously as:

$$r_T = yr_{(L-T)} + xr_{(L)} + yr_{(L+T)}$$

Figure 4 shows five possible types of rainfall model, type 0 - 1 - 0 being the simple lag measure at present employed. Before going as far as actually producing a time-area diagram, the sensitivity of the model was tested on two of these simple triangular distributions. There is no reason why the model should be symmetrical or why it should extend over three data intervals; however, with a catchment lag of one hour and a data interval of half an hour the models illustrated seemed a reasonable proposition. Types 0-1-0, 0.2-0.6-0.2 and 0.33-0.33-0.33 were applied to the flood event of 1st April 1973. In this analysis, present parameter values of L = 1 and k_1 = 4.9 were used in the basic Type I model on Dee at New Inn data. Observed and predicted hydrographs are shown in Figure 5.

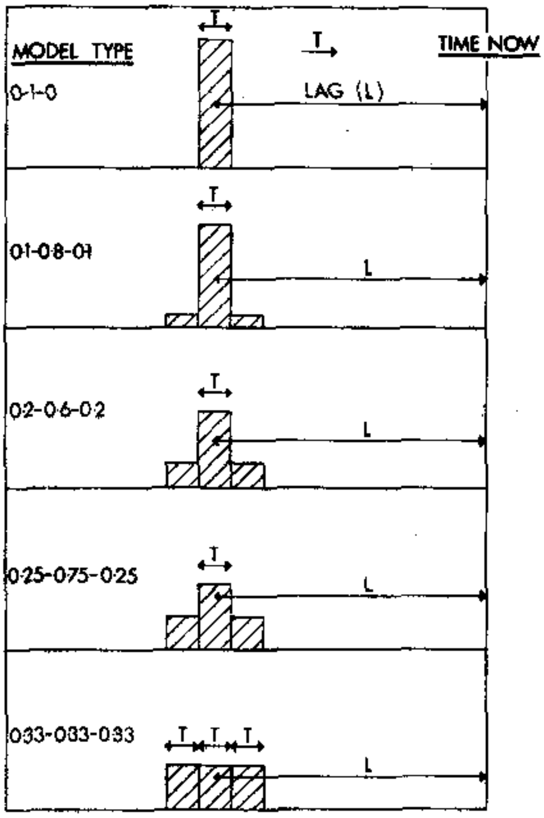


FIGURE 4

Types of rainfall model

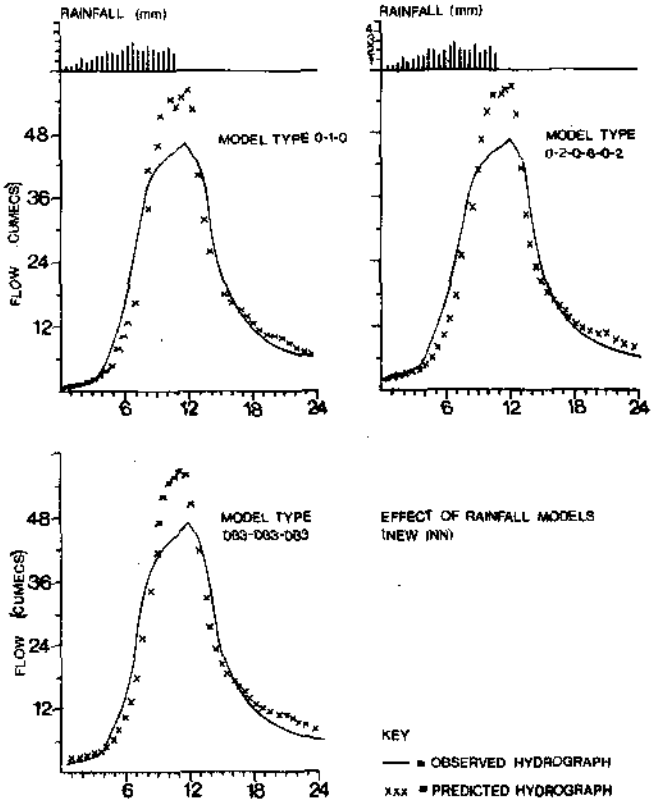


FIGURE 5

Effect of rainfall models (New Inn)

From Figure 5 it can be seen that only minor changes to the hydrograph shape result from incorporation of the rainfall model; the hydrograph peak is progressively smoothed and rounded as the rainfall model becomes flatter. However, there is a danger in taking the smoothing too far since this implies a corresponding reduction in sensitivity to rainfall (ie there is little point in having half-hourly rainfall data if it is going to be averaged out over 1½ hours as in model Type 0.33-0.33-0.33).

Rainfall model Type 0.2-0.6-0.2 does give some smoothing (an improvement on the single lag Type 0-1-0) whilst retaining some sensitivity to the rainfall data. There is also credibility in the physical significance of Type 0.2-0.6-0.2 (which implies that the total rainfall arriving at storage is composed of 0.2 time the half-hourly rainfall from the time interval commencing 1.5 hours ago plus 0.6 times that from 1 hour ago, plus 0.2 times the rainfall falling from 0.5 hours ago) in that it is most like the likely real time-area diagram.

The benefits of rainfall smoothing are not great on this catchment and are really of a cosmetic nature, but as they do improve the hydrograph shape, a Type 0.2-0.6-0.2 model is recommended for the Dee at New Inn and is incorporated in the remaining results of this report.

It was anticipated that the benefits from this type of rainfall smoothing would become more noticeable with an increase in catchment area and two events from the Alwen (137.2 km^2) and Ceiriog (113.7 km^2) catchments were analysed to investigate this point.

Figures 6 and 7 (opposite) show the results of the 0-1-0, 0.2-0.6-0.2 and 0.33-0.33-0.33 type models applied to both floods.

Compared with the smaller (53.9 km^2) Dee at New Inn catchment, improvements in hydrograph prediction appear to be of a similar size and nature (Figure 5). It is possible that a further improvement in prediction might have been possible with a broader based rainfall smoothing model. One possible model, for example, which extends over five time intervals is shown in Figure 8.

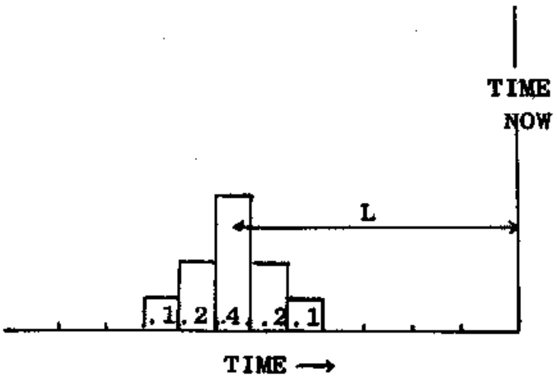


FIGURE 8

Alternative rainfall model

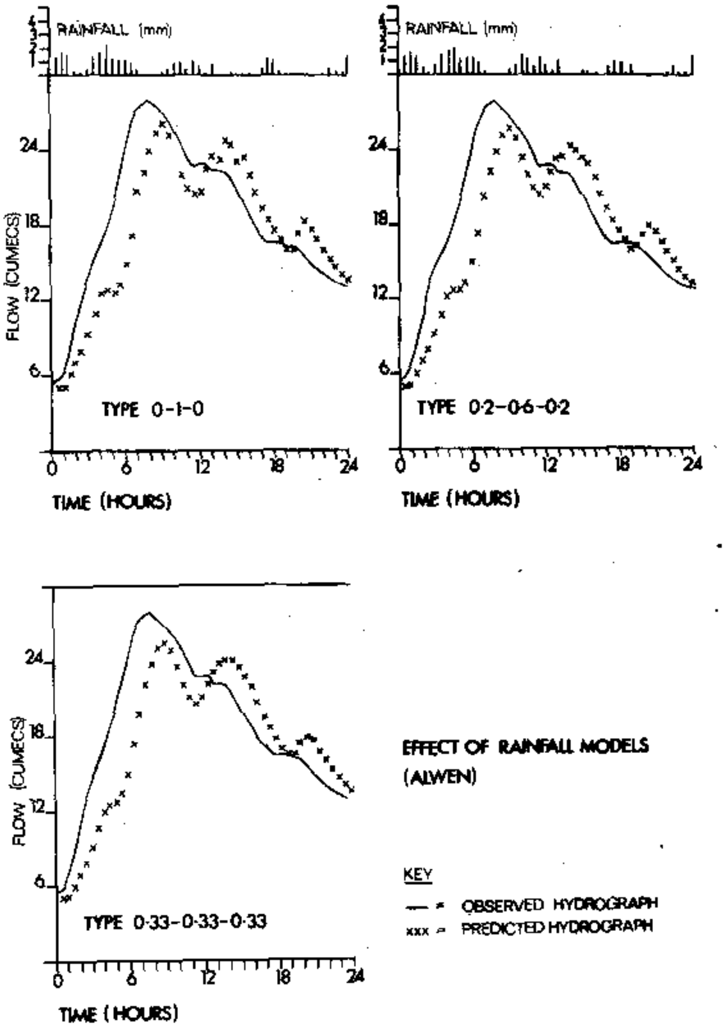


FIGURE 6

Effect of rainfall models (Alwen)

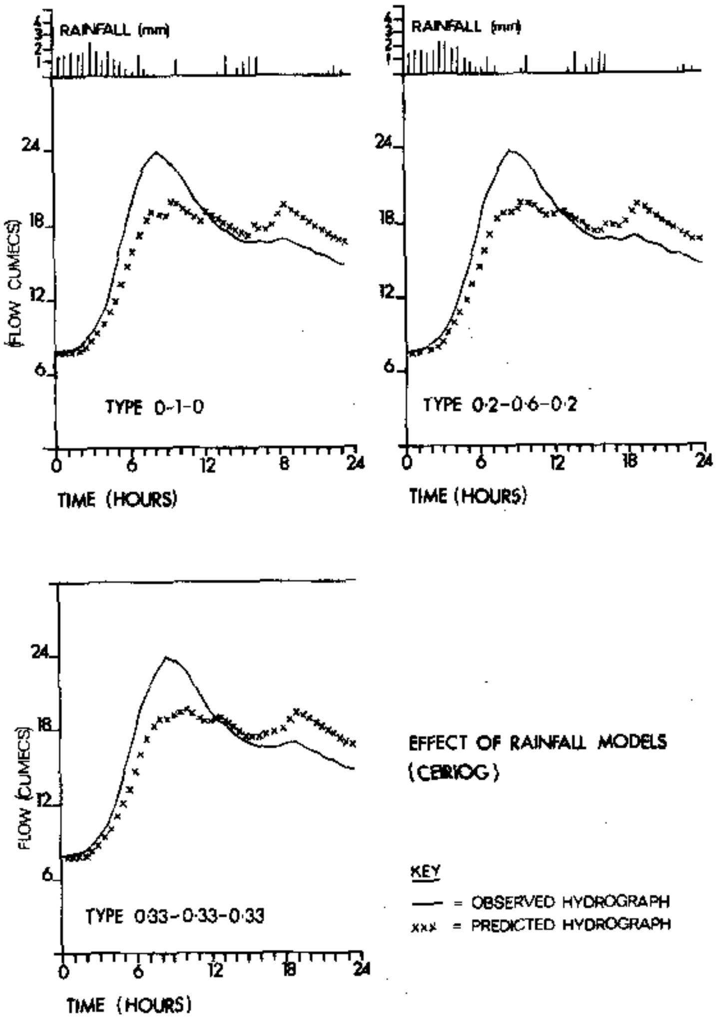


FIGURE 7

Effect of rainfall models (Ceiriog)

However in the light of the small changes resulting from the simpler model it was not considered worthwhile to pursue this point. Nevertheless it is quite possible that in catchments where travel times are longer, investigation of a more sophisticated type of model would prove beneficial.

Therefore, in addition to the Dee at New Inn, the 0.2-0.6-0.2 rainfall model was incorporated on the Alwen and Ceiriog catchments but not on the Hirnant and Gelyn catchments for the following reasons:

(1) With areas of 33.9 km^2 and 13.1 km^2 the Hirnant and Gelyn catchments are very quick to respond to rainfall (lag times of 0.5 hours for both). Smoothing of the rainfall input is not necessarily beneficial as sensitivity is lost.

(2) The catchment lag of 0.5 hours implies that rainfall input to the model should basically come from the previous half hour period. Introducing a 0.2-0.6-0.2 type of model on a lag of 0.5 hours means that 0.2 times the rainfall over the future half hour period is required to make the prediction. In practice this implies that a rainfall forecast is needed. Ordinarily on the system at Bala there is no such forecast and the model therefore assumes zero rainfall. In such a situation the model would be operating below its best. No rainfall smoothing is therefore incorporated on the Hirnant and Gelyn catchments (ie a 0-1-0 type of rainfall input is used).

5. STORAGE PARAMETER, k

5.1 Original derivation

The second of the two recommendations expressed in the introduction states that the existing storage parameter, k, should be replaced by a relationship between k and q (the outflow discharge). This is the subject of the majority of the remainder of this report and is where greatest improvements in model performance have been made. Firstly, however, it is necessary to derive the k vs q relationship. Techniques previously used for deriving k are:

(1) Lambert's original derivation of k is based on a recession curve analysis whereby at a series of time increments, depletion of catchment storage is computed from the changing 'volume' under the recession curve. Catchment storage is plotted against flow for I.S.O. function type II or log flow for I.S.O. function Type I and where the slope of the line gives k_2 and k_1 respectively. Although several k values over a flow range may be determined by this method, it is based on isolated hydrograph recessions and does not produce the best overall hydrograph fit (ie including the rising limb).

(2) McKerchar derived L and k_1 by an optimization technique in which the sum of squares of the residuals between observed and predicted flows were minimised over several months of data. Using this technique separately over winter and summer months, seasonal L and k_1 values were derived with a two part k_1 (incoporating a threshold) for some subcatchment during summer months. The values of k_1 produced were therefore based not only on hydrograph recessions as in (1) above, but also on the rising limb. Optimisation in this way allowed for evaporation to be incorporated in the final k_1 value.

(3) The parameters produced by method (2) above gave the best fitting model over the months for which they were optimised, but for real time use, where the difference is the continual updating by telemetered flow, better forecasts were produced by a set of parameters optimized subjectively on isolated flood events.

5.2 Parameter generation

An additional technique of deriving k from basic rainfall-run-off data has been developed to produce a relationship between k and q. The principle involves rewriting the basic prediction equations for I.S.O. functions I and II to leave k as the unknown term on the left hand side of the equation:

(1) I.S.O. function Type I - $S = k_1 \log_e q$

Equation (10) may be re-written as:-

$$k_1 = \frac{-r\ T}{\log_e\left[\frac{q_o\ (r-q_n)}{q_n\ (r-q_o)}\right]} \qquad \text{(when } r \neq 0\text{)} \qquad \ldots(13)$$

and equation (11) as :-

$$k_1 = \frac{T}{\frac{1}{q_n} - \frac{1}{q_o}} \qquad \text{(when } r = 0\text{)} \qquad \ldots(14)$$

(2) I.S.O. function Type II - $S = k\ q$.

Equation (12) may be re-written as:-

$$k_2 = \frac{-T}{\log_e \left[\frac{q_n - r}{q_o - r}\right]} \qquad \ldots(15)$$

The value of k so derived may be considered as that which would be necessary to give perfect prediction of flow, q_n, from the initial value q_o and rainfall input r. Advantages of this method are threefold:

(1) The parameter is derived in the real-time (or point by point) sense and not by one optimisation over several months of data.

(2) Each value of k may be associated with the initial discharge q_o, at which it occurred. This forms the basis of derivation of a k vs q relationship.

(3) The rising and falling limbs of the hydrograph may be treated separately and an individual k vs q relationship for each part. This introduces another degree of flexibility to the model with minimal complication in its formulation; the benefits are discussed later in Section 8.

5.3 Elimination of unwanted points

In order that q_n be predicted exactly there is considerable scatter in the k parameter produced. However, since there are almost as many k values produced as points in the optimisation data set (16032 for 11 months New Inn data), some may be discarded provided an acceptable reason for doing so can be found. Points were discarded for the following reasons:

(1) k parameter generation Equations (13) and (15) break down when $q_o > r > qn$. Under these conditions there is an attempt to take the logarithm of a negative number. Only relatively few points are eliminated in this way (of the order 0.1%).

(2) Values of k less than zero are generated when the storage principle of the model is violated:

  (a) the flow rises when no rain has been recorded.

  (b) when $r > q_o$, k determines by how much q_n should increase above q_o. If $q_n < q_o$, however, then a negative k value is produced.

(c) When $r < q_o$, k determines by how much q_n should fall below q_o. (But if q_o is in fact less than q_n, a negative k value is generated).

(3) Occasionally, very large values are assigned to k where the apparent cause-and-effect relationship between rainfall and runoff seems unlikely. In recession, for example, when a much smaller drop in flow is recorded than would normally be expected or when a sudden burst of rain produces a minimal increase in flow, a very large k value is generated. It could be argued that a model should be able to predict this, but the aim is to get the best overall fit and inclusion of these large numbers makes it more difficult to establish the general trend of the k vs q relationship. Values of k greater than the arbitrary limit of 80 (for model Type I and II) have been discarded. Points lost by eliminating factors (2) and (3) amount to approximately 2½% of the total.

Figures 9-14 illustrate the scatter of points obtained by generating k_1 and k_2 values from three individual months of data; NOV 72, APR 73 and AUG 73 (a plot of the entire 11 months data would be overcrowded). Figures 9, 10 and 11 represent model Type I, while Type II is shown in Figures 12, 13 and 14. Points for exclusion under criteria (2) and (3) above, have been included in these graphs for illustration. Points obtained from the rising and falling limb are shown separately and points outside the graphical limits of -10.00 and +50.00 are plotted on the graph's perimeter.

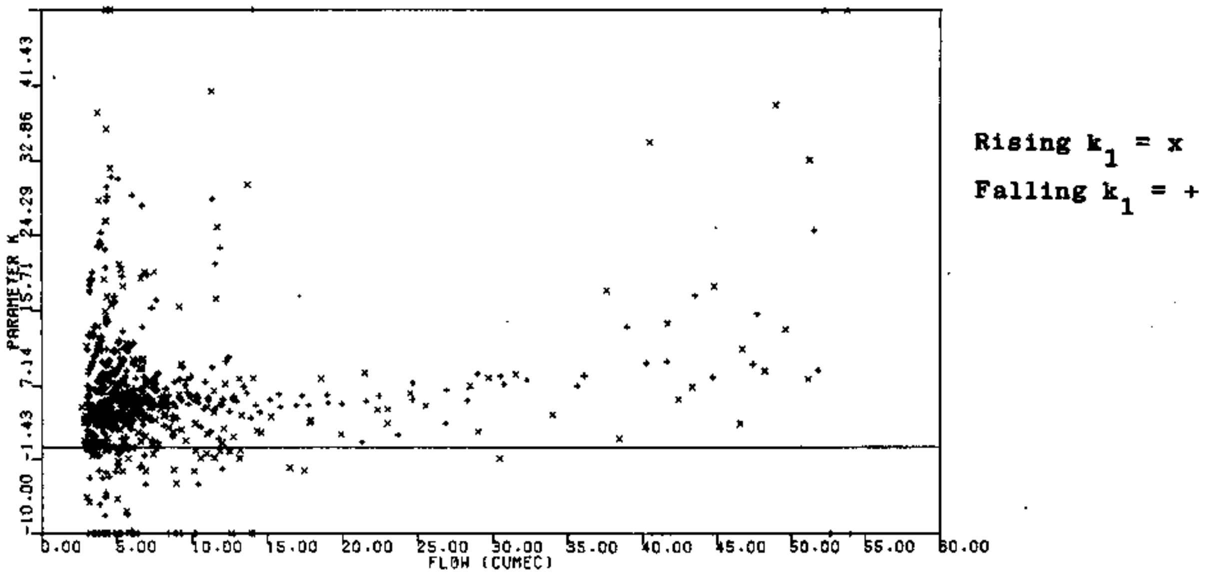


FIGURE 9 Dee at New Inn : k_1 for November 1972

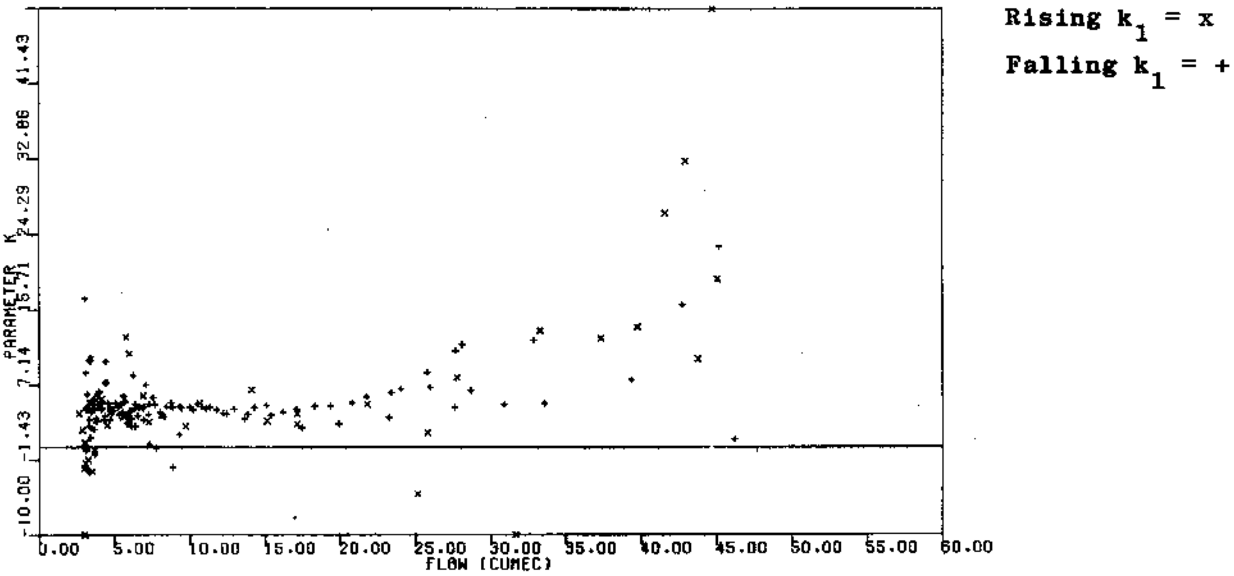


FIGURE 10 Dee at New Inn : k_1 for April 1973

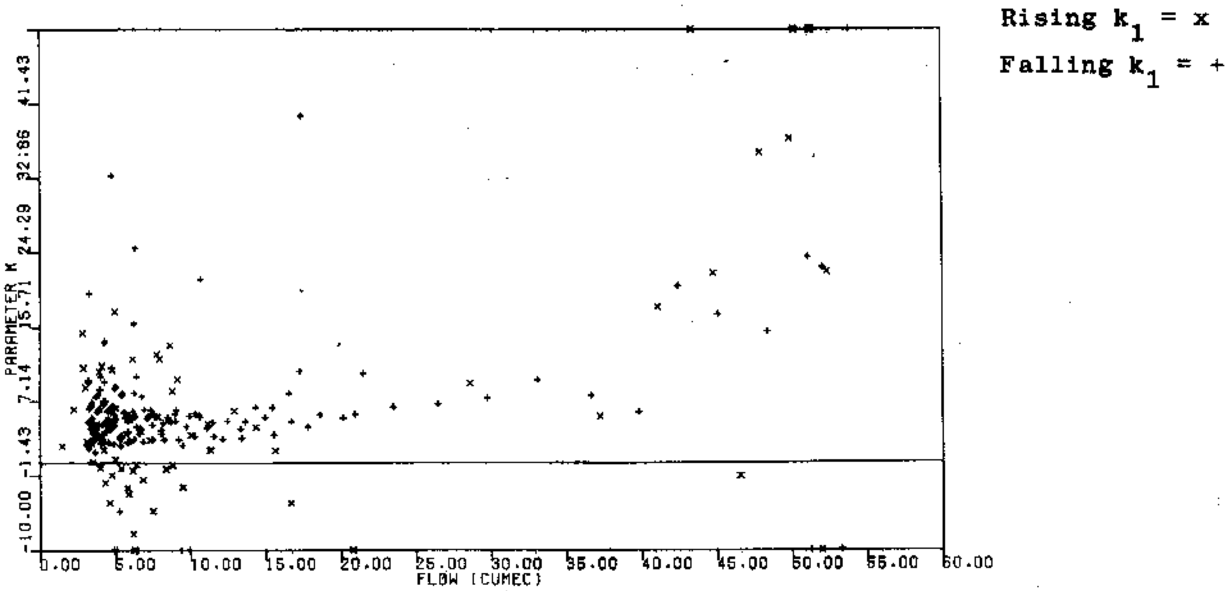
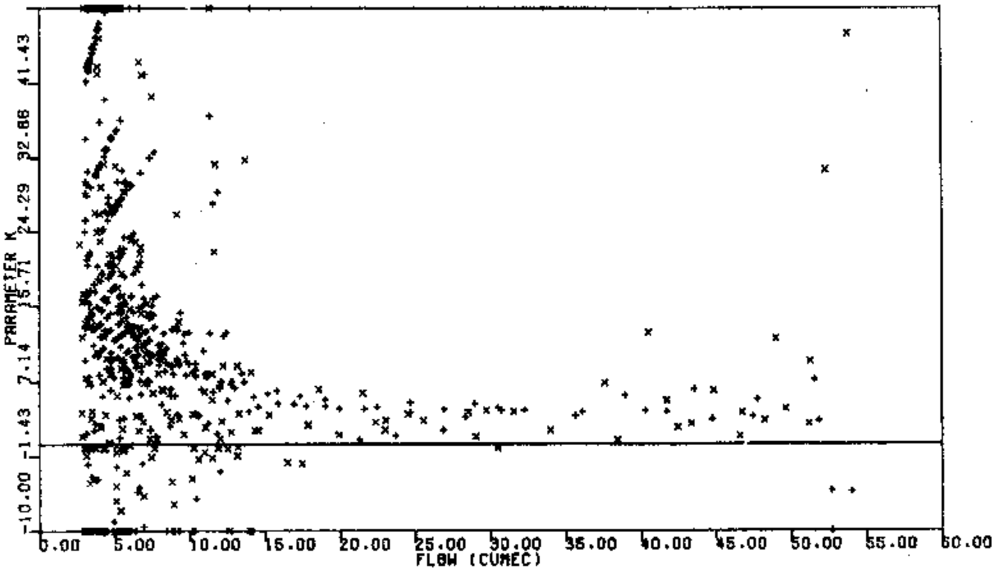


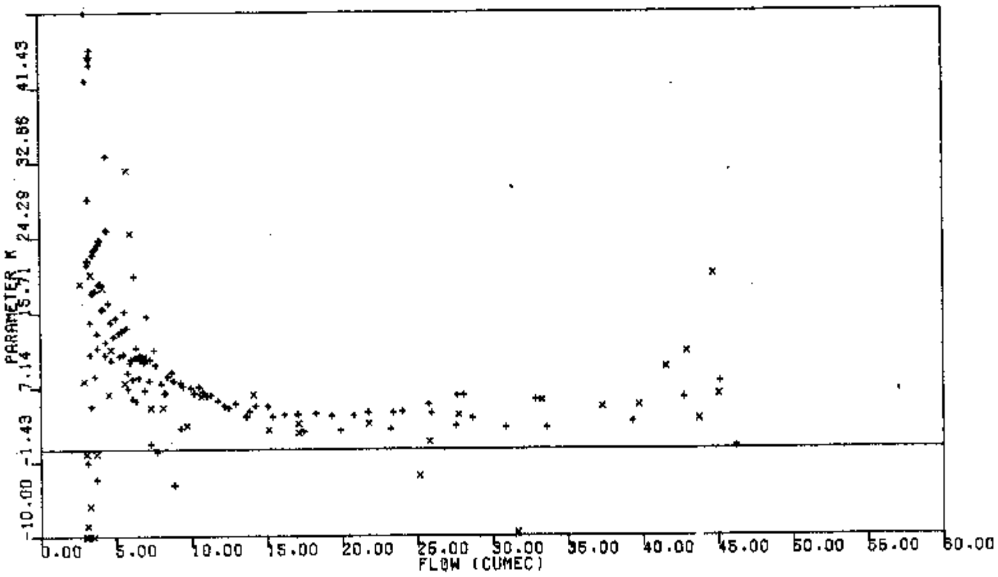
FIGURE 11 Dee at New Inn : k_1 for August 1973



Rising k_2 = x

Falling k_2 = +

FIGURE 12 Dee at New Inn : k_2 for November 1972



Rising k_2 = x

Falling k_2 = +

FIGURE 13 Dee at New Inn : k_2 for April 1973

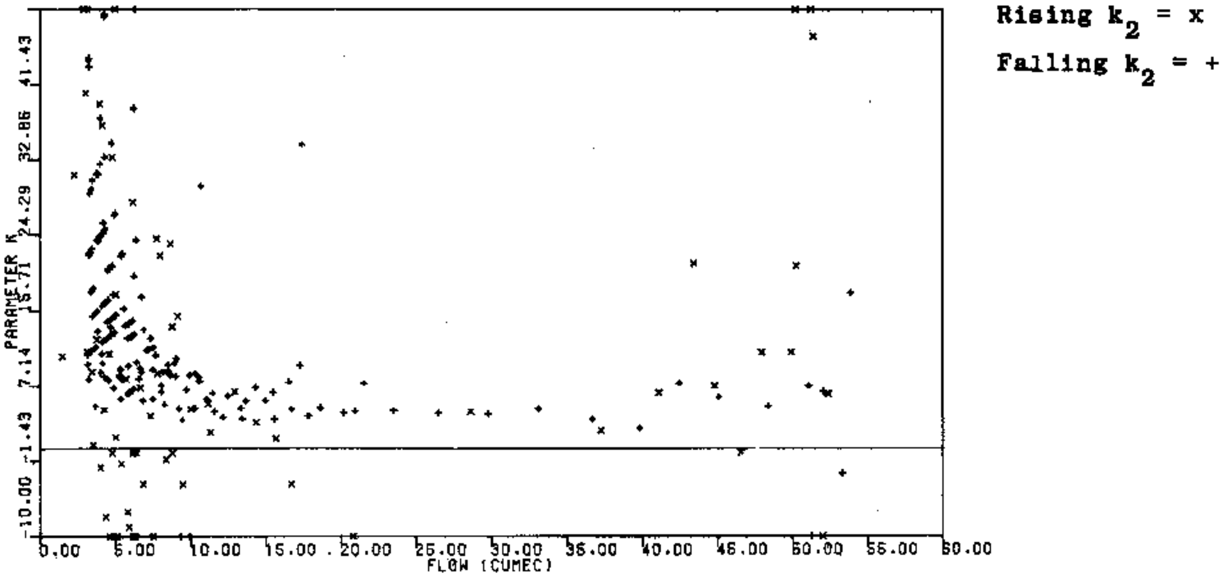


FIGURE 14 **Dee at New Inn : k_2 for August 1973**

5.4 Parameter generation during low flows

In periods of recession where there are very small changes in flow at successive half hourly intervals, two or more adjacent flow measurements can have the same recorded value. This is because the accuracy of the flow measurement station is insufficient to record the small changes in head which occur. During the parameter generation process the following procedure is adopted when one or more successive flow values are detected as being the same.

(1) No parameter (k) is generated until a drop in flow occurs from Q_1 to Q_2

(2) The number of data points with the same value is stored (N)

(3) The eventual drop in recorded flow is divided by N,

$$\frac{Q_1 - Q_2}{N}$$

(4) One parameter is then generated for this period with the following values of q_o and q_n

$$q_o = Q_1$$

$$q_n = Q_1 - \frac{(Q_1 - Q_2)}{N}$$

6. SEASONAL VARIATION

McKerchar (1975) allowed for seasonal variation in catchment response (evaporation, transpiration etc), by deriving sets of model parameters independently for summer (May-October) and winter (November-April) months. For the Dee at New Inn the following were developed:

	k_1	L
Summer	7.9	1.0
Winter	4.9	1.0

In an attempt to determine a seasonal trend for this study, the following analysis was carried out:

(1) The catchment storage parameter, **k**, was derived on a month by month basis using a 0.2-0.6-0.2 rainfall smoothing filter and the optimum lag of 1 hour.

(2) The mean of all k values obtained in any one month was plotted against their respective month for I.S.O. function Type I and II (Figure 15).

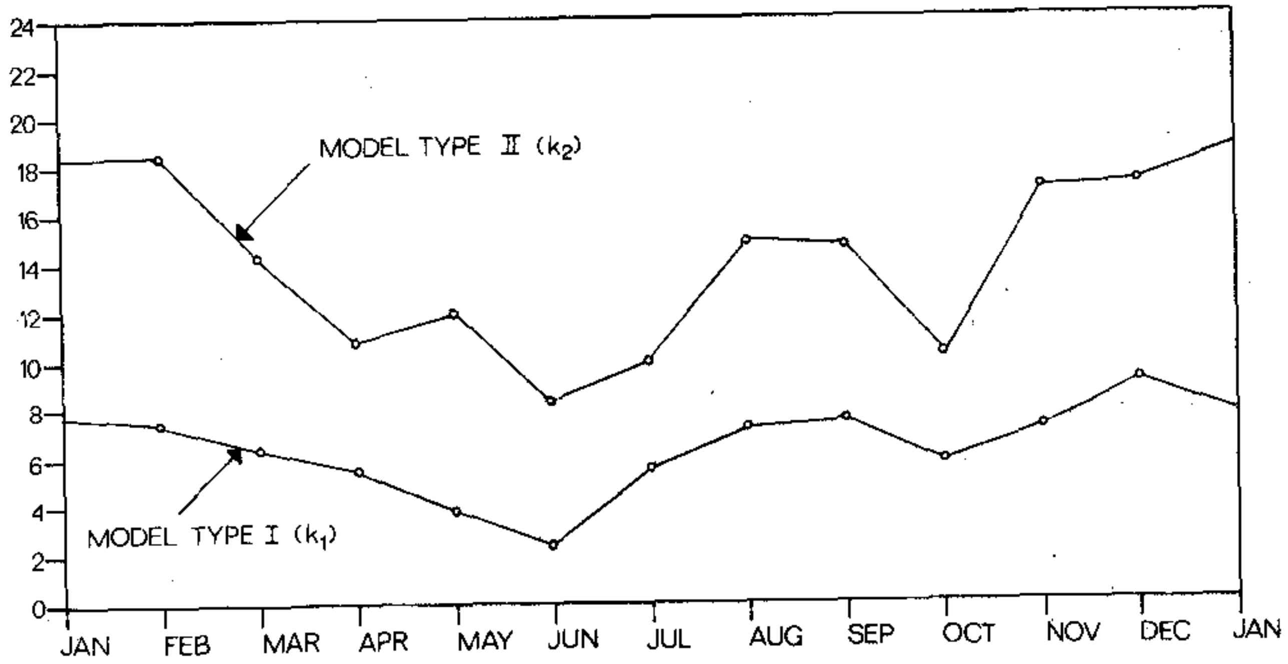


FIGURE 15 Seasonal variation of parameter k

From these results there appears to be a general trend towards a lower mean storage parameter around the months of May and June rising to a maximum in winter months (the opposite to McKerchar's observations). However, since some of these months contained very few flood events, it was considered that there was too little evidence to support this trend and the analysis would need to be repeated for several years to establish it with any certainty. The approach which was adopted therefore, was to use data from November 1972 to September 1973 as a whole and derive the best all year round relationship. In the final analysis this relationship was tested on both winter and summer events to determine whether an adequate hydrograph prediction was being achieved in both instances.

7. DERIVATION OF THE k vs q RELATIONSHIP

Using an assumed lag of 1.0 hours (the derivation of this is discussed later in Section 8.4) and the 0.2 - 0.6 - 0.2 rainfall smoothing process (Section 4), a set of k and q values were generated from the 11 months data as described in Section 5.2, 5.3 and 5.4. The 16032 data points from this period produced approximately 8000 values of storage parameter and associated discharge. These were divided into points from the hydrograph rising limb and hydrograph recession (approxmately 1640 and 6360 points respectively). Both I.S.O. function types (I and II) were considered.

To reduce the considerable scatter and quantity of points into a smooth relationship between k and q, the following technique was applied:

(1) Points were arranged in ascending order of discharge magnitude.

(2) The effect of a change in k on discharge prediction becomes increasingly influential as k approaches zero (ie the difference in discharge prediction for $k = 3$ as opposed to $k = 4$ is more than for $k = 33$ as opposed to $k = 34$). For this reason all k values were replaced by their logarithms.

(3) All points between 0 and 1 cumecs were grouped and the average discharge and average k parameter found for that group.

(4) This was repeated for points between 1 and 2 cumecs, 2 and 3 cumecs etc until the maximum discharge was reached. When there were less than five points within a group, the group was extended to include additional points to make up the

required number. This was necessary to ensure that there were sufficient points to obtain a 'good' average, damping the effects of an occasional spurious point. These results were then anti-logged. The size of the grouping (1 cumec in this case) was chosen to give a reasonable number of points for derivation of the final k vs q relationship.

(5) A three point moving average was run through the grouped and averaged points to provide an initial smoothing to the relationship. The results of this were then plotted (Figures 16-19).

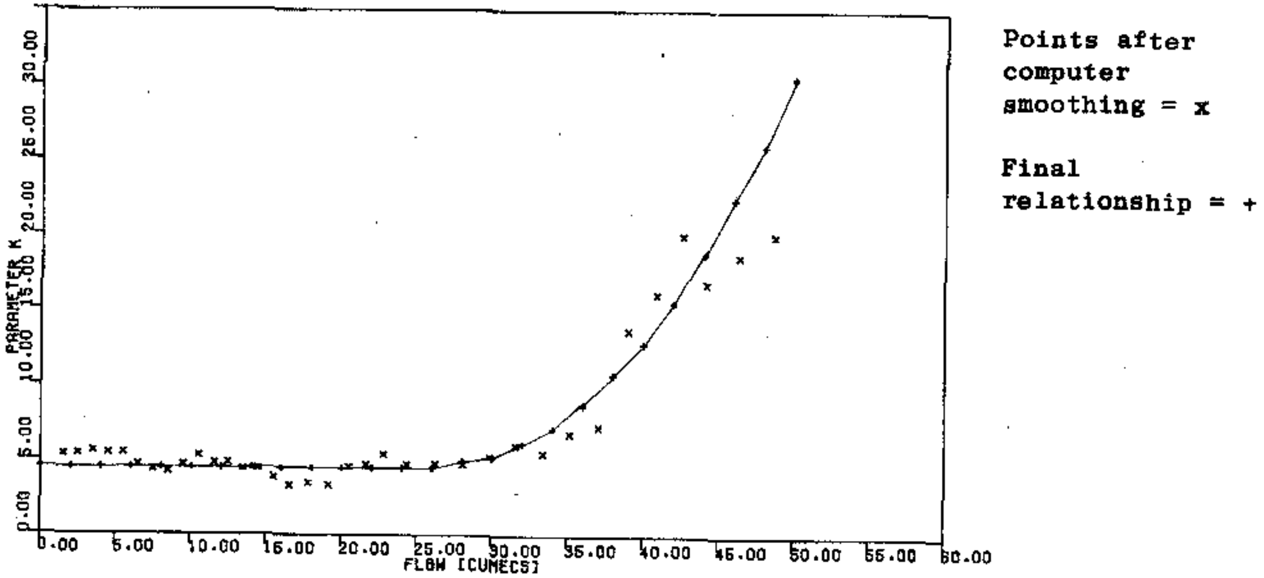


FIGURE 16 Dee at New Inn : k_1 vs q relationship (rising limb)

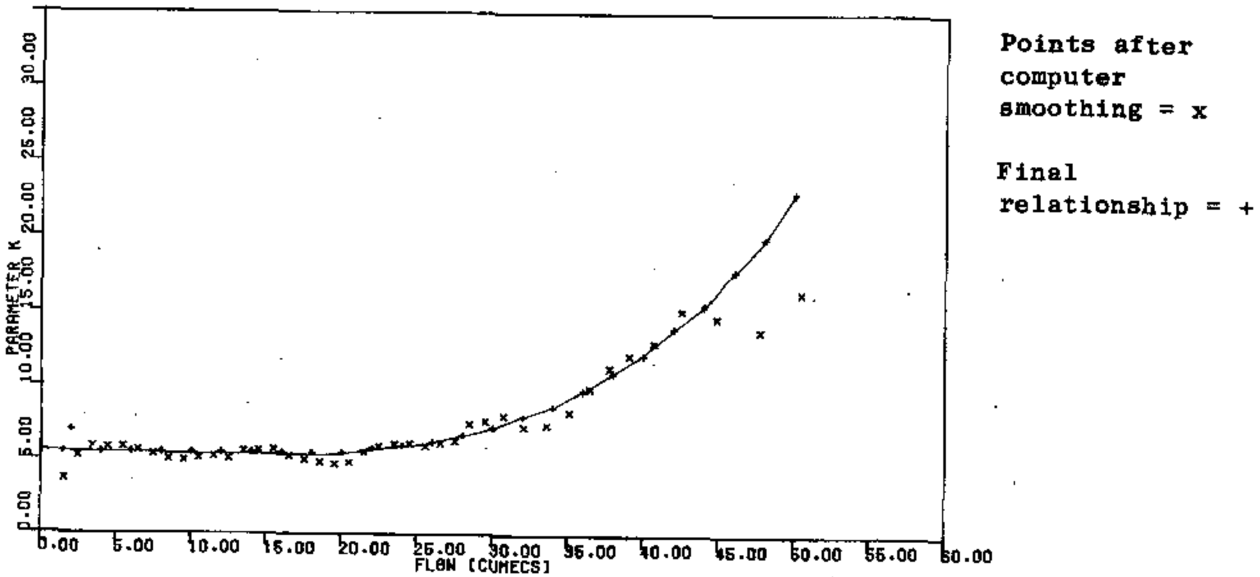
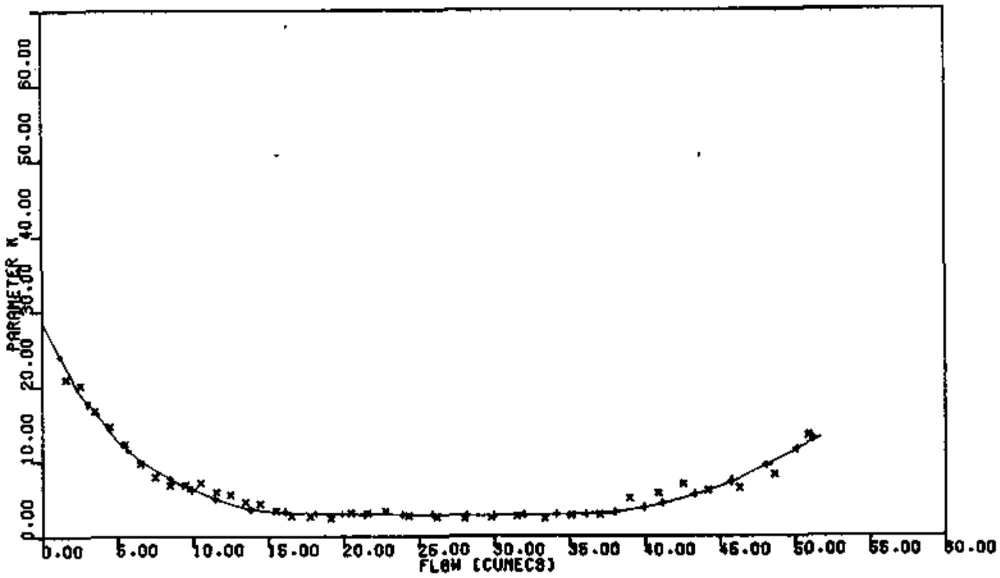


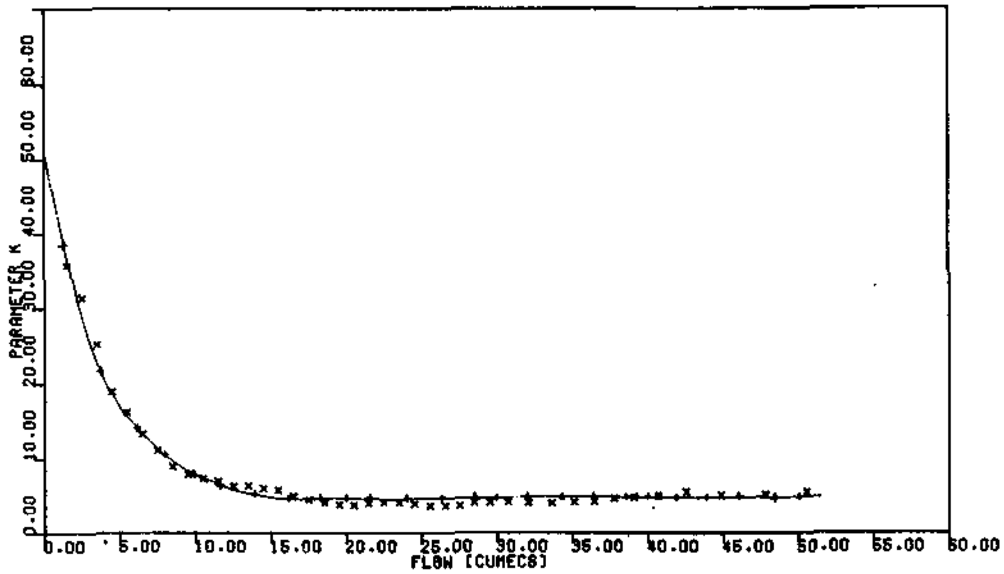
FIGURE 17 Dee at New Inn : k_1 vs q relationship (falling limb)



Points after
computer
smoothing = x

Final
relationship = +

FIGURE 18 New Inn : k_2 vs q relationship (rising limb)



Points after
computer
smoothing = x

Final
relationship = +

FIGURE 19 New Inn : k_2 vs q relationship (falling limb)

(6) Final smoothing was done subjectively by hand (Figures 16-19) and co-ordinates read from the line at successive 2 cumec increments on the discharge axis.

(7) The final k vs q relationship of model type I and II are shown in Figures 20 and 21 respectively. Two lines indicate the separate rising and falling limb components; the third is

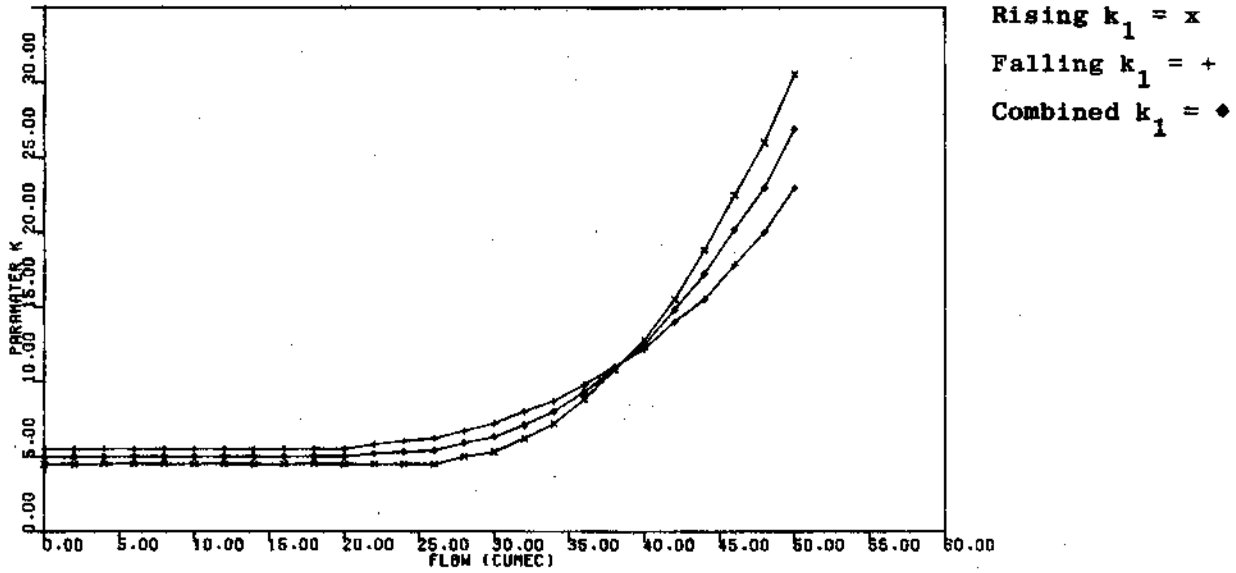


FIGURE 20 Dee at New Inn : parameter k_1 - Type I

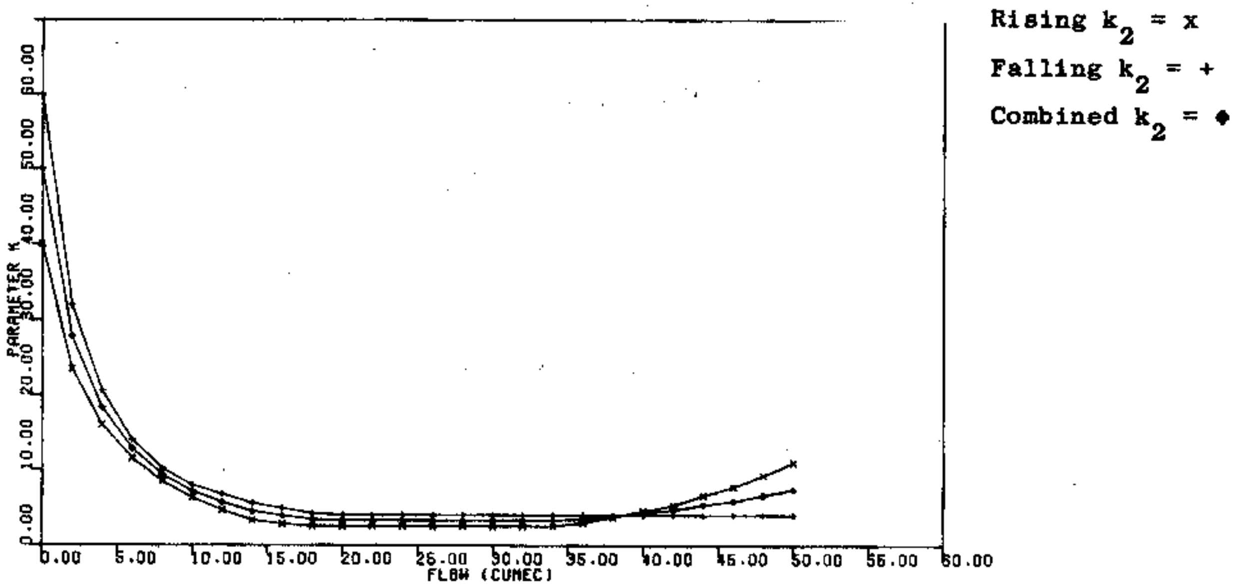


FIGURE 21 Dee at New Inn : parameter k_2 - Type II

the mean of the two - the combined relationship. The latter was used to determine the relative benefit gained by using separate as opposed to a combined relationship.

The k vs q relationships shown in Figures 16 to 21 indicate that:

(1) The basic I.S.O. function assumption (log-linear for Type I, linear for Type II) is in fact correct on segments

of the graphs showing a constant k value, (eg. Figures 16, 17: $q < 25\ m^3/sec$; Figures 18, 19: q between 20 and 35 m^3/sec).

(2) If the graph shows a changing value of k with q (eg Figures 16, 17: $q > 25m^3/sec$), then the basic I.S.O. function assumption is not correct, and the catchment response is modelled appropriately by assuming a 'changing' k value based on q.

(3) Figures 16-21 have been derived using a data time interval of 30 minutes and therefore should strictly only be used with this time step (if k is dependant on q).

8. RESULTS (NEW INN)

8.1 Test Objectives

Having derived a storage-outflow relationship, the following questions need to be answered:

(1) Does the variable parameter Lambert model produce significantly better hydrograph predictions than the fixed parameter version?

(2) Does the division into rising and falling limb k vs q relationships produce a worthwhile improvement in prediction accuracy?

(3) Which of the two forms of I.S.O. function give better results?

(4) Is there a requirement for a seasonal variation in parameter values?

(5) Are any improvements in model prediction on floods within the calibration period, extended to floods which have not been used to determine the best parameter values?

8.2 Test description and results

Seven floods events were selected from the calibration period on the basis that they should represent seasonal variation and a range of peak discharges. An eighth event, from 8th February 1974, was used to test the effectiveness of the k vs q relationship outside the generation period. The events, covering a range of peak flows from 8.4 cumecs to 52.73 cumecs, are listed below in Table 2.

Table 2. Test events for the Dee at New Inn

Start Date	Peak Discharge
9th November 1972	52.73 cumecs
26th January 1973	37.19 cumecs
1st April 1973	46.21 cumecs
12th May 1973	37.58 cumecs
4th August 1973	21.5 cumecs
18th October 1973	48.4 cumecs
27th November 1973	8.4 cumecs
8th February 1974	43.4 cumecs

The comparison was made against the existing fixed parameter model (Type I) with values as given in Table 1 (ie Lag = 1.0 hours, $k_1 = 4.9$ (winter), $k_1 = 7.9$ (summer)).

Each event prediction extends for 24 hours and is assumed to have a 'perfect' rainfall forecast. For each event there are four cases considered:-

(1) Model Type I with fully variable k_1 (ie separate rising and falling limbs), henceforth called the <u>Type I variable</u>.

(2) As in (1) but using the combined k_1 vs q relationship, henceforth called the <u>Type I combined</u>.

(3) Model Type II with fully variable k_2 (separate rising and falling limbs), henceforth called the <u>Type II variable</u>.

(4) As (3) but using the combined k_2 vs q relationship, henceforth called the <u>Type II combined</u>.

Graphs of observed and predicted flow for case (1) above (Type I variable model) may be found in the Appendix. Graphs of the other three (and in the final analysis less satisfactory) model types have not been included to save space. Table 3 summarizes statistics from each event and model type where the convention, error = observed - predicted (discharge, time of peak and volume) has been adopted. For each event the following information is provided

(1) Percentage error at peak

(2) Maximum error on rising limb as a percentage of peak discharge

TABLE 3 Summary of results (Dee at New Inn), given as percentages

EVENT	TYPE I ORIGINAL k_1	TYPE I VARIABLE	TYPE II COMBINED	TYPE II VARIABLE	TYPE II COMBINED	TEST
9-NOV-72	-43.1	4.8	5.0	5.1	4.4	A
	28.7	24.4	30.2	19.7	29.9	B
	- .5	- 0.5	- 0.5	- 0.5	- 0.5	C
	8.3	4.5	7.5	6.7	9.7	D
26-JAN-73	16.1	9.2	20.3	13.2	28.3	A
	17.3	11.0	20.5	19.8	31.6	B
	-.5	0.0	0.0	0.0	- 0.5	C
	12.9	1.5	13.4	6.2	22.6	D
1-APR-73	20.9	- 3.8	- 1.2	- 4.8	- 1.8	A
	55.4	15.4	24.5	10.4	23.0	B
	- .5	0.0	0.0	0.0	0.0	C
	26.7	- 9.5	- 2.3	- 8.9	- .2	D
12-MAY-73	42.6	4.7	9.9	1.2	10.0	A
	49.4	10.6	18.5	- 7.1	17.6	B
	- .5	0.5	0.0	0.5	0.0	C
	32.9	.9	9.8	- 2.9	11.0	D
4-AUG-73	78.4	9.1	31.6	-31.7	4.2	A
	75.3	-10.2	11.8	-56.1	-18.1	B
	0	0.5	0.5	0.5	0.5	C
	48.2	-21.1	0.8	-54.1	-18.4	D
18-OCT-73	-52.1	- 5.7	- 5.5	- 4.4	- 4.1	A
	-25.1	9.3	18.8	15.0	29.6	B
	1.0	- 1.0	- 1.0	- 1.0	- 1.0	C
	- 3.6	-10.5	- 7.7	- 2.3	- 0.4	D
27-NOV-76	17.9	9.7	-21.0	3.2	16.8	A
	20.3	12.7	22.9	-10.9	18.6	B
	- .5	- 0.5	- 0.5	- 0.5	- 0.5	C
	3.4	- 6.1	4.7	-12.8	0.2	D
8-FEB-74	- 9.9	1.0	4.9	2.1	9.5	A
	-10.0	- 8.1	4.8	6.7	14.7	B
	.5	0.5	0.5	0.5	0.5	C
	10.0	1.0	7.9	5.3	16.6	D
AVERAGE * OF ABSOLUTE ERRORS	35.1 (8.9)	6.0 (3.6)	12.4 (5.5)	8.2 (-2.0)	9.9 (8.4)	A
	35.2 (26.4)	12.7 (8.1)	19.0 (19.0)	18.2 (-0.3)	22.9 (18.4)	B
	0.5 (-0.1)	0.4 (-0.1)	0.4 (-0.1)	0.4 (-0.1)	0.4 (-0.2)	C
	18.3 (17.4)	6.9 (-4.9)	6.8 (4.3)	12.4 (-7.9)	9.9 (5.1)	D

TEST
(A = % ERROR AT PEAK
(B = MAX. ERROR ON RISING LIMB AS % OF PEAK
(C = TIMING ERROR (HOURS)
(D = VOLUME ERROR (%)

*Bracketed figures indicate average of actual errors

(3) Timing error of peak discharge

(4) Error in predicting flood volume as percentage of recorded volume.

The following event analysis was produced after considering each flood in turn in conjunction with Table 3 and the appropriate graph from the Appendix.

9 NOV 72 (Figure A1)

A significant improvement in shape and prediction of peak discharge is evident from Figure A1 for model Type I variable over the fixed parameter version. Although not shown, this improvement in shape is extended to model Type II. There is only a small improvement in using the separate rising and falling limb parameter (Table 3).

26-JAN-73 (Figure A2)

Starting with the best, predictions for this event are easily categorised in the following order:-

(1) Type I variable

(2) Type II variable

(3) Original fixed parameter

(4) Type I combined

(5) Type II combined

This implies, for this event, that unless separate parameters are used for the rising and falling limb, no benefit is gained over using the fixed value, $k_1 = 4.9$. However, by using Type I variable, errors in peak discharge are reduced from 16.1% to 9.2%. rising limb errors from 17.3% to 11.0% and volume errors from 12.9 to - 1.5%.

1-APR-73 (Figure A3)

The comments on the event of 9-NOV-72 apply here also, except that the rising limb reproduction is better with Types I and II variable and the peak discharge is predicted better with the combined parameter.

12-MAY-73 (Figure A4)

There is a large improvement in prediction accuracy with this event.

Categorization of models in order of merit gives:

(1) Type II variable

(2) Type I variable

(3) Type I combined, Type II combined

(4) Fixed parameter, $k_1 = 7.9$

Using the best prediction, a Type II variable, peak discharge estimation has improved from 42.6% to 1.2%, rising limb error from 49.4% to - 7.1% and volume error from 32.9% to - 2.9%.

4-AUG-73 (Figure A5)

The original fixed parameter model with $k_1 = 7.9$ produced a very poor prediction for this event with an error in peak discharge of 78.4% and a 48.2% volume error. Optimum peak discharge prediction, rising limb reproduction and volume prediction were, on this occasion, produced by three different models (see Table 3).

18-OCT-73 (Figure A6)

Although there is a reduction in accuracy in the prediction of peak discharge from -0.3% to around -5% (for all variable types) on this event, the hydrograph shape, and in particular the rising limb, is modelled better by all variable parameter types.

27-NOV-73 (Figure A7)

Peak discharge and rising limb error are best produced by a Type II variable model on this event (reduced from 17.9% to 3.2% and 20.3% to -10.9% respectively). Again, the separation of rising and falling limb parameters improves the prediction for model Types I and II.

8-FEB-74 (Figure A8)

This event was taken from outside the calibration period. Overall, the best prediction was made by a Type I variable model, with the error at peak reduced from -9.9% to 1.0%, volume error from 10.0% to 1.0% and there was a slight improvement on the rising limb from -10.0% to 8.1%.

8.3 Summary of results

Having considered each event individually, it appears to be difficult to determine which type of model to use and whether or not it is worthwhile to employ a separate k vs q relationship on rising and falling limbs. A clearer picture emerges, however, when the average of all eight events is taken (see Table 3) to give an overall absolute error at the peak, error on the rising limb, error in timing and volume error for each type of model. From these results on New Inn data, it is clear that the best type of model is based on I.S.O. function Type I and that it is better to have a separate rising and falling limb k vs q relationship. Improvements using the optimum combination - a Type I variable model are:

(1) Average reduction in peak discharge error from 35.1% to 6.0%;

(2) Average reduction in rising limb error from 35.2% to 12.7%;

(3) Average reduction in volume error from 18.3% to 6.9%.

Although the average absolute timing error is given as 0.4 hours, the average timing error (given in brackets alongside the former) ranges from 0.1 to 0.3 hours. The catchment lag of 1 hour is therefore satisfactory, bearing in mind that it can only be defined as some multiple of the basic data interval (0.5 hours).

In answer to the questions posed in Section 9.1, it may now be stated that:

(1) The variable parameter Lambert model does produce significantly better hydrograph predictions than the fixed parameter version.

(2) The division into rising and falling limb k vs q relationship produces an improvement in prediction accuracy. This is particularly true with the model Type I.

(3) Model Type I generally produces better results than model Type II.

(4) There is no noticeable seasonally-based error. It would require several years' data in order to establish a trend with any certainty by using the method described in Section 6.

(5) Improvements gained within the calibration period extend to floods outside that period of which the 8th February 1974 event was an example chosen.

8.4 The catchment lag parameter, L

At the beginning of Section 7 it was stated that for subsequent analysis an assumed catchment lag of 1 hour was to be used. This section, coming after the results have already been discussed, may appear to be out of place. However the final choice of the catchment lag is based on the result analysis summarized in Table 3. The procedure used for obtaining the optimum lag is given below:

(1) k vs q relationships were derived from the basic data for a range of lags likely to contain the optimum value (for the Dee at New Inn, 0, 0.5, 1.0, 1.5 and 2.0 hours were considered).

(2) The eight flood events used in the result analysis were selected with the criteria that they should cover a range of peak discharges and come from summer and winter months.

(3) Using the appropriate k vs q relationship with each lag, the eight events were modelled and the average timing error of these events obtained. Figure 22 shows how the hydrograph prediction for one particular event changes for the five lag times. The optimum catchment lag was indicated by the results with the average timing error of all eight events

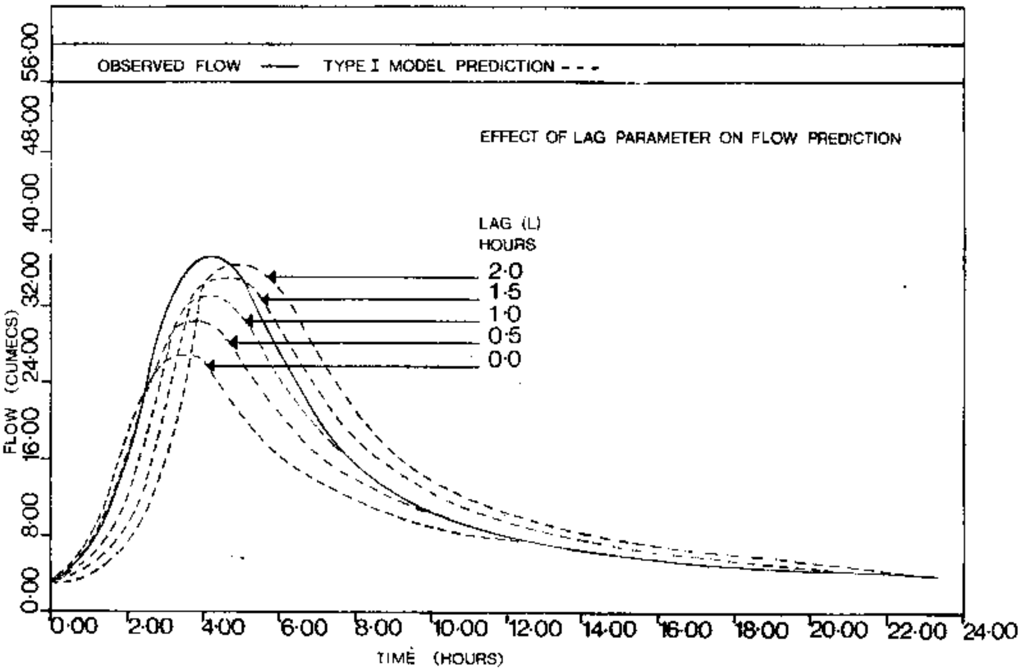


FIGURE 22 Dee at New Inn : variation of lag

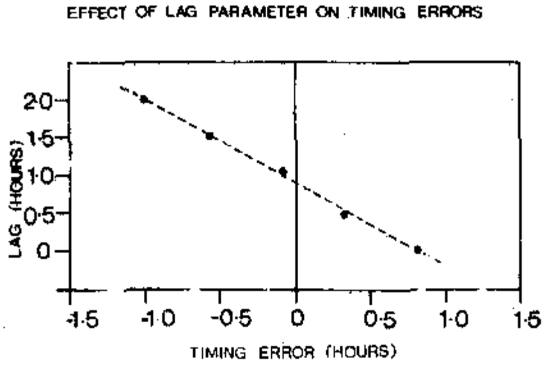


FIGURE 23

Effect of lag parameter on timing errors

closest to zero (Figure 23). From Figure 23 and Table 3 it can be seen that the average timing error for the Type I variable model was -0.1 hours. This figure implies that the best synchronisation of runoff with rainfall on eight representative flood events is achieved with a catchment lag of 1 hour. It should be noted that it is not possible to improve on this because the lag must be a multiple of the basic data interval (0.5hours).

9. RESULTS (ALL CATCHMENTS)

Preceeding sections have been concerned with data and results from the Dee at New Inn subcatchment of the Dee. The variable parameter model thus developed was then applied to the four other gauged subcatchments on the Dee, namely the Hirnant, Ceiriog, Gelyn and Alwen. Between 11 months' and two years' data were used, depending on the availability of a continuous record:

CATCHMENT	FITTING PERIOD
Hirnant	July 72 →June 74
Ceiriog	Nov 72 →Oct 74
Gelyn	Sept 72 →July 73
Alwen	July 72 →Aug 73 (when Alwen reservoir not spilling)

From the New Inn results discussed in Section 8, the Type I (log-linear) model with separate rising and falling parameters emerged as the most satisfactory model. In the light of this it was decided to apply this form of model to the remaining catchments.

In common with the New Inn catchment, eight flood events were selected from the data to encompass a range of peak discharges from both summer and winter months and to have one event from outside the calibration period to ensure that any improvements were maintained after calibration ceased. On each catchment the lag parameter was determined from these events by the procedure described in Section 8.4. The lag parameters for the five subcatchments are given in Table 4 below, togther with the Figure number for the final discharge/storage parameter relationship (the New Inn results are included for completeness).

TABLE 4 Final catchment lag and storage parameter relationship

CATCHMENT	LAG, L (HOURS)	STORAGE PARAMETER k_1
New Inn	1.0	Figure 24
Hirnant	0.5	Figure 25
Ceiriog	2.0	Figure 26
Gelyn	0.5	Figure 27
Alwen	1.5	Figure 28

Table 5 contains a summary of the results for all catchments using the parameters indicated by Table 4. The results are a comparison between the original fixed parameter version of the Lambert model and the new Type I model with separate rising and falling limb parameters incorporating the rainfall smoothing function (Section4).

(1) Hirnant

The variable parameter model gives a significant improvement in prediction over the original fixed parameter version. Although this improvement is not as great as that recorded on the New Inn catchment, the average absolute error on the peak discharge estimation is down from 26.4% to 20.0% and the rising limb error from 33.2% to 22.4%. There is little difference in the timing or volume errors. However, the most significant improvement on this catchment is in the average errors (given in brackets alongside the average absolute errors). For peak discharge estimation these are reduced from 26.2% to 4.5%, for the rising limb from 33.2% to 4.2%, and in volume prediction from 12.4% to 3.6%. This means that there is no overall tendency to consistently over- or under-predict on this group of events.

TABLE 5 Summary of results for all catchments

	ORIGINAL k_1	VARIABLE k_1 TYPE I	TEST
NEW INN	35.1 (8.9)	6.0 (3.6)	A
	35.2 (26.4)	12.7 (8.1)	B
	0.5 (-0.1)	0.4 (0.1)	C
	18.3 (17.4)	6.9 (-4.9)	D
HIRNANT	26.4 (26.2)	20.0 (4.5)	A
	33.2 (33.2)	22.4 (4.2)	B
	0.3 (0.0)	0.3 (0.1)	C
	13.0 (12.4)	14.5 (3.6)	D
CEIRIOG	39.1 (36.4)	17.8 (-0.8)	A
	42.35(40.1)	23.3 (-9.7)	B
	1.1 (-1.0)	0.4 (-0.1)	C
	22.1 (22.1)	22.2 (-16.0)	D
GELYN	19.5 (- 6.3)	19.9(- 5.5)	A
	27.0 (- 7.7)	23.4(-10.2)	B
	0.8 (- .1)	0.7(.2)	C
	14.0 (6.2)	12.7(-1.6)	D
ALWEN	25.2(25.2)	21.1(18.7)	A
	44.9(44.9)	25.4(21.4)	B
	1.7(-1.7)	0.4(0.3)	C
	16.7(9.2)	14.3(-2.5)	D
AVERAGE of ABSOLUTE ERRORS	29.1(18.1)	17.0(4.1)	A
	36.4(27.4)	21.4(2.8)	B
	0.9(-.6)	0.4(0.1)	C
	16.8(16.8)	14.1(-2.3)	D

A = % Error at Peak
B = Maximum error on rising limb as % of Peak
C = Timing Error (hours)
D = Volume Error (%)

(2) Ceiriog

The comments on the Hirnant results given above apply equally to the Ceiriog results. In addition, however, there is an improvement in the timing error of peak discharges from -1.0 to -0.1 hours. This has been achieved by using a catchment lag of 2.0 hours instead of 2.5 hours (Table 1).

(3) Gelyn

This catchment showed the least improvement of all those studied. There was only a small improvement in the rising limb error from 27.0% to 23.4% and a marginal improvement in volume error from 14.0% to 12.7%. The reason for this is possibly due to the fact that the original results with the fixed parameter model were fairly good, it being the best of the five catchments studied.

(4) Alwen

On the Alwen there was a noticeable improvement in rising limb reproduction (from 44.9% to 25.4% error) and the average timing error was reduced from 1.7 hours to 0.3 hours. There was a small improvement in peak discharge estimation from 25.2% to 21.1% and in volume estimation from 16.7% to 14.3%.

The final group of results in Table 5 are an overall average of all events on all catchments. By replacing the existing fixed parameter Lambert model by the Type I separate rising and falling limb parameter model, it can be seen that:

(1) Error in the estimation of peak discharge is reduced from 29.1% to 17.0%;

(2) Error in the rising limb is reduced from 36.4% to 21.4%;

(3) Timing error is reduced from -0.6 hours to 0.1 hours;

(4) Volume error is reduced from 16.8% to 14.1%.

Furthermore, the average errors (given in brackets) on these 40 events are reduced to an insignificant level; 4.1% on peak discharge, 2.8% on the rising limb and -2.3% on volume prediction.

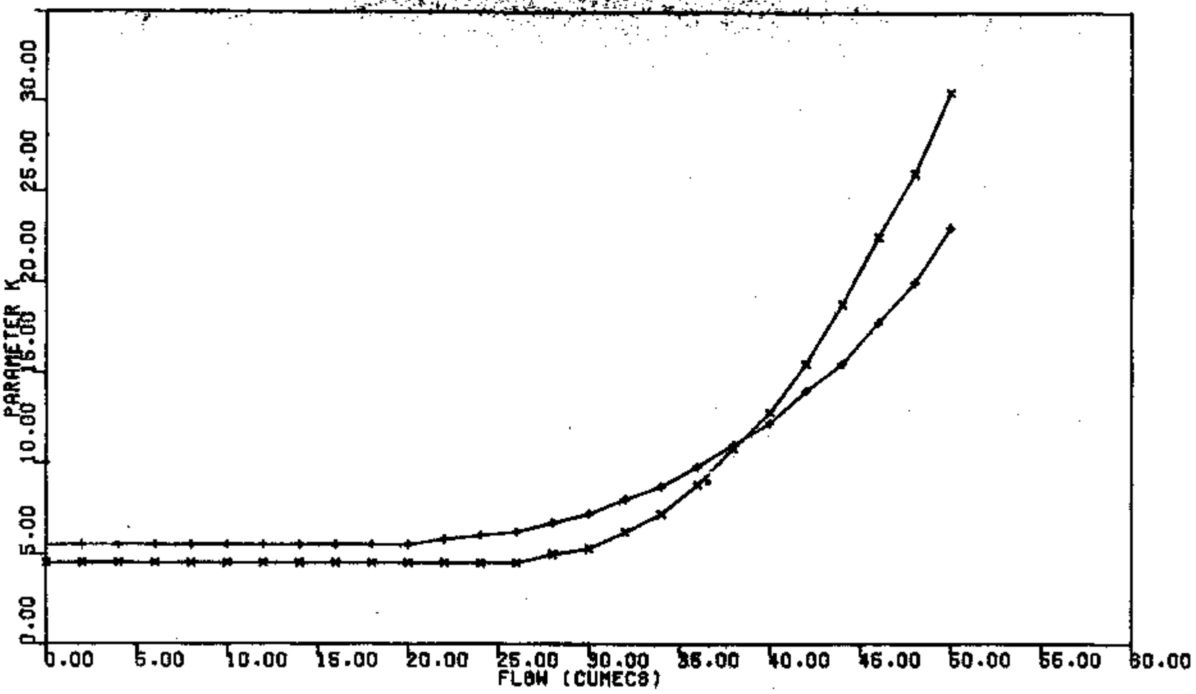


FIGURE 24 New Inn : final k_1 vs q relationship

Rising k_1 = x

Falling k_1 = +

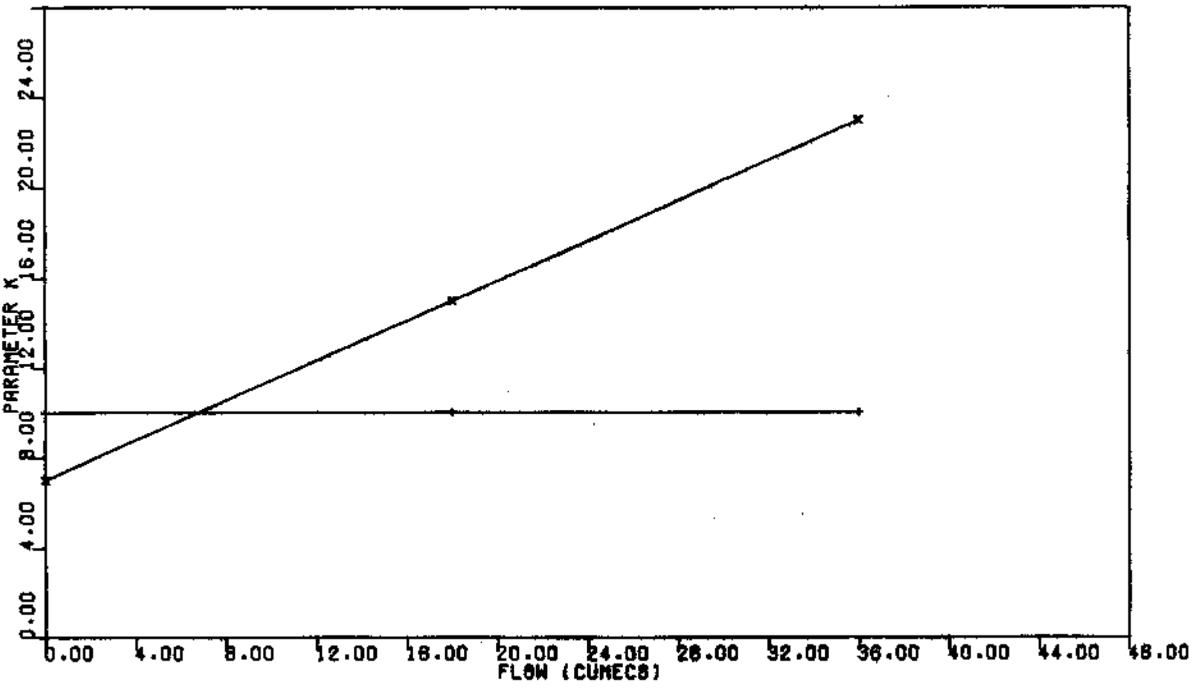


FIGURE 25 Hirnant : final k_1 vs q relationship

Rising k_1 = x

Falling k_1 = +

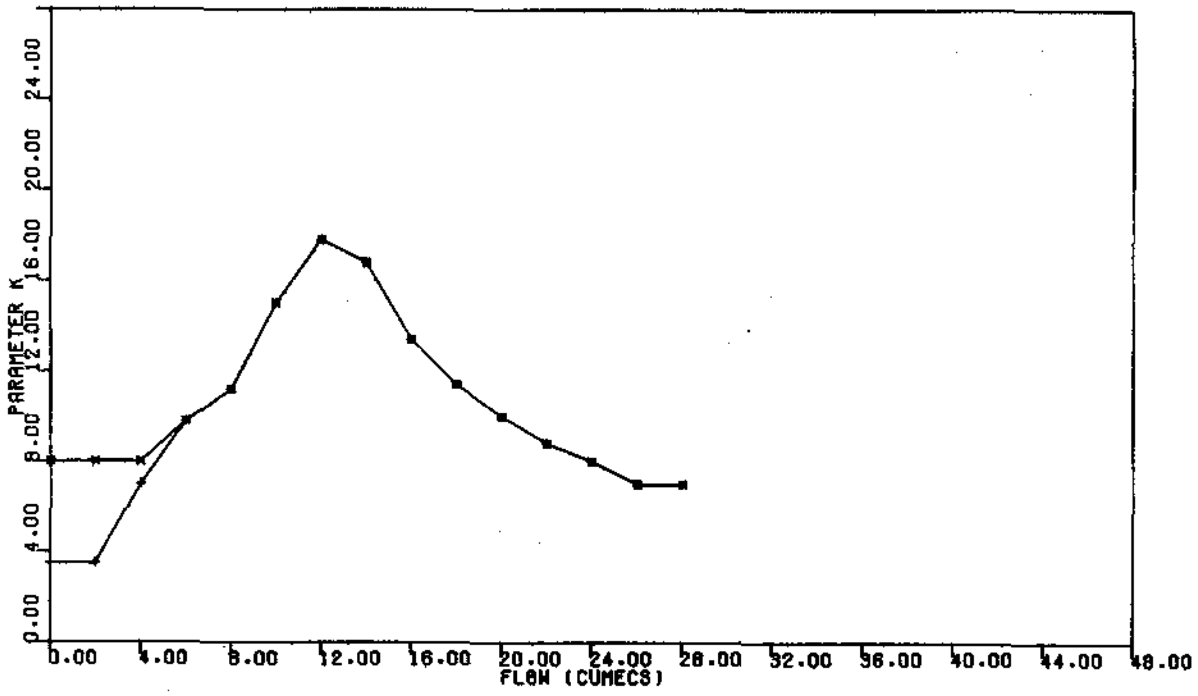


FIGURE 26 Ceiriog : final k_1 vs q relationship

Rising k_1 = x

Falling k_1 = +

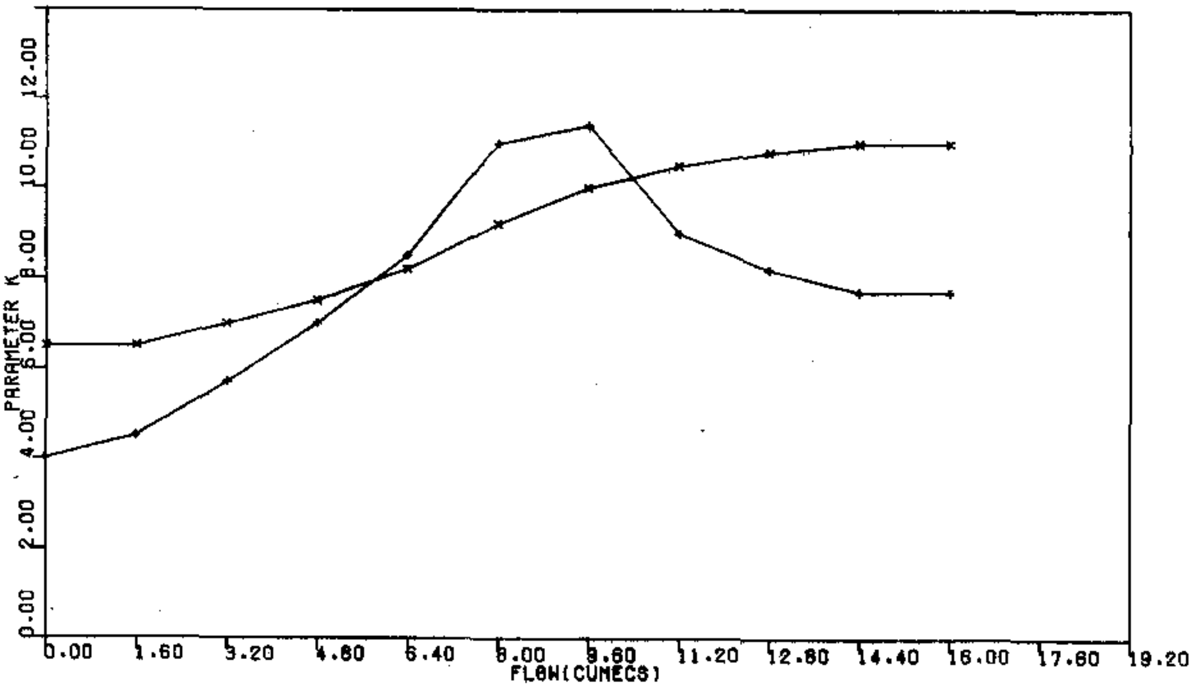


FIGURE 27 Gelyn : final k vs q relationship

Rising k_1 = x

Falling k_1 = +

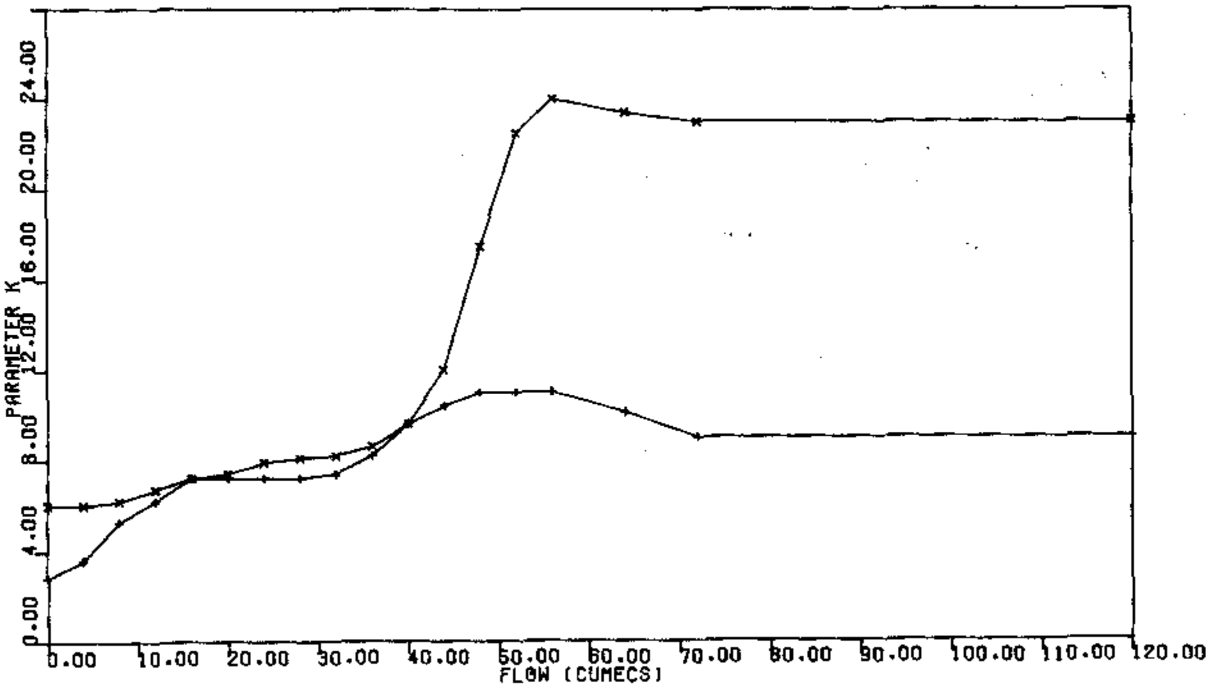


FIGURE 28 **Alwen : final k vs q relationship**

Rising k_1 = x

Falling k_1 = +

10. CONCLUSIONS

The results for the New Inn catchment (Section 8) and the other Dee subcatchments (Section 9) show that a significant improvement in hydrograph prediction can be achieved over the fixed parameter version of the Lambert model by using the Type I model with separate rising and falling limb parameters. A small improvement (in hydrograph shape) is obtained by using a smoothed rainfall input as described in Section 4. For the New Inn catchment the log-linear Type I model gave the best results and this is probably due to the relative confidence with which the smooth line of the k vs q relationship can be drawn through the derived points (Figures 16 and 17). It is much easier to fit the best straight line in the low flow region of the log-linear curve than to fit the best curved line in the same region in model Type II curves (Figures 18 and 19).

Although 11 months was the minimum period used to obtain the k vs q relationship, it is possible to get an approximate relationship from a much shorter period than this. Figure 10, for example, contains information from only one month's data (April 1973) during which there were three flood events. As a first approximation, it would be possible to use these results to draw in a best fit line and later improve on this as more data become available.

In practice, development of the k vs q relationship could be done in two ways:

(1) After every few months or after a significant flood event, manually repeat the analysis described in this report and then run the model from that point in time with the revised relationship.

(2) By having a fully automated procedure whereby the model starts off with an initial assumption of the k vs q relationship (a constant value, for example). Each time the new telemetered data are received, calculate the value of k which required to give a perfect prediction. This is then combined with the existing k vs q relationship to provide instant updating. The form of this relationship would obviously change very rapidly to begin with but after several months it should stabilize apart from some modification after a large flood event.

The rainfall data used in this report are of a very high standard, coming from a dense network of raingauges throughout the subcatchments. Where subcatchment rainfall is not so reliable (perhaps because it comes from just one raingauge) a greater random variation in the values of k generated might be expected. However, if values from the same raingauge(s) used in the derviation of the k vs q relationships are also used for flow prediction, an advantage becomes apparent. Assuming that the raingauge(s) are such that they consistently over (or under) estimate the average catchment rainfall, this bias will automatically be incorporated in the k vs q relationship for the rising limb (as higher or lower k values respectively). The subcatchment model can therefore be considered to be 'tuned' into one or a combination of raingauges for prediction in a particular subcatchment.

ACKNOWLEDGEMENTS

The work described in this report is part of the Institute's research programme into rainfall-runoff models for engineering application and is financed in part by the Ministry of Agriculture, Fisheries & Food. The author acknowledges with thanks the suggestions of A O Lambert (Welsh Water Authority) and J A Cole (Water Research Centre).

REFERENCES

HARVEY, R.A. & LOWING, M.J. 1976 The development and implementation of a real-time flow forecasting system for the River Dee. Applied Hydrology Division Report, Institute of Hydrology, Wallingford.

LOWING, M.J., PRICE, R.K. & HARVEY, R.A. 1975 Real-time conversion of rainfall to run-off for flow forecasting in the River Dee. Dee Weather Radar and Water Management Conference, Chester. Water Research Centre.

LAMBERT, A.O. 1969 A comprehensive rainfall/runoff model for an upland catchment area. *J. Inst. Wat. Engrs,* *23(4)*, 231.

LAMBERT, A.O. 1972 Catchment models based on I.S.O. functions. *J. Inst. Wat. Engrs,* *26(8)*, 413.

McKERCHAR, A.I. 1975 Subcatchment modelling for River Dee forecasting. *Inst. Hydrol. Rep. 29.*

ANON 1978 Dee weather radar and real-time hydrological forecasting project. Rept. by Steering Committee: Central Water Planning Unit, Meteorological Office, Plessey Radar Ltd., Water Data Unit, Water Research Centre.

APPENDIX

This Appendix contains the graphs of the results from the tests described in Section 8.2. The predicted flow in each case was obtained by using a Type I variable model.

EVENT	FIGURE
9th November 1972	A1
26th January 1973	A2
1st April 1973	A3
12th May 1973	A4
4th August 1973	A5
18th October 1973	A6
27th November 1973	A7
8th February 1974	A8

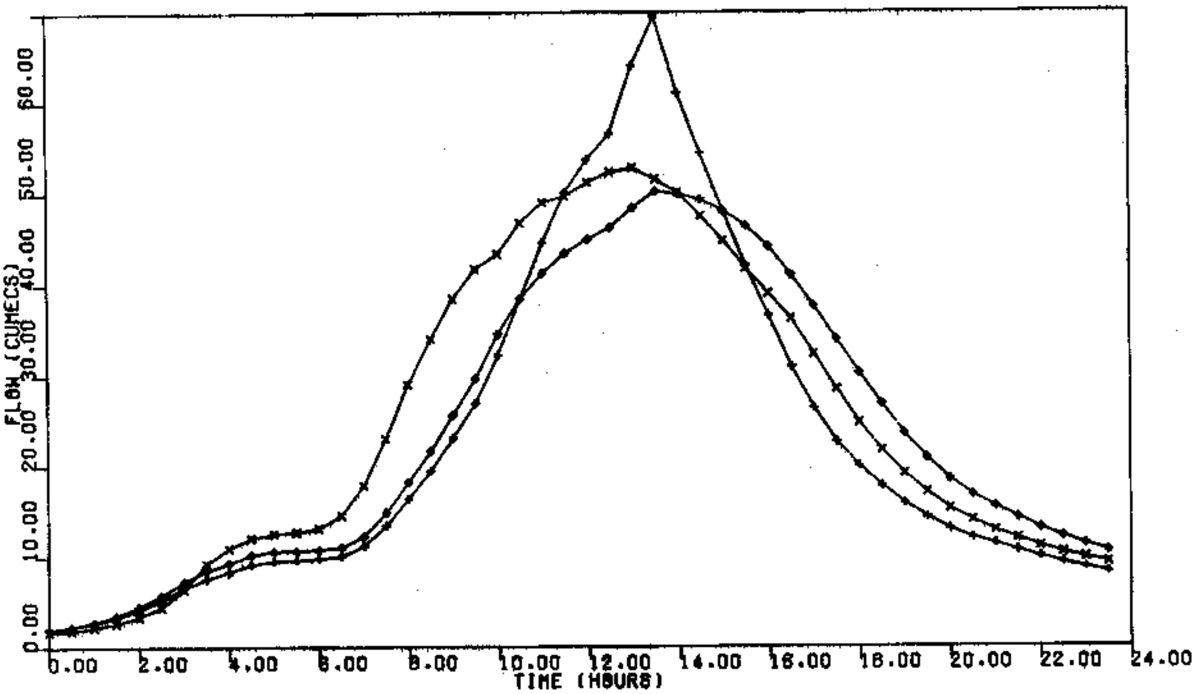


FIGURE A1 Dee at New Inn : 9th November 1972 - Type I

Observed = x k_1 = 4.9 + variable k_1 = ◆

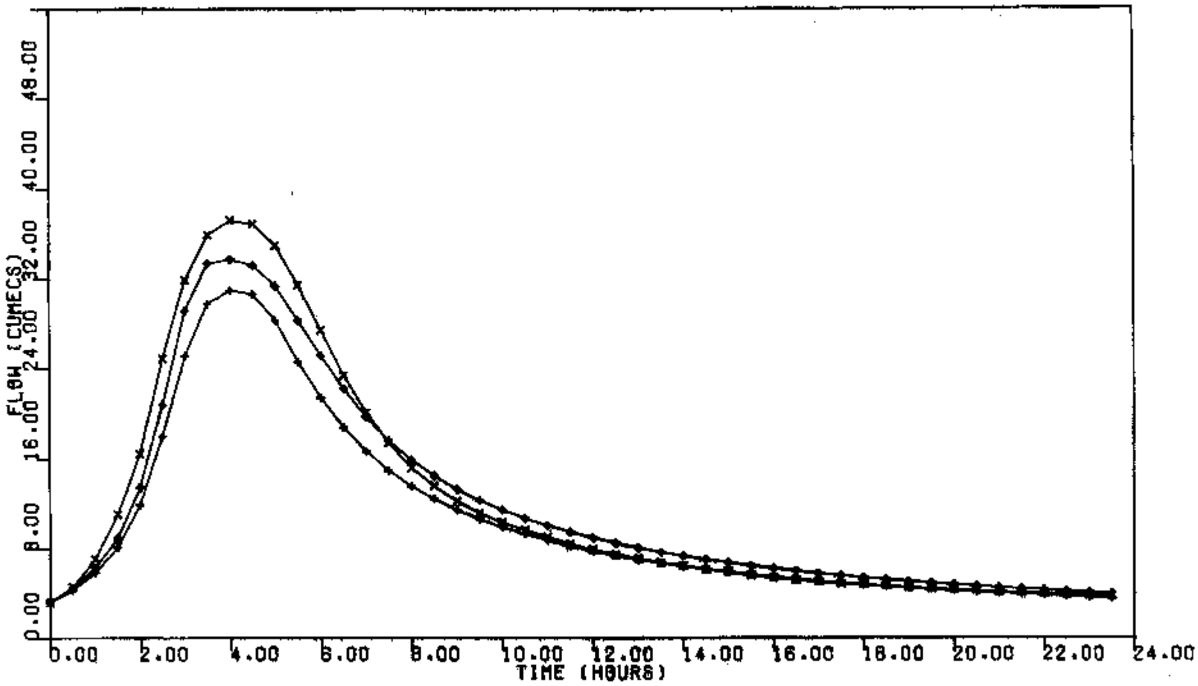
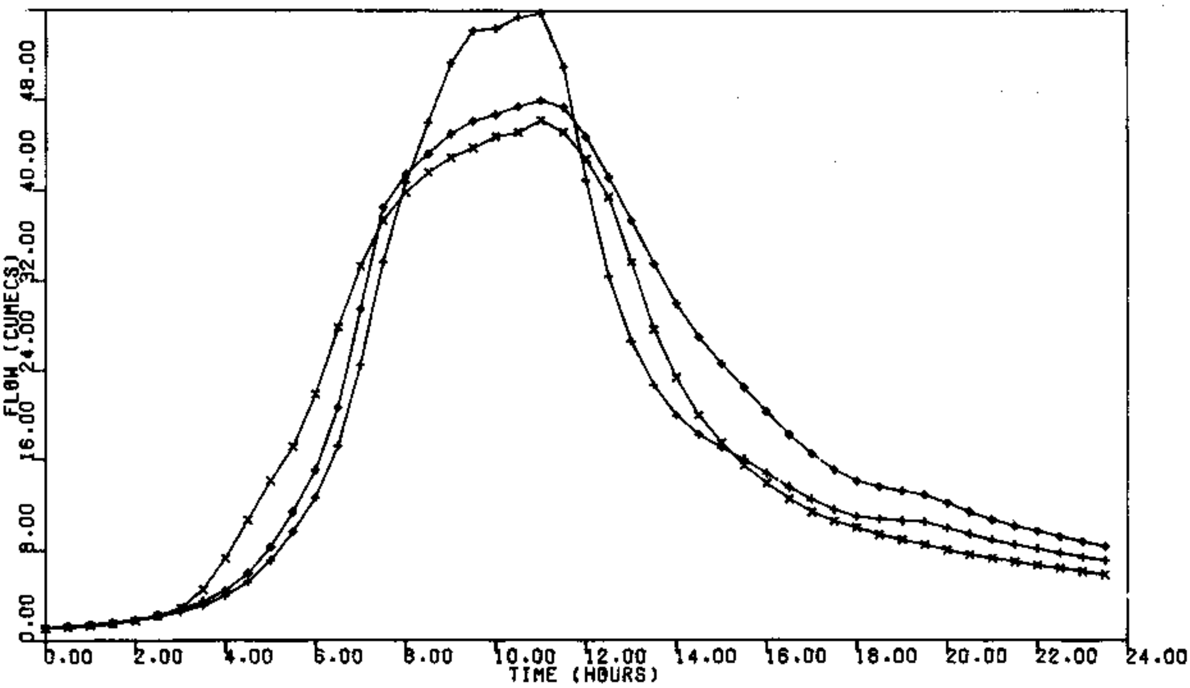


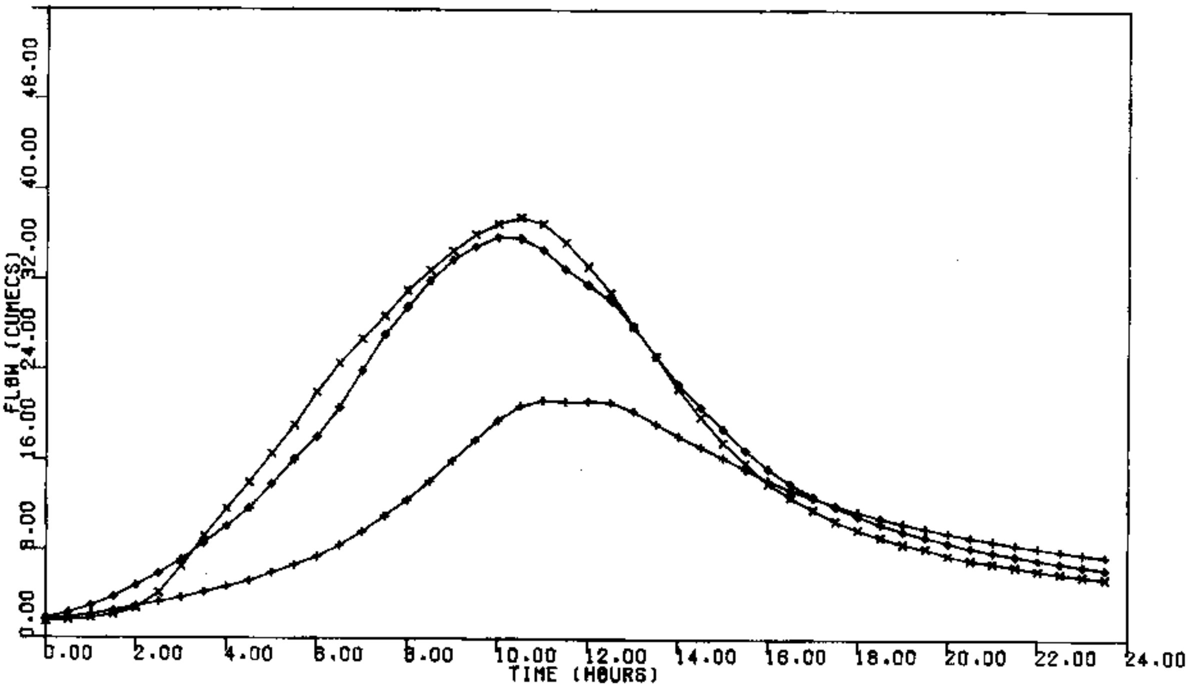
FIGURE A2 Dee at New Inn : 26th January 1973 - Type I

Observed = x k_1 = 4.9 + variable k_1 = ◆



<u>FIGURE A3</u> Dee at New Inn : 1st April 1973 - Type I

Observed = x k_1 = 4.9 + variable k_1 = ◆



<u>FIGURE A4</u> Dee at New Inn : 12th May 1973 - Type I

Observed = x k_1 = 7.9 + variable k_1 = ◆

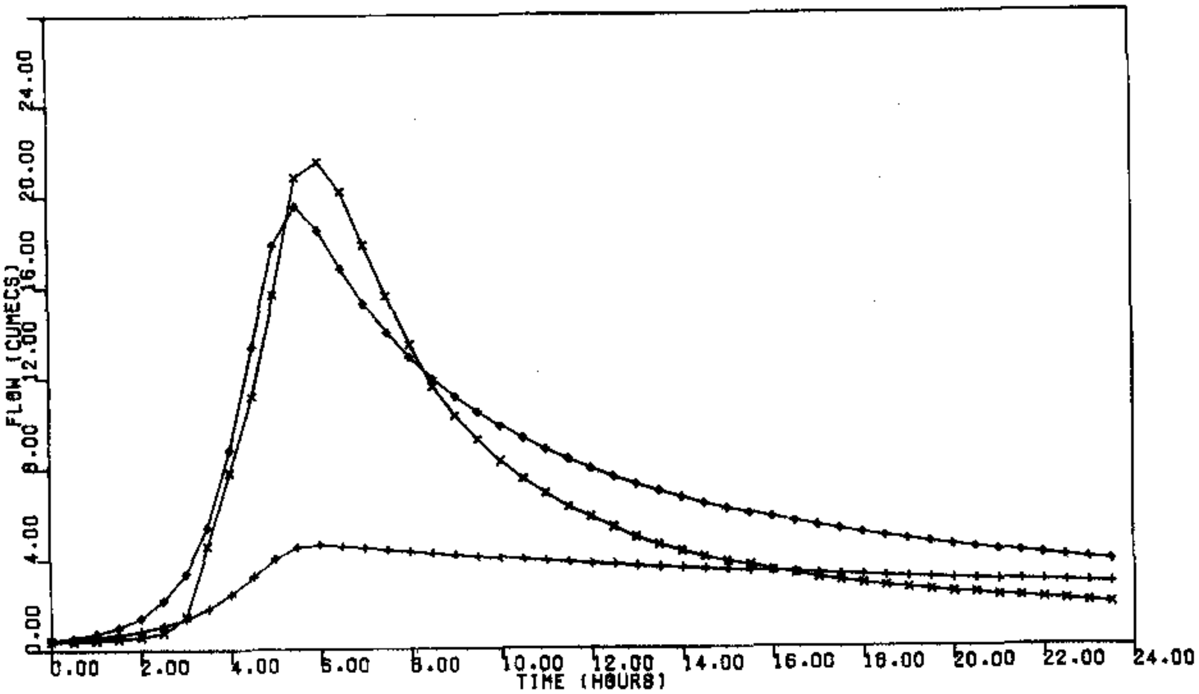


FIGURE A5 Dee at New Inn : 4th August 1973 - Type I

Observed = x k_1 = 7.9 + variable k_1 = ◆

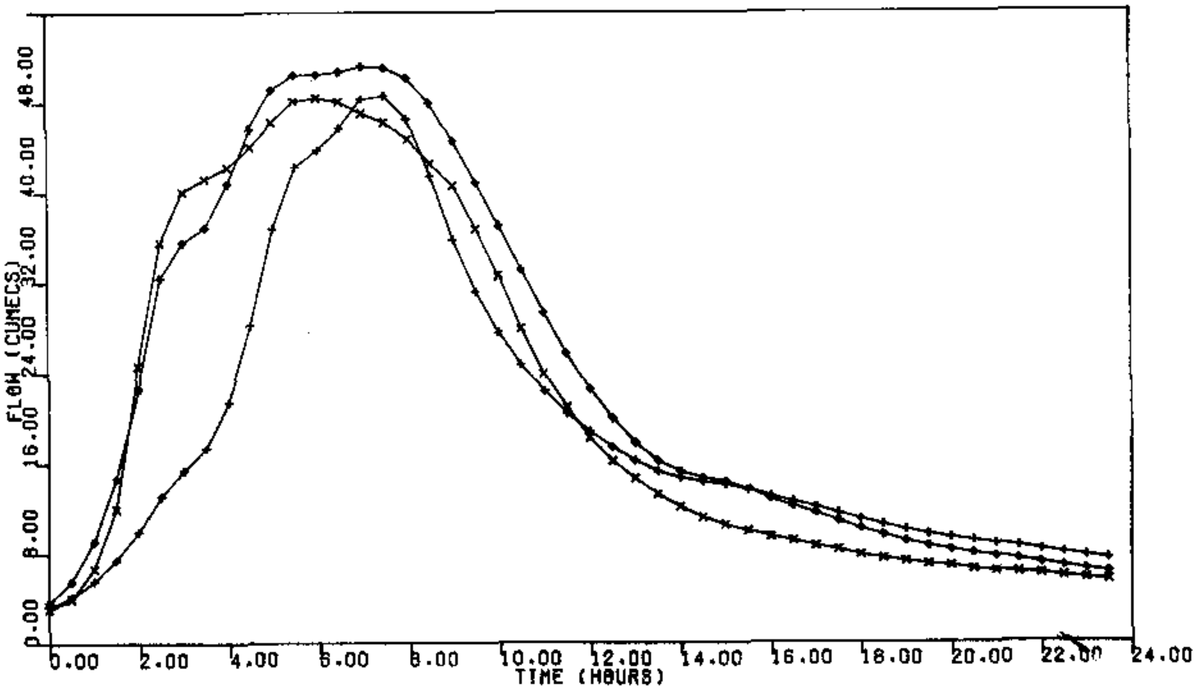


FIGURE A6 Dee at New Inn : 18th October 1973 - Type I

Observed = x k_1 = 7.9 + variable k_1 = ◆

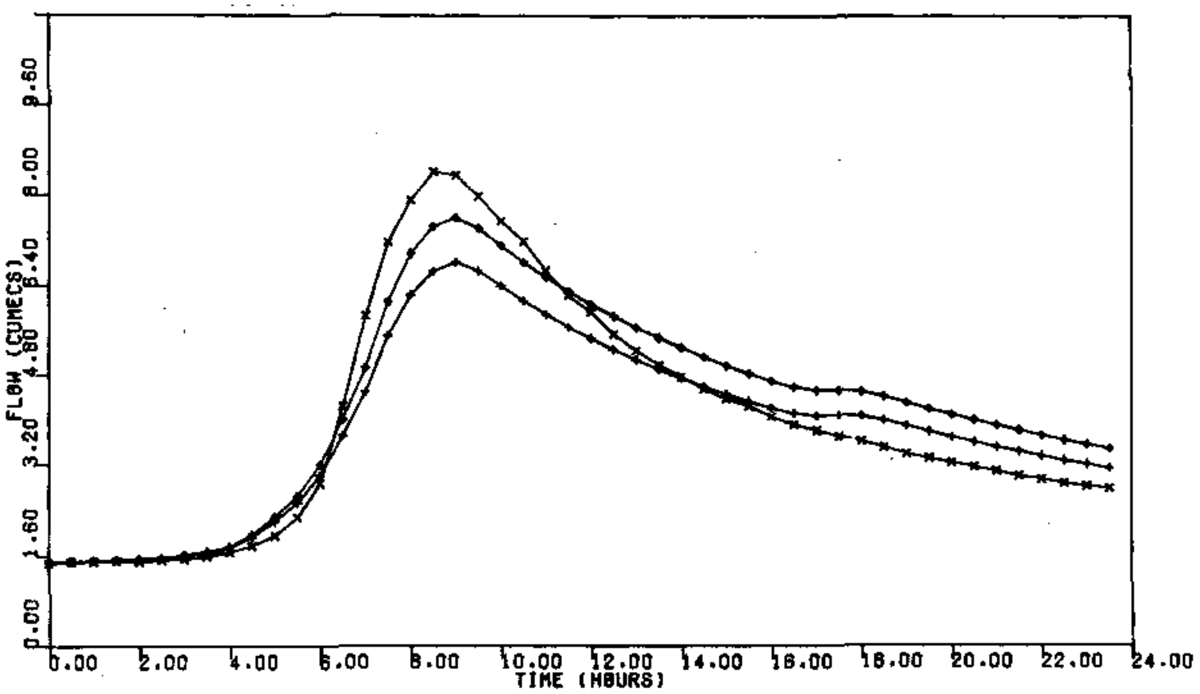


FIGURE A7 Dee at New Inn : 27th November 1972 - Type I

Observed = x k_1 = 4.9 + variable k_1 = ◆

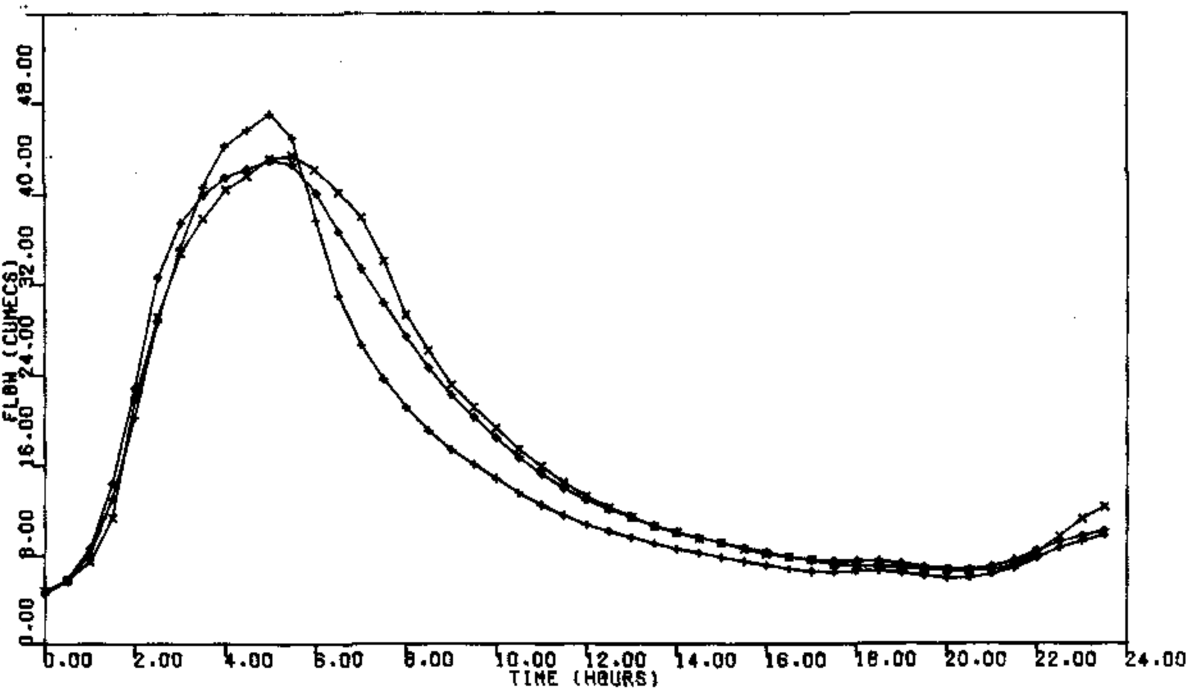


FIGURE A8 Dee at New Inn : 8th February 1974 - Type I

Observed = x k_1 = 4.9 + variable k_1 = ◆