



Research paper

Three-dimensional modelling of drag anchor penetration using the material point method

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ABSTRACT

Drag embedment anchors are a key threat to buried subsea linear infrastructure, such as power/data cables and pipelines. For cables, selecting a burial depth is a compromise between protecting the cable from anchor strike and the increased cost of deeper installation. This paper provides an efficient large deformation, elastoplastic Material Point Method-based soil–structure interaction predictive tool for the estimation of anchor penetration based on Cone Penetration Test (CPT) site investigation data. The tool builds on earlier work by the authors supplemented by three key developments: modelling assemblies of rigid bodies (necessary for articulated anchors), a partitioned domain approach to enable accurate and efficient modelling of long anchor pulls, and an improved means of modelling rotational inertia. The numerical model is calibrated using CPT data and then used to predict the penetration behaviour of two different drag anchors across a range of relative density sands under drained conditions with validation against scaled geotechnical centrifuge physical tests. Numerical simulations both confirm assumptions in, and identify key issues with, the UK Carbon Trust's Cable Burial Risk Assessment (CBRA) approach for estimating anchor penetration. In particular, the results confirm that anchor penetration scales linearly with fluke length but also that the penetration of drag anchors is highly dependent on both the relative density of the sand and the full geometry of the anchor. The numerical model presented in this paper enables site-specific anchor-penetration assessment along cable routes and can be used to evaluate the performance of different anchor designs and sizes in varied soil conditions.

1. Introduction

Subsea cables provide vital arteries for data and power transfer worldwide. In shallow waters these cables are buried below the seabed surface in order to protect them from accidental interaction with anchors/fishing gear that, based on 2024 data, cause around 70% of the reported cable failures (UltramapGlobal, 2024). For power cables the burial depth (or *depth of lowering*) along a cable route can be specified using the UK Carbon Trust's Cable Burial Risk Assessment (CBRA, Carbon Trust, 2015) framework. This approach relies on anchor-seabed penetration factors to estimate the penetration potential of an anchor as a multiplier of its fluke length. However, the CBRA framework provides factors for just two broad soil groups: a factor of 1 for sands/stiff clays and a factor of 3–5 for soft clays, and acknowledges that “*industry as a whole would benefit from further research into anchor penetration in a range*

of seabed types” (Carbon Trust, 2015). The primary motivation for the research presented in this paper is to provide more nuanced seabed penetration factors obtained from simulations of anchor penetration using a numerical model calibrated using routine site investigation data. To achieve this here we extend the large deformation Material Point Method (MPM) soil–structure interaction framework of Bird et al. (2025) to include new features and for the first time use an implicit MPM to study the penetration of real anchor geometries in different seabed conditions.

A variety of methods exist to determine the kinematics of drag anchors in soils, which can broadly be categorised as analytical, experimental, and numerical. One of the first articles to present analytical analysis for the drag anchor trajectory was Neubecker and Randolph (1996b,a), where an energy minimisation process was performed to

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determine the incremental direction of the anchor (incremental translation or rotation). Other notable articles include [Stewart \(1992\)](#) where an analytical force and moment equilibrium is used to determine the increment motion of the anchor. Additionally the hybrid numerical-analytical method of [O'Neill et al. \(2003\)](#) which uses a finite element analysis to first define the load yield locus of the admissible anchor loads in a elasto-plastic domain, through incremental motions of the fluke (horizontal, vertical and rotational). The incremental motion of the anchor is then obtained through force, and moment, equilibrium, of the anchor with the chain tension, which locates the current load point on the yield locus and thus the corresponding incremental motion. Other, more recent analytical methods, include [Liu et al. \(2012\)](#) who derive a closed form kinematic expression for the anchor where the depth at time t is defined by the reverse catenary of the chain and [Ren et al. \(2022\)](#) where a force-moment equilibrium, using similar methods to [Neubecker and Randolph \(1996b\)](#), is performed with a forward-Euler time integration to determine the anchor motion. Analytical models have the benefit that they are very fast compared to numerical methods however, as discussed in [Pretti et al. \(2022\)](#), [Pretti \(2024\)](#), whilst the simplicity of analytical methods makes them attractive, if parameters for the particular anchor and soil conditions are unknown then these formulations cannot accurately, or reliably, make predictions for this non-linear (materially and geometrically) dynamic problem involving multiple bodies.

Numerical methods allow for flexibility in anchor geometry, material behaviour and anchor deployment conditions. Possibly the largest numerical challenge in modelling drag anchors is the very large deformation of the soil, the seemingly most common approach is to use the explicit-in-time Coupled-Eulerian-Lagrangian (CEL) formulation in Abaqus FEA ([Dassault Systèmes Simulia Corporation, 2011](#)), for example see the non-exhaustive list ([Liu and Zhao, 2014](#); [Grabe et al., 2015](#); [Dao and Grabe, 2024](#); [Dou and Yu, 2018](#); [Zhao and Liu, 2016](#); [Zhao et al., 2016](#); [Zhang et al., 2023](#)). The studies cover a wide range of topics, such as investigating the effect of installation velocities on the general kinematics of the anchor for varying soil conditions, [Liu and Zhao \(2014\)](#), or, how an anchor's geometry effects its trajectory ([Dou and Yu, 2018](#)) and also, more recently, real offshore site prediction for the performance of a range of drag anchors in various sands, ([Dao and Grabe, 2024](#)). Two key issues with numerical modelling of anchor penetration are: (i) simulations are expensive in terms of run times and energy use (anchor modelling is inherently 3D and highly non-linear); and (ii) confidence in the results can only be achieved through validation against physical testing. However, published examples of where numerical models are validated against independent results/physical testing is limited. For example, some papers offer no independent validation ([Liu and Zhao, 2014](#)), only validate against other numerical simulations ([Dou and Yu, 2018](#)) or, if validated against physical testing, the validation is limited to a single anchor type and a single seabed state ([Dao and Grabe, 2024](#)). This does not appear to be due to the lack of physical testing data: [O'Neill and Randolph \(O'Neill and Randolph, 2001\)](#) model anchor penetration in a drum centrifuge, [Moore and co-workers \(Moore, 2017; Moore et al., 2021\)](#) use a drum centrifuge for determining anchor penetration characteristics, [Liu et al. \(2010\)](#) experiment on drag anchor designs with various geometries in a 1g setup, [Yan et al. \(2015\)](#) explore the interaction of anchors with rock berms, and [Sharif et al. \(2025\)](#) provide centrifuge, and 1g, tests for anchor behaviour over a range of sand relative densities (very loose to very dense).

In this paper we adopt the numerical modelling framework detailed in [Bird et al. \(2024, 2025\)](#) that allows a single rigid body to interact with a highly deformable material (e.g. the seabed). The latter is represented using the Material Point Method (MPM), which can cope with large deformations in solids much better than the standard Finite Element Method (see [Solowski et al. \(2021\)](#) and [de Vaucorbeil et al. \(2020\)](#) for review articles). Key advances described in [Bird et al. \(2024, 2025\)](#) necessary for the model to function include a new and efficient

contact algorithm which provides a general and consistent representation of the extent of the deformable body without introducing boundary representation in the MPM. However, the approach of [Bird et al. \(2025\)](#) is not immediately applicable to modelling the interaction of drag anchors with the seabed and this paper offers three key numerical advancements that enable the efficient solution of drag anchor penetration problems. Firstly, the ability to model multiple rigid bodies within a single simulation becomes essential when aiming to accurately represent offshore geotechnical assemblies that incorporate hinges or joints. Examples include the skid plates of seabed cable and pipeline ploughs, the interconnected tracks of subsea Remotely Operated Vehicles (ROVs), and the fluke-shank configuration of drag embedment anchors. To address this need, here we extend the approach of [Bird et al. \(2025\)](#) to support assemblies of multiple rigid bodies, enabling the simulation of realistic articulating anchors, as detailed in Section 2.2. The second methodological contribution, also detailed in Section 2.2, is improving the rotational inertia representation of the rigid bodies. A third key development is the introduction of a computationally efficient numerical scheme for simulating realistic anchor pulls. For such simulations to be practically useful, they must produce data over pull lengths many times greater than the anchor's own dimensions, as drag anchors often require substantial horizontal displacement to embed fully, depending on both anchor geometry and soil conditions. Our solution is to partition the deformable problem domain such that the deformation of the soil is solved in a local region linked to the anchor's current position. This approach, which is described in Section 2.3, produces a numerical method where the solution time of each time step is reduced compared to solving the full domain, and independent of the total drag length. It should also be highlighted that, as far as the authors are aware, this is the first time that the MPM has been applied to the modelling of drag embedment anchors and one of a small number of papers¹ that have considered realistic anchor geometries and fluke-shank articulation.

Having described the fundamental components of the MPM approach and the three developments noted above, Section 3 outlines the overall solution procedure, the constitutive model and determines the appropriate size of the partitioned domain in different sand relative densities. Section 4 demonstrates the performance of the MPM predictive tool, starting with calibration of the soil constitutive model using Cone Penetration Tests (CPTs) conducted in a geotechnical centrifuge. CPTs are used for calibration as they are the main site investigation technique along a subsea cable route. The calibrated sand material parameters are then used as inputs to predict the penetration of two different drag anchors in four different homogeneous sand conditions (very loose to very dense) under fully drained conditions. The numerical predictions are validated against penetration depths obtained from scaled physical tests conducted in a geotechnical centrifuge at the University of Dundee ([Sharif et al., 2025](#)), going well beyond the validation presented in any other drag anchor numerical modelling paper in the currently published literature. Once validated, the MPM-based anchor penetration tool is used to investigate the impact of anchor mass on the penetration depth and capacity in different density sands. Section 5 of the paper provides a discussion on the implications of these results for the CBRA framework and the performance of drag anchors more generally before overall conclusions are made in Section 6.

2. Model components

While the developments of the MPM to be described below could be used to model a wide range of complex soil-structure interaction problems, here we will focus on the anchor pull problem. The numerical model comprises (1) a model of the seabed soil and (2) a model of

¹ One other example is the Coupled-Eulerian-Lagrangian study of [Dao and Grabe \(2024\)](#).

the anchor. The former uses the MPM and the approach is based on the three-dimensional implicit dynamic MPM formulation in Bird et al. (2025) details of which are only summarised here. This is a dynamic extension of the quasi-static open-source code AMPLE (A Material Point Learning Environment) code (Coombs and Augarde, 2020) that has its origins in the work of Charlton et al. (2017) and Coombs et al. (2020). The particular variation of the MPM used here is the General Interpolation Material Point Method (GIMPM) due to its ability to reduce cell crossing instabilities under large deformations. As discussed in Bird et al. (2024, 2025) the General Interpolation Material Points (GIMPs) have an associated cuboid domain, the vertices of which are used for contact detection with rigid body surfaces and to apply contact boundary conditions, which are enforced with a penalty method (Wriggers, 2006). This enables an explicit contact formulation based on the GIMP domain and rigid body geometries and resolves the usual lack of a clear definition of the boundary associated with the MPM (Gonzalez Acosta et al., 2021; Pretti, 2024).

The continuum formulation adopted for the soil in the paper is also the same as in Bird et al. (2025). Full details of the large deformation elasto-plastic continuum mechanics approach can be found in Charlton et al. (2017) and Coombs and Augarde (2020). The approach is based on a Hencky material assumption that adopts a linear relationship between logarithmic (or Hencky) strains and the Kirchhoff stress. This combination allows the format of infinitesimal strain constitutive models to be recovered and used within a large deformation (i.e. finite strain) boundary value solution without modification of the stress update algorithm (see de Souza Neto et al. (2008) for details). This approach is widely used across finite element and MPMs, see for example (Charlton et al., 2017; Coombs and Augarde, 2020; Coombs et al., 2025; Coombs, 2023; Bird et al., 2025, 2024; Coombs et al., 2020; Pretti et al., 2024). When the GIMPM and rigid body are in dynamic equilibrium, the weak statement of equilibrium is discretised in time using the Newmark method with the parameters $\gamma = 1$ and $\beta = 1/2$. A FLIP node-to-point mapping is performed (Brackbill and Ruppel, 1986) at the end of the time step to update the material point velocities. Note that due to the adopted values of γ and β , the accelerations of the vertices of the background grid, $\dot{v}_v$, from the previous time step are not required in the next time step. This means that it is not necessary to reconstruct the vertex accelerations in the next time step from material point information at the end of the previous time step or update the material point accelerations at the end of each time step.

The non mesh-matching nature of the MPM means that it can suffer from the *small cut* issue and associated loss of coercivity of the linear system of equations. In the MPM the problem occurs when nodes of the background mesh have very small mass or stiffness contributions but significant residual force values, which results in very large non-physical accelerations or displacements at the boundary of the physical body. This issue was recognised by Sulsky et al. (1995) and motivated the widely used Modified Update Stress Last (MUSL) explicit algorithm. However, there are currently only two general approaches for resolving this issue for implicit MPMs: ghost stabilisation (Burman, 2010; Coombs, 2023) and mesh aggregation (Coombs et al., 2025). Consistent with Bird et al. (2025), in this paper ghost stabilisation is adopted, which adds a penalty term to the mass and/or stiffness matrix that constrains the gradient of the solution across faces of the background mesh near the boundary of the deformable body.

2.1. Numerical framework for soil–structure interaction

For completeness the weak forms for the GIMPM and rigid body are presented here, it is a summary of the formulation from Bird et al. (2024) where further detail on the derivation, implementation and validation can be found, particularly for the contact. The discrete weak form of the deformable body in an updated Lagrangian frame is,

$$\mathbf{A}_{vp} \left([\nabla_x S_{vp}]^T \{\sigma_p\} V_p - [S_{vp}]^T \{b\} V_p + [S_{vp}]^T \{\dot{v}_p\} m_p \right)$$

$$- \mathbf{A}_{p \in P_c} \left(\{\delta f_{N,vp}^o\} + \{\delta f_{T,vp}^o\} \right) = \{0\}, \quad (1)$$

where p in general denotes quantities associated with a material point, V_p and m_p are respectively the material point volume and mass. $\{\cdot\}$ denotes a vector quantity with $\{\sigma_p\}$ as Cauchy stress and $\{b\}$ the body force. The GIMPM basis functions are expressed as $[S_{vp}]$, where $[\cdot]$ is a matrix, they depend on the convolution of the background mesh basis functions and the characteristic function associated with the material point, Bardenhagen and Kober (2004). The set of all material points in contact is defined P_c , where the normal and tangential contact forces acting on the material points are respectively, $\{\delta f_{N,vp}^o\}$ and $\{\delta f_{T,vp}^o\}$.

The engineering machinery used in subsea process often have multiple separate components which interact with the seabed. For the purposes of modelling, each component is represented by a separate rigid body, with multiple bodies connected together with a finite element truss frame. The truss nodes model the kinematics of the rigid bodies with a component's centre of mass set at a truss node. Additionally the machinery is sufficiently large that rotational inertia influences the kinematics. Therefore rotational inertia terms are included to consider the resistance to angular acceleration about a rigid body's centre of mass. The surface of the rigid body is constructed from a continuous set of triangles $\mathcal{T}$, where $\mathcal{T}_e \subseteq \mathcal{T}$ is the set of triangles in contact. As the kinematics are model by truss elements, the contact forces on the surface of the rigid body must be mapped from the contact triangles surface of the rigid body, to the nodes of the truss element e , this is achieved with the mapping $\Xi^e[\cdot]$, see Bird et al. (2025). The discrete weak form for the truss frame is therefore

$$\begin{aligned} & \mathbf{A}_{e \in E} \int_{\mathcal{O}_t(e)} ([\zeta]^T \{f^I\} + \rho^e [\zeta]^T \{\dot{v}^e\} + [\zeta]^T [\partial \theta / \partial x_e]^T [I(x_e)] \{\ddot{\theta}(x_e)\}) dV = \\ & \mathbf{A}_{e \in E} \int_{\mathcal{O}_t(e)} [\zeta]^T \{b\} dV + \mathbf{A}_{e \in E} \Xi^e \left[\int_{\mathcal{O}_t(\mathcal{T}_e)} (\{\delta f_N^o\} + \{\delta f_T^o\}) dV \right] \end{aligned} \quad (2)$$

$\mathcal{O}_t(\cdot)$ is the motion of the rigid body, a truss element is defined e and belongs to the set of all elements E . Additionally, $[\zeta]$ is the matrix of shape functions for the truss element, $\{f^I\}$ is the internal force vector, ρ^e is the material density in e and $\{\dot{v}^e\}$ is acceleration. The third term in the left hand side of Eq. (2) corresponds to the rotational inertia of the truss element in three dimensions, where the angle θ , and its derivatives with respect to time, are a function of the truss element's nodal positions in space x_e . Additionally the rotational inertia matrix $[I(x_e)]$ will evolve as the truss element rotates. The description of the application, first variations and linearisation of this term about one axis, as only one axis is necessary for modelling the anchor for CBRAS, are presented in Section 2.2.

The deformable and rigid body systems, respectively Eqs. (1) and (2) are coupled together through the contact terms. Each time step is solved implicitly using the Newton–Raphson (NR) method combined with a basic line search which chooses the lower residual corresponding to the line search length $\alpha = \{0.1, 1.0\}$ Dennis and Schnabel (1996). It is was found that a line search was required to prevent contact points repeatedly coming in and out of contact during a load step (Dennis and Schnabel, 1996).

The linear system for each NR iteration takes the form

$$\underbrace{\begin{Bmatrix} \{\Delta u_v\} \\ \{\Delta u_b\} \end{Bmatrix}}_{\{K\}^{-1}} = \underbrace{\begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix}}_{\{K\}^{-1}} \underbrace{\begin{Bmatrix} \{f_v\} \\ \{f_b\} \end{Bmatrix}}_{\{f\}} \quad (3)$$

Δu is the incremental solution update and f is the out-of-balance forces of the system. $[A]$ and $[B]$ are matrices from the linearisation of Eq. (1), whilst $[C]$ and $[D]$ are from the linearisation of Eq. (2). The subscript v is used for terms in Eq. (3) to describe the nodes (vertices) of the background grid on which the GIMPM is solved at each time step, the

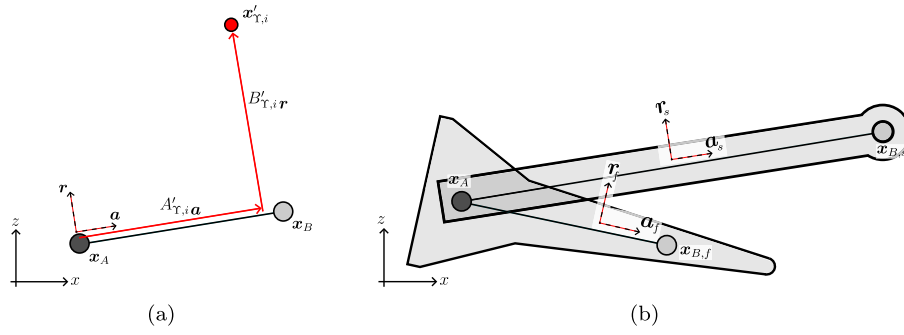


Fig. 1. Rigid body: (a) is a single line element with its normal $\mathbf{a}$ and tangent $\mathbf{r}$ being used to define the point $\mathbf{x}'_{Y,i}$. (b) shows how two line elements are used to define the coordinates of the fluke and shank of an anchor.

subscript b is used to describe the DoF of the truss element frame associated with the rigid bodies. The Dirichlet (i.e. displacement) boundary conditions are background mesh conforming for all of the problems considered in this paper. Therefore any Dirichlet boundary conditions can be applied directly to the vertices of the background mesh, v , using the same technique to strongly enforce Dirichlet constraints as in the finite element method (see Coombs and Augarde (2020) for a MPM example of enforcing displacement boundary conditions in this way). Any rigid body constraints are also imposed directly, where the degree of freedom is set to a prescribed value and eliminated from the linear system. There are no inhomogeneous Neumann boundary conditions (i.e. non-zero surface tractions) in any of the problems solved in this paper.

2.2. Modelling multiple rigid bodies

Having given details of the general components of the numerical model, here we describe in more detail the new features added to enable the anchor pull problem to be efficiently modelled. The contact formulation used here matches that in the previous model (Bird et al., 2025) where the surface of a rigid body is formed from a set of flat triangles $Y \in \mathcal{T}$, which provide the surface component of the *point-to-surface* contact. The corners of all the GIMP domains form the deformable domain's contact surface $p \in P_c$, and provide the point component of the *point-to-surface* contact. A gap function, g_N , obtained from a Closest Point Projection (CPP) algorithm is used to determine whether contact occurs and a penalty law minimises the normal interpenetration. The CPP projects the corner of the GIMP $\mathbf{x}$ onto a point $\mathbf{x}'$ on one of the triangles representing the rigid body. The set of all triangles in contact is defined $Y \in \mathcal{T}_c$.

By definition the rigid bodies cannot deform, they can only rotate and translate as a function of the rigid body's kinematics. Therefore a definition of the rigid bodies' kinematics is required which also defines the positions of the surfaces, necessary for contact. Each triangle has its own local coordinate system ξ^α , where $\alpha \in [1, 2]$, and the position of points on the triangle can be expressed using the usual linear triangle basis functions, $N_i(\xi^\alpha)$, that is

$$\mathbf{x}' = \sum_{i=1}^3 \sum_{\alpha=1}^2 N_i(\xi^\alpha) \mathbf{x}'_{Y,i}, \quad (4)$$

where bold notation is used to define a vector, i refers to the basis number and $\mathbf{x}'_{Y,i} \in \mathbb{R}^3$ are the positions of the triangle's vertices in three dimensional space.

A truss frame finite element method is used to join together the multiple rigid bodies, all of which are in contact with the deformable body. This method was validated for a single rigid body in Bird et al. (2025) where the position of its surface is a function of the nodal positions of a single truss element and is independent of the element's

deformation.² An example is shown in Fig. 1(a) with a truss element which has nodes A and B at positions $\mathbf{x}_A$ and $\mathbf{x}_B$ respectively. From the node positions of the truss the axial and radial directions are also respectively defined,

$$\mathbf{a} = (\mathbf{x}_B - \mathbf{x}_A) / |\mathbf{x}_B - \mathbf{x}_A| \quad \text{and} \quad \mathbf{r} = \mathbf{R} \cdot \mathbf{a}. \quad (5)$$

where $\mathbf{R}$ is a rotation matrix of 90° about the y -axis. To determine, $\mathbf{x}'_{Y,i}$, the position of vertex i of triangle Y , necessary for Eq. (4), the following expression is used

$$\mathbf{x}'_{Y,i} = \mathbf{x}'_{Y,i}(\mathbf{x}_A, \mathbf{x}_B) = A'_{Y,i} \mathbf{a} + B'_{Y,i} \mathbf{r} + \mathbf{x}_A, \quad (6)$$

where $A'_{Y,i}$ and $B'_{Y,i}$ are constants that respectively define how far in the axial and radial direction the point $\mathbf{x}'_{Y,i}$ is from $\mathbf{x}_A$. The positions $\mathbf{x}_A$ and $\mathbf{x}_B$ are the primary variables of the rigid body which are solved for, they fully define the kinematics of the rigid body with the vertex $\mathbf{x}'_{Y,i}$ a function of their position.

For the multiple body case, each rigid body is defined with a truss element, which are connected together permitting rotation of the individual bodies with respect to each other. No interpenetration or contact is permitted hence the surfaces from different rigid bodies are non-intersecting, meaning their interiors are disjoint. As an example, an anchor comprising two components, the fluke f and the shank s , is shown in Fig. 1(b). Each are modelled as rigid bodies with the surfaces constructed from triangles, $Y \in \mathcal{T}_s$ and $Y \in \mathcal{T}_f$, where the set of all triangles is defined as $\mathcal{T} = \mathcal{T}_s \cup \mathcal{T}_f$. The trusses in Fig. 1(b) share the node A and have their own axial and radial directions which are used to define the position of their respective surfaces. The primary unknowns for this system are the positions $\mathbf{x}_A$, $\mathbf{x}_{B,s}$ and $\mathbf{x}_{B,f}$.

Appropriate representation of a structure's rotational inertia is key in predicting its kinematic behaviour if there is any unconstrained rotation. This is particularly the case for anchors which can undergo large rotations, at various rates, and often can be several metres long and tens of tonnes.³ Within the framework of the truss frame's translation Degrees of Freedom (DoF) there are two options for modelling the inertia: Option 1 is to place a series of points around the Centre of Mass (CoM) at a correct radius with a distributed mass to represent the rotational inertia, as in Bird et al. (2025); Option 2 is to define a rotation, and acceleration, as a function of the translational DoF. Option 2 is used here as it is exact but requires more work to determine its linearisation whereas, Option 1 is inexact and could have stability issues if a large number of stiff truss members are used.

Although the approach adopted in this paper is general and can be applied to any rigid body undergoing soil-structure interaction, the subsequent explanation focuses on the specific case of an articulated

² The stiffness of the element is sufficiently high that only arbitrarily small deformations occur.

³ For an example on the range of commercial anchor size see the Sotra product page (Chain & Anchor Sotra, 2025).

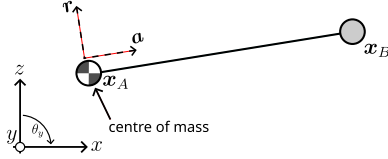


Fig. 2. Rotational inertia: An example of a truss element with angular acceleration defined at the centre of mass x_A .

drag anchor that can rotate about a single axis. Considering Fig. 2, to include rotational inertia a description of the angular acceleration around the centre of mass is required, this can be achieved using the end points of the line element, x_A and x_B . The points x_A and x_B define the axial direction of the element. The rotational inertia term which contributes to the rigid body's weak form, and respectively its linearisation, takes the form

$$\delta f^\theta = I_{yy} \delta \theta \ddot{\theta} \quad \text{and} \quad \Delta \delta f^\theta = I_{yy} \Delta \delta \theta \ddot{\theta} + I_{yy} \delta \theta \Delta \ddot{\theta}$$

Using the notation $(\cdot)^{AB} = (\cdot)_B - (\cdot)_A$ the rotation, angular velocity and acceleration can be defined

$$\theta = \tan^{-1} \left(\frac{x_3^{AB}}{x_1^{AB}} \right), \quad \dot{\theta} = \frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial x^{AB}} \cdot \frac{\partial x^{AB}}{\partial t} \quad (7)$$

and

$$\ddot{\theta} = \frac{\partial}{\partial t} \left(\frac{\partial \theta}{\partial x^{AB}} \cdot \frac{\partial x^{AB}}{\partial t} \right) = \frac{\partial x^{AB}}{\partial t} \cdot \frac{\partial^2 \theta}{\partial x^{AB^2}} \cdot \frac{\partial x^{AB}}{\partial t} + \frac{\partial \theta}{\partial x^{AB}} \cdot \frac{\partial^2 x^{AB}}{\partial t^2} \quad (8)$$

$$= v^{AB} \cdot \frac{\partial^2 \theta}{\partial x^{AB^2}} \cdot v^{AB} + \frac{\partial \theta}{\partial x^{AB}} \cdot \dot{v}^{AB} \quad (9)$$

with the variation, $\delta \theta$, and linearisations $\Delta \ddot{\theta}$ and $\Delta \delta \theta$ provided in Appendix.

In practice, it is often also necessary to constrain the relative rotation between the rigid bodies in an assembly. In the case of a drag anchor with a pivoting shank and fluke, the opening angle of the anchor is constrained by the physical geometry of the connection. In this paper a penalty approach is used to introduce these types of constraint. An example is shown in Figs. 3(a) and 3(b), where the angle θ is limited by including a penalty spring between nodes A and B. To determine when the penalty stiffness should be active the distance between nodes is calculated for the maximum allowable value of θ , this distance is defined L_θ . L_θ is also defined as the undeformed length of the penalty spring. During a simulation if the distance between nodes A and B is less than L_θ there is no additional penalty force between nodes A and B (the case shown in Fig. 3(a)). When the distance is greater than L_θ and additional penalty force is introduced between nodes A and B to limit the angle θ (the case shown in Fig. 3(b)).

2.3. A partitioned domain approach

The previous sections have described the first two developments in the new model and in this section the third is detailed. With respect to drag anchor pulls, it is never known *a priori* how far an anchor will need to be dragged in order to achieve its ultimate penetration depth. This can lead to two potential problems in numerical modelling: (i) the length of the domain may be insufficient for the anchor to achieve its ultimate depth, resulting in an analysis rerun with a longer domain; or (ii) the length of the domain may be longer than required to obtain the ultimate depth, resulting in wasted computational effort. To mitigate these issues a new *partitioned domain* approach is introduced here. The aim is to make the solve time per load step, for a given refinement, invariant to the total drag length with minimal effect on the anchor trajectory. The method is designed so it is possible to include variation in material properties and surface topology. In the simulation of an anchor pull there are two distinct stages of loading: firstly the anchor

is placed on the seabed and embeds slightly under self-weight. The seabed itself will be under stress due to gravity. Secondly the anchor is translated and the forces on the anchor arise from contact between the rigid bodies and the soft deformable domain. Here we model this by undertaking a two stage analysis. We define a large deformable material domain using the MPM but use it only in one of the two analysis stages.

Stage 1: A quasi-static simulation to determine the stress state and material deformation under gravitational load, Fig. 4(a); and

Stage 2: A dynamic simulation of the anchor being pulled through the deformed material Fig. 4(b).

Stage 1 is solved for the entire domain, which ensures that the initial stress state, and displacement, are correctly determined for features such as, variations in material properties and surface topology. Stage 1 calculates the initial material state that is used as an input to Stage 2. In Stage 2 a domain around the anchor is defined which is a partition of the original larger domain. Material points which are in the partitioned domain are set to be active whilst material points outside the domain are set to be inactive and are fixed. As the anchor moves through the domain the partitioned domain moves with it. During movement, material points will leave the partition becoming inactive, whereas other material points, with their initial state calculated in Stage 1, will enter the partitioned domain and become active in the analysis. The overall simulation cost is a function of the mesh used and the GIMP resolution inside the partitioned domain, and hence a greater resolution around the anchor can be achieved for the same computational cost compared to considering the full domain.

For Stage 1 the domain is large but with small gradients in stress, therefore a mesh with large elements is used to reduce computational cost. Since Stage 2 will involve high stress gradients around the anchor, a mesh with significantly smaller elements is used, as shown in Fig. 4(b). Therefore although the meshed domain in Stage 2 is smaller, the number of DoF of the background mesh is likely to be larger than that of Stage 1, hence a load step in Stage 2 is likely to be more expensive than Stage 1. The boundary conditions in Stages 1 and 2 are also different, for Stage 1 all surfaces, except the top surface, have roller boundary conditions applied so that the material can move freely under gravity. For Stage 2 all the boundary conditions, except the top surface, are fixed. The motivation for setting the boundary conditions as fixed is to prevent step changes in the material displacement for material that is either entering, or leaving, the partitioned domain. These boundary conditions can obviously be adjusted to suit other physical problems.

3. Modelling approach

In this section we set out specific details of the model of a drag anchor pull and validate the partitioned domain approach enabling long anchor pulls. This is necessary in order to define the partitioned domain size for the full CPT-based predictive process to the prediction of anchor trajectories which is covered in Section 4.1.

3.1. Solution procedure

The procedure for predicting and recording the anchor trajectory is not trivial. This is particularly important in the context of validating against the centrifuge anchor pulls of Sharif et al. (2025), Section 4.2, as it is necessary to make sure the numerical procedure and recording of the anchor depth is as close to the experiment as possible.

There are three key stages in an analysis: (a) initial stress state of the soil under self weight, (b) placing of the anchor on the soil surface and initial settlement under self weight and (c) anchor drag. In addition, an analysis requires the generation of an initial distribution of material points, which could be uniform or graded depending on

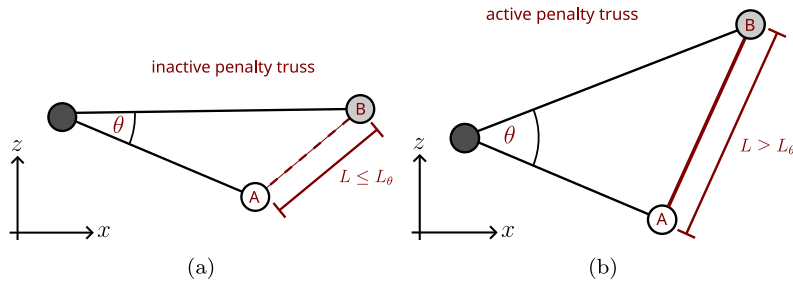


Fig. 3. Limiting opening angle: When the opening angle is less than the limit the penalty spring is inactive (a) but when greater than a limit it is active (b).

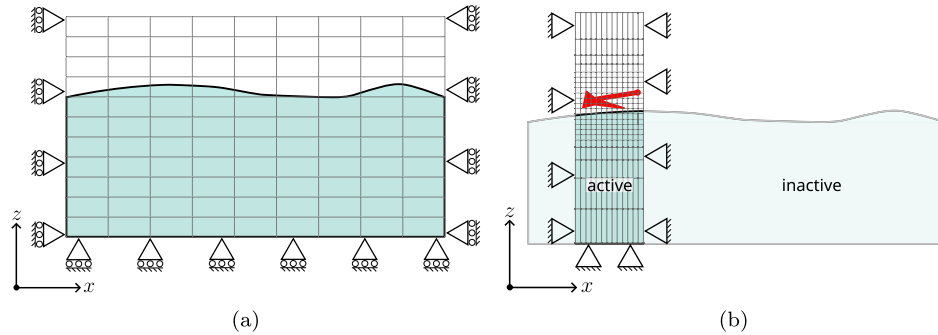


Fig. 4. Partitioned domain: The mesh and initial material configuration for stage 1 is shown in (a) with the subsequent deformed material, new mesh and boundary conditions, and anchor (in red) for stage 2 shown in (b).

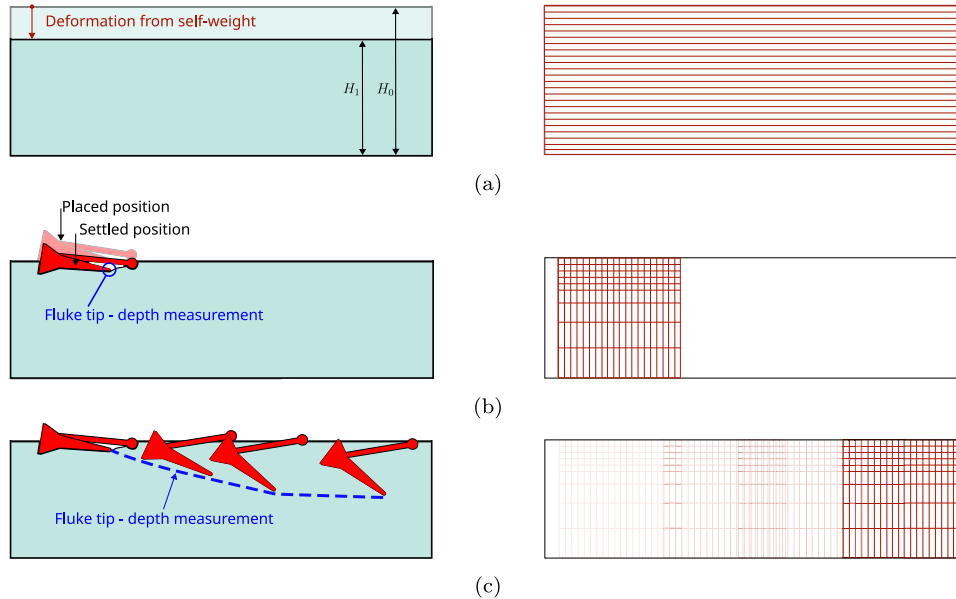


Fig. 5. Anchor solution and recording procedure: The three stages and their corresponding meshes with: (a) Settling of the material under self-weight; (b) Placement and settling of the anchor on the surface; and (c) Anchor pull and recording of fluke tip position.

the problem being solved. The first stage, Stage (a) shown in Fig. 5(a), is solved under quasi-static conditions to determine the initial stress state and displacement of the material points prior to an anchor test. Stage (a) corresponds to Stage 1 of the partitioned domain approach, see Fig. 4(a). Roller boundary conditions are on all surfaces, except the top. Since the material is homogeneous and has a flat upper surface, a stress variation only exists in the vertical direction. Therefore the corresponding mesh, shown on the right of Fig. 5(a), has element side lengths 0.1 m in the vertical direction whilst being only 1 element thick in the remaining directions.

The next two steps are shown respectively in Figs. 5(b) and 5(c). Both stages correspond to Stage 2 of the partitioned domain approach,

see Fig. 4(b). Fixed boundary conditions are applied on all surfaces, except the top where the material has a homogeneous Neumann interface and on the centreline/plane of symmetry where a symmetric (or roller) boundary condition is applied. On the right of Figs. 5(b) and 5(c) an example of how the mesh through the domain as the anchor travels is also provided. The nature of the mesh used during (b) and (c) will depend on the anchor position and orientation.

Stages (b) and (c) are solved using dynamic equilibrium, with the aim to represent as closely as possible the anchor pull procedure in the centrifuge (Sharif et al., 2025). In the centrifuge the anchor is placed on the sand sample, then the rotational speed of the centrifuge

is slowly increased to the required g level (the spin up phase). Once at the correct g , the anchor is then pulled along the sample. The numerical Stage (b) is designed to replicate the penetration of the anchor from self-weight during the spin up phase and is solved as a dynamic problem, which has the benefit of using the anchor's inertia to make the system determinate. The solution has the potential to be otherwise if solved as a quasi-static problem as not all parts of the anchor will be in contact with the sand from the first iteration of the first load step. These components will be unable to resist gravity, resulting in an ill conditioned system and large displacements making the non-linear solution difficult to solve. However, an issue with solving the problem dynamically is that there will be time dependent settling and vertical oscillation. To mitigate the effect of these oscillations on the drag embedment Stage (c), the anchor is only pulled once all components of the anchor velocity are below 10^{-3} m/s. In Stage (c) the anchor is pulled via a linear truss element attached to the shank pad eye. This truss element is designed to replicate the steel wire that is attached to the shank in the centrifuge experiment. There are two key points here in terms of recording the kinematics and depth of the anchor penetration, which are based on the assumption that the fluke tip will achieve the greatest penetration depth:

1. The zero offset for the anchor penetration depth is the depth of the fluke at the end of Stage (b).
2. The recorded depth of the anchor is *always* the fluke tip.

The purpose is to be consistent with the results of Sharif et al. (2025), where the depth is only recorded once the spin up phase is complete and the penetration depth is determined from the fluke tip.

3.2. Constitutive and interface behaviour

The material model used to represent the sand in all of the analyses in this paper is a Hencky material (as described earlier) combined with a perfectly-plastic non-associated flow yield surface defined by a frictional cone with a Willam and Warnke (1974) deviatoric section.⁴ The constitutive parameters for this material model are obtained from Brinkgreve et al. (2010) empirical relationships based on the relative density of the sand. The approach for going from a CPT trace to prediction of anchor penetration depth can be summarised as:

1. Estimate a relative density for the CPT trace and obtain material properties from Brinkgreve et al. (2010).
2. Run numerical CPT and obtain cone resistance trace.
3. If the trace compares well, material properties are found; if the difference is large, increase or decrease the assumed relative density, update the material properties and go back to (2) to rerun the CPT.
4. Use the calibrated material properties in the anchor simulations and determine the anchor trajectory.

The scope of these validations is limited to sands with a homogeneous relative density. For the remaining sections the material properties are defined using Brinkgreve et al. (2010) empirical equations, as provided in Table 1. It is acknowledged that the calibration against CPT data is potentially non-unique in that a different set of constitutive parameters could be used to obtain a very similar CPT trace. However, in order to match the CPT results, only the relative density, R_D , value in the relationships in Table 1 is modified until a good match with the centrifuge CPT result is achieved. Deriving all of the variable material

Table 1

Empirical equations to determine the material data for relative density R_D , where the R_D value in the equations varies between 0 and 1, i.e. a relative 82% would be $R_D = 0.82$. Gravitational acceleration is denoted by $g = 9.81$ m/s².

Reference Young's modulus (E_{50}^r)	=	$60R_D$	MPa
Unit weight ($\rho^0 \times g$)	=	$(19 + 1.6R_D) - 9.81$	kN/m ³
Friction angle (ϕ)	=	$28 + 12.5R_D$	°
Dilation angle (ψ)	=	$\max(-2 + 12.5R_D, 0)$	°
Stiffness exponent (m_E)	=	$0.7 - 0.3125R_D$	–

properties from one parameter results in a unique value of R_D to match to the experimental CPT.

The Willam and Warnke (1974) Lode angle dependent deviatoric section requires the specification of the deviatoric radius under triaxial extension normalised by that under triaxial extension, $\bar{\rho}_e$. In order to not introduce an additional constitutive parameter, in this paper $\bar{\rho}_e$ is obtained from the friction angle, ϕ , by assuming that the Willam-Warnke section matches the Mohr–Coulomb criteria at the compression and extension meridians (Coombs, 2011)

$$\bar{\rho}_e = \frac{3 - \sin(\phi)}{3 + \sin(\phi)}. \quad (10)$$

It is also assumed that the Young's modulus of the material varies with depth, where the initial Young's modulus for the point p at initial depth d_p is calculated using the formula by Schanz et al. (2019) assuming a negligible cohesion

$$E_{50} = E_{50}^r \left(\frac{\sigma_v K_0}{p^r} \right)^{m_E} \quad \text{with} \quad \sigma_v = g \rho d_p, \quad (11)$$

where $K_0 = 1 - \sin(\phi)$ is the coefficient of earth pressure at rest (Jaky, 1944), $p^r = 100$ kPa is the reference pressure, E_{50}^r is the reference Young's modulus (see Table 1) and $\sigma_v = g \rho d_p$ is the assumed vertical stress at d_p . Additional material parameters, which are constant for all relative densities are a Poisson's ratio of $\nu = 0.25$ and a cohesion of $c = 300$ Pa. As with Bird et al. (2024), the Young's modulus, and all other material properties, remain unchanged throughout the simulation. In this work the material response is assumed to be based on that of a Hencky material, with a linear relationship between elastic logarithmic strains and Kirchhoff stress, which is bounded by the yield surface described earlier. This means that the non-linear elastic response typical of sand behaviour is approximated by adopting the E_{50} value as the constitutive model's Young's modulus, an approach also recommended in the PLAXIS handbook (PLAXIS, 2010).

It is acknowledged that the constitutive model used to represent the sand behaviour in this paper is relatively simple and it would be unable to capture the full complexity of the stress-path response of sands. Several advanced sand models are available that can capture this complex behaviour. However, the trade-off with these advanced models is that they usually have numerous constitutive parameters that require an appropriate range of specialist laboratory element tests for calibration, or empirical relationships that relate the parameters to easy to measure quantities, before they can be applied. The numerical method developed in this paper is deliberately as simple as possible in terms of the required numerical and constitutive parameters and has adopted a constitutive model that can be calibrated from site investigation data available along a subsea cable route, namely CPTs. It has been shown that simple elasto-plastic models, that correctly capture the failure state of the sand, are able to accurately predict large deformation soil–structure interaction problems (Bird et al., 2025, 2024). Due to the very large deformation nature of these problems, any soil in close proximity to the anchor (or other rigid body) will very quickly reach its ultimate state. It is this critical aspect that any adopted constitutive model must capture in order to make reasonable predictions for large deformation soil–structure interaction problems.

Friction between the rigid and the deformable bodies is modelled using Coulomb's friction law which captures the stick–slip behaviour

⁴ The Willam-Warnke deviatoric section (Willam and Warnke, 1974), although originally designed for concrete, is very similar to other functions used for frictional soils, such as Lade-Duncan, Matsuoka-Nakai, Gudehus, Bhowmik-Long, etc. (see Bardet (1990) for a comparison). Critically it includes dependency on both the Lode angle and the intermediate principal stress.

in the tangential direction, described with the coefficient of friction μ . The formulation is subject to the Karush-Kuhn-Tucker conditions which are only approximately enforced using a penalty formulation. The normal contact is also captured by a penalty formulation and therefore the Signorini-Hertz-Moreau conditions are also only approximated. The contact formulation, and implementation description, for contact between the GIMPs and the rigid body are described in 2D and 3D in Bird et al. (2024, 2025). The normal and tangential penalty constants for all simulations take the form,

$$\epsilon_N = 50E \times A \quad \text{and} \quad \epsilon_T = 25E \times A, \quad (12)$$

respectively, where the constants 50 and 25 have units 1/length, in 2D A is a characteristic length of the GIMP and in 3D A is a characteristic area. It is important to note that other contact formulations exist, all with their own advantages and disadvantages, for example see the book by Wriggers (2006). However it was found in Bird et al. (2024, 2025) that the penalty method, with these parameters, was a good compromise between accuracy, algorithmic stability and implementation difficulty.

3.3. Partitioned domain validation

The aim of this section is to validate the partitioned domain approach for drag anchors and to identify a partitioned domain size which is sufficiently large, for all relative densities, so the boundaries do not influence the result. Therefore the drag anchor trajectory is recorded and compared for the different domain sizes and relative densities. Additionally the computational cost with drag distance is also recorded, although the primary aim of the method is to make the costs per load step invariant to the total length, it will be shown that a small increase in performance can be achieved by having a smaller partitioned domain.⁵

3.3.1. Partitioned domain dimensions

To validate the partitioned domain approach the dimensions for the partitioned domain need to be defined based on the geometry and location of the rigid body.

Fig. 6 illustrates the methodology behind defining the partitioned domain's lengths, and also the regions of uniform refinement, of the background mesh, around the partitioned domain.

All the dimensions of the domain are defined in terms of the dimensions of a box which bounds the rigid body, as an example in Fig. 6 an AC-14 anchor has been used.

The steps to defining the domain are as follows:

1. Identify the bounding box which surrounds the rigid body and identify its length H_x , width H_y and height H_z .
2. From the bounding box side lengths, $H_{(\cdot)}$, determine the distance from the edge of the box to the partitioned domain's boundary $L_{(\cdot),(\cdot)}$.
3. From $H_{(\cdot)}$ determine the regions of uniform refinement $U_{(\cdot),(\cdot)}$.

For the anchor simulations in this paper the partitioned domain approach only affects the x and z directions with the y -dimensions of the mesh already defined in Section 3.1 ($L_y = 5$ m and $U_y = 2$ m). From a numerical perspective the anchor problem is symmetric in the $x-z$ plane and no roll can occur. Therefore on the box the centre line of the anchor is marked, this corresponds to the plane where symmetric (roller) boundary conditions are applied. The distance of a partitioned domain boundary away from a face of the box, $L_{(\cdot),(\cdot)}$, is a function of the respective box size $H_{(\cdot)}$. Inside the box the mesh is uniform,

Table 2

Validation of partitioned domain: Partitioned domain lengths.

	Test 1	Test 2	Test 3
$L_{x,1}$ (m)	$1.25H_x$	$1.5H_x$	$2H_x$

Table 3

Anchor design: Total mass, rotational inertia (rot. inertia) and geometric properties of the anchor components, where the CoM position and length and measured from the pivot point.

	Mass (kg)	Rot. inertia (kgm ²)	CoM position (m)	Length (m)
Fluke	6583.2	1100	0.131	1.7
Shank	2116.7	1350	1.272	3.3

additionally in the regions marked $U_{(\cdot),(\cdot)}$ (their size is also a function of $H_{(\cdot)}$) the mesh is also uniform.

For partitioned domain sizes the function which describes the region of uniform element sizes is kept constant

$$U_{x,1} = U_{x,2} = \frac{1}{2}H_x \quad \text{and} \quad U_{z,1} = U_{z,2} = \frac{1}{2}H_z.$$

Furthermore the domain size behind the anchor is also kept constant as this has minimal loading, $L_{x,2} = H_x/2$. The region above the anchor, $L_{x,1}$, is set to be sufficiently high to always include all the material above the anchor, and below the anchor $L_{z,2}$ always extends to the bottom of the full domain. Therefore the partitioned domain size which is varied is $L_{x,1}$, for a loose sand (32% relative density) and a very dense sand (88% relative density) with their setup and definition were outlined in Section 3.2. The range of simulations is shown in Table 2.

3.3.2. Anchor design

The anchor centrifuge tests of Sharif et al. (2025) use a model scale AC-14 anchor, here a full scale 8.7 tonne anchor is used, with a schematic for the anchor shown in Figs. 7(a) and 7(b).

The full anchor is represented⁶ but since a line of symmetry exists down the centre line of the anchor, Fig. 7(a), it is only necessary to model half the material domain, achieved with a symmetric boundary condition. This assumption necessitates restraining the motion of the rigid body so it is only free to displace in the x and z (drag and penetration) directions and rotate about the y (pitching) axis. To be consistent as only half the anchor is modelled, the anchor mass and rotational inertia also need to be halved. The full mass, rotational inertia and the position of the centre of mass (CoM) from the hinge point are provided in Table 3 with the anchor having a maximum opening angle of $\theta = 35^\circ$.

The wire, shown by the thick red line in Fig. 7(c), is modelled with a truss element and has a length of 9.5 m, stiffness of 10^{10} N/m, the wire has a nominal mass of 0.1 kg to represent the negligible mass of the wire compared to the anchor in the centrifuge experiment. The end of the truss element not attached to the anchor, the pull point, has a prescribed velocity of $\{v\} = \{0.1 \ 0 \ 0\}^T$ m/s. The initial height of the pull point is set to coincide with the initial surface of the soil ($z = 8$ m in all of the examples in this paper). The coefficient of friction acting at the sand-anchor interface is assumed to be 0.3 as a representative value of the coefficient of friction between smooth steel and dry sand (Uesugi and Kishida, 1986).

3.3.3. Material point discretisation

To validate the partitioned domain approach and determine an appropriate domain size, two different relative density sands were considered, 32% and 82%, with the material properties defined by

⁵ All timing tests were performed on 40 cores of a 2× AMD EPYC 7702 chipset using 200 GB of 256 GB RAM.

⁶ The anchor's fluke and shank are represented by a triangulated surface mesh with 372 and 188 facets respectively.

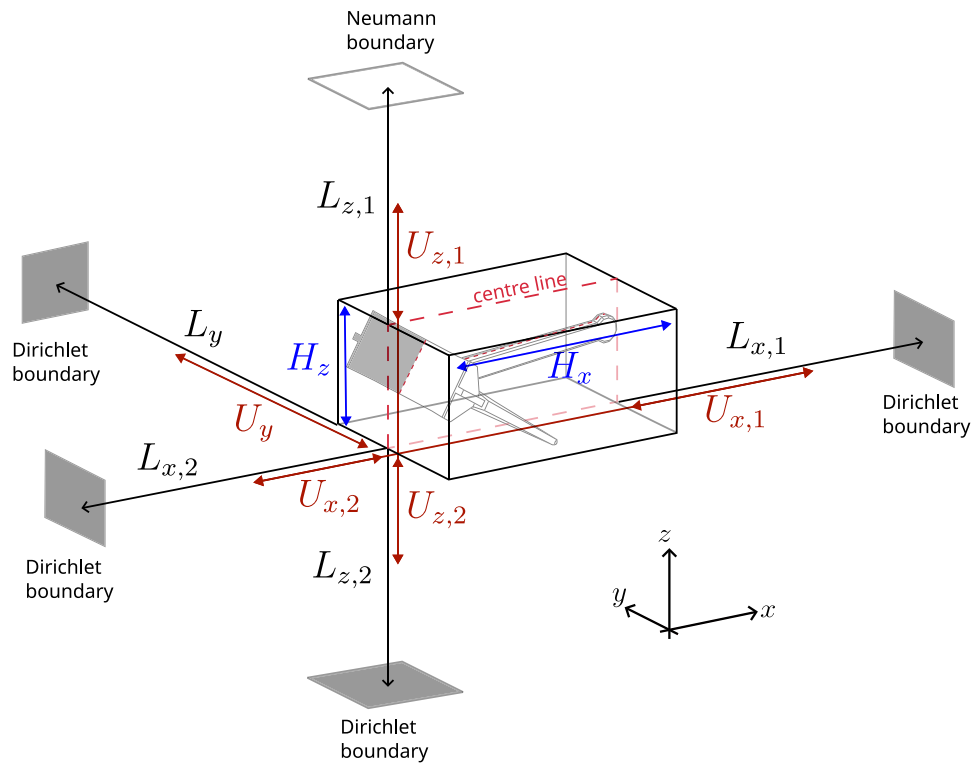


Fig. 6. Dimensions for the partitioned domain.

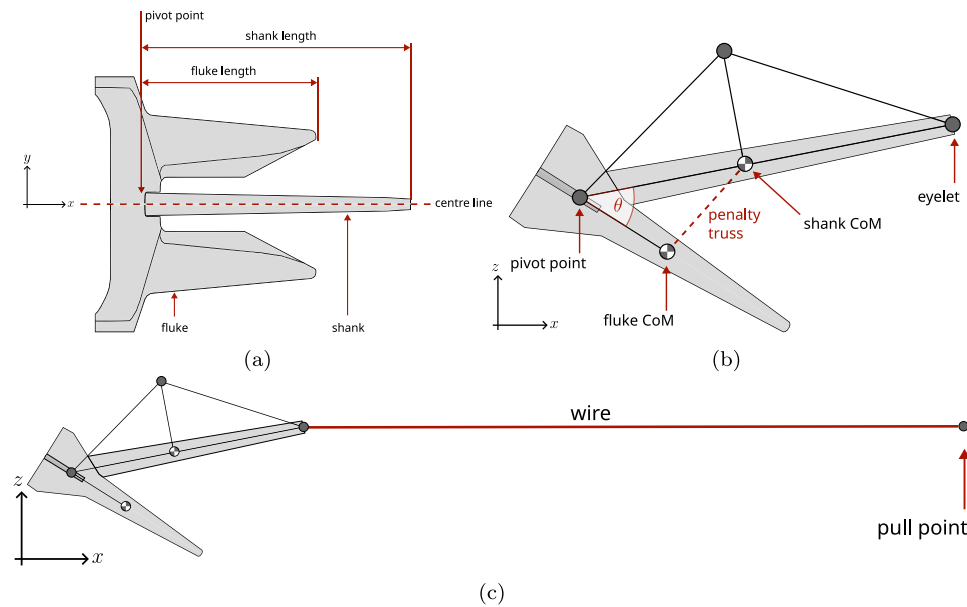


Fig. 7. Anchor design: Top down schematic of the AC-14 anchor used for analysis is shown in (a) with the side profile in (b) and extended profile in (c).

the equations in Section 3.2. The mass and stiffness ghost stabilisation (Coombs, 2023) parameters are set to $\gamma_M = \rho/4$ and $\gamma_K = E/10$, respectively, where ρ and E are the volume weighted average density and Young's modulus of the material points that occupy the elements that share the element boundary where the stabilisation is applied. The stages to solving and recording the anchor trajectory defined in Section 3.1. In total the anchor was dragged a distance of 7 m.

As discussed in Section 2.3, an initial distribution of GIMPs for the full domain is required. For this validation, a Cartesian mesh is generated which is sufficiently large to encompass the full anchor trajectory with GIMPs which are evenly distributed in a $2 \times 2 \times 2$ grid

within each element. An indicative mesh is shown in Fig. 8 which shows the exterior mesh. Fig. 8 shows two regions: the red region contains uniform elements with side lengths of 0.1 m, and in the blue region the element side lengths are increased by a factor of 1.3, starting with side length 0.1 m, in the y -direction between adjacent elements. The total domain size is $L_x = 25$ m, $L_y = 5$ m, $L_z = 8$ m with $U_y = 2$ m. Fig. 8 provides a schematic of the problem setup prior to the anchor pull.

3.3.4. Results and discussion

The results for the three partitioned domain tests, Table 2 are shown in Figs. 9(a) and 9(b). The first observation of the anchor trajectory,

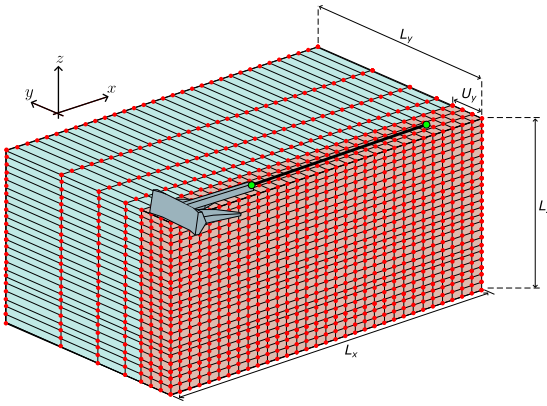


Fig. 8. Partitioned domain validation: Initial distribution of the background grid nodes for the GIMP setup.

Fig. 9(a), is that the 32% relative density sand has consistent results for all three domain sizes whereas 82% agrees well for the two larger domain sizes with the smallest domain size an outlier. The probable cause of the greater dependence on $L_{x,1}$ for the 82% relative density case is the higher stiffness and strength of the material, resulting in a larger required $L_{x,1}$ to avoid boundary effects. There are differences in the results for $L_{x,1} = 2H_x$ and $L_{x,1} = 1.5H_x$ with an 82% relative density. However, the differences are small, especially in terms of the final penetration depth where there is only an 0.8% difference between the $1.5H_x$ and $2H_x$ cases. In terms of how domain length affects the solution we therefore consider the $2.0H_x$ case to be converged for the two cases; employing a larger partitioned domain length will add unnecessary computational cost for a marginal increase in accuracy. Overall the results show that having $L_{x,1} = 2H_x$ is sufficient for the range of materials considered here, and this value will be adopted in all of the subsequent analyses in this paper. Lower relative density cases, such as the 32% case reported here, could use a smaller partitioned domain size, but $L_{x,1} = 2H_x$ will be adopted across all cases for consistency.

The timing results are shown in Fig. 9(b), the primary observations are: (i) the 88% relative density sand is consistently more expensive than the 32%; (ii) for this range of relative density there is not a strong correlation between the computational cost and the domain size. The reason for the lack of correlation is that the difficulties of solving a large deformation contact problem with an iterative solver dominate the solution time, rather than the number of GIMPs and Number of Degrees of Freedom (NDOF). This presents itself either as large number of iterations for the linear solver, a large number of NR-iterations, or, having to restart the load step with a reduced time increment. An example of such a case is at distance 3.5 m of 88% $L_{x,1} = 2H_x$ where the wall clock time continuous to increase but the distance pulled is unchanged. Combining these observations with the observations of Fig. 9(a) shows that for the necessary domain size length of $L_{x,1} = 2H_x$ there is no compromise on in terms of solution time. Therefore this domain size will be used for the remaining anchor simulations.

Inspection of the hydrostatic stress for the 88% relative sand provides evidence as to why the depth profiles in Fig. 9(a) vary with $L_{x,1}$ but also why increasing $L_{x,1}$ leads to convergence in the depth profile. To inspect the hydrostatic stress state the following parameter, similar to Dao and Grabe (2024), is used

$$\alpha = \frac{\Delta I_1}{I_{1,0}} \quad (13)$$

where $I_{1,0}$ and ΔI_1 are the initial and change in the first stress invariant of Cauchy stress. Three stress plots are presented Figs. 10, 11, and 12, which respectively correspond to $L_{x,1} = 2H_x$, $L_{x,1} = 1.5H_x$ and $L_{x,1} = 1.25H_x$. The plots show the material points and are coloured

with respect to four α ranges, $\alpha \in [5, 10]$, $\alpha \in [10, 20]$, $\alpha \in [20, 30]$ and $\alpha > 30$. The Figures show that as $L_{x,1}$ is increased the α -plots become more similar. $L_{x,1} = 2H_x$ and $L_{x,1} = 1.5H_x$ have the most similar α -plot, with a difference of $0.5H_x$, whereas the comparison between $L_{x,1} = 1.5H_x$ and $L_{x,1} = 1.25H_x$ shows a significant difference in α -plots despite a smaller change of $0.25H_x$. At $x = 12$ m $L_{x,1} = 1.25H_x$ a vertical front for the range $\alpha \in [5, 10]$. Additionally high α values are observed along the surface in front of the anchor for $L_{x,1} = 1.25H_x$ which are not observed for $L_{x,1} = 1.5H_x$. Below the eyelet of the shank for $L_{x,1} = 1.25H_x$ (marked with the red arrow) there also appears to be an increase in the volume of for $\alpha \in [5, 10]$ compared to the other two cases. The similarities and differences in the surface for $\alpha > 2$ for the three figures correspond to the similar behaviour in the depth profile in Fig. 9(a), with $L_{x,1} = 2H_x$ and $L_{x,1} = 1.5H_x$ being the most similar. Since the results for depth are still marginally changing from Fig. 11 to Fig. 10 the value of $L_{x,1} = 2H_x$ is used for the Anchor prediction validation in Section 4.2.

4. CPT-based anchor penetration prediction

In this section CPT results are used to calibrate the material parameters in the numerical model by comparing the numerical trace for cone resistance with depth with the traces obtained by the centrifuge CPT for four sand densities, Davidson et al. (2022), Cerfontaine et al. (2020), which were used in the centrifuge anchor pulls. The implicit-axisymmetric CPTs were validated in Bird et al. (2024), Coombs et al. (2025) for a range of numerical parameters, therefore for this set of computations the numerical parameters are kept constant whilst the relative density is varied. Each numerical CPT therefore had a wall clock time of approximately 15 min (Bird et al., 2024), run as a serial computation in MATLAB,⁷ meaning the material calibration was a relatively fast process.

The implicit quasi-static GIMP analysis of a CPT by Bird et al. (2024) demonstrated good agreement between numerical predictions and experimentally measured q values across of a range of relative densities where the relative density was known *a priori*. However, in non-lab conditions the relative density of the sand is not known and often only a q -trace is provided (in reality offshore geotechnical data availability can be even more limited (Macdonald et al., 2023)). This necessitates a framework which first estimates the material properties from limited CPT data, and second, uses these estimated properties to predict anchor trajectory and ultimate penetration depth at the CPT location.

The material properties are defined using Brinkgreve et al. (2010) empirical relations, see Section 3.2. The relative density parameter is used to generate the material properties to obtained agreement between the numerical and experimental results, however the general trace of the CPT is not unique for a given set of parameters and a combination of other parameters could also generate similar results. Hence the relative density determined using the numerical CPT is not a determination of the true relative density, this is particularly important to note at the extreme values of relative density since it is likely in these regions the empirical relations of Brinkgreve et al. (2010) were derived from fewer data points.

The mass and stiffness ghost stabilisation (Coombs, 2023) parameters are set to $\gamma_M = \rho/4$ and $\gamma_K = E/10$, respectively, where ρ and E are the volume weighted average density and Young's modulus of the material points that occupy the elements that share the element boundary where the stabilisation is applied.

⁷ All computations were computed on a machine with Intel Core i7-8665U (4 cores, 8 threads) with 32 GB RAM running Ubuntu 24.04 LTS.

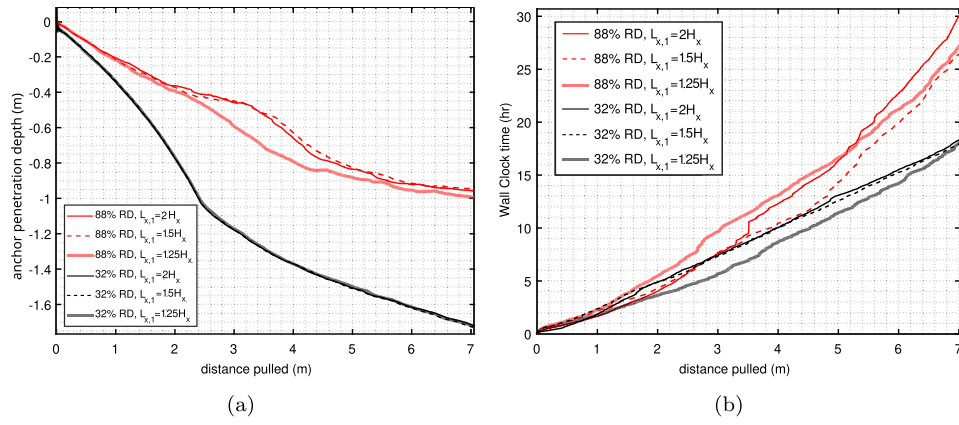


Fig. 9. Partitioned domain validation: The penetration depth of the anchor with drag distance is shown in (a) with the corresponding cumulative time for the simulation shown in (b).

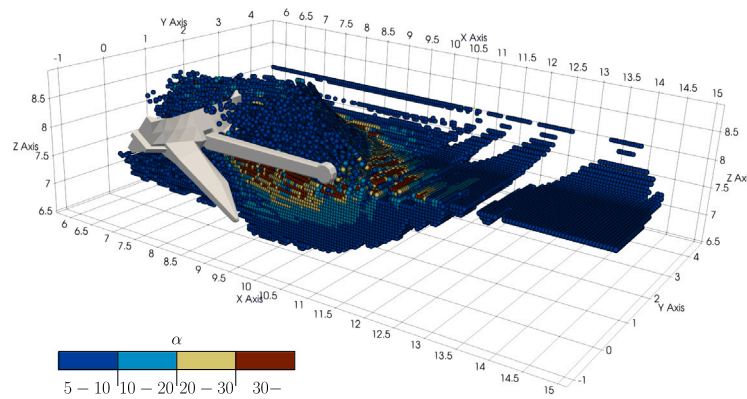


Fig. 10. Validation of partitioned domain: Visualisation of the anchor at a pull distance of 7 m when $L_{x,1} = 2H_x$.

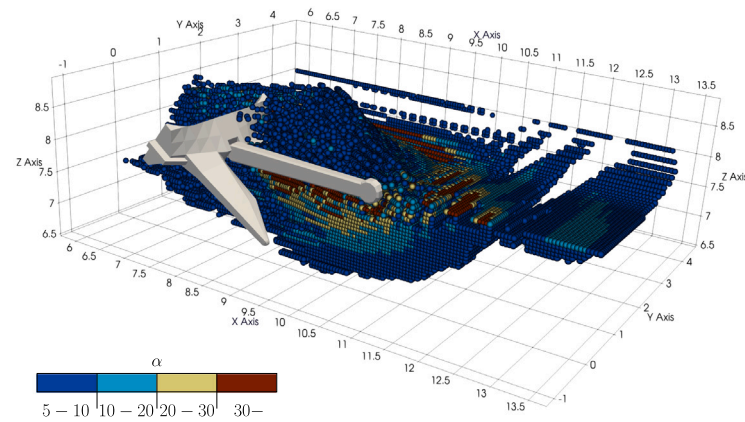


Fig. 11. Validation of partitioned domain: Visualisation of the anchor at a pull distance of 7 m when $L_{x,1} = 1.5H_x$.

4.1. CPT calibration

Physical testing

To obtain the CPT data, a custom designed 16 mm diameter CPT probe with a 60 degree conical tip was used in geotechnical centrifuge experiments conducted at the University of Dundee, using the same procedure as outlined in Davidson et al. (2018). The experiments were conducted at 50g in dry sand. Penetration occurred at 20 mm/min, with the CPT cone resistance measured by a pair of strain gauges mounted on the tip of the probe, and displacement was measured by a draw wire transducer. For each of the relative densities reported, a sand bed

was created using a slot type pluviator using HST95 sand, which is commonly used for physical modelling at the facility. Density pots and the bulk density of the soil were used to confirm the target relative densities of very loose (25%), loose (38%), medium dense (55%) and very dense (82%) had been achieved.

Numerical setup

The material equations are described in Section 3, here the geometry, mesh, boundary conditions, and GIMP properties are defined for the CPT simulations. A diagram of the overall setup is provided in Fig. 13(a) which shows the geometry, boundary and indicative mesh. The

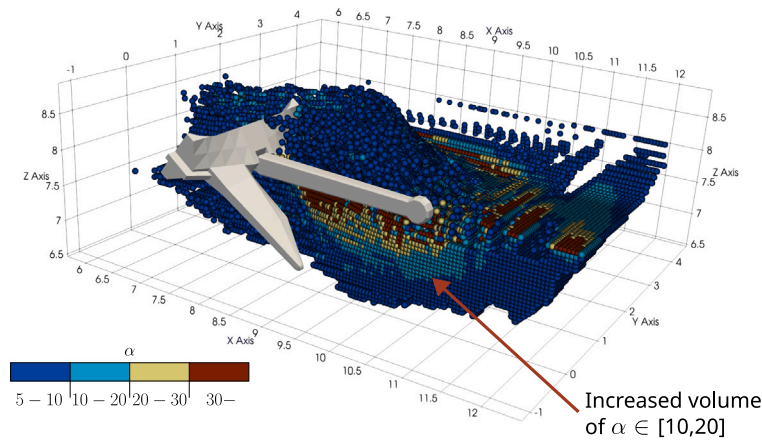


Fig. 12. Validation of partitioned domain: Visualisation of the anchor at a pull distance of 7 m when $L_{x,1} = 1.25H_x$.

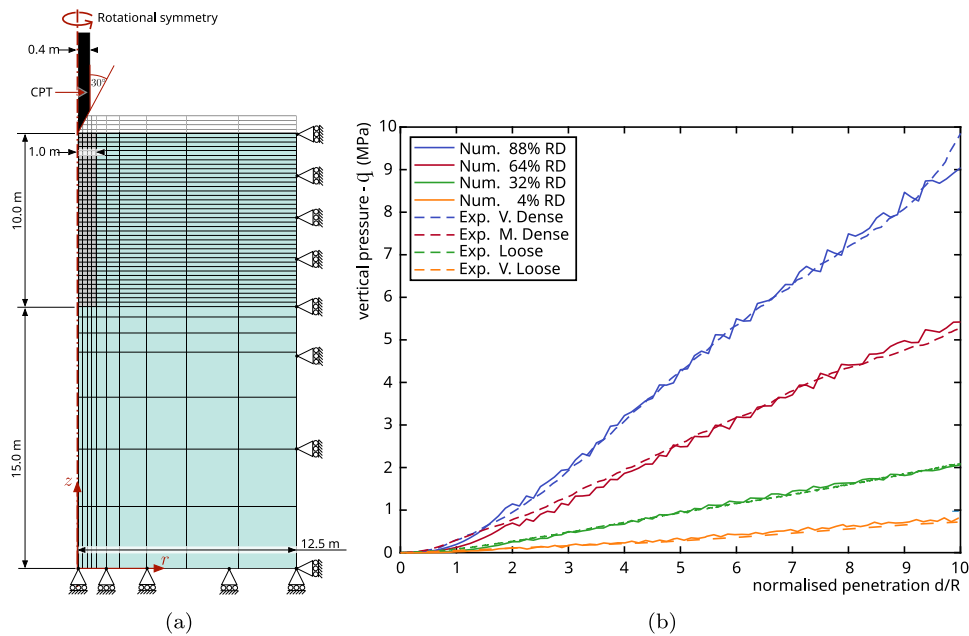


Fig. 13. CPT calibration: (a) is a diagram of the overall geometry and setup of the numerical CPT with an indicative, and (b) showing the final calibration of numerical cone resistance, against normalised penetration, for a range of laboratory CPT results.

CPT has a radius of 0.4 m and will penetrate into the sand 4.0 m, the coefficient of friction between the CPT and sand is 0.3. In Fig. 13(a) the region in which the CPT will penetrate is marked in grey, it has a refined and uniform mesh where the elements have a side length of 0.1 m. Outside the grey region the element side length of adjacent element is increased by a factor of 1.3. Initially each element throughout the mesh has 3^2 material points.

The simulation procedure is to first apply the gravitational load, then over 100 load steps the CPT is moved vertically downwards over the distance of 4.0 m. The load on the tip of CPT, from contact with the GIMPs, is recorded and then divided by the cross sectional area to give the cone tip resistance q MPa. The cone resistance is then used for the calibration procedure outlined in Section 3 for the four sands used by Sharif et al. (2025) in the anchor centrifuge tests.

Results and discussion

The result of the calibration procedure for the four sand Relative Densities (R_D s) is provided, with the results shown in Fig. 13(b) with the results summarised in Table 4. Inspection of the Fig. 13(b) shows the relatively simple material model is able to sustain a very good level

Table 4

CPT calibration: Summary of the calibrated Relative Density (R_D) values versus the experimental description.

Exp. description	Very dense	Medium dense	Loose	Very loose
Calibrated R_D	88%	64%	32%	4%

of agreement of a large range of normalised penetration, no adjustment of the material parameters is required over the penetration depths considered. The oscillations in the MPM response is due to the finite size of material points causing incremental changes in the tip load as they come into contact with the cone. When calibrating the MPM model the soil cohesion and Poisson's ratio are kept fixed at $c = 300$ Pa and $\nu = 0.25$, respectively, and only the R_D value is modified to fit the physical testing CPT result. All of the other constitutive properties are obtained from the relative density-dependant relationships in Table 1. This results in a process where there is a unique R_D value, and therefore dependent constitutive parameters, that provides the best fit to the physical testing CPT result. It is acknowledged that the relative densities used in the numerical model are different to those reported

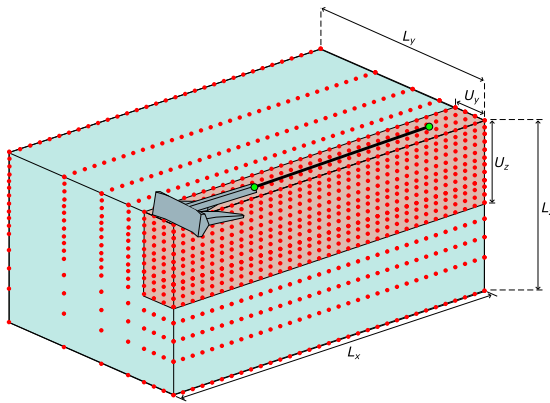


Fig. 14. Anchor penetration: Schematic of the initial distribution of background grid nodes.

from the physical tests, with the very loose case showing the most significant discrepancy. This is likely due to the empirical relationships summarised in Brinkgreve et al. (2010) being *best fits* to a range of sands and focused on sands in the loose-dense range of relative densities. We are not implying that the tested very loose sand has a relative density of 4%, but rather than relationships in Table 1 require an R_D value of 4% to match the physical testing CPT results. It is worth noting that the calibrated R_D value for the medium dense case used in the MPM simulations, 64%, is very similar to that adopted by Dao and Grabe (2024) who used a relative density of 63% when tuning their CEL-based AC-14 anchor simulation to match the medium dense case from Sharif et al. (2023, 2025), albeit with a different constitutive model.

4.2. Anchor penetration prediction

Setup and scope

The material properties for each of the densities in Table 4 are used to calculate the anchor trajectories and final penetration depths of the AC-14 anchor, completing the prediction procedure which started with a physical cone resistance trace. The coefficient of friction acting between the anchor and the sand is set to 0.3.

The setup for this problem is similar to the validation of the partitioned domain, Section 3.3, the only difference is that the total length of the domain is set to $L_x = 50$ m and the volume of the initially uniformly distributed GIMPs is reduced from being the total depth of the domain to $U_z = 4$ m, Fig. 14, such that $L_z = 8$ m. In the z -direction, below the uniform region, the adjacent node spacing was increased by a factor of 1.3.

The final penetration depths of the AC-14 are compared to those obtained in the centrifuge by Sharif et al. (2025) for four homogeneous sands with the relative densities determined in the previous section based on the CPTs. In the centrifuge the anchor is pulled by a thin wire and as such has no catenary, this is replicated in the simulations through the use of a truss finite element which connects the anchor eyelet to the pull point. This is so that the numerical model can be validated against the centrifuge data. Adding a representation of chain, either through modelling the chain links (Dao and Grabe, 2024) as rigid body representations, or including it as an empirical expression similar to the analytical method of Neubecker and Randolph (1996b), would be possible and we acknowledge that it could change the results. However, as noted by Sharif et al. (2025), the use of the wire is meant to represent the case of maximum anchor burial and a study into the effect of different wire/chain geometry is beyond the scope of this paper.

Given the results of the partitioned domain validation, the parameters for the partitioned domain and the volume of uniform refinement

Table 5

Anchor penetration: Partition domain parameter values.

Dimension	$U_{x,1}, U_{x,2}$	U_y	$U_{z,1}, U_{z,2}$	$L_{x,1}$	$L_{x,2}$	L_y
Value (m)	$H_x/2$	2	$H_x/2$	$2H_x$	$0.5H_x$	5

about the anchor are provided in Table 5. The material model, and setup, is consistent with the CPT calibration in Section 4.1, no alteration to the relative density, and therefore material parameters, are made from those summarised in Table 4 and there are no further parameters or complexity added to the material model. Last, the simulations are stopped if over 2 m of drag the depth of the anchor does not increase by 10 mm. 2 m was considered as it is greater than the length of the fluke, 1.7 m, and so therefore represents the total process of sand moving over and around the fluke. The depth value is taken as this is considered to be small enough, with respect to 2 m, that no more significant penetration will occur.

Results and discussion

The results for the anchor penetration are presented and compared to the final experimental penetration depths, are shown in Fig. 15. It must be emphasised that the results in this section are predictions of the anchor behaviour; no numerical or material data tuning has been performed to get results that agree.

In general the trend of the results matches that of the experimental data, for a high relative density the penetration is less and the difference between the final penetration depth increases with decreasing relative density. Furthermore the comparison of the results for the middle relative densities are very good, for the medium dense sand the absolute difference is 0.1 m and the loose sand is 0.2 m (both correspond to $\approx 8\%$ relative difference). Good agreement is also achieved for the very loose sample with a predicted final penetration of 3.5 m compared to the experimental result of 4.1 m, corresponding to an difference of 14%, or relative to the anchor length, a fraction of 0.35 of the fluke length. Last, the 88% has an absolute difference of 0.3 m, but with a higher relative difference of 43%, compared to the final penetration depth. The reason for this discrepancy could be due to a number of factors, including: adopting a perfectly plastic yield envelope which neglects the change of the friction angle of the sand between initial yielding and the ultimate state; not accounting for the pressure dependent nature of the sand stiffness; and/or differences in the kinematics between the experimental and numerical anchor for the very dense sand, which are explained below.

Fig. 16 shows the AC-14 anchor pulled in a sand of 88% relative density at a 12 m pull distance, the shank is laying nearly horizontally at 0.5° to the undeformed surface of the sample whilst the fluke is at 36.14° degrees to the horizontal. The result is that nearly all the penetration is due to the fluke inclination. This is in contrast to the experimental results where the penetration is due to burying of the fluke at a near horizontal angle of 5° to the surface. The difference in the kinematics could potentially be caused by the assumption of a frictionless pin between the fluke and the shank in the numerical model. The physical tests are conducted in a geotechnical centrifuge which place the anchor assembly, and therefore the joint between the fluke and the shank, under large forces which will result in some frictional resistance to the anchor opening. There is also the potential for sand to be *trapped* in the connection during the centrifuge test that again could provide additional resistance to shank-fluke rotation. It is acknowledged that this is a hypothesis and work is ongoing to extend the numerical framework to include friction in the articulated joint.

The kinematic behaviour of the anchor is very different in very dense and loose sand conditions. This can be seen by observing the differences between Fig. 16 and Fig. 17, where the latter shows the anchor position in the very loose sand after 17 m of drag. In the very loose case, the shank and fluke are inclined at 15° and 20° to the horizontal, respectively. The result for opening angle and, fluke and shank

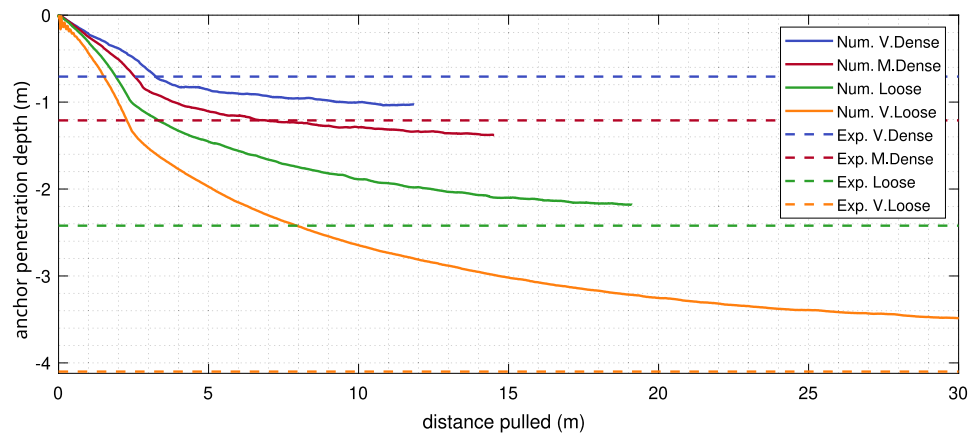


Fig. 15. AC-14 anchor penetration: Comparison of anchor trajectories with the final depth of the experimental results.

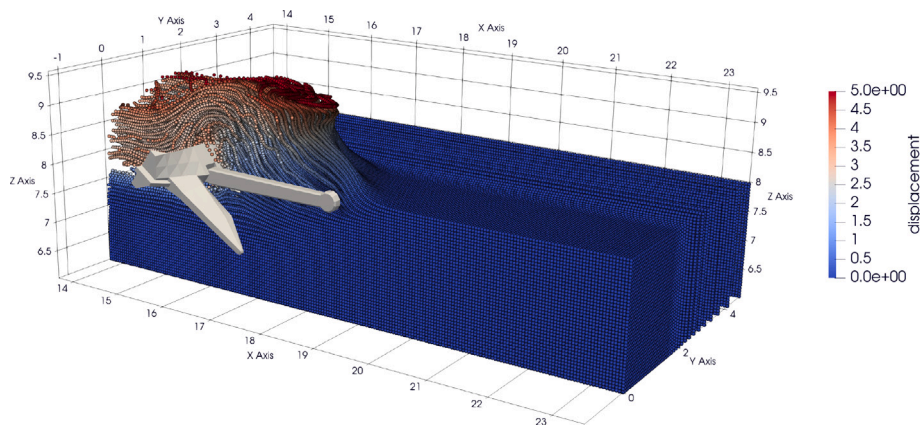


Fig. 16. AC-14 anchor penetration: anchor position for the very dense sand at a 12 m pull distance.

angle, both to horizontal with drag distance are shown respectively in Figs. 18(a)–18(c). For the four densities the only common response is the opening angle in Fig. 18(a) which all quickly reach the maximum, with the lowest density first, of 35° . For the fluke and shank angle, Figs. 18(b) and 18(c) show that for all densities a convergence in the angle is achieved with drag distance however the behaviour of the anchor is different. For the very dense sample the shank angle reduces towards zero with drag distance, whereas when the density is reduced the angle increases with distance, additionally the ultimate value shank angle also increases with reducing relative density. The opposite is true for the fluke angle, where for the lower densities the fluke is becoming more parallel to the horizontal. However, for the high densities the fluke angle is largest and changes marginally, with the majority of the change being oscillatory.

In general the response of the anchor for the range of sands considered follows the description in Chapter 3 of Puech (1984) where a number of full scale experiments were performed on articulated (shank and fluke are free to rotate) anchors. In summary the shank angle stays approximately parallel to the wire/chain, see Fig. 18(c), which is pulling the anchor whilst the fluke is initially angled down in the soil causing penetration. For the loose samples as the anchor is pulled the angle of the fluke creates a resultant forces which pushes the anchor deeper into the sample, the anchor subsequently rotates so that the fluke becomes more parallel to the horizontal whilst the shank becomes more vertical. Whereas for the denser samples the majority of the penetration is caused by the fluke angle. This is also noted in Puech (1984), particularly when the soil is very dense and the anchor is “tripped” causing the anchor to fully open with only the fluke tip embedding.

4.3. Influence of anchor size

Setup and scope

The scope of this numerical study is to explore the impact of anchor size (or mass) on the penetration depth in uniform sands. Two sand densities are considered (loose and very dense), the length dimensions of the 8.7 tonne AC-14 anchor, used in the rest of the paper, are scaled by a factor of 0.5, 1.5 and 2.0 times their original values. Overall the anchors were dragged 6 times their fluke length with their dimensions, and initial penetration due to self weight, shown in Table 6.

All simulations used the same mesh as in Section 4.2 but with their element size and overall dimensions scaled by the corresponding factor in Table 6, the pull velocity is also scaled by the same factor. The only parameter which was not scaled was the time step increment.

All depth and pull distance results in this section are normalised with respect to the fluke length.

Results and discussion

The scaled anchor penetration results for the loose and very dense case are shown in Fig. 19. The figure shows that regardless of scale, the normalised penetration depths achieve a very similar penetration, this is particularly true for the loose case where the penetration curves nearly all overlap exactly. For the very dense case the penetration of the fluke happens at different normalised drag distances. However, once the fluke has penetrated into the sand the normalised depths for all scales are very similar, see the normalised depth for the drag distance in the range 5.5–6.0. The initial penetration of the anchors due to self weight is provided in Table 6, and shows that initial penetration does not scale proportional to the fluke length. An extreme example is the

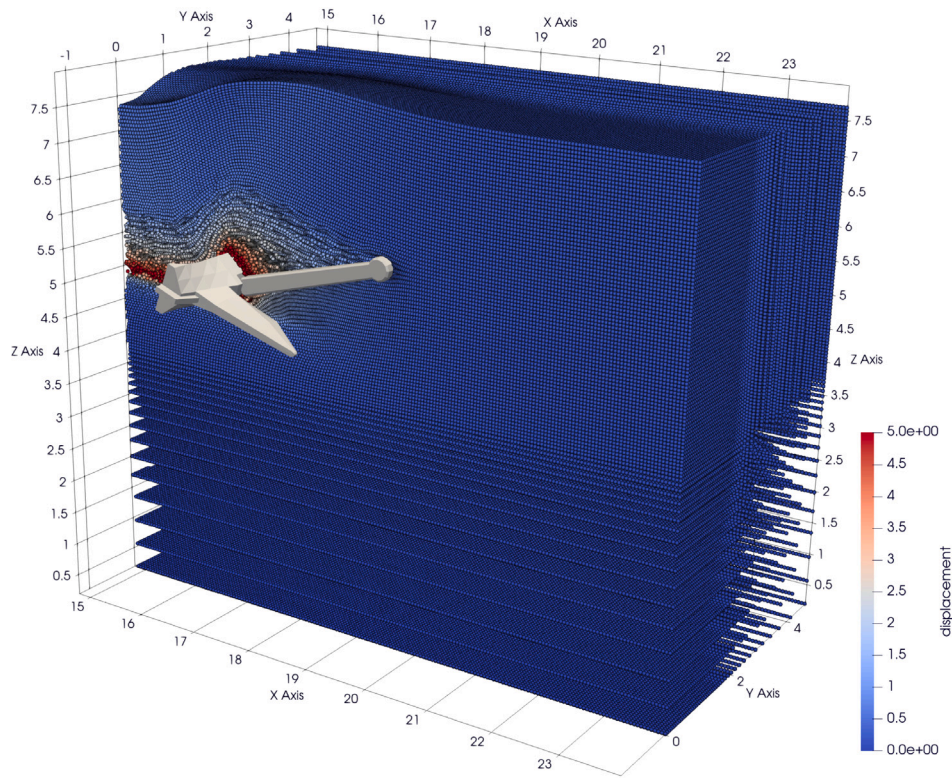


Fig. 17. AC-14 anchor penetration: anchor position for the very loose sand at a 17 m pull distance.

Table 6

AC-14 anchor fluke lengths, l_f , mass and initial embedment under self weight, d_{in} , in loose and very dense conditions.

Scaling factor	l_f (m)	Mass ($\times 10^3$ kg)	Loose d_{in} (m)	v. dense d_{in} (m)	Loose d_{in}/l_f	v. dense d_{in}/l_f
0.5	0.85	1.09	0.052	0.028	0.061	0.033
1.0	1.70	8.70	0.102	0.050	0.059	0.029
1.5	2.55	29.36	0.203	0.130	0.079	0.051
2.0	3.40	69.60	0.451	0.234	0.131	0.068

comparison between the 2 and 0.5 scaling factors in the loose sand condition, where the initial penetration due to self weight is 8.67 times larger, despite the fluke only being 4 times larger. Therefore when considering anchor penetration depths, the influence of the anchor size on the initial penetration should be taken into account as it has the potential to increase the overall penetration depth by 5%–10% depending on the sand state and the size of the anchor. It is interesting that for the scaling factors of 0.5 and 1.0, the normalised initial penetration is very similar and for both densities with 0.5 having a larger normalised penetration, whilst the opposite trend is seen for the larger anchors. This is due to the non-planar geometry of the fluke which results in different anchors having different contact patch sizes with the sand which deforms to support the weight of the anchor.

A similar response is seen in the angles related to the anchor, see Fig. 20. For the loose case, the opening, shank and fluke angles follow almost the same trend. For the very dense case, the trends are not the same however their values at approximately the normalised pull distance of 5, and further, are very similar. The response of the normalised depth for the very dense case is reflected in the change in the angles, for example at the scale 0.5, the depth starts to increase substantially at a drag distance of 4.5 m (see Fig. 19), caused by the increase in the opening and fluke angles at a drag distance of 4.5 m seen in Fig. 20. The delay in the normalised penetration of this scale is caused by the delay in the rotation of the anchor, but when the anchor

starts to rotate, and therefore penetrate the seabed, the overall response is very similar.

Fig. 21 provides the drag force normalised by the length of the fluke squared (i.e. the relative change of the anchor area) versus the drag distance normalised by the fluke length for the four anchor sizes in loose and very dense sand. The consistency of the loose results demonstrates that the drag force scales with the area of the anchor and although there are variations in the very dense results, which are consistent with penetration depth and angle results discussed previously, the final normalised force is common across the anchor sizes.

4.4. Influence of anchor geometry

The previous analyses in this paper focused on an AC-14 anchor. This section investigates the impact of changing the anchor geometry by comparing the AC-14 results with the penetration depths obtained from a 7.5 tonne Hall's stockless anchor (named after John Francis Hall, 1854–1897), another widely used shipping anchor. All the mesh and material parameter values are the same as in Section 4.2, the coefficient of friction acting at the anchor-sand interface is 0.3. The penetration results of the Hall anchor are compared to experiments obtained using the same experimental setup as Sharif et al. (2025).

Setup and scope

The Hall anchor is dragged through 4 homogeneous sands, defined in the CPT calibration, Section 4.1, as *very dense*, *medium dense*, *loose* and *very loose*. The schematic of the anchor pull is shown in Figs. 22(a)–22(c), where the wire for this simulation has a length of 9 m (different to the AC-14 at 9.5 m) and has a mass of 0.1 kg to represent the negligible wire mass in the centrifuge experiment. Note that the maximum opening angle of the Hall anchor is 45°.

The corresponding total mass, rotational inertia (rot. inertia) and geometric parameters (corresponding to Fig. 22) are shown in Table 7. Finally the pull point position is set to coincide with the initial surface of the soil ($z = 8$ m in all of the examples in this paper) and has a prescribed velocity of $\{v\} = \{0.1 \ 0 \ 0\}^T$ m/s.

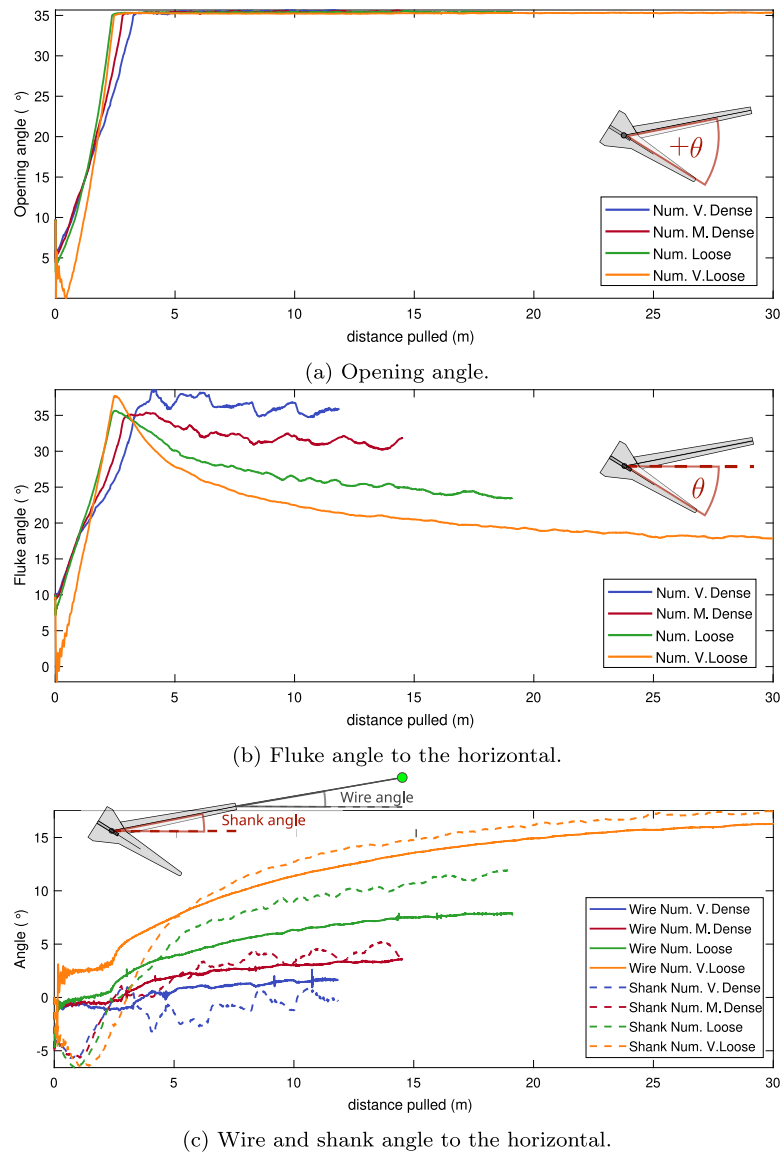


Fig. 18. AC-14 anchor penetration: Opening angle (a) and, fluke (b) and, wire and shank angle (c), both to horizontal for the AC-14 anchor with drag distance.

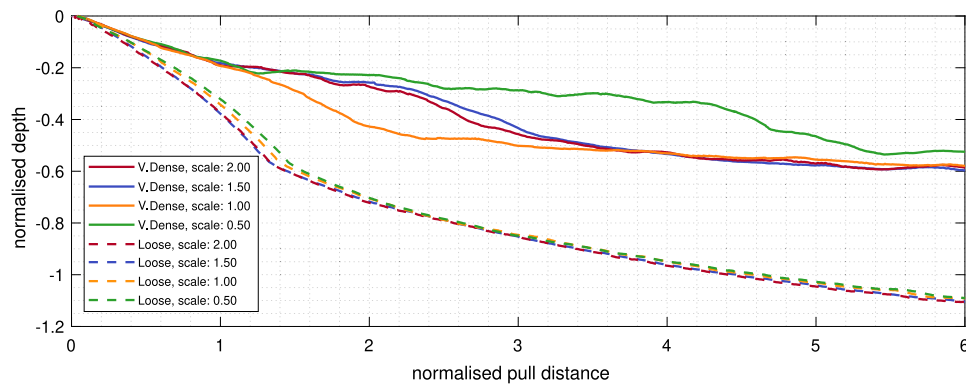


Fig. 19. AC-14 scaled anchor penetration: scaled penetration depths for loose and very dense sands. Results are normalised with respect to the fluke length.

Results and discussion

The MPM-predicted Hall anchor penetration depths are shown in Fig. 23, with the equivalent experimental data also shown for the final penetration depth. The opening, shank-horizontal and fluke-horizontal angles are also shown in Figs. 24(a)–24(c). In all cases the opening

angle quickly transitions to the maximum opening angle of the Hall anchor.

The differences between the numerical predictions and experimental results for the Hall anchor are similar to those with the AC-14 anchor. As with the AC-14 results, the differences, especially in the

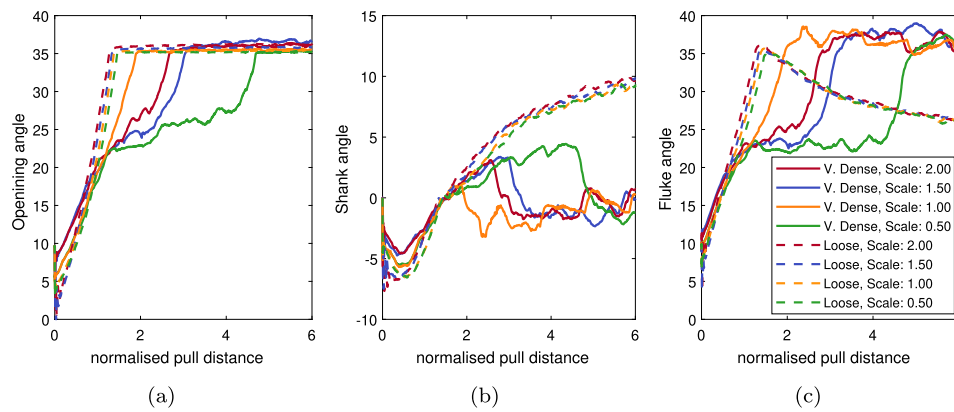


Fig. 20. AC-14 scaled anchor: (a) opening angle, (b) shank angle and (c) fluke angle against normalised pull distance. Results are normalised with respect to the fluke length.

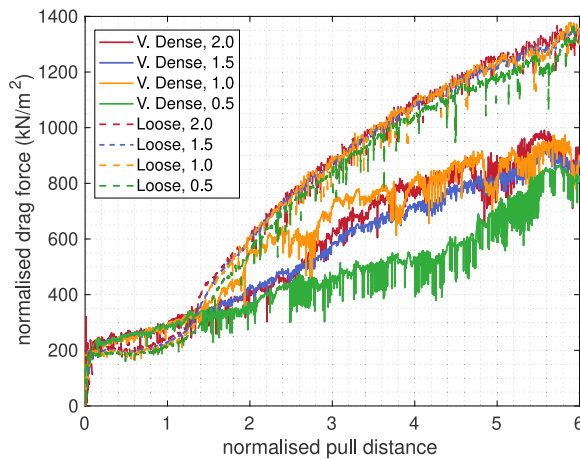


Fig. 21. AC-14 scaled anchor: normalised drag force versus normalised drag distance.

Table 7

Hall anchor: Total mass, rotational inertia (rot. inertia) and geometric properties of the anchor components, where the CoM position and length are measured from the pivot point.

	Mass (kg)	Rot. inertia (kgm ²)	CoM position (m)	Length (m)
Fluke	5688.4	1376	0.232	1.75
Shank	1811.6	1643	1.328	3.23

very dense case that over predicts the level of penetration, could be due to the lack of friction in the articulated connection between the fluke and the shank. Including friction in this joint would resist the fluke-shank opening and likely result in a smaller opening angle, a less steep fluke and therefore less anchor penetration in the very dense case. Good agreement is achieved for the other sand densities, with discrepancies of 12%, < 1% and 1% for the medium dense, loose and very loose cases, respectively.

The shank angle of the Hall anchor (Fig. 24(c)) is different to the AC-14 shank angle (Fig. 18(c)) in the same soil condition. In very dense and dense sand conditions, the Hall anchor reaches a steady state with the shank tipped forwards the back of the anchor higher than the padeye, whereas the AC-14 anchor transitions to a positive shank angle in all cases. This is due to the large flat plates at the back of the fluke on the Hall anchor that also limit the penetration depth compared to that achieved by the AC-14 anchor. For loose and very loose conditions the Hall anchor transitions to a positive shank angle with the back of the shank fully embedded in the seabed. For a qualitative comparison of

the Hall anchor in different soil conditions, see Fig. 25 which shows the steady state anchor position in very loose and very dense sands. The differences between the AC-14 and the Hall anchors highlight the importance of anchor geometry on the ultimate penetration depth in different soil conditions.

The validation of the penetration depth for the Hall anchor further supports the predictive capability of the proposed methodology, it shows the method is robust to a large range of sand relative densities and different anchor designs.

5. Discussion and practical implications

5.1. Anchor performance

It was shown in Section 4.3 that the capacity of a drag anchor scales with the square of the relative change in the anchor fluke length (i.e. linearly with the relative change of the anchor area). However, it is more typical to express the holding capacity of an anchor in terms of anchor mass (see for example (Vryhof, 2018)). Fig. 26 provides the holding capacity for different masses of AC-14 anchor in loose and very dense sand conditions. In both cases the holding capacity scales with the mass of the anchor to the power of 0.7. This value is slightly lower than that suggested for Vryhof Stevin® and US Navy Stockless anchors in Naval Civil Engineering Laboratory (1987) who suggest a power of 0.8 based on field trials. However, these field trials bound the numerical modelling result and provide additional validation of the MPM simulations presented in this paper. The numerical results also go some way in addressing the point raised by Randolph and Gourvenec (2011) that design curves, such as those presented in Fig. 26, have been historically based on “simple extrapolation of the ultimate holding capacity measured in small-scale tests to larger anchors”. The results confirm that the sand relative density shifts the position of the ultimate capacity, which is intrinsically linked to the anchor penetration depth, which is dependent on the sand state. This is an important point as it is typical that design charts combine sand and hard clay into a single performance line, often with separate lines for medium and soft clay. The results in Fig. 26 suggest that this may not be appropriate for all anchor designs. However, changing the relative density does not appear to change the exponent linking the capacity to the anchor mass. Discrete points are also included on Fig. 26 for the 7.5 tonne Hall anchor in very loose to very dense sand conditions and the 8.7 tonne AC-14 anchor in very loose and dense sand conditions. These discrete points confirm the trends of very dense and loose AC-14 data and fall within the field trial data (Naval Civil Engineering Laboratory, 1987).

Drag anchor performance is also often presented in the form shown in Figs. 27(a) and 27(b) showing percentage of ultimate holding capacity versus percentage of ultimate penetration depth (in the figure for the

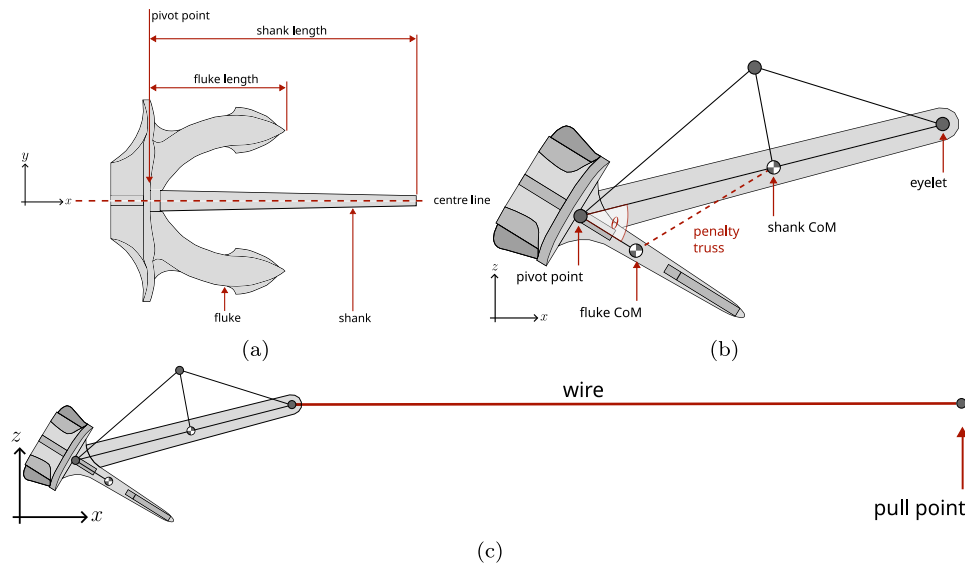


Fig. 22. Hall anchor: (a) top down schematic, (b) side profile and (c) extended profile. The maximum opening angle is 45° .

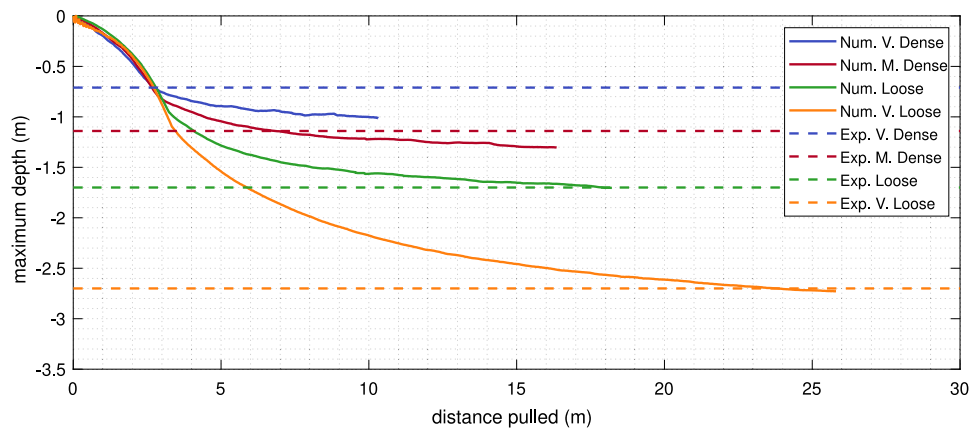


Fig. 23. Hall anchor penetration: Comparison of anchor trajectories with the final depth of the experimental results.

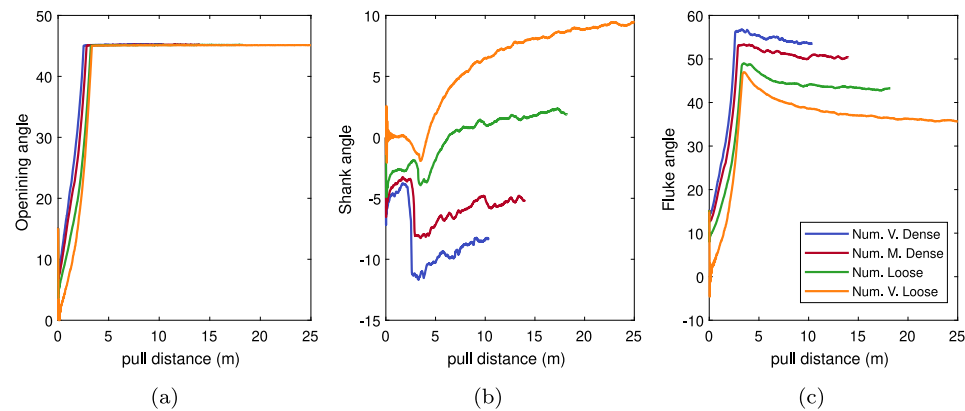


Fig. 24. Hall anchor: The opening (a), shank (b) and fluke angle (c) against pull distance.

AC-14 and Hall anchors in particular). The final absolute values for the penetration depths and holding capacities are provided in Table 8. The MPM results are included for very loose to very dense sand conditions and compared against data from Vryhof (2018) for the Stevin[®] Mk3. The response of the AC-14 and Hall anchors are similar. However,

the shape of the AC-14 and Hall anchor curves are different to the Stevin[®] Mk3 anchor, which generates more of its percentage capacity with a lower proportion of its penetration depth. This highlights the importance of anchor-specific performance charts versus utilisation of standard curves.

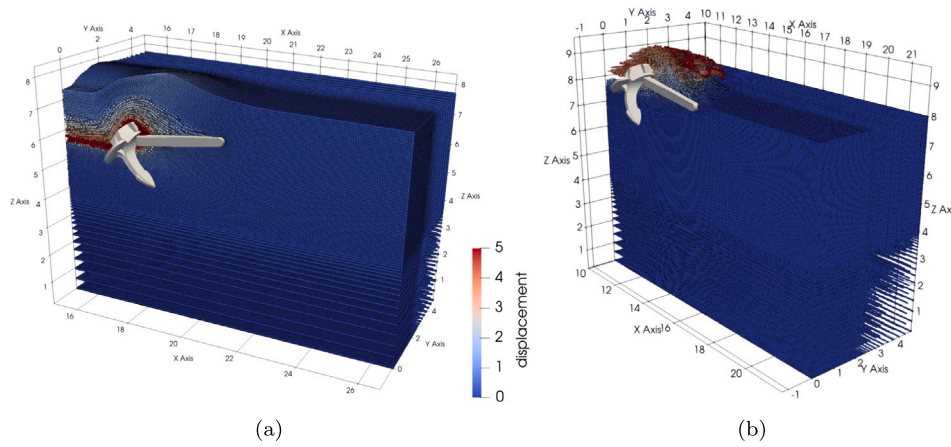


Fig. 25. Hall anchor: anchor position for the very loose at a drag distance of 15 m (a) and very dense sands at a drag distance of 9 m (b).

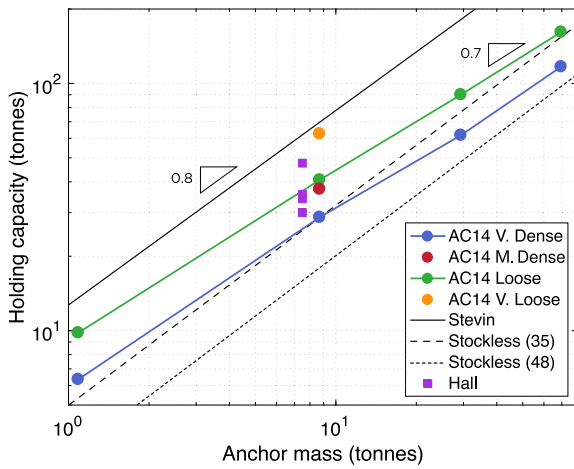


Fig. 26. Anchor performance: holding capacity with anchor mass. AC-14 and Hall numerical results compared with Stevin[®] and US Navy Stockless (35° and 48° fluke angles) lines from *Naval Civil Engineering Laboratory (1987)*.

Table 8

Ultimate Holding Capacity (UHC) and maximum penetration, d_p , for the AC-14 and Hall anchors. Initial penetration under self weight is not included.

Sand state	AC-14		Hall	
	UHC (kN)	d_p (m)	UHC (kN)	d_p (m)
v. dense	283.5	1.03	294.3	1.01
m. dense	360.0	1.38	341.0	1.30
loose	491.5	2.21	323.7	1.70
v. loose	601.4	3.51	479.7	2.73

5.2. CBRA implications

The UK Carbon Trust's Cable Burial Risk Assessment (Carbon Trust, 2015) assumes that the penetration depth of a drag anchor can be approximated as

$$d_p = S_f \sin(\theta_{sf}) l_f, \quad (14)$$

where S_f is the seabed factor (taken as 1 for sands in the CBRA guidance), θ_{sf} is the shank-fluke maximum opening angle (or 45° if the maximum opening angle is not known) and l_f is the fluke length. The results presented in this paper highlight two key shortcomings of the UK Carbon Trust's CBRA assumptions in terms of anchor penetration:

1. Eq. (14) assumes that the shank is horizontal at the seabed surface and the anchor is fixed at its maximum opening angle. The numerical results have shown that the shank is only horizontal for the AC-14 anchor in very dense sands. In other AC-14 anchor cases the shank-fluke joint is below the seabed surface and the shank inclined to the horizontal. In dense and very dense sands the Hall anchor finds equilibrium with the shank pitched forwards, whereas loose sand conditions result in a positive shank angle. These results challenge the geometric assumptions in Eq. (14).
2. The CBRA guidance document gives a constant seabed factor of $S_f = 1$ for sands, suggesting that an anchor will achieve the same penetration depth irrespective of the sand relative density. This is incorrect based on the numerical results of this section and the physical testing of Sharif et al. (2025). This constant seabed factor is linked to Point 1, where it is assumed that the shank is horizontal and at the seabed surface through the anchor drag process.

The anchor scaling tests have shown that the penetration depths associated with the anchor drag process vary linearly with fluke length, l_f , confirming the dependency in Eq. (14). These tests have also highlighted that initial penetration of the anchor under self weight is not linearly dependent on the fluke length; this initial self weight penetration is 5%–10% of the penetration achieved during the drag process over the tested range of scaling factors.

Whilst acknowledging the shortcoming of the CBRA equation, the MPM anchor penetration results represent a chance to provide new sand density dependent seabed penetration factors. Table 9 provides equivalent CBRA seabed factors for the AC-14 anchor calculated from

$$\bar{S}_f^{(\theta_{sf})} = \frac{d_p}{l_p \sin(\theta_{sf})}, \quad (15)$$

where d_p is taken as the penetration depth obtained from the MPM analyses during the drag process, based on a fluke length of $l_f = 1.7$ m (i.e. the results shown in Fig. 15). Values are tabulated using the actual maximum opening angle for the anchor in the MPM simulations (35° in all cases) and 45°, as the default value in the CBRA guidance document. The back-calculated seabed factors for an AC-14 anchor vary between 1.1 and 3.6 for the 35° opening angle and 0.9 and 2.9 for the 45° opening angle, which are in good agreement with the physical testing (Sharif et al., 2025) derived seabed factors, which have a range of 0.6 to 3.4 assuming a 45° opening angle.

Table 10 provides equivalent CBRA seabed factors for the Hall anchor. In this case the opening angle of the anchor from the MPM simulations was 45° for all sand relative densities, the same as the CBRA default value. The back-calculated seabed factors for the Hall

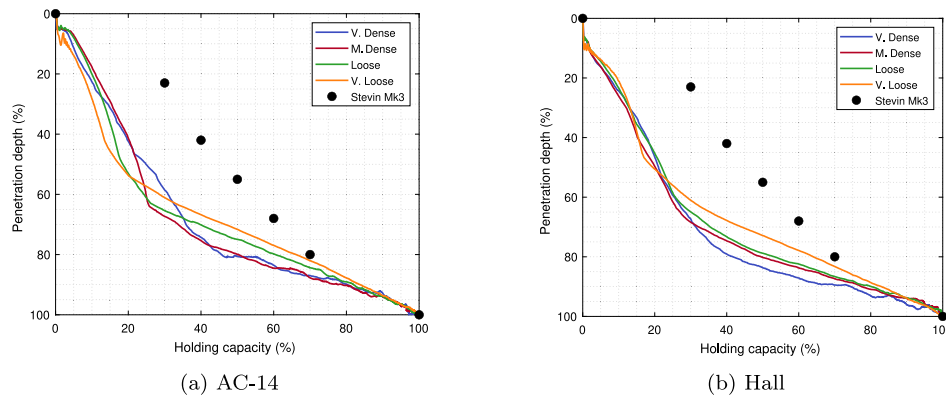


Fig. 27. Anchor performance: percentage penetration depth with holding capacity in different sand conditions. Final absolute values for the penetration depth and holding capacity are provided in Table 8.

Table 9

CBRA AC-14 seabed factors for different sands based on anchor penetration, d_p , from the MPM analyses with an 8.7 tonne anchor with a fluke length of $l_f = 1.7$ m. Initial penetration under self weight is not included.

Sand state	d_p (m)	d_p/l_f	$S_f^{(35^\circ)}$	$S_f^{(45^\circ)}$
v. dense	1.03	0.61	1.06	0.86
m. dense	1.38	0.81	1.42	1.15
loose	2.21	1.30	2.27	1.84
v. loose	3.51	2.06	3.60	2.92

Table 10

CBRA Hall seabed factors for different sands based on anchor penetration, d_p , from the MPM analyses with an 7.5 tonne anchor with a fluke length of $l_f = 1.75$ m. Initial penetration under self weight is not included.

Sand state	d_p (m)	d_p/l_f	$S_f^{(45^\circ)}$
v. dense	1.01	0.57	0.82
m. dense	1.30	0.74	1.05
loose	1.70	0.97	1.37
v. loose	2.73	1.56	2.21

anchor vary between 0.8 and 2.2, which are significantly less than the AC-14 35° opening angle results. The penetration factors for the two anchors with a 45° opening angle are similar in medium dense and very dense sands, but the Hall anchor has less penetration potential in loose sands, likely due to the large plate behind the fluke-shank interface, which is not present on the AC-14 anchor (see Figs. 7 and 22). The back-calculated seabed factors for the AC-14 and Hall anchors suggest that the seabed factor of $S_f = 1$ from the CBRA guidance document is non-conservative beyond the very dense case. Note that the initial penetration under self weight is not included; in this case the seabed factors would increase by $\approx 5\%$ if taken into account. It should also be noted that these seabed factors are considered to be the worst case scenario as they are based on fully drained conditions. This is consistent with the state of the art in the numerical modelling published literature, which also assumes fully drained conditions (Dao and Grabe, 2024). The work of Brown et al. (2025) has shown that rate effects can reduce the penetration depth, especially in the case of loose sands. However, the exact drag speed of an anchor when it crosses a subsea cable would not be known *a priori* when undertaking an CBRA and providing worst case seabed factors facilitates a conservative approach.

6. Conclusions

This paper presents for the first time the application of the Material Point Method (MPM) to the prediction of drag anchor penetration in fully drained sand conditions. This is achieved by extending the work of Bird et al. (2025) in three areas to enable computationally tractable

simulations of geotechnical problems where a rigid body, formed of multiple parts, is dragged a long distance through a highly deformable domain. In particular, the partitioned domain approach proposed in this paper makes the computational cost per time step independent of the total drag distance and it has been shown that if the partitioned domain is sufficiently large the anchor trajectory is unaffected by the reduced domain size. The other two advancements included in this paper that are required to model drag anchors are: allowing for assemblies of rigid bodies; and an improved way of modelling rotational inertia of the rigid bodies. A CPT-based anchor penetration approach was also developed, where material properties of the sand seabed are calibrated using physical CPTs. This sand condition is then used to predict the trajectory and the maximum penetration depth of a drag anchor. The framework was validated across a range of relative density sands using geotechnical centrifuge experimental results on two articulated anchor designs, achieving good predictions for the ultimate penetration depths in most cases. The proposed method is predictive, in that no numerical, or material, parameter tuning is performed once the material properties have been set based on CPT data.

Variations in the anchor size in the numerical analysis have confirmed that the penetration depth and the holding capacity scale with the fluke length and the fluke length squared, respectively, and that the holding capacity depends on the sand state as well as the anchor mass. The simulations have also revealed two key issues with the CBRA anchor penetration estimation equation, namely the assumption: (i) of a constant seabed factor of 1 irrespective of the sand state; and (ii) that the shank is horizontal and at the seabed surface. Back analysis of the simulation results suggest seabed penetration factors between 0.8 and 3.6 depending on the sand state, assumed opening angle and the anchor type, highlighting the potentially non-conservatism of the CBRA suggested penetration factor for sands. However, it should be highlighted that the results in this paper represent the worst case scenario of a slow pull in fully drained sand and that anchor penetration is reduced when anchors are pulled at higher rates in saturated sands, especially in loose conditions (Brown et al., 2025). The seabed factors found in this paper are also only applicable to the specific seabed conditions and anchor types considered as part of this study.

In summary, this paper provides a framework for predicting anchor penetration, which has been shown to provide good predictions over a very wide range of sand relative densities. The underlying material model can be calibrated using site investigation data that is routinely available along a cabling route, which opens the door to performing site specific analysis for anchor penetration within the CBRA framework.

CRedit authorship contribution statement

Robert E. Bird: Writing – review & editing, Writing – original draft, Validation, Software. **William M. Coombs:** Writing – review &

editing, Writing – original draft, Supervision, Project administration, Funding acquisition, Conceptualization. **Michael J. Brown:** Writing – review & editing, Validation, Supervision, Funding acquisition. **Charles E. Augarde:** Writing – review & editing, Writing – original draft, Supervision, Funding acquisition. **Yaseen U. Sharif:** Data curation. **Giuliano Pretti:** Writing – review & editing. **Catriona Macdonald:** Funding acquisition. **Duncan Stevens:** Data curation. **Gareth Carter:** Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. Determining $\delta\theta$, $\Delta\ddot{\theta}$ and $\Delta\delta\theta$

In this section the definitions of $\delta\theta$, $\Delta\ddot{\theta}$ and $\Delta\delta\theta$ are presented. To make the derivation clearer the following abuse of notation, *applied only in this section*, is used for the difference of position, velocity and acceleration for the truss nodes A and B, see Fig. 2:

$$\mathbf{x} := \mathbf{x}^{AB} = \mathbf{x}_B - \mathbf{x}_A \quad (\text{A.1})$$

$$\mathbf{v} := \mathbf{v}^{AB} = \mathbf{v}_B - \mathbf{v}_A \quad (\text{A.2})$$

$$\dot{\mathbf{v}} := \dot{\mathbf{v}}^{AB} = \dot{\mathbf{v}}_B - \dot{\mathbf{v}}_A \quad (\text{A.3})$$

Before defining the linearisation of $\ddot{\theta}$, it is necessary to define the first and second derivatives of θ , respectively

$$\frac{\partial\theta}{\partial\mathbf{x}} = \begin{bmatrix} -x_3/(x_1^2 + x_3^2) \\ x_1/(x_1^2 + x_3^2) \end{bmatrix} \quad (\text{A.4})$$

and

$$\frac{\partial^2\theta}{\partial\mathbf{x}^2} = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \quad \text{where} \quad \begin{aligned} A &= (2x_1x_3)/(x_1^2 + x_3^2)^2 \\ B &= -(x_1^2 - x_3^2)/(x_1^2 + x_3^2)^2. \end{aligned} \quad (\text{A.5})$$

Additional the rotational velocity and acceleration have the respective forms

$$\dot{\theta} = \frac{\partial\theta}{\partial t} = \frac{\partial\theta}{\partial\mathbf{x}} \cdot \frac{\partial\mathbf{x}}{\partial t} \quad (\text{A.6})$$

where t is time and

$$\ddot{\theta} = \mathbf{v} \cdot \frac{\partial^2\theta}{\partial\mathbf{x}^2} \cdot \mathbf{v} + \frac{\partial\theta}{\partial\mathbf{x}} \cdot \dot{\mathbf{v}} \quad (\text{A.7})$$

A.1. Variation of θ

As θ is a function of only $\mathbf{x}$ its variations takes the form,

$$\delta\theta = \frac{\partial\theta}{\partial\mathbf{x}} \cdot \frac{\partial\mathbf{x}}{\partial t} \delta t = \frac{\partial\theta}{\partial\mathbf{x}} \cdot \delta\mathbf{x} \quad (\text{A.8})$$

where $\partial\theta/\partial\mathbf{x}$ is defined in Eq. (A.4).

A.2. Linearisation of $\delta\theta$

The linearisation of $\delta\theta$, defined in Eq. (A.8), is

$$\Delta\delta\theta = \Delta\left(\frac{\partial\theta}{\partial\mathbf{x}} \cdot \delta\mathbf{x}\right) = \Delta\left(\frac{\partial\theta}{\partial\mathbf{x}}\right) \cdot \delta\mathbf{x} \quad (\text{A.9})$$

where

$$\Delta\left(\frac{\partial\theta}{\partial\mathbf{x}}\right) = \frac{\partial^2\theta}{\partial\mathbf{x}^2} \cdot \frac{\partial\mathbf{x}}{\partial t} \Delta t = \frac{\partial^2\theta}{\partial\mathbf{x}^2} \cdot \Delta\mathbf{x} \quad (\text{A.10})$$

and $\partial^2\theta/\partial\mathbf{x}^2$ is defined in Eq. (A.5).

A.3. Linearisation of $\ddot{\theta}$

The linearisation of $\ddot{\theta}$ is defined $\Delta\ddot{\theta}$ and it takes the form

$$\Delta\ddot{\theta} = \Delta\left(\mathbf{v} \cdot \frac{\partial^2\theta}{\partial\mathbf{x}^2} \cdot \mathbf{v}\right) + \Delta\left(\frac{\partial\theta}{\partial\mathbf{x}} \cdot \dot{\mathbf{v}}\right) \quad (\text{A.11})$$

$$= \Delta\mathbf{v} \cdot \frac{\partial^2\theta}{\partial\mathbf{x}^2} \cdot \mathbf{v} + \mathbf{v} \cdot \frac{\partial^2\theta}{\partial\mathbf{x}^2} \cdot \Delta\mathbf{v} + \mathbf{v} \cdot \Delta\left(\frac{\partial^2\theta}{\partial\mathbf{x}^2}\right) \cdot \mathbf{v} \quad (\text{A.12})$$

$$+ \Delta\left(\frac{\partial\theta}{\partial\mathbf{x}}\right) \cdot \dot{\mathbf{v}} + \frac{\partial\theta}{\partial\mathbf{x}} \cdot \Delta\dot{\mathbf{v}}. \quad (\text{A.13})$$

where $\Delta(\partial\theta/\partial\mathbf{x})$ is defined in Eq. (A.10) and the remaining linearisation of $\mathbf{v}$, $\dot{\mathbf{v}}$, and $\Delta(\partial^2\theta/\partial\mathbf{x}^2)$ are provided below. The expressions for $\dot{\mathbf{v}}$ and $\mathbf{v}$, and their subsequent linearisation, take the Newmark form since the Newmark method is used to integrate time. The linearisation of acceleration has the form

$$\dot{\mathbf{v}} = \dot{\mathbf{v}}_{n+1} = \frac{\mathbf{x}_{n+1} - \mathbf{x}_n}{\beta \Delta_n t^2} - \frac{\mathbf{v}_n}{\beta \Delta_n t} - \dot{\mathbf{v}}_n \left(\frac{1}{2\beta} - 1\right) \quad (\text{A.14})$$

$$\Delta\dot{\mathbf{v}} = \Delta\dot{\mathbf{v}}_{n+1} = \frac{\Delta\mathbf{x}_{n+1}}{\beta \Delta_n t^2} = \frac{\Delta\mathbf{x}}{\beta \Delta_n t^2} \quad (\text{A.15})$$

where Δ_n is the time integration increment. The linearisation of the velocity is

$$\mathbf{v} = \mathbf{v}_{n+1} = \gamma \frac{\mathbf{x}_{n+1} - \mathbf{x}_n}{\beta \Delta_n t} + \mathbf{v}_n \left(1 - \frac{\gamma}{\beta}\right) + \Delta_n t \dot{\mathbf{v}}_n \left(1 - \frac{\gamma}{2\beta}\right); \quad (\text{A.16})$$

$$\Delta\mathbf{v} = \Delta\mathbf{v}_{n+1} = \gamma \frac{\Delta\mathbf{x}_{n+1}}{\beta \Delta_n t} = \gamma \frac{\Delta\mathbf{x}}{\beta \Delta_n t}. \quad (\text{A.17})$$

Last the linearisation of the term $\partial^2\theta/\partial\mathbf{x}^2$ is expressed as

$$\Delta\left(\frac{\partial^2\theta}{\partial\mathbf{x}^2}\right) = \frac{\partial}{\partial\mathbf{x}} \left(\frac{\partial^2\theta}{\partial\mathbf{x}^2}\right) \cdot \Delta\mathbf{x} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \text{where} \quad (\text{A.18})$$

$$\begin{aligned} A &= \begin{bmatrix} -2x_3(3x_1^2 - x_3^2) & 2x_1(x_1^2 - 3x_3^2) \\ (x_1^2 + x_3^2)^3 & (x_1^2 + x_3^2)^3 \end{bmatrix}^T \cdot \Delta\mathbf{x} \\ B &= \begin{bmatrix} 2x_1(x_1^2 - 3x_3^2) & 2x_3(3x_1^2 - x_3^2) \\ (x_1^2 + x_3^2)^3 & (x_1^2 + x_3^2)^3 \end{bmatrix}^T \cdot \Delta\mathbf{x} \\ C &= \begin{bmatrix} 2x_1(x_1^2 - 3x_3^2) & 2x_3(3x_1^2 - x_3^2) \\ (x_1^2 + x_3^2)^3 & (x_1^2 + x_3^2)^3 \end{bmatrix}^T \cdot \Delta\mathbf{x} \\ D &= \begin{bmatrix} 2x_3(3x_1^2 - x_3^2) & -2x_1(x_1^2 - 3x_3^2) \\ (x_1^2 + x_3^2)^3 & (x_1^2 + x_3^2)^3 \end{bmatrix}^T \cdot \Delta\mathbf{x} \end{aligned}$$

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