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Estimating geothermal gradient from seismic velocities using rock physics models

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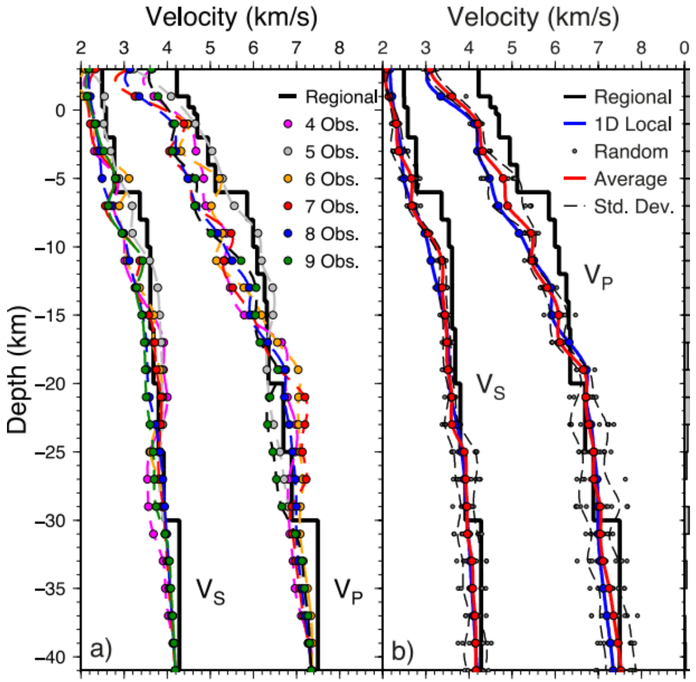
Introduction

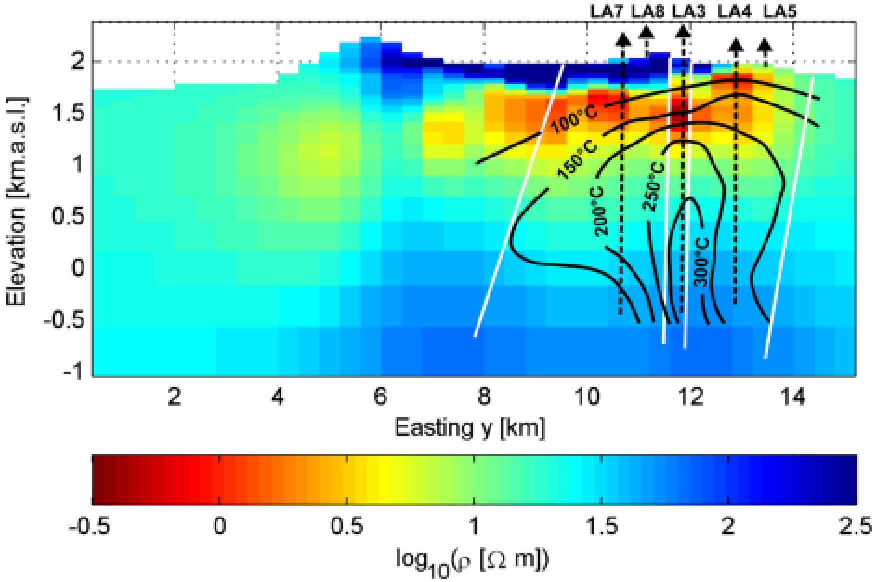
- Aluto is a stratovolcano in the Central Main Ethiopian Rift, part of the East African Rift.
- Geophysical measurements e.g. passive seismicity, MT surveys have been conducted.
- Image subsurface structures and regions of partial melt and over-pressured gases, detect hydrothermal and magmatic reservoirs.

Wilks et al. (2020)



1D seismic velocities estimated from earthquake data

Samrock et al. (2015)

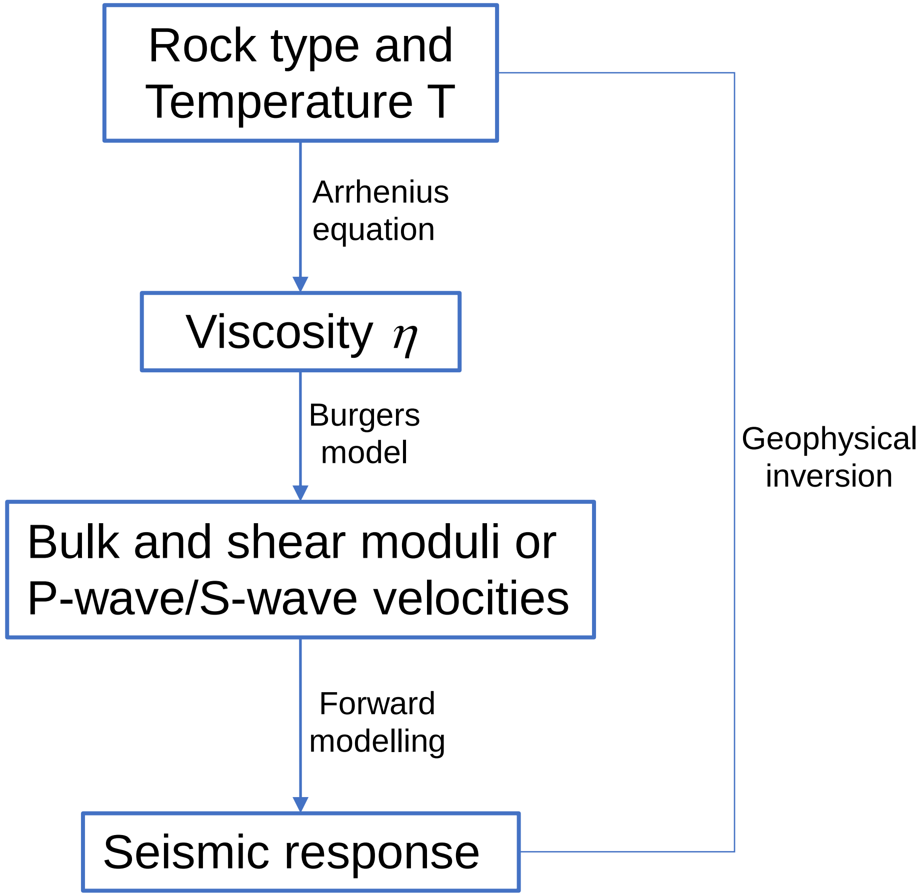


Resistivity mapped from magnetotelluric (MT) surveys

Motivation

- Investigate rock physics models relating rock properties with temperature.
  - **Burgers model**: describe the linear viscoelastic effect for a dry hot rock frame at very high temperatures
  - **Arrhenius equation**: establish link between viscosity and temperature
  - **Chapman et al. (2002) squirt flow model**: calculate the relaxed and unrelaxed moduli of a rock.

- Estimate temperature parameters from seismic response or resistivity.



Burgers model

Carcione et al.(2006) suggested the use of the Burgers model to describe the linear viscoelastic creep for ductile media, where stress is not only linearly related to strain, but also depends on the time variation rate of strain.

Creep function

$$\chi(t) = \left(\frac{t}{\eta} + \frac{1}{\mu_0}\left[1 - \left(1 - \frac{\tau_\sigma}{\tau_\epsilon}\right)\exp\left(-\frac{t}{\tau_\epsilon}\right)\right]\right) H(t).$$

Frequency-dependent complex shear modulus

$$\mu_B = [\mathrm{F}(\dot{\chi})]^{-1} \longrightarrow \mu_B(\omega) = \frac{\mu_0(1 + i\omega\tau_\epsilon)}{1 + i\omega\tau_\sigma - \frac{i\mu_0}{\omega\eta_B}(1 + i\omega\tau_\epsilon)}$$

Complex velocities

$$v_P(\omega) = \sqrt{\frac{K + 4\,\mu_B/3}{\rho}} \qquad v_s(\omega) = \sqrt{\frac{\mu_B}{\rho}}$$

Phase velocities

$$c = \left[Re\left(\frac{1}{v_s}\right)\right]^{-1}$$

Shear attenuation

$$\frac{1}{Q} = \frac{Imag(\mu_B)}{Real(\mu_B)}$$

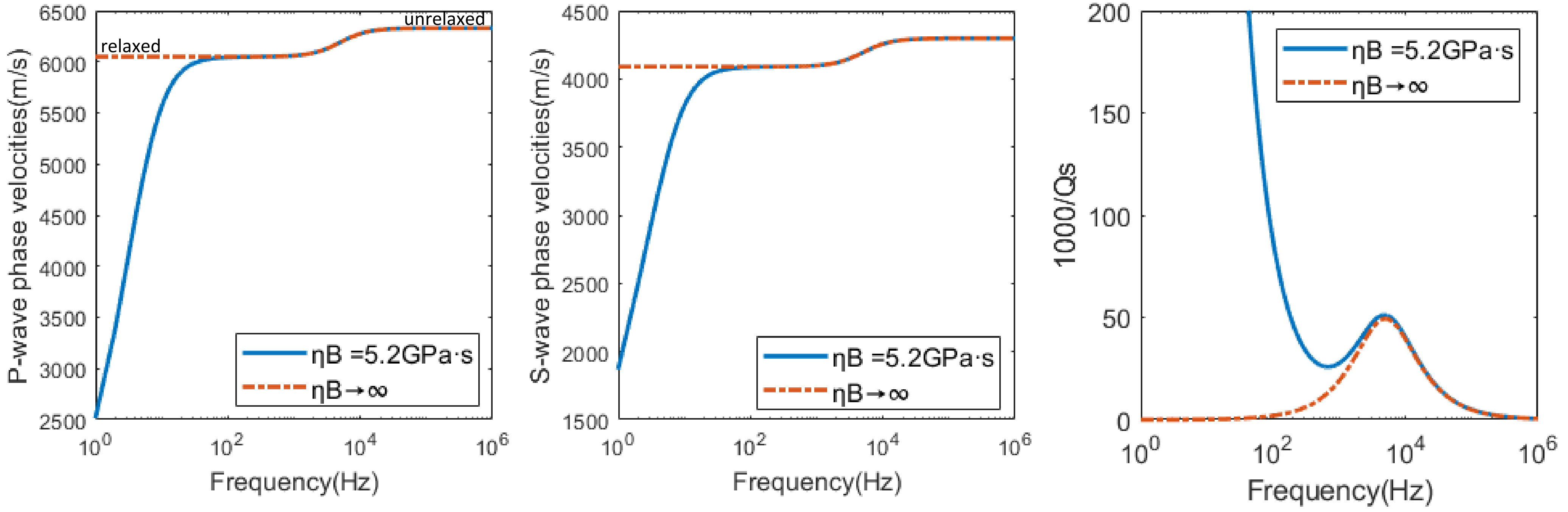
where *t* is time and *H*(*t*) is the Heaviside step function. τ_σ and τ_ϵ are seismic relaxation times for shear deformations, μ_0 is the relaxed shear modulus, and η_B is the Burgers flow viscosity, describing the ductile behaviour related to shear deformations.

Parameters used for the Burgers model, density and velocities are from Mavko et al. (2009) for quartz.

Quartz Model

grain	Vp_relaxed (m/s)	Vp_unrelaxed (m/s)	Vs_relaxed (m/s)	Vs_unrelaxed (m/s)	Density (kg/m³)	t_ϵ(s)	t_σ (s)
quartz	6050	6356	4090	4205	2650	0.0172	0.0147

Velocities and attenuation vary with frequency for the two cases.



The velocities and attenuation curves for Burgers model at η_B = 5.2GPa·s and elastic model $\eta_B \rightarrow \infty$, respectively.

Elastic model: typical high and low frequency bounds for the real part of moduli.

Burgers model: high attenuation at seismic band (low frequencies) due to highly viscous property of the melting rock.

The Arrhenius equation

According to Carcione and Poletto (2013), viscosity η can be expressed by the Arrhenius equation as a function of absolute temperature T,

$$\eta = \frac{\sigma_0}{2\dot{\epsilon}} \qquad \dot{\epsilon} = A_\infty \sigma_0^n \exp(-E/RT)$$

where σ_0 is the octahedral stress given in MPa, $\dot{\epsilon}$ is the steady-state creep rate, A_∞ and *n* are constants, which determine the viscosity at infinite temperature together with σ_0, *E* is the activation energy, or the energy barrier for the rock to melt. R = 8.3144 J/mol/K is the universal gas constant.

Octahedral stress

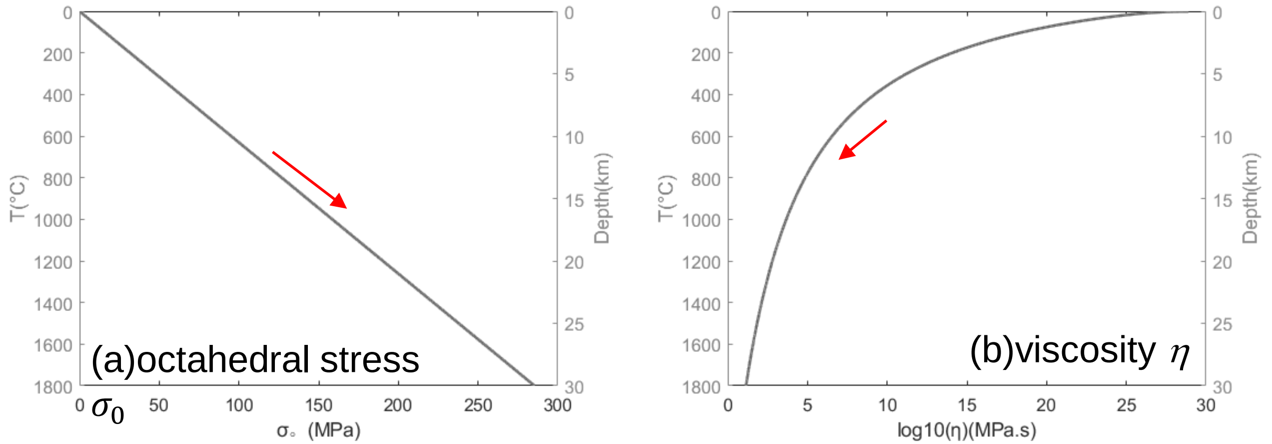
$$\sigma_0 = \frac{1}{3}\sqrt{(\sigma_v - \sigma_h)^2 + (\sigma_v - \sigma_H)^2 + (\sigma_h - \sigma_H)^2} \qquad \sigma_v(z) = \bar{\rho} g z \qquad \sigma_H = \frac{\upsilon \sigma_v}{1-\upsilon} \qquad \sigma_h = \xi \sigma_H$$

where $\sigma_v, \sigma_H, \sigma_h$ are the vertical lithostatic stress, the maximum and minimum horizontal tectonic stresses respectively. $\bar{\rho}$ and υ are average density and Poisson's of the overburden.

Constant overburden density and geothermal gradient

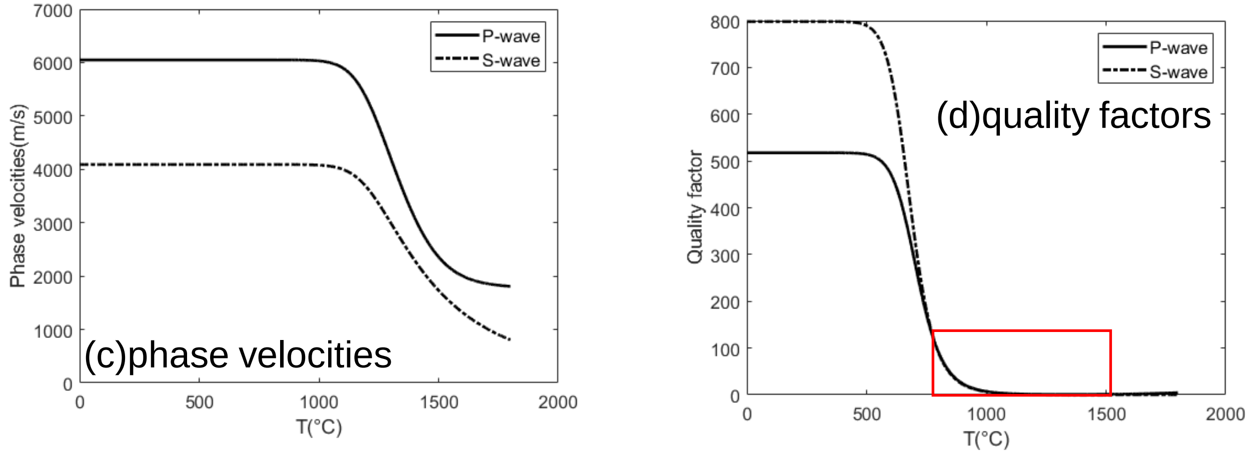
Parameters used for the Arrhenius equation from Carcione and Poletto (2013).

A_∞ ($(MPa)^{-n}s^{-1}$)	E (kJ/mol)	$\bar{\rho}$ (Kg/m^3)	ξ	n	G (°C/km)	υ
10^{-2}	134	2650	0.8	2.6	60	0.2

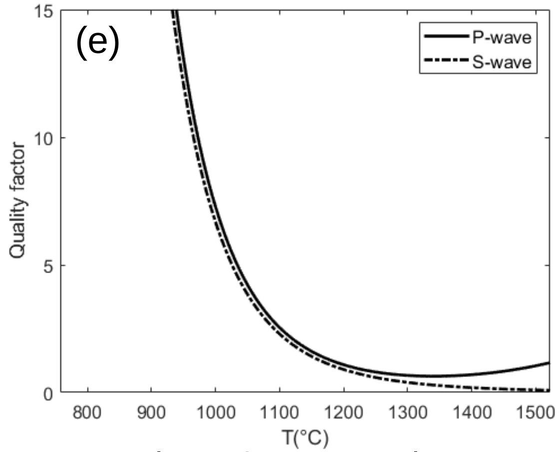


Octahedral stress σ_0 and viscosity η as a function of depth and temperature.

(a) Octahedral stress linearly increases with temperature,
(b) Viscosity decreases with temperature.
(c) Phase velocities do not change with temperature before 1100°C, after which a melting zone occurs.
(d) P-wave and S-wave Quality factors start to decrease at a relatively lower temperature at 750°C.



P-wave and S-wave phase velocities (left) and quality factors (right) as a function of temperature for a constant overburden density and constant geothermal gradient model.



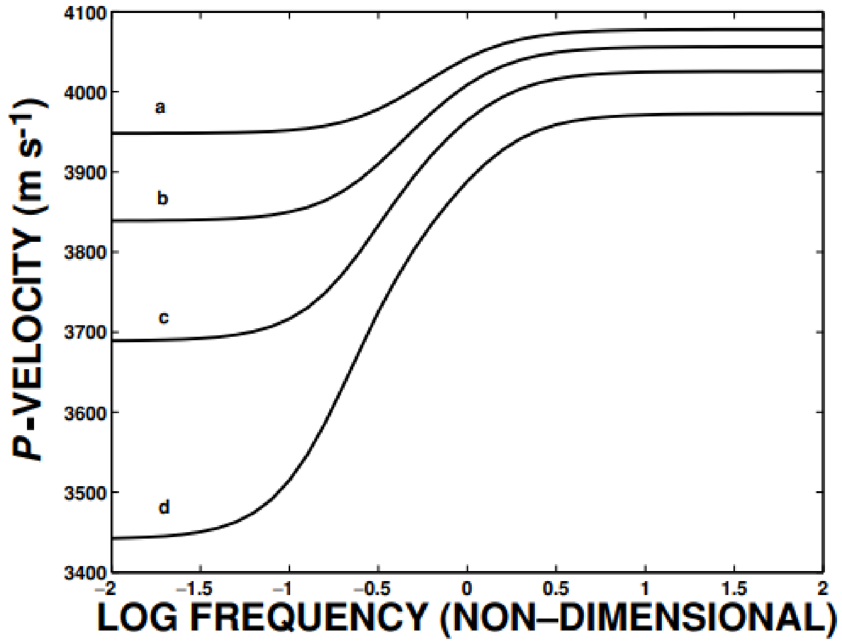
Enlarge the rectangular area

max(Qp)=517.5
min(Qp)=0.6

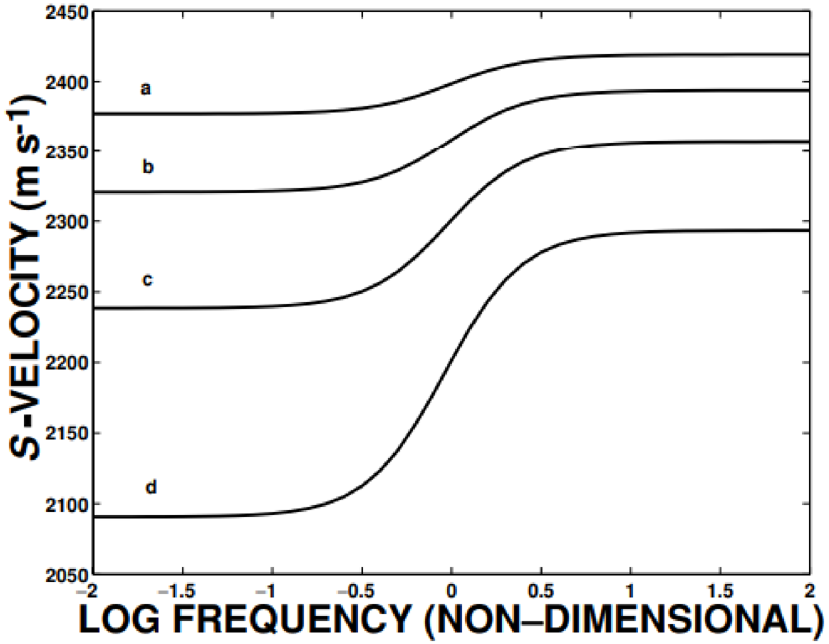
max(Qs)= 798.2
min(Qs)= 0.02

Chapman et al. (2002) squirt flow model

Chapman et al. (2002) model is used to calculate relaxed (low frequency limit) and unrelaxed (high frequency limit) bulk and shear moduli for Burgers model at each depth. These parameters are used to decide the time scale parameters $\tau_\sigma, \tau_\epsilon$.

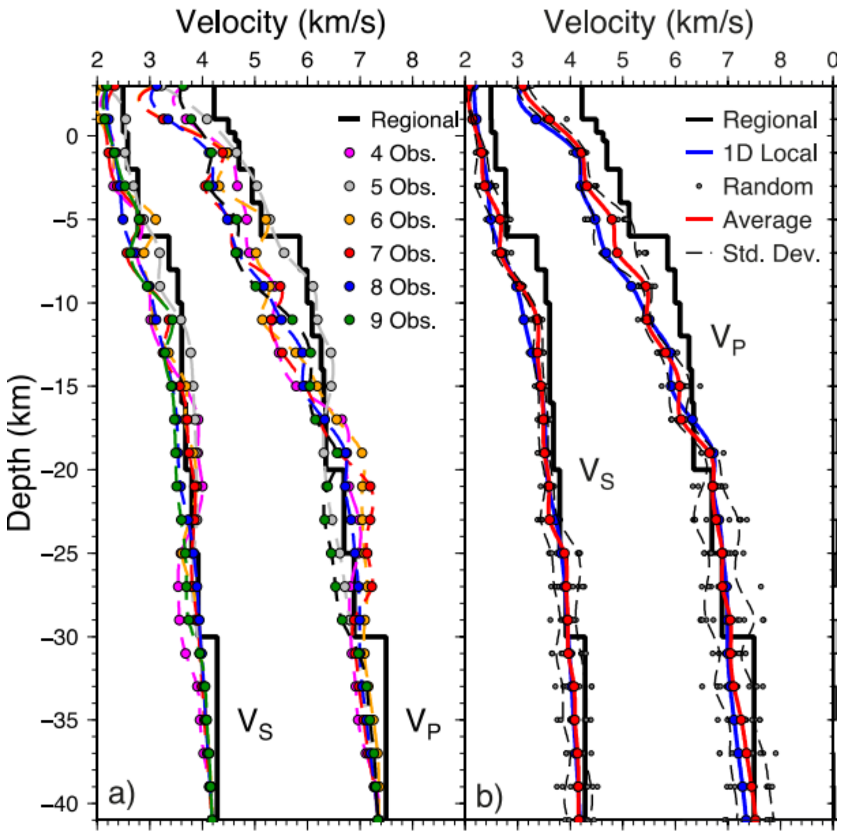


P-wave velocity as a function of log($\omega\tau$) for effective stresses of: (a) 40 MPa, (b) 30 MPa, (c) 20 MPa and (d) 10 Mpa.

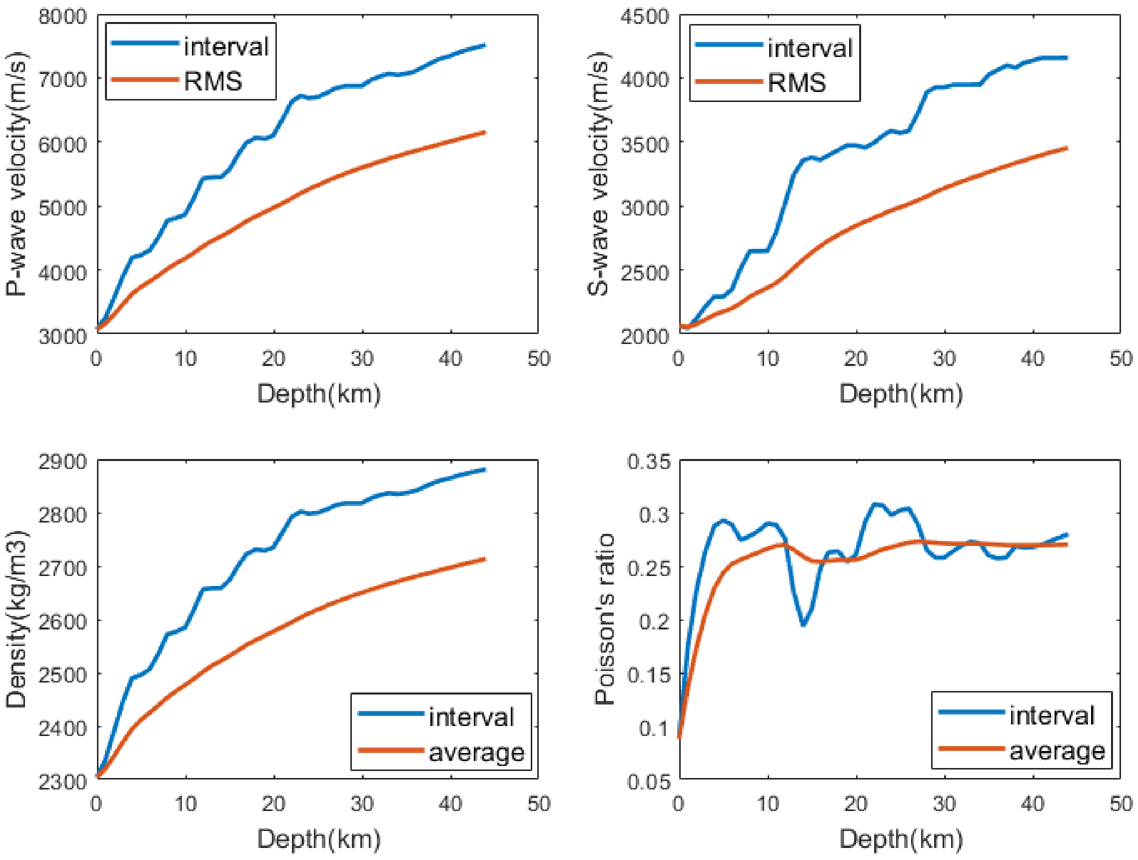


S-wave velocity as a function of log($\omega\tau$) for effective stresses of: (a) 40 MPa, (b) 30 MPa, (c) 20 MPa and (d) 10 Mpa.

Application to Aluto geothermal field



Wilks et al. (2020)



Gardner et al. (1984) density-velocity relation,

$$\rho = 1.74V_p^{0.25}$$

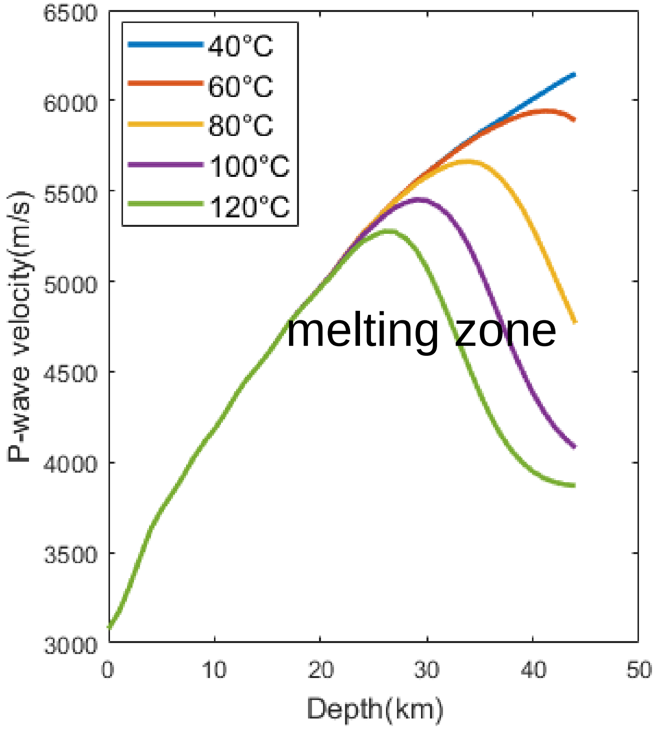
where the unit of ρ is g/cm^3, and the unit of V_p is km/s.

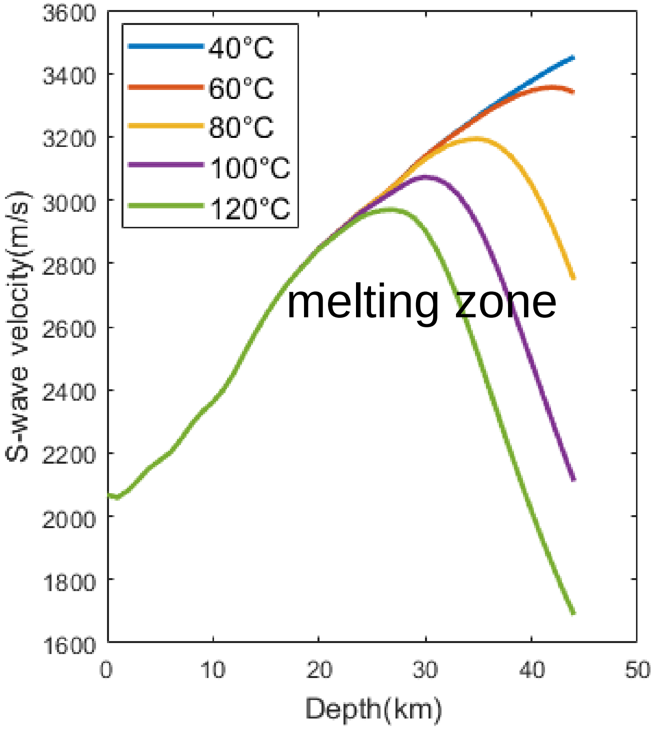
Poisson's ratio $$\boldsymbol{v} = \frac{1}{2}\frac{(\boldsymbol{V_P}/\boldsymbol{V_S})^2 - 2}{(\boldsymbol{V_P}/\boldsymbol{V_S})^2 - 1}$$

- Wilks et al. (2020) estimated the P and S wave velocities.
- Density and Poisson's ratio of overburden are then estimated from velocities.

geothermal gradient G

A_∞ $((MPa)^{-n}s^{-1})$	E (kJ/mol)	ξ	n	G (°C/km)	f (Hz)
10^{-6}	134	0.8	2.6	varying	1



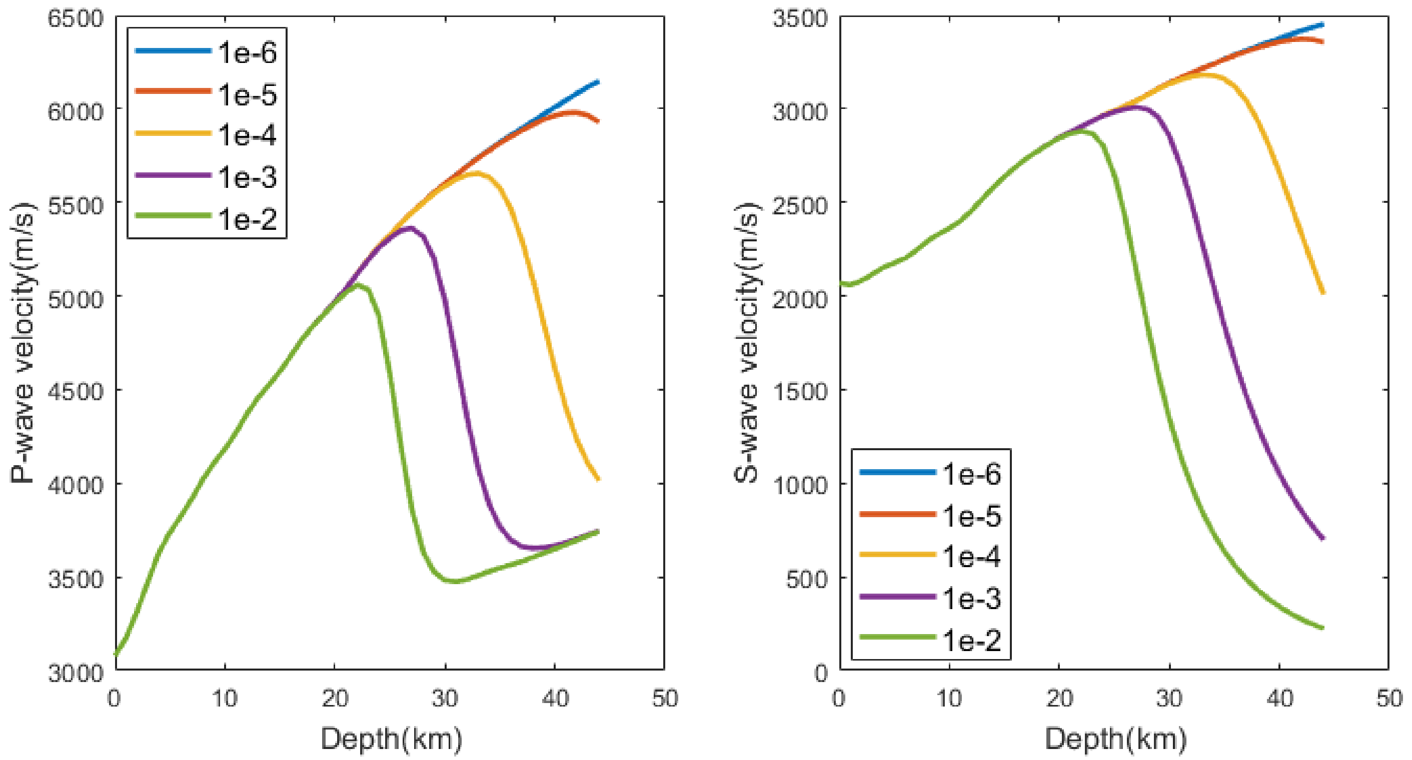


- Both velocities increase with depth up to about 25 km, which is different from that of a constant overburden density value.
- Both velocities are still not sensitive to temperature before 25km. A higher geothermal gradient value leads to a shallower melting zone.

The calculated P-wave and S-wave velocities vary with geothermal gradient using Burgers model and Arrhenius equation.

A_∞

A_∞ ((MPa)$^{-n}$s^{-1})	E (kJ/mol)	ξ	n	G (°C/km)	f (Hz)
varying	134	0.8	2.6	40	1

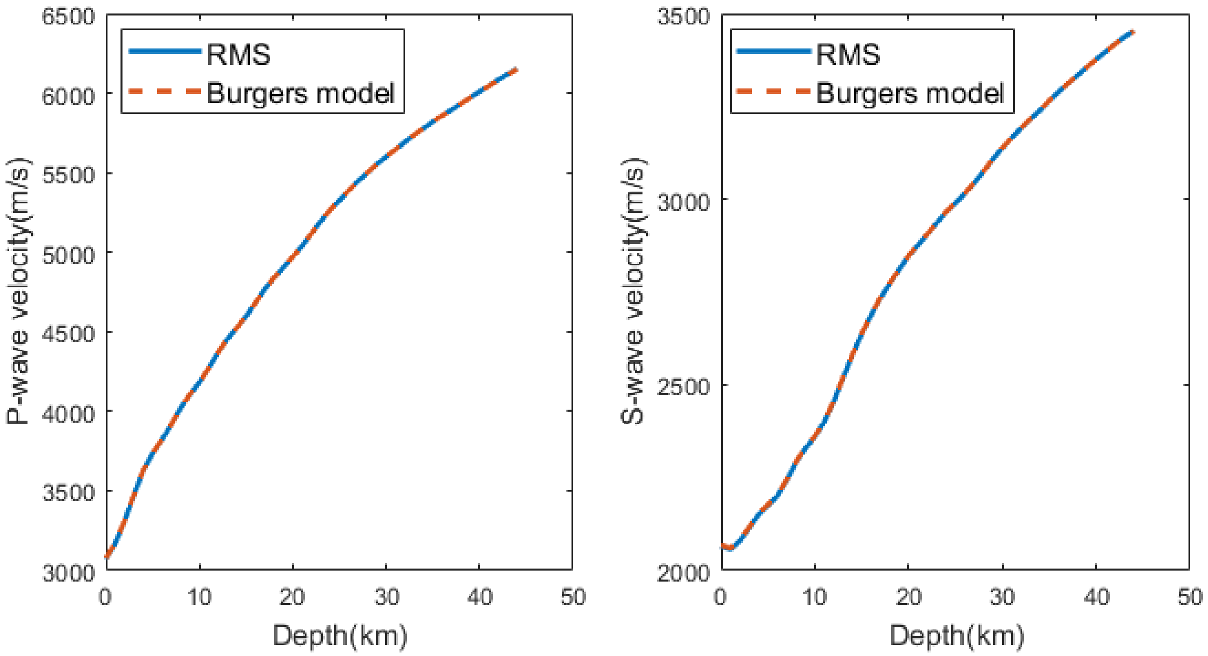


The calculated P-wave and S-wave velocities vary with A_∞ using Burgers model and Arrhenius equation.

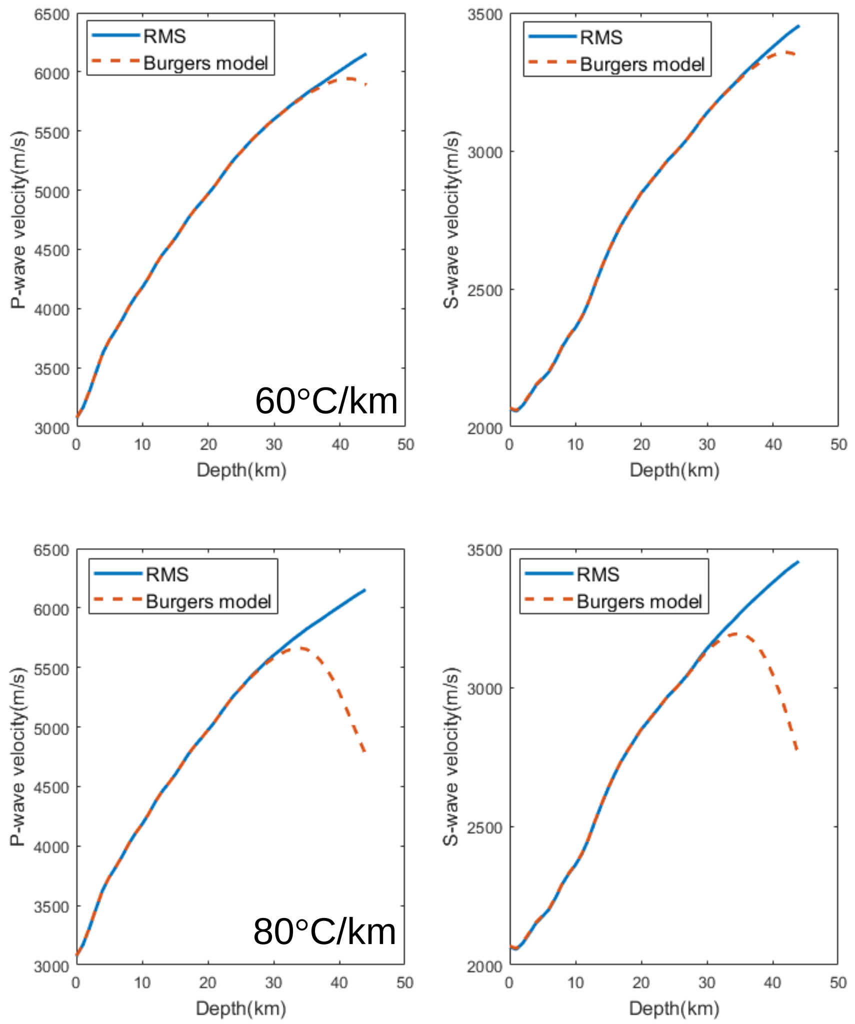
- A higher A_∞ value leads to a shallower melting zone.
- When A_∞ = 0.01 after the melting zone, the P-wave velocity increases with depth again.
- A higher A_∞ can also reduce the melting temperature to a lower value.

Fitting result

A_∞ ((MPa)$^{-n}$s^{-1})	E (kJ/mol)	ξ	n	G (°C/km)	f (Hz)
10^{-6}	134	0.8	2.6	40	1



The fitting result estimates the geothermal gradient G=40°C/km.



Conclusions

The Burgers model is used to describe the linear viscoelastic effect of a dry hot rock frame at very high temperatures. A combination with the Arrhenius equation allows geothermal gradient to be estimated from seismic velocities. The Chapman et al.(2002) model is used to calculate the relaxed and unrelaxed velocities of rock at different depths, which determine the time scale parameters of stress and strain.

From the application in Aluto geothermal field, we can conclude that a combination of the three models has the potential to estimate regional geothermal gradient deep to the lower crust and upper mantle if we know the P-wave and S-wave velocities from the surface to a certain depth of interest.

Future studies will be focused on applying rock physics models to estimate subsurface temperature distribution and local geothermal gradient caused by magma chambers.

Acknowledgements

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Link to the report:

Reviewing the relations of seismic velocities and electrical resistivity with the temperature of high-enthalpy geothermal reservoirs with an example from an East African rift volcano

https://nora.nerc.ac.uk/id/eprint/536212/