SOLUTION OF THE GYPSUM CLIFF (PERMIAN, MIDDLE MARL) BY THE RIVER URE AT RIPON PARKS, NORTH YORKSHIRE

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SUMMARY

Gypsum in the cliff on the west bank of the River Ure at Ripon Parks is being dissolved by the river. The solution rate has been determined from the solution of a large fallen block in the river and is close to laboratory experimental values. In the centre of the river and around the gypsum block flow rates of 1 m/s have been measured. Laboratory based experiments suggest that such rates would produce undercutting of massive gypsum at about 1.7 m/year. However, flow rates adjacent to the cliff are substantially reduced and much slower rates of undercutting, around 0.10 to 0.18 m/year, are predicted. Observed minimum rates of undercutting are close to these values. The massive gypsum at the southern end of the cliff appears to have been undercut up to 5 m in 50 to 60 years, i.e., between 0.10 and 0.08 m/year, and the northern part of the cliff has been undercut a similar distance in around 100 years.

I. INTRODUCTION

Gypsum readily dissolves in water, a process that has been studied under laboratory conditions by James and Lupton (1978). The solution rates have been related to the design of hydraulic structures (James and Kirkpatrick 1980). James and Lupton (1978) showed that the solubility and specific rates of solution influenced the enlargement of fissures or the removal of disseminated particles in dam foundations. However, their paper was theoretical for direct visible evidence of solution phenomena is hard to find in nature. Nevertheless, the results of underground solution are obvious in many areas as shown by numerous sink holes developed in gypsum and by the collapse of overlying strata. Such phenomena are particularly well developed in the area around Ripon in North Yorkshire (Tute 1870; Smith 1972). In view of such evidence of large-scale solution, the presence at Ripon Parks (SE 307753; Figure 1) of a large natural cliff section exposing gypsum, which is subject to direct contact with water of the River Ure, would appear to be anomalous.

The authors agreed that this gypsum cliff might provide a unique, readily observable example of gypsum solution and allow comparisons of the actual and theoretical solution and retreat rates of a gypsum cliff. It was also hoped to throw some light on the history of the cliff and to explain its continued presence. It was agreed that the rate of gypsum solution and retreat of the cliff might be calculated, albeit somewhat crudely, from the equations derived by James and Lupton (1978). It was first predicted that such a gypsum cliff should retreat at between about 0.1-1.5 m/year depending upon river flow velocities and if the course of the river persisted in following the line of retreat. This retreat rate is sufficiently high to suggest that it may be directly observable over a period of several years. Field work and research were undertaken between 1977 and 1981 to measure the solution rates of the cliff.

II. GEOLOGY

The gypsum cliff at Ripon Parks has been known to geologists for over a century. It is shown on the Primary Geological Survey by A. C. G. Cameron and H. H. Howell in 1880 and was mentioned by Tute (1868,1870,1884) and by Fox-Strangways (1886). Kendall and Wroot (1924) briefly described and illustrated the section and it is mentioned by Versey (1948, p. 55), but the first detailed description of the section and its structural complexity was given by Forbes (1958). Further details were given by Smith (1974, p. 380).

The generalised stratigraphical sequence in the area is shown in Figure 1. A detailed section of the gypsum at Ripon Parks (Figure 2) was measured at the southern end of the cliff, where three main lithological divisions were recognised.

The lowest exposed part of the sequence consists of up to 7.10 m of massive grey alabastrine and porphyroblastic gypsum which is colour banded and in places finely laminated, with interbedded grey gypsiferous mudstone partings in the bottom 1.20 m. Sporadic, mainly concordant, fibrous gypsum veins are also present. This part of the sequence forms the southern end of the cliff.

The middle 2.83 m of the sequence are mainly composed of red and grey marl and mudstone with numerous thin beds of grey alabastrine and porphyroblastic gypsum, and thin beds of pink and grey gypsiferous mudstone. Nodular gypsum is also present and there are abundant concordant and discordant fibrous gypsum veins.

The top 3.73 m of the sequence consist mainly of brownish red and pink marl and mudstone with, near the base, several thin beds and lenses of buff-coloured gypsiferous dolomite, which locally contain chalcopyrite. Abundant concordant fibrous gypsum veins are also present. This division occurs mainly in the northern half of the cliff.

The above sequence is disposed in a series of tight folds which are illustrated in Plates 1 and 2, and also by Kendall and Wroot (1924, p. 269) and Forbes (1958). These folds expose all the lithological divisions to the action of the river and differential erosion and solution occur. The origin of these folds, discussed by Forbes (1958), is uncertain; they may result from tectonic movements, glacial overriding of the section, periglacial phenomena, foundering following solution of underlying evaporites, or, as favoured by the authors, from the expansion resulting from the hydration of anhydrite to gypsum.

Kendall and Wroot (1924) and Forbes (1958) regarded the beds exposed in the cliff as belonging to the Upper Marl of the local Permian succession. The present authors believe, following Tute (1884) and Smith (1974), that they should be referred, on both lithological and stratigraphical grounds, to the Middle Marl (Figure 1). The presence of dolomite beds, a probable 'feather edge' of the Kirkham Abbey Formation (Smith 1974), are further evidence in support of this conclusion. The massive gypsum at the base of the section is a likely correlative of the Hayton Anhydrite.

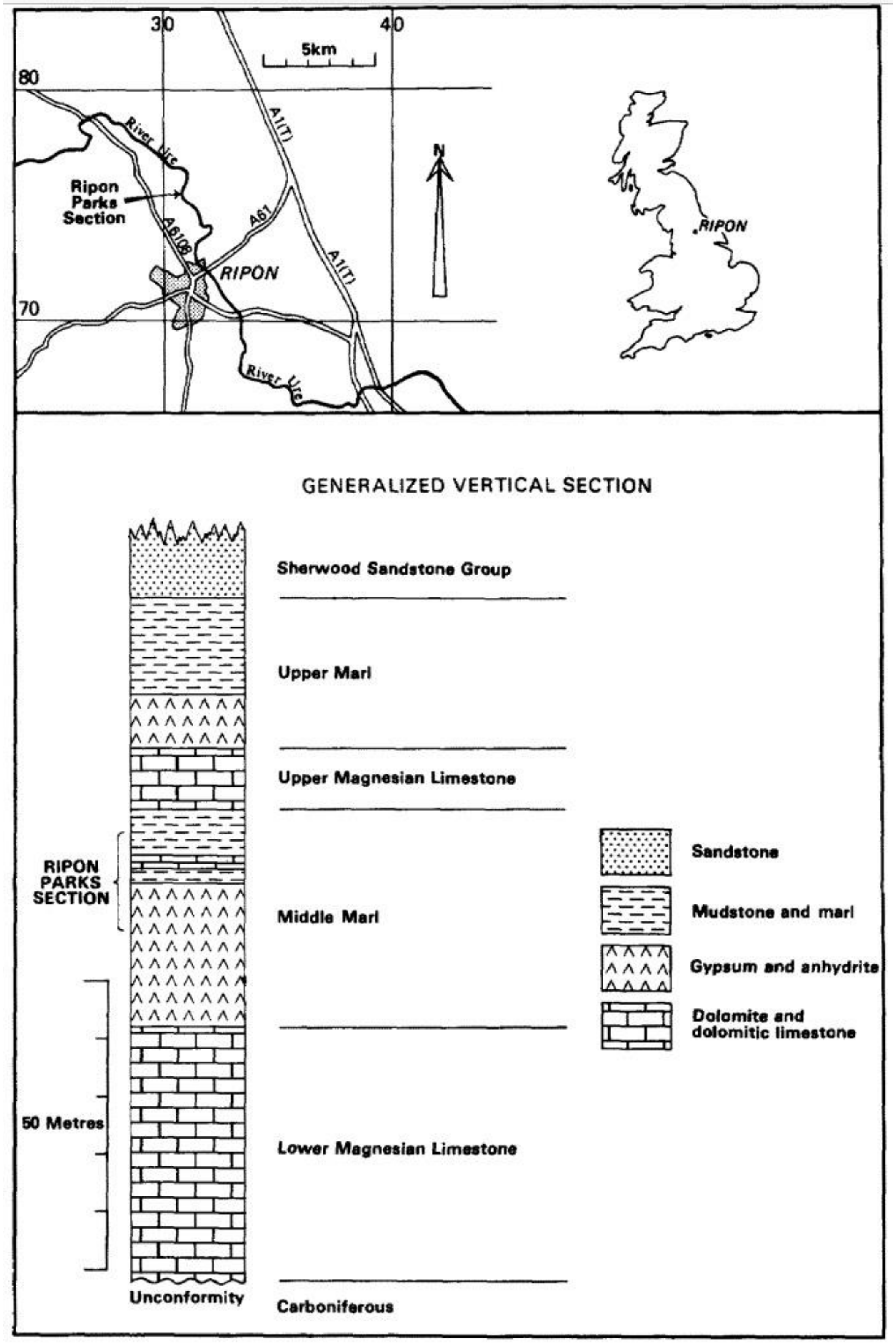

Figure 1. Location map and generalized vertical section.

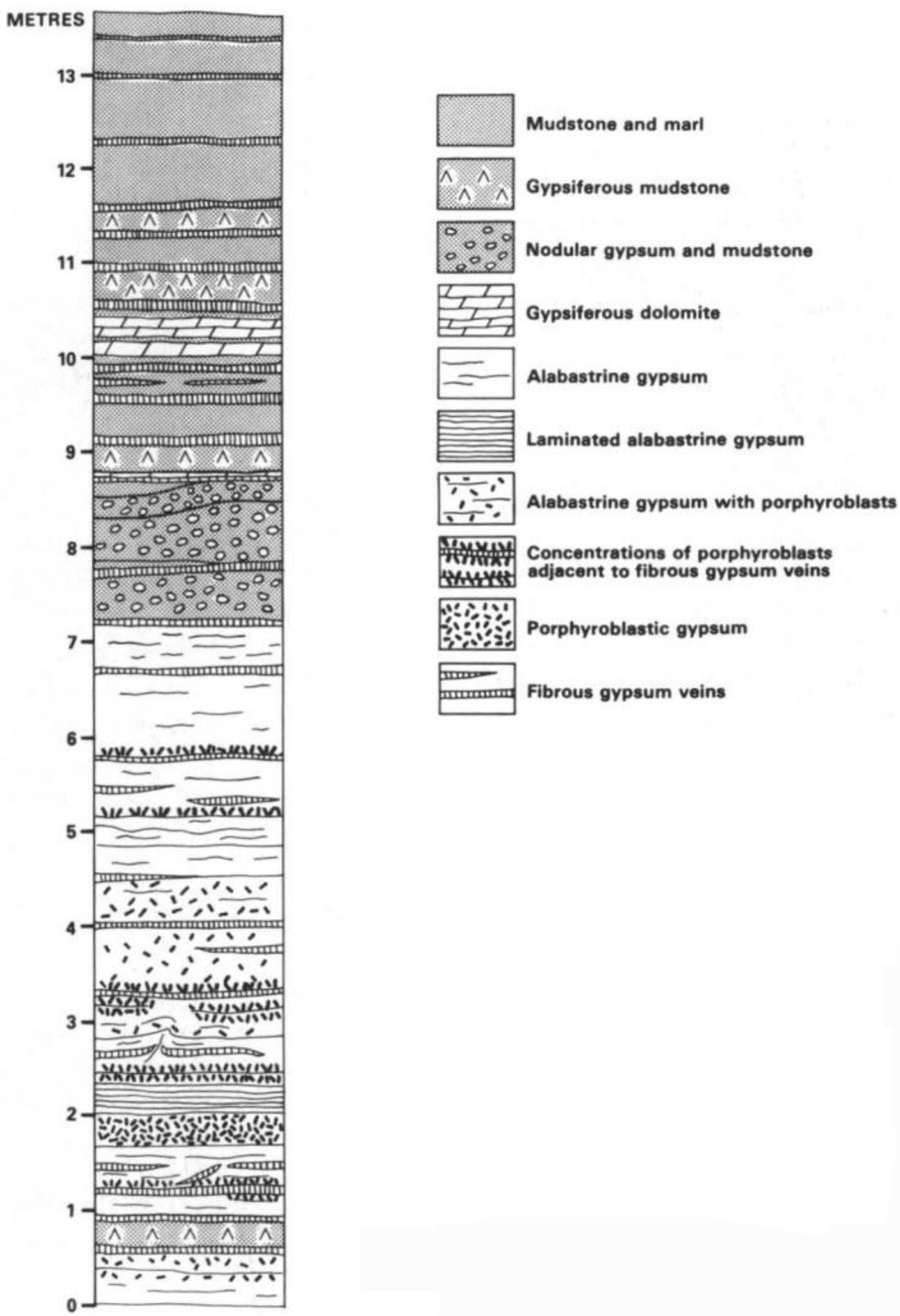

Figure 2. Detailed lithological sequence of the Ripon Parks gypsum cliff.

III. OBSERVATIONS

From March 1978 onwards visits to the site have been made by the authors at approximately six-monthly intervals. On each occasion the massive gypsum at the southern end of the cliff has been photographed and on four occasions observations and photographs were taken from the east bank. These observations have highlighted the active solution of the massive gypsum cliff by the river water. Analysis of this water shows that it contains very little calcium sulphate (less than 30 mg/1) and will dissolve gypsum almost as though it were pure water. Additional, though less important, solution agents are the concentrated flow of rain water down the face in gullies and the flow of groundwater through joints and fissures within the massive gypsum.

The following factors, which were expected to affect the rate of solution and erosion of the bank, have been assessed.

1. Variations in water level. Where the river has undercut the gypsum bank the notch can be up to 5 m deep. Between low-water level and very low-water level the back face of the notch is sub-vertical, but from low-water level to normal winter level the top of the cut slopes and rises at a shallow angle to the horizontal towards the river. Above normal water level, to the highest water level, solution is slight and restricted to the formation of a slightly dimpled surface (Plate 5).

2. Variations in flow velocities, as well as turbulence and eddies. The experimental work of James and Lupton (1978) suggested that water flow rate adjacent to the gypsum surface has a great effect on solution rate. The theoretical rate of solution at a flow of 1 m/s is five times greater than at 0.2 m/s and nearly 18 times greater than at 0.05 m/s (see Appendix). Observations of river flow show that in the main channel the surface water velocity is typically 1-2 m/s. However, 0.2 m from the cliff the surface flow velocities are generally lower, typically between 0.2 and 0.5 m/s and, within a few millimetres of the dissolving surface, probably as low as 0.05-0.1 m/s (Schlichting 1961). The river flow velocities below the surface are thought to be virtually the same as at the surface. It is the flow in the zone close to the dissolving surface which controls solution. Assuming a river temperature of 8°C such flow rates should produce, according to the equations of James and Lupton (1978), cliff retreat of the 0.18 m/year (see Appendix). If the cliff were subject to water flowing, as in the centre of the river, at 1 m/s, it should be undercut at a rate of 1.7 m/year.

The flow adjacent to the cliff generally occurs as eddies and gyres with small areas of stagnant water between adjacent circulations. This stagnant water next to the gypsum is a poor solvent because it rapidly becomes saturated with calcium sulphate. Turbulent water, conversely, produces an enhanced solution rate as shown by the experiments of James and Kirkpatrick (1980). These variables depend on the constantly changing positions of pools and gravel banks and are therefore impossible to quantify, but they do qualify the theoretical predictions.

3. Solution by groundwater. The massive gypsum at the southern end of the cliff shows abundant evidence of solution by groundwater. Water seepages, which are considerable in winter months, issue from small caverns and joints above river-level, indicating that there is sub-surface water-movement within the gypsum. Signs of etching and solution of the rock (dimpled surfaces, Plate 5) are everywhere developed on joint surfaces and on cavern walls. It is evident that by this process the joints are actively being enlarged locally to form small caves. This process is aided by tree roots which force their way down joint planes. Thus prior to solution by river water, the joint surfaces of the massive gypsum have been opened out by groundwater. Groundwater action is less pronounced in those parts of the cliff made up of alternating gypsum, marl and mudstones which do not show such well-developed jointing.

4. Collapse of the cliff. The cliff is actively undercut up to 5 m (Plate 6) and collapse of the cliff face has been observed in recent years.

Between November 1977 and March 1978, a large block and associated loose debris detached themselves from the gypsum cliff near the position marked A in Plate 1. The main block was roughly a three-metre cube (Plates 3 and 4). It rested in the river two to three metres from the cliff, and was estimated to have weighed 60 tonnes. By October 1979 the block and associated debris had largely disappeared (Plate 4). Water velocities very close to the block were about 1 m/s and the flow was turbulent in places. Calculations based on the solution of this block are consistent with the solution rates of gypsum determined by laboratory experiment and, as shown more fully in the Appendix, the block was reduced to one-eighth of its original volume in 14 months compared with 16 months predicted from a simple model. In this example the main block had fallen clear of the face and undercutting of the cliff locally continued. However, in one place smaller blocks have protected the face from river action and currently are being dissolved away. These fallen blocks may be undercut several times before the adjacent part of the cliff face is again affected. In this manner rates of cliff migration, considerably less than the theoretical amounts, might be observed. However, it should be noted that Mr C. Slater of Ripon, a regular visitor to the site for the past 25 years, recalls only the one major fall of gypsum blocks in 1977-78, described above. This indicates that during that period collapse of the cliff was not an important feature and, as discussed below, is consistent with the view that undercutting of the gypsum only began between 50 and 60 years ago.

Between October 1979 and March 1980, a slip occurred from above the massive gypsum cliff causing a quantity of boulder clay and a tree to fall into the river. This slip was incipient in October 1979 and the crack at the top of the failure plane was evident. North of this locality the top of the cliff shows evidence of past boulder clay slips and incipient failure of the cliff face. Several trees have bends in their trunks and, since mature trees will not bend, it is estimated that these trees slipped between 15 and 50 years ago. The slipped boulder clay of 1979-80 fell between the cliff and the remnants of the gypsum block that collapsed in 1977-78, thus temporarily protecting part of the gypsum cliff from solution and slowing the rate of undercutting. Slips that deposit boulder clay into the river without failure of the gypsum cliff are less effective in protecting the face. The slipped material at present would be deposited in 1.5 to 3 m of water and readily carried away by the river. Much of the gypsum for at least 40 m north of point C and south of point A (Plates 1 and 2), currently is obscured and protected by slipped glacial debris.

Plate 1 (top). The gypsum cliff at Ripon Parks, April 1980, Panoramic view looking west from A northwards to B (See Figure 3).

Plate 2 (bottom). The gypsum cliff at Ripon Parks, April 1980, Panoramic view looking west from B northwards to C (See Figure 3)

Plate 3. View of fallen gypsum blocks looking towards the north-west, March, 1978.

Plate 4. View of the residues of the fallen gypsum blocks looking towards the north-west. October, 1979.

Plate 5. Effects of solution of the gypsum cliff at Ripon Parks. Block of dissolving gypsum at the bottom of the cliff, just to the right of point A in Figure 3, showing the typical dimpled surface caused by the solution of gypsum by water.

Plate 6. Undercutting of the gypsum cliff approximately at point A in Figure 3.

5. Lithological variations in the sequence. Gypsum in all the forms present (alabastrine, alabastrine with porphyroblasts, porphyroblastic, and fibrous) dissolves equally readily. Solution etches the surfaces, which soon become typically dimpled, even after a fairly short time under water. Gypsiferous mudstone breaks down fairly easily. The marls and mudstones degrade readily to a soft clay which washes out from between the beds of gypsum and gypsiferous mudstone and may be removed more quickly than pure gypsum. The dolomite bands are relatively insoluble, being 150 and 200 times less soluble than gypsum.

Important differences were noted in the way that the southern part of the cliff, made up dominantly of massive gypsum, collapses and retreats compared to the remainder in which thin beds of gypsum, marl and mudstone alternate. In recent years the massive gypsum has failed by the collapse of large joint-bounded blocks. In the remainder of the cliff, where groundwater solution and joint opening is less important and there is repeated alternation of lithologies, large scale sudden failures do not appear to take place. Selective solution and erosion of the various lithologies causes the more resistant bands to stand out from the less resistant beds. This produces a highly characteristic surface with alternating thin ridges and furrows (Plates 1 and 2). Deepening of the furrows eventually causes the harder beds forming the ridges to fracture and break down. The dimensions of the fallen fragments are unlikely to exceed a few centimetres. This type of collapse is on a small scale and is probably of very common occurrence; it occurred during the authors' visits.

6. Erosion of sediment in the river. Most of the face at Ripon Parks is on the outside of the gentle curve in the river and the water below the face, as measured by Dr J. H. Powell, is generally between 1.5 and 3 m deep. The bed of the river contains sand, gravel and boulders and in times of flood this bed load will abrade the submerged base of the cliff. However, observations show that from the bottom of the river to near the normal low water level, the back face of the undercut notch is steep or near vertical and is dimpled with the typical surface solution pattern. Thus, it would appear that erosion by suspended sediment is slight compared with the solution of gypsum. This is also illustrated by the cobbles and boulders of gypsum found in the river, which always display the dimpled solution surface, no smooth well rounded gypsum cobbles having been found. The gypsum dissolves so readily that the pebbles and cobbles do not survive long enough to be abraded, and they are not found very far downstream from the cliff.

IV. HISTORICAL EVIDENCE

The observations described above show that the gypsum cliff is subject to attack by the river. There is clear evidence of undercutting of the cliff resulting from solution, which, from the time taken for a fallen gypsum block to dissolve, is proceeding at rates close to values predicted from laboratory experiments. Historical evidence was sought to see how closely predicted rates of cliff retreat matched observed values.

Comparison of the 6 inches to 1-mile topographical maps held by the Institute of Geological Sciences (Leeds) suggests that the cliff as a whole cannot have retreated more than a few metres since 1853 (Figure 3). In 1853 the cliff was protected from erosion by banks of gravel and alluvium which were removed prior to the 1928 survey. The date of this removal which allowed the river to approach the cliff is unknown but, from the recollections of Professor H. C. Versey (Personal communication, 1981), it was prior to 1913. From the evidence of the Primary Survey 6 inches to 1-mile geological maps made in 1880 some alluvium was then present, protecting the contorted marls and gypsum, at the northern end of the cliff. No mention of undercutting of this part of the cliff was made in Fox-Strangways' notebooks (around 1865) and the authors assume that the undercutting of up to 5 m now seen here, has taken place since 1880. The inferred rate of retreat (0.05 m/year) is probably a minimum value because small-scale collapse has been noted in this part of the cliff (p. 444) and the total amount of undercutting may be somewhat greater than seen at present. The authors know of no major historical collapse of this northern part of the cliff, the slightly recumbent anticline (mid-way between B and C; Plate 2) looking the same in the photographs of Kendall and Wroot (1924) and of Forbes (1958) as it does now.

Despite the removal of protective banks of gravel and alluvium by at least 1913, the massive gypsum, at present so well displayed at the southern end of the cliff, may not have been attacked by the river until fairly recently. No exposure of massive gypsum was mentioned by Fox-Strangways in his field notebook of about 1865, and Kendall and Wroot (1924, p. 269), though they noted that the contorted marl with gypsum bands, at present seen to the north, made no mention of massive gypsum hereabouts. Professor Versey (Personal communication, 1981) recalls that this end of the cliff was covered largely by 'slipped marl and considerable vegetation' until the early 1920's when the first exposures of gypsum appeared. By 1929 or 1930, according to Professor J. E. Hemingway, deep water was flowing at the foot of the massive gypsum exposure (Personal communication, 1981). This suggests that the 5 m of undercutting seen in this part of the cliff has developed in 50 to 60 years, i.e. at a rate of 0.08-0.10 m/year, assuming that no ancient relics of undercutting were present initially. These observed rates of undercutting are close to those values (0.10-0.18 m/year) deduced above from employing the inferred present flow rates adjacent to the cliff in the equations of James and Lupton (1978). The observations of Mr C. Slater, that no collapse of the gypsum cliff in the 20 or so years prior to 1977, suggests that only in recent years has the cliff been sufficiently undercut for it to begin to fail.

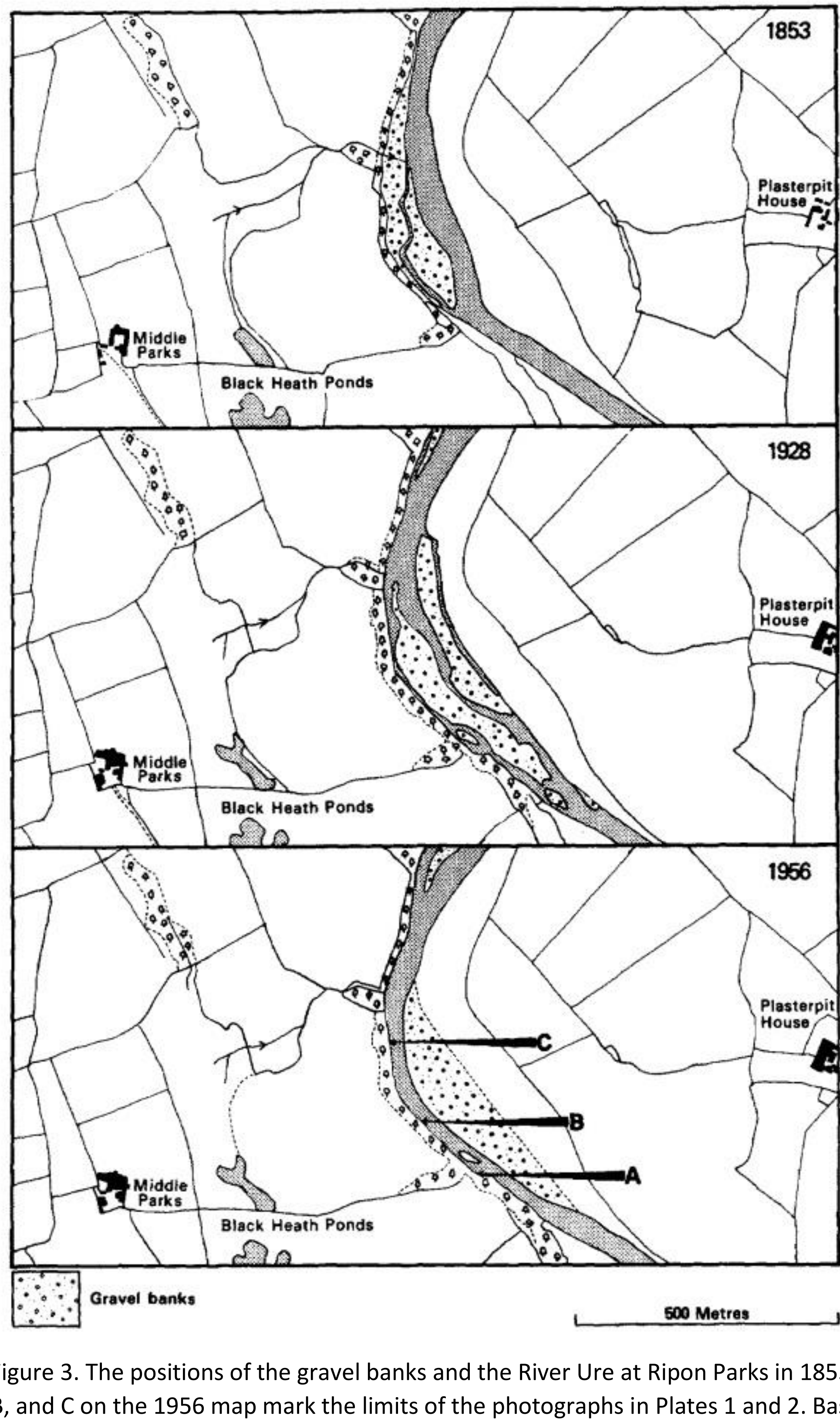

Figure 3. The positions of the gravel banks and the River Ure at Ripon Parks in 1853, 1928 and 1956. A, B, and C on the 1956 map mark the limits of the photographs in Plates 1 and 2. Based upon the 1853, 1928 and 1956 Ordnance Survey 6 inch to 1-mile maps with permission of the Controller of Her Majesty's Stationery Office. Crown copyright reserved.

V. CONCLUSIONS

Solution rates of gypsum predicted from laboratory experiments have been verified by the solution of fallen gypsum blocks by the River Ure at Ripon Parks. A rough three metre cube of gypsum subject to high water flow velocities around it (about 1 m/s) dissolved to about one-eighth of its original volume in 14 months compared with 16 months predicted from a simple model. The rate of undercutting of the cliff is expected to be much less. Under non-turbulent conditions the solution rate is greatly influenced by the water flow velocity which, 0.2 m from the main cliff, is only about 0.2-0.5 m/s and within a few millimetres is probably only 0.05-0.1 m/s. These rates of flow in a perfect system may be expected to undercut gypsum at between 0.10 and 0.18 m/year although local turbulence could increase these values. Observed rates of undercutting are close to these values. For most of the last century the cliff was protected from river erosion. Following the removal of this barrier, slipped debris protected the massive gypsum at the southern end of the cliff until the 1920's. This part of the cliff, therefore, has since undercut approximately 5 m at a rate of 0.08-0.10 m/year. The northern part of the cliff has undercut by a similar amount in about 100 years, giving a minimum rate of 0.05 m/year.

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APPENDIX: RATE OF SOLUTION OF LARGE GYPSUM BLOCK

A large block of gypsum in the form of a rough cube (edge 3 m), fell from the cliff into the river between November 1977 and March 1978. In March 1978 it lay in the river at an angle so that only one corner was immersed (Plate 3). By August 1978 it was about half immersed and sitting in a roughly square position on the river bed. By October 1979 (Plate 4) it was estimated that about one-eighth of the original volume of the block was left, and by March 1980 it had virtually disappeared.

Any calculation of the rate of dissolution of the block is bound to be approximate for the following reasons:

1. The block was not a perfect cube with smooth sides and the authors were unable to measure its dimensions accurately.

2. Its depth of immersion in the river was dependent on river levels which varied greatly during the period.

3. The river flow velocities around the block are not known accurately and they affect the solution rate constant of gypsum.

4. Most important of all, the manner in which the dissolving block disintegrated and settled below the water surface was no doubt irregular, and small pieces probably fell from it to dissolve separately. Nevertheless, the calculation is worth making to see whether the predicted and observed rates of disappearance are of the same order of magnitude.

For purposes of calculation an idealised model is taken, assuming that half the block was immersed in August 1978 and that this bottom half dissolved first. Thereafter the top half sank to the bottom of the river and dissolved to leave about one-eighth of the volume of the original block intact by October 1979 (Plate 3). It is further assumed that the bottom of the block is effectively sealed to the river bed and that the top of the block is not covered by water.

Using the equations and data provided by James and Lupton (1978) the first and third equations in the section 'Dissolution Mechanisms' (pp. 250-251) are applied. Taking ϕ in the third equation as equal to 1 and changing the sign in front of $\frac{dM}{dt}$ so that the latter now represents the rate of loss of solid material, this gives:

$$-\frac{dM}{dt} = KA\,(c_s\text{-}c) \qquad (1)$$

where K is the dissolution rate constant, A is the exposed area and c_s is the solubility of gypsum at 8°C (2.42 kg/m^3). c is the concentration of gypsum in solution which in this case is approximately zero so that $(c_s - c) \approx c_s$

It is further assumed that the flow velocity of water adjacent to the block is 1.0 m/s. From James and Lupton (1978, table 2 and Figure 10) a value of K at 8°C of 5.3 X10 -5 m/s can be obtained.

$$\text{Hence } -\frac{dM}{dt} = 2.42\text{x}5.3\text{X}10^{-5}A \qquad (2)$$

To start, a calculation is made of the time to dissolve the lower half of the block which has been assumed to be square in plan with sides equal to L and height $\frac{L}{2}$.

To integrate (2) a relationship between A and M, the mass of the block, is required. It is obtained by assuming that even as the block dissolves, it remains square in plan with side 1. Then $M = l^2 \frac{L}{2}\,\rho$ where ρ is the density of gypsum and $A = 2lL$

Thus $A = 2\,\rho^{-½}\,(2L)^{½}M^{½}$. Using (2) this gives:

$$t = \frac{-1}{2.42x5.3x10^{-5}} \cdot \frac{\rho^{½}}{2\,(2L)^{½}} \int_{M_2}^{M_1} M^{-½}\mathrm{d}M \qquad (3)$$

Where M_1 and M_2 are the initial and final masses. In the present case $M_1 = \rho\frac{L^3}{2}$ and $M_2 = 0$.

Integration of (3) gives:

$$t = t_1 = \frac{1}{2.42x5.3x10^{-5}} \cdot \frac{\rho^{½}}{2\,(2L)^{½}} \cdot 2M_1^{½} = \frac{\rho L}{2.42x5.3x10^{-5}x2} \qquad (4)$$

Similarly, the time for the partial solution of the top half of the block to a volume one-eighth of the total volume of the original block is equal to 0.50t_1. Thus, the total time (t_1+t_2) required to dissolve seven-eighths of the volume of the original block taking L=3m and ρ as 2320 kg/m^3 is

$$1.50t = \frac{1.5x3x2320}{2.42x5.3x10^{-5}x2} = 4.07x10^7 \text{ seconds} = 16 \text{ months}$$

From visual observations the block appeared to dissolve to about one-eighth of its original volume in 14 months. The agreement is reasonably good considering the uncertainties in the manner of dissolution of the block.

Equation (1) can also be used to predict the rate of solution occurring at the cliff face, provided that the flow velocity can be estimated. Measurements of flow velocities were made about 0.2 m from the cliff face and gave values of between 0.2 and 0.5 m/s. However, the rate of solution of the gypsum is essentially determined by the flow velocities within a few millimetres of the dissolving surface. These could not be measured by the authors, but conventional hydraulics (Schlichting 1961) suggest that the flow velocities within a few millimetres of the surface are between 0.05 and 0.1 m/s

The corresponding values of K for different flow velocities at 8°C are given below and taken from James and Lupton (1978).

Water Flow Velocity (m/s)	$K \times 10^5$ (at 8°C)	Rate of Cliff Face Retreat
(m/s)	(m/s)	(m/s)
0.00	0.073	0.025
0.05	0.300	0.10
0.10	0.541	0.18
0.20	1.046	0.34
0.50	2.607	0.86
1.00	5.301	1.70

The rate of cliff face retreat is calculated from equation (1) replacing dM by $-\rho A \frac{dl}{dt}$

Where $-\frac{dl}{dt}$ is the linear rate of solution or retreat of the cliff face.

Hence $-\frac{dM}{dt} = -\rho A \frac{dl}{dt} = K A c_S$

therefore $-\frac{dl}{dt} = \frac{K c_S}{\rho}$ where c_S = 2.42 kg/m^3 (at 8°C) and ρ = 2320 kg/m^3

Substituting the various values of K yields the corresponding values for retreat of the cliff face. The rate of retreat at 0.05 - 0.1 m/year cover a similar range to the observed rates of retreat, i.e. 0.08-0.10 m/year.

Note: Further information on the gypsum cliff and the dissolution of it is contained in:

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