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Summary. Transfer function estimates are required to provide information regarding the lateral variation of conductivity structure within the Earth. Conventional techniques assume that the effect of external field characteristics on transfer function estimates can be removed by a least-squares procedure. This assumption is examined at three mid-latitude stations. It is found that, at all three stations, source field characteristics do affect the transfer function estimates. The source field effects increase with both latitude and period. The source field contribution is minimized by the determination of the transfer function from a weighted mean of estimates of transfer functions obtained from individual records. The weighting function is the vertical field partial coherence and it is, furthermore, necessary to provide a wide distribution of this parameter if a valid transfer function estimate is to be obtained.

1 Introduction

The process of electromagnetic induction in the Earth provides a method of probing the conductivity structure of the Earth's interior. By separating magnetic variations into parts of internal and external origin we can determine the electromagnetic response of the Earth to a particular external field variation. In the frequency range $2\ \mathrm{c.yr}^{-1}$–$0.25\ \mathrm{c.day}^{-1}$ an expansion in terms of spherical harmonic coefficients allows an effective separation of the parts of internal and external origin (Banks 1969). The procedure is effective because the external field variations in the above frequency range occur on a global scale and their morphology can be adequately quantified. The response functions in this frequency range are considered diagnostic of the radial variation in conductivity.

Response functions in the period range from several hours to several minutes derive from storm/substorm associated external field variations. The response functions are obtained from correlation (least-squares) procedures. The response or transfer functions so obtained are considered diagnostic of lateral variations in conductivity. Transfer functions are often calculated under the assumption of spatial uniformity of the external field variations. In regions where this assumption is not valid, the transfer function ultimately becomes a function of the wavelength of the external field. The purpose of the present study is to examine the validity of the above assumption at mid-latitudes and to examine methods of minimizing external field contributions to estimates of transfer functions.

2 The single-station transfer function

A transfer function estimate allows the separation of recorded magnetic fields into parts of internal and external origin (Schmucker 1970; Banks 1973). Three relationships may be assumed between Fourier components at a particular frequency:

$$H_a = T_{HH}H_n + T_{HD}D_n + T_{HZ}Z_n$$

$$D_a = T_{DH}H_n + T_{DD}D_n + T_{DZ}Z_n$$

$$Z_a = T_{ZH}H_n + T_{ZD}D_n + T_{ZZ}Z_n.$$

Subscripts a and n refer to anomalous (deriving from lateral variations in conductivity) and normal parts. Horizontal component transfer functions can only be obtained using inter-station methods. In the absence of simultaneous recording a single-station transfer function that forms an internal response estimate of the vertical field can be obtained. Three simplifying assumptions are introduced which allow the total fields ($F = F_a + F_n$) recorded at a single-station to be used (Banks 1973):

(1) That if Z_n is small compared with Z, T_{ZZ} can be ignored.
(2) That there is no correlation of Z_n with H_n and D_n.
(3) That both H_a and D_a are small in comparison to the total observed fields so that H_n and D_n can be replaced by H and D respectively.

Under these circumstances, the vertical field response estimate becomes:

$$Z_c = A \cdot H + B \cdot D + E. \tag{1}$$

Z_c is an estimate of Z_a whose validity depends on the validity of the above three assumptions. The relationship is determined in a least-squares sense such that the error function E is minimized. Using high S/N inputs, E can be considered to be Z_n i.e. that part of Z that does not *consistently* correlate with the right side of equation (1). On this basis, E is often taken to be a measure of the 'goodness of fit' of the single-station transfer function (A, B). A related quantity is the vertical field partial coherence (R^2)

$$R^2 = 1 - E/S_{ZZ}$$

where S_{ZZ} is the total power in the Z component. R^2 is the coherence between the observed vertical field and the vertical field predicted by equation (1).

Assumption (3) cannot be examined using single-station methods, although the effect of increasing H_a and D_a is to increase $|(A, B)|$ (Beamish 1977). Assumptions (1) and (2) relate to the magnitude and consistency of Z_n within the observed vertical fields. Z_n is given by:

$$Z_n = c.\left(\frac{\partial H}{\partial x} + \frac{\partial D}{\partial y}\right) \tag{2}$$

where c is a complex function of skin-depth and (x, y) are geomagnetic axes (Schmucker 1970). In the absence of lateral variations in conductivity, a determination of c affords information on the radial distribution of conductivity (Schmucker 1970; Lilley & Sloane 1976). In the determination of the transfer function (A, B) using equation (1), Z_n is treated as noise and is minimized. From equation (2) it can be seen that the magnitude of Z_n

is proportional to the spatial scale or wavelength of the external field variations. For uniform external fields Z_n is zero. Z_n is greatest when the external field varies on a horizontal scale which is shorter than the skin-depth of the variation field within the Earth (Schmucker 1970).

Magnetic substorms and continuously disturbed records are often used in the determination of equation (1); they provide magnetic variations generated by independent current systems and contain adequate power levels over a wide range of frequencies. Although the substorm mechanism is complex, its magnetic signature as a function of latitude has been both observed and modelled (Kisabeth & Rostoker 1971). From these results it is evident that Z_n will increase with latitude from mid-latitudes towards the auroral oval as the gradient of the horizontal disturbance field increases. A non-zero Z_n results from the finite scale-length of the external field variations. A substantial Z_n requires each term in equation (1) to be a function of the wavelength of the external field variations. For substorm fields, the requirement for this dependence to be acknowledged will increase with increasing latitude towards the auroral oval.

Since Z_n derives from the gradient in the horizontal field (equation (2)) we must recognize that the least-squares procedure used in the fitting of equation (1) can produce a Z_c that incorporates Z_n either in part or in full. It has been assumed that the fitting of equation (1) over sufficient quantities of data allows this unwanted correlation to be minimized. This is only true if a major portion of the data sets used possess $Z_n \sim 0$. In this case, the perturbations in (A, B) due to external fields possessing a substantial Z_n can be effectively minimized by a least-squares procedure. In cases where the major portion of the data sets possess a substantial Z_n, two different effects could be observed in the fitting of equation (1). If Z_n, and hence wavelength, is consistent between the available data sets, the amount of Z_n appearing within Z_c would be consistent; the determination of (A, B) by least-squares would then be effective. If, however, Z_n varies between the available data sets, the stability of (A, B) becomes a function of the variability of the external field wavelengths. The least-squares procedure in this case would not be meaningful since the wavelength dependence would have to be acknowledged.

If the single-station transfer function (A, B) is to provide an accurate estimate of lateral conductivity structure it is clearly necessary to identify the extent of source field effects at a particular station and to minimize them if possible.

3 Data analysis

For several years the Institute of Geological Sciences has operated magnetometer stations within the UK and Scandinavia. The instruments are 3-axis rubidium vapour magnetometers with a sampling interval of 2.5 s and a resolution of 0.025 nT (Riddick, Brown & Forbes 1976). The data are recorded digitally on cassette and subsequently transcribed into computer compatible format. Three of these stations with almost complete data sets spanning the year 1977 were chosen to provide a geomagnetic latitude range from 54.2° to 60.0°. Full geographic and geomagnetic coordinates for the three stations are listed in Table 1.

Two types of procedure used in transfer function analysis have been identified by Banks (1975). The first procedure involves the use of individual transient variations. The second procedure involves the use of continuously disturbed sections of record. Stretches of continuous disturbance tend to be the more common at UK latitudes and computational procedures based on such records are employed in the present study.

To provide data in the conventional Geomagnetic Deep Sounding period range, 12 hr record lengths were used. The 2.5 s data were resampled at 1 min intervals providing 720

Table 1. Station coordinates used in the study.

Station	Mnemonic	Geographic co-ordinates Latitude	Longitude	Geomagnetic co-ordinates* Latitude	Longitude
Lerwick	LE	60.13	-1.18	60.03	-135.42
Nurmijarvi	NU	60.51	24.66	56.47	-112.35
York	YO	53.95	-1.05	54.22	-138.64

* Eccentric geomagnetic.

data points per record in the *H, D* and *Z* components. To provide consistent data, the records were chosen from the five International Disturbed Days (IDD) per month. Such a selection ensures good S/N over a wide frequency range. For each IDD, two records were obtained spanning local day and night-time respectively. Forty IDD days were used providing 80 12 hr records at each station.

Transfer function estimates were determined using the Unit Vector method of Everett & Hyndman (1967), spectral estimates being formed using the technique of complex demodulation described by Banks (1975). Each data record was divided into bands of frequencies whose central frequency varied linearly with period. Spectral bands containing Fourier coefficients with periods 128–64, 64–32, 32–16, 16–8 and 8–4 min were used. They are referred to as Bands 1–5 respectively. Each demodulated estimate provides two degrees of freedom and individual demodulation estimates within a given band are independent. The procedure used provides, for each 12 hr record, 32, 50, 98, 128 and 256 degrees of freedom within Bands 1–5 respectively.

4 Effects of degrees of freedom

4.1 MINIMUM DEGREES OF FREEDOM

The transfer function (A, B) is a statistically determined quantity and requires sufficient degrees of freedom (ν) to be determined accurately. For a given record length and frequency band, the ν must be greater than a certain minimum if the resulting transfer function is to be valid. This applies in particular to the longest period band. If insufficient ν are available, the record length must be increased.

Coherence is a quantity particularly sensitive to ν. Insufficient ν will result in an unreasonably high correlation or coherence between two variables. The minimum ν required to obtain a valid (A, B) was studied using the vertical field partial coherence R^2. A frequency band centred on 60 min was chosen. The ν were altered by adjustments to the width of the frequency band. The distribution of R^2 determined for each record over the 80 data records was then studied. Fig. 1 is a comparison of the distribution of R^2 at YO (lowest latitude station) for $\nu = 18$ and 26. The distribution for $\nu = 18$ is seen to be heavily biased towards high coherencies as a result of the insufficient ν within each record. The distribution of R for $\nu = 26$ is considered reasonable since the distribution was not significantly altered by an increase in ν. The frequency bands used in the present study are considered to provide sufficient ν for a valid estimate of the transfer function from a 12 hr record.

4.2 EFFECTS OF DEGREES OF FREEDOM ON TRANSFER FUNCTION ESTIMATES

The use of complex demodulation in conjunction with the Unit Vector method allows all the records or selected subsets obtained at a particular station to be used in the fitting of equation (1). We are thus in a position to study the effects of degrees of freedom on (A, B) at the three selected stations.

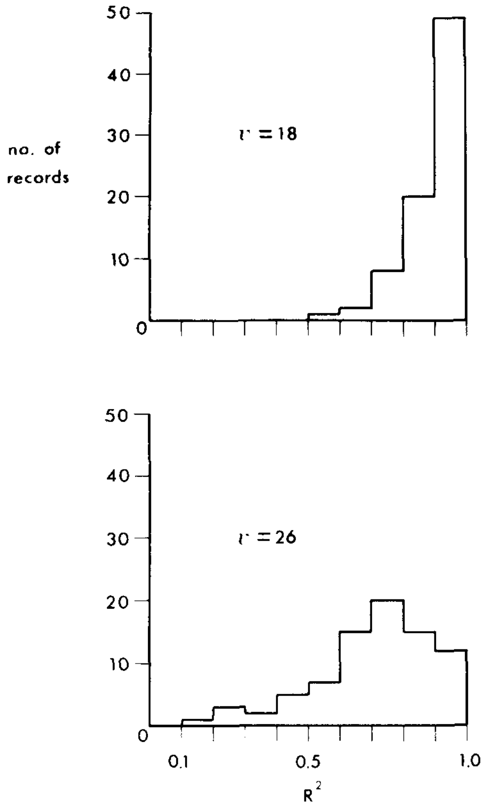

Figure 1. Distribution of vertical field partial coherence R^2 of individual records from total of 80 records using degrees of freedom (ν) $\nu = 18$ and 26 at YO. Band is centred on a period of 1 hr.

The procedure used was to calculate (A, B) from between 1 and 40 records thus providing estimates with given ν which are necessarily multiples of the ν of individual records for a particular frequency band. The scheme adopted is shown in Table 2. The scheme provided a number of estimates of (A, B) for each ν. For each frequency band, the estimates for each ν were averaged and the standard deviation calculated. The standard deviation so determined can be taken to represent the scatter of (A, B) due to differing source field characteristics. The scatter is therefore a measure of the reproducibility of (A, B) for a given number of degrees of freedom used in the analysis.

Table 2. Scheme for studying effects of degrees of freedom.

Number of records	Number of estimates of (A,B) obtained	Number of degrees of freedom in each Band:				
		1	2	3	4	5
1	80	32	50	98	128	256
2	40	64	100	196	256	512
3	20	96	150	294	384	768
4	20	128	200	392	512	1024
5	15	160	250	490	640	1280
10	8	320	500	980	1280	2560
20	4	640	1000	1960	2560	
40	2	1280	2000			

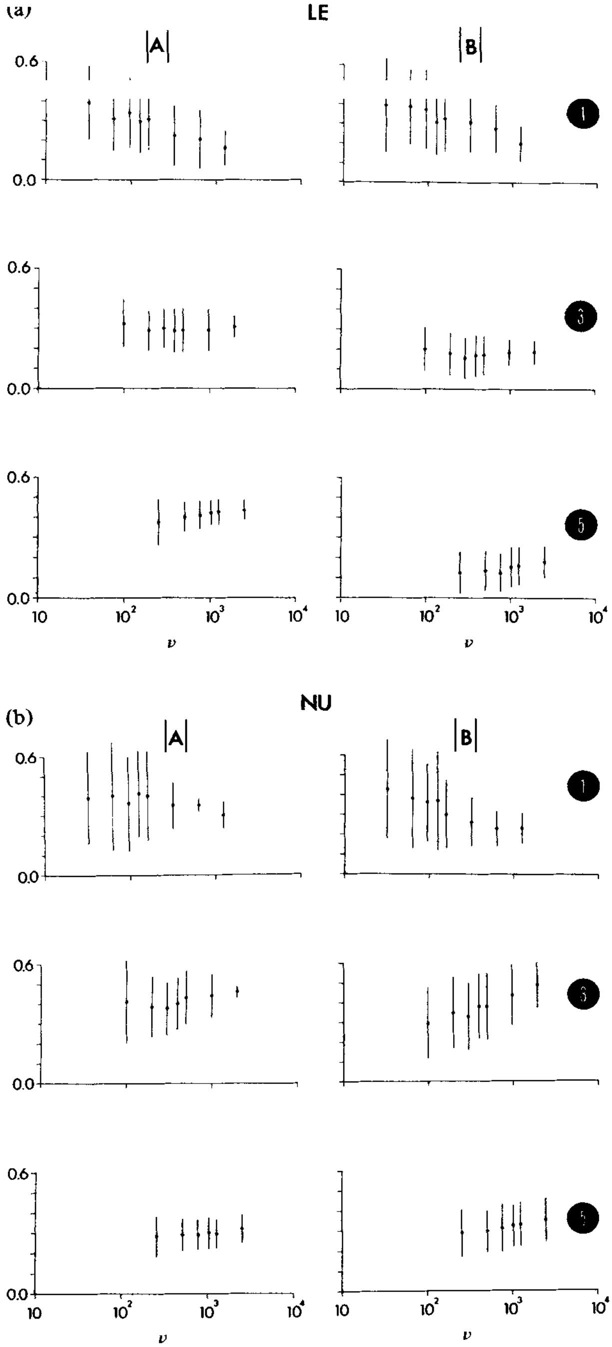

Figure 2. Variation of transfer function estimate $|(A, B)|$ with degrees of freedom (ν) for three period bands. Band 1: 128–64 min, band 3: 32–16 min, band 5: 8–4 min: (a) At LE, (b) at NU, (c) at YO.

The transfer function (A, B) is complex and only the variation in $|(A, B)|$ for bands 1, 3 and 5 is considered. Fig. 2(a) shows the variation of $|A|$ and $|B|$ with ν at the northernmost station LE. $|(A, B)|$ exhibit different characteristics with ν for the three bands considered. In band 1 $|(A, B)|$ decrease, in band 3 $|(A, B)|$ are approximately constant and

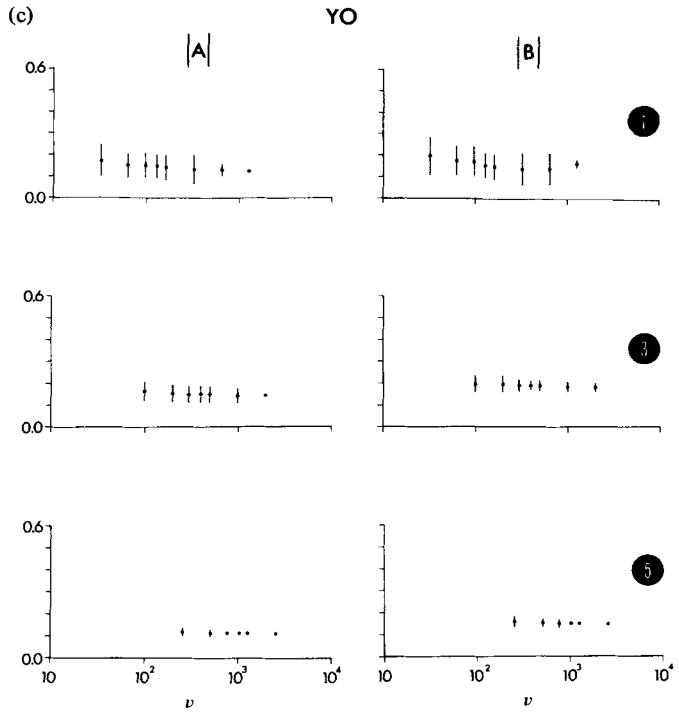

Figure 2

in band 5 $|(A, B)|$ show a slight increase with increasing ν. Fig. 2(b) shows the results obtained at NU. Again $|(A, B)|$ exhibit different characteristics with increasing ν in the three bands. In band 1 $|(A, B)|$ tend to decrease while in bands 3 and 5 a slight upward trend is observed. Fig. 2(c) shows the results obtained at the lowest latitude station YO. A lack of trends in $|(A, B)|$ with ν for any of the three bands is evident. In addition, the scatter in $|(A, B)|$ are much reduced from each band at YO when the results are compared with those at the higher latitude stations.

In the absence of source field (Z_n) effects we expect (A, B) to be independent of ν and the reproducibility of (A, B) to be high. Only the results at the lowest latitude station (YO) for bands 3 and 5 appear to represent a period range and latitude free from source field effects. At longer periods and higher latitudes; source field effects, their spatial wavelengths and the variability of some are required to explain both the trends observed and the lack of reproducibility of the transfer function estimates.

4.3 EFFECTS OF DEGREES OF FREEDOM ON TRANSFER FUNCTION ERRORS

The Unit Vector method of obtaining transfer function estimates provides error estimates $(\delta A, \delta B)$ of the determined transfer function (A, B). The effect of ν on the likely errors $(\delta A, \delta B)$ at the three stations is shown in Fig. 3 for bands 1, 3 and 5. The data organization and averaging procedure is identical to that used previously (Table 2). The variation of the mean percentage error $(\delta X \cdot 100/X)$ is shown.

At all three stations the errors for the three bands show a distinct separation. For a given ν, the accuracy of (A, B) increases with decreasing period indicating that source field effects increase with increasing period. Only in the shortest period band (5) do the error estimates

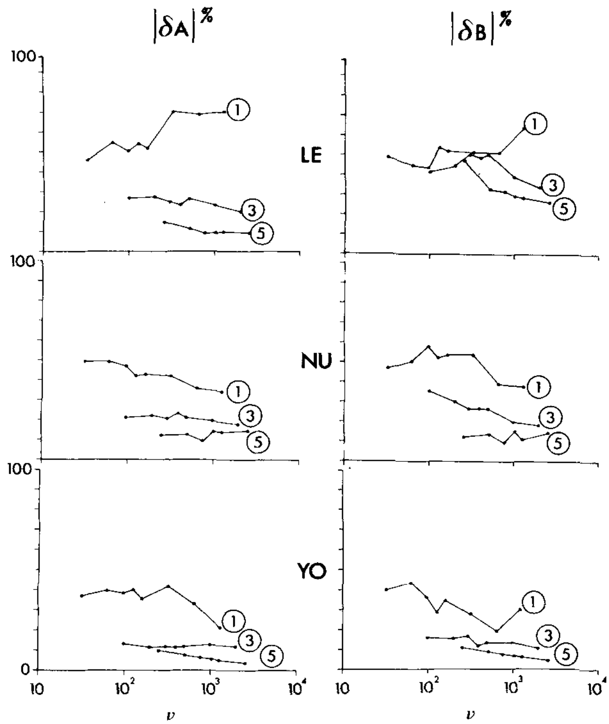

Figure 3. Variation of mean percentage error in transfer function estimate with degrees of freedom (ν) for three period bands at LE, NU and YO. Band 1: 128–64 min, band 3: 32–16 min, band 5: 8–4 min.

approach a reasonable level of less than 10 per cent. With the exception of band 1 at the northernmost station LE, the overall trend is for the error to decrease with increasing ν.

The errors $|(\delta A, \delta B)|$ can be shown to be given approximately by:

$$|\delta A|^2 = \frac{E^2}{S_{\rm HH}(1 - R_{\rm DH}^2)}, \quad |\delta B|^2 = \frac{E^2}{S_{\rm DD}(1 - R_{\rm DH}^2)} \tag{3}$$

(Everett & Hyndman 1967; Banks 1975). Here $S_{\rm HH}$ and $S_{\rm DD}$ are the total powers in the H and D components and $R_{\rm DH}^2$ is the coherence between D and H. As a consequence of the terms $S_{\rm HH}$ and $S_{\rm DD}$ we can expect $(\delta A, \delta B)$ to decrease with increasing ν. This is observed with the exception of band 1 at the highest latitude station LE. The effect at LE could be the result of an increase in $R_{\rm DH}^2$ with latitude. The distributions of $R_{\rm DH}^2$ determined for individual records for the 80 records at each station in band 1 are shown in Fig. 4. All the distributions are biased towards low values of $R_{\rm DH}^2$ and there is no obvious latitudinal effect. Examination of the power levels of the records used shows that $S_{\rm HH}$ and $S_{\rm DD}$ increase with latitude over the period range considered. We are left to conclude that the error function E is the controlling influence on the accuracy with which (A, B) can be determined.

It follows from equations (3) that the power ratio $S_{\rm HH}/S_{\rm DD}$ will determine the error ratio $\delta A/\delta B$. The ratio $S_{\rm HH}/S_{\rm DD}$ is governed by the distribution in azimuth of the major axes of the horizontal field polarization ellipses. In the case of demodulated records, the polarization ellipses are readily determined (Banks 1975). The normalized distribution of azimuths in 10 degree sectors for bands 1, 3 and 5 at the highest (LE) and lowest latitude (YO) stations is shown in Fig. 5. The distributions are determined from the 80 records

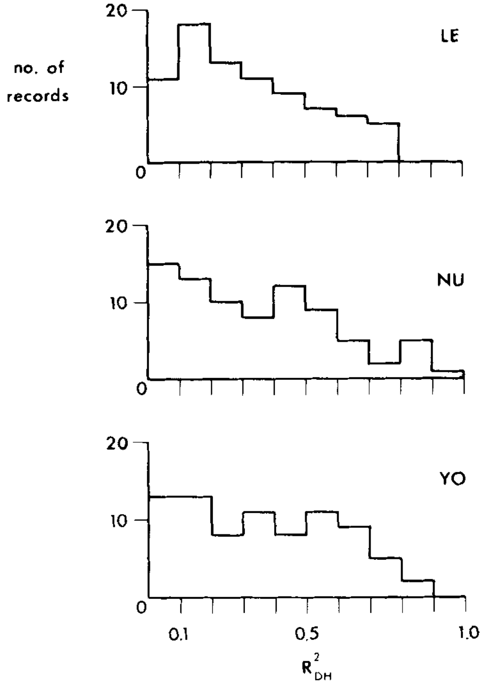

Figure 4. Distribution of coherence between H and D components R^2_{DH} of individual records from total of 80 records at LE, NU and YO for band 1: 128–64 min.

available at each station. At the lowest latitude station, YO, there is a clear bias towards N–S orientations. This bias is much reduced at the highest latitude station, LE, where, in band 1, all orientations of horizontal field azimuth have an equal probability of being recorded.

4.4 EFFECTS OF DEGREES OF FREEDOM ON R^2

The behaviour of (A, B) can be controlled by the behaviour of E (or R^2) as outlined in Section 2. If E is found to increase (R^2 to decrease) with increasing degrees of freedom, the result will be to decrease $|(A, B)|$.

The vertical field partial coherence R^2 was determined according to the data organization and averaging procedure of Table 2. The resulting variation of R^2 with ν at the three stations for bands 1, 3 and 5 is shown in Fig. 6. In bands 1 and 3, at all stations, R^2 is found to decrease with increasing ν. Over individual records (estimates at the lowest ν) a high R^2 may be obtained due to a persistent and coherent external field gradient (see discussion by Cochrane & Hyndman 1974). Increasing ν by the introduction of other complex substorm records allows external fields with $Z_n = 0$, Z_n (consistent) or Z_n (inconsistent) characteristics to be introduced. Under these conditions, if the same R^2 is maintained with increasing ν, source fields of the $Z_n = 0$ or Z_n (consistent) type can be considered present. If, on the other hand, R^2 is found to decrease with increasing ν, source fields of the Z_n (inconsistent) type must be present.

The observed decreasing trends in R^2 at all stations for periods greater than 32 min indicates that $Z_n \neq 0$ and that the external field variations used in the analysis do not possess any predominant wavelengths. The gradients observed in Fig. 6 can be considered a measure

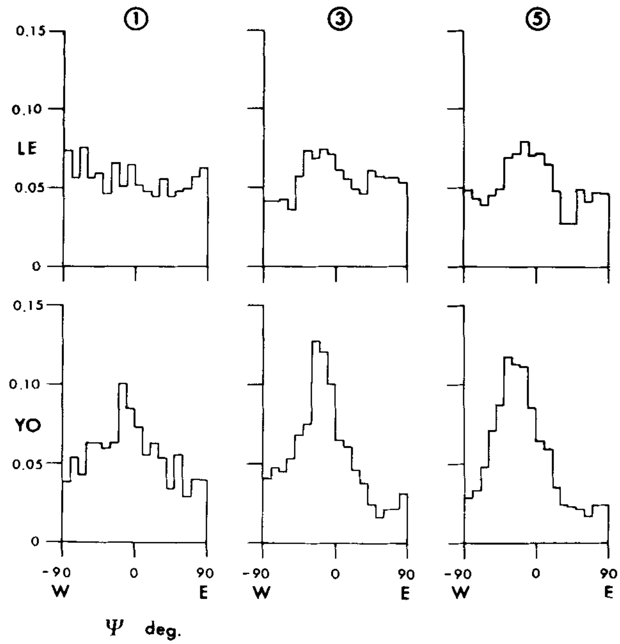

Figure 5. Normalized distribution of azimuth Ψ of horizontal field polarization ellipses at LE and YO for three period bands. Band 1: 128–64 min, band 3: 32–16 min, band 5: 8–4 min. Degrees of freedom are band 1: 5120, band 3: 7840, band 5: 20480.

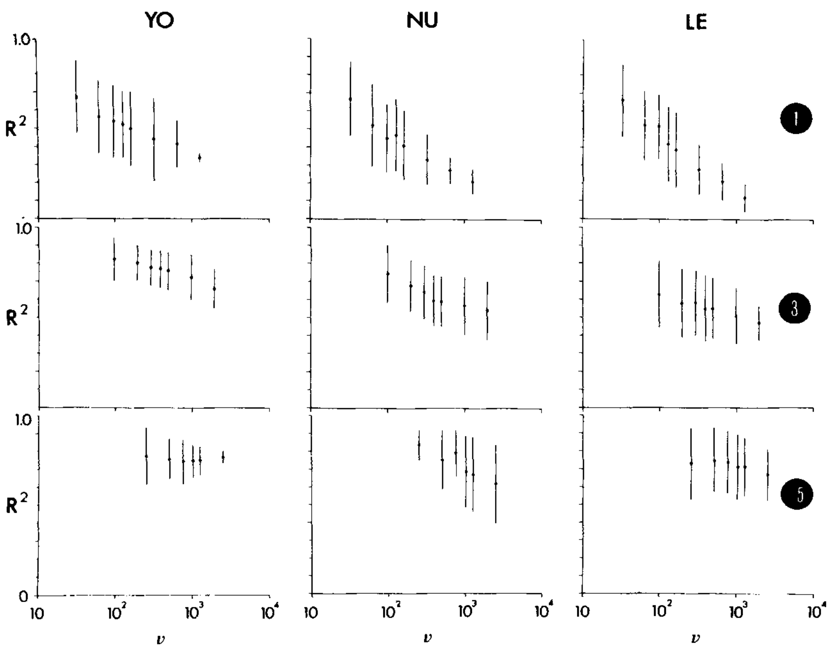

Figure 6. Variation of vertical field partial coherence R^2 with degrees of freedom (ν) for three period bands at YO, NU and LE. Band 1: 128–64 min, band 3: 32–16 min, band 5: 8–4 min.

of the variability of the wavelengths of the source fields considered. This variability is most severe at the longest period (128–64 min) of the analysis and at the highest latitude station.

Within the period range of the analysis, source field effects on the estimate of the transfer function (A, B) have been identified at all three selected stations. The effects appear to increase with latitude and period. A more detailed quantitative analysis of the effects would require the selection of individual wavelength features from the records. Such a procedure would require observations on a scale greater than the inherent wavelengths. In the face of such a formidable task it is simpler to enquire if there are alternative empirical selection procedures that can be applied.

5 The use of R^2

Although coherence as a quality criterion for the selection of records is discussed by Bennett & Lilley (1971) and Cochrane & Hyndman (1974) nowhere does it appear to have been applied rigorously in order to seek transfer functions free from source field effects. The previous discussion has shown how the behaviour of $(A \pm \delta A, B \pm \delta B)$ and R^2 with ν can provide various measures of the magnitude of source field effects at a particular station and within a particular period band. In order to understand the effects still further, the behaviour of $(A \pm \delta A, B \pm \delta B)$ and R_c^2 (the R^2 obtained from combined records) with R_i^2 (the R^2 obtained from individual records) is studied.

The normalized distribution of R_i^2 determined from individual 12 hr records for the 80 records available at each station for bands 1–5 is shown in Fig. 7. There is no obvious variation in the distributions of R_i^2 for any band with latitude. The main effect observed is with period; the distributions for the shorter period bands all appear biased towards higher predicted coherencies.

Individual records of a given R_i^2 are next selected and used to redetermine $(A \pm \delta A, B \pm \delta B)$ and R_c^2. The availability of records of a given R_i^2 at a particular station varies as shown in Fig. 7. Where a sufficient number of degrees of freedom exist, a selection

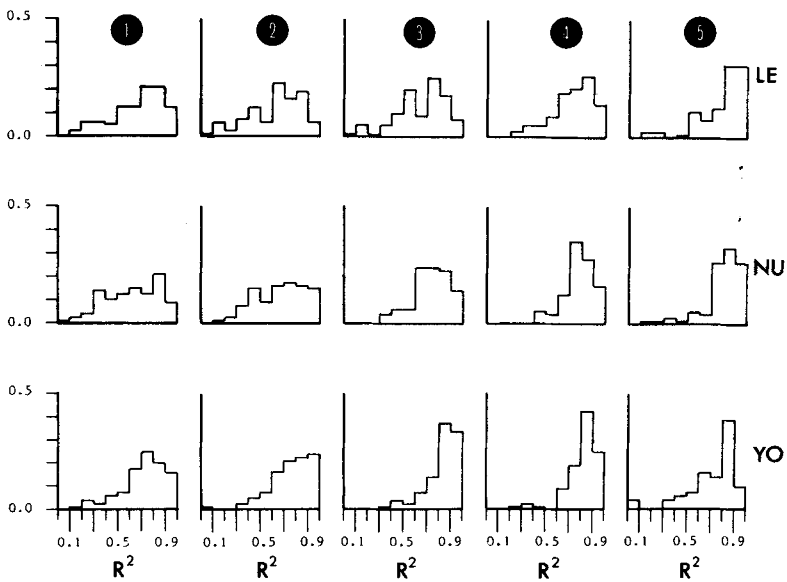

Figure 7. Normalized distribution of vertical field partial coherence R^2 of individual records from total of 80 records at LE, NU and YO for five period bands. Band 1: 128–64 min, band 2: 64–32 min, band 3: 32–16 min, band 4: 16–8 min, band 5: 8–4 min.

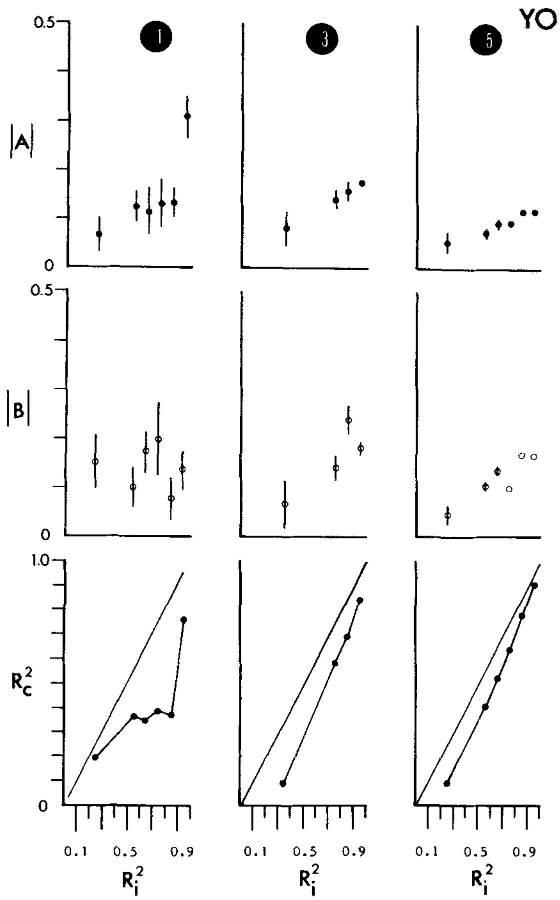

Figure 8. Variation of transfer function estimate $|(A, B)|$ and R_c^2 as a function of the individual record partial coherence R_i^2 used in the determination at YO. Band 1: 128–64 min, band 3: 32–16 min, band 5: 8–4 min.

increment in R_i^2 of 0.1 has been used. Due to the lack of records exhibiting a low R_i^2, the selection increment is necessarily extended for low R_i^2. Figs 8–10 show the variation in $(A \pm \delta A, B \pm \delta B)$ and R_c^2 in the period bands 1, 3 and 5 for the selected increments of R_i^2. The number of degrees of freedom used in each determination is given in Table 3. In the discussion of the results presented in Figs 8–10, it should be remembered that ν can affect the determinations of $(A \pm \delta A,\ B \pm \delta B)$ and R_c^2 as discussed previously. Hence it is anticipated that some variability due to the different νs used within a given period has been introduced.

At a given station, for a period band with no source field affects and noise free data, three effects would be expected:

(1) $|(A, B)|$ would increase with increasing increment value of R_i^2,
(2) $|(\delta A, \delta B)|$ would decrease with increasing increment value of R_i^2,
(3) R_c^2 would follow the diagonal line shown in the variation of R_c^2 with R_i^2 in Figs 8–10.

These effects follow from equations (1) and (3) in which incremental changes in E (or R_i^2) would require corresponding changes in $(A \pm \delta A, B \pm \delta B)$.

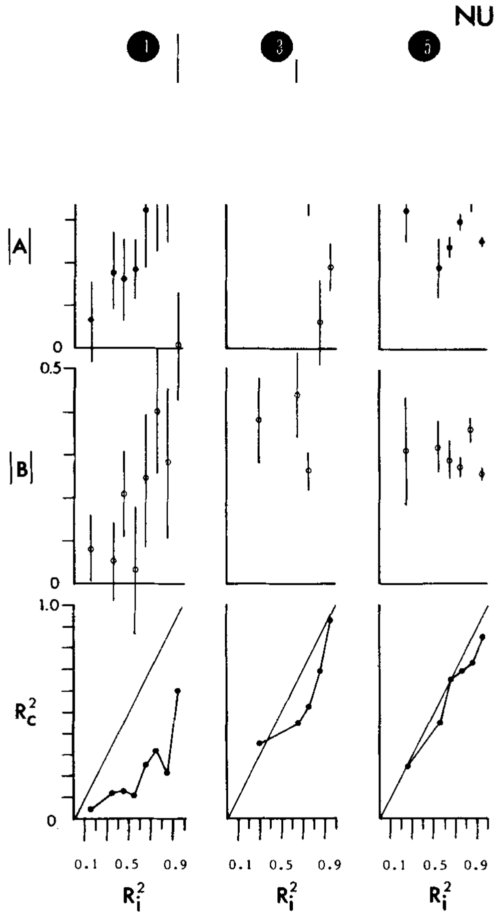

Figure 9. Same as Fig. 8 at NU.

If Z_n (consistent) records are used we expect a variation in R_c^2 below the ideal diagonal and running parallel to it. The departure from the ideal diagonal would be a measure of Z_n at that particular station. If Z_n (inconsistent) records are used we expect a complex variation of R_c^2 with R_i^2 that would be a function of the differing wavelengths present in the individual records used in the determination of R_c^2.

The determinations at the lowest latitude station YO (Fig. 8) are perhaps the most readily understood since Z_a is small. The behaviour of R_c^2 suggests that Z_n small or consistent records have been used in bands 3 and 5. It is also found that $|(A, B)|$ increase and $|(\delta A, \delta B)|$ decrease with increasing increment value of R_i^2. The results confirm the previous results of $|(A, B)|$ and R^2 against ν at YO (Figs 2(c) and 6) which suggested that these bands were free from source field effects. The variation of R_c^2 in band 1 at YO shows a clear source field effect. R_c^2 for the increment $0.9 < R_i^2 \leq 1.0$ is a high value which falls dramatically for values of $R_i^2 < 0.9$. This behaviour is also reflected in $|A|$. It appears that records possessing $R_i^2 > 0.9$ can result from a contribution from Z_n arising due to predominantly N–S spatial gradients.

Of the determinations of R_c^2 at NU (Fig. 9) only bands 3 and 5 appear to represent Z_n (consistent) conditions. However, although close to the ideal diagonal they show a degree of variability (i.e. not parallel to the ideal) that suggests that Z_n (inconsistent) conditions are

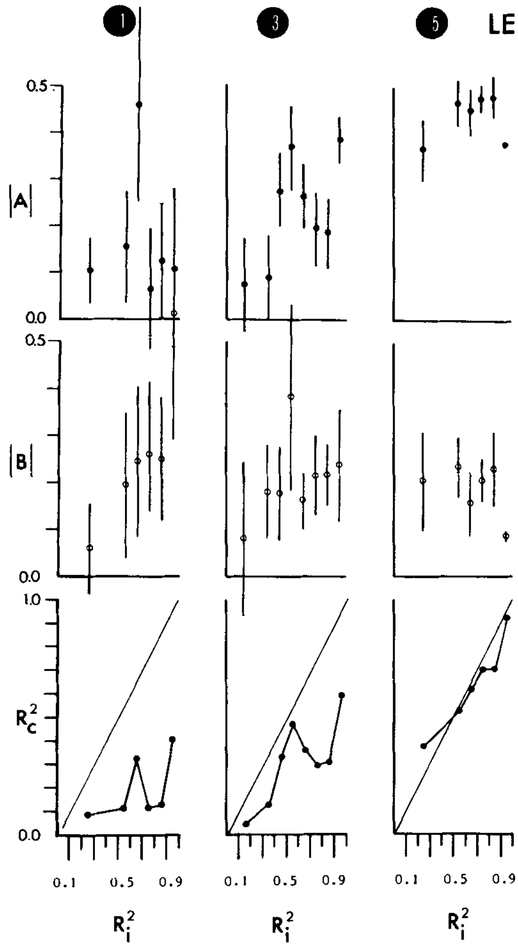

Figure 10. Same as Fig. 8 at LE.

Table 3. Degrees of freedom used in the transfer function determinations of Figs 8, 9 and 10.

R_i		1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
YO	Band 1	408	512	640	448	192			352			
YO	Band 3	2500	2500	1078				1076				
YO	Band 5	2048	2560	2560	2560	1536			2560			
NU	Band 1	224	544	320	284	320	256	352		192		
NU	Band 3	1078	1764	1862	1862				1274			
NU	Band 5	2560	2560	2560	768	1024			1280			
LE	Band 1	320	544	544	320	320			512			
LE	Band 3	588	1372	1960	686	1568	784		980			
LE	Band 5	2560	2560	2560	1536	2304			1280			

also present. This has been indicated previously by the determinations of $|(A, B)|$ and R^2 against ν for the same bands (Figs 2(b) and 6). The source field effects previously indicated in band 1 at NU are apparent in the degree of variability observed in R_c^2. Z_n (inconsistent) effects on $|(A \pm \delta A, B \pm \delta B)|$ in all three bands at NU are clearly complex. At NU Z_a is large; in cases where $Z_a \geq Z_n$ the identification of source field effects in $|(A \pm \delta A, B \pm \delta B)|$ is difficult.

None of the three R_c^2 determinations at Le (Fig. 10) can be considered to represent Z_n (consistent) determinations. This is in agreement with the indications from previous results. At Le Z_a is large and the identification of obvious source effects in $|(A \pm \delta A, B \pm \delta B)|$ is again difficult.

One feature to emerge from Figs 8–10 is that in band 1 at all stations Z_n (inconsistent) conditions are present. R_c^2 between the increments $0.9 < R_i^2 \leq 1.0$ and $0.8 < R_i^2 \leq 0.9$ falls from a high to a low value. It seems clear from the behaviour of $|A|$ at YO that the determination of (A, B) from records with $R_i^2 > 0.9$ is due to Z_n appearing within Z_c and that Z_n arises from a predominantly N–S spatial gradient. Similar effects are observed in band 1 at NU (Fig. 9). In this case Z_n is due to spatial gradients in both N–S and E–W directions. In band 1, at the highest latitude station LE (Fig. 10), Z_n can be considered to arise from predominantly E–W spatial gradients.

6 The determination of (A, B)

Having examined the use of R^2 (or E) in identifying source field effects, we are in a position to restate its role in the transfer function relationship of equation (1). For data records free from source field effects, $E = 0$ for data with high S/N. If S/N is low, E will define the reliability of the transfer function estimate. For large S/N and Z_n (consistent) records, E may define Z_n. For Z_n (inconsistent) records, E is a function of the distribution of Z_n within the available data records.

In the first two cases, the determination of (A, B) can be made using all the available data records as input since (A, B) and E are not functions of the degrees of freedom of the analysis. Alternatively, estimates of (A, B) from individual records can be combined using a weighting factor $(1-E)^2$ as suggested by Klein & Larsen (1978) or $(R^2)^2$ in terms of the present analysis. Such a procedure is justified since, in this case, $|(A \pm \delta A, B \pm \delta B)|$ and E (or R^2) behave as in bands 3 and 5 at YO (Fig. 8).

In the third case of Z_n (inconsistent) records we know from the results of Section 4.2 that we must minimize the introduction of differing Z_n. This effectively means using individual record determinations of (A, B) and combining them. Also in the case of Z_n (inconsistent) records, the wavelength dependence of equation (1) should be acknowledged. From the results of the previous section, the only clear control we have over the wavelength dependence appears to be the variation of R_c^2 from records with R_i^2 in the intervals $R_i^2 > 0.9$ and $R_i^2 < 0.9$. It has been demonstrated that the higher R_i^2 records produce an estimate of (A, B) influenced by the correlation properties of Z_n. Such records presumably contain shorter wavelength features of the external field variations.

The applicability in the use of a weighting factor of $(R^2)^2$ to produce an estimate of (A, B) from distribution of individual estimates of (A, B) is controlled by the available distribution of R_i^2 such as those shown in Fig. 7. In cases where Z_n (inconsistent) records occur, a distribution of records heavily biased towards $R_i^2 > 0.9$, the use of a weighting factor of $(R^2)^2$ would result in the introduction of a source field contribution. This is demonstrated in Fig. 11. From the 80 available determinations of $(A \pm \delta A, B \pm \delta B, R_i^2)$ at each station, a final estimate of (A, B) was obtained from individual determinations using

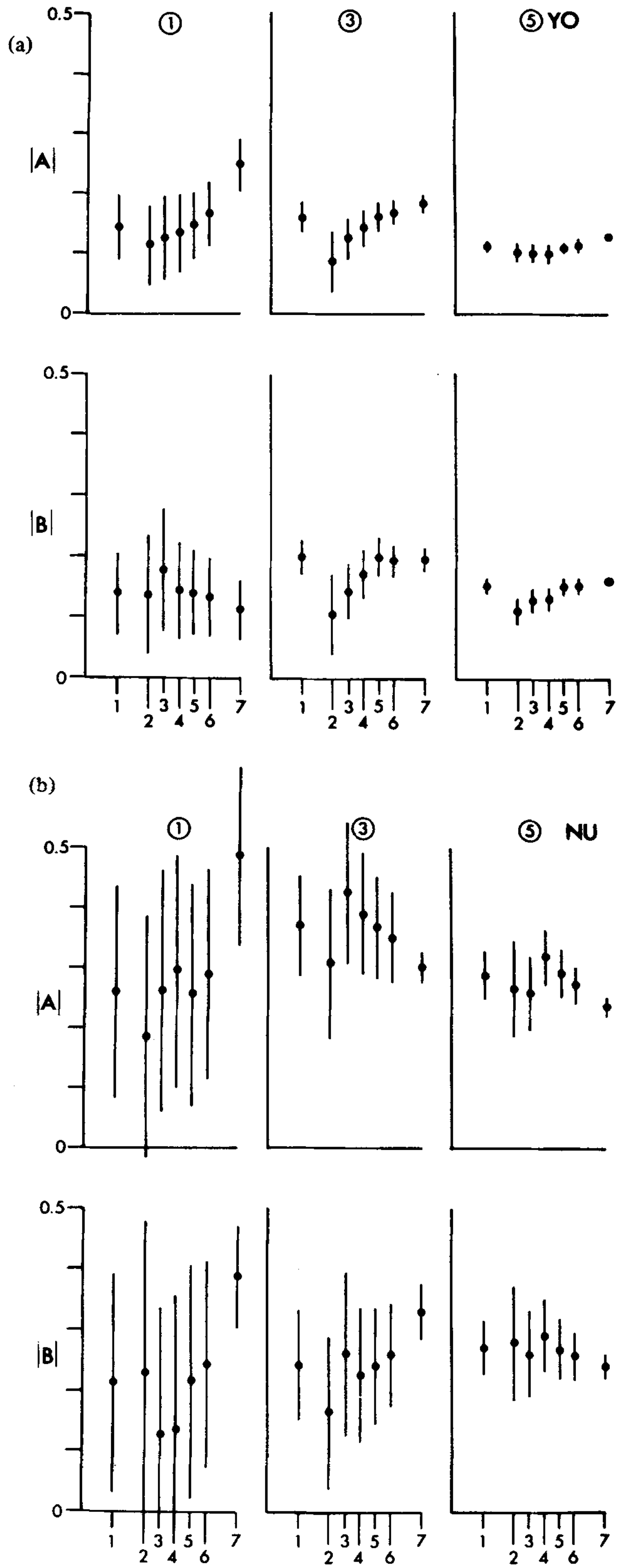

Figure 11. Variation of the *weighted* transfer function estimate $|(A, B)|$ as a function of seven selection intervals of R_i^2: 1, $0.5 \leqslant R_i^2 \leqslant 0.9$; 2, $0.0 \leqslant R_i^2 \leqslant 0.6$; 3, $0.0 \leqslant R_i^2 \leqslant 0.7$; 4, $0.0 \leqslant R_i^2 \leqslant 0.8$; 5, $0.0 \leqslant R_i^2 \leqslant 0.9$; 6, $0.0 \leqslant R_i^2 \leqslant 1.0$; 7, $0.9 \leqslant R_i^2 \leqslant 1.0$. (a) At YO, (b) at NU, (c) at LE.

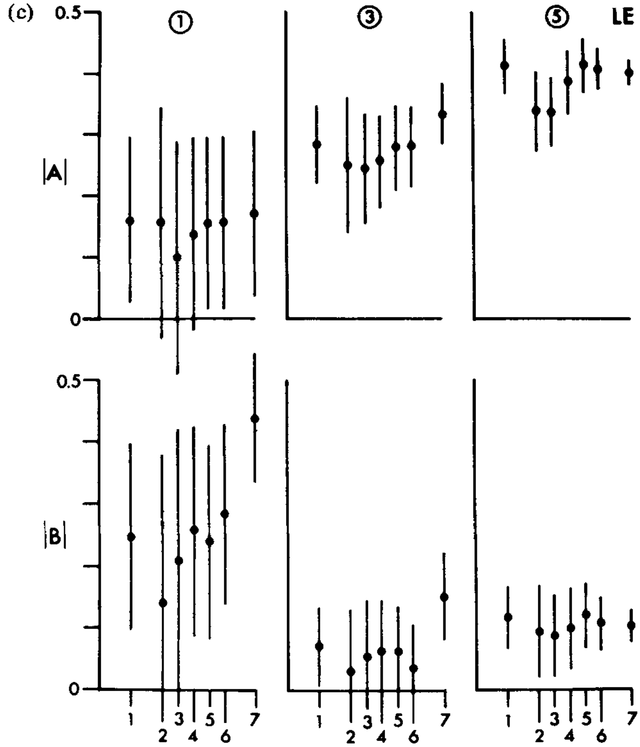

Figure 11

$(R^2)^2$ as a weighting factor. This was done using seven ranges of R_i^2: 1, $0.5 \leq R_i^2 \leq 0.9$; 2, $0.0 \leq R_i^2 \leq 0.6$; 3, $0.0 \leq R_i^2 \leq 0.7$; 4, $0.0 \leq R_i^2 \leq 0.8$; 5, $0.0 \leq R_i^2 \leq 0.9$; 6, $0.0 \leq R_i^2 \leq 1.0$; 7, $0.9 \leq R_i^2 \leq 1.0$. The results from increment 7, $0.9 \leq R_i^2 \leq 1.0$, confirm that source field effects can be emphasized by such an unequal distribution. If the distribution of R_i^2 is biased towards low values then again an inadequate estimate of the transfer function is obtained. This is demonstrated in Fig. 11 by range 2 of R_i, $0.0 \leq R_i^2 \leq 0.6$, which in many cases produces a significantly low value of $|(A, B)|$. The need for a wide distribution of R_i^2 when Z_n (inconsistent) effects are present is clear. For the distributions available in the present study the ranges $0.5 \leq R_i^2 \leq 0.9$ and $0.0 \leq R_i^2 \leq 1.0$ produce similar transfer function estimates. More severe discrepancies would be expected using more limited data sets.

It was demonstrated in Section 4.3 that the error function E is the controlling influence on $(\delta A, \delta B)$. The large errors in the determinations of (A, B) in band 1 in Fig. 11 reflect the presence of Z_n rather than reliability. Using the procedures outlined above we can only minimize the variability of Z_n and its contribution to (A, B). The ultimate procedure when Z_n (inconsistent) records only are available would involve the selection of individual wavelength features. If such a procedure could be devised, it would further allow the introduction of equation (2) into the analysis procedure.

Since we have identified certain statistical features of R^2 with source field effects it was considered whether an alternative prior selection procedure based on geomagnetic indices would be adopted. The records used, although selected from IDD, corresponded to time intervals possessing a wide range of geomagnetic activity. No correlation was found between R_i^2 and either K_p or D_{st} indices. In the light of the arguments presented here it is suggested that while the use of geomagnetic indices can identify the magnitudes of global disturbance fields they may not be the most useful criterion in identifying disturbance field wavelengths.

7 Conclusions

The preceding study has identified source field effects on transfer function estimates and has related them to the finite wavelengths of external field variations at mid-latitudes. It appears that the source field effects arise from the varying contributions of Z_n in the least-squares fitting of the transfer function relationship.

Provided sufficient records are available, the presence of source field effects can be identified as a variation in $(A \pm \delta A, B \pm \delta B)$ and the vertical field partial coherence R^2 with the number of degrees of freedom used in the analysis. On this basis it is found that source field effects increase with both latitude and period. Only at the lowest latitude station studied, in the period range 32–4 min, can the determination of (A, B) be considered independent of the contributions from external field characteristics. At higher latitudes and longer periods it is found that source field configurations or wavelengths are variable. The results suggest that such spatial variability tends to be confined to the N–S direction at the lowest latitude station while it appears predominantly in the E–W direction at the highest latitude station. Further results from additional stations would be required to claim this as a general result.

At mid-latitudes the transfer function (A, B) is best determined from a weighted mean of estimates of (A, B) from individual records. The weighting procedure, if it is to be effective, must take into account the distribution of the vertical field partial coherence. In extreme cases, a limited or inadequate distribution of the vertical field partial coherence, will result in an inadequate estimate of the transfer function.

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