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Key Points:

- There is a fractal scaling relationship between river inlet abundance and lake area
- Inlet abundance increases with lake area and shape factor, drainage density, and junction angle
- Inlet-area scaling rules correctly predict the relationship between water residence time and lake area

Supporting Information:

Supporting Information may be found in the online version of this article.

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The Fractal Scaling Relationship for River Inlets to Lakes

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Abstract Scaling relationships provide simple rules for understanding complex hydrographic patterns. Globally, river inlet abundance varies among lakes by about three orders of magnitude, but few scaling relationships describe this aspect of lake-river connectivity. In this study, we describe a simple theoretical scaling relationship between lake surface area and river inlet abundance, and test this theory using data from Scandinavia. On average, the number of inlets increases by 67% for each doubling of lake area. However, lakes of vastly different areas can have the same number of inlets with relatively small variations of drainage density, lake shape, or junction angle - characteristics that can often be linked to specific geological processes. Our approach bridges the gap between the detailed understanding of geomorphic processes and large-scale statistical relationships, and engenders predictions about additional patterns including the relationship between lake area and water residence time.

Plain Language Summary Lakes and rivers are often connected, but the patterns and consequences of this connectivity are poorly described, because lakes and rivers are studied separately. In this study, we develop and test simple rules that describe lake and river connectivity. Specifically, we focus on predicting the number of river inlets, which varies among lakes from 0 to almost 1,000. The following are the factors controlling inlet abundance: lake area, lake shape, the average distance between rivers, the average angle that rivers intersect lakes, and topographic complexity. We make several predictions based on these rules, including that glacial lakes should have relatively high connectivity with rivers compared to lakes with other geologic origins. We also use these rules to predict the relationship between lake area and the age of water within lakes. Collectively, our results and predictions demonstrate how simple rules like those developed in our study can enrich the understanding of inland waters.

1. Introduction

Lakes are integral components of river networks, constraining hydrological, biological, and chemical processes at both the ecosystem and network scales (Gardner et al., 2019; Jones, 2010; Schmadel et al., 2018). However, research has often focused on rivers and lakes in isolation and not from an integrated perspective that reflects the intimate relationship between these systems (Gardner et al., 2019; Jones, 2010). In particular, there is a need to develop scaling relationships that describe the morphology of river-lake networks (Gardner et al., 2019). Such relationships are central to the up-scaling approaches that are widely used to generalize understanding aquatic patterns and processes at regional to global scales (Downing, 2009).

Hydrological scaling relationships are primarily based on characteristics that are easy to quantify on maps, including lake surface area and river order (Downing, 2009; Strahler, 1957). For example, there are well-known scaling relationships between river abundance, mean river segment length, and mean upstream contributing area relative to river order (Strahler, 1957). For lakes, abundance, perimeter, volume, and mean depth scale predictably with surface area (Cael et al., 2017; Kent & Wong, 1982; Seekell et al., 2013). Some characteristics of lakes on river networks have been assessed. For example, lake abundance decreases, lake size increases, and spacing between lakes increases as river order increases (Gardner et al., 2019). Globally, the abundance of river inlets varies among lakes by about three orders of magnitude, but this aspect of lake-river connectivity is not described by these existing scaling relationships (Mark, 1983).

In this study, we describe a simple theoretical scaling relationship between lake surface area and the number of river inlets based on the principle of line intercepts of topographic features. We evaluate how lake and landscape characteristics effect river inlet abundance based on this theoretical scaling relationship. Finally, we test this theory with data from Scandinavian lakes and rivers. Collectively, these analyses provide simple rules for understanding lake-river connectivity at regional and global scales.

2. Theory

Most of Earth's lakes are small (<1 km^2) and of glacial origin, especially lakes in the high northern latitudes (Meybeck, 1995). The formation processes of these lakes are typically sudden catastrophic events (Timms, 1992). Therefore, we assume there is not a strong systematic relationship between the river networks and lake shorelines. Under these conditions, a line-intercept approach can be taken, where the lake shoreline is a traverse line and the expected number of river inlets per lake on the river network is the lake shore length (P, km) divided by the mean orthogonal distance between rivers (Mark, 1974, 1983; Wentworth, 1930). Drainage density (D_d, km^{-1})—the total river length (km) divided by watershed area (km^2)—is the inverse of the mean orthogonal distance between rivers (Mark, 1974). Therefore, the expected number of river inlets (N) for a lake is:

$$N = D_d P \tag{1}$$

Shore length is related to lake surface area (A, km^2), a shape factor C, and a scaling exponent which is the shoreline fractal dimension (D) divided by two:

$$P = CA^{D/2} \tag{2}$$

The shape factor must be $C \geq 2\pi^{0.5}$, the value for a perfectly circular lake (Cheng, 1995). The exponent $D/2$ is geometrically limited to the range $0.5 \leq D/2 \leq 1$, where $D = 1$ is a smooth shoreline and $D = 2$ is a shoreline so convoluted that it is a space filling curve (Cheng, 1995; Seekell, 2018). Substituting the perimeter-area relationship for perimeter in Equation 2 gives the following scaling relationship between lake surface area and the number of river inlets:

$$N = D_d C A^{D/2} \tag{3}$$

The line intercept approach assumes that line intersects features at 90° angles (Wentworth, 1930). Comparisons of river flow paths to lake shorelines suggest that this is often not the case (e.g., Schmadel et al., 2018), and hence these measurements require a multiplicative correction factor which is the sine of the junction angle (Wentworth, 1930). While the junction angles for rivers are well studied, we do not know of analogous studies for river-lake junctions. Therefore, we assume that all junction angles (θ, radians) are equally likely:

$$\frac{1}{\pi / 2} \int_0^{\pi/2} \sin \theta d\theta = \frac{2}{\pi} \tag{4}$$

Applying this correction factor, the expected number of inlets is:

$$N = \frac{2}{\pi} D_d C A^{D/2} \tag{5}$$

Hence, the expected number of inlets is a function of lake area adjusted by a few control parameters that can easily be estimated for a collection of lakes in a given region. Like other scaling relationships, inlet-area scaling is expected to hold as an average among many lakes. The relationship can be fit statistically as a power-law regression, where N is the dependent variable, $\frac{2}{\pi} D_d C$ is the intercept, A is the independent variable, and $D/2$ is the power-exponent.

3. Empirical Analysis

3.1. Study Location and Data

We evaluated the inlet-area scaling relationship for 106 Scandinavian lakes, primarily from the mountainous border region between Sweden and Norway (Table 1). Surface areas and perimeters were extracted

Table 1
Characteristics of Lakes Used in the Empirical Analysis

Characteristic	Median	Range
Lake surface area (km^2)	0.14	0.01–3.78
Lake perimeter (km)	2.18	0.49–16.33
Drainage density (km^{-1})	1.17	0–4.47
Number of inlets	2	0–26
Elevation (m)	729	170–1,401
Watershed area (km^2)	3.32	0.05–232

from digitized 1:50,000 scale maps from the Swedish Mapping Agency Lantmäteriet and the Norwegian Water Resource and Energy Directorate (Lindmark, 2021). The locations and characteristics of individual lakes are given by Lindmark (2021). Watershed area was calculated using the approach described by Klaus et al. (2019), which is based on standard extraction techniques optimized for Scandinavian data sources. We counted river inlets and measured river lengths based on map blue lines that represent flowing waters (both rivers and streams, but referred to simply as rivers throughout this study). These corresponded well to channel networks visible in satellite imagery during spot checks. Drainage density was calculated as the ratio of total river length to watershed area. Lakes with clear human influence, such as dams, were not included in our analysis.

3.2. Data Analysis

We first evaluated the basic patterns formed by our scaling relationship by calculating the expected number of inlets across the typical range of variability for each parameter (Table 2). Specifically, we solved for specific numbers of inlets ($N = 1, 10, 20$) based on combinations of $\frac{2}{\pi} D_d C$ versus A. We did these calculations twice, once with $D = 1$ and once with $D = 4/3$, to illustrate the effect of variation in fractal dimension.

Next, we calculated the fractal dimension by regressing perimeter by area, and compared this to the result achieved by regressing the number of inlets by area. The dimensions should be the same if the scaling equation outlined above is correct. We used generalized linear models with a log-link for both regressions. This is important for the inlet-area relationship because the expectation, and not the dependent variable itself, is log-transformed. Hence, there is no issue of including lakes without inlets which are very common on the landscape and would be problematic for traditional approaches that require log transformation of the dependent variable (Mark, 1983; Seuront, 2010). Overdispersion in the inlet-area relationship was minor and did not have a material impact on our analysis (Text S1).

Finally, we calculated the expected number of inlets for our study lakes based on our scaling relationship. This analysis is meant to demonstrate that the parameters of our statistically fit inlet-area relationship capture the physical meaning described by our theory. We used an average shape factor and fractal dimension derived from the perimeter-area relationship (Cheng, 1995; Rex & Malanson, 1990; Seuront, 2010).

Our analysis was conducted using R version 4.0.2 with the boot and CAR packages (Canty & Ripley, 2020; Fox & Weisberg, 2019; R Core Team, 2020). We report confidence intervals based on bootstrapping ($n = 9{,}999$ replications) that are bias corrected and accelerated.

4. Results

Because of the inlet-area scaling relationship's multiplicative form, the number of inlets can remain constant across several orders of magnitude of lake area, with only small differences in drainage density or shape among lakes (Figure 1). These types of patterns are apparent in our study lakes. For example, there

Table 2
Typical Values for Parameters in the River Inlet-Lake Area Scaling Relationship

Parameter	Typical value	Source
Drainage density (D_d)	0–5 km^{-1}	Luoto (2007); Schneider et al. (2017)
Lake shape factor (C)	$> 2\pi^{0.5}$	Hamilton et al. (1992); Cael and Seekell (2016)
Lake surface Area (A)	0.01–10 km^2	Cael and Seekell (2016)
Fractal dimension (D)	$1/2 \leq D/2 \leq 2/3$	Kent and Wong (1982); Seekell (2018)

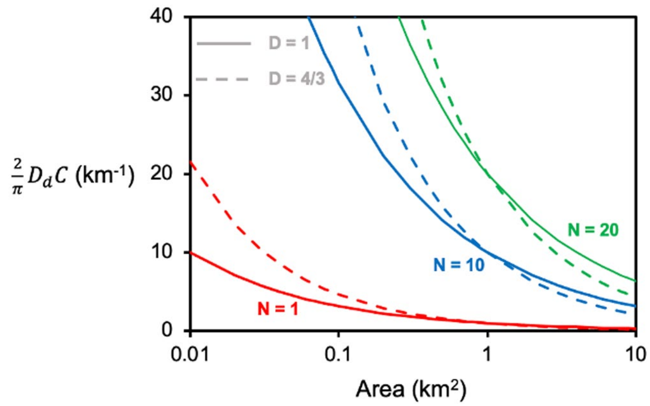

Figure 1. General patterns from the inlet-area scaling relationship. *N* is the number of inlets. The ordinate is the product of drainage density, lake shape factor, and a correction factor for river-lake junction angle.

is more than 70-fold variation in lake area for lakes with one inlet, but $\frac{2}{\pi}D_dC$ only varies by a factor of nine, primarily reflecting a variability in D_d (coefficient of variation = 0.53) and not the shape factor (coefficient of variation = 0.20). The effect of fractal versus smooth perimeters depends on the lake size (Figure 1). For smaller lakes (e.g., <1 km^2), greater drainage density or lake shape factor is needed to have the same number of inlets of an equally sized lake with a smooth perimeter. The opposite is true for larger lakes (e.g., >1 km^2).

The scaling exponent for perimeter-area relationship was $D/2 = 0.65$ (95% CI = 0.59–0.70) (Figure 2a). The scaling exponent for the inlet-area relationship was $D/2 = 0.67$ (95% CI = 0.58–0.75) (Figure 2b). A complete overlap among the confidence intervals for $D/2$ offers strong support for the proposition that the perimeter-area and inlet-area relationships share a fractal dimension. The intercept from the perimeter-area scaling ($\log_e(C) = 2.12$, 95% CI = 2.05–2.20) is almost exactly the same as the natural logarithm of the mean of the shape factors ($\log_e(C) = 2.13$, 95% CI = 2.09–2.19). The intercept of the inlet-area relationship ($\log_e \left(\frac{2}{\pi}D_dC\right) = 2.15$, 95% CI = 1.97–2.29) was higher than expected ($\log_e \left(\frac{2}{\pi}D_dC\right) = 1.85$, 95% CI = 1.70–2.01) based on our calculations, but with overlapping at 95% confidence intervals.

The predicted versus observed number of inlets generally fit well along a 1:1 line when calculated directly (Figure 3**).** The mean difference between predicted and observed values was 0.17 inlets, which is much less than the median (2) and mean (3.66) number of inlets for our study lakes. Since our analysis for Figure 3 has no free parameters, the variance in number of inlets is clearly dominated by variance in lake area, because this varies by more than two orders of magnitude (coefficient of variation = 1.65), whereas drainage density only varies between approximately 0-5 (coefficient of variation = 0.80). If we had used a lake-specific shape factor, it would have contributed the least to the variation in the number of inlets (coefficient of variation = 0.27). The correction factor for junction angle (Equation 4) is critical to having accurate predictions. When we repeated our calculations to predict the number of inlets without the correction factor, the average difference between predicted and observed number of inlets increased from 0.17 to 2.35—a substantial difference given that the median and mean number of inlets was 2 and 3.66, respectively.

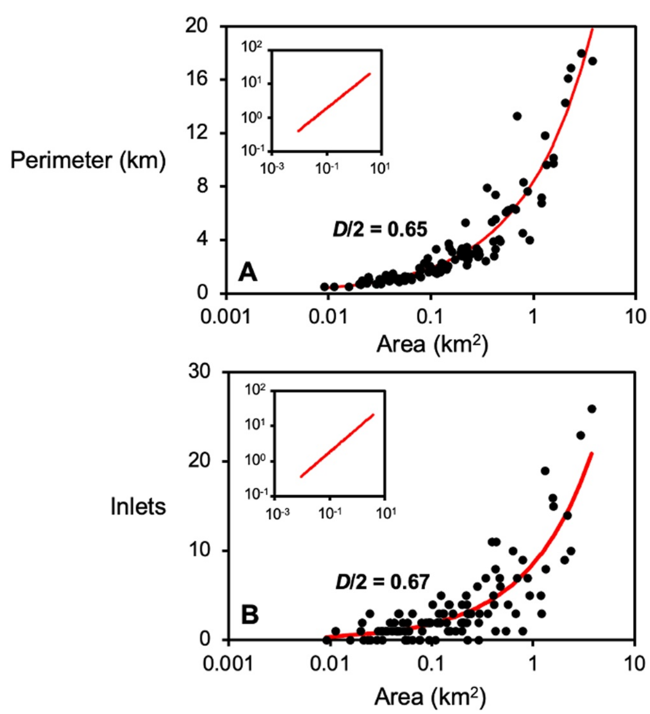

Figure 2. (a) The perimeter-area relationship for the study lakes. (b) The inlet-area relationship for the study lakes. For both panels, the insets display the expectation as a power-law.

5. Discussion

Scaling relationships provide simple rules for understanding hydrographic patterns at regional and global scales. Our study contributes to this understanding by identifying the basic factors that are statistically related to the variation in the number of river inlets among lakes. Prior research has typically focused on rivers or lakes in isolation, whereas our study provides an integrated perspective that reflects the close relationship between these systems. Lake surface area, drainage density, and lake shape are the primary factors determining the variation in the number of river inlets among lakes, with lake area and shape having the highest and lowest importance, respectively. The landscape fractal dimension moderates the influence of lake surface area. Hence, the scaling relationship developed in our study explicitly integrates fluvial, lacustrine, and landscape characteristics and processes such that it can be used both to synthesize current understanding about integrated lake-river systems and to develop new testable hypotheses for future studies.

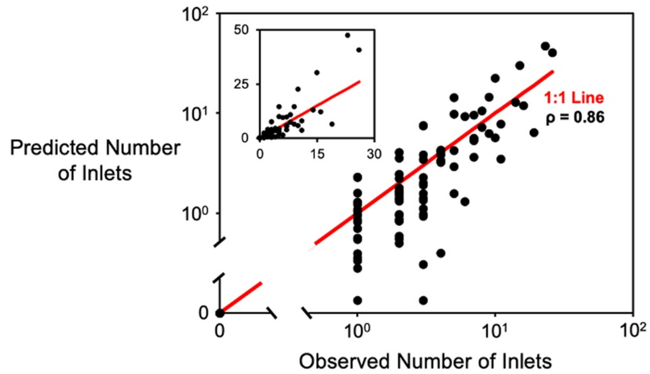

Figure 3. Predicted versus observed number of river inlets on $\log_{10}$ axes. The inset displays the data and 1:1 line on the original scale. The Spearman correlation between predicted and observed is $\rho = 0.86$ (95% CI = 0.78–0.92), and is same for the panel and inset because it is based on ranks. The mean difference between predicted and observed is 0.17 inlets. The calculations have no free parameters, hence are deterministic and not a function of statistical fitting (e.g., Figure 2).

Area is the dominant factor creating the variation in inlet abundance among lakes simply on the basis that it is the most variable of the relevant lake, river, and landscape characteristics. The power-law function form of the inlet-area scaling relationship indicates constant relative change—i.e., there is a 67% increase in the number of inlets for each doubling in surface area, when all else is equal. Since the power exponent is $D/2 < 1$, the total number inlets should primarily be concentrated among smaller lakes, which are far more abundant than large lakes (Cael & Seekell, 2016; Meybek, 1995; Seekell & Pace, 2011; Seekell et al., 2013; Verpoorter et al., 2014). This pattern is similar to the pattern of shore length and lake abundance, but contrasts the patterns for volume and area which are concentrated among large lakes (Cael et al., 2017; Seekell et al., 2013). The concentration of volume, but not inlets among large lakes, is consistent with the observation that larger lakes have longer water residence times than smaller lakes (Brooks et al., 2014). A first-order estimate based on scaling relationships provides additional insights into this pattern. If volume (V) is at steady state and completely mixed, residence time (T) is $T = V / Q$, where Q is inflow. Assuming most hydrological input is from channelized flow and that this is proportional to the number of inlets, residence time should be proportional to lake surface area raised to a power. While water residence times are not available for our study lakes, we found the relationship $\frac{V}{Q} \sim A^{0.53}$ for 907 drainage lakes for which residence times were available (Text S2). This is identical to the value expected from the combined results of our inlet-area and a previously published volume-area scaling analysis (Text S2). Non-channelized surface runoff contributes significantly to the water inflow of some lakes, but this will probably not cause significant deviations from this predicted scaling relationship because, as identified in our study, the inlet-area and perimeter-area relationships have the same scaling exponent and hence are integrated within the same residence time-area scaling relationship.

Drainage density is the second most important factor constraining the numbers of river inlets. Intuitively, our analysis indicates that regions with more rivers (higher drainage density) should have more river inlets than regions with fewer rivers (lower drainage density). Unlike lake area which has no geographic pattern from the regional to continental scale, drainage density has a strong regional pattern that reflects slope, lithology, and climate (Lapierre et al., 2015, 2018; Schneider et al., 2017). Hence, we hypothesize that, while lake-to-lake variations in the number of inlets are dominated by differences in lake area, regional-to-region variations in the mean number of inlets should more strongly reflect variations in drainage density.

The inlet-area relationship is fractal ($D = 1.34$), a characteristic that reflects the fractality of lake shorelines. The consequence of this fractality is that large lakes (>1 km^2) have more inlets than they otherwise would have if the shorelines were perfectly smooth (i.e., if $D = 1$). How does this fractality originate? In general, the dynamic processes creating fractal patterns are poorly understood for lakes when compared to other landforms (Cael & Seekell, 2016; Mandelbrot, 1983; Seekell et al., 2013; Turcotte, 2007). One conceptual model used to explain patterns of lake size and abundance is that depressions are randomly located on the landscape, with hypothetical flooding to outlets sills used to identify the location of lakes (e.g., Cael & Seekell, 2016; Downing & Duarte, 2009; Goodchild, 1988). Connected depressions represent lakes on river networks, with overlapping regions merging to become multi-basin lakes (Cael & Seekell, 2016). This is analogous to the processes that give rise to fractality in percolation theory (Cael & Seekell, 2016; Cael et al., 2015). Percolation theory predicts that if lakes connected by streams are considered independent features with separate surface areas and perimeters, the shoreline fractal dimension for a large number of lakes will tend to $D = 4/3$ (Cael & Seekell, 2016). This is how lake areas and perimeters were measured in our study, and our measurements ($D = 1.3$, 95% CI = 1.18–1.4) are very close to this theoretical value. This consistency between our empirical analysis and predictions from percolation theory is suggestive that lake fractality arises as a statistical inevitability, emergent from the interaction randomness with simple constraints dictated by the nature of water and lake basins.

The inlet-area scaling relationship predicts that lakes with higher shape factors have more river inlets than lakes with low shape factors. The shape factor is a key parameter for the description of lake morphometry due to its role in relating area to perimeter, but it is often not reported even though it is calculated while evaluating perimeter-area scaling (e.g., Cael & Seekell, 2016; Cael et al., 2015; Hamilton et al., 1992; Kent & Wong, 1982; Seuront, 2010). This is somewhat unsatisfactory because lake shape is related to the geologic processes responsible for lake formation (Blair, 1986; Scheffers & Kelletat, 2016; Timms, 1992; Text S3). For example, lakes originating from collapse (e.g., dolines/sink holes) or explosion (e.g., volcanic craters) are typically circular ($C = 2\pi^{1/2}$). Triangular lakes ($C = 6 / 3^{1/4}$) are found in dune fields subject to aeolian processes when sand is deposited around obstacles. Square lakes ($C = 4$) are sometimes found in permafrost regions where ice wedges are present. The lakes in our analysis have high shape factors (mean $C = 8.33$), which is typical of glacial lakes compared to lakes with other origins, and reflects the elongation, embayment, and merging of basins in glacially scoured regions like Scandinavia (Timms, 1992). The high shape factor for glacial lakes indicates that, on average, inlets should be more abundant on glacial lakes compared to lake types with lower shape factors such as volcanic crater lakes (Text S3). This context enriches the understanding of hydrographical patterns that is engendered by scaling relationships because it integrates more detailed knowledge of geologic processes with the large-scale statistical relationships that are created by randomness and invariance, and described by the fractal dimension.

The junction angle between rivers and lakes is critical for predicting inlet abundance. Specifically, our predictions were very accurate when a correction factor for junction angle (mean error = 0.17 inlets) was included, and very inaccurate when not including the correction (mean error = 2.35 inlets). We assumed that all junction angles are equally likely, but evidence from river networks suggests that some angles may be more likely than others, and that mean junction varies among regions (e.g., Hooshyar et al., 2017). We do not know of any analogous measurements for river-lake junctions, but we can predict from the inlet-area scaling relationship that regions with lower mean junction angles (i.e., closer to 0°, for the smaller of the supplementary angles) will have fewer inlets than regions with higher mean junction angles (i.e., closer to 90°).

6. Conclusion

Our study describes basic rules for understanding how lake, river, and landscape factors influence lake-river connectivity. In particular, we show that numbers of river inlets primarily reflect lake area, while drainage density, lake shape, junction angle, and landscape fractal dimension have a secondary influence. Our inlet-area scaling relationship, when examined collectively with other scaling rules, provides further insights into hydrographic patterns. Specifically, scaling patterns for water residence time derive directly from inlet-area and volume-area relationships. Our analysis highlights several gaps in the understanding of lake morphometry and lake-river connectivity within broader hydrologic networks. In particular, relatively little is known about lake shape factors and lake-river junction angles, two factors that are particularly well-suited to bridge the gap between the detailed understanding of geologic processes and large-scale statistical relationships. Overall, our study both advances the basic understanding of the factors constraining lake-river connectivity and delineates an agenda for future research on this topic.

Data Availability Statement

The data and code used in this study are archived on Zenodo: doi.org/10.5281/zenodo.4612170

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