

1 **An integrated modelling approach for assessing the effect of multiscale**
2 **complexity on groundwater source yields**

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11 **ABSTRACT**

12 A new multi-scale groundwater modelling methodology is presented to simulate pumped water
13 levels in abstraction boreholes within regional groundwater models, providing a robust tool for
14 assessing the sustainable yield of supply boreholes and improving our understanding of
15 groundwater availability during drought. A 3D borehole-scale model, which solves the Darcy-
16 Forchheimer equation in cylindrical co-ordinates to simulate both linear and non-linear radial
17 flow to a borehole in a heterogeneous aquifer, is embedded within a Cartesian grid, using a
18 hybrid radial-Cartesian finite difference method. The local-scale model is coupled to a regional
19 groundwater model, ZOOMQ3D, using the OpenMI model linkage software, providing a flexible
20 and efficient tool for assessing the behaviour of a groundwater source within its regional
21 hydrogeological context during historic droughts and under climate change. The advantages of
22 the new method are demonstrated through application to a Chalk supply borehole in the UK.

23

24 **KEYWORDS**

25 Water resources, groundwater, sustainable yield, multiscale modelling, OpenMI

26

27 **SOFTWARE AVAILABILITY**

28 Name of software: SPIDERR; Developer: Dr Kirsty Upton (kirlto@bgs.ac.uk); Availability: On

29 request; Program language: MATLAB

30

31 **DECLARATION OF INTEREST**

32 We wish to confirm that there are no known conflicts of interest associated with this publication

33 and there has been no significant financial support for this work that could have influenced its

34 outcome.

35

36 **1. INTRODUCTION**

37 Groundwater plays a significant role in providing water for domestic, agricultural and industrial

38 use worldwide (WWAP, 2015), but around 20% of the world's aquifers are estimated to be over-

39 exploited (Gleeson et al., 2012). As pressures on global water resources increase due to

40 climate change and population growth, the sustainable management of groundwater resources

41 becomes more critical. Preventing over-exploitation and subsequent reductions in long-term

42 groundwater availability at the aquifer scale requires an understanding of the relationship of

43 demand for groundwater abstraction with the environmental role of groundwater in delivering

44 ecosystem services within the context of long-term average recharge. Over shorter time-scales,

45 effective water resource management also requires an understanding of the sustainable yield of

46 individual abstraction boreholes during drought. This is particularly important in aquifers where

47 there are seasonal groundwater level fluctuations and where groundwater abstractions impact

48 on other wells and the surrounding environment. Water resource managers therefore require

49 tools to allow evaluation of short- and long-term groundwater availability, at both the aquifer
50 scale and at the scale of individual boreholes, and how external stresses such as climate
51 change and increasing demand might impact the availability of groundwater at these different
52 spatial and temporal scales.

53

54 Numerical groundwater models provide invaluable tools to investigate the response of aquifers
55 to different environmental stresses (Singh, 2014). There are several examples in the literature of
56 numerical models used for simulating the groundwater level response in a borehole. The Multi-
57 Node Well package (Konikow et al., 2009) enables the representation of boreholes that are
58 open to multiple aquifers in MODFLOW (McDonald & Harbaugh, 1988). This scheme uses the
59 steady-state equation of flow towards a borehole (Thiem, 1906) to correct for the difference in
60 head between that simulated on a low resolution Cartesian grid and that in a borehole. The
61 main advantage of this approach is that it is a computationally efficient way of including
62 abstractions in regional groundwater models because it does not refine the model mesh around
63 the borehole. However, it is often found that heterogeneity and hydrogeological complexity close
64 to boreholes control their behaviour (Rushton & Rathod, 1988; Tamayo-Mas et al., 2018; Upton
65 et al., 2019). Consequently, most studies that have aimed to simulate fluctuations in within-
66 borehole water levels and their performance accurately have applied numerical 'radial flow'
67 models (Rushton & Booth, 1976; Connorton & Reed, 1978; Rushton & Weller, 1985; Rathod &
68 Rushton, 1991; Rushton, 2006; Konikow et al., 2009; Mathias & Todman, 2010; Mansour et al.,
69 2011; Upton et al., 2019). These models are typically based on a radial grid structure to
70 simulate flow converging to an abstraction borehole. They represent the small-scale features
71 and groundwater flow processes that are required to reproduce the groundwater level response
72 within an individual borehole (typically at spatial scales of less than 1m), including the effects of
73 borehole storage, well casing and screening, vertical aquifer heterogeneity, seepage face

74 development, and non-Darcian flow. Radial flow models do not generally provide an adequate
75 representation of regional groundwater flow processes. In contrast, there are numerous
76 examples in the literature of distributed groundwater models, which represent large-scale
77 features and processes such as regional recharge patterns and geological heterogeneity, that
78 have been applied to regional water resource assessment and management problems in
79 various geographical, climatic, and hydrogeological settings (Rushton et al., 1989; Salmon et
80 al., 1996; Ebraheem et al., 2004; Al-Salamah et al., 2011; Jackson et al., 2011; Shepley et al.,
81 2012; Sherif et al., 2012; Eissa et al., 2013; Mohamed et al., 2016). Regional models typically
82 assign abstraction boreholes to single grid cells, which may be on the scale of several tens,
83 hundreds, or even thousands of meters, leading to large discretization errors in the vicinity of
84 the borehole. There are few examples in the literature of multi-scale numerical groundwater
85 models that allow a detailed representation of individual abstraction boreholes within regional-
86 scale groundwater models (Abrams et al., 2016; Feinstein et al., 2016).

87
88 Numerical groundwater models typically use finite difference (FDM), finite volume (FVM), or
89 finite element methods (FEM) to solve the governing partial differential equation (Wang &
90 Anderson, 1982). These methods take different approaches to simulating flow across multiple
91 scales. In widely applied FDM groundwater modelling codes such as ZOOMQ3D (Jackson &
92 Spink, 2004) and earlier versions of MODFLOW (McDonald & Harbaugh, 1988), local grid
93 refinement (LGR) methods (von Rosenberg, 1982; Szekely, 1998; Jackson, 2000; Mehl & Hill,
94 2002; Mehl & Hill, 2004) allow a more accurate solution in regions with rapidly changing
95 hydraulic gradients, such as in the vicinity of abstraction boreholes, but are limited to square or
96 rectangular grids. Outside the field of groundwater modelling, a hybrid radial-Cartesian FDM has
97 been applied to simulate production wells in petroleum reservoir models (Akbar, 1974;
98 Mrosovsky, 1974; Pedrosa Jr & Aziz, 1986; Gottardi, 1990). In this method, one or more grid

99 cells of the regional model is replaced by a radial grid allowing grid refinement down to the
100 diameter of the well. The conductance term of the irregular shaped blocks at the boundary
101 between the radial and Cartesian grids is determined by substituting the irregular block with a
102 fictitious radial or rectangular block with the same volume and hydraulic properties. This
103 conserves mass but assumes that potential is uniform across the irregular volume, flow is
104 orthogonal across the outer boundary of the irregular block, and the principal directions of
105 permeability are aligned with the co-ordinate axes thus not dealing with full tensor anisotropy.

106

107 The FVM and FEM, which are based on unstructured grids, allow a smooth transition between
108 coarse and fine grid scales without complex boundary conditions at a grid interface, as is the
109 case in the LGR and hybrid radial-Cartesian methods. These methods provide greater
110 geometric flexibility but grids can be more time-consuming to construct and may result in
111 models that are computationally more demanding to solve. The FVM based on a two-point flux
112 approximation, often referred to as the control volume finite difference (CVFD) method, has
113 recently been applied in MODFLOW-USG (Panday et al., 2013) and MODFLOW 6 (Hughes et
114 al., 2017; Langevin et al., 2017). It has also been used in multi-scale petroleum reservoir
115 (Rozon, 1989; Heinemann, 1991; Palagi, 1993; Palagi, 1994; Palagi & Aziz, 1994) and
116 atmospheric models (Ringler et al., 2008; Ju et al., 2010; Ringler et al., 2013). The CVFD
117 method requires grids to be locally orthogonal and, like the FDM, does not account for full-
118 tensor anisotropy, although this is accommodated in MODFLOW 6 through the XT3D package
119 (Provost et al., 2017). The multi-point flux approximation FVM and FEM can incorporate full-
120 tensor anisotropy, and are applied on fully unstructured grids providing the highest level of
121 geometric flexibility. The FVM methods have been applied to multi-scale problems in petroleum
122 reservoir, atmospheric and oceanic modelling (Aavatsmark et al., 1996; Ding & Jeannin, 2001;

123 Aavatsmark, 2002; Chen et al., 2003; Mundal et al., 2010; Neale et al., 2010), while the FEM is
124 used in the FEFLOW groundwater modelling code (Diersch, 2002).

125

126 Hiebert *et al.* (1993) and Fung *et al.* (1994) compare the discretization schemes described
127 above (refined Cartesian, hybrid radial-Cartesian, refined CVFD, as well as a hybrid radial-
128 CVFD scheme) for simulating near-well processes in regional petroleum reservoir models. They
129 show that a radial scheme provides the greatest accuracy and efficiency for simulating near-well
130 processes, compared with the refined Cartesian or CVFD schemes and that, while the hybrid
131 radial-CVFD grid provides greater flexibility and accuracy across the boundary than the hybrid
132 radial-Cartesian grid, the results from these two schemes are comparable. Comparison of the
133 two-point and multi-point flux approximation FVMs show that the multi-point scheme is less
134 computationally efficient and only necessary for highly anisotropic simulations (Mundal et al.,
135 2010).

136

137 The review above does not contain an exhaustive summary of all potential methods for
138 modelling groundwater from the regional to the local (borehole) scale, but covers the most
139 commonly applied numerical approaches. Alternative approaches, such as the analytical
140 element, mesh-free, and lattice Boltzmann methods have gained increased attention in the
141 literature but have had limited uptake within the groundwater community (Li et al., 2002; Anwar
142 et al., 2008; Anwar & Sukop, 2009b; Anwar & Sukop, 2009a; Bandilla et al., 2009; Wen et al.,
143 2014).

144

145 When considering the specific example of representing individual boreholes within a regional
146 groundwater model, the task of constructing, solving, and calibrating a large-scale model
147 containing refined sub-regions around multiple abstraction boreholes would be both challenging

148 and time-consuming. An alternative approach is to link or couple two different models,
149 specifically developed for simulating groundwater flow processes at different scales: a regional
150 model for representing catchment-scale processes and a small-scale model for simulating flow
151 processes local to a borehole. Model linkage standards, such as OpenMI (Moore & Tindall,
152 2005; Gregersen et al., 2007) and the Community Surface Dynamics Modeling System
153 (Peckham et al., 2013), provide such methods for linking models at different scales in a
154 relatively straightforward way. OpenMI was developed in response to the EU Water Framework
155 Directive, which requires an integrated, catchment-scale approach to water resource
156 management. The OpenMI standard was therefore designed to enable different types of
157 catchment models, representing processes at different spatial and temporal scales, to run
158 simultaneously, exchanging data at run-time. It has been widely applied for linking different
159 types of environmental models that are designed, constructed and calibrated to simulate
160 processes at different scales (Betrie et al., 2011; Elag et al., 2011; Janssen et al., 2011;
161 Safiolea et al., 2011; Liao et al., 2012; Butts et al., 2013; Castronova & Goodall, 2013;
162 Castronova et al., 2013; Goodall et al., 2013; Shrestha et al., 2013; Zhou et al., 2013; Shrestha
163 et al., 2014; Zhu et al., 2016).

164

165 This paper presents a novel methodology for linking a radial flow model with a regional
166 groundwater model for simulating the groundwater level response in an abstraction borehole
167 within its regional context. The multi-scale model provides a tool for water resource managers
168 to investigate the behavior of individual abstraction boreholes within their regional context, and
169 to run predictive scenarios to evaluate the potential impact of climate change and increasing
170 demand on the sustainable yield of the borehole and its surrounding environment. The existing
171 finite difference radial flow model, SPIDERR (**S**imulating **P**umping Boreholes with a **D**arcy-
172 **F**orchheimer **R**egional-**R**adial Flow Model), described in detail by Upton et al. (2019), represents

173 the local features and flow processes required to reproduce the response in an abstraction
174 borehole, as described above. This radial flow model is linked to the finite difference ZOOMQ3D
175 regional groundwater modelling code through OpenMI. There are several advantages of this
176 multi-scale modelling method for applications to water resource management: it makes use of
177 existing regional groundwater models, which underpin groundwater resource assessment and
178 represent a major investment; models of individual boreholes can initially be developed as
179 standalone models without the need to run in a large regional simulation, making the calibration
180 process more efficient; individual borehole models can then be quickly and easily linked to an
181 existing regional groundwater model without significant time and effort required to re-grid and
182 re-calibrate large regional-scale models (this is particularly important where multiple boreholes
183 need to be assessed within a single regional model); the local- and regional-scale models can
184 be run on different time-steps and over different time periods, allowing a finer temporal
185 resolution in the local-scale model; and there is potential for this multi-scale groundwater model
186 to be linked with other environmental models as part of an integrated approach to catchment
187 management. This method still requires handling of the boundary between the radial and
188 Cartesian models, which is achieved using the hybrid radial-Cartesian finite difference method
189 described above.

190

191 The multi-scale modelling framework is described in Section 2.1, including the hybrid radial-
192 Cartesian coupling method and the linkage of the local- and regional-scale models through
193 OpenMI. Section 2.2 provides an evaluation of the methodology, which is then applied to a
194 groundwater source in the Chalk aquifer in southern England (Section 3) to demonstrate the
195 ability of the multi-scale model to accurately reproduce the response in an abstraction borehole
196 within a regional-scale groundwater model. In the discussion (Section 4) we outline the key

197 benefits and limitations of the methodology, as well as potential opportunities for future
198 development and application.

199

200 **2. METHODS**

201 **2.1 Model Development**

202 The multi-scale modelling framework links a Darcy-Forchheimer radial groundwater flow model,
203 SPIDERR (Upton et al., 2019), with the ZOOMQ3D regional groundwater modelling code
204 (Jackson & Spink, 2004) in two key stages, which are described in detail below: (1) within
205 SPIDERR, the grid of the radial flow model is coupled to a Cartesian grid using the hybrid radial-
206 Cartesian finite difference method; (2) the SPIDERR flow model is then linked to ZOOMQ3D
207 using OpenMI.

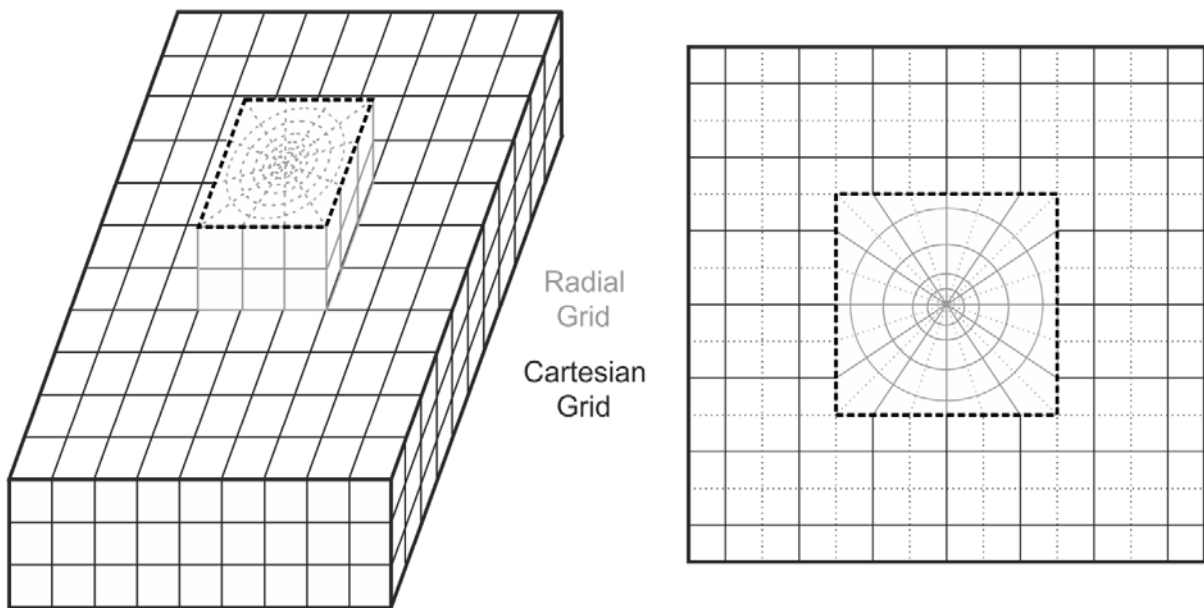
208

209 2.1.1 Radial-Cartesian Coupling

210 The radial groundwater flow model, SPIDERR (Upton et al., 2019), is a 3D finite difference
211 model based on the Darcy-Forchheimer equation for simulating linear and non-linear flow to
212 boreholes in complex, heterogeneous aquifers. It has been shown to reproduce the
213 groundwater level response in abstraction boreholes where drawdown is influenced by
214 variations in borehole storage with depth, vertical heterogeneity and non-linear flow. SPIDERR's
215 radial grid is coupled to a Cartesian grid using the hybrid radial-Cartesian finite difference
216 method (Pedrosa Jr & Aziz, 1986). As shown in Figure 1, the radial grid effectively replaces one
217 or more cells of the Cartesian mesh, where the location of the borehole, which is at the center of
218 the square radial mesh, corresponds to a single node on the Cartesian mesh. This requires that
219 the radial grid replaces a square number of Cartesian grid cells. On the radial grid, the spacing
220 of nodes increases logarithmically in the radial direction providing greater resolution close to the
221 borehole, where the water table has greatest curvature and is most dynamic. Each node on the

222 boundary of the radial grid is linked to a single node on the Cartesian grid. The radial grid must
 223 therefore comprise at least four slices in the cylindrical direction. The radial and Cartesian grids
 224 are equivalent in the vertical direction, i.e. they have the same number of layers, and these are
 225 laterally continuous across the model domain. The current model requires the Cartesian grid
 226 cells to be square and the angle of each radial slice to be equal, however it would be simple to
 227 update the model code to accommodate rectangular grids.

228



229

230 *Figure 1 Conceptualization of the hybrid radial-Cartesian method used in the SPIDERR*

231

232 The continuity equation for transient groundwater flow through a porous medium can be
 233 expressed in cylindrical (r, θ, z) and Cartesian (x, y, z) coordinates as:

234
$$\frac{1}{r} q_r + \frac{\partial q_r}{\partial r} + \frac{1}{r} \frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z} = S_s \frac{\partial h}{\partial t} + N \quad (1)$$

235
$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = S_s \frac{\partial h}{\partial t} + N \quad (2)$$

236

237 where q is specific discharge [LT^{-1}], S_s is specific storage [L^{-1}], t is time [T], h is hydraulic head
 238 [L], and N is a volumetric source or sink per unit volume [T^{-1}]. In SPIDERR, specific discharge
 239 (q) is calculated by the Darcy-Forchheimer equation to simulate linear and non-linear
 240 groundwater flow along each direction (l):

$$241 \quad q + \beta q^2 = -K \frac{\partial h}{\partial l} \quad (3)$$

242
 243 where β is the non-linear Forchheimer parameter [L^{-1}T] and K is hydraulic conductivity [LT^{-1}].

244 Equation 3 reduces to Darcy's Law when $\beta = 0$.

245
 246 SPIDERR is based on a finite difference approximation to the continuity equation, whereby the
 247 vertical dimension is replaced by a series of layers, hydraulic head is integrated over the
 248 saturated thickness of each layer (b) [L], and vertical flow is calculated as a leakage term [LT^{-1}].
 249 Specific storage is replaced by a storage coefficient (S) [-], where $S = S_s b$, except for the layer in
 250 which the water table is situated in an unconfined aquifer, where $S = S_s b + S_y$ and S_y is the specific
 251 yield [-] of the aquifer layer. Equations 1 and 2 therefore reduce to the following ordinary
 252 differential equations with respect to time, where i, j , and k are the node locations along the
 253 cylindrical (r, θ, z) and Cartesian (x, y, z) directions in Equations 4 and 5, respectively:

$$254$$

$$255 \quad S_{i,j,k} \frac{\partial h_{i,j,k}}{\partial t} = \frac{r_{i+\frac{1}{2},j,k} b_{i+\frac{1}{2},j,k} q_{i+\frac{1}{2},j,k} - r_{i-\frac{1}{2},j,k} b_{i-\frac{1}{2},j,k} q_{i-\frac{1}{2},j,k}}{r_{i,j,k} \left(r_{i+\frac{1}{2},j,k} - r_{i-\frac{1}{2},j,k} \right)} +$$

$$256 \quad \frac{b_{i,j+\frac{1}{2},k} q_{i,j+\frac{1}{2},k} - b_{i,j-\frac{1}{2},k} q_{i,j-\frac{1}{2},k}}{r_{i,j,k} \Delta \theta} + q_{i,j,k+\frac{1}{2}} + q_{i,j,k-\frac{1}{2}} - N_{i,j,k} \quad (4)$$

257

$$\begin{aligned}
258 \quad S_{i,j,k} \frac{\partial h_{i,j,k}}{\partial t} &= \frac{b_{i+\frac{1}{2},j,k} q_{i+\frac{1}{2},j,k} - b_{i-\frac{1}{2},j,k} q_{i-\frac{1}{2},j,k}}{\left(x_{i+\frac{1}{2},j,k} - x_{i-\frac{1}{2},j,k}\right)} + \\
259 \quad &\frac{b_{i,j+\frac{1}{2},k} q_{i,j+\frac{1}{2},k} - b_{i,j-\frac{1}{2},k} q_{i,j-\frac{1}{2},k}}{\left(y_{i,j+\frac{1}{2},k} - y_{i,j-\frac{1}{2},k}\right)} + q_{i,j,k+\frac{1}{2}} + q_{i,j,k-\frac{1}{2}} - N_{i,j,k} \quad (5)
\end{aligned}$$

260

261 Mathias *et al.* (2008) showed how Equation 3 is solved for specific discharge, which is used to
262 calculate q in the horizontal directions in Equations 4 and 5; vertical flow is assumed to be linear
263 and is calculated as shown by Upton *et al.* (2019).

264

265 Equations 4 and 5 are used to approximate groundwater head at each grid node on the radial
266 and Cartesian meshes, respectively. The grid nodes in the irregularly shaped cells on the
267 boundary between the two meshes (shaded grey in Figure 2), which have a curvilinear surface
268 on the radial grid boundary and a rectilinear surface on the Cartesian grid boundary, are
269 handled using the hybrid radial-Cartesian method outlined by Pedrosa & Aziz (1986). The
270 hydrogeological properties and volume of the boundary cells remain constant and flows to/from
271 the boundary nodes (i.e. nodes A, C, D and F in Figure 2) are calculated by substituting the
272 irregular cell with a fictitious radial or Cartesian cell of equivalent volume (Figure 2). Flow
273 between the boundary nodes and adjacent radial nodes (A and B in Figure 2a) is assumed to be
274 radial and is calculated by substituting the irregular cell with a fictitious radial cell, whereby the
275 new outer radius ($r_{i+1/2}^*$) is calculated from the area (a) of the irregular cell by:

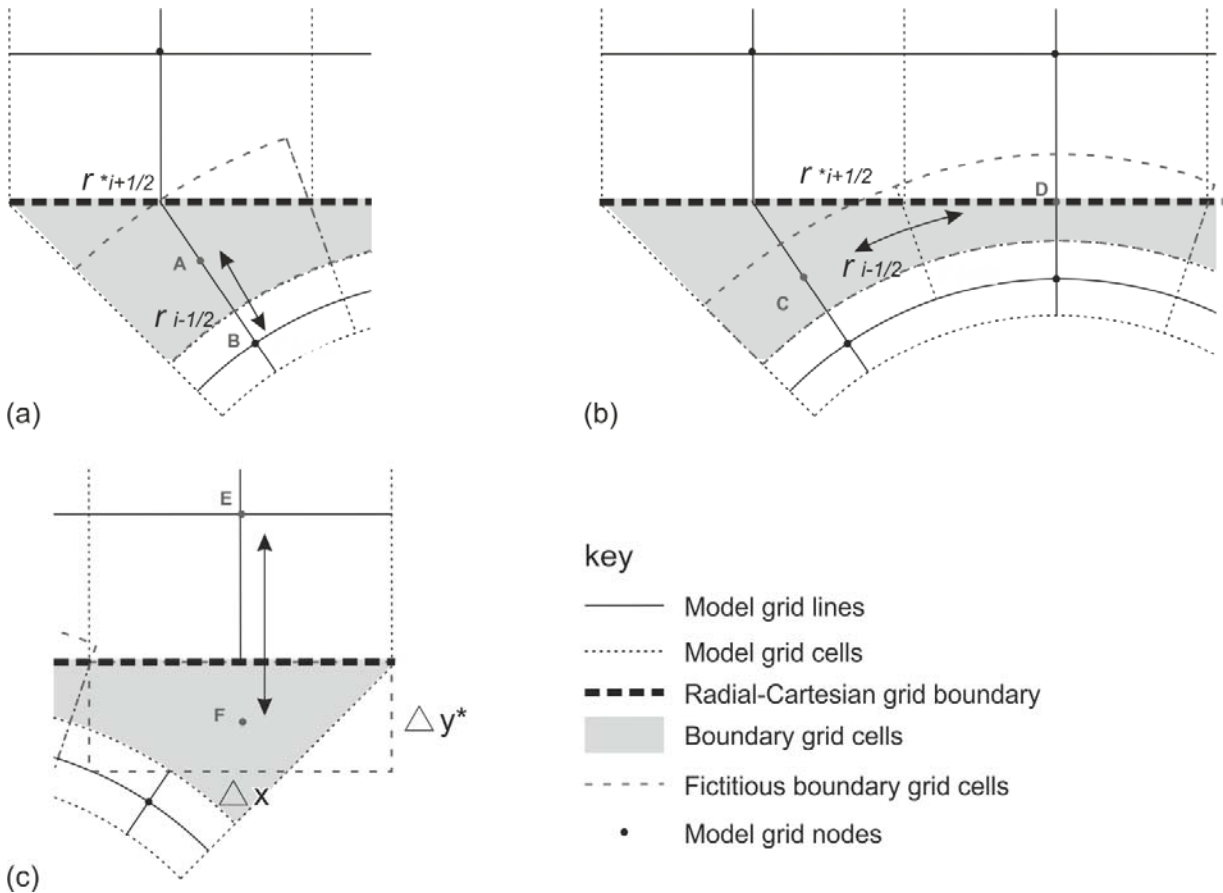
$$276 \quad r_{i+\frac{1}{2}}^* = \sqrt{\frac{2a}{\Delta\theta} + r_{i-\frac{1}{2}}^2} \quad (6)$$

277

278 The apparent radius (r^*) of node A is calculated as the logarithmic mid-point between $r_{i+1/2}^*$ and
 279 $r_{i-1/2}$. $r_{i-1/2}$ and $r_{i+1/2}^*$ are then substituted into Equation 4 for $r_{i,j,k}$ and $r_{i+1/2,j,k}$, respectively. Flow
 280 between adjacent boundary nodes (C and D in Figure 2b) is also assumed to be radial and is
 281 calculated by substituting the irregular cells with fictitious radial cells, whereby the shifted outer
 282 radius ($r_{i+1/2}^*$) is also calculated by Equation 6 but a and $\Delta\theta$ apply to the total area and angle of
 283 the two cells. Flow between the boundary node and adjacent Cartesian node (E and F in Figure
 284 2c) is assumed to be orthogonal and is calculated by substituting the irregular cell with a
 285 fictitious Cartesian cell, whereby the new length (Δy^*) of the Cartesian cell, and thus position of
 286 the grid node (F), is calculated from the area (a) of the irregular cell by:

287
$$\Delta y^* = \frac{a}{\Delta x} \quad (7)$$

288



289

290 *Figure 2 Conceptualisation of the radial-Cartesian boundary in SPIDERR: (a) radial flow between the*
291 *boundary cell and adjacent radial cell; (b) circumferential flow between boundary cells; (c) Cartesian flow*
292 *between boundary cell and adjacent Cartesian cell.*

293

294 This hybrid finite difference method conserves flow across the boundary between the radial and
295 Cartesian grids but assumes that groundwater head is uniform within the boundary cell and that
296 flow is orthogonal across the outer boundary of the irregular cell. The errors associated with
297 these assumptions are explored in Section 2.2. The method also allows for horizontal and
298 vertical heterogeneity, but does not deal with full tensor anisotropy.

299

300 The SPIDERR flow model is coded using the MATLAB software package (Mathworks, 2012).
301 The set of non-linear ordinary differential equations is solved using the stiff differential equation
302 solver ode15s, which uses adaptive time-stepping (Shampine & Reichlet, 1997).

303

304 2.1.2 Model Linking in OpenMI

305 The second stage in developing the multi-scale modelling framework involves linking the
306 SPIDERR flow model with the ZOOMQ3D regional groundwater modelling code using OpenMI.
307 In this linked composition, a radial flow model can quickly and easily be placed within a regional
308 groundwater model, with the regional model providing a boundary condition for the smaller-
309 scale model, as described below.

310

311 ZOOMQ3D is based on a finite difference approximation to the continuity equation, whereby
312 specific discharge is calculated by Darcy's law (see equation 3). It incorporates local grid
313 refinement, allowing problems to be solved across different scales on a Cartesian grid. As is the
314 case for SPIDERR, ZOOMQ3D is able to represent multiple aquifer layers with varying hydraulic

315 parameters and confined and unconfined conditions. ZOOMQ3D also incorporates spatially and
316 temporally varying recharge and river-aquifer interaction, and has been used to model several
317 regional-scale aquifers in the UK for groundwater resource assessment purposes (Jackson et
318 al., 2005; Jackson et al., 2011; Jones et al., 2012; Le Vine et al., 2016).

319

320 The OpenMI standard (Moore & Tindall, 2005; Gregersen et al., 2007) is a software component
321 interface definition that can be used to link multiple different software components, including
322 numerical models, databases, and analytical or visualization tools. When implemented, OpenMI
323 compliant components can be configured to run simultaneously, exchanging data during
324 computation. ZOOMQ3D was already OpenMI compliant, but the SPIDERR flow model
325 (MATLAB) code had to be reconfigured into the structure required by OpenMI. This requires the
326 solution algorithm, or model “engine”, to be separated from code that initializes and terminates
327 the simulation. This results in three principal program functions: (1) *Initialize*, which defines the
328 set-up of the model; (2) *PerformTimeStep*, which solves the governing flow equation for a single
329 time-step, and; (3) *Finish*, which writes output files and terminates the program. The
330 restructured SPIDERR and ZOOMQ3D codes were each compiled as a dynamic-link library
331 (dll); in the case of SPIDERR, the MATLAB compiler was used to compile the model as a C dll,
332 including the MATLAB Compiler Runtime (MCR), allowing it to be deployed without a MATLAB
333 licence. The OpenMI Software Development Kit (SDK) is used to write wrappers for the models,
334 allowing the dlls to be accessed as OpenMI-compliant linkable components. OpenMI version 1.4
335 was used for this application and the OpenMI Graphical User Interface was used to link and run
336 components. OpenMI provides functionality for linking multiple models, potentially allowing more
337 than one axisymmetric model grid to be represented in a single regional model. Although this
338 has not yet been implemented with SPIDERR and ZOOMQ3D, it would allow interaction
339 between multiple boreholes to be simulated within a single regional-scale model.

340

341 The models are linked spatially by providing the location of the outer boundary of SPIDERR to
342 ZOOMQ3D through the OpenMI interface. The Cartesian grids of the two models must be
343 equivalent such that a single node on the SPIDERR model boundary links directly to a single
344 node in ZOOMQ3D and, as mentioned above, the SPIDERR model currently only
345 accommodates a square Cartesian mesh. This is achievable due to the local grid refinement
346 capabilities of ZOOMQ3D. Due to the gridding requirements of SPIDERR, this also means that
347 the location of the abstraction borehole in SPIDERR directly corresponds to a grid node in
348 ZOOMQ3D, which is specified through the OpenMI interface. The layer elevations of the two
349 models must be equivalent (SPIDERR can read these from the ZOOMQ3D input files), however
350 layers can be vertically refined in SPIDERR to allow greater vertical resolution in the vicinity of
351 the borehole. Consequently, a single ZOOMQ3D layer can be associated with multiple layers in
352 SPIDERR. The linking of the models in this way relies on the assumption that flow across the
353 boundary between ZOOMQ3D and SPIDERR is horizontal and Darcian.

354

355 The dynamic linking of the models is based on a one-way flow coupling, whereby ZOOMQ3D
356 provides volumetric flows across the outer boundary of the SPIDERR Cartesian grid and
357 provides recharge fluxes across the upper layer of SPIDERR. Under this one-way flow coupling,
358 SPIDERR does not pass a boundary condition back to ZOOMQ3D; instead the regional model
359 continues to perform calculations for the area occupied by the local model using equivalent
360 parameters and stresses as SPIDERR to maintain consistency at the boundary between the two
361 models. SPIDERR therefore passes the following variables to ZOOMQ3D at run-time through
362 OpenMI: (1) the pumping rate at the borehole, which can be updated during a simulation based
363 on the modelled groundwater head in the borehole by SPIDERR's abstraction management
364 module; (2) transmissivity and storage, which vary during a simulation based on fluctuation of

365 the water table. Table 1 provides more detail of the data exchange process between the two

366 models.

367

Exchanged Variables: ZOOMQ3D to SPIDERR	
Volumetric boundary flows	Passed directly from a ZOOMQ3D layer to the corresponding SPIDERR layer if no vertical refinement. If a ZOOMQ3D layer is refined in SPIDERR, boundary flows are partitioned and weighted according to the transmissivity of each layer at each time-step.
Lumped recharge fluxes	Including: recharge from rainfall, leakage to or from rivers or springs, additional point abstractions or discharges. Applied to the upper active layer in SPIDERR. Passed directly from ZOOMQ3D to the Cartesian grid of SPIDERR; partitioned across corresponding radial nodes in SPIDERR.
Exchanged Variables: SPIDERR to ZOOMQ3D	
Pumping rate at abstraction borehole	Passed directly from the abstraction borehole in SPIDERR to the corresponding abstraction node in ZOOMQ3D.
Model parameters	Including: horizontal and vertical hydraulic conductivity, specific storage and specific yield. With no vertical refinement in SPIDERR: <ul style="list-style-type: none"> • On the Cartesian grid of SPIDERR, parameter values are passed directly from SPIDERR to ZOOMQ3D • On the radial grid of SPIDERR, where multiple radial nodes are associated with a single ZOOMQ3D node, parameter values are averaged prior to being passed to ZOOMQ3D. With vertical refinement in SPIDERR:

	<ul style="list-style-type: none"> • Transmissivity (T) and storativity (S) of a single ZOOMQ3D layer is equal to the total T and S of all corresponding SPIDERR layers; • $K_{x,y}$ and S_s are calculated based on the total saturated thickness; • S_y (for unconfined layers) in ZOOMQ3D is taken from the uppermost corresponding layer in SPIDERR; • K_z in ZOOMQ3D is calculated as the weighted mean of K_z over all corresponding SPIDERR layers.
--	---

368 *Table 1 Exchange items between SPIDERR and ZOOMQ3D when run as linked models through OpenMI*

369

370 The exchange of data through the OpenMI interface at run-time is achieved using a pull-driven
371 approach, which is based on a single linkable component being assigned as the driver of the
372 multi-component composition. A function in OpenMI calls the driver component to update (i.e.
373 proceed to the next time-step) at which point it requests all necessary data from all other linked
374 components. This causes OpenMI to update the other linked components, which perform a
375 time-step and provide data back to the driver so it can also proceed in time. If there is a two-way
376 linkage between components, the driver provides an estimate of the required data based on the
377 previous time-step so the second component can update and return the required computed
378 values to the driver. Each linkable component provides a time-stamp at each time-step so the
379 interface knows when all components have reached the same point in time. In this linked
380 composition, in which SPIDERR is the driver, it is therefore possible for the two models to run at
381 different temporal resolutions and over different time periods. SPIDERR, which uses adaptive
382 time stepping, can run on a smaller time-step than ZOOMQ3D and data is transferred between
383 the two models through OpenMI at the smallest common time-step. ZOOMQ3D will wait for
384 SPIDERR to complete all sub time-steps, before both models proceed.

385

386 **2.2 Model Evaluation**

387 2.2.1 Radial-Cartesian Coupling

388 A vast number of grid configurations and model parameterisations could be run to evaluate the
389 hybrid radial-Cartesian coupling method used in SPIDERR. In this case, the model was
390 evaluated against the Papadopoulos-Cooper solution for simulating drawdown in a large
391 diameter well in a confined aquifer (Papadopoulos & Cooper, 1967). Several grid configurations
392 were set up to analyse model error related to the radial-Cartesian boundary in SPIDERR for
393 different levels of radial and Cartesian grid refinement (Table 2 and Figure 3). The results
394 presented here are in no way exhaustive, and many more grid configurations could be tested.
395 The results presented below help to illustrate the value and limitations of the methodology but
396 further testing of different grid configurations under different aquifer conditions (e.g. unconfined,
397 vertically heterogeneous, or non-Darcian flow) would be valuable.

398

399 SPIDERR was run as a single layer, homogeneous, confined model with a 0.1m diameter
400 borehole located at the center of a 20 × 20 km Cartesian grid. The model was run over a period
401 of 50 days with an abstraction rate of 1000 m³day⁻¹, a transmissivity of 500 m²day⁻¹ and a
402 storativity of 0.005. Three different Cartesian grids were used, with square cells 250, 500, and
403 1500 m wide. In each of these, three different radial grids were also applied containing 4, 12 or
404 20 radial slices, which have different radial extents as shown in Figure 3. Simulated drawdown
405 at the abstraction borehole and at the node in the central-slice on the radial-Cartesian boundary
406 was compared with the analytical solution (Figure 4). The percentage error, calculated at the
407 end of the simulation from drawdown at the abstraction borehole and drawdown at the central-
408 slice node on the radial-Cartesian boundary, is shown in Table 2.

409

410 The results show that a four-slice radial model produces relatively high errors at the boundary,
411 which are also visible at the abstraction borehole. The timing of the deviation from the analytical
412 solution at the borehole varies with the extent of the radial grid, with the effects of the boundary
413 seen earliest in the four radial slice / 250 m Cartesian set-up as the boundary is closest to the
414 borehole in this configuration. Refinement on the Cartesian grid generally improves the error at
415 the boundary and has a small impact on the error at the borehole. Errors in simulated drawdown
416 are significantly reduced by increasing the resolution on the radial grid from four to 12 radial
417 slices (see Table 2). For the 12-slice model, errors are slightly higher at the corner boundary
418 nodes because the cell area is larger, therefore the assumption of uniform head across the cell
419 is less valid. Table 2 suggests that increasing the radial refinement from 12 to 20 slices appears
420 to increase the error at the boundary; however, the 20-slice model has a larger radial extent
421 than the 12-slice model with the same number of radial nodes, therefore a coarser resolution at
422 the boundary so the configurations are not directly comparable. For each of the validation runs
423 shown in Table 2 and Figure 4 the model was set-up with 40 nodes along the radial dimension.
424 For the 20-slice radial model with a 1500 m Cartesian grid resolution, the error at the boundary
425 and abstraction borehole can be reduced further to 2.29% and 0.05%, respectively, by
426 increasing the number of radial nodes from 40 to 80.

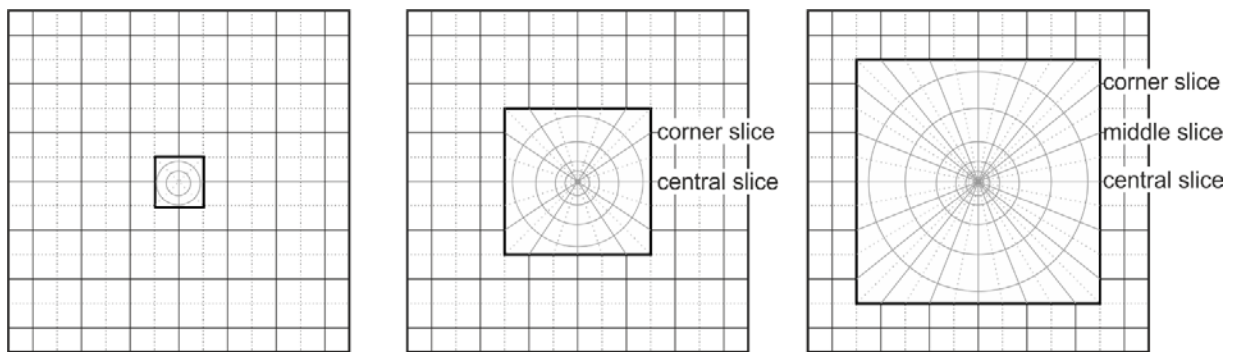
427

428 The boundary errors are dependent on an interplay between refinement on the Cartesian and
429 radial grids however, the validation results indicate that errors related to the coupling
430 methodology are most sensitive to the number of radial slices in the SPIDERR model with an
431 increase from four to 12 slices providing the greatest reduction in error. As mentioned above,
432 further testing of different grid configurations for specific model and aquifer setups would be
433 recommended.

Test	Cartesian Grid Resolution (m)	Number Radial Slices	Radial Boundary Extent (m)	Error at the abstraction borehole (%)	Boundary Error for Central Slice (%)
1	250	4	125	2.23	6.27
2	250	12	375	0.07	0.25
3	250	20	625	0.27	1.33
4	500	4	250	2.23	8.03
5	500	12	750	0.03	0.06
6	500	20	1250	0.18	1.96
7	1500	4	750	2.24	14.1
8	1500	12	2250	0.18	2.14
9	1500	20	3750	0.16	6.95

434 Table 2 Grid configurations to evaluate SPIDERR model error

435



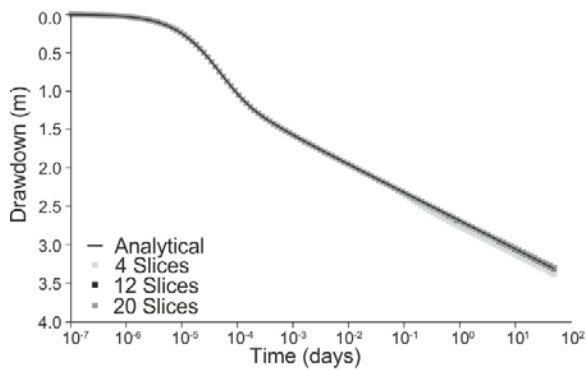
436 (a)

(b)

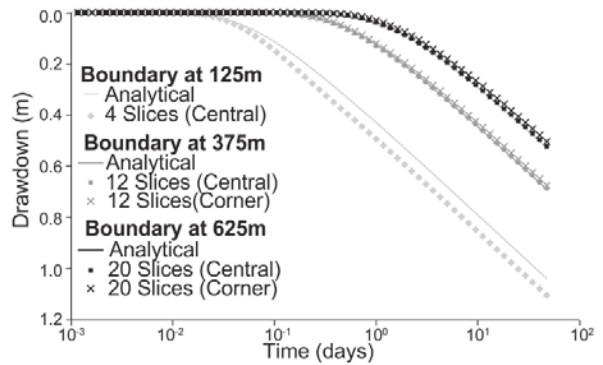
(c)

437 Figure 3 Grid configurations for SPIDERR validation: (a) 4 slice radial model; (b) 12-slice radial model; (c)

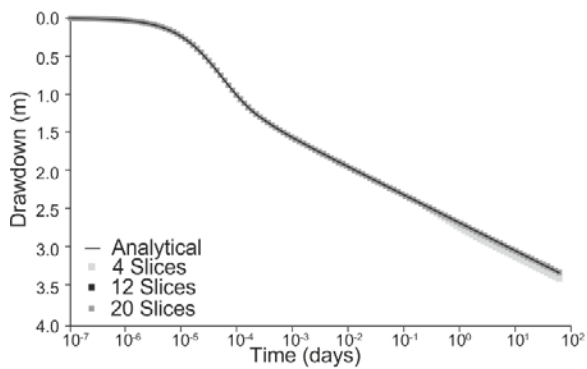
438 20-slice radial model.



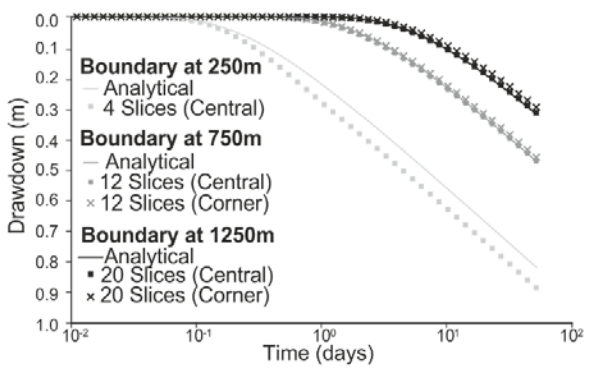
a) Cartesian resolution: 250m
Number radial nodes: 40
Number radial slices: 4, 12, 20



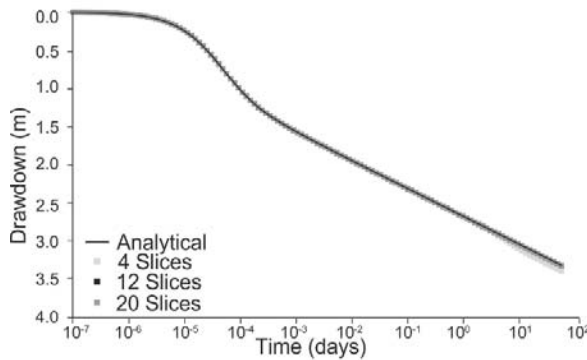
d) Cartesian resolution: 250m
Number radial nodes: 40
Number radial slices: 4, 12, 20



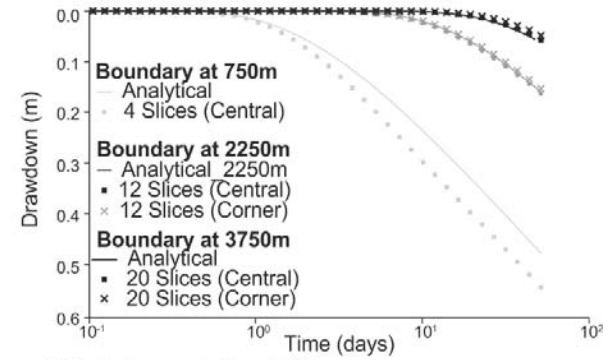
b) Cartesian resolution: 500m
Number radial nodes: 40
Number radial slices: 4, 12, 20



e) Cartesian resolution: 500m
Number radial nodes: 40
Number radial slices: 4, 12, 20



c) Cartesian resolution: 1500m
Number radial nodes: 40
Number radial slices: 4, 12, 20



f) Cartesian resolution: 1500m
Number radial nodes: 40
Number radial slices: 4, 12, 20

Figure 4 Simulated drawdown compared with the Papadopoulos-Cooper solution at the abstraction borehole (a-c) and on the radial-Cartesian boundary (d-f) for different levels of radial and Cartesian grid refinement.

2.2.2 Model Linking in OpenMI

The method for linking SPIDERR and ZOOMQ3D through OpenMI was evaluated by comparing a standalone SPIDERR model, standalone ZOOMQ3D model, and coupled composition for an aquifer of increasing complexity. The standalone SPIDERR model was set up as for Test 2 above (i.e. a 12 slice radial grid with a radial extent of 375m, coupled to a 20 × 20 km Cartesian grid at a resolution of 250 m). The standalone ZOOMQ3D model had an equivalent Cartesian grid with an abstraction node located at its center. The standalone models were initially compared for the series of aquifers outlined in Table 3. For each simulation the models were run over a period of 50 days on a daily time-step with a constant abstraction rate of 1000 m³/day. In simulations four and five, vertical heterogeneity and anisotropy were introduced in the multi-layer SPIDERR models by varying horizontal hydraulic conductivity (Kh) with depth and reducing vertical hydraulic conductivity (Kv) relative to Kh (Table 3). ZOOMQ3D was kept as a single-layer homogeneous model. Specific storage and specific yield (for the unconfined simulations) were kept constant at $2.5 \times 10^{-4} \text{ m}^{-1}$, and 0.1, respectively, in both models.

Simulation	Aquifer Type	SPIDERR				ZOOMQ3D		
		Nbr Layers	Layer Thickness (m)	Kh (m/d)	Kv (m/d)	Nbr Layers	Layer Thickness (m)	Kh (m/d)
1	Confined	1	20	25	-	1	20	25
2	Unconfined	1	20	25	-	1	20	25
3	Unconfined	3	4	25	25	1	20	25
			6	25	25			
			10	25	25			
4	Unconfined	3	4	100	25	1	20	25

			6	12.5	25			
			10	2.5	25			
5	Unconfined	3	4	100	0.25	1	20	25
			6	12.5	0.25			
			10	2.5	0.25			

Table 3 Model setup for evaluation of the SPIDERR-ZOOMQ3D coupling methodology

Comparison of the standalone SPIDERR and ZOOMQ3D models shows that the inconsistencies between the two models, related to the more detailed discretization of the borehole and aquifer in SPIDERR, are relatively local to the borehole. As would be expected, ZOOMQ3D is unable to reproduce drawdown at the borehole itself, which is represented in the regional model as a single Cartesian node with dimensions of 250 × 250 m (Figure 5a). Discretization errors can also be seen at a distance of 500 m from the borehole in ZOOMQ3D, and these errors are more significant in the unconfined simulations (Figure 5b). However, at a distance of 1000 m from the borehole the two standalone models are largely consistent, even where significant vertical heterogeneity and anisotropy are introduced in the unconfined multi-layer SPIDERR model. Increasing anisotropy causes larger vertical hydraulic gradients to develop in the aquifer, but again these are local to the borehole (Figure 6). For these simulations vertical hydraulic gradients, and thus vertical flow, are reduced to zero at the boundary of the radial grid within the SPIDERR model, satisfying the assumption of horizontal flow for linking with ZOOMQ3D.

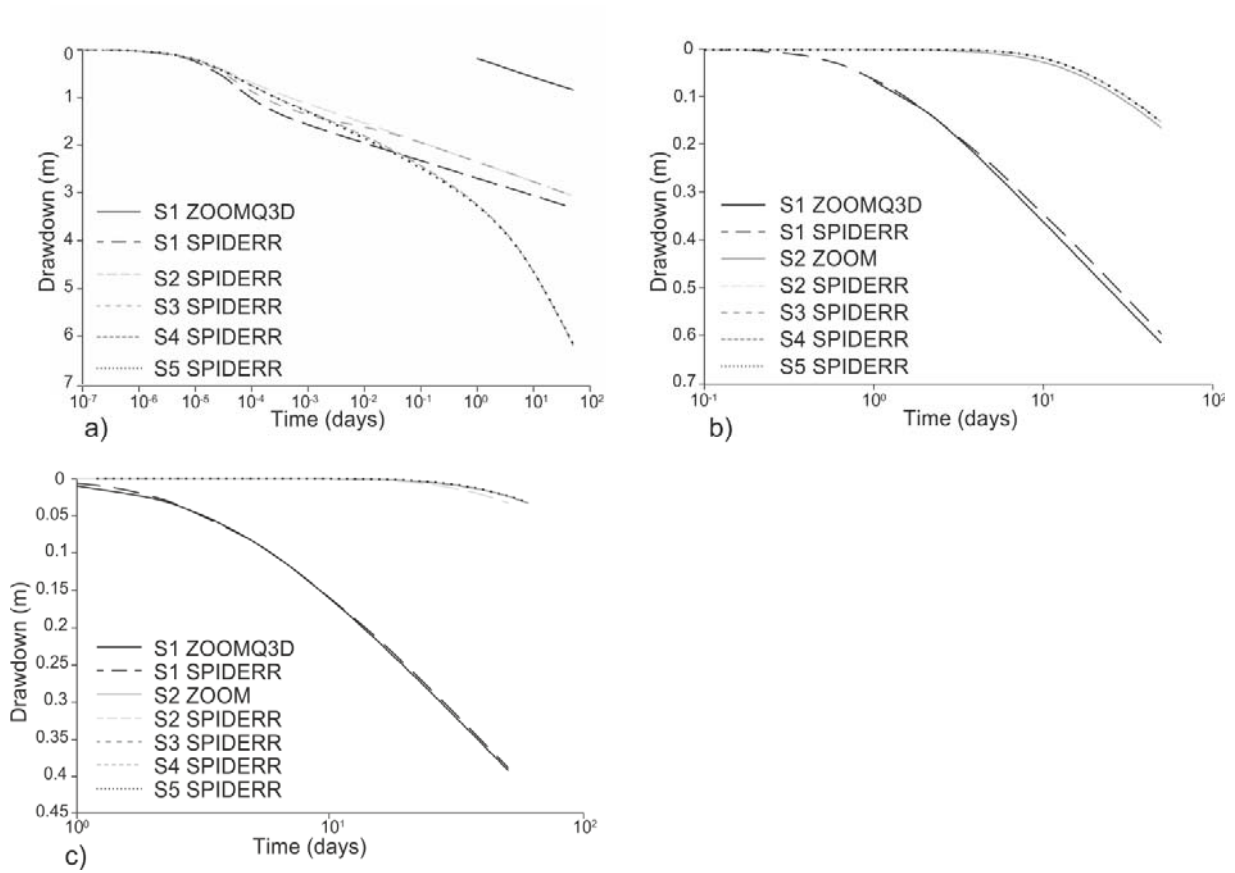


Figure 5 Simulated drawdown in standalone SPIDERR and ZOOMQ3D models for simulations 1-5, as outlined in Table 3, at: (a) the abstraction borehole; (b) 500m from the borehole; (c) 1000m from the borehole

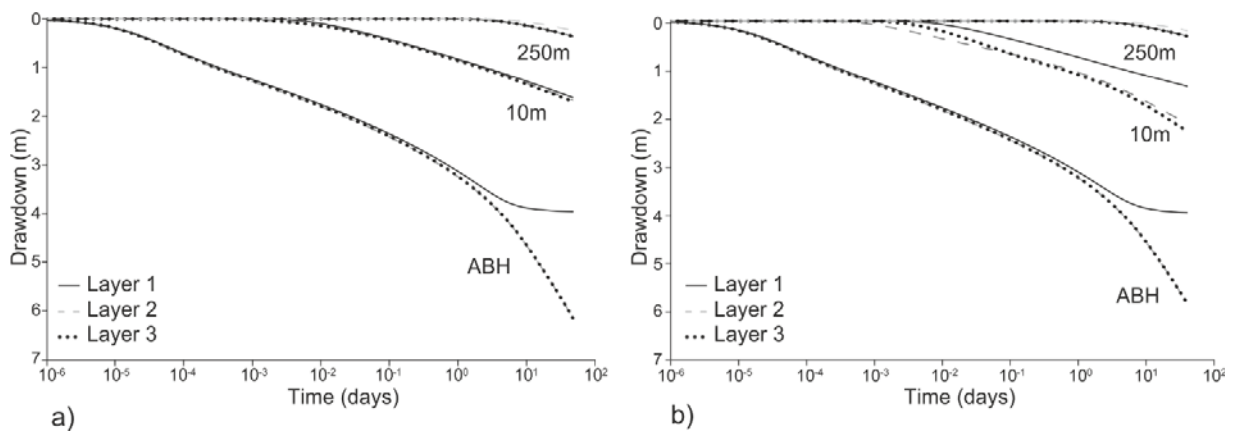


Figure 6 Simulated drawdown in each layer (Layer 1, 2 and 3) of the standalone SPIDERR model at the borehole (ABH) and at 10 m and 250 m from the ABH for (a) simulation 4; (b) simulation 5.

A linked composition was set up for simulation 5. This used a SPIDERR model covering a 1500×1500 m area with a 250 m resolution Cartesian grid and 12-slice radial grid. The results of the standalone simulations indicated that at this SPIDERR model boundary (i.e. a radial distance of 750 m from the borehole) the two models were consistent and flow in the multi-layer SPIDERR model was horizontal. The SPIDERR model was therefore linked to the 20×20 km ZOOMQ3D grid through OpenMI and the models were run over the same 50-day time period, exchanging data on a daily time-step. The results of this simulation are compared with the equivalent standalone SPIDERR model at various points on the radial and Cartesian grids, showing good agreement (Figure 7). In this coupled simulation, the abstracted volume is initially derived from well storage, before water is released from storage on the radial grid. The cone of depression reaches the boundary of the radial SPIDERR grid after approximately one day, at which point the Cartesian grid starts to provide water to the borehole. After six days, flow across the SPIDERR model boundary increases, with ZOOMQ3D contributing just under one third of the total abstraction rate by the end of the simulation.

As discussed in Section 2.2.1, there are a vast number of model set-ups that could be run to evaluate the methodology. The results presented here show that the linked composition reproduces the standalone SPIDERR model results even when a significant amount of flow is passed across the ZOOMQ3D-SPIDERR boundary and the SPIDERR model is vertically refined compared with the regional model. This indicates that the methodology is valid for representing the refined area around a borehole within a regional groundwater model. However, further evaluation could be undertaken to test the methodology under different aquifer conditions.

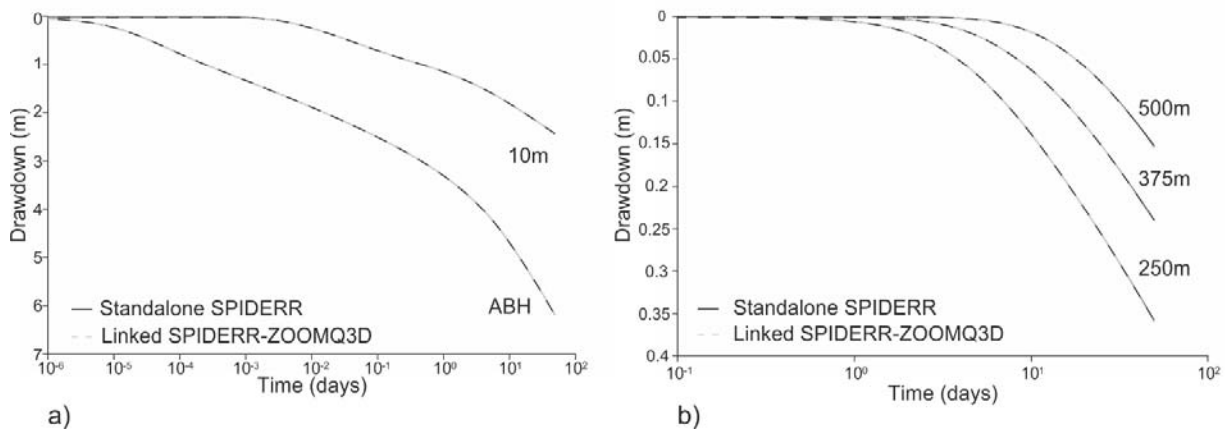


Figure 7 Comparison of standalone SPIDERR and linked SPIDERR-ZOOMQ3D simulations for simulation 5 (i.e. refined, heterogeneous, and anisotropic SPIDERR model) at: (a) the abstraction borehole and 10 m distance from the abstraction borehole; (b) 250m and 375m from the borehole on the radial grid, and 500m from the abstraction borehole on the Cartesian grid.

3. MODEL APPLICATION

To further evaluate the methodology, the linked modelling framework was applied to a public water supply borehole (Borehole A) located in the River Thames catchment in southern England. This groundwater source was chosen for evaluation purposes as it is a relatively simple single-borehole source, for which pumping test and operational data are available, and it is located within an existing regional groundwater model, described below, that has been widely used for groundwater resource assessment.

3.1 Study Area

The exact location of Borehole A, which is operated by Thames Water Utilities Ltd., cannot be given for confidentiality reasons but is situated in the Chalk aquifer – a principal aquifer in the UK, accounting for up to 70% of the total public water supply in parts of southern England (Allen et al., 1997; Butler et al., 2012). The Chalk is a very fine-grained limestone with a low permeability matrix and highly permeable fracture network, which is often enhanced by

dissolution and provides the main mechanism for groundwater flow in the aquifer (Allen et al., 1997; MacDonald, 1998). Vertical and lateral variations in transmissivity and storativity are controlled by fracture development in the Chalk, which is greater at shallow depths and in valleys and dry valleys. Transmissivity estimates from pumping test data range from $50 \text{ m}^2\text{day}^{-1}$ in interfluvial areas to $500\text{-}2000 \text{ m}^2\text{day}^{-1}$ in valleys and dry valleys. These variations have a strong influence on the yield of abstraction boreholes, which often decrease non-linearly as groundwater levels fall (Williams et al., 2006; Butler et al., 2009). Borehole A is drilled to a depth of 55 m into the unconfined Chalk aquifer, penetrating the Lewes Nodular and underlying New Pit Chalk Formations (Mortimore, 1986; Robinson, 1986). The Lewes Nodular Chalk has a higher flint content than the underlying New Pit Chalk and an important hard band, the Chalk Rock, marks the boundary between the formations. Geophysical logging of the borehole indicates that more than 75% of the total inflow occurs within the upper 32 m of the borehole (i.e. from the Lewes Nodular Chalk), with significant inflows at depths of 22 m and 32 m associated with fracturing and the Chalk Rock, respectively. Borehole A is located within the valley of the River Kennet, a major tributary of the River Thames. Groundwater level data from observation boreholes near Borehole A show relatively small fluctuations and shallow hydraulic gradients, indicative of a highly transmissive groundwater system. This was confirmed by a Jacob straight line analysis of a five-day constant rate pumping test at Borehole A, which gave transmissivity estimates of more than $2000 \text{ m}^2\text{day}^{-1}$. Borehole A is licensed to abstract at an average rate of $2500 \text{ m}^3\text{day}^{-1}$.

3.2 Model Setup

3.2.1 ZOOMQ3D Regional Groundwater Model

Borehole A is located within the ZOOMQ3D regional groundwater model of the Marlborough and Berkshire Downs. This model is described in detail by Jackson et al. (2011; 2015). The

existing regional groundwater model had a variable mesh resolution of between 100 m and 2 km, but only covered the area of Borehole A at the coarsest resolution. Consequently the grid was refined to 500 m over the River Kennet catchment and then to 250 m around Borehole A. This allowed greater accuracy in the positioning of the abstraction borehole in the regional model and a more detailed representation of the River Kennet, which is approximately 500 m from the borehole. The regional groundwater system is represented in the model by three layers, which were informed by 3D geological modelling of the lithostratigraphy of the Chalk group. In the vicinity of Borehole A, the model layers correspond well to the geophysical logs of the borehole, with Layer 1 representing the Lewes Nodular Chalk Formation and Chalk Rock, which contribute the majority of inflow, and Layer 2 representing the New Pit Chalk Formation, the base of which is 5 m below the base of the borehole. Comparison of modelled and observed groundwater levels from the nearest observation borehole, located 1.7 km from Borehole A, showed good agreement (Figure 8). No further development or calibration of the regional groundwater model were therefore undertaken for this study.

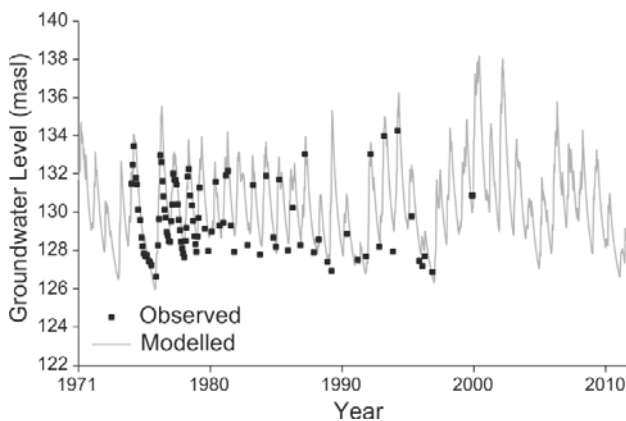


Figure 8 Modelled and observed groundwater levels at closest observation borehole to Borehole A.

3.2.2 SPIDERR

The SPIDERR flow model was initially calibrated as a standalone model to the five-day constant rate pumping test data from Borehole A. This first model had a 10 km radius, a 0.96 m diameter

open borehole at its center and an outer fixed head boundary. The radial dimension was discretized using 51 nodes spaced logarithmically between the borehole and outer boundary, the circumferential dimension was split into 12 slices, and the vertical dimension was represented as three layers using the layer elevations from the regional groundwater model. The borehole was extended to the bottom of layer 2 and was pumped using the abstraction rates shown in Figure 8. These do not incorporate the early-time fluctuations in abstraction rate pumping, which were related to issues with the electricity generator running the borehole pump.

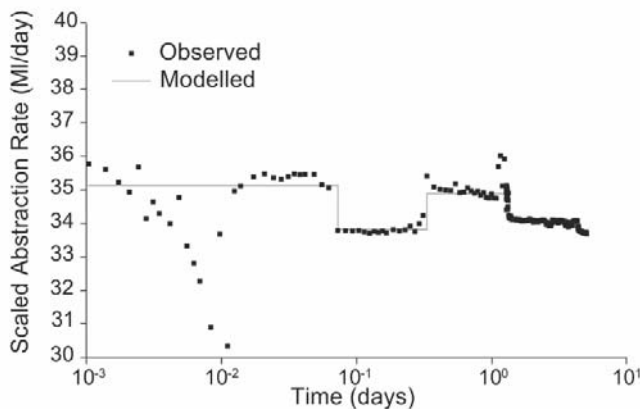


Figure 9 Observed and modelled abstraction rates for the five-day constant rate pumping test at Borehole A. Abstraction rates have been scaled for confidentiality reasons. Data provided courtesy of Thames Water Utilities Limited.

An initial model run (SFM Run 1) was undertaken using the regional model parameters in the vicinity of Borehole A, which were then calibrated to observed groundwater levels from the abstraction borehole and a nearby observation borehole located 15 m away. The calibration process was informed by a brief sensitivity analysis, which indicated the model results were most sensitive to the hydraulic conductivity and Forchheimer parameter in layer 1. All other parameters were therefore kept constant and these two parameters were adjusted to fit the observed groundwater levels in the abstraction and observation borehole. The results of the final calibration (SFM Run 4) are shown in Table 4 and Figure 10. The transmissivity of the fully

saturated model from SFM Run 4 is just under 2000 m² day⁻¹, which is close to the value derived from the Jacob straight-line analysis of the constant rate test data.

	K_h(1) m day⁻¹	K_h(2) m day⁻¹	K_h(3) m day⁻¹	S_y(1) -	S_s(1-3) m⁻¹	β(1) m⁻¹ day
Regional model parameters (SFM Run 1)	58	19	1	0.03	5 × 10 ⁻⁷	0
Calibrated parameters (SFM Run 4)	76	19	1	0.03	5 × 10 ⁻⁷	0.075

Table 4 Initial and calibrated parameters of the standalone SPIDERR Flow Model

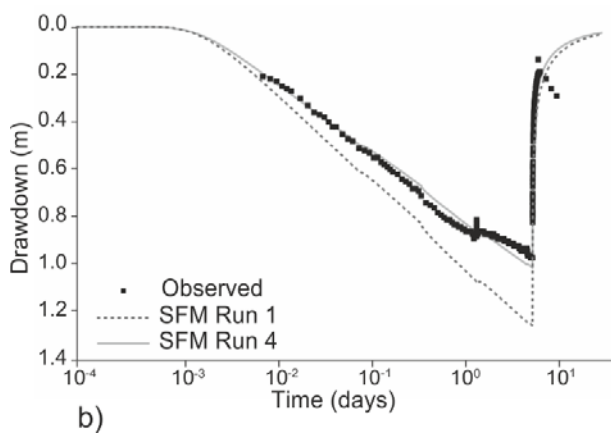
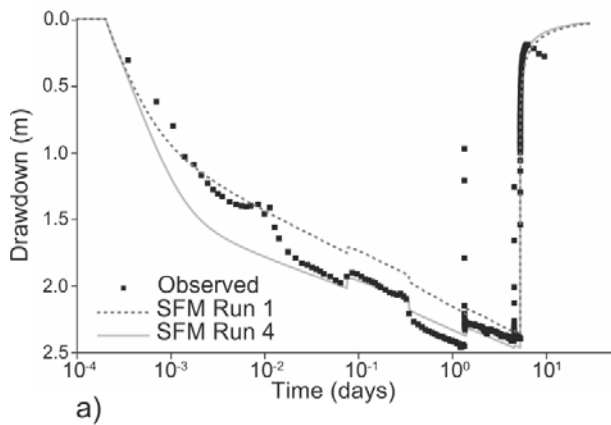


Figure 10 Observed and modelled drawdown in Borehole A during the five-day constant rate pumping test: (a) at the abstraction borehole; (b) at the observation borehole. Observed data provided courtesy of Thames Water Utilities Limited.

3.2.3 Coupled Model

The SPIDERR model was coupled with the ZOOMQ3D regional model as shown in Figure 11. Borehole A sits at the center of the SPIDERR grid and corresponds to a single node in the regional model. Around the abstraction borehole, the regional model and Cartesian grid of the SPIDERR model have a resolution of 250 m and the SPIDERR grid extends over an area of 1500×1500 m. Nine Cartesian nodes of the SPIDERR grid are replaced by a 12 slice radial model with a radius of 375 m. The three vertical layers are equivalent in the two models. The parameters from the calibrated standalone radial model (Table 4) are applied in the SPIDERR model and transferred to ZOOMQ3D through the OpenMI linking process, updating the parameters of the regional model within the boundary of the local-scale model. The coupled model is run through OpenMI on a daily time-step over the historic simulation period 1971 to 2012. Daily abstraction rates are available for Borehole A from 2003, when it first became operational. Weekly abstraction rates are available for a decommissioned borehole located 15 m from Borehole A (now used as an observation borehole) from 1971 to 2003. The two abstraction data sets are combined into a weekly time-series (shown from 2003-2012 in Figure 12) and applied at Borehole A in the SPIDERR model. Abstraction rates are passed from SPIDERR to the borehole node in ZOOMQ3D through the linking process in OpenMI, which also transfers the following daily volumetric data from the regional model to the local-scale model: flows across the SPIDERR boundary, distributed recharge, and leakage between the river and aquifer nodes along the River Kennet.

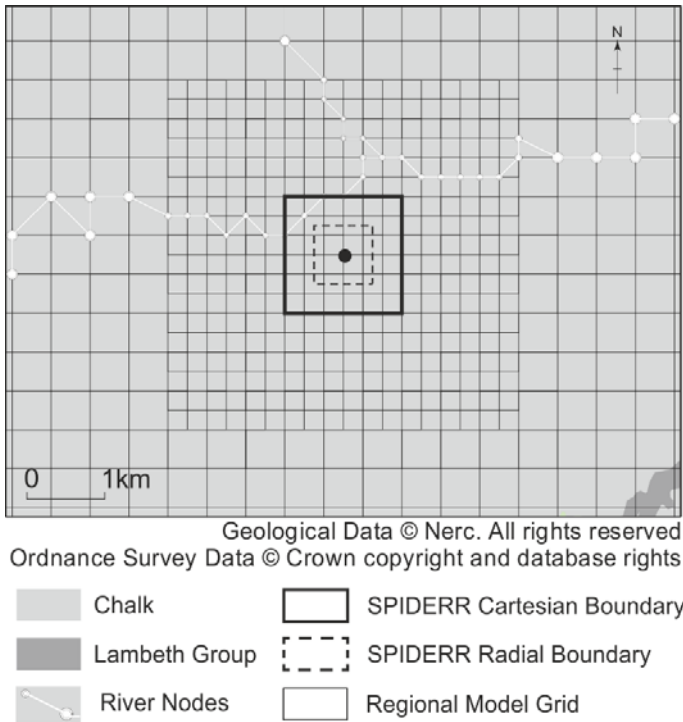


Figure 11 Coupled model composition around Borehole A

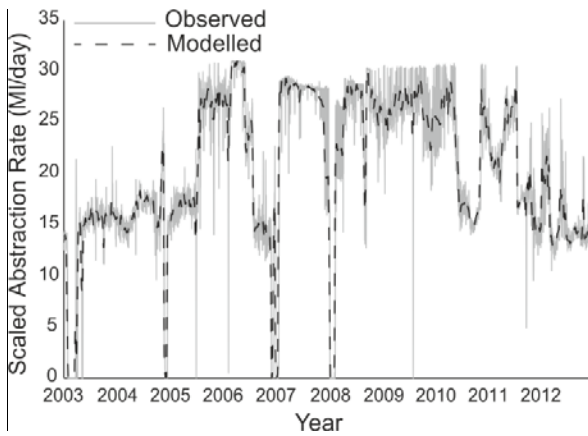


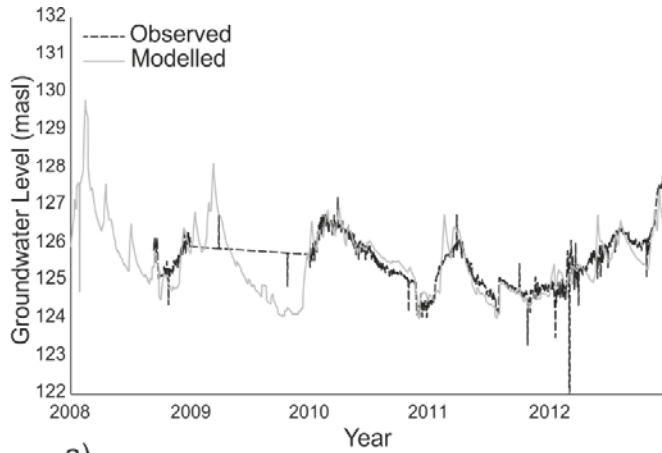
Figure 12 Operational (observed) and modelled abstraction rates over the simulation period. Abstraction rates have been scaled for confidentiality reasons. Observed data provided courtesy of Thames Water Utilities Limited.

3.3 Results

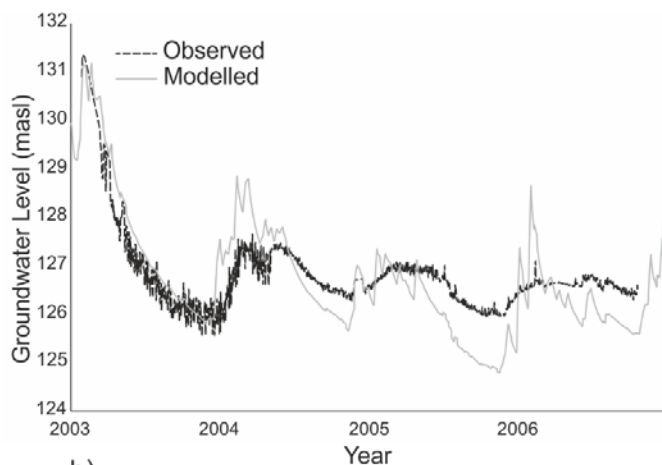
The results of the linked simulation are compared with groundwater levels monitored in Borehole A, which were available from 2008-2012 (Figure 13a). Simulated groundwater levels are also compared with observed levels from the decommissioned borehole, which was monitored over the period 2003-2006 (Figure 13b). Figure 13a shows very good agreement between modelled and operational water levels within the abstraction borehole. Daily or sub-daily fluctuations related to abstraction are not represented due to use of the weekly averaged abstraction rates, however these are generally small compared with the seasonal fluctuations, which are reproduced well by the model. In the observation borehole, simulated groundwater levels match the average observed groundwater level but show a greater degree of fluctuation than the observed time-series. This is most likely due to storage in the decommissioned borehole, which has a large diameter of 2.7 m and is represented in the model as a normal aquifer node with a specific yield of only 0.03, but this could be explored further through a sensitivity analysis.

Consistency between the two models, thus accuracy of flows passed from the regional- to the local-scale model, was evaluated by comparing simulated groundwater levels at equivalent points on the Cartesian grids in SPIDERR and ZOOMQ3D. The difference between groundwater levels in the two models was less than 0.1 m at all locations, providing confidence in the coupling procedure. Furthermore, the additional computational burden associated with the coupled composition was not prohibitive. The standalone regional model, which consists of 3118 grid nodes, required a run-time of 518 minutes while the coupled composition, which contains an additional 1299 computational nodes, required 862 minutes to run. The increased run-time was therefore proportional to the increased number of nodes in the coupled

composition, which allows significant refinement around the abstraction borehole compared to the standalone regional model.



a)



b)

Figure 13 Simulated and observed groundwater levels at (a) the abstraction borehole (Borehole A), and (b) the decommissioned borehole located 15 m from Borehole A. Observed data provided courtesy of Thames Water Utilities Limited.

4. DISCUSSION

The SPIDERR flow model has previously been shown to reproduce drawdown in complex heterogeneous aquifers through application to a borehole in a fractured chalk aquifer, which

displays vertical heterogeneity (in both the borehole and aquifer) and non-Darcian flow (Upton et al., 2019). The hybrid radial-Cartesian model developed and applied in this study allows the borehole-scale model to be linked with a regional-scale groundwater model through OpenMI. While there have been recent advancements in multi-scale groundwater modelling that allow groundwater processes to be simulated across multiple scales, such as MODFLOW-USG (Panday et al., 2017), this methodology provides a novel way of simultaneously representing regional and local-scale groundwater processes. Unlike existing models, it allows a much more detailed representation of the borehole-scale processes and features, including non-Darcian flow, which can significantly impact drawdown and borehole yields, particularly in fractured aquifers.

The key limitations of the methodology are related to errors across the radial-Cartesian boundary and SPIDERR-ZOOMQ3D boundary; however evaluation against analytical solutions and a standalone radial flow model show that these errors can be minimized with appropriate grid refinement and construction, which is made easy by the two model codes. Errors across the radial-Cartesian boundary within SPIDERR are particularly minimized by refinement in the circumferential direction, with a 12-slice radial model providing the optimum grid set-up. Refinement on the Cartesian grid and along the radial dimension of the radial grid were found to have less influence on the accuracy of the solution in SPIDERR. Linking SPIDERR with ZOOMQ3D through OpenMI requires the two models to be equivalent at the grid boundary and is based on the assumption that flow is largely horizontal and Darcian. These requirements can be met by positioning the boundary of the SPIDERR model at an appropriate distance from the abstraction borehole such that the effects of non-Darcian flow and vertical head gradients due to drawdown around the borehole are negligible, and discretization errors related to the representation of the abstraction borehole in ZOOMQ3D are also negligible. This will be site-

specific, but test simulations indicate that the effects of vertical heterogeneity and anisotropy may be fairly local (e.g. \ll 250 m, see Figure 6) to the abstraction borehole. The extent of the SPIDERR model in a linked composition can be determined by test simulations using standalone models, or through calculation of the approximate extent of the cone of depression of the borehole from long-term average abstraction and recharge.

While the methodology presented here provides less flexibility in grid construction compared with other multi-scale methods, such as the control-volume finite difference or finite element method, the simplicity of this method ensures it is very user-friendly and efficient to solve. The use of OpenMI to link the two models means that a standalone radial flow model can be developed and calibrated to an individual abstraction borehole, ideally through the use of pumping test data, prior to coupling with a regional model. This is easier than trying to calibrate a small area of interest in a regional-scale model, particularly if the method is to be applied to multiple sources. It also makes the solution process more efficient as individual sources can effectively be switched on and off in the regional model when they are not required. The methodology allows existing groundwater models, which often represent significant investments, to be used without any complicated or time-consuming changes to the grid structure and the multi-scale groundwater model can be quickly and easily linked with other OpenMI compliant codes for water resources management. This has already been done by Foster et al. (2017), who used a linked SPIDERR-ZOOMQ3D composition to investigate the impacts of well yield on irrigation economics.

The application of the methodology to a relatively simple single-source abstraction site in the Chalk aquifer of southern England demonstrates the ability of the model to reproduce the operational groundwater level time-series in a supply borehole. Further evaluation of the

methodology will require application to more complex abstraction sites with, for example, greater degrees of aquifer heterogeneity or multiple boreholes. The potential for linking multiple borehole models with a single regional groundwater model provides an opportunity to investigate interference effects between neighbouring abstraction boreholes, and impacts of multiple groundwater abstractions on other parts of the environment, e.g. rivers or wetlands. The linked methodology also provides the opportunity to run predictive scenarios to investigate the potential impacts of climate change or increasing demand on the sustainable yield of supply boreholes, which is a critical part of water resource management, particularly for strategically important aquifers, or those vulnerable to over-exploitation.

5. CONCLUSIONS

The linked modelling methodology presented in this paper provides a tool for effective groundwater resources management, whereby groundwater sources can be simulated within their regional context to enable short- and long-term groundwater availability to be assessed at the scale of the individual abstraction borehole and wider aquifer. The methodology links a borehole-scale non-linear radial flow model – SPIDERR (Upton et al., 2019) – to ZOOMQ3D, a regional groundwater modelling code, using OpenMI. SPIDERR represents the small-scale features and processes required to simulate the groundwater level in a pumped borehole, including features of the borehole itself, aquifer heterogeneity, and non-Darcian flow (Upton et al., 2019), while ZOOMQ3D represents the regional processes that can also influence the sustainable yield of an abstraction borehole, including large-scale aquifer heterogeneity, distributed recharge and discharge to rivers and springs. The methodology is shown to reproduce operational groundwater levels at a supply borehole in the Chalk aquifer in the UK, and has many potential applications for both water resource management and environmental

management more widely, particularly in the context of climate change in increasing global demand for water.

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