MAPPING THE DEPTH OF A VALLEY GLACIER BY RADIO ECHO SOUNDING

By A. C. WAGER

ABSTRACT. The depth of glaciers can be calculated from the length of time taken for the echo of a radio pulse to return from the bottom surface. Most previous work was done with the equipment mounted in an aircraft but this is not a suitable method for surveying small glaciers. Determination of the exact position of the aircraft is difficult and echoes from the valley walls obscure the echoes from the glacier bottom. With the equipment mounted on a sledge these problems are avoided, navigation is by markers on the glacier, and a very detailed survey can be carried out. Previous sledge-borne sounding was mainly concerned with developing equipment and technique. In contrast, this paper describes sounding carried out with existing equipment to map the depth of an Antarctic valley glacier. Because the glacier bed sloped, the point from which the echo returned may not have been directly below the observer. A practical method of calculating the coordinates of the actual reflecting point was developed and applied to the results. The average correction applied to echo range was −3 m. and to surface position 24 m. Isopleth maps of ice thickness and bedrock elevation were based on corrected data together with a contour map of surface elevation.

SINCE the early development of radio echo sounding for ice-depth measurement, the emphasis has been mainly on airborne work (Evans, 1967). Ice depths over most of Antarctica were unknown and airborne survey was the most attractive way of obtaining a general understanding of the land forms beneath the ice. However, the airborne technique cannot be used successfully for detailed studies of valley glaciers. Echoes from the walls interfere with surface and bottom echoes, and particularly on small glaciers which are traversed by an aircraft within a very few seconds, the accuracy of navigation becomes a limiting factor. Attempts have been made to obtain a good network of depth measurements on some temperate valley glaciers using a sledge-borne sounder, but results have been limited owing to the high absorption of radio energy in the relatively warm ice (Davis and others, 1973). Other surveys on colder ice masses (Bailey and others, 1964; Bailey and Evans, 1968; Clough and Løken, 1968; Goodman, 1970) have obtained more continuous measurements but the emphasis has been on developing technique rather than on detailed survey.

The sledge-borne method is valuable on cold glaciers and during 1972 the opportunity was taken to survey the depth of Spartan Glacier (lat. 70°59′S., long. 68°20′W.) in Alexander Island off the southern end of the Antarctic Peninsula (Figs. 1 and 2). This glacier was one on which detailed mass- and heat-balance studies were being made as a contribution to the International Hydrological Decade (Anonymous, 1970). The glacier did not terminate on ice-free ground but instead flowed into an ice shelf, so that ice depths had to be known before mass flux out of the study area could be determined.

EQUIPMENT AND TECHNIQUE

The echo sounder used was an S.P.R.I. Mark 4 designed principally for airborne echo sounding (Evans and Smith, 1969) and the technical specification is given below.

Transmitter	
Carrier frequency	35 MHz
Peak power	500 W (available)
Rise time	60 nsec.
Pulse repetition interval	40 μsec.
Pulse length	240 nsec.
Range calibration wave form	100 nsec., at 1 μsec. intervals

Fig. 1. Spartan Glacier (centre foreground) flows into the ice shelf which fills George VI Sound. Behind are the mountains of Alexander Island. (Photograph by the U.S. Navy for the U.S. Geological Survey.)

Receiver	
R.F. gain	92 dB
Calibrated in put attenuator	0–120 dB in 1 dB steps
Overall system performance	140 dB (160 dB less 20 dB attenuation of transmitter output)

Information Output

Information was displayed on two Tektronix type 321 oscilloscopes. A monitor oscilloscope displayed received signal strength along the y-axis and time along the x-axis. A recorder oscilloscope started an x-sweep when the transmitter emitted a pulse. The trace was normally at a very low intensity but it was modulated by the received signal so that the trace brightened when echoes arrived. This display was photographed on film moving continuously perpendicular to the sweep direction (Fig. 3). The film was annotated each minute with a range-calibration wave form and a character display giving the time and the receiver attenuator settings.

All the apparatus was mounted on a 3·7 m. Nansen sledge (Fig. 4) which was towed by a motor sledge. The total weight of sledge and equipment was about 250 kg., which limited the

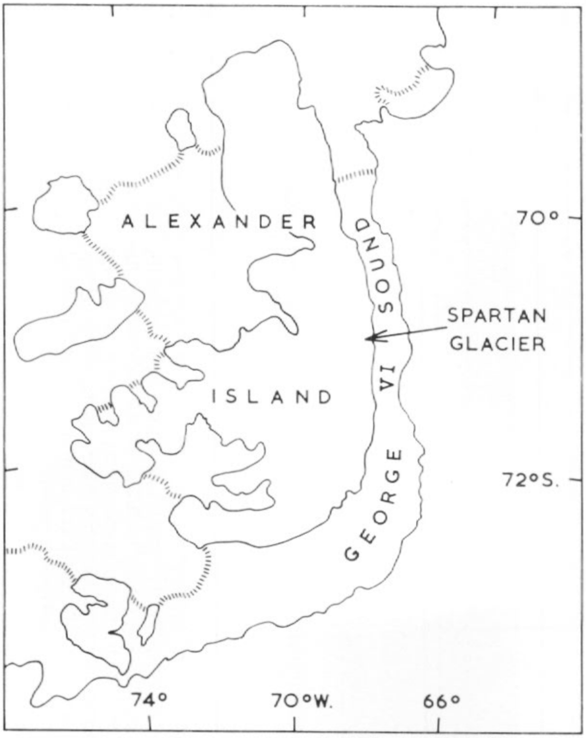

Fig. 2. Map of Alexander Island showing the location of Spartan Glacier.

steepness of slopes on which echo sounding could be carried out. The sledge was towed at constant speed along lines of survey stakes spaced at 180 m. intervals. The recording film was marked by briefly closing the camera shutter as each stake was passed. The time of passing each stake was noted and later the marks on the film were correlated with stake numbers. Runs were also made between stake lines in order to fill in gaps in the ice-depth survey. In order to simplify navigation, traverses were made in straight lines between stakes and speed was kept as constant as possible. Recording film was developed daily so that faults could be corrected. A two-bath developer was selected which did not require careful control of temperature and developing time and this proved convenient for processing in the field.

The shallowest depth of ice that could be sounded was limited by the time taken for the receiver to recover from the saturation caused by the transmitter pulse. In an attempt to reduce the coupling between transmitter and receiver, two separate aerials were used, one at the back of the sledge for the receiver and one at the front for the transmitter. A 20 dB attenuator was placed in the transmitter aerial lead in order to reduce the radiated power and consequently the overloading of the receiver. These precautions considerably reduced the length of time taken for the receiver to recover and allowed depths as shallow as 70 m. to be sounded. In spite of the much reduced power output from the transmitter, performance was still adequate for the deepest ice encountered on the glacier (220 m.). When working on the shallowest ice, it was essential for the operator to be on the sledge monitoring the receiver output. The receiver gain could be decreased as the echo strength was seen to be rising in shallower ice. In this way the receiver recovery time was reduced and the echo could still be distinguished from the transmitter pulse when sounding ice only 70 m. thick. This process is illustrated at the left-

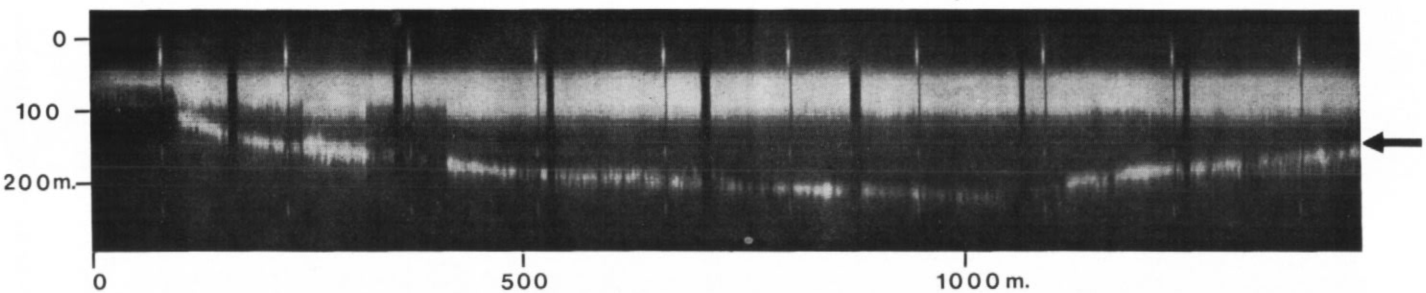

Fig. 3. Radio echo sounding record from the profile marked S–T on Fig. 11. The arrow on the right marks the leading edge of the bottom echo. The light band above is caused by overloading of the receiver by the transmitted pulse. It appears to start at a depth of 40 m. because the receiver was switched off during the emission of the pulse. Dark vertical bars extending across the film are event marks made as the sledge passed stakes on the glacier. The other vertical marks are from automatic annotation of the film in which 1 μsec. calibration marks, time and attenuator settings were displayed each minute.

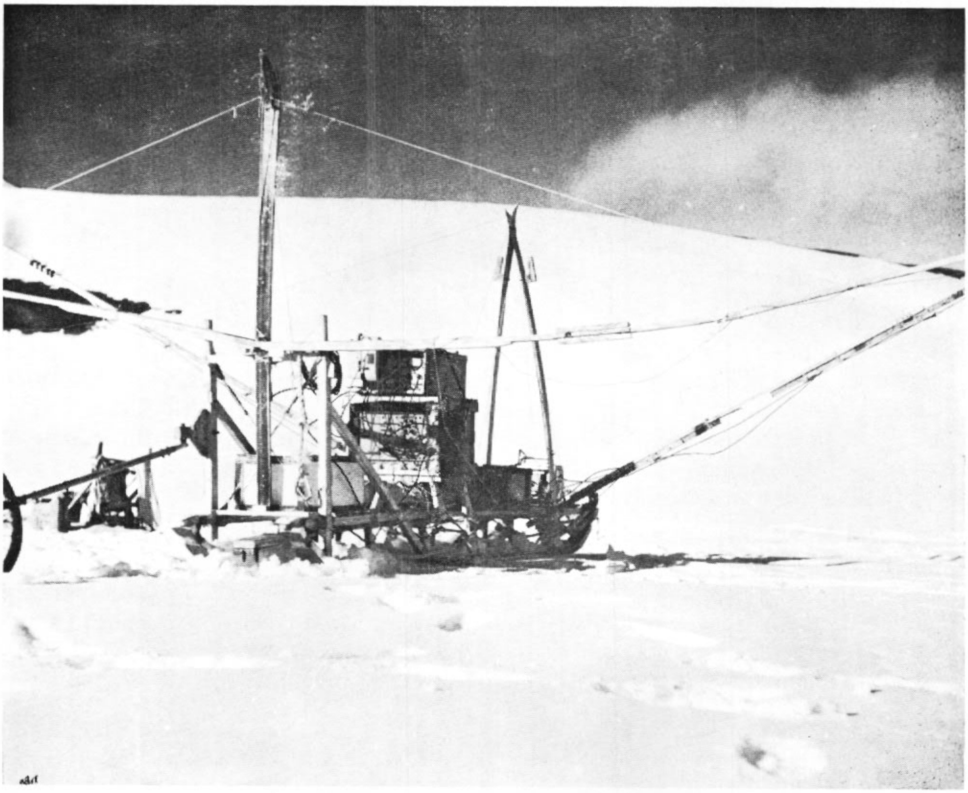

Fig. 4. Nansen sledge equipped with the S.P.R.I. Mark 4 radio echo sounder.

hand end of the record shown in Fig. 3. Receiver attenuator settings varied from 20 dB over the deepest parts of the glacier to 60 dB close to the edges. These settings corresponded to overall system performances of 120 and 80 dB.

Topographic Survey

In order to obtain a map of the bottom surface of the glacier, it was necessary to know the surface elevation as well as the ice depth at each point. The positions of 14 stakes on the glacier were determined by theodolite resection using six permanent survey stations located on ice-free ground around the glacier. From these positions, vertical and horizontal angles were measured to 97 other stakes. The horizontal angles were used to calculate coordinates of stakes by intersection and the vertical angles were used to obtain height differences by vertical triangulation. This survey defined the absolute position on the glacier of 111 points and served as the basis of a surface contour map (Fig. 10).

Reduction of Data

Navigational data consisted of marks on the recording film (Fig. 3) which identified stakes as they were passed. The distance on the film between successive marks and the corresponding distance on the map were divided into the same number of intervals. The number of intervals chosen depended on how fast the range was changing and corresponded to spacings on the ground of 35–50 m. Errors in navigation introduced by this simple method of reduction were caused by variations in the speed of the sledge and deviations from a straight track between stakes. If the speed changed by a fraction v, when the sledge was half the travel time between two stakes a distance d apart, the error introduced at the half-way point by assuming a steady velocity is $dv/4$. If the course changed by $\phi°$ midway between two stakes, the error in position introduced by assuming a straight course is $d \tan \frac{\phi}{2}/2$. An error, dt, in position on the surface caused an error in the measured range given by

$$\mathrm{d}r = \mathrm{d}t \tan \theta, \tag{1}$$

where θ is the bottom slope. The total error in the range is given by

$$\mathrm{d}r = \frac{d}{4} \tan \theta \left(v^2 + 4 \tan^2 \frac{\phi}{2}\right)^{\frac{1}{2}}. \tag{2}$$

Speed changes greater than 10 per cent and direction changes greater than 5° are unlikely to have occurred at any time, and in general the changes were much less. Also, 90 per cent of the points used had bottom slopes of less than 20°. Along the stake lines, these values gave a "worst case" dr of less than 4 m. at the maximum stake spacing of 180 m. Some of the runs between adjacent stake lines were as long as 1,000 m., but these were near the middle of the glacier where bottom slopes were rarely greater than 10°. In this case, the maximum error in range was unlikely to have exceeded 10 m.

The delay time was measured from the film, and had to be corrected for an instrumental error which caused a discrepancy between the start of the calibration sequence and the initiation of the transmitter pulse. The correction was obtained by reference to those parts of the film record where the transmitter pulse leading edge could be seen. Different echo-sounding runs passing the same point were compared in order to check that the instrumental error was eliminated by the reduction method used. The results are shown in Table I.

TABLE I. COMPARISON OF MEASURED ICE DEPTHS

Stake number	*Date*	*Time*	*Depth* (m.)	*Mean depth* (m.)	*Standard deviation* (m.)
	16 April 1972	16.48	180		
B94		17.00	185	183	2
	19 April 1972	15.07	185		
		16.37	182		
	16 April 1972	16.11	208		
B10	17 April 1972	14.44	202	208	5
		17.18	212		
	26 April 1972	16.52	211		
	16 April 1972	16.04	208		
B12	17 April 1972	14.47	214	213	8
		17.15	206		
	26 April 1972	16.41	224		
	15 April 1972	16.09	149		
B96	16 April 1972	16.50	146	146	3
		16.58	143		

In converting delay times to ranges, the velocity of radio signals in ice was taken to be 169 m./μsec. (Robin and others, 1969, p. 459–61). There was much melting at the surface of the glacier during the summer and much of the winter snow accumulation was converted to superimposed ice. There was therefore no region of firn over most of the glacier and the allowance for firn in the conversion to range given by Robin and others (1969, p. 461–62) was not made.

Calculation of Ice Depths

The range measured by the first echo may not correspond with the ice depth immediately below the observer. The echo may have come from a point situated to one side of the observer, in which case the range measured was a slant range. The actual position from which a given echo returned can be calculated from the measured range and from its variation with position as deduced from nearby measurements.

Harrison (1970) reduced airborne echo-sounding results by means of a two-dimensional reconstruction. His method relied on the directional properties of an aerial which was assumed to receive echoes only from points along the track of the aircraft. He deduced the rate of change of range with distance along track from adjacent measured ranges. The directivity of the aerial system used on the sledge was not measured but Fig. 5 shows the directivity of the same aerial when it was mounted beneath the wings of an aircraft. The beam width was 60° for a reduction in gain of 5 dB. When the aerial was mounted across the sledge, the beam width was increased because the outer ends of the transmitter aerial had to be raised to prevent them striking the ground and breaking. Fig. 5 shows that echoes received from 30° to one side of the sledge were attenuated by less than 5 dB relative to echoes received from directly below. This difference in echo strength was not detectable on the recording film. It is unlikely that echoes were received from angles greater than 30° because critical reflections take place at this angle (Robin and others, 1969, p. 455). The sledge-borne aerials were therefore considered as isotropic radiators and an extension to three dimensions of the two-dimensional reconstruction (Harrison, 1972, p. 35) was used. Each echo was treated as a reflection from a single point on the glacier bed. The position of the reflecting point was given by

$$x = s - r\left(\frac{\partial r}{\partial s}\right)_t \tag{3}$$

$$y = t - r\left(\frac{\partial r}{\partial t}\right)_s \tag{4}$$

$$z = r\left(1 - \left(\frac{\partial r}{\partial t}\right)_s^2 - \left(\frac{\partial r}{\partial s}\right)_t^2\right)^{\frac{1}{2}}, \tag{5}$$

where $(s, t, 0)$ were the coordinates of the observer and (x, y, z) were those of the point reflector. The observed range was r.

Before applying these equations the effect of surface slope was examined. From Fig. 6 it can be seen that the deconvolution factor z/r becomes

$$\cos\theta - \sin\theta\tan\alpha, \tag{6}$$

where α and θ are the surface and bottom slopes in the direction of the reflecting point. Fig. 7 shows the effect of surface slope on the deconvolution factor for varying values of the bottom slope. From Fig. 8, it can be seen that the surface slope was generally much smaller than the bottom slope. Additionally, the value of α in Equation (6) was the slope in the direction of the reflecting point beneath the glacier. This slope was always less than the steepest slope value given in Fig. 8 because the surface and bottom slope contours were not parallel. The value of $\tan\alpha$ is given by

$$\tan\alpha = \tan\alpha_s\cos\Omega, \tag{7}$$

where α_s is the steepest slope angle and Ω the angle between surface and bottom contours. It can be seen from the maps that Ω was often large (45–90°) in those places where α was large. Analysis of a sample of observed ranges (31 points) gave an average value of $1-\frac{z}{r}$ of 1·8 per cent (S.D. 2·8 per cent) when surface slope was neglected, and an average additional correction for surface slope of only 0·9 per cent (S.D. 0·8 per cent).

The average correction applied for surface slope was 1·2 m. and it rarely exceeded 5 m. In general, when θ was small the surface-slope contribution to the deconvolution factor was

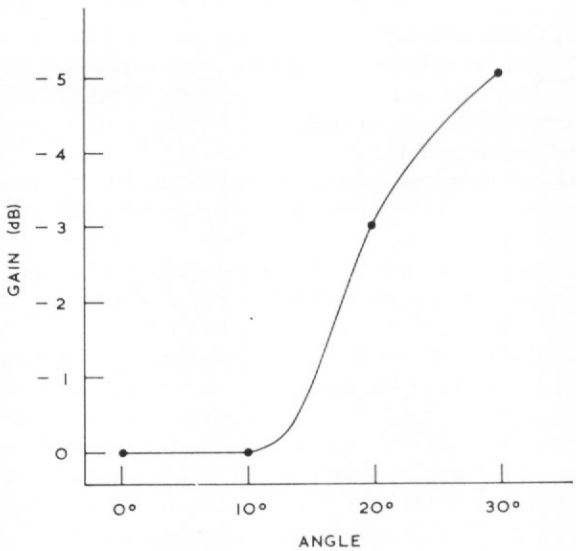

Fig. 5. Decrease in gain of the aerial plotted against the angle from which radio echoes were received. The angle is measured from the vertical in the plane perpendicular to the direction of motion. The aerial was mounted beneath the wings of an aircraft flying at an altitude of 900 m. above calm sea. Signal strength was 80 dB above noise at nadir. Data obtained by B. M. E. Smith (personal communication).

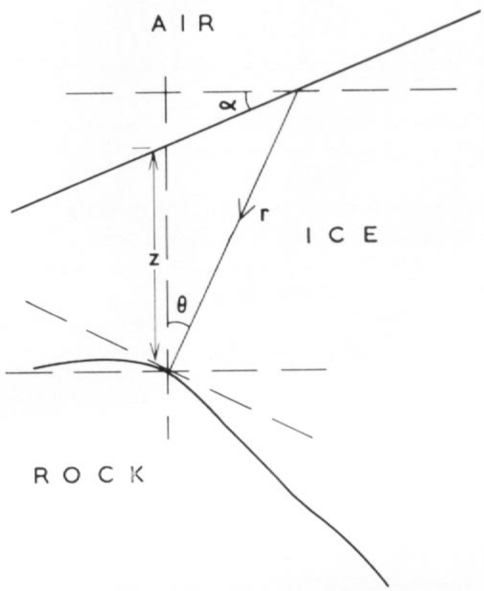

Fig. 6. Relationship between echo range, ice thickness, surface slope and bottom slope of a glacier.

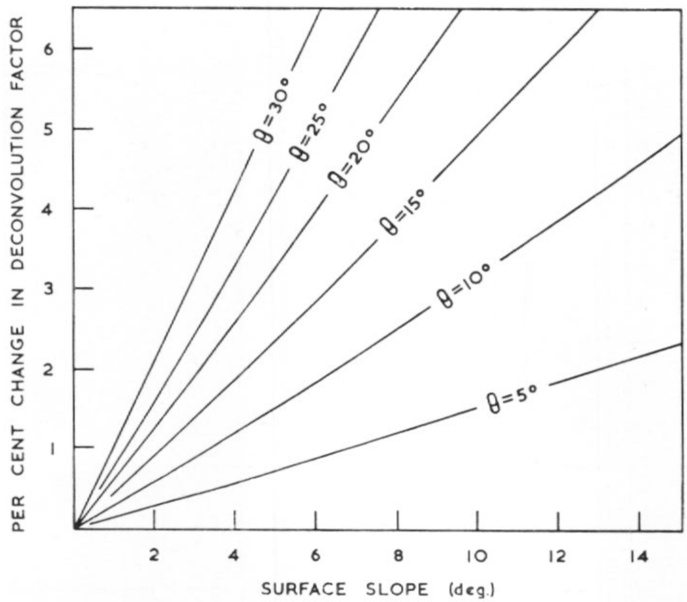

Fig. 7. Effect of surface slope α on the deconvolution factor z/r plotted for varying values of bottom slope, θ. Change in the deconvolution factor has been calculated as $100 \sin\theta \tan\alpha/(\cos\theta - \sin\theta \tan\alpha)$.

dominant but not significant. When θ was large, the bottom-slope term was much larger than the surface-slope term which was thus neglected. This relationship will generally hold for ice masses surveyed by radio echo sounding so that, unless the resolution of the equipment is very much greater than that used here, surface-slope effects may safely be neglected. This avoids the use of complicated deconvolution equations which would be difficult to apply to large sets of data.

Calculation of the position of a reflecting point using Equations (3)–(5) required the terms $\left(\frac{\partial r}{\partial s}\right)_t$ and $\left(\frac{\partial r}{\partial t}\right)_s$ in addition to measured range and the position of the observer. These terms are the rate of change of measured range with position along the two coordinate axes. A computer programme was used to interpolate between observed ranges to obtain range values in a regular matrix. The four values from the matrix nearest to each observed point were used to derive two orthogonal rates of change of range. These were substituted for $\left(\frac{\partial r}{\partial s}\right)_t$ and $\left(\frac{\partial r}{\partial t}\right)_s$ in Equations (3)–(5) to obtain the position of the point reflector.

The mean change in depth resulting from the deconvolution was —3 m. and the mean change in position was 24 m. The negative change in depth does not imply that the deconvoluted profiles were shallower than the uncorrected profiles because changes in horizontal position must be taken into account (Fig. 9). The profile in Fig. 9 was chosen as an example because it ran approximately along the line of steepest bottom slope, so that horizontal displacements occurred mainly along and not across the profile, allowing the two profiles to be directly compared.

The limitation of this method of deconvolution lies in the interpolation of ranges. The components of change of echo range along track were always accurate since they were based on closely spaced points, but components across the track had to be derived from points on other profiles which were generally farther away. The errors introduced in this way are difficult

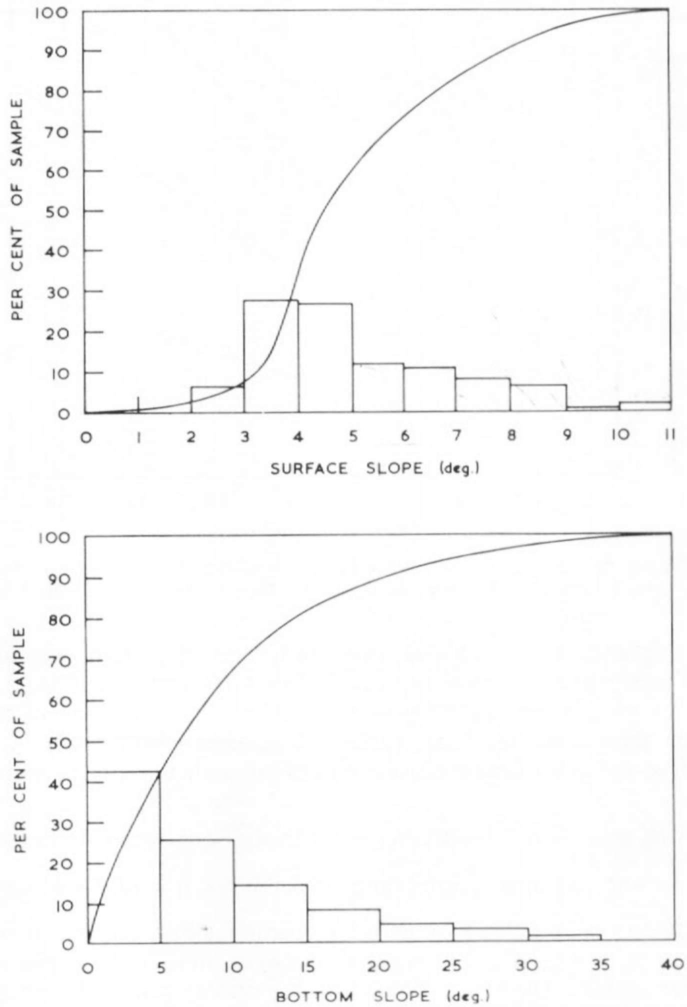

Fig. 8. Distribution of surface and bottom slopes.

Top. Fraction of surface slopes falling into 1° intervals. The solid line shows the fraction of sample less than a given value. The sample contained 110 values.

Bottom. Fraction of bottom slopes falling into 5° intervals. The solid line shows the fraction of sample less than a given value. The sample contained 554 values.

to estimate but for a 5° error in bottom slope the deconvolution factor changed by 6 per cent at the maximum possible bottom slope of 34° and 1·5 per cent for a more typical bottom slope of 10°. Bottom slope over most of the area was less than 10° (Fig. 8) and it was therefore unlikely that interpolated values were in error by more than 5°. As a check on serious errors in the interpolation of bottom slopes, those angles which were calculated to be near the critical bottom slope (34°) were examined. In the set of 554 deconvoluted points, bottom-slope angles greater than 35° were found on only three occasions, and only ten angles were above 30° which was considered by Robin and others (1969, p. 455) to be the practical limit for reflections.

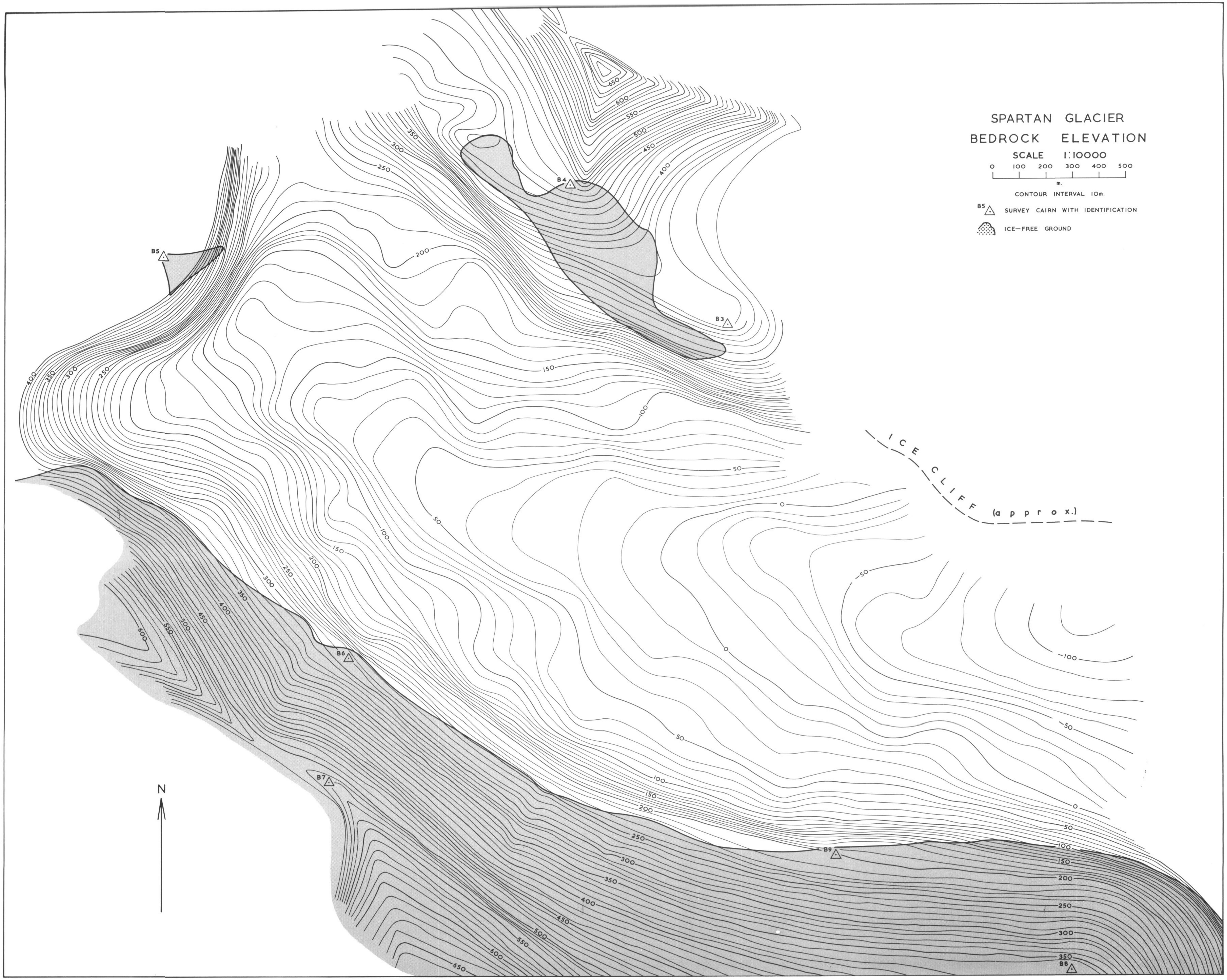

Fig. 12. Bedrock elevations beneath permanent ice were calculated by subtracting ice depths obtained by radio echo sounding from surface elevations. Spot heights were obtained from vertical triangulation by theodolite and contours off the glacier were interpolated from a plane-table survey. Heights are in metres above estimated mean sea-level.

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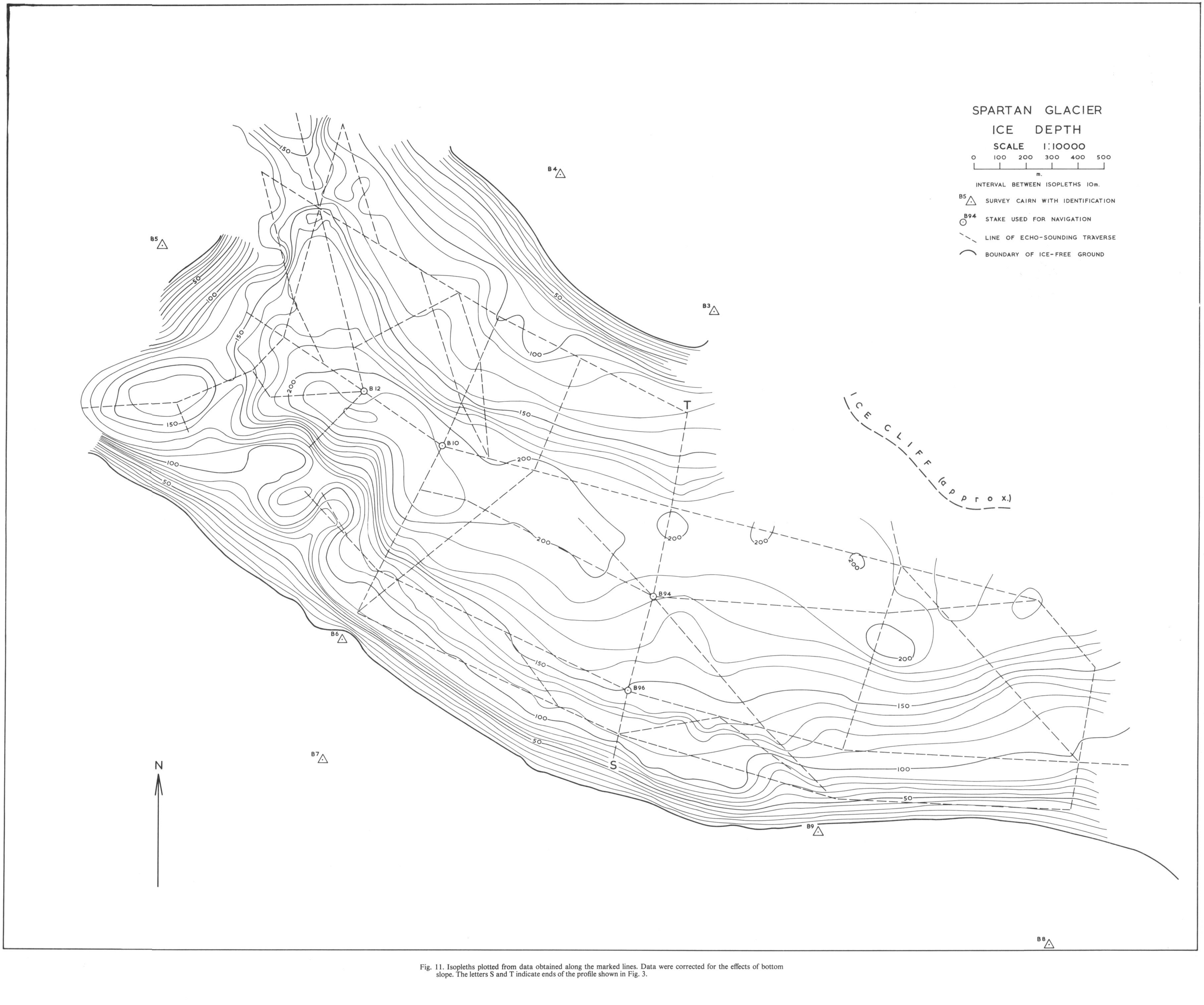

Fig. 11. Isopleths plotted from data obtained along the marked lines. Data were corrected for the effects of bottom slope. The letters S and T indicate ends of the profile shown in Fig. 3.

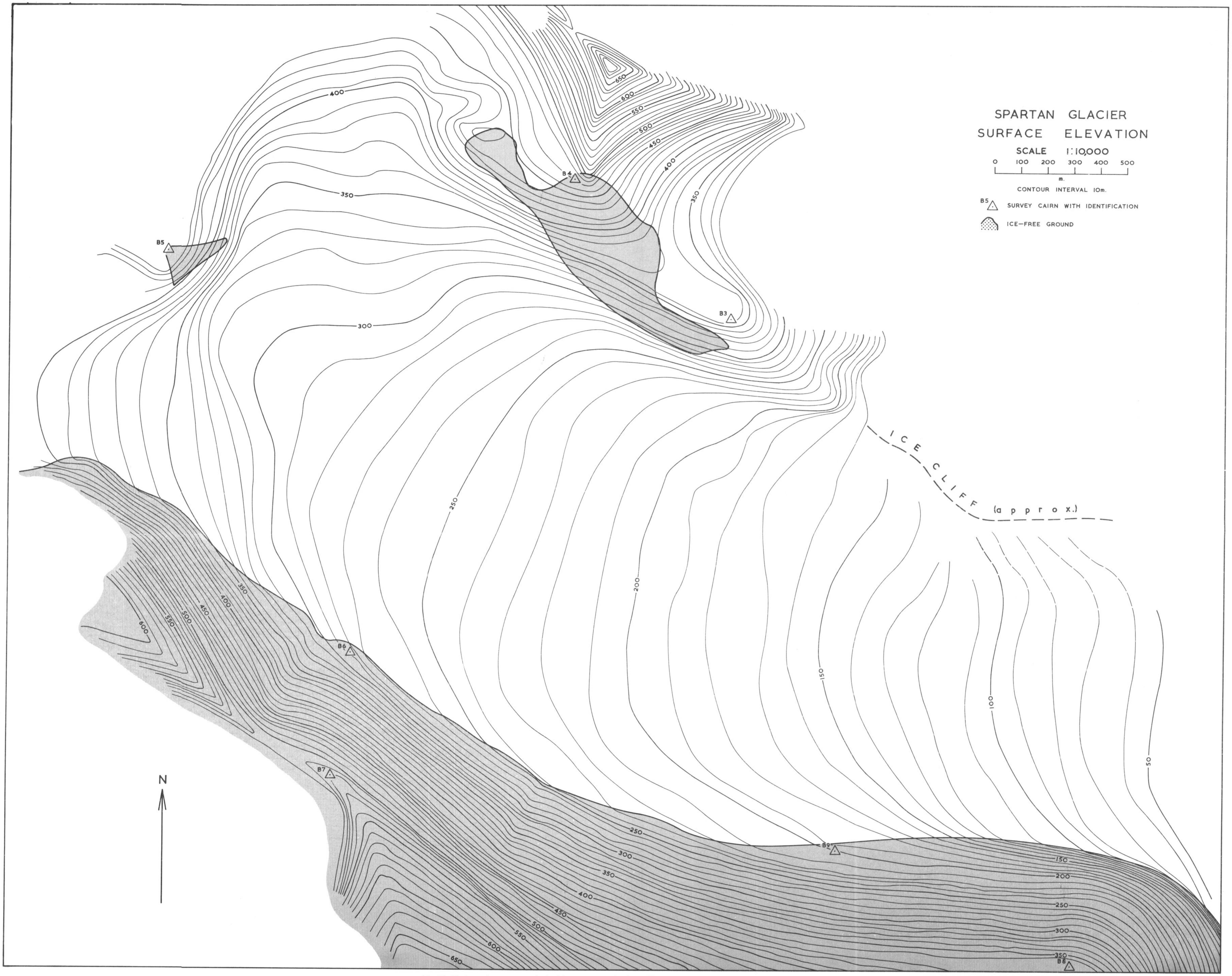

Fig. 10. Glacier surface contours obtained from a survey of stake positions. Contours off the glacier were estimated from a plane-table survey and from spot heights obtained by vertical triangulation. Heights are in metres above estimated mean sea-level.

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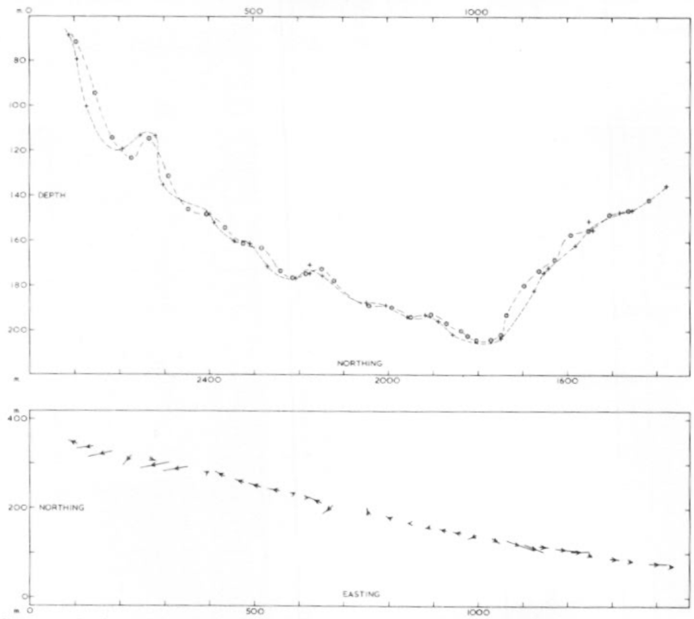

Fig. 9. Effects of deconvolution on a typical profile (S–T in Fig. 11).

Top. Vertical displacement of the reflecting point from position is indicated by the observed range. Circled dots represent observed echo ranges and crosses represent deconvoluted depths.
Bottom. Horizontal displacement of the reflecting point from position of the observer. The arrow points from the observer towards the reflecting point.

The main sources of error were:

	Max.	*Mean*
i. Navigation	10 m.	3 m.
ii. Film reading	8 m.	4 m.
iii. Surface slope	5 m.	2 m.
iv. Bottom slope	12 m.	2 m.
v. Velocity of propagation		2 m.

The probable maximum error in depth at a single point was therefore 18 m. In order to minimize the effect of errors in the depth of single points, the isopleths (Fig. 11) were smoothed over a distance spanning several data points. This means that information about changes in depth over distances less than 50 m. has been discarded because much of it was probably due to random error. It has been replaced by a mean trend which is more consistent with the features of the glacier.

Systematic errors in the velocity of propagation of radio waves in ice may have affected the calculated depth by up to 1·5 per cent (Robin and others, 1969, p. 469). Surface-slope effects contributed a mean error of −2 m. since the surface slope was generally in the same direction as the bottom slope.

Fig. 12 is a contour map of bedrock for which data were obtained by subtracting deconvoluted depths from the surface elevations (Fig. 10). Ice depths could not be measured on the steep northern wall of the glacier and bedrock elevations above the area of ice-free ground were estimated by assuming that ice depths there were small.

Vertical sections (Fig. 13) have been drawn from these maps to indicate the general shape of the glacier.

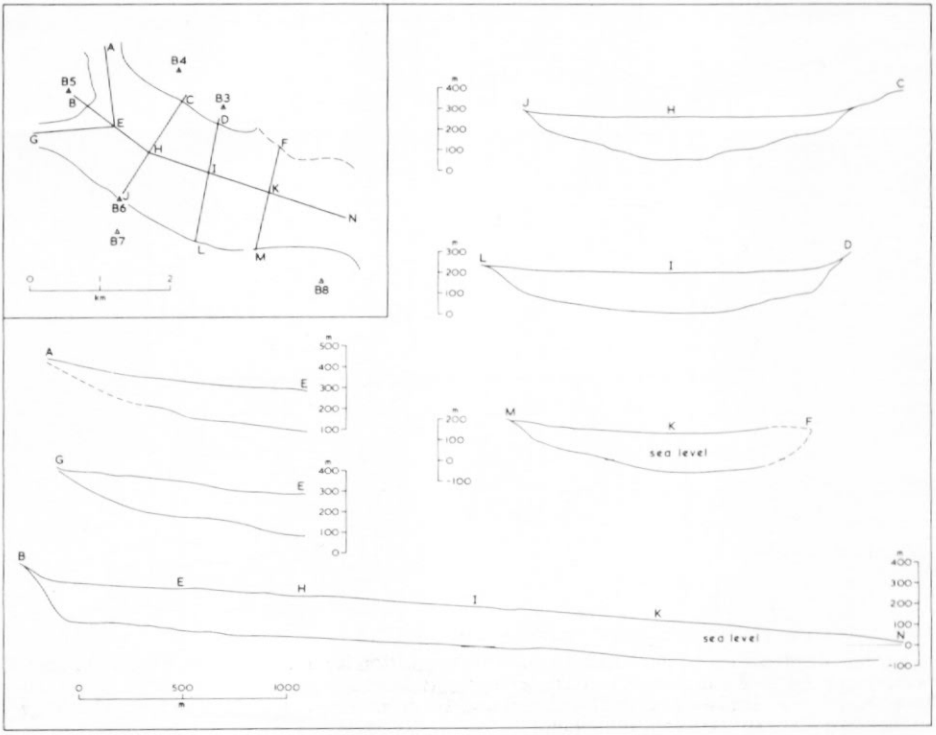

Fig. 13. Vertical sections of Spartan Glacier. The location of sections is shown in the inset. Survey cairns are marked B3 to B9. Horizontal and vertical scales are the same for all sections.

Acknowledgements

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