The beauty of equations

Tom Anderson

What makes something beautiful? Out of curiosity, I googled 'famous beautiful painting' and one of the images that came up was 'The Scream' by Edvard Munch. My wife saw the painting during a visit to Oslo and so I asked her if she thinks it is beautiful. She replied with a resounding 'No!', but maybe whoever paid $120 million for it in 2012 has a different opinion. A rather less extravagant example is provided by the postcard shown in Figure 1 which was posted in 1932 and shows the Sultan's palace in Selangor, Malaysia. I collect Malaysian postcards as a hobby and find this one particularly beautiful because, almost a century ago, producing colour cards of this kind took immense skill. Making this card involved printing in black using the collotype method (based on exposing gelatin to light passed through a photographic negative), in combination with hand colouring of individual printing plates by applying dots of different densities (stipple) using pen and lithographic ink. Its beauty, at least for me, lies not only in its appearance but also in the skill with which it was accomplished.

Figure 1 *Postcard of the Sultan's palace in Selangor, Malaysia (posted in 1932)*

Beauty, simplicity and usefulness

Simple things are often considered to be the most beautiful. For example (quoting Eleanora Duse), 'If the sight of the blue skies fills you with joy, if a blade of grass springing up in the fields has power to move you, if the simple things of nature have a message that you understand, rejoice, for your soul is alive.' Mathematicians often see beauty in simple equations because simplicity abstracts ideas into the purest dependencies between terms. The simpler the equation, the more evident is the nature of the dependency. Perhaps the most famous example is Einstein's $E = mc^2$. Furthermore, simple (and thereby beautiful) equations and theories are sometimes viewed as being closest to the truth. For example, the English chemist Rosalind Franklin remarked that Watson and Crick's discovery of the double-helix structure of DNA 'was too pretty not to be true'. The renowned physicist Paul Dirac even remarked that 'It is more important to have beauty in one's equations than to have them fit the experiment.'

Simplicity in equations, i.e. mathematical description using the purest dependencies, is likewise a virtue for me, but only when it is meaningful and can be usefully applied to further our understanding of the world that surrounds us. For example, here is a beautiful pair of equations that I derived some time ago to calculate the ratio of bacterial production (*BP*) to primary production (*PP*) in the ocean (at steady state):

$$\frac{BP}{PP} = \frac{\gamma + \phi_\infty}{1/\omega - \phi_\infty} \qquad (1)$$

$$\phi_\infty = \frac{\phi_1}{1 - (1 - \phi_1)\,k_c} \qquad (2)$$

The *BP* : *PP* ratio can be used to indicate the importance of microbial pathways as a sink for organic carbon in the ocean and depends on the supply of dissolved organic carbon (DOC) to fuel *BP* via phytoplankton and zooplankton. The calculation involves only four parameters: the DOC released by phytoplankton normalised to *PP* (γ), the fraction of carbon processed by zooplankton during a single grazing event that is transferred to DOC (ϕ_1), the zooplankton net production efficiency (k_c) and bacterial gross growth efficiency (ω).* Variable ϕ_∞ is the DOC released via grazers at successive trophic levels (ordered into an infinite series for practical application). I see beauty not only in the simplicity and elegance of the equations, but also in their underlying meaning and their consequent usefulness.

*Note that the model, as published, also had parameters for viral lysis which I have excluded here because calculated *BP*/*PP* was insensitive to them.

I nevertheless discount any necessary relation between simplicity, beauty and truth. Some of the assumptions and equations that go into marine ecosystem models are oversimplified, even 'dysfunctional', and such equations are in no way beautiful to me. Biological organisms and ecosystems are complex entities in terms of physiology, behaviour and interactions among individuals. Complex models that describe these processes and interactions may therefore be seen as beautiful, in the same way that a symphony involves harmony between many different musical instruments. Usefulness is the key. An interesting non-mathematical example is provided by Urhobo pottery. Most of us think of vases and other ceramics as beautiful if they have appealing shapes or are decorated with alluring

Figure 2 *Oni Oche pot of the Urhobo people. (By courtesy of Mansfield Ceramics, Gulgong, Australia)*

colours and motifs, such as Japanese Imari porcelain. For the Urhobo people of southern Nigeria, however, the beauty of a pot such as the Oni Oche (Figure 2) is not due to a pleasing shape or embellished appearance but, rather, it is expressed through its functionality in relation to the purpose for which it is made. This 'beautiful' pot was made for storing liquid, with a narrow neck and rounded belly that slopes gradually to the base.

The eye of the beholder

I suggest that we marvel at, and see beauty in, the wonders of the world such as the Pyramids and Taj Mahal, not only because of their appearance, but also because of the tremendous effort and skill that went into building them. The Danish author Isak Dinesen, best known for her book *Out of Africa*, once wrote: 'Probably one can say that all beautiful, noble, or brilliant works are of use, or that everything that proves to be useful or beneficial has its own beauty.' This may be so, but only in the eye of the beholder. For example, I consider the engine in my car to be most useful, but do not see any beauty in it. This is despite the fact that it is a feat of engineering that marks the achievement of thousands of people over many decades. It is simply not within my sphere of interest. In similar fashion, I do not expect you to necessarily see beauty in my equations for calculating *BP*/*PP*. For starters, I provided here only minimal ecological context with little about the assumptions behind the equations and, moreover, it may be that doing math is not your favourite pastime.

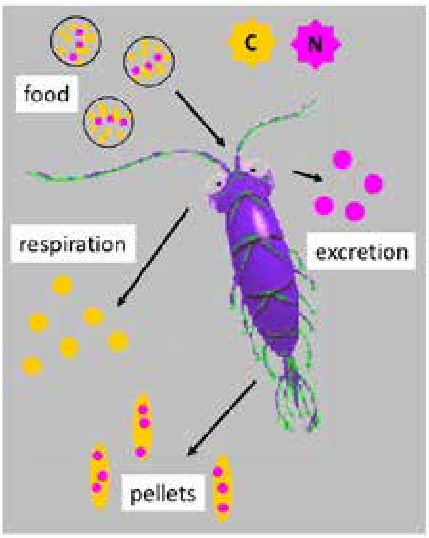
In many instances, beauty is an acquired characteristic, requiring time and experience, and we all have different interests in life. The best choice of venue for a day out with my wife, for example, would not be the Math Museum (Figure 3)! Models and equations are, however, an integral part of scientific research and it therefore behoves modellers, including myself, to make their models as transparent and accessible as possible to the rest of the scientific community. This need not require users to delve deeply into the equations and their derivation as it is exploring the concepts that matters most, along with their quantitative testing in the context of available data. Two great marine ecosystem modellers spring to mind in this regard: John Steele and Mike Fasham. Both enjoyed nothing more than reaching out to, and interacting with, biologists and ecologists, sharing ideas and, through the combination of observations and theory, advancing our understanding of the structure and function of marine ecosystems.

Figure 3 *Formula Appreciation Class (© Sidney Harris)*

The beauty of discovery

The role of equations and models in science is often not well understood. To some, modelling is about taking existing knowledge, formalising it mathematically and making predictions such as how marine ecosystems will respond to future changes in climate. Making these kinds of predictions is, of course, an entirely worthwhile and important activity. Modelling, however, can provide so much more in terms of contributing to knowledge and the advancement of science in general. Equations organise thinking, and models provide a framework for carrying out numerical experiments to rigorously test cause and effect. An obvious example that springs to mind is Mike Fasham's use of a simple ecosystem model to test various hypotheses about the factors that regulate the distinctive high-nutrient low-chlorophyll (HNLC) ocean ecosystems, including iron limitation, grazing and light. Models may suggest new observations and experiments – as elegantly put by John Steele: [By] 'forcing one to produce formulas to define each process and put numbers to the coefficients, reveals the lacunae in one's knowledge … to suggest further field or experimental work'. These experiments suggest new hypotheses and so the advance of science continues through the cycle of theory and observation.

Figure 4 *Cycling of carbon and nitrogen in food by zooplankton. (Adapted from a cartoon by the Zoology Dept, University of Guelph)*



As I wrote in a previous *Ocean Challenge* article, 'the beauty of science … is that it is a voyage of discovery, with the unexpected and unexplained lying in wait at every turn.' Models should surprise us, at least occasionally, and the equations involved are most beautiful when they contribute to this voyage. I recently developed a model that predicts how invertebrates, notably zooplankton, use the carbon and nitrogen in their food for growth and metabolism (Figure 4), and how this is influenced by temperature. My co-authors and I were surprised by its predictions. We had hypothesised that in a warming environment zooplankton will require increasingly carbon-rich diets because increasing energetic costs of metabolism will be mainly fuelled by compounds such as carbohydrates. In fact, however, we ended up concluding that the hypothesis is false, namely that 'the nutritional requirements of invertebrate consumers may change little, if at all, at elevated temperature'. I especially enjoyed pondering and discussing the results with my colleague in Southampton, Dan Mayor, and we couldn't but help remark that, with the model predictions in front of us, it all seemed so logical and beautiful.

Further reading

Anderson, T.R. (2010) The beauty of science. *Ocean Challenge* **17**(1), 4–5.

Anderson, T.R. and H.W. Ducklow (2001) Microbial loop carbon cycling in ocean environments studied using a simple steady-state model. *Aquat. Microb. Ecol.* **26**, 37–49.

Anderson, T.R., D.O. Hessen, M. Boersma, J. Urabe and D.J. Mayor (2017) Will invertebrates require increasingly carbon-rich food in a warming world? *Amer. Nat*. **190**, 725–42.

Elizabeth, A.O. (2018) The concept of beauty in Urhobo Pottery. *Ceramics – Art and Perception* **109**, 96–101.

Fasham, M.J.R. (1995) Variations in the seasonal cycle of biological production in subarctic oceans: A model sensitivity analysis. *Deep-Sea Res. I*, **42**, 1111–49.

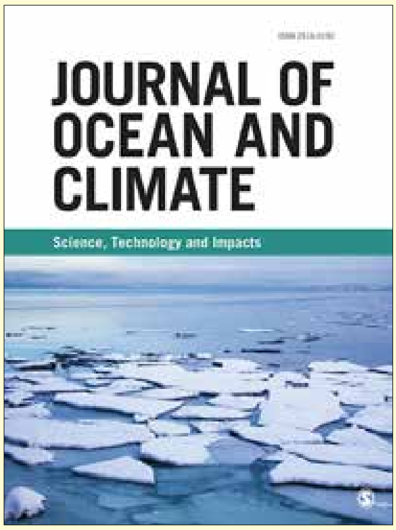
McAllister, J.W. (1996) *Beauty and Revolution in Science*. Cornell Univ. Press, 231pp.

Steele, J.H. (1974) *The Structure of Marine Ecosystems*. Harvard Univ. Press, 128pp.

Tom Anderson is a senior research scientist in the Marine Systems Modelling group at the National Oceanography Centre, Southampton. His interests include Earth System science, marine biogeochemical cycles, stoichiometry, the microbial loop and dissolved organic matter, model complexity, and the methodology and philosophy of science.
tra@noc.ac.uk

Special Collection of articles in *Journal of Ocean and Climate: Science, Technology and Impacts*

At the Challenger Conference 2018 a partnership was established with the SAGE peer-reviewed, open access *Journal of Ocean and Climate: Science, Technology and Impacts* (https://journals.sagepub.com/home/ocs). Authors of abstracts for the Challenger Conference are encouraged to submit a journal paper to form part of a Special Collection in the journal.



Article Processing Charges (APCs) are waived for this Special Collection. When submitting your paper, please indicate that it is to be considered for this Special Collection by selecting 'Special Collection: Challenger Conference 2018' as the article type. For full details about the Special Collection and how to submit your paper please read the Special Collection Call for Papers https://journals.sagepub.com/page/ocs/call-for-papers/special-collection/challenger-society-conference

The submission deadline is August 31, 2019.

MASTS Annual Science Meeting
Bold science to meet grand challenges

MASTS will hold its 9th Annual Science Meeting on 2–4 October 2019 (Wednesday–Friday) at the Technology and Innovation Centre, University of Strathclyde, Glasgow. Everyone is welcome.

This cross-disciplinary meeting will examine the science we need to meet the modern challenges which face our marine waters and seas. It will bring together members of the marine science community, with the aim of promoting and communicating research excellence and forging new scientific collaborations. The cross-disciplinary nature of the event as well as the high calibre of the selected talks means that scientists can broaden their knowledge in marine science as well as benefit from expertise and ideas gained in a range of fields other than their own.

Science presentations and e-poster sessions will take place on the first two days (Wednesday and Thursday, 2 and 3 October), together with plenary sessions and opportunities to network. On the third day, the venue will host a number of meetings and workshops.

Please get in touch if you would be interested in running a special session on 2 or 3 October, hosting a workshop on 4 October, or exhibiting at the event.