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Key Points:
- Nonextensive statistics are shown to provide a proper description of the frequency distribution of geomagnetically induced currents (GICs) amplitudes
- Distributions of GICs amplitudes follow *q*-exponential Tsallis distributions worldwide
- GICs for extreme events are likely to be observed at least once over a period of 10 solar cycles

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The Tsallis statistical distribution applied to geomagnetically induced currents

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Abstract Geomagnetically induced currents (GICs) have been long recognized as a ground effect arising from a chain of space weather events. GICs have been measured and modeled in many countries, resulting in a considerable amount of data. Previous statistical analyses have proposed various types of distribution functions to fit long-term GICs data sets. However, these extensive statistical approaches have been only partially successful in fitting the data sets. Here we use modeled GICs data sets calculated in four countries (Brazil, South Africa, United Kingdom, and Finland) using data from solar cycle 23 to show a plausible function based on a nonextensive statistical model of the *q*-exponential Tsallis function. The fitted *q*-exponential parameter is approximately the same for all locations, and the Lilliefors test shows good agreement with the *q*-exponential fits. From this fit, we compute that the likely numbers of extreme GICs events over the next ten solar cycles are 1–2 for both Finland and United Kingdom, at least one for Brazil and less than one event for South Africa. Our results indicate that the nonextensive statistics are a general characteristic of GICs, suggesting that the ground current intensity has a strong temporal correlation and long-range interaction.

1. Introduction

Geomagnetically induced currents (GICs), first described by *Barlow* [1849] as anomalous currents in telegraphic wires, are currently understood as a ground effect arising from a chain of events in the Sun-Earth system. Highly disturbed solar wind (SW) plasma propagates through the interplanetary medium and eventually impinges on the Earth's magnetosphere causing magnetic disturbances detectable on the ground. The perturbed magnetospheric-ionospheric coupling can produce intense currents systems that can induce strong geoelectric fields at the Earth's surface, which in turn generate currents that can flow through grounded technological infrastructure as it offers low-resistance pathways [*Lanzerotti*, 2013; *Zanetti et al.*, 1994].

GICs amplitudes in high-voltage power networks are controlled by a combination of geophysical conditions and network parameters. The geophysical conditions include the amplitude of the magnetic field variations and the local ground conductivity structure, while the latter include the network's topology, number of nodes, and the resistance of transmission lines and transformer grounding connections. GICs strength measured during geomagnetic storms can vary from hundreds of amperes at high latitudes (>60° north or south) [e.g., *Pulkkinen et al.*, 2008, 2012] to several tens of amperes at low-to-middle geomagnetic latitudes (10°–60° north or south, in geomagnetic coordinates), see, e.g., *Ngwira et al.* [2008], *Caraballo et al.* [2013], *Barbosa et al.* [2015a], and *Carter et al.* [2016, 2015].

GICs amplitudes may be determined either through direct measurements (i.e., the electric current measured directly in the power network) or model estimations (i.e., the electric current determined from ground-based magnetic field data). Space weather events are mainly related to disturbances in the solar wind and its interaction with the Earth's magnetosphere, and the disturbance storm time index (*Dst*) is used to categorize a geomagnetic storm when it exceeds a given limit value. The physical environment involved with geomagnetic storms, mainly SW plasma and also the magnetospheric plasma, has often been described as a nonequilibrium system [e.g., *Burlaga and F.-Viñas*, 2004; *Burlaga et al.*, 2007; *Balasis et al.*, 2009; *Balasis and Eftaxias*, 2009; *Balasis et al.*, 2011; *Tsallis*, 2012; *Livadiotis and McComas*, 2013]. Accordingly, *Balasis and Eftaxias* [2009] and *Balasis et al.* [2008] show that the description of the Sun-Earth coupling by means of *Dst* time series can describe different magnetospheric states when considering the level of the complexity in the nonequilibrium Sun-Earth system during a geomagnetic storm. The question of which statistical distribution should be used to describe the amplitudes of GICs remains open.

We propose an approach to this problem in the context of nonequilibrium systems. The geoelectric field driving GICs is related to the geomagnetic field time derivative. Since the magnetic field is a continuous function in time, and thus also its derivative, one should expect that geoelectric field and GICs data set would be time correlated. Noting that the Boltzmann-Gibbs statistics are not able to describe nonrandom data, we suggest the application of nonextensive statistics, namely, Tsallis statistics [*Tsallis*, 1988, 1998], to describe the GICs time series during the intense geomagnetic storms which occurred in the solar cycle 23. More specifically, we apply a generalization of the Boltzmann-Gibbs (BG) power law distribution, namely, the q-exponential Tsallis frequency distribution function. We use goodness of fit as a technique to show that the q-exponential function can robustly represent the data sets. The GICs data sets used in this study are derived from both direct and indirect measurements and modeling from four power networks distributed in countries covering mid- to high latitudes (Brazil, South Africa, United Kingdom, and Finland).

In section 2.1 we review the extensive statistical distribution as presently applied to GICs data sets. In section 2.2 we introduce the nonextensive Tsallis statistics as may be applied to GICs. Section 3 presents the methodology applied to the investigation and also the description of the different GICs data sets. In section 4 we present the q-exponential Tsallis statistical results and discussion. Finally, in section 5, we summarize the main conclusions of this work.

2. Statistical Analysis of GICs

2.1. The Extensive Boltzmann-Gibbs Statistics Applied to GICs

The main concept of Boltzmann-Gibbs entropy is extensivity, i.e., the entropy of a system is proportional to its size or to the number of elements of the system. Thus, the entropy of a separated subsystem in thermodynamic equilibrium is lower than or equal to the final entropy of the complete system. This concept and its derived statistics can be applied to nearly uncorrelated systems. The distribution function derived from the maximization of entropy which ensures the equilibrium state for BG statistics, leads to exponential and Gaussian distributions. Therefore, these are the distribution functions for an extensive system.

By assuming that BG entropy is valid during active space weather scenarios, several authors have investigated the distribution of geoelectric field amplitudes. *Langlois et al.* [1996] analyzed 500 days of data from the Hydro-Quebec region in Canada and proposed a power law probability distribution function for the intensity of geomagnetic fields. *Pulkkinen et al.* [2008, 2012] used geomagnetic field records from 23 magnetometers at high latitudes and found that the lognormal distribution provided an approximation to the data. However, *Pulkkinen et al.* [2008] aimed to infer a statistical distribution for extreme values of GICs, but they noted that both the chi-squared and Shapiro-Wilk statistical tests showed that the lognormal distribution does not fit the data satisfactorily.

More recently, *Myllys et al.* [2014] used the International Monitor for Auroral Geomagnetic Effects (IMAGE) magnetometer array to model the distributions of the occurrence of the geoelectric field at a given location and found that it was necessary to adjust two different power law functions to reproduce the complete data set. Those previous works used direct and modeled geoelectric field measurements, obtained during considerable time intervals at different locations. Their main purpose was to obtain the statistical distribution function which best matched their data. However, those results show that statistical functions based on extensive entropy fail to determine a statistical distribution function capable of describing the complete GICs data sets, for long-term observations, regardless of where the infrastructure is installed.

Considering that all previous attempts postulated heavy-tailed distributions, we propose an alternative to circumvent this issue. There is strong evidence that the coupled system composed by the SW and the Earth's magnetosphere-ionosphere behaves as a low dimensional chaotic system. This has been evidenced by *Vassiliadis et al.* [1990] in the study of the *AE* index time series that led to a fractal dimension of 3.6; as a conclusion, the disturbed Sun-Earth system, such as observed during geomagnetic storms, can be considered in a nonequilibrium state. Moreover, several phenomena studied by *Karakatsanis et al.* [2013], *Pavlos* [2012], and *Pavlos et al.* [1999] support the hypothesis of the SW-magnetosphere-ionosphere as a nonlinear dissipative system leading to the development of anomalous diffusion processes in the magnetosphere-ionosphere electrical circuit during disturbed times. Those phenomena includes ion-cyclotron waves and strong topological distortions in the Earth's magnetic field, acceleration of energetic particles and their precipitation into the ionosphere, enhancement of ionospheric currents, among others. Accordingly, in this scenario we can consider GICs as a phenomenon close related with the nonlinear response of the magnetosphere-ionospheric current system; thus, an out-of-equilibrium statistical formalism may be used to describe the GICs amplitude distribution function. We suggest the introduction of a generalized exponential such as the Tsallis q-exponential [*Tsallis*, 1998, 2009] which takes into account the possible nonlinearities in the generation process of geoelectric fields (and subsequent GICs). The detailed understanding of the complete physical processes involved with the magnetospheric out-of-equilibrium system are beyond the scope of this study. As the q-exponential is part of the family of heavy-tailed distributions, it can approximate the lognormal or power law distributions of the extreme values (i.e., in the tail region) and it is also shown to fit the low-to-medium GICs amplitudes, which is not possible with the previously discussed distributions. In this sense, it is always possible to partially approximate a q-exponential by a power law or lognormal distribution.

2.2. Nonextensive Tsallis Statistics Applied to GICs

The physical systems in which long-range interactions or long-term memory effects are observed, such as the Sun-Earth system, can be considered as open systems (with exchange of particles and energy from one subsystem to the other). The generalized Tsallis statistical mechanics function has been proposed as a more convenient way to describe those systems trapped into nonequilibrium conditions which eventually reach quasi-stationary states [*Tsallis*, 1998, 2009; *Balasis et al.*, 2011; *Tsallis*, 2012; *Tirnakli and Borges*, 2016].

By definition, thermodynamic equilibrium implies thermal, mechanical, and chemical equilibrium. Since none of these conditions are satisfied for the coupled Sun-Earth system during geomagnetic storms, we consider the geoelectric field induced during storm times as a parameter of a system in a quasi-equilibrium state. The q-exponential distribution function, which are associated with such states, are derived from maximizing the nonadditive Tsallis entropy (S_q),

$$S_q = k\frac{1 - \sum_{q=1}^{W} p_i^q}{q - 1} \tag{1}$$

where k is Boltzmann's constant, W is the number of states in a system, p_i is the probability of the ith microscopic state, and q is the entropic index (note that BG entropy is the special case for $q \to 1$). Note, q is not an integer as might be implied by the equation (1). The Tsallis entropy differs from BG entropy by the nonadditive entropy. Systems that follow nonextensivity statistics are characterized by long-range interaction, memory effect and nonequilibrium thermodynamic [*Tsallis*, 2009]. The probability function resulting from the maximization of the Tsallis entropy is the q-exponential distribution (i.e., a generalization of the exponential distribution),

$$P(x) = \exp_q(-Bx) \tag{2}$$

where B is positive and the q-exponential can be written as:

$$\exp_q(-Bx) = [1 - (1 - q)Bx]^{\frac{1}{1-q}}. \tag{3}$$

B and q are the distribution parameters which characterize the analyzed data set. B has units of the inverse of the physical magnitude of the system, i.e., A^{-1}. As the B magnitude decreases, the probability of observing higher values of x increases. The q parameter measures the degree of nonextensivity of a given phenomenon (i.e., the level of the systems complexity). In this sense, as $q \to 1$ in equation (1), S_q tends to BG entropy. Additionally, the function can be normalized and then ascribed a physical meaning for values of $1 \leq q \leq 3$.

If the probability distribution of a given variable y is known, then the cumulative distribution function evaluated at x provides the probability of y having a value lower or equal to x. It can be demonstrated that the evaluation of a Tsallis cumulative distribution function results in another q-exponential function. Thus, we use this property to evaluate q and B in equation 3 in such a way that these parameters can give useful physical information about the system, once we fit the normalized inverse cumulative frequency (NICF) for the direct and modeled GICs obtained in each of the four power networks (see below). Past research, such as

Barbosa et al. [2015b], working on a Brazilian power network, showed that the *q*-exponential function fit the majority of GICs data (∼99%) measured during large geomagnetic storms ($Dst \leq -100$ nT) in the solar cycle 23. In this work, we investigate if similar nonextensivity of GICs distributions is also observed elsewhere in the world.

3. Methodology

We compute GICs in the high-voltage networks of Brazil, South Africa, United Kingdom, and Finland for the 90 largest geomagnetic storms ($Dst \leq -100$ nT) during solar cycle 23 (see list by *Pandey and Dubey* [2009]). Each of the data sets are arranged as the NICF of GICs and fitted to a Tsallis distribution with the fitting parameters determined by least squares inversion. A Lilliefors (L) statistical test is used to determine their significance. The L test is a modification of Kolmogorov-Smirnov (KS) statistical test applied when the fitted function parameters are derived from the data set [*Wilks*, 1995]. The test is based on the difference between the accumulated relative frequency, recorded in a sample of size *n*, and the values given by the investigated frequency distribution, in the present case, a Tsallis distribution. The maximum calculated difference (d_n) is then compared to the critical value (C_α) calculated for a confidence level of 95%. Thus, the result of the L test calculated in this manner gives true if $d_n \leq C_\alpha$; otherwise, it is false.

3.1. Description of the GICs Data Sets

Modeled GICs values from each country were obtained by considering all the large geomagnetic storms ($Dst \leq -100$ nT) which occurred during solar cycle 23 (1997–2007), comprising 90 storms as detailed in *Pandey and Dubey* [2009]. We assume the time series obtained from these data to be statistically representative of the distribution which describes the GICs intensity. In addition, we note the GICs data sets were obtained from different mathematical techniques for computing the geoelectric field, though all are based on the method of *Lehtinen and Pirjola* [1985] for the network modeling. For statistical purposes, we impose a minimum threshold on the computed GICs values arising from geomagnetic storms based on the specific characteristics observed in each country. Given these inputs, we assume that the data are sufficiently independent for the generality of the statistical distribution tests to be applicable. We briefly describe how the data sets from each country were obtained. Note that the Brazilian and South African data sets come from a single node, while the United Kingdom and Finnish data sets represent dozens of network nodes.

3.1.1. Brazilian Power Network

The modeled GICs from the Itumbiara substation (18.4°S, 49.1°W) in the 500 kV power network located in the central Brazil were used for this study. The induced geoelectric field was first calculated using the plane wave and *Lehtinen and Pirjola* [1985] methods, from minute mean values of the horizontal components of the geomagnetic field measured at Vassouras observatory, (22.4°S, 43.6°W), the closest magnetic observatory to the power network. The ground conductivity model used consists of four layers obtained from magnetotelluric surveys by *Bologna et al.* [2001]. The power network topology was changed during the solar cycle 23, and the modeled GICs take this into account. For more details see *Barbosa et al.* [2015a] and *Barbosa et al.* [2015b].

3.1.2. South African Power Network

The local plane wave method was applied using a ground conductivity model derived by *Ngwira et al.* [2008] to determine the geoelectric field at the Grassridge substation (33.7°S, 25.6°E). South African minute mean geomagnetic data were obtained from the Hermanus observatory, (34.4°S, 19.2°E). A major portion of the South African high-voltage power transmission system runs roughly from southwest in Cape Town to northeast in Johannesburg. The network has several interconnecting lines which branch off to other major economic zones. The Grassridge substation, which is about 800 km east of Cape Town, lies at the end of one of the interconnecting lines running approximately north-south [see *Ngwira et al.*, 2011, Figure 1]. To compute the GICs at the Grassridge substation, we used the network coefficients introduced by *Ngwira et al.* [2008]. Although power networks are constantly evolving, a large part of the South African bulk power transmission system has not changed significantly over the last decade.

3.1.3. United Kingdom Power Network

Values of the geoelectric field in the United Kingdom were computed using the geomagnetic minute mean data for the three days surrounding the peak of each of the 90 largest storms for solar cycle 23. The thin-sheet method of *Vasseur and Weidelt* [1977] was used to calculate the geoelectric field. The thin-sheet model is driven by magnetic field variation as measured at the three United Kingdom observatories (Lerwick, Eskdalemuir, and Hartland) interacting with a simple 2-D conductance model of the land and seas and a

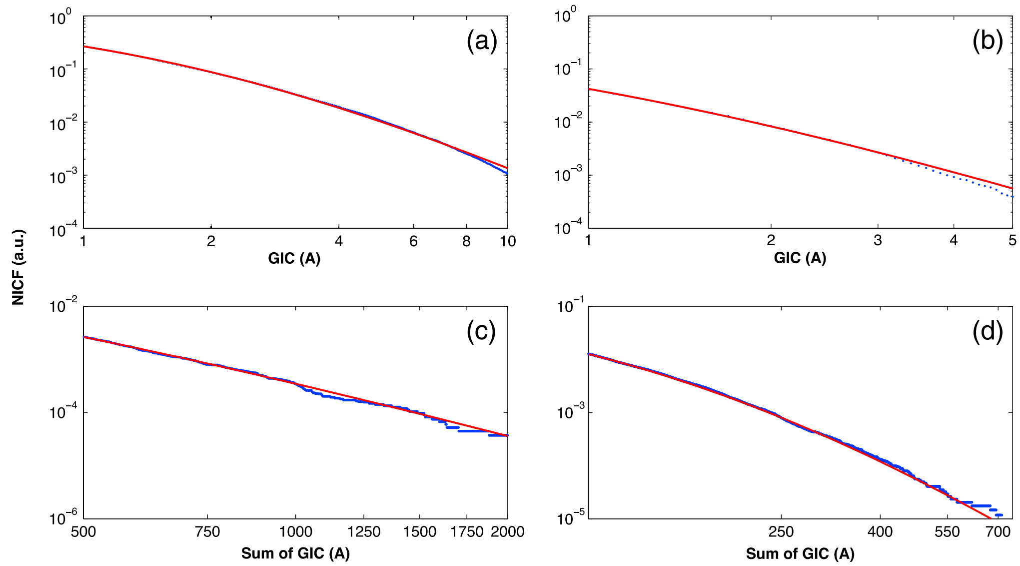

Figure 1. Normalized inverse cumulative frequency (NICF) distribution of the GICs calculated for the magnetic storms during solar cycle 23. (a) Fitted parameters for GICs in Brazil are between 1 and 10 A ($q=1.27$, $C=1.33$, $B=2.01$ A^{-1}). (b) Fitted parameters for GICs in South Africa are between 1 and 5 A ($q=1.27$, $C=0.83$, $B=4.57$ A^{-1}). (c) Fitted parameters for sum of GICs in United Kingdom are between 480 and 2.000 A ($q=1.27$, $C=0.33$, $B=0.02$ A^{-1}). (d) Fitted parameters for sum of GICs in Finland are between 85 and 974 A ($q=1.16$; $C=0.22$, $B=0.04$ A^{-1}).

1-D model of conductivity at depth [see *Beggan et al.*, 2013]. The high-voltage network consists of 426 nodes representing the 400 kV and 275 kV lines. The resulting GICs at each node were computed by the method of *Lehtinen and Pirjola* [1985]. The sum of the absolute GICs in the all 426 nodes of the power network at each minute is used as a proxy for the entire grid.

3.1.4. Finnish Power Network

We modeled GICs in the Finnish 400 kV power network using parameters that were valid for the grid in 1978–1979. Although the grid has changed since that time, this model provides a quantitative and comparable measure to assess long-term GICs activity. We used as input the geomagnetic recordings in 1997–2007 from the IMAGE magnetometer network, World Data Centre for Geomagnetism (Edinburgh), and International Real-time Magnetic Observatory Network (INTERMAGNET). The geoelectric field was calculated using the conductivity model proposed by *Viljanen et al.* [2014] and the plane wave method. The method proposed by *Lehtinen and Pirjola* [1985] was used to calculate GICs. For details of the modeling method and the grid, see *Viljanen et al.* [2012]. The quantity used in this paper is the sum of absolute GICs values as minute mean at all 17 nodes of the power network.

4. Results and Discussion

4.1. Fitting the *q*-Exponential Tsallis Distribution

The NICF was calculated for the GICs data from 90 storms when $Dst < -100$ nT during solar cycle 23, for each country (Figure 1). In Brazil and South Africa, GICs data were computed for a single node (Figures 1a and 1b). The limited GICs data sets for these two countries are due to the difficulty of obtaining information about the power networks. In Finland and United Kingdom, the sum of GICs is used as proxy for the entire networks. Figure 1 shows the NICF for the GICs intensity and the q-exponential statistical function that best fits each data set. The GICs data sets were processed taking into account a valid range of statistical representation, i.e., we establish a threshold to the GICs value per location for which the intensity measured could be associated to geomagnetic storms. In this way, it was chosen as 1 A per substation for Brazil, South Africa, and United Kingdom and 5 A for Finland. The fitted results show the q-exponential Tsallis functions are able to fit more than 99% of the available data, regardless the power network location considered.

Table 1 summarizes the best fit function parameters for each data set. The fits were calculated independently for each data set and it was found that $q\approx1.3$ for all locations. As the q-exponential is able to describe the probability distribution of these data sets, we suggest that the GICs generation mechanism arises from physical processes which are properly described by nonextensive statistics. Hence, as a consequence, these

Table 1. Fitting Parameters for Each Studied Country[a]

Country	Latitude Range (deg)	q	B (A^{-1})	R	Sample Size	C_α ($\alpha = 0.1$)	d_n Values
Brazil	−18 to −20	1.27	2.01	0.999	40,000	6.1×10^{-3}	2.2×10^{-3}
South Africa	−22 to −34	1.27	4.57	0.999	40,000	6.1×10^{-3}	3.2×10^{-3}
United Kingdom	50 to 60	1.27	0.02	0.999	40,000	6.1×10^{-3}	1.8×10^{-3}
Finland	55 to 65	1.16	0.04	0.999	40,000	6.1×10^{-3}	2.2×10^{-3}

[a]Geographic location, parameters (q and B) of the Tsallis distribution function, correlation coefficient (R), amount of data used in the selected regions, and parameters (C_α and d_n) from L test.

mechanisms are associated with complex physical phenomena responsible for generating GICs at the respective latitudes. The q-parameter is a proxy for the systems complexity. Besides the lack of a complete understanding of the physical processes leading to nonextensive GICs amplitudes, we interpret that the similarity of the q-parameter values suggest that GICs observed at each location might indicate a similar degree of correlation between the nonlinear physical processes occurring in the Sun-Earth coupled system which generate GICs.

Tsallis distribution parameter B can be interpreted as a proxy for the intensity associated to GICs in different locations. The small-scale spatial phenomenon such as the local ionospheric current systems (e.g., auroral electrojets) and ground conductivity have a major contribution in magnifying local magnetic field and thus geolectric field, which results in more intense GICs variations within a power network. In all four investigated locations, the GICs intensity follow a q-exponential Tsallis statistical distribution with 95% of confidence level using the L test, with $d_n \leq \chi_n$.

The aforementioned memory effect means that data representable by a Tsallis statistical distribution exhibit a temporal correlation. Accordingly, GICs measurements depend on past disturbances in the geomagnetic field as modeled by first principles and thus shows that data are temporally correlated. The present statistics were derived from minute mean data which introduce an inherent smoothing effect on the GICs signal and hence in the statistics. It would be desirable to use higher temporal resolution data in order to pick the short period variations to improve the statistics. However, the present statistical analysis can be useful for estimating the probability of significant GICs events within a given period of time.

4.2. Application of *q*-Exponential Function to Extreme GICs Values

Since the q-exponential distribution function has been shown to be suitable for describing GICs intensity globally, we calculate the likely number of occurrences of the maxima and extreme GICs values over 10 solar cycles for Brazil and South Africa and, likewise for the sum of GICs in the case of United Kingdom and Finland power networks. The extreme GICs values are chosen for all locations as twice that of the maximum GICs modeled in those countries. Table 2 shows the four q-exponential functions parameters calculated for each country. To obtain the number of likely events through 10 solar cycles, we consider the sample size (i.e., 40,000 events) to be the total number of events for one solar cycle. We assume that on average, the GICs values measured in each case should be similar for the period of 10 solar cycles (i.e., approximately 100 years). Our results suggest that the largest GICs values measured at the low-latitude node are likely to be observed several times over a 10 solar cycle period, while a GICs value twice this size is rare. For the sum of GICs over several power grid nodes (i.e., 17 nodes for Finland and 426 nodes for United Kingdom) GICs values twice the size of the recorded maximum are likely to be observed more than once in a period of 10 solar cycles.

Table 2. Number of Likely Events (NLE) Calculated Over 10 Solar Cycles for the Maximum of (Sum Of) GICs Observed in One Solar Cycle and Extreme GICs (eGICs)

Country	Max GICs	Max Sum of GICs (A)	NLE	eGICs	Sum of eGICs (A)	NLE
Brazil	30	−−	13	60	−−	1.0
South Africa	15	−−	5 − 6	30	−−	< 1
United Kingdom	−−	2000	14 − 15	−−	4000	1 − 2
Finland	−−	1000	15 − 16	−−	2000	1 − 2

5. Conclusions

We have analyzed the global behavior of the nonextensivity of GICs distributions using geomagnetic field data measured during solar cycle 23. The GICs arising in high-voltage power networks were examined for Brazil, South Africa, United Kingdom, and Finland for the 90 largest geomagnetic storms ($Dst \leq 100$ nT) which occurred between 1997 and 2007. In all cases, geomagnetic data of minute mean cadence were used to compute GICs values. The data sets were processed in order to produce a statistically representative sample, and minimum thresholds were established to provide a valid GICs intensity range. We applied the q-exponential Tsallis function to fit the processed data set for each country. The goodness of fit was tested by the correlation coefficient, which presented values above 0.999 for the four different locations. Finally, the q-exponential statistical distribution was tested by using the L test and the result was considered significant for all different countries.

We computed the likely number of maximum and extreme events assuming that on average, GICs and sum of GICs will have a similar distribution function for the next 10 solar cycles. We found that sum of GICs for extreme events are likely to be observed at least once over a 10 solar cycles time period for high-latitude power networks. The modeled extreme GICs for midlatitudes are likely to be observed about once in Brazil, over the same period. The results of these analyses show that the nonextensive statistics are a general characteristic of GICs, suggesting that the ground current intensity has a strong temporal correlation and a long-range interaction, such as with the SW-magnetosphere-ionosphere coupling. However, the detailed understanding of the physical processes leading to the observed q-exponential distribution functions is beyond the scope of this study and needs to be considered more detailed in the future. The distribution of GICs intensity has non-Gaussian behavior, which has implications for other geophysical effects. We propose that the use of Tsallis statistics may provide new tools for geophysical research and space weather monitoring and forecasting.

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