Institute of Hydrology

Working Note No.6

The timing of low flow events

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Executive Summary

For the purpose of the GREAT-ER project the Institute of Hydrology is required to produce flow estimates with the necessary confidence for use by non hydrologists. To improve the confidence of flow estimates the necessity to incorporate both the dependency of the seasonality of low flows on the hydrogeology and rainfall, and the available local hydrometric data was recognised.

The assumption that flows are temporally coherent over all parts of the upstream network underlies the existing low flow estimation techniques. This is unlikely to be the case in large catchments given the diversity of geology and distribution of rainfall across such catchments. The implications of this assumption for the GREAT-ER project would be an underestimation of dilution at the catchment scale. Previous work, undertaken by the Institute of Hydrology, has demonstrated that differences in the timing of low flows can be identified at a monthly resolution.

As GREAT-ER is focussing on the daily variation in flows, Working Note No. 6 aims to determine whether the timing of the Q95 flow differs in geologically and climatically heterogeneous catchments at a daily resolution, and to quantify the effect of the differences in the timing on Q95 estimates.

A number of analyses were undertaken using case study catchments and UK wide catchments. The use of case study catchments enabled a preliminary investigation of the dependency of the timing of the Q95 on climate and hydrogeology. A larger set of UK wide catchments were used to investigate the regional variations in the timing of low flows and to quantify the Q95 under different assumptions of the timing of low flows.

The analyses identified differences in the day of occurrence of the Q95 flow, which were shown to be a function of variations in climate and geology. However, when considering the inter-annual variability in the timing of the Q95 flow, these differences were not statistically significant in most nearest neighbour catchments.

The Q95 estimates, derived under the assumption of temporal coherence, are mostly smaller than those estimates derived under the assumption that the low flows occur at the time of one of the upstream catchments. 96% of the estimates derived under the assumption of temporal coherence are within a factor of three of the estimates derived under the assumption that the low flows occur at the time of one of the upstream catchments. Those differences which are larger than a factor of three are associated with wetter and more impermeable catchments. This is due to the large variability that is observed in the daily flows over a small period of time.

The recommendation of the study is that, since the timing is not significantly different at a daily resolution in these wet and impermeable catchments, it is not practicable to incorporate the effect that the flow variability exerts on the estimated Q95. However, the timing of the Q95 at a monthly resolution, which has previously been found to be significant, should be incorporated into the hydrological model.

Contents

1. Introduction

In the GREAT-ER proposal, it was recognised that to produce flow estimates with the necessary level of confidence for use by non hydrologists in water quality modelling activities, it would be necessary to incorporate into the existing low flow estimation methods:

1. the dependence of the seasonality of low flows on hydrogeology and rainfall;
2. the available local hydrometric data.

In estimating low flow statistics at a site, the existing model (Gustard *et al*, 1992) contains the implicit assumption that the temporal distribution of flows is coherent over all parts of the upstream network. This implies spatial and temporal coherence of rainfall, evaporation and, most importantly, catchment response (principally hydrogeological response in the case of UK low flows). Whilst these conditions may potentially be met in small catchments it is unlikely to be the case in larger catchments. For example; within the UK Thames basin (9948 km^2), given the diversity of geology and the temporal variations in the phasing and magnitude of rainfall distributions across the basin, it is highly improbable that all gauging stations within the basin will measure flows corresponding to the same percentile exceedance at the same instant in time. Even assuming that rainfall is temporally coherent across the catchment the permeable catchments within the basin will experience their low flows later in the year than the impermeable catchments.

Within the GREAT-ER project, the implications for the water quality modelling would potentially be an underestimation of dilution at the catchment scale due to the variations in the timing of the Q95 within the river network. Therefore, it is essential that the seasonality of low flows and their dependence on geology and the seasonal nature of rainfall is accounted for in the estimation of the flow regime.

It has previously been demonstrated (Bullock *et al*, 1994) that on a monthly time scale variations in the timing of low flows occurs between different hydrogeological units. The monthly time resolution, which was used in the study, is a useful management unit but is not a hydrologically significant one. Therefore, three principal objectives of the study, discussed in this Working Note, were to:

1. Determine whether the timing of the Q95 flow differs significantly in geologically and climatically heterogeneous catchments at a daily resolution, as the fate modelling activities are being undertaken at this resolution.

2. Quantify the effect of the differences in the timing of low flows on the precision of the estimated Q95 flows.

3. Provide recommendations of the necessity for and/or feasibility of incorporating the probability of occurrence of low flow events into the development and application of regional multivariate hydrological modelling.

In order to meet these objectives three analyses were undertaken:

1. A small number of case study catchments were used in a preliminary investigation of the dependency of the timing of low flows on climate and hydrogeology. This analysis is presented in Chapter 2.

2. A larger set of catchments were used to build on the preliminary investigation to investigate the regional variations in the timing of low flows. Together, analyses one and two enabled the requirements of objective one to be met. This analysis is presented in Chapter 3.

3. The larger set of catchments were used to quantify the Q95 under different assumptions of the timing of low flows and thus determine the precision of the estimated Q95 flows derived using the current assumption of temporal coherence. This third analysis enabled the requirements of objective two to be met. This analysis is presented in Chapter 4.

2. Preliminary investigation of the timing of low flows

2.1 INTRODUCTION

Seven case study catchments were selected so that a preliminary investigation of the variations in the timing of low flows could be made. The selected case study catchments, given in Table 2.1, have the following properties:

(i) they are of good hydrometric quality and are relatively natural;

(ii) they have 20 years of coherent period of record (1976 to 1995), with less than 31 days missing for any one month throughout the period of record;

(iii) The catchments are hydrogeologically homogenous, defined as 75% of the catchment area comprising of two or less similar hydrological response classes (based on a hydrological classification of soils). The range of hydrological response is typical of that for the UK;

(iv) The catchments have either low (<850mm) or high (>1000mm) Standard period (1941 - 70) Average Annual Rainfall (SAAR).

Table 2.1 *Hydrogeologically representative catchments*

Station number	LFG	Geology	SAAR (41-70) (mm)	Area (km^2)	BFI
39019	AB	Chalk	737	234.1	0.97
39043	BA	Chalk	800	295	0.95
44006	AA	Chalk	1098	12.4	0.87
55029	AA	Old Red Sandstone	1001	354	0.59
41025	AA	Weald Clay	806	91.6	0.23
39054	AA	Weald clay	827	31.8	0.24
55026	AA	Metamorphised Silurian sediments	1618	174	0.36

The catchments are described in detail below.

Stations 39019, Lambourn at Shaw, 39043, Kennet at Knighton, and 39054, Mole at Gatwick Airport, are all located in the Thames catchment of southern England. The flows at station 39019 are occasionally influenced by a downstream sluice and also by the West Berkshire Groundwater Scheme which has occasionally provided low flow support to the Lambourn. Artificial influence is otherwise limited in this rural catchment and the flow pattern is baseflow dominated. The flow regime at station 39043 is also base flow dominated although flows are slightly diminished by groundwater abstraction. Some of the pre 1980 flows are uncorrected and there is some bypassing during floods. The catchment is mainly rural but there is some urban growth in the valley. There is very little disturbance to the responsive, natural flow regime at station 39054. Gatwick Airport is not in the catchment, thus the catchment is predominantly rural with only a small export of water.

Stations 55029, Monnow at Grosmont, and 55026, Wye at Ddol Farm, are situated in south Wales. Station 55029 is a natural catchment draining the relatively permeable Old red Sandstone of the Black Mountains. Station 55026 is characteristic of a wet upland impermeable catchment. The flow regime is unaffected by the large water supply reservoir which flows from the Elan valley.

Station 41025, Loxwood Stream at Drungewick, is an impervious clay catchment located in the south-east of England. The abstractions and discharges have a negligible impact on overall runoff, but there is occasional anomalous behaviour at low flows.

Station 44006, Sydling Water at Sydling St. Nicholas, is a predominantly rural, Lower Chalk catchment situated in the south west of England. The station is modular under all flow conditions.

The flow duration curve (FDC) graphically represents the relationship between any given discharge and the percentage of time that the discharge is exceeded. The FDC reflects the geology of the catchment. For example, a permeable catchment has a characteristically low gradient FDC, reflecting the relatively low flow variability of flows about the median flow. Conversely, an impermeable catchment has a characteristically steep FDC, indicative of a flashy flow regime and a high variance of daily flows. The standardisation of discharge by mean flow facilitates comparisons between catchments by reducing the differences in the location of the flow duration curve which are caused by differences in the mean annual runoff.

Figure 2.1 shows the annual FDCs associated with each of the case study catchments. The gradients of the FDCs of the catchments generally increase with increasing impermeability, indicating that greater variability exists in the daily flows in impermeable catchments, as would be expected. The FDCs are typical of the type of catchments that they are assumed to represent.

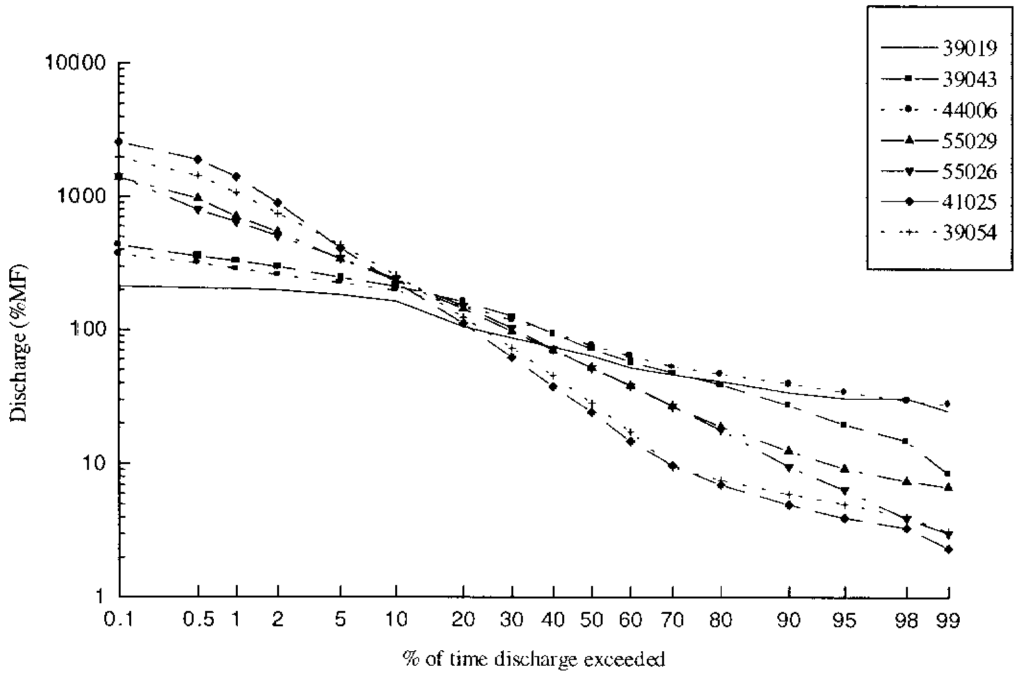

Figure 2.1 *Annual flow duration curves*

An investigation of the timing of low flows was made using these case study catchments. Since the objective of this study was to determine the effect of different permeability and/or rainfall in upstream catchments on the timing of the downstream Q95, hypothetical flow time series were derived by combining the time series of paired catchments with differing hydrogeological and/or rainfall characteristics. Linear combinations of the flow time series were made in the ratios of 100%:0%, 75%:25%, 50%:50%, 25%:75%, and 0%:100% to represent different stages in the transition of a hypothetical catchment with the characteristics of the first catchment in the pair to one with the characteristics of the second.

The paired catchments are given in Table 2.2. Pairs one and two were intended to act as control catchments. Only minimal changes in the timing of the Q95 flow would be expected as the catchment progressed from 100% of catchment one to 100% of catchment two, as in each case the pairs are both climatically and hydrogeologically similar. The stations in pairs three and four were used to investigate the effect of an decreasingly permeable catchment, in areas of low and high rainfall respectively. Pairs five and six were used to identify the effect of increasing rainfall in permeable and impermeable catchments respectively.

Table 2.2 *Paired catchments*

Pair	Station 1	Station 2	Investigation
1	39043	39019	Two dry permeable catchments
2	39054	41025	Two dry impermeable catchments
3	39043	41025	Decreasing permeability in catchments with low rainfall
4	55029	55026	Decreasing permeability in catchments with high rainfall
5	39043	44006	Increasing rainfall in permeable catchments
6	39054	55026	Increasing rainfall in impermeable catchments

The time of occurrence of the Q95 and standard deviation were derived for each of the time series associated with the pairs. These were plotted and analysed graphically to determine the effect of decreasing permeability or increasing wetness. The mean day of occurrence of the two paired catchments was statistically analysed to determine whether on average the Q95 occurred on significantly different days. Details of this analysis are given in Section 2.2.

2.2 ANALYSIS OF THE TIMING OF THE Q95 IN THE CASE STUDY CATCHMENTS

For every time series associated with each of the paired catchments the day of the annual Q95, i.e. the day on which the 18th lowest flow occurred, was extracted for each year over the period of record. Calculating the mean day of the annual Q95 over the period of record presents the problem that the 31st of December and the 1st of January have adjacent values in the time series but will not be considered as such if day numbers are used (Bayliss, A. & Jones, R.).

For example, if the Q95 occurs on days 20, 258 and 293 in three consecutive years then the arithmetic mean of the day of occurrence is 190, thus implying that on average the Q95 occurs in the middle of the year. This is clearly a mis-representation of when the Q95 actually occurs. To avoid this problem the following approach was adopted (Mardia, 1972) to calculate the mean day of occurrence of the

Q95 and the corresponding standard deviation.

Mean Q95 day of occurrence

The day number is expressed as an angle, where:

$$\theta_i = \left(\text{day of occurrence.} \frac{2\pi}{\text{LENYR}} \right) - \text{ADJUST} \tag{2.1}$$

where:

$$\text{ADJUST} = \frac{1}{2}\left(\frac{2\pi}{365} \right)$$

LENYR is 365, or 366 for a leap year.

The x-axis is chosen as an arbitrary starting point and all angles are calculated in an anti-clockwise direction from this point (Figure 2.2). The occurrence of the Q95 values are assigned to the day on which they occur, but in reality the flow is the mean of the flows logged at various times within that day. Therefore, an adjustment (ADJUST) is made to each value so that the Q95 is represented by the angle occurring at its mid point.

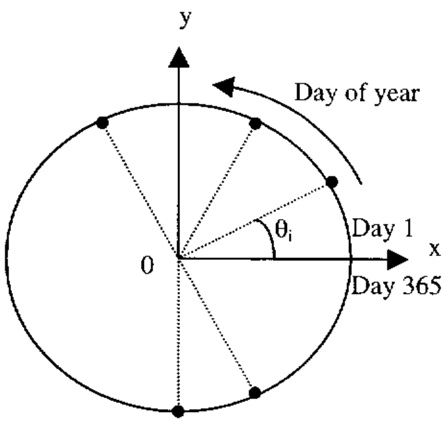

Figure 2.2 *Calculating the mean Q95 day using directional statistics (Source: Bayliss and Jones, 1993)*

Following the approach adopted by Mardia (1972), the mean Q95 day is obtained by representing all the days as weights of unit mass, sited on the circumference of a circle of unit radius, and then calculating the centroid of these weights.

Thus the mean of the x-co-ordinates and the mean of the y-coordinates are calculated:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} \cos\theta_i \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} \sin\theta_i \tag{2.2}$$

Thus, the mean direction is given by:

$$\bar{\theta} = \tan^{-1}\left(\frac{\bar{y}}{\bar{x}}\right) \tag{2.3}$$

If $\bar{\theta}$ is negative then 2π should be added to $\bar{\theta}$. Finally, the mean direction is converted back to a day number using:

$$MQ95D = \left(\bar{\theta}.\frac{365}{2}\right) + 0.5 \tag{2.4}$$

A half is added to the MQ95D to compensate for the earlier adjustment (ADJUST) during the conversion of day numbers to angles. The value is rounded to the nearest day.

Standard deviation of the mean Q95 day of occurrence

The 'mean resultant' $\bar{r}$ gives some indication of the spread of data:

$$\bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2} \tag{2.5}$$

If $\bar{r}$ is close to unity then the data displays a strong direction, which is likely to indicate strong seasonality in the case of the day of occurrence of the Q95 flow. If $\bar{r}$ is close to zero then the data are not strongly seasonal and the value of MQ95D is less meaningful.

The standard deviation, S_o, of circular data is defined by Mardia (1972) as:

$$S_o = \sqrt{-2\ln\bar{r}} \tag{2.6}$$

This standard deviation, in radians, can be converted to a standard deviation in days about the mean Q95 day, MQ95D, by:

$$SDQ95D = S_o.\left(\frac{365}{2\pi}\right) \tag{2.7}$$

This value is rounded to the nearest day.

The resultant mean day and standard deviation were plotted as a function of the degree of permeability or wetness of a catchment and analysed graphically to determine the effect of decreasing permeability or increasing wetness.

A two sample T-test was used to determine whether on average the Q95 occurred on significantly different days in the paired catchments, listed in Table 2.2. The assumption is made that the two independent samples originate from Normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$. Therefore, the hypothesis that the two populations have the same mean was put forward. The null hypothesis $H_0{:}\mu_1=\mu_2$ was tested, i.e. the mean day of occurrence of the Q95 is equal in the two catchments associated with each pair. Since the population variances were unknown there was a choice of two test statistics, based on the equality of the variances of the two sample populations.

Significance test statistics

The first, given in equation 2.8, is used when the population variances are unknown but equal.

$$T_1 = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \tag{2.8}$$

Where: $$s_p^2 = \frac{(n-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$ i.e. the pooled variance

$\bar{x}_1$ and $\bar{x}_2$	are the two sample means
$\bar{s}_1^2$ and $\bar{s}_2^2$	are the two sample variances
n_1 and n_2	are the sample sizes

The second, given in equation 2.9, is used when the variances are unequal and known.

$$T_2 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \tag{2.9}$$

To determine the equality of the variances of two populations, and thus enable the correct choice of test statistic, an F-test was undertaken. The assumption was made that the two independent samples originate from Normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, and the population mean and variances were unknown. Thus to test the null hypothesis that H_0: $\sigma_1^2 \neq \sigma_2^2$ the value of $F=s_1^2/s_2^2$ was compared to the tabulated value of the F-distribution with n_1-1 and n_2-1 degrees of freedom. Since the F-distribution is tabulated for the upper tail only, for a 5% significance test the larger variance based on n_1 observations was used as the numerator. The observed F value was then compared with the tabulated F value at the 2.5% point. If $F \leq F_{\alpha/2}\,(n_1 - 1, n_2 - 1)$ then it was assumed that the variances were not significantly different.

The sample number for all stations was 20, except for station 41025 where n = 17 and station 39054 where n = 19. Therefore, depending on the sample sizes, the critical values for comparing the variances of the catchments in each pair were:

$F_{19,19}(2.5\%) = 2.58$
$F_{18,19}(2.5\%) = 2.60$
$F_{19,16}(2.5\%) = 2.75$

2.3 RESULTS AND DISCUSSION OF THE CASE STUDY INVESTIGATIONS

For each of the pairs of catchments the mean day of occurrence and the corresponding standard deviation were plotted for each of the linear combinations representing the transition of a catchment with the response of the first catchment in the pair to that of the second. Thus, Figures 2.3a to f are graphical representations of the mean day of occurrence of the Q95 and corresponding standard deviation as a function of catchment geology or rainfall.

T-tests were also undertaken to identify whether the mean day of occurrence of the of the Q95 was different for catchments one and two in each pair. The critical values for comparing the means of the catchments in each pair, depending on the sample size of each station within a pair, are:

$t_{35}(5\%) = 2.032$
$t_{37}(5\%) = 2.027$
$t_{38}(5\%) = 2.025$

The t-statistics are given in Table 2.3.

Table 2.3 *t-statistics for differences between mean day of occurrence*

Pair	Stations	t-statistics
1	39043, 39019	0.15
2	39054, 41025	0.09
3	39043, 41025	2.78
4	55029, 55026	3.94
5	39043, 44006	1.96
6	39054, 55026	3.51

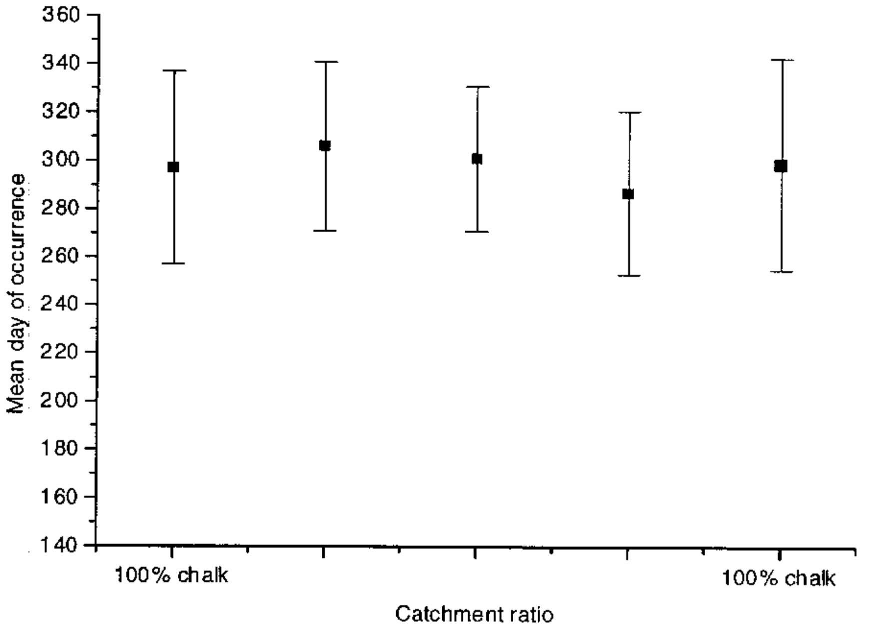

Figure 2.3a **Mean day of occurrence of Q95 ± 1 SD for catchments with high permeability and low rainfall (Pair 1)**

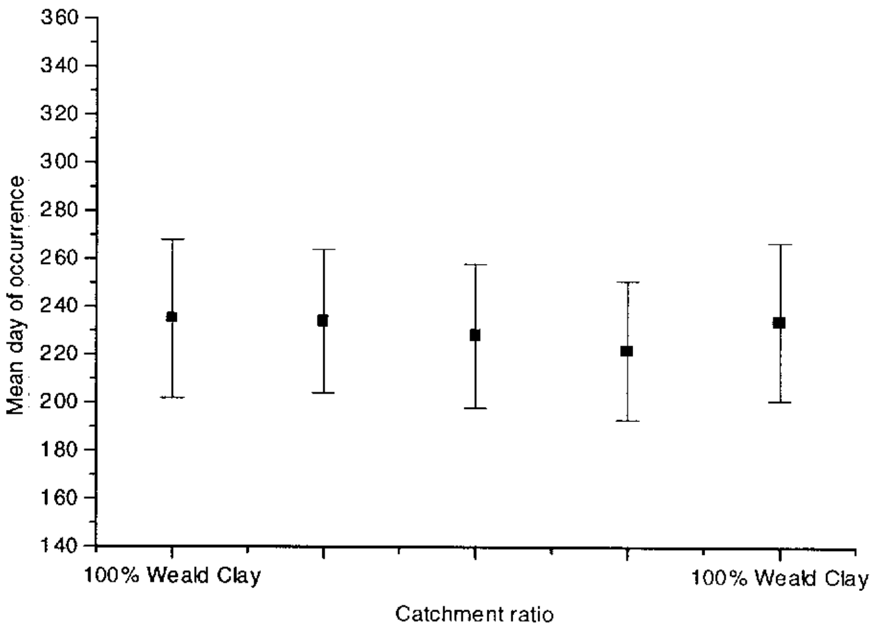

Figure 2.3b **Mean day of occurrence of Q95 ± 1 SD for catchments with low permeability and low rainfall (Pair 2)**

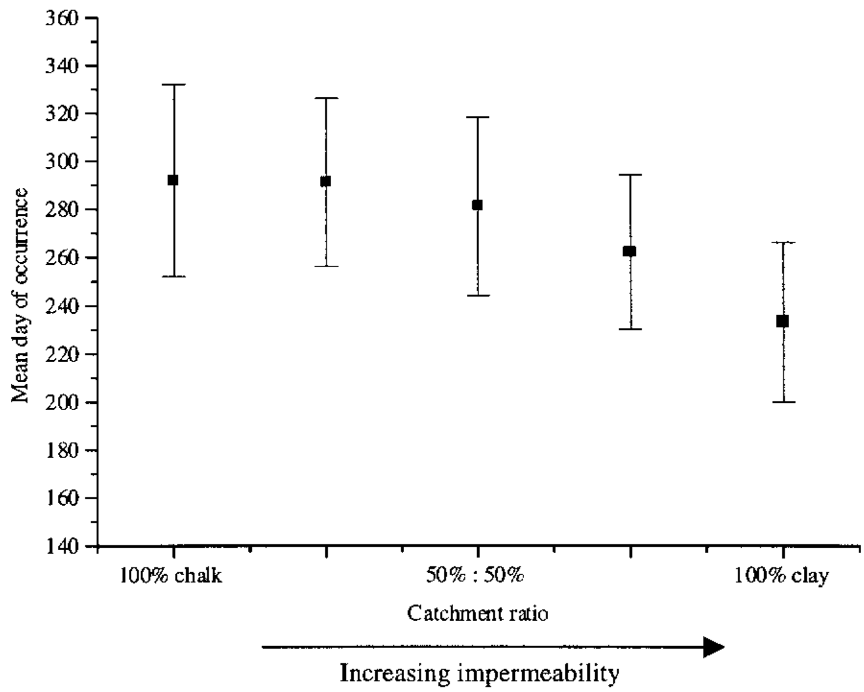

Figure 2.3c **Mean day of occurrence of Q95 ± 1 SD for catchments with low rainfall and decreasing permeability (Pair 3)**

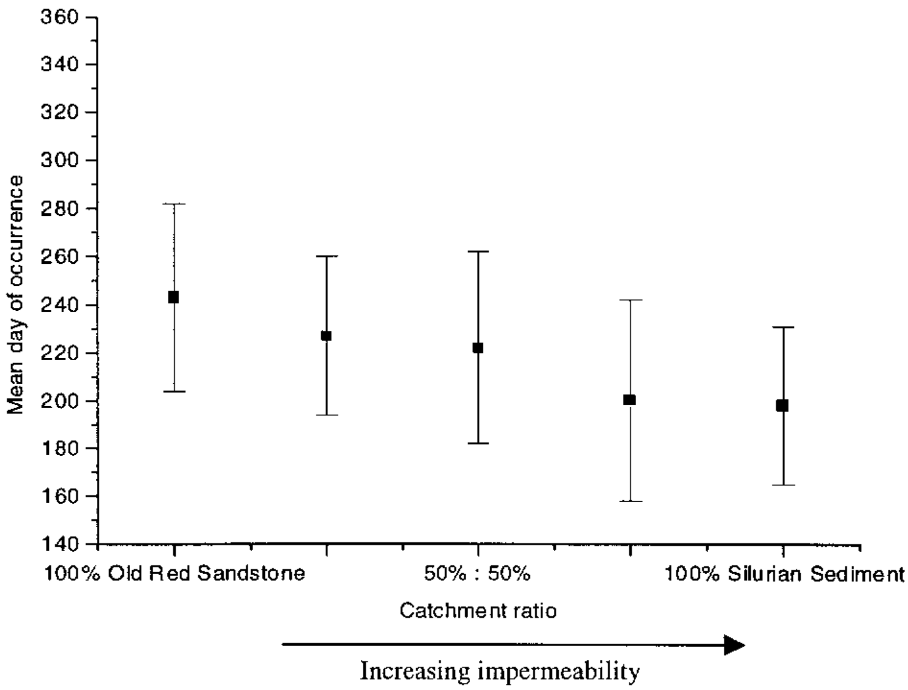

Figure 2.3d **Mean day of occurrence of Q95 ± 1 SD for catchments with high rainfall and decreasing permeability (Pair 4)**

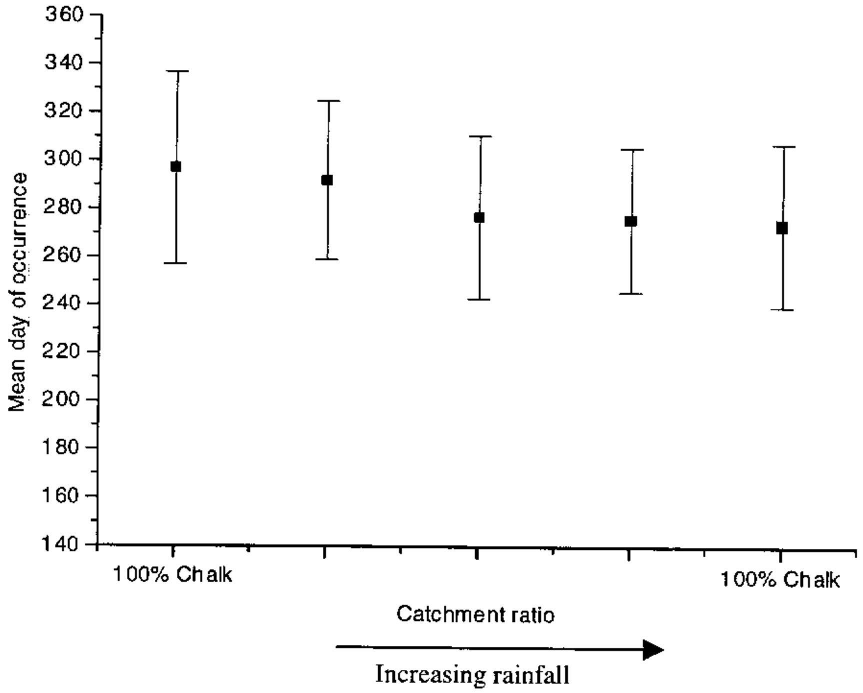

Figure 2.3e **Mean day of occurrence of Q95 ± 1 SD for catchments with high permeability and increasing rainfall (Pair 5)**

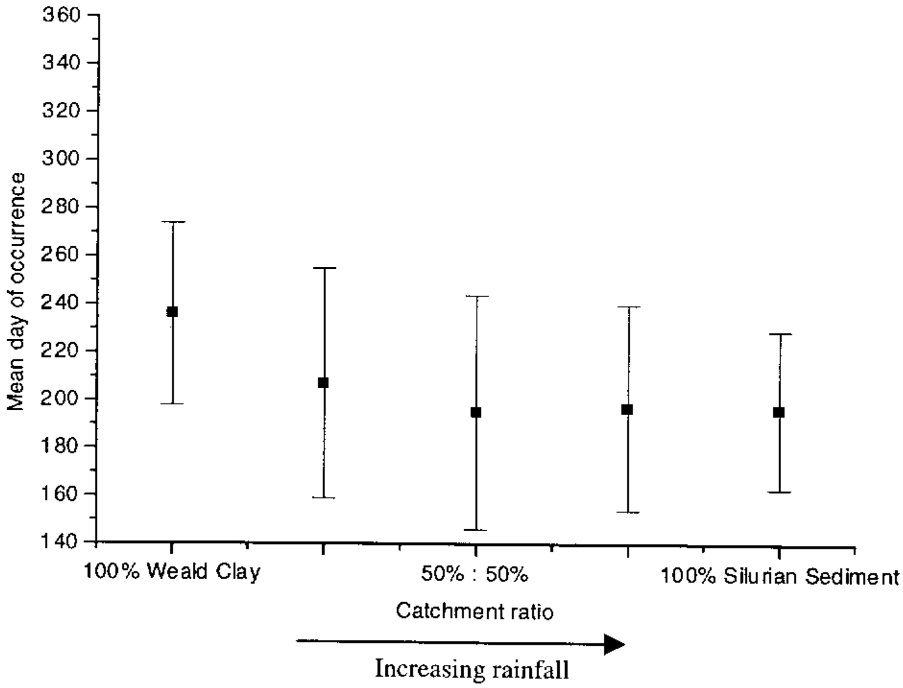

Figure 2.3f **Mean day of occurrence of Q95 ± 1 SD for catchments with low permeability and increasing rainfall (Pair 6)**

1. Control catchments

Figures 2.3a and b show the mean day of occurrence of the Q95 and the corresponding standard deviations for the dry chalk and dry clay catchments. There appears to be very little difference in the timing, as expected in catchments with similar characteristics. The t-statistics of 0.15 and 0.09 confirmed that the mean day of occurrence associated with the two pairs of catchments are not significantly different.

2. Decreasing permeability in wet and dry catchments

Dry

Figure 2.3c shows that the Q95 occurs progressively earlier as the permeability decreases in the dry catchment pairing. The rate at which the mean day becomes earlier is slower than the rate of increase in percentage area of clay. This is due to the buffering effect that the groundwater discharge has on the river flows. This even occurs when only a small percentage of the contributing catchment consists of a permeable geology. It should be noted that the confidence intervals around the mean day overlap at either extreme. The t-statistic of 2.78 indicated that the mean day in the 100% chalk catchment is significantly different to the mean day in the 100% clay catchment.

Wet

A similar pattern is evident in Figure 2.3d as permeability decreases in wet catchments. However, the rate at which the mean day becomes earlier is more constant than that observed in the dry catchments. This may be attributable to the different characteristics of the Chalk and Old Red Sandstone catchments used in pairs three and four to represent permeable catchments. Whilst both are associated with large storage capacities, the Old Red Sandstone has a higher transmissivity and is consequently more responsive to rainfall. Therefore, the groundwater buffering effect may be less important due to the flashier response of the Old Red Sandstone catchment and the greater contribution to flow that rainfall has in wetter catchments. The t-statistic of 3.94 showed that the mean day in the 100% Old Red Sandstone catchment is significantly different to the mean day in the 100% Silurian Sediment catchment.

3. Increasing wetness in permeable and impermeable catchments

Permeable

Figure 2.3e shows that the Q95 occurs earlier as the catchment becomes progressively wetter. Is likely to be attributable to the complex hydrogeology of the chalk and the fact that the two catchments are associated with different chalk outcrops, which are known to behave differently (Bullock *et al*, 1994).The t-statistic of 1.96 showed that the mean day in the dry chalk catchment is not significantly different to the mean day in the wet chalk catchment. This suggests that rainfall may not significantly influence the timing of the Q95.

Impermeable

Figure 2.3f demonstrates that a similar effect occurs in the impermeable catchments as they become wetter. However, the effect is more noticeable perhaps due to the greater influence that rainfall has on river flows in impermeable catchments. The t-statistic of 3.51 indicated that the mean day in the dry impermeable cathment is significantly different to the mean day in the wet impermeable catchment.

Contrary to the permeable catchment, this result implies that rainfall may influence the timing of the Q95 in impermeable catchments.

3. Investigation of regional differences in the timing of low flows across the UK

3.1 INTRODUCTION

The paired catchment analysis was useful for investigating the dependency of the differences that may occur in the timing of low flows on climate and geology. However, the hypothetical catchments were derived by combining catchments with geological and rainfall characteristics of opposite extremes and sometimes from different regions of the UK. In reality such extremes are unlikely to occur in a catchment.

Therefore, a second subset of 388 natural catchments of good hydrometric quality and flow records covering the period 1976 to 1995 were selected. This second set was used within a regional analysis to determine whether the patterns observed within the paired catchments are replicated across the UK.

3.2 ANALYSIS

3.2.1 Analysis of the timing of the Q95

Using the procedure outlined in Section 2.2, the mean day of occurrence of the Q95 and corresponding standard deviation were calculated for each of the 388 catchments. The mean day of occurrence of the Q95 and the mean resultant are plotted for each catchment on a UK map. The mean resultant is used as opposed to the standard deviation in days in order to give visual weight to those stations which have strong seasonality of low flow events. A visual analysis of the map was undertaken in the context of the distribution of geological units throughout the UK.

Linear stepwise multivariate regression analysis was used to identify the most important variables which determine the mean day of occurrence of the Q95. Potential controlling variables were BFI, SAAR and catchment area. The dependent variable, day of occurrence, was regressed against the independent variables, BFI, SAAR and catchment area.

Stepwise regression was utilised, which is based upon the forward selection of variables, with a backward look at each stage. The first variable that produces the optimum one variable subset (i.e. the variable with the largest R^2) is identified, to which the second variable is added, such that the largest increase in R^2 occurs. Following each addition of a variable a backward elimination may be implemented.

Similar to the determination of the existence of significant differences in the timing of the Q95 in the paired case study catchments (Section 2.2), each of the 388 stations was paired with its nearest neighbour and analysed to determine whether significant differences occur in the timing of the Q95 between nearby catchments. The choice of T-test was based on the equality of the variances, as discussed in Section 2.2.

3.3 RESULTS AND DISCUSSION OF THE VARIATIONS IN TIMING ACROSS THE UK

Figure 3.1 shows the mean day of occurrence of the Q95, as indicated by the direction of the tail, and the strength of the seasonality, as indicated by the length of the tail. In association with Figure 3.2, which shows the distribution of Low Flow HOST Groups in the UK, it was possible to observe regional differences in the timing of the Q95 that occur as a result of regional variations in hydrogeology. Most of the Q95 flows occur between the 1st of July and the 1st of October. In the north west, where catchments are characterised by the impermeable geologies associated with LFHGs 9 and 10, the flows occur in early July, whilst those which occur after October can be associated with LFHG 1, which is primarily comprised of chalk. More generally, Figure 3.1 indicates that there is more variability in the timing of the low flows between gauging stations in areas of more permeable geology, which demonstrates the complex controls of hydrogeology in permeable systems. However, the at station variability, indicated by the length of the tail, is small. This reflects the strong seasonality of low flow events in all regions of the UK.

The results of the multivariate regression analysis, which was undertaken to determine the variables which influence the timing of the Q95, are given in Table 3.1. It is evident that the dominant factor in influencing the day of occurrence of the low flow statistics is the BFI.

Table 3.1 *Multivariate regression results*

Significant variables	Partial R^2	Model R^2	Standard Error	Parameter estimate
INTERCEPT				221.71
BFI	0.4776	0.4776		77.83
SAAR	0.1759	0.6534		-0.0293
AREA	0.0167	0.6701	16.12	-0.0049

BFI accounts for almost 50% of the variability observed in the mean day of occurrence, whilst SAAR accounts for less than 20%. Therefore, a linear regression between the mean day of occurrence and BFI was undertaken. Figure 3.3 shows the linear relationship between BFI and the mean day of occurrence of the Q95. The trendline represents the mean day of occurrence that would occur at a given BFI. The standard error for the distribution around the mean at any given BFI is equal to approximately 20 days. It was therefore possible to estimate the difference in BFI required for the associated mean days of occurrence to be significantly different using the following test statistic:

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \qquad (3.1)$$

where: $\bar{x}_1$ and $\bar{x}_2$ mean day of occurrence of Q95

σ_1^2 and σ_2^2 variance associated with $\bar{x}_1$ and $\bar{x}_2$

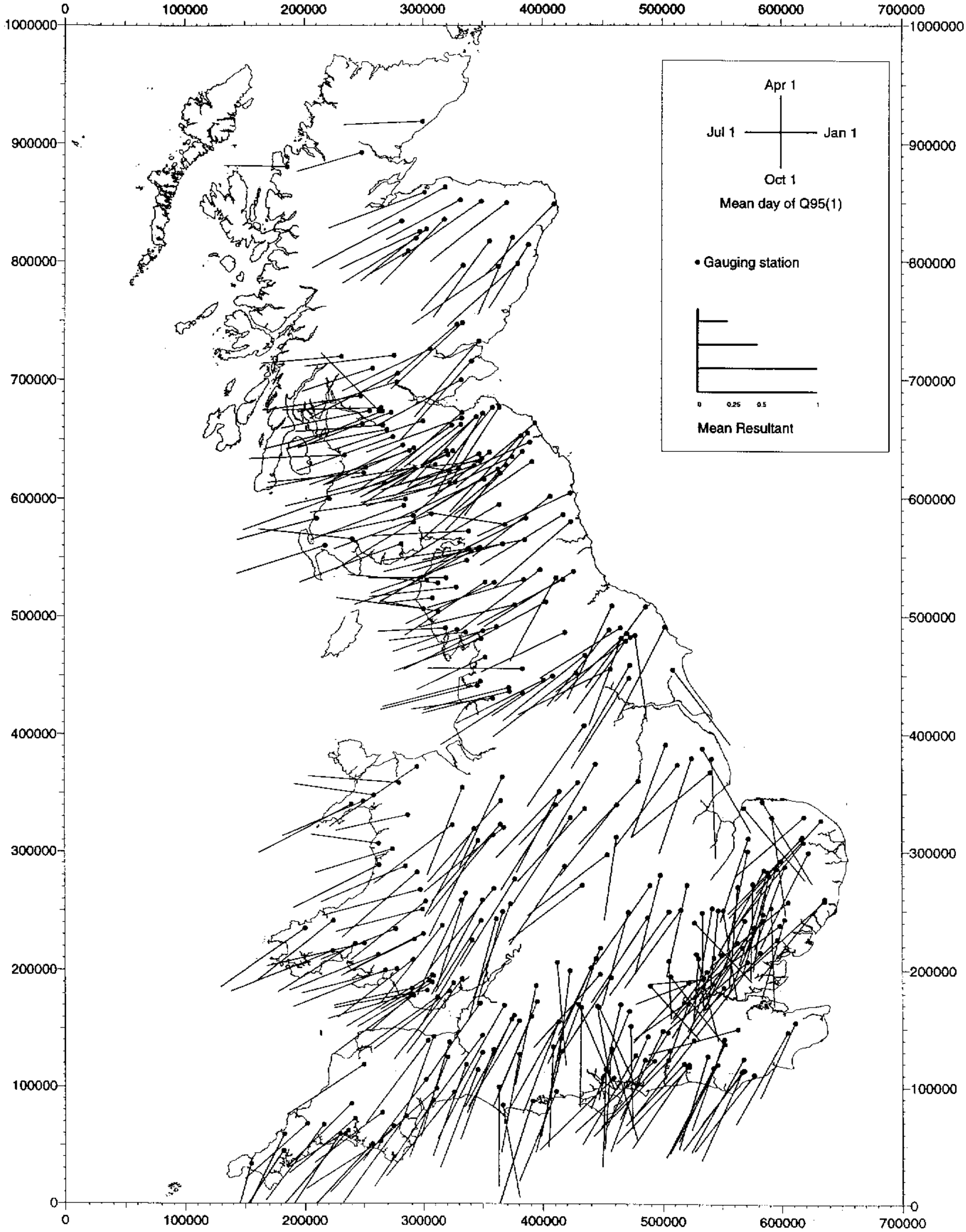

Figure 3.1 *Mean day and seasonality of occurrence of Q95*

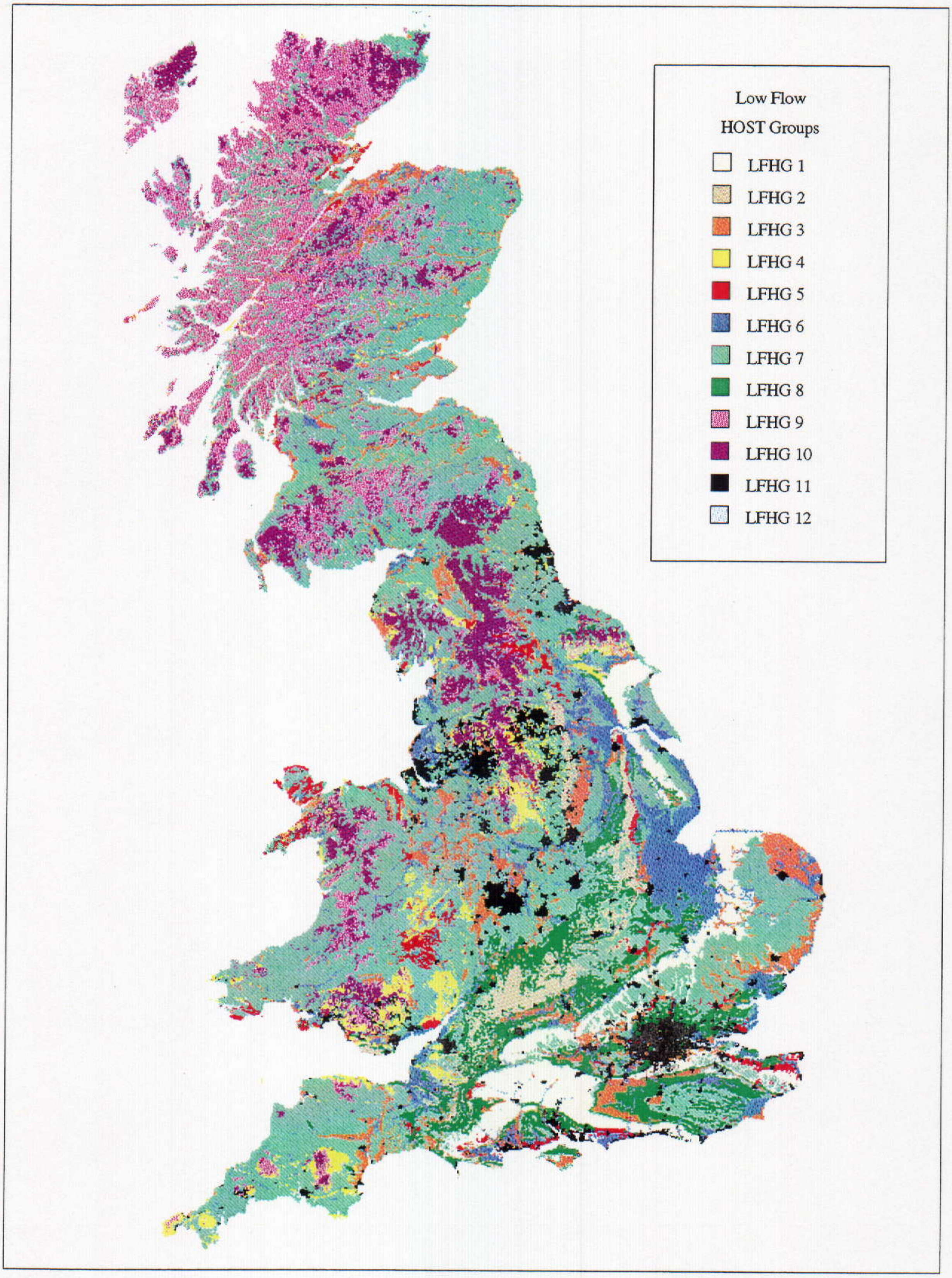

Figure 3.2 *Distribution of LFHGs in Great Britain*

At the 5% significance level, if T exceeds 1.96 then the mean days of occurrence are significantly different. A BFI of 0.2 has a corresponding mean day of occurrence of 194, as calculated from the regression equation given in Figure 3.3.

σ_1^2, σ_2^2 and $\bar{x}_2$ are all known and were therefore substituted into equation 3.1 to determine $\bar{x}_1$:

$$1.96 = \frac{\bar{x}_1 - 194}{\sqrt{20.24^2 + 20.24^2}}$$

Therefore, $\bar{x}_1$ is equal to 250 which, given the pooled variance, is significantly different (95% confidence) from day 194. The BFI associated with day 250 was determined from the regression equation, and is equal to 0.72. Therefore it may be implied, with 95% confidence, that on average a difference in BFI of 0.52 is required before a significant difference is observed in the mean day of occurrence. This compares to the approximate range of BFI values in the UK of 0.2 to 0.98, which represents a difference of 0.78.

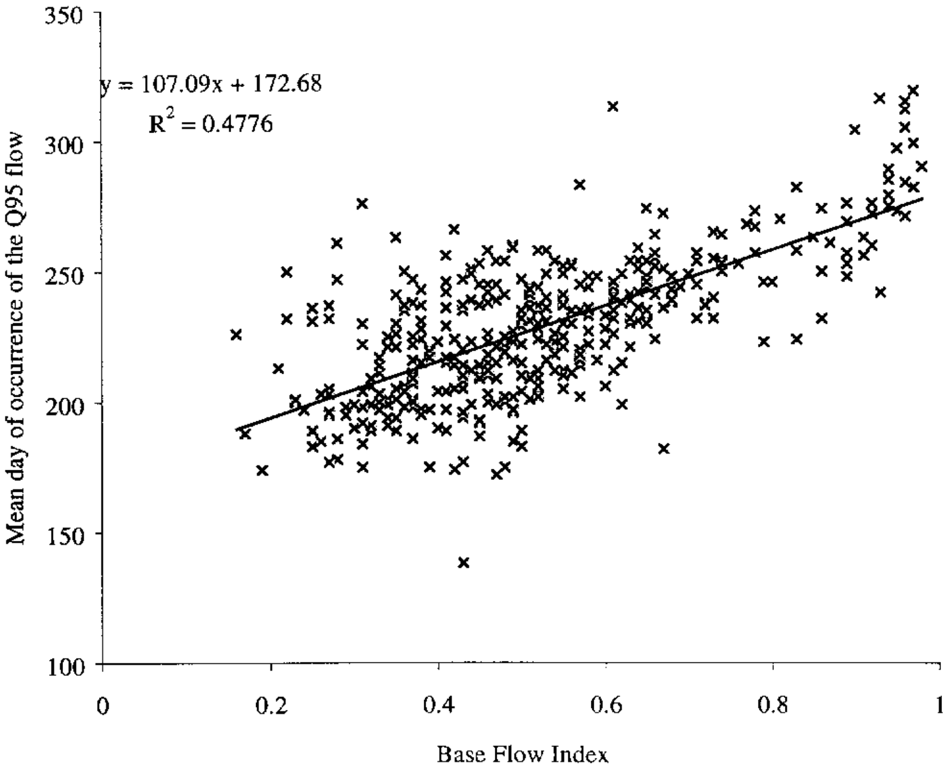

Figure 3.3 *Linear relationship between BFI and mean day of occurrence of the Q95 flow*

The nearest neighbour analysis was undertaken to determine the variations in the timing of the Q95 that are likely to occur over small distances. Therefore, the distances between the nearest neighbour stations was calculated to ensure that the pairs of stations were within a small distance of each other. The frequency distribution of the distance between these nearest neighbour stations is given in Figure 3.4 and shows that approximately 90% of the paired lay within a distance of 20km of each other.

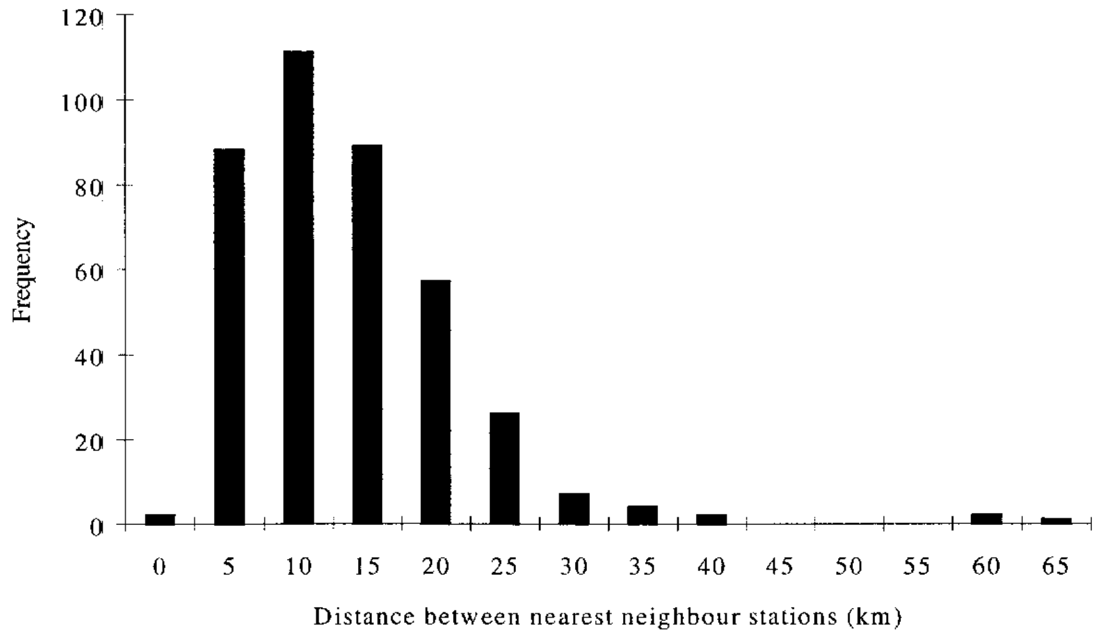

Figure 3.4 *Frequency distribution of distances between nearest neighbour stations*

The T-tests, which were used to test for a significant difference in the mean day of occurrence of the Q95 associated with the pairs of nearest neighbour stations, indicated that only two pairs of stations had significantly different mean days of occurrence. The two pairs of stations (36003 & 36005, 76011 & 23011) with significantly different days are detailed in Table 3.2.

Table 3.2 *Information of paired stations*

Nearest neighbour stations	SAAR (mm) (1941-1970)	Catchment area (km^2)	BFI	Distance between nearest neighbour stations (km)
36003	602	53.9	0.63	6.48
36005	602	156	0.46	
23011	1401	58.8	0.33	17.60
76011	1163	1.5	0.19	

The first pair of catchments represent the two driest catchments in the UK, where the effective rainfall is approximately 100mm. Therefore, the timing of the Q95 flow is more likely to be driven by climatic factors as opposed to the hydrogeological response. The second pair of catchments may have significantly different days of occurrence of the Q95 as a consequence of the large differences in the size of the catchments.

There are two possible explanations for the lack of significant differences in the mean day of occurrence in the remaining pairs of catchments:

1. There are only small differences in SAAR and BFI between the paired catchments. Approximately 90% of the paired catchments have a difference in BFI of less than 0.2 and a difference in SAAR of less than 350mm.

2. The magnitude of the variability around the mean day of occurrence, associated with each gauging station, is large compared to the magnitude of the difference between the mean day of occurrence of the paired catchments.

4. Quantification of the Q95 flow in the UK

4.1 INTRODUCTION

In Chapter 3 it was shown that significant differences in the mean day of occurrence of the Q95 do not generally occur in catchments with different rainfall and hydrogeological characteristics. This is due to the small differences in SAAR and BFI that occurs between nearby stations and also due to the magnitude of the variability around the mean day of occurrence. Since the ultimate concern of this study is to determine the effect of assuming that the Q95 is temporally coherent over all parts of the upstream network the Q95 was estimated for each pair of nearest neighbour catchments using various assumptions of the timing.

4.2 ANALYSIS

The Q95 was estimated for the hypothetical catchments, which derived by taking the nearest neighbour stations (Chapter 3), using the following three assumptions:

1. the Q95 occurs at the same time in both catchments;
2. the Q95 occurs on the mean day of occurrence of catchment one in both catchments;
3. the Q95 occurs on the mean day of occurrence of catchment two in both catchments.

In each case the Q95 was expressed as runoff with units of $mmyr^{-1}$, thus minimising the influence of catchment area on the scale of the runoff processes in each catchment. Under assumption one, the Q95 flows in each catchment were simply summed. Under assumption two, the Q95 in catchment one was added to the average flow that occurred in catchment two on the mean Q95 day of catchment one, and vice versa for assumption three.

Since assumption one is used in current low flow estimation procedures, the Q95 derived under this assumption was used as a benchmark to determine the percentage difference between the Q95 derived using assumption one and using assumptions two and three. Where a large percentage difference occurred in the Q95 estimates, the catchments were investigated to determine reasons for the differences.

4.3 RESULTS AND DISCUSSION OF THE QUANTIFICATION OF THE Q95

Figure 4.1 shows the frequency distribution of the percent difference between the Q95 derived using assumption one and the Q95 derived using assumptions two and three.

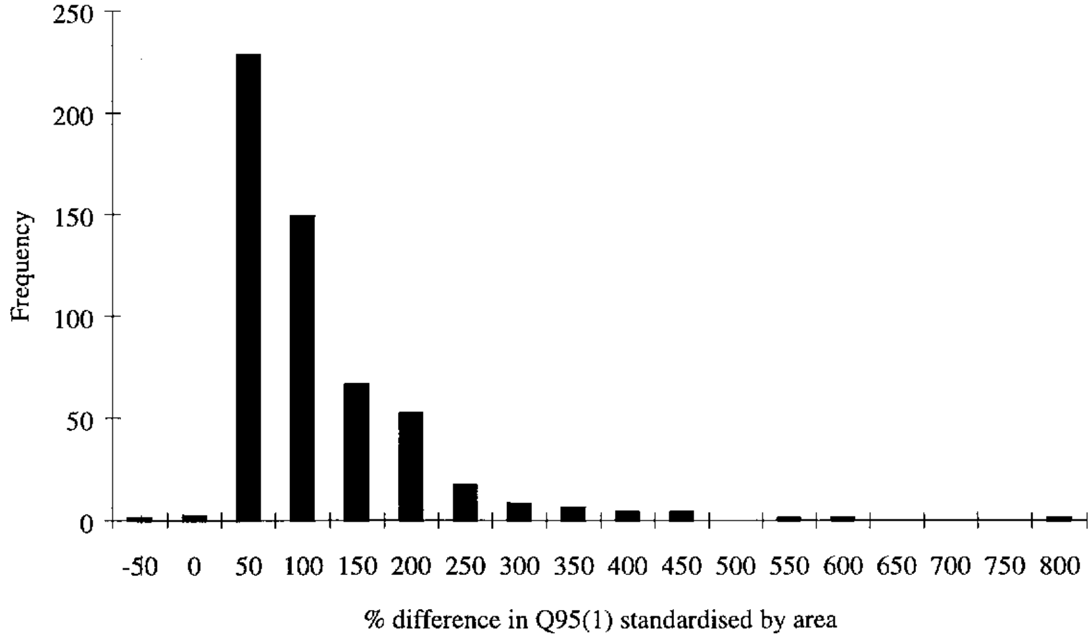

Figure 4.1 *Frequency distribution of differences in Q95*

Figure 4.1 shows that the distribution of percent differences in the Q95 is highly skewed, which indicates that most of the differences are small. The median percent difference is 56.19, and the 68% and 95% confidence limits are (25.86, 154.76) and (12.39, 324.47) respectively. It should be noted that only three of the percent differences are negative, which indicates that the assumption of temporal coherence usually results in an underestimate of the Q95.

Since the GREAT-ER project is concerned with a target accuracy of a factor of three, the pairs of catchments were split into two subsets based on differences in the Q95 of (1) less than 300%, and (2) more than 300%. Approximately 96% of the percent differences lie within a factor of three. Approximately 96% of the nearest neighbour pairs displayed a difference in Q95 of less than 300% (i.e. a factor of three). The mean BFI and SAAR values were derived for the catchments in both classes in order to identify whether the larger differences are associated with a certain type of catchment. Table 4.1 presents the results and implies that those nearest neighbour stations where a difference of more than 300% was observed in the estimates are generally wetter and more impermeable catchments.

Table 4.1 *Average BFI and SAAR for catchments with a < 300% and > 300% difference in estimated Q95*

Characteristic	Difference < 300%	Difference > 300%
BFI	0.54	0.37
SAAR (mm)	1057	1604

5. Conclusions

1. Differences in the timing of the mean day of occurrence of the Q95 flow can be identified. Significant differences do occur in the timing of low flows between catchments with different hydrogeological and rainfall characteristics. Low flows occur earlier in impermeable catchments, although there is some evidence of low flows also occurring earlier in wet permeable catchments.

2. However, this difference in timing at a daily resolution is not observed in the majority of nearest neighbour stations due to their similarity in SAAR and BFI and also the magnitude of the variability around the mean day.

3. The assumption of temporal coherence of low flows results in mostly smaller estimates of Q95 than those derived under the assumption that the low flows occur at the time of one of the upstream catchments.

4. 96% of the Q95 estimates derived using the assumption of temporal coherence are within a factor of three of the Q95 estimates derived the assumption that the low flows occur at the time of one of the upstream catchments. Those differences which are larger than a factor of 3 are generally observed in wetter and more impermeable catchments.

5. Assuming temporal coherence of the Q95 flow over all parts of the upstream network results in the largest underestimates of the Q95 in impermeable and wet catchments due to the large variability in daily flows that occurs over a small time scale. Since the timing is not significantly different at a daily resolution in such catchments, it is not practical to incorporate the effect that the flow variability exerts on the estimated Q95.

6. The recommendations of the study are to incorporate the timing of the Q95 at a monthly resolution, which Bullock *et al* (1994) found to be a significant unit of time.

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