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A NOTE ON SOME TROUBLESOME HARMONIC CONSTITUENTS

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NATURAL ENVIRONMENT RESEARCH COUNCIL

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A note on some troublesome harmonic constituents

A variety of investigations in 1968 and 1969 have indicated the inconstancy of certain of the orthodox harmonic constants. This note considers the probable source of some of the observed variability. The starting point is the set of harmonic analyses performed by I.C.O.T. on Harwich observations during the period 1954-1961.

1. The data

Figure 1 shows the scatter in H (amplitude plotted radially) and g (phase lag) of selected terms in the Harwich analyses. These latter comprise 21 analyses, one year each, at irregular intervals.

M_2 and S_2 exhibit amplitude changes but no angular changes. All others display varying degrees of scatter, of which the largest are those for $2N_2$ and L_2.

Moreover, certain terms, notably $2N_2$ and L_2, reveal a time dependent pattern, rather than a scatter, which on further examination reveals regular variations in H, or g, or both.

Much of the observed data thus suggest regular perturbations to certain of the standard terms which can be explained in one or both of the following ways :-

(a) Imperfect interpretation of the harmonic development of the tide generating potential.

(b) Shallow water terms of nearly the same frequency as the standard term.

2. Imperfect interpretations of the harmonic development.

The standard work on this subject is Doodson's 1921 paper (re-printed by the I.H.B. in 1954). His schedule uses the following notation to identify individual lines in the spectrum of the equilibrium tide. The angular speed of a line, expressed in the form

$$a\tau + bs + ch + dp + eN + fp_1,$$

where τ = local mean lunar time reduced to angle

s = moon's mean longitude

h = sun's mean longitude

p = longitude of moon's perigee

N = -N, where N is the longitude of the moon's ascending node

p_1 = longitude of sun's perigee

gives rise to the shorthand notation for an argument number

$$a(5+b)(5+c)(5+d)(5+e)(5+f)$$

The coefficient a is always positive, being the species number; the base 5 is used to avoid negative quantities, since b to f inclusive may be negative but rarely less than -5.

Thus the term N_2 of speed $2\tau - s + p$ has the argument number 245 655.

Since we must deal with imperfect observations in tidal analysis, we must always accept the restriction that we cannot identify neighbouring lines of speeds σ and $\sigma + \Delta\sigma$ unless the span of data analysed is at least equal to the "synodic period" of the two lines, given by $2\pi/\Delta\sigma$. Put another way, in terms of argument numbers, for unit change in b, c, d, e, or f we must operate on the following minimum spans of data :-

Unit change in	Span
b	27·3 days
c	365·2 days
d	8·85 years
e	18·6 years
f	209 Julian centuries

Acknowledging the difficulties in obtaining sufficient data for " p_1 - splitting", we could use the approach practised by the German Hydrographic Institute and achieve "N-splitting". For many reasons, however, not least the scarcity of 19 years of observations at U.K. stations, normal practice is to standardise on " h -splitting", i.e. the analysis of 1 year's data.

The traditional (and generally effective) way of dealing with terms separated only by 1 or more units of p and N is to select the dominant term, and by using the amplitude relationships explicit in the harmonic development, to represent the perturbing influences of the minor terms by what have come to be called the <u>nodal factor</u> (f) and <u>angle</u> (u).

Thus, in Doodson's schedule we have terms

	Rel. amp.
245 556	14
245 645	-648
245 655	17387

The third is the dominant line, denoted <u>N_2</u>. The second is the minor line which represents the nodal variation of N_2, and appears in tidal analysis and prediction as the "f and u of N_2". The first term was rightly ignored by Doodson when compiling his f, u tables, as being negligibly small.

The relative amplitudes shown above are associated with a general coefficient (G) which is a function of latitude. Doodson's schedule also contain other terms associated with a different coefficient (G') dependent upon latitude in quite a different way. For example, within the N_2-group there is the term with argument number 245 755 and relative amplitude 11, but associated with G'. Even if this were not small it could <u>not</u> be incorporated in the f and u of N_2; although the relationship between G and G' is known for the equilibrium tide, this relationship cannot be assumed valid for the real tide.

3. Shallow water terms

Most astronomical terms coincide with one or more shallow water terms. This has long been appreciated, and some allowance made.

An outstanding example is $\underline{\mu_2}$; in the schedule this appears as lines 237 545 (minor) and 237 555 (major). A large shallow water term $\underline{2MS_2}$ also exists as line 237 555, arising from the interaction between M_2 and S_2. This can be expected to be accompanied by side lines arising from the side lines of M_2 and S_2. Thus the harmonic constants for "μ_2" should be derived by analysing for line 237 555, and using f and u corrections which take into account the relative contributions from the astronomical μ_2 and the shallow water $2MS_2$.

This has not, in fact, been the practice; examination shows that the differences between the f's and u's of the two components are quite small, and the equilibrium values have therefore always been used. Nevertheless, the scatter in the μ_2-diagram of Figure 1 is suggestive.

On the other hand, the very small astronomical line 227 655 is always assumed to produce the coincident shallow water line MNS_2, and the nodal corrections are derived accordingly.

Although Doodson's judgement in selecting the appropriate nodal corrections has not previously come under serious criticism, the cases of $2N_2$, L_2 and to a lesser extent some others, has compelled me to investigate the possibility of separating coincident astronomical and shallow water terms.

4. Separation of an astronomical and a shallow water term

In the extreme case, when only one year's data is available for analysis, separation is only necessary when

(a) their speeds differ by p or a multiple thereof, or

(b) their nodal corrections are significantly different.

The problem is the same in both cases; one of the two components must be deduced in order to compute the other. Since our knowledge of the precise laws of generation of shallow water tides is quite imperfect, the only practical solution is to deduce the astronomical tide. In theory this should be possible by assuming that both the amplitudes and phase lags of the astronomical tides are functions of their angular speeds (this corresponds to the "credo of smoothness" used by Munk and Cartwright in their response method). If these functions can be established from the analytical results for the major astronomical lines, the smaller ones contaminated by shallow water tides can be deduced.

Before describing an application of this technique, it is necessary to indicate the specific pairs of lines worthy of consideration in the 2 c.p.d. band. These are given in Table 1.

Table 1

Astronomical tide	Shallow water tide	Remarks
	235 555 ($2MK_2$)	$2MK_2$ is probably more important than
235 755 ($2N_2$)	235 755 (MNL_2)	MNL_2
237 555 (μ_2)	237 555 ($2MS_2$)	Nodal corrections slightly different
245 655 (N_2)	245 655 ($2ML_2$)	Nodal corrections appreciably different
247 455 (ν_2)	247 455 (MLS_2)	Nodal corrections appreciably different
255 555 (M_2)	255 555 (OK_2)	Nodal corrections appreciably different
263 655 (λ_2)	263 655 ($2M\nu_2$, SNM_2)	Nodal corrections slightly different
265 455 (L_2)	265 455 ($2MN_2$)	Nodal corrections appreciably different
273 555 (S_2)	273 555 ($2M\mu_2$)	Nodal corrections appreciably different
275 555 (K_2)	275 355 ($2M(2N_2)_2$)	Nodal corrections appreciably different

(a) Amplitudes (H) as a function of angular speed (σ).

For any constituent X_2 we plot $H(X_2)/\bar{H}(X_2)$ against σ, where $\bar{H}$ denotes a suitable multiple, fixed for a given species, of the relative amplitude given by Doodson. Figure 2 illustrates the results for one analysis each for Harwich, Southend, Newlyn and Aberdeen.

Although some consistency appears amongst the major constituents, there is clearly a problem in choosing key plots and joining them. Only those constituents for which the astronomical component is substantially greater than the shallow water component can be accepted for this role, and experience suggests M_2, S_2, N_2 and K_2. However, Cartwright (1968) has shown that the radiational component of S_2 in British waters approximates to 20% of the astronomical; this is supported by Figure 2, and rules out S_2. Similarly K_2 is not suitable, and we are therefore reduced to a linear relationship based on M_2 and N_2 only. However, the shallow water contribution to N_2 ($2ML_2$) may also be significant. Until more is known about the frequency dependency of amplitude relationships it seems best to assume it to be zero, and to deduce $H(X_2)$ from

$$H(M_2) \times \frac{\bar{H}(X_2)}{\bar{H}(M_2)}$$

(b) Phase lags (g) as a function of agular speed (σ).

Figure 3 represents the plot of g against σ, and a realistic linear relationship emerges for all major terms.

(c) Procedure.

Consider the combination $2N_2/2MK_2$. Then the orthodox analysis gives an H and g which can be represented by

$$f H \cos(V + \sigma t + u - g) = f_1 H_1 \cos(V_1 + \sigma t + u_1 - g_1) + f_2 H_2 \cos(V_2 + \sigma t + u_2 - g_2)$$

where suffix 1 refers to the astronomical component ($2N_2$) and suffix 2 refers to the shallow water component ($2MK_2$). For our purposes σ can be assumed the same in all terms; further, $f = f_1$, $V = V_1$ and $u = u_1$.

Put $fH = R$, $g - (V + u) = \delta$.

Then $R \cos(\sigma t - \delta) = R_1 \cos(\sigma t - \delta_1) + R_2 \cos(\sigma t - \delta_2)$.

Given R, δ from analysis, and R_1, δ_1 by deduction, it is a simple matter to compute R_2, δ_2 and hence H_2, g_2.

An example for the most recent Harwich analysis is given in Table 2.

Table 2. Separation of $2N_2/2MK_2$. Harwich analysis Central Day 1.1.1968

$H(M_2) = 4{\cdot}40$ $\quad \bar{H}(2N_2)/\bar{H}(M_2) = 0{\cdot}0253$

Suffix		H	f	R	g	V	u	δ		$R\cos\delta$	$R\sin\delta$
-	From analysis	0·134	0·966	0·129	205	218	-1	348	→	0·126	-0·027
1	Deduced	0·110	0·966	0·106	268	218	-1	51	→	0·067	0·082
2		0·103	1·202	0·124	78	135	5	298	←	0·059	-0·109

This indicates that "$2N_2$" consists of two terms of almost the same amplitude, thus :-

$2MK_2$	235 555	H = 0·10, g = 78
$2N_2$	235 755	H = 0·11, g = 268.

As confirmation, a detailed least squares solution of all the Harwich analyses gave

$2MK_2$	H = 0·10, g = 127
$2N_2$	H = 0·10, g = 272

5. Proposals for treating certain harmonic terms

Referring to Figure 1, we can now select the troublesome constituents and decide what, if any, treatment can be adopted.

(a) $2N_2$ It is clearly advantageous to treat this as two terms, $2N_2$ and $2MK_2$, differing in speed by $2p$. The separation should be effected as above, and the results used in predictions.

(b) L_2 The perturbation of L_2 seems attributable to the existence of $2MN_2$; they are of identical speeds but with appreciably different nodal variations. For Lowestoft 1968, using the separation technique described, we find

"L_2" =	L_2	+ $2MN_2$
0·28,331	0·12,355	0·22,147

The perturbations of Figure 1 are now seen to be the result of associating the nodal variations of L_2 with a larger and out-of-phase $2MN_2$.

(c) ν_2 If the small perturbations are attributed to MLS_2, we find

"ν_2" =	ν_2	+ MLS_2
0·21,297	0·16,300	0·05,337

Since the astronomical ν_2 dominates MLS_2, there is little to be gained by this separation. The same seems likely to be true of the other combinations of Table 1, in which the two nodal corrections are appreciably different, namely

N_2 and $2ML_2$

S_2 and $2M\mu_2$

K_2 and $2M(2N_2)_2$

(d) μ_2 This is perturbed by $2MS_2$, one of the largest of the 2 c.p.d. shallow waters :-

"μ_2"	=	μ_2	+	$2MS_2$
0·25,77		0·14,275		0·40,82

Since the difference between the nodal variations of the two components is small, probably the best procedure is to acknowledge the dominance of $2MS_2$ at the majority of ports, and instead of separating the lines to use the $2MS_2$ nodal variations in analysis and prediction.

(e) M_2 The amplitude perturbation of M_2 is sufficiently well determined to warrant further investigation, and this will continue. It will clearly be difficult to make assumptions about this, the fundamental term.

(f) M_1 The variations in this term have not been plotted in Figure 1 since the phases varied too widely. They are, in fact, quite systematic, and have arisen from an error by Doodson when compiling his f, u tables from the schedule. They are also associated with errors in his tables of V.

This error was drawn to Doodson's attention by W. Horn of the German Hydrographic Institute by letter in 1947. This correspondence has only recently been discovered by the writer, who can only imagine that Doodson did not consider M_1 to be sufficiently large, in general, to merit the extensive work involved in correcting the tables and all past analyses.

Since that time Horn has published new tables for f and u, known as j and v. In general they differ little from those traditionally used by I.C.O.T.; in addition to M_1, L_2 is probably the most important affected (j and v were used in the preceding $L_2/2MN_2$ calculations).

I.C.O.T. proposes to adopt Horn's tables as soon as possible, and to correct the M_1 errors both in analysis and prediction.

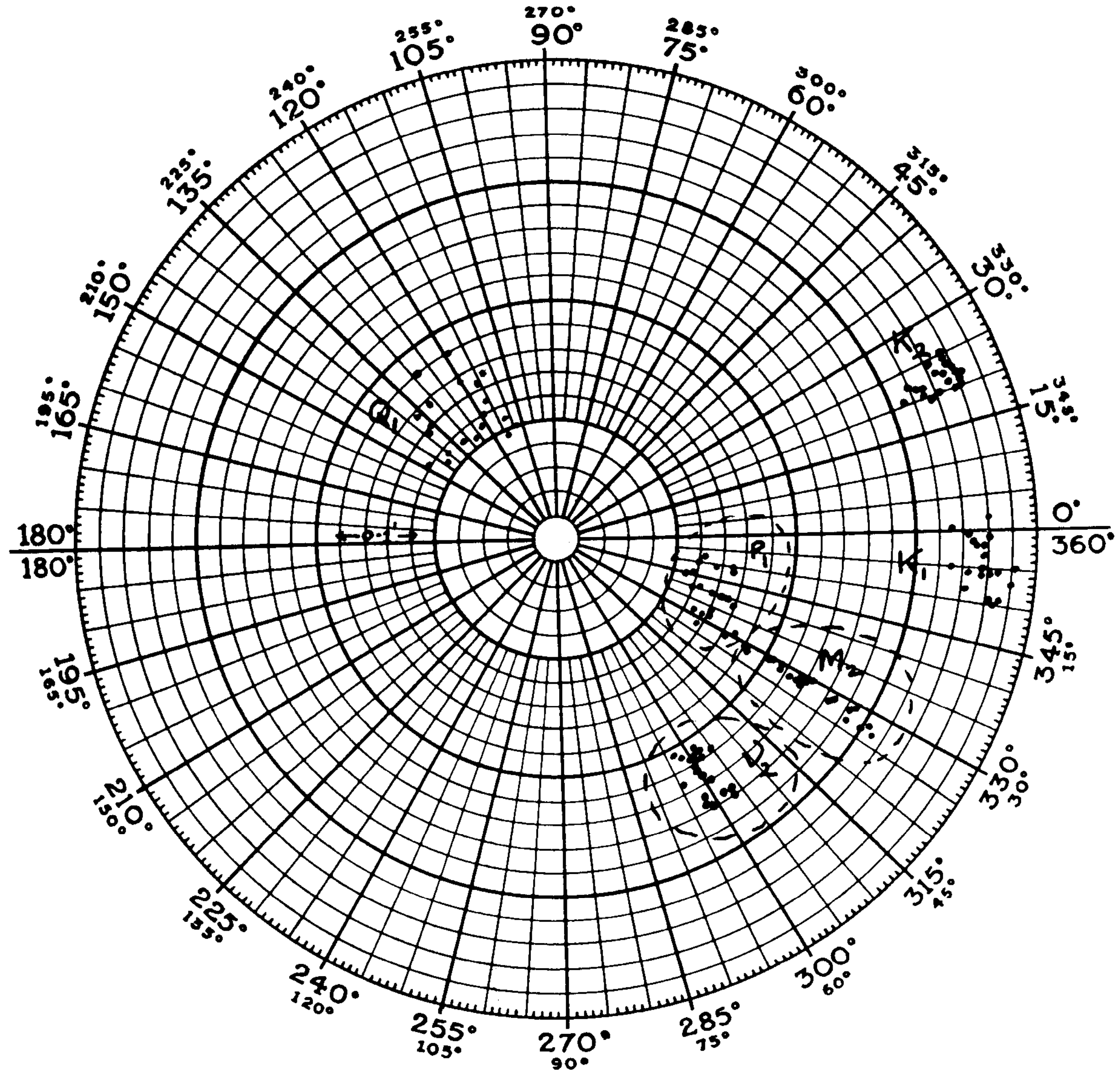
6. Concluding remarks

This note has been written with three purposes in mind.

Firstly, to suggest an explanation for some of the grosser variations in the Harwich "constants". Secondly, to indicate how some astronomical and shallow water terms of the same (or almost the same) frequency can be separated even if only one analysis of one year's data is available. Thirdly, to explain, in fairly simple language (not least for I.C.O.T. staff concerned with routine analysis and prediction) some of the background to theory and practice in this field; it is fair to remark, in passing, that much of this sort of information is not explicitly on paper but held in the heads of a small and dwindling number of people.

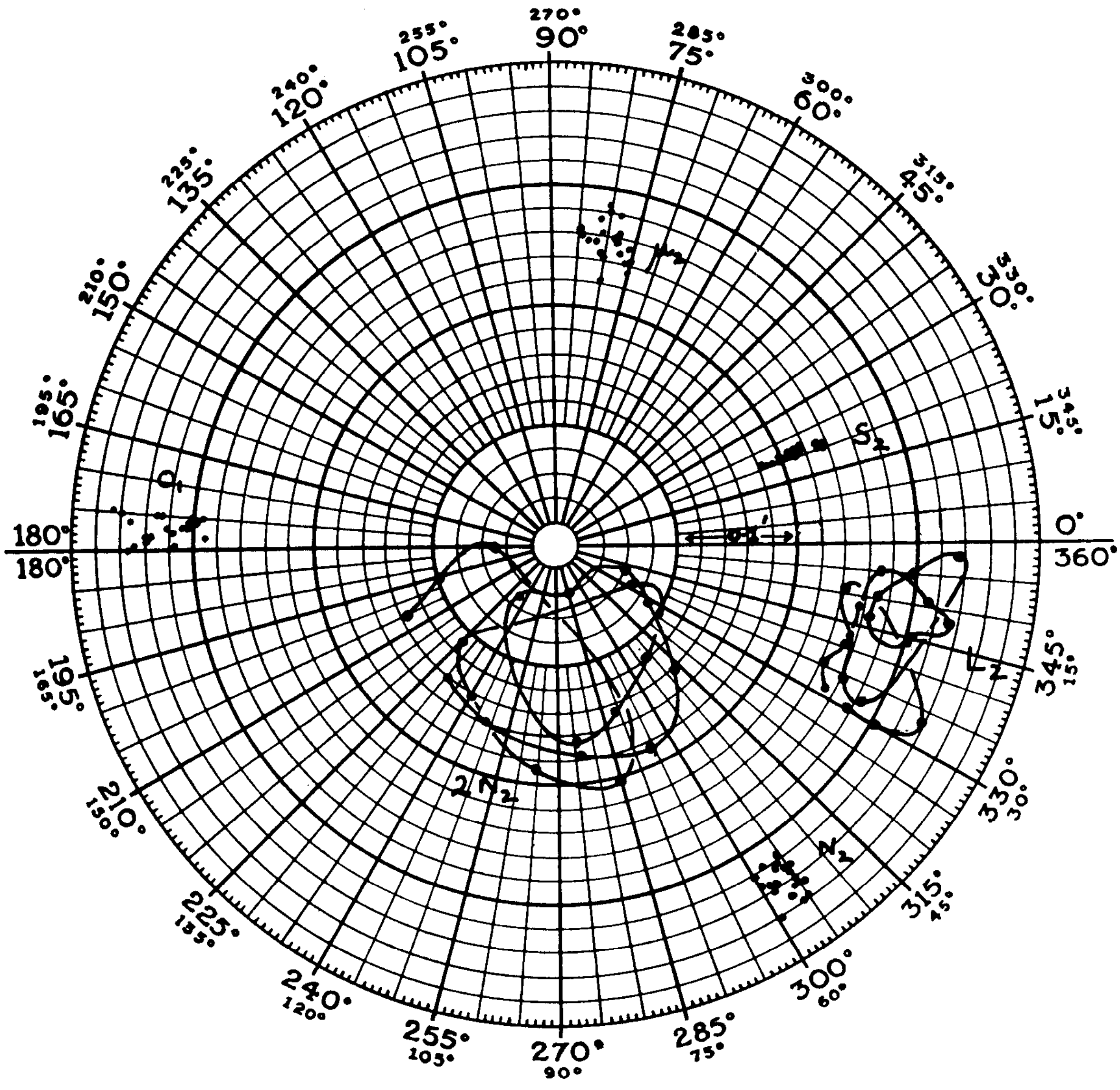
The note should not be interpreted as heralding a radical improvement in the accuracy of tidal predictions. However, tests performed on Harwich data clearly indicate that if the proposals in § 5 are put into effect, the coherent energy in the 2 c.p.d. band of residuals should be reduced by at least 0·05 ft^2.

Finally, this note does not materially affect the still unresolved major problem of tidal analysis and prediction, which is that of the overall relative merits of the harmonic and response methods.



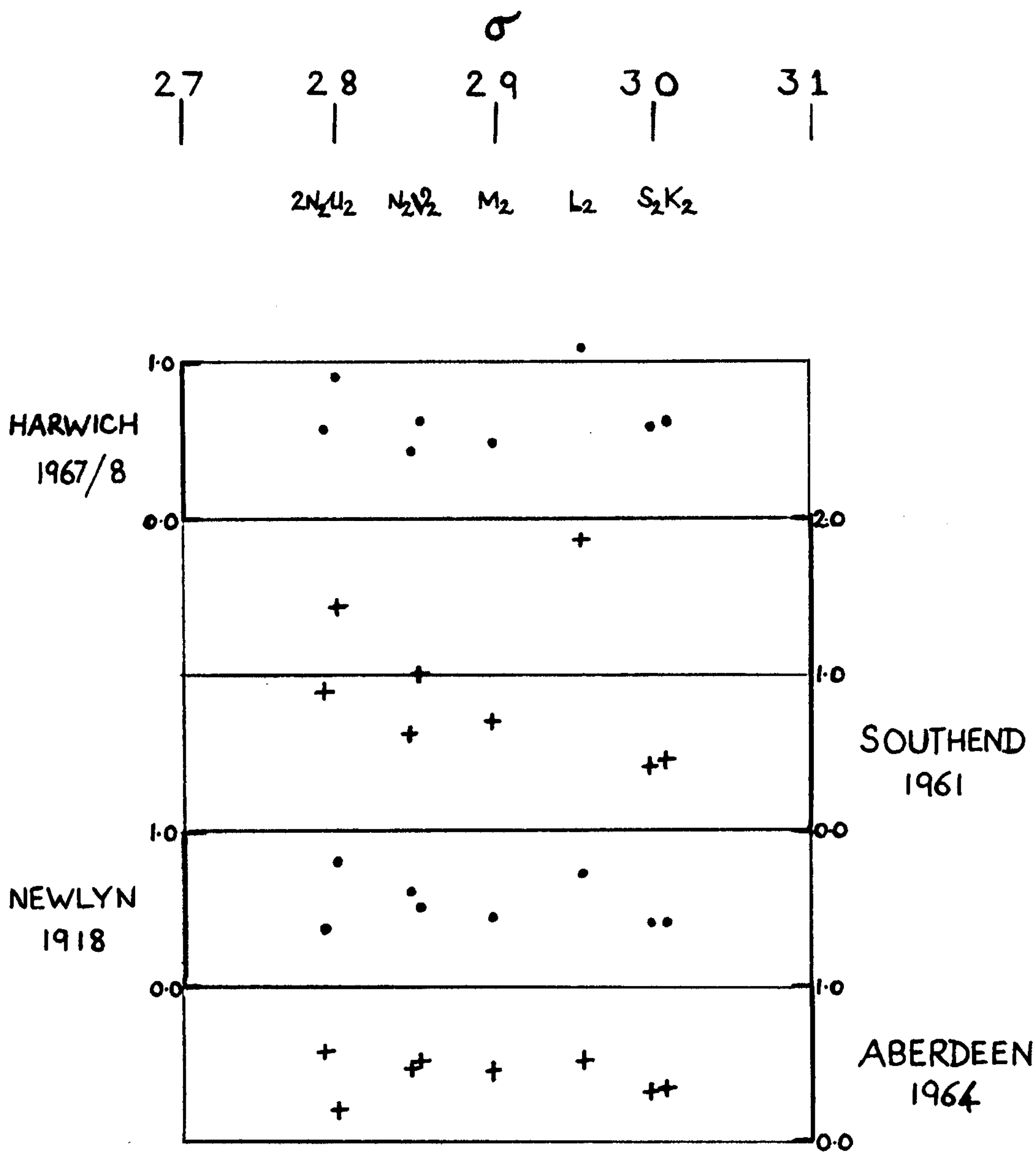
Variations in Harmonic Constants at Harwich

Figure 1a



Variations in Harmonic Constants at Harwich

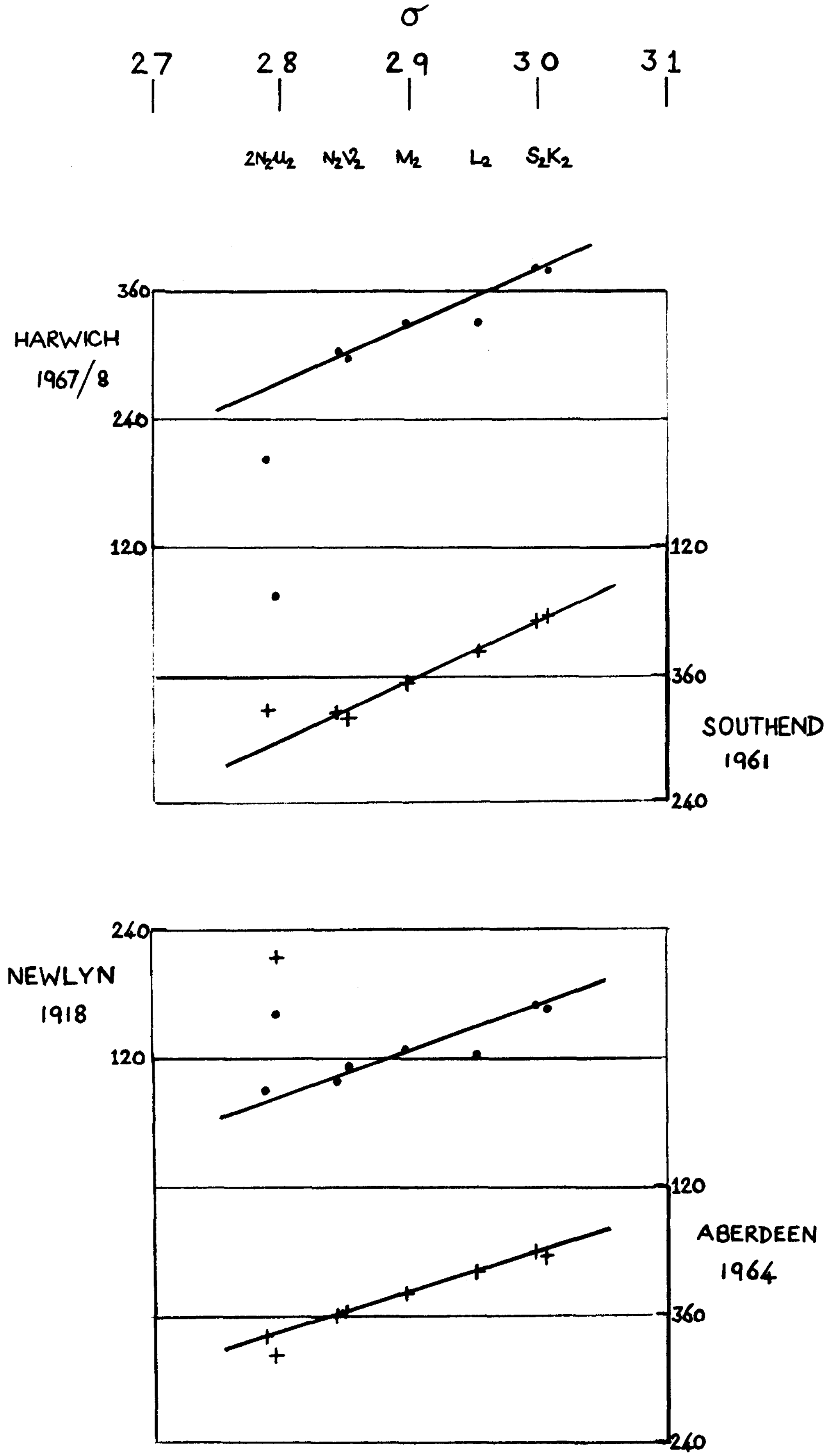
Figure 1b



Ratio of observed ($\bar{H}$) to equilibrium ($\bar{H}$) amplitude

as a function of angular speed (σ).

Figure 2



Phase lag (g) as a function of angular speed (σ).

Figure 3