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ESTIMATES OF THE EFFECT OF BARRAGES ON THE TIDES IN THE SOLWAY FIRTH USING ANALYTICAL MATHEMATICS

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NATURAL ENVIRONMENT RESEARCH COUNCIL

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This Report was prepared before the Tidal Institute became the Institute of Coastal Oceanography and Tides.

1. Introduction

This study was commissioned by Messrs. Babtie, Shaw and Morton. Its aim was to provide a quick, cheap and therefore approximate estimate of the possible changes in the mean tidal conditions in the Solway Firth consequent upon the introduction of a barrier near Newbie.

These conditions necessitate many assumptions being made regarding the physical nature of the estuary. In addition, the hydrogrpahy and tidal regime of the area are very poorly documented. Results can therefore only be considered to be approximations. A full scale and detailed investigation, using numerical techniques and some observational material, would be desirable.

The work has been concentrated upon the mean tidal condition, i.e. upon the lunar semidiurnal component, M_2.

2. Case I Fresh water reservoir barrage.

The observed data (⊙ in the Figure 1) were deduced from Admiralty Tide Tables, Volume 1. The phase difference between the time of maximum height and time of maximum current at Workington for ebb current was taken as 111 degrees. In order to prove the analytical model it was found necessary to divide the whole estuary, from the sea end to the barrage site into two parts called hereafter the outer compartment and the inner compartment. The laws for the mean breadth b, the mean depth h and the frictional parameter k were taken as follows :-

$$\text{Outer Compartment} \quad \begin{cases} b = b_0 e^{ax} \\ h = h_0 x^2 \\ k = 3 \times 10^{-4} \ / \ \text{sec} \end{cases}$$

$$\text{Inner Compartment} \quad \begin{cases} b = b_1 x^{2\cdot 37} \\ h = h_1 x^{0\cdot 75} \\ k = 9 \times 10^{-4} \ / \ \text{sec} \end{cases}$$

Here b_0 , h_0 , a , b_1 , h_1 are constants.

The solutions in the outer compartment were complex confluent hypergeometric functions and in the inner compartment complex Bessel functions. At the interface of the two compartments the velocities and the amplitudes were made equal so ensuring the continuity of the two solutions. The values of the frictional parameter k and constant indices (2·37 and 0·75) were slightly adjusted in order to reproduce the curves agreeing with the observed points. In this way the full curves for the amplitude of M_2 height and M_2 velocity were obtained.(Figure 1). The phase difference between the maximum height and the maximum current was found to be 109 degrees at the sea end, increasing gradually to 118 degrees at the barrage site.

To simulate the barrage, the following conditions were imposed :-

i) Velocity = 0 at the barrage site

ii) Amplitude (of height) at Workington = Amplitude at Workington without barrage.

The results are shown in Figure 1. by dotted lines. There is an increase of 12% in the amplitude at the barrage site. The increase in the inner compartment is also about 12%. In the outer compartment as we go seawards, the amplitude gradually decreases to the value without barrage at Workington. Seaward of Workington the amplitude is expected to remain the same as in the absence of a barrage. The velocity is about 1·5 ft / sec in the outer compartment and very small about 0·1 ft / sec in the inner compartment, becoming zero at the barrage site.

It is emphasised that the above results for the amplitude and velocity in the presence of the barrage are for a theoretical estuary, which is closed at the sides, so that breadth b is the same for every height. For the actual estuary the increase in the amplitude at the barrage will be less than 12%, possibly 8%. Similarly the velocity in the inner compartment could be more, possibly of the order of $\frac{1}{2}$ ft / sec ; but it is clear that near the barrage the velocity will be small, of the order of 0·1 ft / sec for some distance (about 7 miles) from the barrage. This would presumably encourage siltation near the barrage site.

Almost the same conclusions hold for the other semidiurnal components.

3. Case II Tidal Power Barrage

In this case the outer compartment solution for

$$b = b_0 e^{ax}$$
$$h = h_0 x^2$$
$$k = 3 \times 10^{-4} / \text{sec}$$

was found sufficient to prove the model. For given values of amplitude, of velocity and of phase difference between the time of maximum height and maximum velocity, at Workington, each of these variables increase towards the tidal power barrage end in an expected way.

To simulate the barrage, the following conditions were imposed :-

i) Velocity = 0 at the barrage site near Dubmill Point

ii) Amplitude at Whitehaven = Amplitude at Whitehaven without barrage.

The results are shown in Figure 2. It is seen that the amplitude at the barrage increases by 3% from the value without barrage for no discharge at barrage. For 75% discharge the increase will be about 2%.

The velocity near the barrage in this case behaves in a different way from that of case I. In case I, starting with zero velocity at barrage it increases very slowly and remains of the order of 0·1 ft / sec for some distance (of about 7 miles) from the barrage; but in the present case starting from zero velocity at the barrage it attains the value 1·3 ft / sec after a distance of about 7 miles.

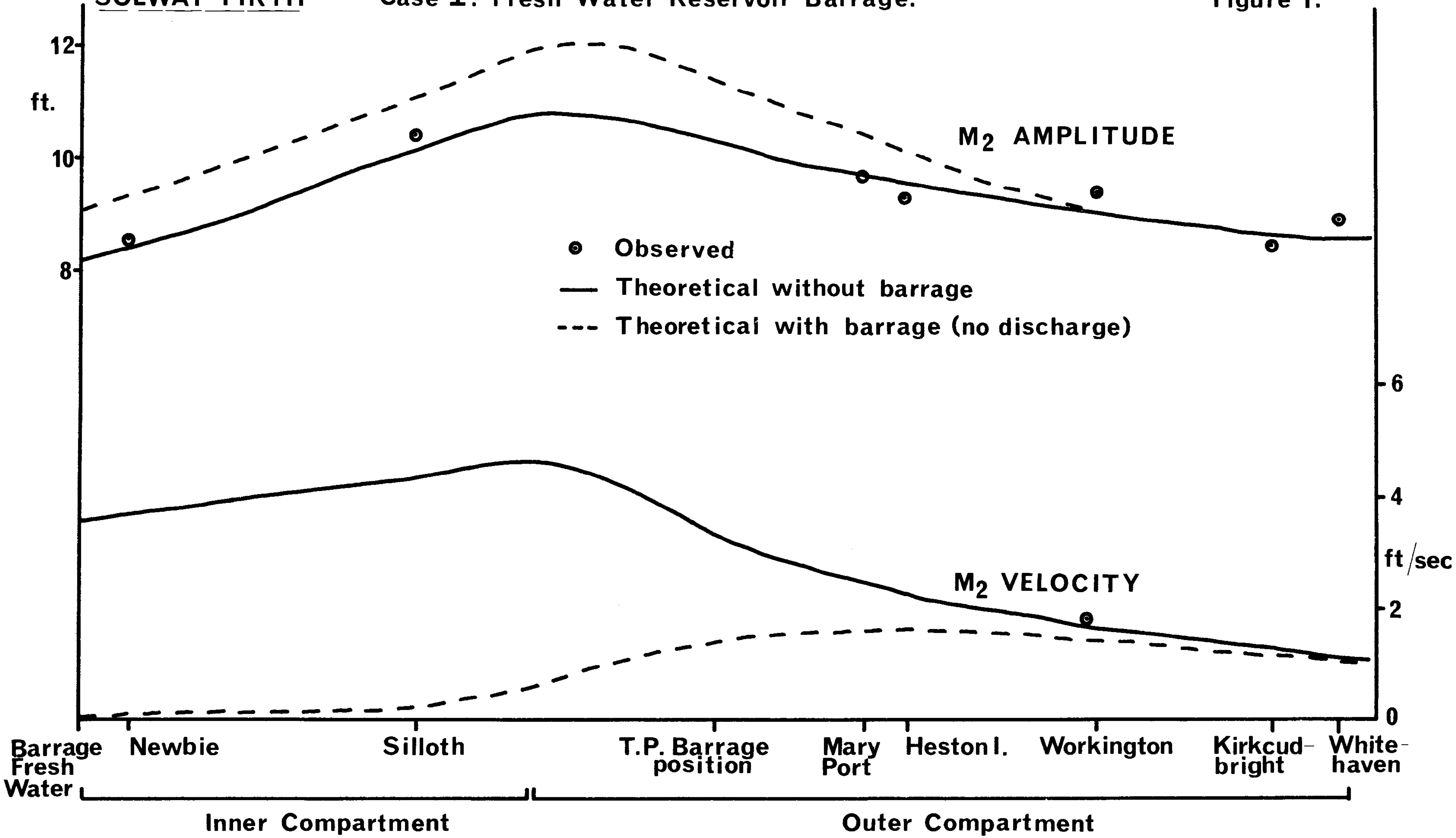
In previous calculations concerning the effect of this tidal power barrage, the laws for the breadth and depth were the same as here but the value of k was taken as equal to $2{\cdot}1 \times 10^{-4}$ / sec in place of $3{\cdot}0 \times 10^{-4}$ / sec in the present case. With the lower value of k the velocity and amplitude, in calculations for the proving of the model, increased more rapidly than expected. Cutting off a velocity of 4 ft / sec at the barrage site (as in the previous calculation) as compared to 3·2 ft / sec in the present case, led to a 13% reduction in amplitude at the barrage. The present result now seems to be the more probable.

Almost the same conclusions hold for the other semidiurnal components.

SOLWAY FIRTH

Case I: Fresh Water Reservoir Barrage.

Figure 1.



SOLWAY FIRTH CASE II: TIDAL POWER BARRAGE FIGURE 2.