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Differences between Mean Tide Level and Mean Sea Level

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Abstract

This paper discusses the differences between Mean Tide Level (MTL) and Mean Sea Level (MSL) as demonstrated using information from a global tide gauge data set. The roles of the two main contributors to differences between MTL and MSL (the M4 harmonic of the M2 semidiurnal tide, and the combination of the diurnal tides K1 and O1) are described, with a particular focus on the spatial scales of variation in MTL-MSL due to each contributor. Findings from the tide gauge data set are contrasted with those from a state-of-the-art global tide model. The study is of interest within tidal science, but also has practical importance regarding the type of mean level used to define land survey datums. In addition, an appreciation of MTL-MSL difference is important in the use of the historical sea level data used in climate change research, with implications for some of the data stored in international databanks. Particular studies are made of how MTL and MSL might differ through the year, and if MTL is measured in daylight hours only, as has been the practice of some national geodetic agencies on occasions in the past.

1. Introduction

This paper discusses the differences between Mean Tide Level (MTL) and Mean Sea Level (MSL) using data from a worldwide set of tide gauges. MTL is defined as the average of Mean High Water (MHW) and Mean Low Water (MLW) over a period such as a year, while MSL is the arithmetic mean of regularly sampled (e.g. hourly) measurements of sea level during that period. It will be seen that at many locations the two quantities are the same to within ~1 cm. However, in some areas, particularly shallow-water locations, they can differ by many cm. In these cases, it is important for a sea level researcher firstly to appreciate that there is a significant difference, and then to understand the reasons for the difference as far as possible.

There are both scientific and practical reasons to have more understanding of the MTL-MSL difference. A scientific reason is an interest in understanding which aspects of the tide contribute to the difference, and it will be shown below that there is more to this topic than usually appears in the text books. There are several practical reasons. For example, mean levels derived from tide gauge data are often used as geodetic datums for land survey purposes. Therefore, at locations where MTL and MSL differ significantly, it is important that they are not confused in the definitions of the land datum. A further complication stems from some geodetic agencies historically determining sea level-related datums only from high and low water measurements made in daylight, when the sampling of highs and lows can lead to a different MTL than would be obtained from measurements made around the clock.

A second practical reason concerns values of MSL and MTL that are archived in international databanks such as the Permanent Service for Mean Sea Level (PSMSL, Holgate et al. 2013) and which are used in studies of sea level variability and long-term change. Although the accuracy of an annual mean value is difficult to estimate, and will differ between locations and the tide gauge technology used, it is usually considered to be ~1 cm (see Table 2.2 of Pugh and Woodworth 2014). Consequently, if measurements of MSL and MTL are mixed in a sea level time series, and if the two quantities differ by significantly more than 1 cm, then there will be false interannual variability introduced into the record. One answer to this problem is never to mix the two quantities in such records. However, if long records are required for study, then it will often be tempting for a researcher to combine the two so as to obtain the longest possible composite time series. In these cases, it will be necessary to know which years of data are MSL and MTL and to understand, and correct for, the difference as far as possible.

The following Section provides some 'text book background' to the differences between MTL and MSL. Section 3 describes the tide gauge data available for study and the analysis methods used. The main results are presented in Section 4, while Section 5 describes some case studies stemming from the results. Validation of the findings using datum information from the US tide gauge network is given in Section 6. In Section 7, the analysis is broadened to consider MTL differences from MSL across the world ocean, using a state-of-the-art global tide model. Section 8 considers how the seasonal cycle of sea level measured using MTL information may differ from that using MSL, while Section 9 investigates how MTL measured only in daylight hours differs from that measured around the clock. Implications of the findings for the PSMSL databank are discussed in Section 10. Section 11 presents conclusions from the work.

2. Text Book Background

There has been a long history of attempts to understand the difference between MTL and MSL from first principles (Doodson and Warburg 1941; USC&GS 1952; Rossiter 1958; Pugh and Woodworth 2014; Simon 2015). Most text books discuss a predominantly semidiurnal tidal regime and explain the difference in terms of the M4 harmonic of the main M2 constituent:

$$MTL - MSL = H_{M4} \cos(2g_{M2} - g_{M4}) \tag{1}$$

(plus terms involving M8, M12 etc.), where H_{M4} is the amplitude of M4 and assuming $H_{M2} >> H_{M4}$, and g_{M2} and g_{M4} are the phase lags of M2 and M4 respectively. The way that M4 results in a finite MTL-MSL can be readily appreciated from Figure 1(a), in which M4 produces more positive high and low waters than would have occurred with M2 alone, and so a positive MTL-MSL, and Figure 1(b), in which more negative high and low waters result, together with a negative MTL-MSL. An important thing to note is that M2 and M4 are phase-locked so the patterns shown in these figures will be the same for every tide.

However, such a discussion omits consideration of parts of the world coastline where the tide is either 'mixed and mainly diurnal' or 'diurnal' (Figure 2). This includes the coasts of Antarctica, Caribbean, Gulf of Mexico, North Pacific, South China Sea, Gulf of Carpentaria and Western Australia. These locations have large K1 and O1 components; the former is both lunar and solar in origin while the latter is purely lunar (Pugh and Woodworth 2014). When these two tides are large and have a similar amplitude, they can beat together to generate, what can be parameterised as, an M1 carrier wave within an envelope with a period of 27.3 days, the phase of M1 flipping by 180°

between the two halves of the envelope (Figure 3). This combination of K1 and O1 into an M1 is the same process that results in the contribution of diurnal tides to asymmetries in tidal probability distribution functions (Woodworth et al. 2005).

Any M2 in the record will combine with this pure M1 to produce distortions in the combined signal, and thereby differences between MTL and MSL, in a similar way to the M4 combination with a pure M2 shown by Equation (1). The MTL above MSL that results from normal M1 and M2 sinusoids can be written as:

$$MTL = MTLA + MTLB$$

$$MTLA = \frac{H_{M1}}{2}\left[\sin x \sin \vartheta - \sin y \cos \vartheta\right]$$

$$MTLB = \frac{H_{M2}}{2}\left[\cos X - \cos Y\right]$$

where $x = \frac{r}{2}\sin\vartheta$, $y = \frac{r}{2}\cos\vartheta$, $X = r\sin\vartheta$, $Y = r\cos\vartheta$, $r = \frac{H_{M1}}{2H_{M2}}$ and $\vartheta = \frac{g_{M2}}{2} - g_{M1}$.

(2)

It can be seen that, if the phase lag of M1 changes by 180°, then MTLA, MTLB and MTL are unchanged. This means that, in the case of the effective M1 that originates from K1 and O1, there will be an equal contribution to MTL in both halves of the envelope. In parts of the world coastline with diurnal or mixed tides, this contribution from the diurnal tides to MTL-MSL can be considerably larger than that of the M4 contribution of Equation (1). (The division of g_{M2} by 2 in Equation (2) initially looks odd, as one would expect everything to be unchanged if g_{M2} increments by 360°, whereas a $\frac{g_{M2}}{2}$ change of 180° must change the signs of x, y, X and Y. This is an artefact of M1 spanning two M2 cycles, one sign being relevant to one of the two high and low waters.)

Another text book cliché is the statement that M6 (and M10 etc.) cannot contribute to an MTL-MSL difference. This statement is correct in the ideal situation when only M2 and M6 are considered. However, in practical situations when one has an M6 with significant amplitude, there is almost certainly also a significant M4. Therefore, consideration of M6 cannot be made in the context of distorting a pure M2 signal, but rather the further distortion of an already distorted curve due to M4. In these cases the additional contribution of M6 can be large as demonstrated further in Section 5.1.

This paper addresses some of these issues at different locations around the world. In some of the examples discussed below, the contribution of M4 (or M4 plus M6) can be seen to simply add to that from K1 plus O1 (i.e. the effective M1) to produce the overall MTL-MSL difference. In addition, there will be contributions from other pairs of constituents that can combine in similar ways but usually with a lesser importance. For example, Q1 and J1 can also combine to produce an M1 carrier wave. All of these smaller contributions (e.g. esoteric ones involving third-diurnals) can best be studied via simulations rather than analytically. However, an important point to make is that all the individual contributions are never truly independent but combine in the complexity of the tidal curve to produce the overall MTL-MSL.

3. Data and Methods

Our tide gauge information comes from a data set called GESLA-2 that has been assembled from a number of national and international sea level databanks by Philip Woodworth (National Oceanography Centre, UK), John Hunter (University of Tasmania, Australia), Marta Marcos (University of the Balearic Islands, Spain), Melisa Menéndez (University of Cantabria, Spain) and Ivan Haigh (University of Southampton, UK). It is an update and extension of the GESLA (Global Extreme Sea Level Analysis) data set used by Menéndez and Woodworth (2010) and others. Although there are many individual contributions to GESLA-2, over a quarter of its station-years are provided by the research quality data set of the University of Hawaii Sea Level Center. The data set already has a reasonable geographical distribution although it is planned to add other data to it in the future as they become available.

The data set presently contains 36877 station-years of information from 1308 station records with some stations having alternative versions of the records provided by different sources. Therefore, there are 28 years in an average record, although the actual number of years varies from only 1 at several short-lived sites, to 167 in the case of Brest, France. More information on the data set, and the data itself, may be obtained from www.gesla.org.

Each year of data was subjected to a tidal analysis using a standard set of 63 constituents (Appendix 1). Sixty of these harmonic constituents (i.e. the 63 minus the MSL, called Z0 in tidal terminology, and the annual and semiannual components, called Sa and Ssa) obtained from each year were then used to generate 1-minute tidal predictions for a given ‘reference year’, which was chosen to be 2011 on the basis that the main semidiurnal and diurnal tides in that year will have amplitudes within the nodal cycle that are approximately average (see Table 4.3 of Pugh and Woodworth 2014 and Errata corrections to the Table in http://www.cambridge.org/9781107028197.) The 1-minute values were then used to determine the high and low turning points of the tide, and thereby MHW, MLW and MTL.

Because the harmonic constants used for the 1-minute predictions did not include Z0, then the MTL calculated in this procedure will correspond to the MTL-MSL one would obtain using the original data itself. In addition, because the harmonic constants did not contain Sa and Ssa, the seasonal cycle of the calculated MTL will correspond to the difference between the cycles obtained using monthly MTL and MSL computed from the original data.

Each year of data in a particular tide gauge record will thereby provide a separate estimate of MTL-MSL for 2011, from which one can find the median MTL-MSL that we consider to be the best estimate of this quantity. The median selection procedure guards against possible measurement errors in the sea level record in particular years, and therefore in the derived harmonic constants. The standard deviation of MTL-MSL obtained from the individual years of data provides an assessment of how well the MTL-MSL best estimate is known. Implicit in the method is an assumption that there is no long-term change in the tide (or even short-term ones due to dredging, for example). Such changes are known to have occurred at many locations (e.g. Woodworth 2010) but are in general much smaller than could affect the MTL-MSL computed here. At locations where the tide is changing rapidly then a more sophisticated analysis would be required.

One may ask why one wants to work this way, using tidal predictions derived from each year of data, rather than determine MTL and MSL from the data itself. The main reason is that individual years of data can be incomplete, or there can be measurement errors that are not detected by quality control procedures. In addition, the record will contain an amount of non-tidal sea level variability due to storm surges etc. that will vary from year to year. The approach of using predictions derived from these data provides sea level time series that are complete and can be studied as being completely astronomical tidal in origin. Comparisons between MTL-MSL obtained using this tidal prediction approach and values published by the National Oceanic and Atmospheric Administration (NOAA), obtained directly from sea level measurements over their 'tidal datum epoch' of 1983-2001, are given in Section 6.

Another research option is to use databanks of harmonic tidal constants, such as that maintained for some years by the International Hydrographic Organization (IHO). One-minute predictions can be produced directly from those constants, instead of undertaking a tidal analysis of each year of tide gauge data as in this study. However, this option has several disadvantages. The IHO databank is no longer maintained, and its older versions are known to contain many errors in tidal amplitudes and phases. Different numbers of tidal constants are given for each station, based on tide gauge recordings of different durations (sometimes only a few days or weeks, in other cases over a year) at different times. Alternatively, there are a number of national and specialist databanks of harmonic constants, but some of those do not make their data freely available for research, and assembling them all into a quasi-global set would have presented as many difficulties as collecting the GESLA-2 data set. Most importantly, whatever their source, harmonic constants are difficult to verify, if anomalous findings on MTL-MSL occur, without access to the original data from which they were computed. The use of multiple years of original data, as we have used in this analysis, allows constants to be produced each year and compared for consistency.

In spite of these reservations, we have made use of the IHO databank below (with caution) to check and extend some of our findings from GESLA-2. In particular, we have used IHO information for India for which GESLA-2 contains no information.

Another option is to make use of global or regional tide models. We have employed one such model below to simulate the contributions to MTL-MSL from M2, M4, K1 and O1. However, there is no model available that can reliably simulate all the shallow water constituents (M6 etc.) that would be required in an ideal model study.

4. Results

The MTL-MSL best estimate obtained using the method described above will have uncertainties for several reasons:

(1) The harmonic constants determined from each year of data will vary slightly due to the many sources of variability in the ocean, resulting in slightly different predictions for 2011 and thereby MTL-MSL. Therefore, the best estimate for MTL-MSL difference will depend to some extent on which years of data happen to be available. Its uncertainty can be estimated from the standard deviation of the values of MTL-MSL obtained for each year.

(2) The best estimate will vary with the chosen reference year (e.g. 2011) due to nodal (and to a lesser extent perigean) variations in those constituents that are responsible for the MTL-MSL difference. This variation will be greatest where diurnal contributions are primarily responsible for the difference, owing to their nodal variations in amplitude and phase being larger than for semidiurnal constituents (Pugh and Woodworth 2014). This aspect is investigated further in Section 5.5.

(3) The MTL-MSL difference will vary depending upon the choice of constituents employed in the harmonic analysis. The set of constituents that has been used in the present analysis (Appendix 1) will provide a good parameterisation of the tide using individual years of data at most locations. However, at locations where a multitude of shallow water terms are

required to better simulate the local tide, we would expect the determined MTL to be less accurate. This is explored further in Section 5.1.

With these reservations in mind, Figure 4(a) shows the distribution of best estimates of MTL-MSL at the stations in our data set. To some extent the map is a reflection of the tidal range, as the MTL-MSL difference will tend to be larger when M4 and/or K1 and O1 are larger. Figures 4(b-d) focus on N America, NW Europe and Asia/Australasia respectively. Inspection of these maps shows some spatial continuity. For example, the US west coast values are mostly small and positive (<1 cm), while those of the east coast are small and negative (Figure 4b). A small enclave of positive MTL-MSL exists at approximately 41°N around Newport and Providence, Rhode Island and Woods Hole, Massachusetts. Positive values are obtained along the coast of southern Africa, while negative ones are found in west Africa (Figure 4a). Values change from consistently negative to consistently positive travelling north along the Pacific coast of South America (Figure 4a). Values in Japan are almost all positive (Figure 4d).

On the other hand, there are areas where large differences occur over short distances. The UK coast and that of northern France to the Netherlands provide examples, with more consistent negative values obtained further south around the Bay of Biscay (Figure 4b). This spatial complexity in NW Europe is discussed further in Section 7.

Even though values change significantly over short distances in some areas, it is important to note that this is not due to some kind of measurement noise. Given the large amount of data that has been used to determine each MTL-MSL best estimate, all of them are undoubtedly individually correct. The standard deviation of MTL-MSL for 2011, determined from each year of data, exceeds 0.5 cm at only 65 of the 1245 stations in Figure 4. These are largely locations where the average MTL-MSL is large, and others are where the tide is known to be calculated less accurately owing to suspected timing errors in the measurements. The largest standard deviation of 3.8 cm, at Charchanga in Bangladesh, is an example of the latter.

The maps of Figure 4 include all stations and are made using all turning points (high and low waters) obtained from the 1-minute predictions for 2011 derived from the year of data corresponding to the MTL-MSL 'best estimate'. There is no other way in which a global distribution can be computed. However, it will be seen in Section 5 that on certain occasions for some stations there are turning points that 'common sense' would dictate should not be included in the calculation of MTL. In particular, there are a small number of stations with many more turning points than the ~705 highs and ~705 lows one would expect in a semidiurnal regime each year; these will be locations with double or triple high or low waters. Consequently, unless otherwise stated, the histograms and statistics presented below have been obtained only from stations where 710 or fewer turning points occurred in 2011 (the choice of 710 is not a critical one). Figure 5 histograms the MTL-MSL differences with this selection, and statistics are provided in Table 1. Most values have a magnitude below 1 cm and so can be considered 'small' for the reasons given above. However, there is a long tail with 20% greater than ±1.60 cm and 5% greater than ±6.55 cm.

For each station, three further sets of 1-minute tidal predictions were made, making use of the set of harmonic constituents determined from the year of data corresponding to the MTL-MSL best estimate. These additional predictions employed M2 plus M4 only, or M2 plus M4 and M6, or M2 plus K1 and O1, in order to investigate the main individual contributions to MTL-MSL. Statistics for these contributions are also shown in Table 1.

If one subtracts these estimated individual contributions from the best estimate MTL-MSL, then one obtains the statistics shown in lines 5-7 of Table 1. Line 5 shows that, as would be expected from

Equation (1), M2 plus M4 does a good job in explaining some of the MTL-MSL difference shown on line 1 and in reducing the tail. M2 plus M4 and M6 does even better. M2 plus K1 and O1 also reduces the tail in the distribution as shown on line 6. Line 7 shows that M2 plus M4 and M6, together with M2 plus K1 and O1, explains the tail more completely, such that only 5% of MTL-MSL values remain unexplained by more than ± 1.56 cm.

Figure 4(a) shows that there are no records from India in the GESLA-2 set. Therefore, in this case (and with the reservations explained above) we have made use of tidal constants in the IHO data set, generated 1-minute predictions for 2011, and followed the same procedure as for the analysis of GESLA-2 data. Of the 183 sets of constants available from this region, 59 were rejected as not fulfilling a minimum requirement of containing information on the 5 main constituents (M2, M4, M6, K1 and O1) that are likely to contribute to an MTL-MSL difference.

Figure 6 shows that MTL-MSL differs by only ~1 cm, and is positive, for the west coast of India apart from the Gulf of Khambaht in the north, where several values of ~5 cm or more are found, and further north in the Gulf of Kutch, where there are examples of MTL-MSL of approximately -5 cm. These two gulfs are regions of local tidal resonance (Shetye 1999; Nayak and Shetye 2003). On the east coast, values of MTL-MSL are mostly small and positive, although there are several locations with larger positive values and others with slightly negative values. Sri Lanka also has slightly negative values. The IHO data set has only sparse information from the northern Bay of Bengal to compare to the strongly negative values shown from GESLA-2 in Figure 4(a). The latter come from stations in Bangladesh, where there are large tidal amplitudes and shallow waters that are likely to generate large and localised MTL-MSL variations (the quality of the data in these records, especially to do with timing, is a particular issue.)

Maps corresponding to Figure 4, but made from IHO information, are included in the Supplementary Material (Figure SM1). Of the 4135 sets of constants available globally, 2866 were rejected for not containing information for the 5 main constituents, and 22 others from early off-shore bottom pressure records were also removed. There are many features in common between Figures 4 and SM1, which contain a similar number of stations and so complement each other. However, there are also differences, for example from New Zealand or the north coast of France for which we believe Figure 4 to be more reliable. We have not been confident enough to make more use of the IHO information.

5. Case Studies

This section presents several case studies which illustrate aspects of the difficulty of determining MTL at certain stations. The first, from Liverpool, explains the importance of M6 to MTL-MSL, most tidal text books having suggested that M6 plays no role. The next case studies, from Weymouth, Adak and Bintulu, demonstrate the difficulty of defining MTL in situations where the tidal curve does not have its classical semidiurnal form. The final case study considers the importance of the choice of reference year.

5.1 Liverpool

Liverpool provides an interesting case study of a location with a large and predominantly semidiurnal tide, in which several topics concerning contributions to MTL-MSL can be investigated. A first topic is a demonstration of the contribution of M6, using constants for M2, M4 and M6 from Amin (1982). These three constituents have amplitudes of 312.8, 23.4 and 5.3 cm respectively.

The top part of Table 2 gives the times of high and low water over a day defining the 'tide' by the M2 sinusoid alone. The real phase lags for each constituent have been used in this example and are taken as relative to time zero. The first low water occurs at time 0.206 days, followed by high, low and high waters spaced equally apart by 0.259 days. Now, imagine adding M6 to this M2. Because M6 has three times the speed of M2, it will contribute an opposite amount at low and high tide (± 1.6 cm in this example). The resulting MTL-MSL can be calculated as the average of the four M2 heights over the tidal day and the four contributions from M6, giving zero in this case.

Next, imagine that the tide contains M4 as well as M2. As shown in Figure 1, M4 will modify the simple sinusoid of M2, so that the heights of low and high waters are changed and their times are no longer equally spaced, as shown by the middle part of Table 2. The unequal spacing in time means that the contribution from M6 at an M2+M4 low water is not simply the negative of that at a high water.

Finally, the bottom part of Table 2 shows the heights and times of low and high waters, calculating the tide from all three constituents. It can be seen that M6 has reduced by about a third the MTL calculated from M2 and M4 in the middle part of Table 2 (or alternatively using Equation 1). Therefore, in practice, M6 can indeed make a contribution to MTL.

A second topic in this case study concerns the best estimate of MTL-MSL at Liverpool. Many years of experience with Liverpool data (e.g. an unpublished study in the NOC archives of 8 years of Liverpool hourly values by A.T. Doodson in the 1930s) showed that MTL-MSL is about 0.19 feet (6 cm), which is consistent with the 6.5 cm one would calculate using the 110 constituents of Amin (1982). However, if only the shorter set of constituents in Appendix 1 is used, one obtains a smaller value of 4.4 cm. Most of the difference can be traced to contributions from several constituents such as M8 that are not in this set. This demonstrates that the best estimates of MTL-MSL that we have calculated in this paper, based on the set of constituents in Appendix 1, may not be as accurate as examining MTL-MSL from 18.6 years of real data at locations such as Liverpool where the tide is large and there is a complexity of higher harmonics.

A third topic is to do with tides measured at different locations in estuaries. The Liverpool harmonic constants mentioned above were derived from measurements at Princes Pier on the Liverpool waterfront. However, since the early 1990s, measurements have been made instead at Gladstone Dock, which is 3 miles downriver. The latter comprise the Liverpool information in the GESLA-2 data set. Although the tide is predominantly semidiurnal throughout the Mersey estuary, and M2 and M4 have similar amplitudes at Princes and Gladstone, the phase lag of M4 decreases from approximately 214° at the waterfront to 202° at Gladstone, while the M2 phase lag remains at about 320° (Lane 2004). Equation (1) shows that the change in M4 phase lag is sufficient to reduce MTL-MSL at Gladstone to about half its value at Princes Pier. Meanwhile, the M6 phase lag increases by over 20°, and altogether these tidal differences result in MTL-MSL at Gladstone that is about a half of that at the waterfront. Similar changes in MTL can be expected to occur over short distances in other estuaries.

Further up estuaries, the methods used in the present study are not appropriate as the tidal curve is not parameterisable as a sum of harmonics (Godin 1999; Pugh and Woodworth 2014 Chapter 6). For example, a tidal analysis of data from Avonmouth in the Bristol Channel shows large positive and negative residuals at low tide which are not well understand (primarily because they are of little interest with regard to flooding) but, nevertheless, contribute to the observed MLW and thereby MTL. Such features are clearly not represented in the tidal constants for the site and thereby in the MTL one obtains using the present method. At certain times, the tide gauge at this location bottoms out at low tide (Proctor and Flather 1989) and no method will work in this case.

5.2 Weymouth

When shallow-water processes result in the amplitude of M4 becoming larger than ¼ of that of M2 then, for certain combinations of phases ($2g_{M2} - g_{M4}$), there can develop double high or low waters. Figure 7 shows schematic examples with M4 amplitude 30% of that of M2. Figure 7 (a,c) shows that double lows and highs are obtained for ($2g_{M2} - g_{M4}$) of 0 and 180° respectively, while for Figure 7(b,d), corresponding to 90 and 270°, there are no double waters. This figure is included in many text books on tides e.g. Figure 2.27 of Parker (2007) or Figure 6.4 of Pugh and Woodworth (2014). M6 can also contribute to double highs or lows when its amplitude exceeds 1/9 that of M2, and other quarter- and sixth-diurnal constituents will also play a role.

Double high or double low waters are a feature of the tide on the coasts of southern England, northern France, Belgium and the Netherlands. Southampton provides the most famous example of double high waters (Figure 6.1 of Pugh and Woodworth 2014), which are also found at ports such as Le Havre in France or Den Helder on the northern Netherlands coast. However, just to the west of Southampton, at Portland or Weymouth, and at Hook of Holland on the southern Netherlands coast, one finds that the tide contains double low waters instead of double highs.

As an example, Figure 8 presents the tidal curve during spring tides at Weymouth, demonstrating the double low waters. An important question then is, how does one determine MHW and MLW, and thereby MTL? Common sense would suggest that MHW is determined from the major highs, and not by including the small high waters that occur between the double low pairs. Similarly, one would imagine that MLW is calculated from the lower of the two lows, or possibly from the average of the two lows. But how does one know? The point is that any reported MTL has to be accompanied by a statement, not only of the year or other period it is measured over, but also by an explanation of how it was computed.

In this case, if one defines MHW and MLW using the 'common sense' approach (i.e. ignoring the small high water and the higher of the two low waters) then the resulting MTL will be essentially the same as that which would have been obtained with only the M2 and M4 components of the Weymouth record. The same situation applies for the French and Dutch stations with double highs and lows. In these cases, the contributions to MTL from M4 can be very large (order ± 20 cm) and so it is essential to compute it correctly. We return to this issue in Section 10.

5.3 Adak

Adak in the Aleutian Islands provides another example of the difficulty of unambiguously defining MTL. The tide here is predominantly diurnal. Figure 9 shows the curve for 9 February 2011 with the major low and high waters at the start and end of the day (in GMT). However, in the 3 hours just before and after midday, the curve can be seen to contain a small oscillation with a minor high and low water. Should these be included in the calculation of MHW and MLW? If one does include them, then clearly both quantities, and also MTL-MSL, will be biased towards zero.

The NOAA web site provides a list of the high and low waters at Adak identified by their software (http://tidesandcurrents.noaa.gov/waterlevels.html selecting Adak and HL data for 9 February 2011). The turning points at the start and end of the day are flagged but not the ones in the middle. This suggests that NOAA does not consider them as proper turning points, which would again be a common sense approach. But at what point does one accept minor high and low waters as real ones?

It seems that the NOAA software follows a procedure similar to that suggested by Hicks (1980), in which he recommended that high and low waters should not be considered as true turning points when they differ in height by less than 0.1 ft (approximately 3 cm) from an adjacent low and high water. The modern software also requires that adjacent high and low waters be separated by at least 2 hours (NOAA 2009). However, these are somewhat *ad hoc* criteria, and the practice of different tide gauge agencies may well be different. Therefore, it is important that each agency provides detailed documentation of its data processing procedures.

5.4 Bintulu, Malaysia

Bintulu in Malaysia is where the simple model of explaining MTL-MSL difference in terms of contributions from the 5 main constituents is least successful (by 7.4 cm). This location has some similarity to Adak. It is a predominantly diurnal location with amplitudes of K1 and O1 of 44 and 36 cm respectively, while that of M2 is 19 cm, and M4 and M6 are 1.7 and 1.1 cm respectively. Unusually, there are large 3rd diurnal tides with MO3 and MK3 amplitudes of 3.7 and 3.6 cm respectively.

Inspection of the record shows predominantly diurnal tidal variations, the amplitude of which reduces to small values at 'diurnal neaps'. High waters are seen to be sometimes double (as for Southampton discussed above but now on top of a diurnal rather than a semidiurnal tide) and sometimes triple, while low waters are normal diurnal. At the diurnal neaps, there are sometimes multiple oscillations that could individually count as high or low water turning points. Consequently, MHW, MLW and MTL at this location are once again hard to define.

5.5 Nodal Variation in MTL-MSL due to choice of Reference Year

The importance of the choice of reference year that has been used in this analysis (2011) can be explained in the following two simulations of a tide containing only M2, K1 and O1 constituents with amplitudes of 100, 25 and 25 cm respectively. Two runs with different sets of phase lags were made (the exact values are not important). Figure 10 (a,b) shows for each run the MTL values that result for different choices of reference year.

An important first remark to make is that these MTL-MSL nodal variations arise from the sampling of the semidiurnal and diurnal tides at high and low waters (MHW, MLW, MTL), and must not be confused with the nodal component in the tidal potential which leads to a long period tide in both MSL and MTL (Woodworth 2012).

In run 1, the relative phases of M2 and the effective M1 were such that a non-zero MTL-MSL results with an average value of approximately -0.8 cm. However, because the amplitudes of O1 and K1 vary by 18.7% and 11.5% over a nodal cycle (18.61 years), with maximum values in 1969, 1987, 2006, 2025 etc., then the resulting MTL-MSL is approximately 20% larger in those years. Similarly, MTL-MSL will be 20% smaller for 1978, 1997, 2015 etc. Figure 10(a) shows that this is indeed the case, with 2011 being an approximately average year.

Figure 10(b) presents a different situation. Here, the relative phases of M2 and the effective M1 are such that an MTL-MSL results that is close to zero. However, there are still variations every nodal cycle around the mean value due to variations in the amplitudes and phase lags of each constituent. In this case, 2011 is not an 'average year'.

The K1/O1 contribution is the main term which varies with the nodal cycle. However, similar considerations apply to the M4 contribution: the amplitude of M4 varies over 18.61 years by approximately twice the 3.7% variation of M2 (or about 8%). Consequently, one may conclude that in any future study of this topic based on the above method, it would be best to average MTL-MSL findings over 18.61 'reference years' for stations in any tidal regime, as is the procedure usually adopted when determining MTL-MSL difference using the original tide gauge measurements (e.g. by NOAA over a tidal epoch). Such an 18.61 year approach was not used in the present study as it would have been considerably more demanding computationally. Instead, the approach of using 2011 as a 'nodal average' reference year has been found to be almost as good, as demonstrated by the findings of the next section.

6. Validation Using NOAA Data

To our knowledge, the only tide gauge agency that routinely provides detailed information on MSL, MTL and other tidal datums in its network is NOAA (https://tidesandcurrents.noaa.gov/datum_options.html). Their database contains datum information from many sites, of which 71 are also in the GESLA-2 set. The NOAA datums are derived from analysis of observed sea levels over the National Tidal Datum Epoch (1983-2001), which is a different approach to datum determination to our use of predictions for 2011 based on data for all available years. Nevertheless, the two approaches give similar results for MTL-MSL (Figure 11(a)). Correlation of the two sets gives a coefficient of 0.983. The NOAA datums confirm the 'Newport enclave' on the US east coast in Figure 4(b) mentioned above.

Another datum that can be investigated with both our tidal predictions and the NOAA database is Diurnal Tide Level (DTL). This is defined at the average of Mean Higher High Water (MHHW) and Mean Lower Low Water (MLLW), where 'higher' and 'lower' refer to the higher of the two high waters and lower of the two low waters respectively during a 'tidal day'. In the above notation, DTL can be written schematically as:

$$\begin{aligned} HHW &= H_{M2}\cos(r\sin\vartheta) + \left|H_{D1}\cos\vartheta\right| \\ LLW &= -H_{M2}\cos(r\cos\vartheta) - \left|H_{D1}\sin\vartheta\right| \\ DTL &= \left(MHHW + MLLW\right)/2 \end{aligned} \tag{3}$$

where D1 in (3) refers to what can be called the 'daily harmonic' (comprised of K1, O1 and many other diurnal constituents) that occurs on a particular day (cf. Pugh and Woodworth 2014, Section 6.3), while the absolute value in the second part of HHW and LLW ensures that the daily harmonic contribution to MHHW and MLLW does not average to zero over the year.

At some US locations, the historical sea level record consists of tabulations of DTL instead of MSL or MTL (e.g. Maul and Martin 1993). The difference DTL-MSL clearly depends on the diurnal inequality and can be very large where diurnal tides are large, such as the northern part of the Pacific coast of the continental US and Alaska. Figure 11(b) provides a comparison between DTL-MSL obtained from our tidal prediction method and from NOAA observations over the tidal datum epoch. The two can be seen to agree well (correlation coefficient 0.991) with DTL lying below MSL at most of the NOAA tide gauge locations.

There are few other published values of MTL and MSL obtained from the same tide gauge data which would allow an independent check on MTL-MSL. Two editions of the Publications Scientifiques of the International Association of Physical Oceanography (IAPO 1959,1963) contain both values for approximately a dozen stations in India for several years around 1960. In most cases, the reported values of MTL-MSL compare well to estimates using the methods of the present paper, apart from two stations in the Hooghly River (Diamond Harbour and Calcutta) where differences between the two of about 6 cm are found. These are two other examples of the difficulty of computing MTL-MSL in estuaries. De Bruyn (1900) gives a value for MTL-MSL at Delfzijl, Netherlands (-19.3 cm) that agrees exactly with that estimated in this study.

7. Global Distributions of the Main Contributions

Instead of using tide gauge data (or the harmonic constants derived from them) to study the various contributions to MTL-MSL, one can also make use of tide models. In the present study, we have used the Technical University of Denmark DTU-10 model (Cheng and Andersen 2010). The model has 10 constituents of which eight were derived by the authors (K1, O1, P1, Q1, K2, M2, N2 and S2) and two (S1 and M4) were taken from the GOT4.7 model of Richard Ray which is a successive update of the model of Ray (1999). The amplitude and phase lag of each constituent are provided on an 1/8° grid.

The DTU-10 model is derived from over two decades of satellite altimeter data, in common with a number of other presently available tide models (Stammer et al. 2014). These models all represent the open ocean tide excellently, and for our purposes can be considered as almost identical, but they are known to contain significant (and model-dependent) uncertainties near the coast, where tide gauges are located in estuaries and harbours and where shallow-water constituents such as M4 and M6 will vary considerably over short distances (see a discussion of coastal tides by Ray et al. 2011).

Therefore, while the model is claimed to contain an M4 component, and while that might be a faithful representation of M4 in the open ocean, where it could have an amplitude of 1-2 cm or more (e.g. Ray 2007), one cannot expect the model to represent M4 adequately at the coast. On the other hand, one knows that K1 and O1 (and their resulting M1) will have a much larger spatial scale than M4, and therefore the model should be able to simulate the M2 plus K1 and O1 contribution to MTL-MSL adequately at the coast. No global models that we know of include an M6 component.

One can make use of the model to generate 1-minute predictions for 2011 as for the tide gauges, so as to obtain some insight into the spatial scales of the separate M2 plus M4, and M2 plus K1 and O1 contributions to MTL-MSL. Figure 12 (a-b) shows the distributions for each of these two contributions, while Figure 12(c) shows that for the two combined.

The contribution from M2 plus M4 (Figure 12a) is small for most of the deep ocean, apart from regions such as west of Gibraltar where it is ~1 cm. Considerably larger values (that are difficult to inspect in detail at this global scale) are found on the NW European continental shelf, Patagonian Shelf, East China Sea and Hudson Bay. The component arising from M2 plus K1 and O1 (Figure 12b) can be much larger in the deep ocean, with larger values (several cm) around Antarctica and in the northern Arabian Sea and N Pacific. Short spatial scale patterns can be seen near to some (but not all) M2 amphidromes such as near to the Philippines and to the west of California. When added together (Figure 12c), the large-scale deep ocean pattern is largely that of the M2 plus K1 and O1 contribution, while patterns on the shelves are largely those of the M2 plus M4 contribution.

The global distributions of Figures 12(a-c) were made adequately by sampling the DTU-10 1/8° grid every 1°. However, in order to focus more generally on the coastal MTL-MSL, instead of the deep ocean, the original 1/8° grid was employed at ocean grid points that were adjacent to one or more land points (Figure 12d). This coastline map has much the same information as Figure 12(c), but shows better the short spatial scale variations along all coastlines, especially those of the SW Atlantic, SE Asia and the Pacific coast of N America. In particular, in the latter can be seen a small section of high positive MTL-MSL near to Vancouver Island that is also seen in the GESLA-2 data

(Figure 4b) and IHO information (Figure SM1(b)). On larger scales, the same patterns can be seen in Figures 4(a) and 12(d) along the coastlines of the Americas, Africa and Australasia.

The examination of MTL-MSL changes over short distances in shelf areas also requires the use of the original 1/8° grid. Figure 12(e) shows MTL-MSL for NW Europe which may be compared to the values derived from tide gauge data in Figure 4(c). The model demonstrates the short spatial scale complexity that occurs in shelf areas, primarily due to the M2+M4 component, and confirms much of the spatial pattern obtained from the tide gauges. For example, the reason for the change from negative to positive values at the western extremity of Britanny in Figure 4(c) can be understood from the band of positive values extending southeast from the Celtic Sea shown in Figure 12(e). On the other hand, the model fails to reproduce the large positive values in the eastern Irish Sea, preferring large negative ones.

8. The Seasonal Cycle of MTL-MSL

The 1-minute predictions for 2011 that were generated for each station, as described in Section 3, were calculated from a set of harmonic constituents that did not contain the annual and semiannual components of MSL (Sa and Ssa). Therefore, the high and low waters, from which the annual MTL discussed above has been computed, can also be used to investigate the extent to which the seasonal cycle of sea level calculated from MTL differs from that obtained from MSL. Several studies have investigated the seasonal cycle of sea level using monthly means from the PSMSL (e.g. Tsimplis and Woodworth 1994). Most of these monthly mean values will be MSL but a small number are MTL and consequently it is of interest to ask what biases there may be between them.

Monthly MHW and MLW values were calculated from the predictions, and thereby monthly values of MTL were obtained, enabling the amplitudes and phases of Sa and Ssa in MTL-MSL to be determined. Figure 13(a) shows that in most cases the annual amplitude in MTL-MSL is less than 1 cm, although the distribution has a tail of 23 stations in excess of that amount. Many of the stations in the tail can be identified as those with large amplitudes in the constituents MA2 and MB2, which represent the annual variation in the M2 tide (Figure 13b). The semiannual amplitudes are approximately 50% larger than the annual values (Figure 13c) with semiannual amplitudes for 91 stations in the tail above 1 cm. It is more difficult to identify its source, but Appendix 1 shows that there are a number of other constituents that can modulate seasonally those we have discussed above. In particular, P1 can have an amplitude of many cm and will modulate K1 over half a year.

This interpretation of the sources of the annual and semiannual components is confirmed by generating predictions for 2015 and 2019 instead of 2011. In each case, the amplitudes of the annual cycles of MTL-MSL so obtained were found to be the same to within 1 cm for all stations, as would be anticipated if MA2 and MB2 simply modulate M2 each year. Larger differences of up to 2.5 cm were obtained between the amplitudes of the semiannual cycles in the three individual years, as might be expected when constituents such as P1 and K1 vary over the nodal cycle.

9. MTL Measured in Daylight Only

A further question concerns whether there are major differences between MTL determined from high and low waters measured around the clock and those recorded only in daylight hours. In fact, there seem to be only small differences at most locations. However, it is useful to have an appreciation of why there are differences at all.

The tidal literature contains many examples of MTL measured only in daylight. It is known that the small number of valuable sea level records that extend back to the 18th century are derived from measurements of high and low waters (or sometimes high waters only) obtained in different ways at different times. Brest provides a particularly important example of such time-dependent recording. During the years 1711-1835, there were different percentages of high and low waters recorded, sometimes during both day and night and at other times during daylight only (see Figure 8.2 of Pouvreau 2008). Low water measurements at night were generally rarer than high water measurements in daytime. Another historical example is that from the First Geodetic Levelling of England and Wales in the 19th century, when the Ordnance Survey made measurements of high and low waters during daylight hours only in order to determine the apparent gradients of sea level (i.e. MTL) around the coast (James 1861).

The Publications Scientifiques of IAPO were, in effect, the predecessor of the PSMSL databank. In the first of this series (IAPO 1939), one finds a list of the acceptable ways for measuring sea level (MSL, MTL etc.) that includes MTL measured in daylight as one possible option (measurement method type 5b). In fact, from inspection of the series of IAPO reports, it seems that the only data measured that way to have been included in the databank were 19th century values from the Netherlands, and these data were claimed to have been corrected for 'shallow water effects', as were Dutch values of MTL obtained from measurements at all hours (see Section 10).

It is difficult to determine accurately any long-term biases in MSL/MTL trends due to the time-dependent (daylight or around the clock) sampling of MTL such as occurred at Brest (Pouvreau 2008 provides a discussion of this topic). However, one can make simpler tests using the 1-minute predictions for 2011 obtained above. Figure 14 shows the difference between MTL determined from data obtained at all hours and MTL obtained from data only in daylight hours for the stations in the GESLA-2 set. Daylight was defined as the period between sunrise and sunset through the year at the latitudes and longitudes of each station using the algorithms of Yallop (1996). It can be seen that most differences are less than 1 cm but there is a small tail that reaches 6.7 cm for Moulmein, Myanmar.

The Moulmein difference occurs because of an unusually large S1 component (amplitude 11.2 cm). We suspect this is due to some kind of instrumental problem rather than a real ocean signal (Pugh and Woodworth 2014 discuss how temperature effects and incorrectly mounted tide gauge charts can generate false S1 signals). Nevertheless, this example serves to provide an interesting case study. K1 has similar amplitude (17.8 cm) and it turns out that these two constituents can explain the daylight bias, by beating together to generate an R1 carrier wave within an envelope with a period of 2 years. In effect, S1 provides the average daylight bias and K1 its seasonal dependence. The Supplementary Material provides a schematic example which provides further explanation.

Figure 14 shows that there are other locations where there are large differences between MTL measured at all hours and in daylight only. These are also associated with apparently large S1. Several of these are from NW Australia, Indonesia, SE Asia and British Columbia where the S1 ocean tide is known to be large and associated primarily with diurnal air pressure loading (Ray and Egbert 2004). (The possibility of associated strong local winds, or sea breezes, has also been suggested as a contributor.) However, 'large' in the case of Ray and Egbert (2004) usually means amplitudes around 2 cm, and not the much larger values shown in Figure 14 which we suspect are at least partly instrumental in origin.

10. Implications for MTL Data Stored in the PSMSL Databank

The PSMSL databank includes 52 sea level records with a possible mixture of monthly MTL and MSL information, with 38 in the Revised Local Reference set that is used for research into sea level change. The two largest subsets are from the Indian subcontinent (Pakistan, India, Sri Lanka) with 14 station records, and the Netherlands and Belgium with 10 records. As regards the former, the GESLA-2 data set that we have used in this paper has little data from this region on which to base conclusions as to the importance of MTL-MSL difference (Figure 4). However, by using tidal constants from the IHO data set (Figure 6), it seems that the differences will be mostly small. The DTU-10 model in Figure 12 also suggests that the difference will be only 1 or 2 cm around India with largest values in the northern Bay of Bengal.

Therefore, for stations from the Indian subcontinent in the PSMSL databank, it would appear to be reasonably safe to combine MTL and MSL information in the same time series analysis (i.e. to within ~1 cm difference), apart from the areas around the Gulfs of Khambaht and Kutch, Mumbai and Calcutta mentioned above. Certainly, information could be combined for relatively open ocean locations such as Sri Lanka or the Andaman Islands.

Stations from the Netherlands, with their double high and low waters, were discussed in Section 5.2. MTL-MSL in this region is likely to be large (one or two decimetres) and the case study showed that the method used to compute MTL must be known if MTL-MSL is to be estimated accurately. IAPO (1939) indicates that 19th century MTL data were provided to the PSMSL measured in daylight only (measurement type 5b) using tide poles and around the clock (type 5a) using 'recording gauges'. In both cases, it is stated that 'shallow water corrections' were applied to make them comparable to MSL measured at regular intervals through the day. One can only assume that these corrections were made accurately so as to be comparable to MSL as recorded later, and certainly inspection of Dutch MTL/MSL time series shows no obvious decimetric offsets between periods of different measurement.

The remaining stations in the 52 subdivide into a small number in individual countries: Australia (5), UK (4), Portugal (4), Germany (3) etc. MTL-MSL values for German stations computed in this study are large (several centimetres to a decimetre), but have been found to be in good agreement with those obtained by the 'k factor method' of Wahl et al. (2010). On the other hand, those for open ocean locations (e.g. Christmas Island in the Australia subset or Azores stations in the Portugal subset) have small MTL-MSL.

In early 2016, the PSMSL adjusted monthly and annual mean values in its databank that had been MTL so as to correspond to MSL as closely as possible. In addition, an explanation of how the adjustments were made was added to the station documentation in its web pages (also see News Item of 3 February 2016 in www.psmsl.org). A reasonable question is whether confusion between MSL and MTL information in the PSMSL data set could have affected any previously published sea level studies. However, given that the vast majority of data in the PSMSL is MSL, and that MTL-MSL differences are less than 1 cm at most locations, then any confusion is unlikely to have had any impact on studies of global- or regional-average change (e.g. Church et al. 2013).

Nevertheless, there could have been incorrect conclusions drawn from the use of individual records at locations where MTL-MSL is large. For example, Table 3 of Tsimplis et al. (2005) reported an anomalously high trend of 6.5 mm/year at Cuxhaven, Germany for 1954-2000 that was almost certainly an artefact of data up to 1984 being MTL with MSL thereafter; MTL measurements were standard at stations in the German Bight until recently (see Wahl et al. 2010). In addition, Figure 2 of Unnikrishnan and Shankar (2007) showed the record for Kandla, India with anomalously low values in its early years, that were MTL rather than the MSL in the later part of the record.

Any confusion will in general have manifested itself in medium-length or short records, or sections of long records, rather than in the longest records that are often used for studies of long-term sea level accelerations (e.g. Hogarth 2014). That is because the sections of MTL information in those long records are short and/or MTL-MSL is anyway small. For example, there are sections of MTL data for 1903-1911 in the overall record spanning 1858-1983 for Liverpool (Georges/Princes Pier), 1807-1835 within 1807-2014 for Brest, and 1931-1958 within 1878-2010 for Mumbai, where MTL-MSL is approximately 65, -23 and 31 mm respectively. The main impact on the overall sea level trends is for Brest, where the MTL data are at the start of the record, and the adjustment results in an overall trend now 0.09 mm/year less than previously estimated. The Liverpool trend is now 0.07 mm/year greater, while that for Mumbai is 0.04 mm/year less than previously. These small changes have roughly the same magnitude as one standard error on the trend.

As well as consideration of secular trends, interannual variability in sea level could have been represented incorrectly in studies that have included records containing both MTL and MSL information. However, at the time of writing, no publications are known that could have been seriously impacted in this respect. It is worth mentioning that many PSMSL users will already have been aware of these issues and made appropriate allowances in their analyses of sea level trends and variability (e.g. the long Brest and Liverpool records shown by Woodworth et al. 2011).

11. Conclusions

There were several objectives for the present study. One objective was to collect sufficient information on the differences between MTL and MSL so as to alert sea level scientists (should it be necessary) to the potential errors that could be made when using the wrong mean levels as geodetic datums or as parts of time series of sea level change. It has been seen that, when MTL measurements are used to define a geodetic datum, it is essential to document the epoch of the measurements and to recognise that MTL will differ from MSL and the difference will vary over a nodal period (Section 5.5). Ideally, any such measurements should take place over a ‘tidal epoch’ such as that of NOAA, although the use of an ‘average nodal year’ in the present study has been proved to be almost as good. It has also been shown that, when sets of MTL and MSL data are combined in sea level records, then their differences can potentially contribute to errors in sea level trends and false interannual variability.

A second objective was to understand the reasons for the difference between MTL and MSL as far as possible, by determining those tidal constituents that play the most important roles. The fact that M4 plays a major role was already well known. However, one purpose was to point to the importance of diurnal constituents, a fact that is largely missing from text books on tides. (A reviewer pointed to a good discussion of this topic in USC&GS 1952).

The study has shown that there are indeed large scale patterns in the global distribution of MTL-MSL. This was to be expected as maps of K1 and O1 for the global ocean have been available for many years, and their high amplitudes (many decimetres at some locations) and large spatial scales might have been expected to contribute to the large scale MTL-MSL pattern. More recently, global maps of M4 have also become available and that component has been shown to also have a large spatial scale in the deep ocean.

However, as one approaches the coast, the shallow water components become a more important consideration and are increasingly spatially variable. This is evidenced for example by the migration from double low water tides at Weymouth to double high water tides at Southampton only 50 miles apart on the south coast of England. Our findings on the shorter spatial scales of variation in MTL-MSL as obtained from tide gauge data (Figure 4c), which we interpret as being primarily due to the contribution of M4 (or M4 plus M6), are consistent with what is already known of the spatial variations of the constituents themselves (Figure 12d). As one enters estuaries and rivers, as discussed above for Avonmouth, then the methods used in this paper are no longer appropriate as the tidal curve is no longer describable in terms of a set of harmonics; in these cases, one would have to resort to determining MTL-MSL directly from the measurements.

One could argue that by examining how MTL-MSL depends upon the various tidal constituents is simply ‘playing with cosines’ in a parameterisation of the tide, and does not necessarily lead to an improved understanding of the physical processes involved in generating those constituents in the first place. This applies in particular to discussion of the coastal tides. However, to investigate the spatial variation of MTL-MSL near to the coast in greater detail would require access to larger amounts of tide gauge data together with more detailed numerical modelling of coastal tides.

Additional objectives were to assess how MTL-MSL might vary through the year, and how MTL might be affected when high and low water turning points were observed only in daylight. At most locations, the seasonal MTL-MSL values and differences between MTL and daylight-only MTL have been found to be only ~1 cm, although several locations where larger differences were found have provided interesting case studies. If future ‘data archaeology’ exercises result in historical measurements of MTL being discovered that have been made with an irregular (although necessarily well-documented) sampling, then any possible biases in the measurements could be investigated by the methods described in this paper.

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Figure Captions

1. Schematic of a sinusoid curve with the frequency of M2 (red) to which another sinusoid, with an amplitude 10% that of M2 and with the frequency of M4 (blue), is added to make the total curve (black). The peak of M2 is defined as 0 hours. In (a) the combination of phase lags ($2g_{M2} - g_{M4}$) is zero, so M4 also peaks at 0 hours, in (b) it is 180° and M4 has a minimum at 0 hours.
2. The tidal form factor, defined by the ratio of the amplitudes of the two main diurnal and semidiurnal constituents (K1, O1, M2 and S2), $F = \frac{(H_{K1} + K_{O1})}{(H_{M2} + H_{S2})}$. The amplitudes used to make this map were taken from the Technical University of Denmark DTU-10 global tide model (Cheng and Andersen 2010). Tidal regimes where F is below 0.25 are normally said to be 'semidiurnal', F between 0.25 and 1.5 is said to be 'mixed and mainly semidiurnal', F between 1.50 and 3 is 'mixed and mainly diurnal', while F greater than 3 is 'diurnal'.
3. Schematic of sinusoids with frequencies of K1 and O1 (green and blue respectively), and with the same amplitudes, that add to make the black curve with a frequency of M1 within an envelope of 27.3 days. For comparison, the red lower curve shows a normal M1 sinusoid with twice the K1 or O1 amplitude. Comparison of the black and red curves shows that the phase of the black M1 flips by 180° between the two halves of the envelope.
4. Best estimates of MTL-MSL derived from a global set of tide gauges. (a) Global distribution. (b) North America. (c) North West Europe. (d) Asia and Australasia. Note that the colour scale in (c) differs from that in (a,b,d).
5. Histogram of the MTL-MSL differences. Bins with one or more entries are shown by a black dot.
6. Distribution of MTL-MSL in India and neighbouring countries derived from tidal constants in the databank of the International Hydrographic Organization.
7. Schematic illustration of a combination of M2 (red) and M4 (blue), as for Figure 1 but with M4 amplitude now 30% of M2, with the total curve in black. In (a-d) the combination of phase lags ($2g_{M2} - g_{M4}$) has values of 0, 90, 180 and 270°.
8. The tidal curve at spring tides at Weymouth on the south coast of England.
9. The tidal curve for 9 February 2011 at Adak, Aleutian Islands.
10. Values of MTL-MSL arising from a combination of M2, K1 and O1 as a function of reference year using runs with different choices of relative phases (see text).
11. (a) MTL-MSL at 71 US tide gauges computed by the method described in this paper compared to values derived by NOAA from sea level observations over the tidal datum epoch 1983-2001. Note that the point for Anchorage at (-24.4,-34.4) is not included in the plot limits. (b) DTL-MSL obtained by the two methods. The points for Anchorage, Friday Harbor and Seattle at (-48.3, -57.5), (-21.6, -20.5) and (-30.0, -29.2) respectively are not included in the plot limits.
12. (a) MTL-MSL determined using the method described in this paper using only the M2 plus M4 constituents in the DTU-10 model. (b) MTL-MSL determined from the M2 plus K1 and O1 constituents in the DTU-10 model. (c) MTL-MSL determined as the sum of (a) and (b). The 1/8° model fields were sampled every 1° for (a-c). (d) MTL-MSL determined from M2, M4, K1 and O1 for the global coastline computed at 1/8°. (e) As (c) but computed at 1/8° for the NW European continental shelf. Note the different colour scales in each case.
13. (a) Histogram of the annual amplitude of MTL-MSL. Bins with one or more entries are shown by a black dot. (b) Annual amplitude of MTL-MSL compared to the sum of the amplitudes of MA2 and MB2. (c) Histogram of the semiannual amplitude of MTL-MSL.
14. Histogram of the differences between MTL determined at all hours and in daylight only. Bins with one or more entries are shown by a black dot.

Table 1: Statistics of |MTL-MSL| (cm) showing the median, 80- and 95-percentile and maximum values.

	Using MTL-MSL derived from:	Median	80-percentile	95-percentile	Maximum
1	60 tidal constituents from the global set of tide gauges	0.56	1.60	6.55	24.95
2	M2 plus M4 only.	0.25	1.06	5.10	24.82
3	M2 plus M4 and M6 only.	0.25	1.08	5.16	24.70
4	M2 plus K1 and O1 only.	0.15	0.78	1.83	9.38
5	[1] – [2]	0.30	0.93	1.91	11.20
6	[1] – [3]	0.29	0.90	1.78	6.96
7	[1] – [3] – [4]	0.11	0.39	1.56	7.43

Table 2: High and Low Waters from M2, M4 and M6 at Princes Pier, Liverpool

	Low Tide Time (days)	Height (cm)	High Tide Time	Height	Low Tide Time	Height	High Tide Time	Height	MTL
Tide defined by M2 alone	0.206	-312.8	0.465	312.8	0.723	-312.8	0.982	312.8	0.0
Time since previous low or high tide (days)			0.259		0.259		0.259		
M6 contribution at these times		-1.6		1.6		-1.6		1.6	
Tide defined by M2 and M4	0.218	-309.2	0.454	322.7	0.736	-309.2	0.972	322.7	6.7
Time since previous low or high tide (days)			0.236		0.282		0.236		
M6 contribution at these times		-3.7		-0.4		-3.7		-0.4	
Tide defined by M2, M4 and M6	0.221	-313.1	0.458	322.6	0.739	-313.1	0.975	322.6	4.8
Time since previous low or high tide (days)			0.237		0.281		0.237		
M6 contribution at these times		-4.0		0.3		-4.0		0.3	

Appendix 1. Names and speeds (deg/solar hour times 10^7) of the tidal constituents in the standard set used in this study.

Name	Speed
ZO	0
SA	0410686
SSA	0821373
MM	5443747
MSF	10158958
MF	10980331
2Q1	128542862
SIG1	129271398
Q1	133986609
RO1	134715145
O1	139430356
MP1	140251729
M1	144920521
CHI1	145695476
PI1	149178647
P1	149589314
S1	150000000
K1	150410686
PSI1	150821353
PHI1	151232059
TH1	155125897
J1	155854433
SO1	160569644
OO1	161391017
OQ2	273416965
MNS2	274238337
2N2	278953548
MU2	279682084
N2	284397295
NU2	285125831
OP2	289019669
M2	289841042
MKS2	290662415
LAM2	294556253
L2	295284789
T2	299589333
S2	300000000
R2	300410667
K2	300821373
MSN2	305443747
KJ2	306265120
2SM2	310158958
MO3	429271398
M3	434761563
SO3	439430356
MK3	440251729
SK3	450410686
MN4	574238337
M4	579682084
SN4	584397295
MS4	589841042
MK4	590662415
S4	600000000
SK4	600821373
2MN6	864079380
M6	869523127
MSN6	874238337
2MS6	879682084
2MK6	880503457
2SM6	889841042
MSK6	890662415
MA2	289430356
MB2	290251728

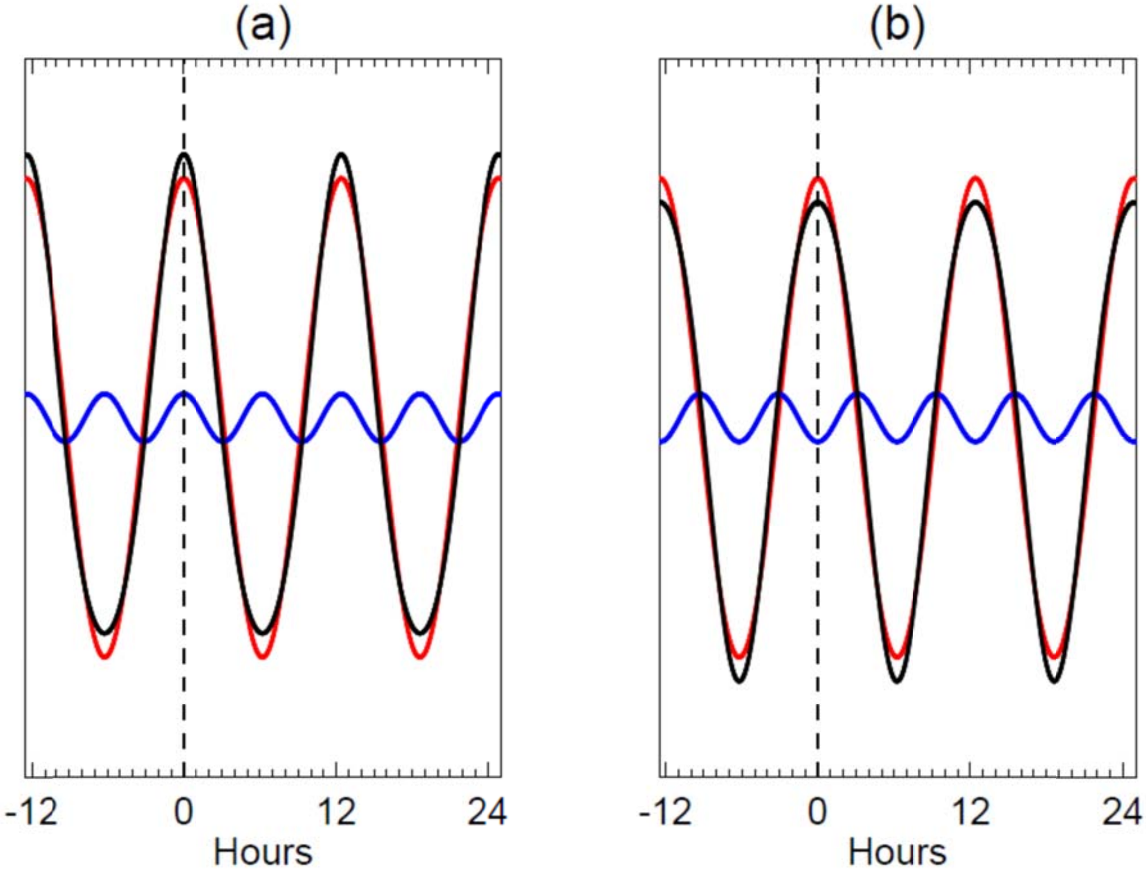


Figure 1

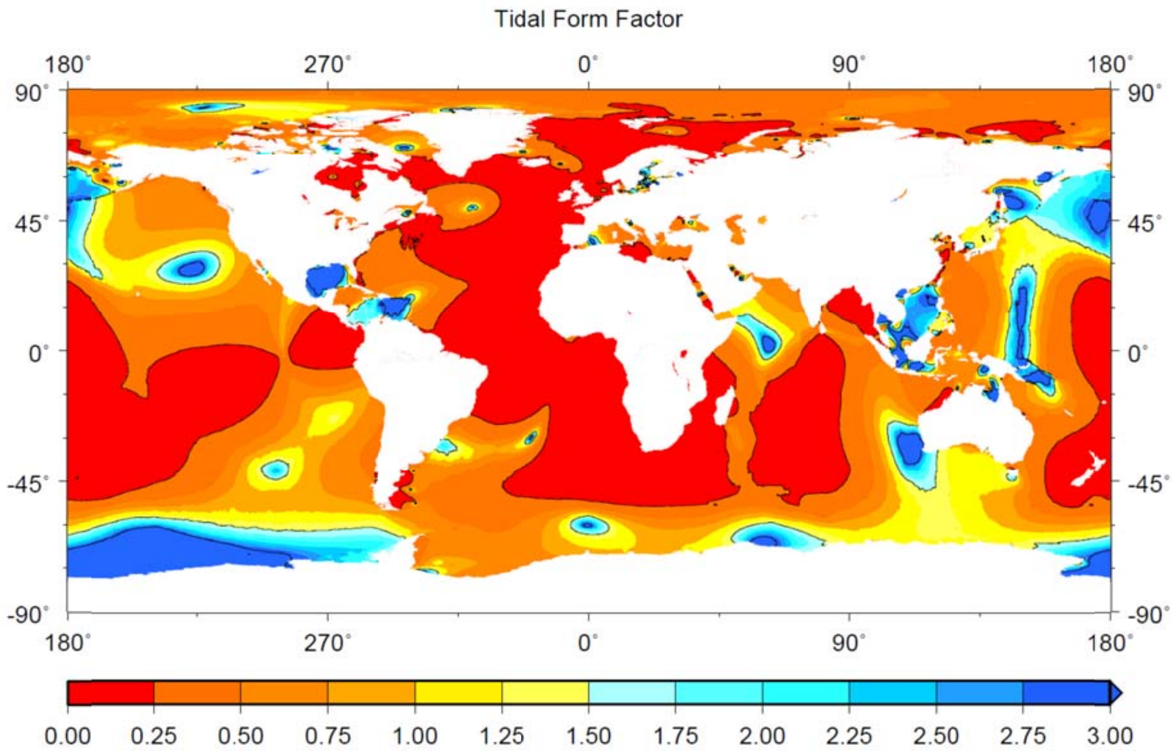


Figure 2

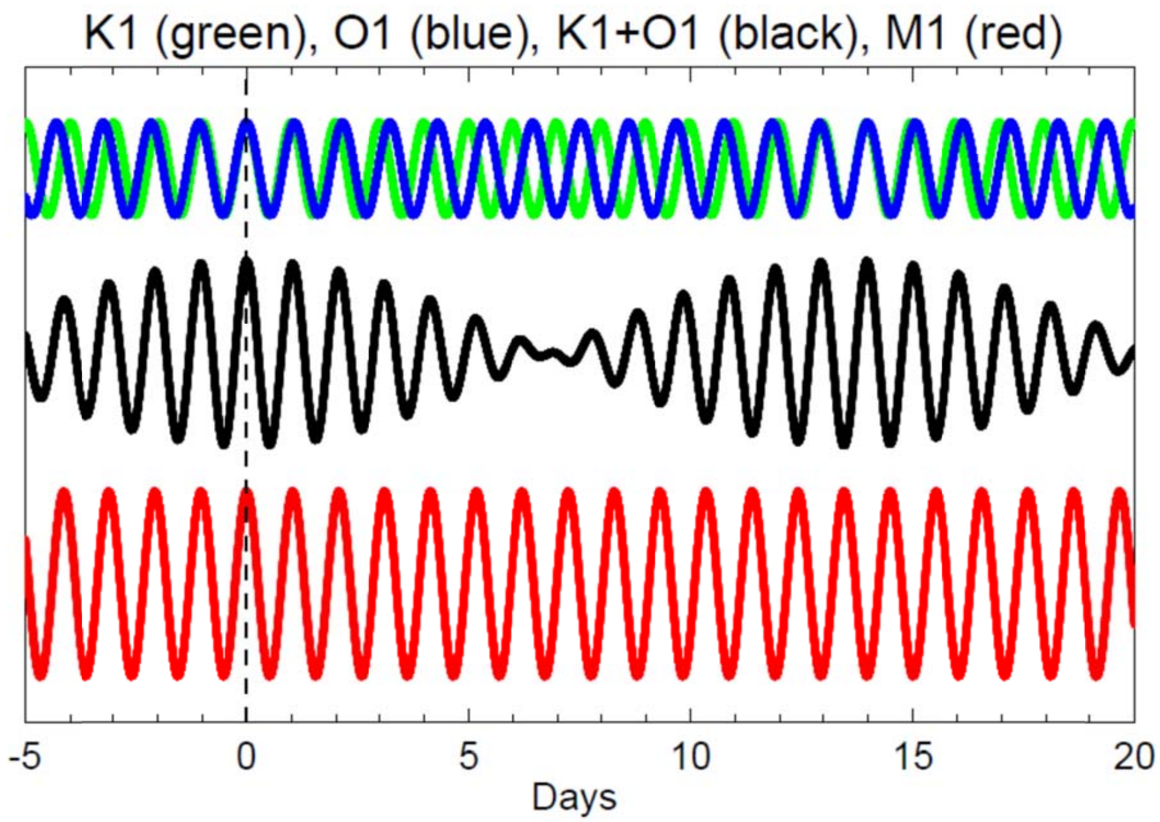


Figure 3

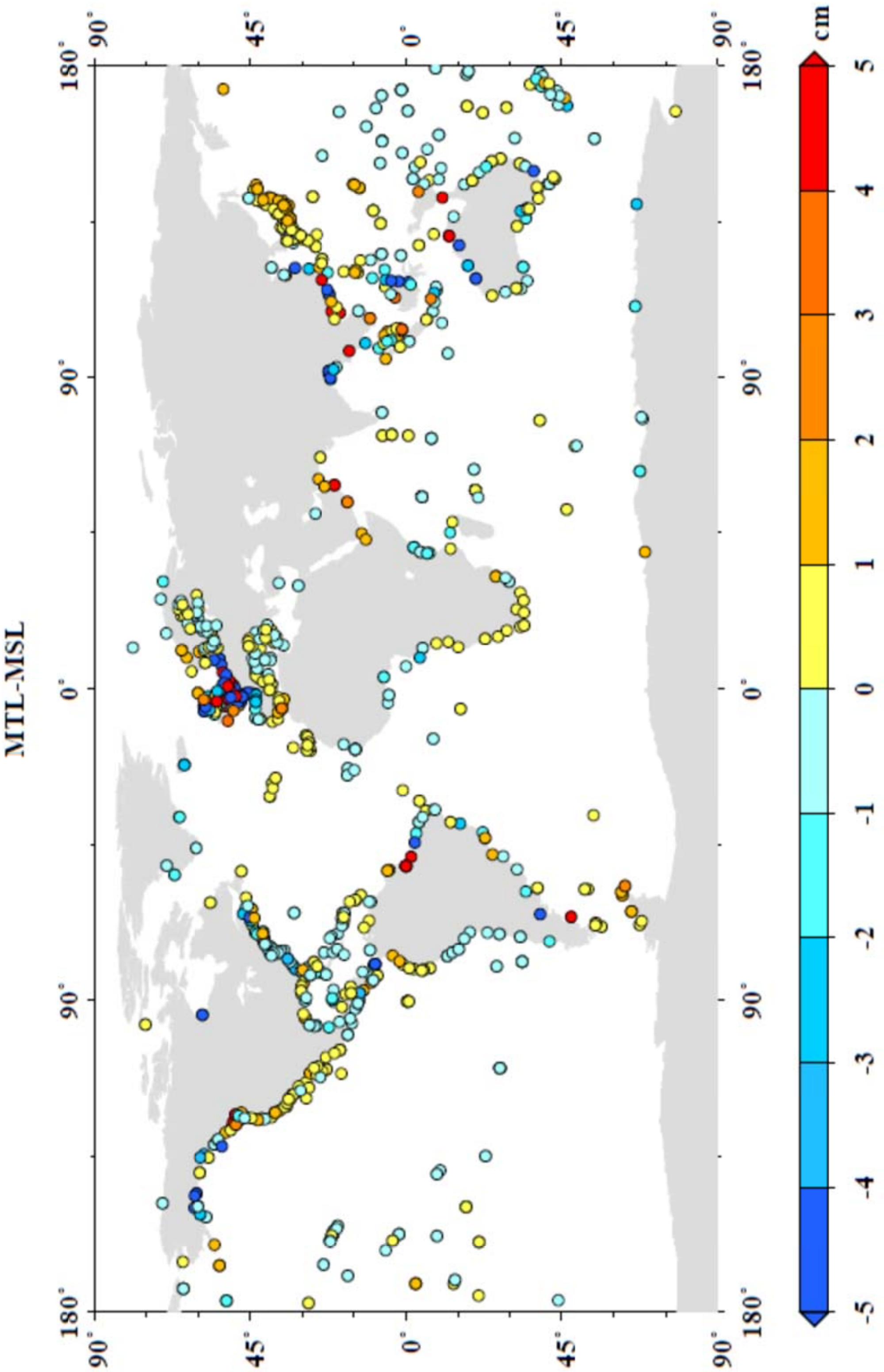


Figure 4a

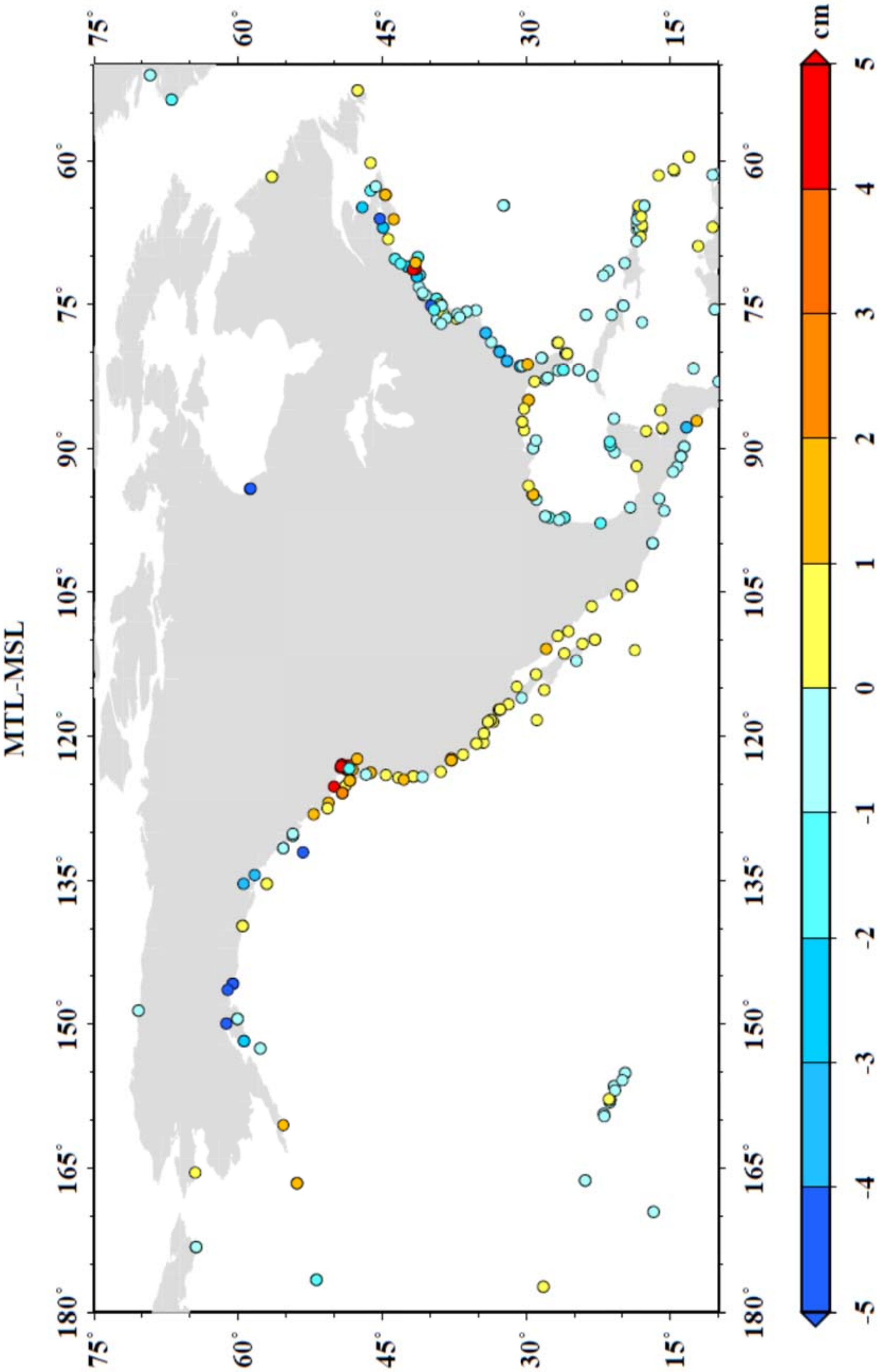

MTL-MSL
75°
60°
45°
30°
15°
180°
165°
150°
135°
120°
105°
90°
75°
60°
cm
-5
-4
-3
-2
-1
0
1
2
3
4
5

Figure 4b

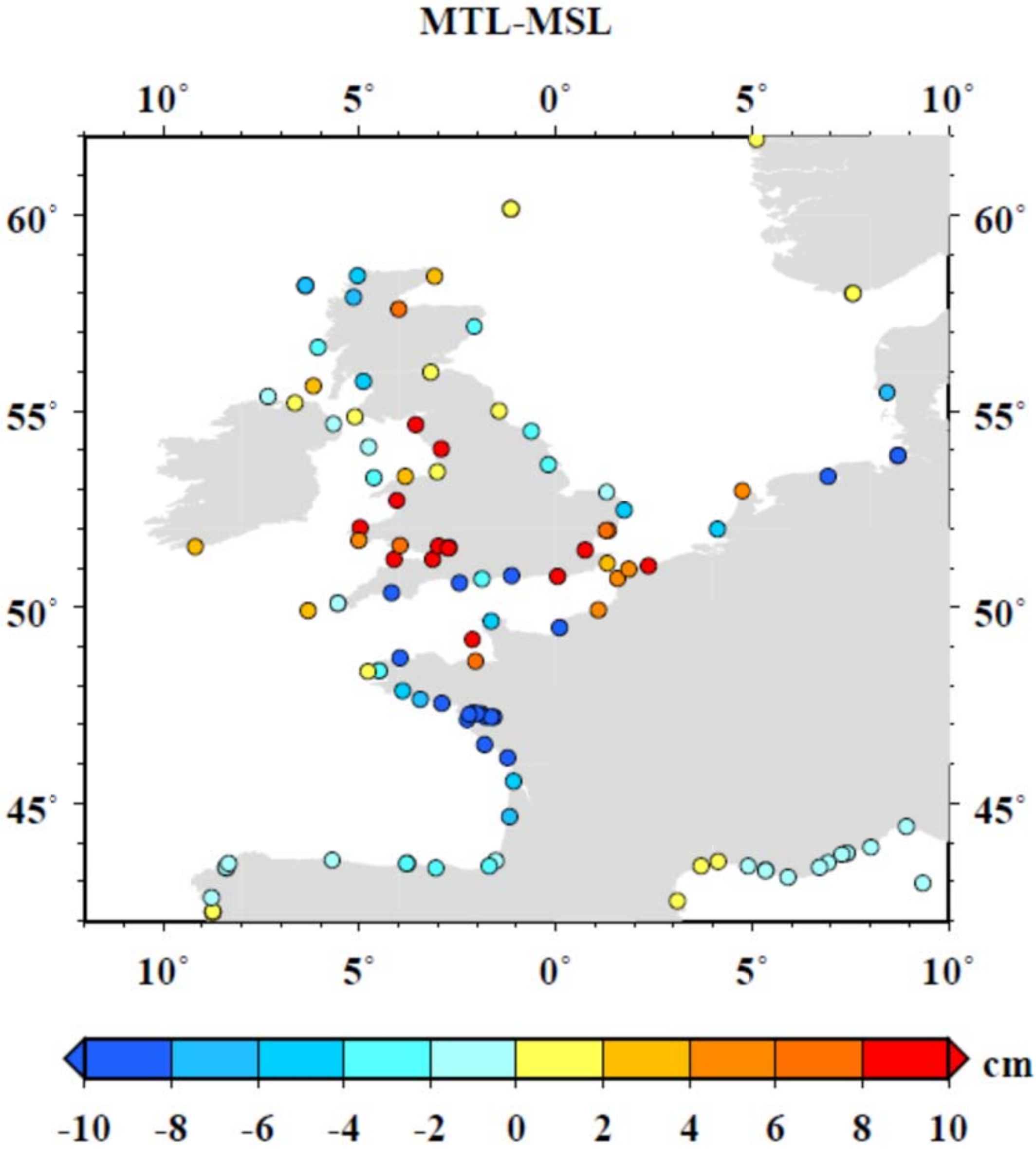


Figure 4c

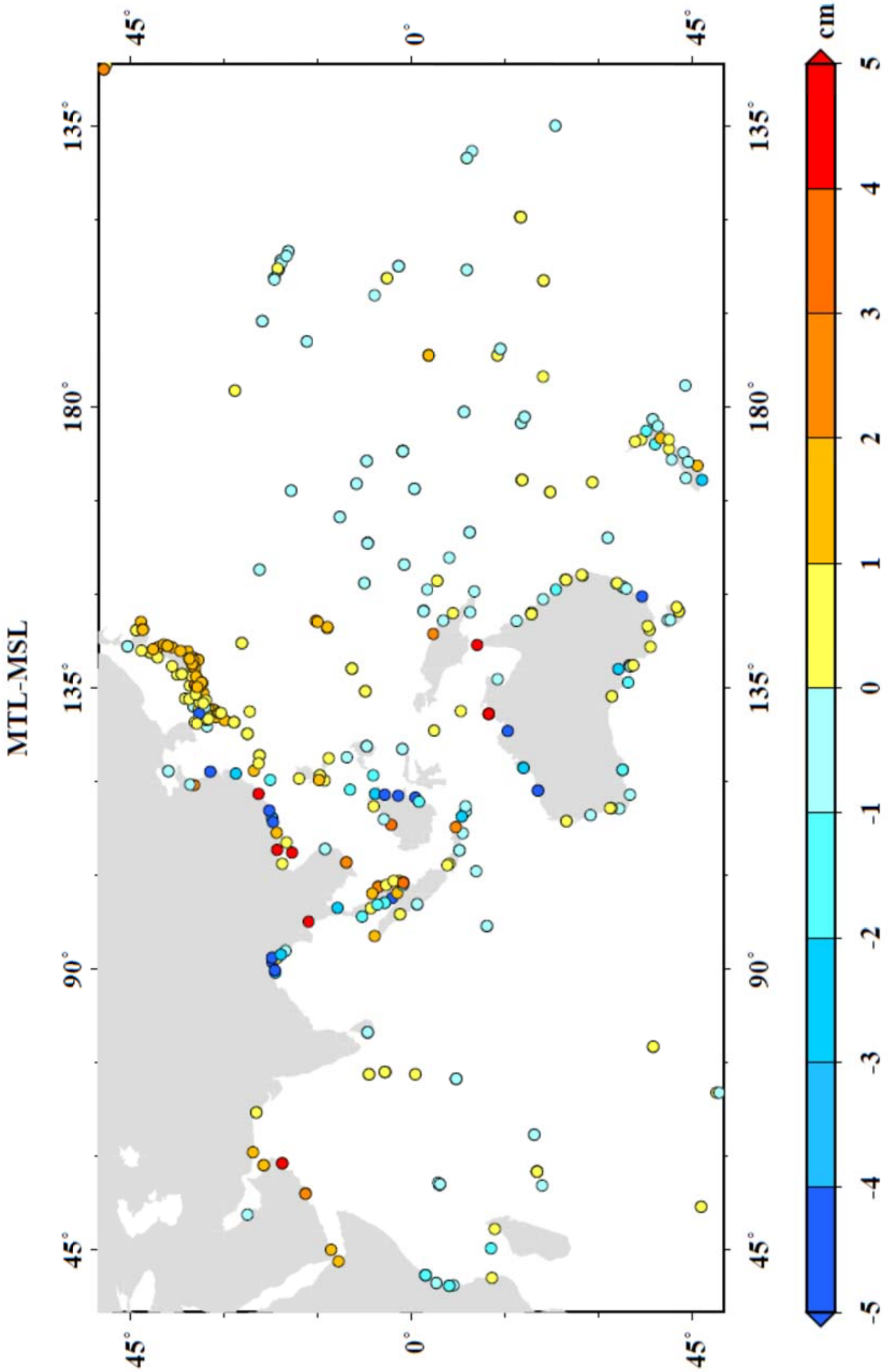


Figure 4d

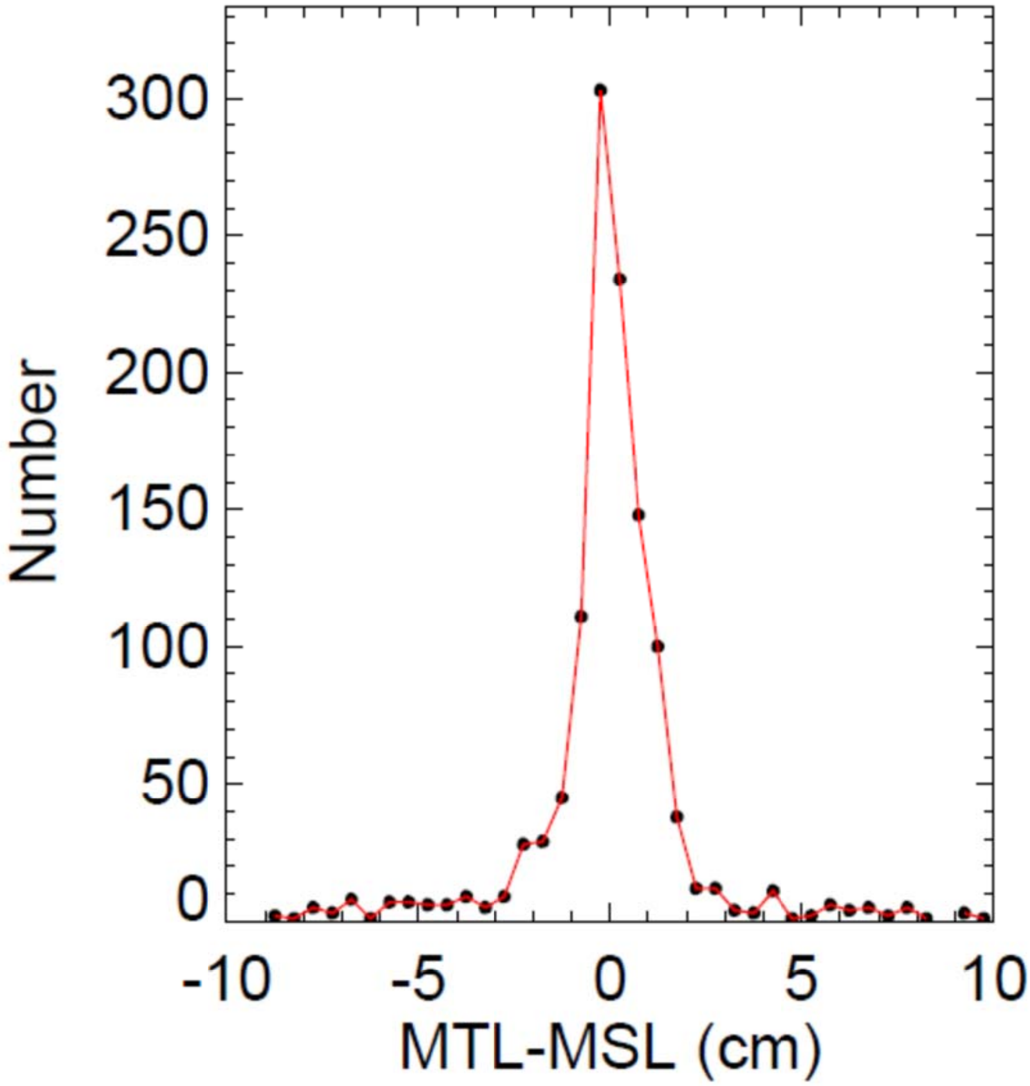


Figure 5

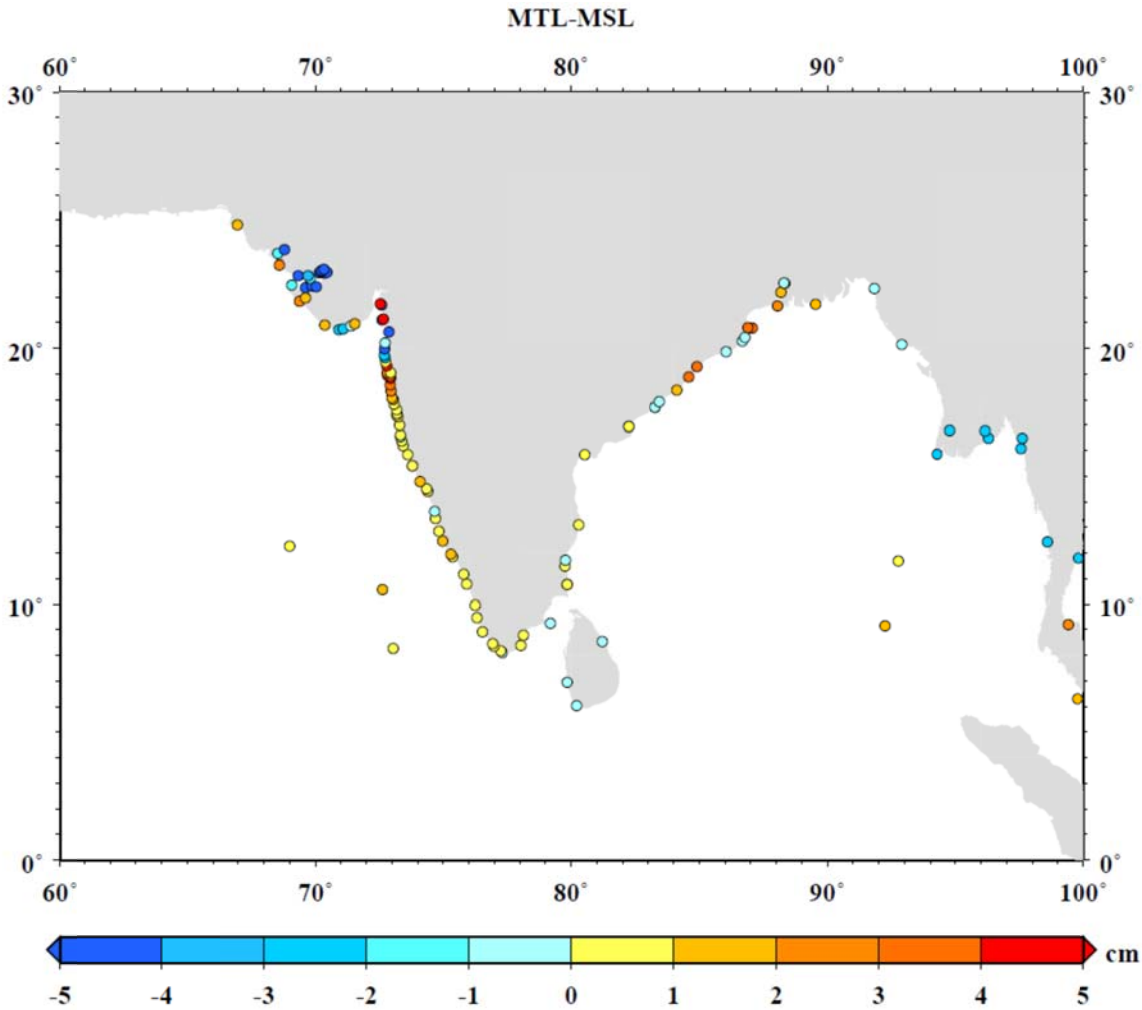


Figure 6

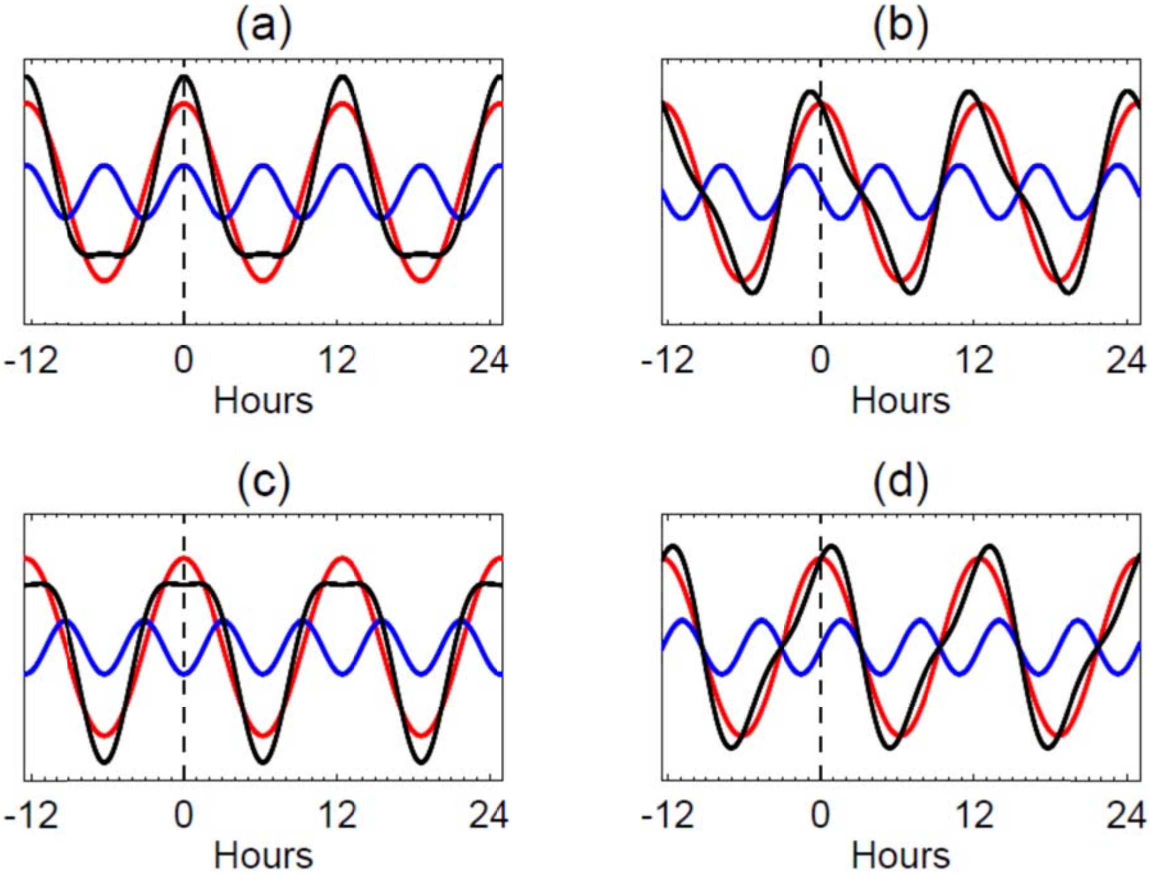

(a)
-12
0
12
24
Hours
(b)
-12
0
12
24
Hours
(c)
-12
0
12
24
Hours
(d)
-12
0
12
24
Hours

Figure 7

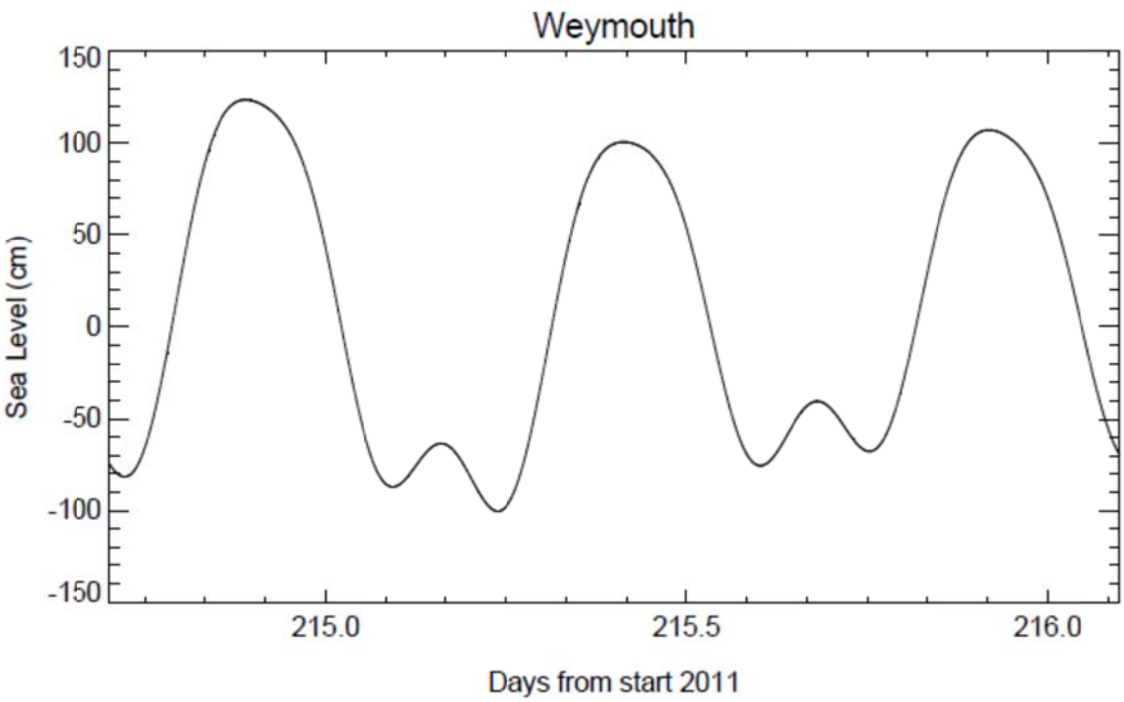


Figure 8

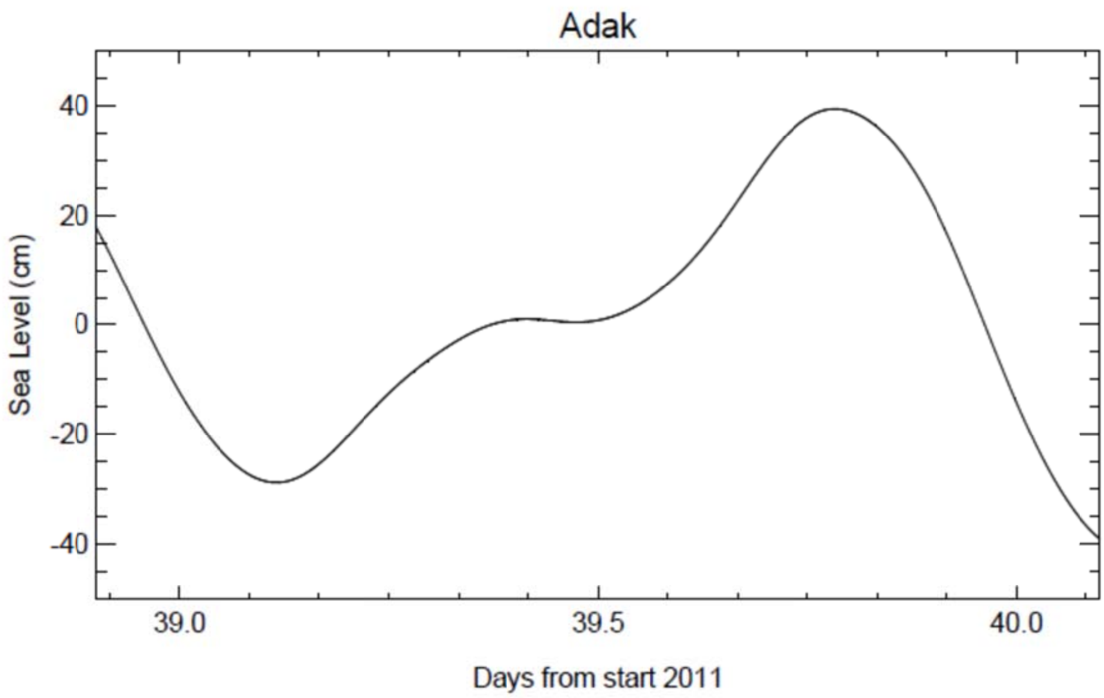
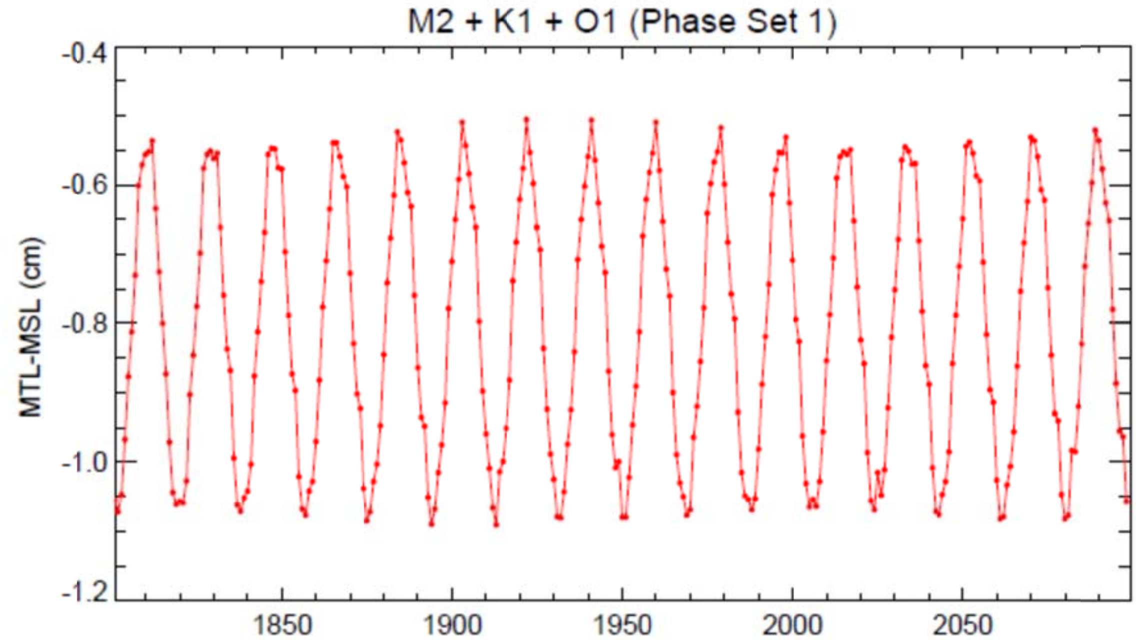


Figure 9


M2 + K1 + O1 (Phase Set 1)
MTL-MSL (cm)
-0.4
-0.6
-0.8
-1.0
-1.2
1850
1900
1950
2000
2050

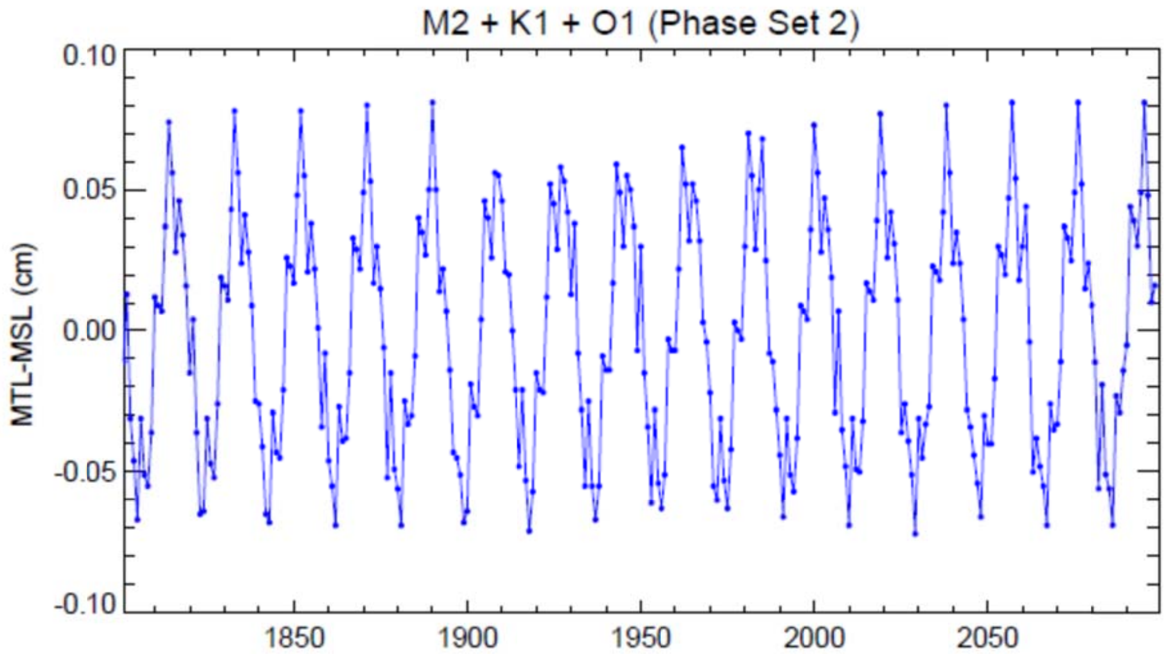
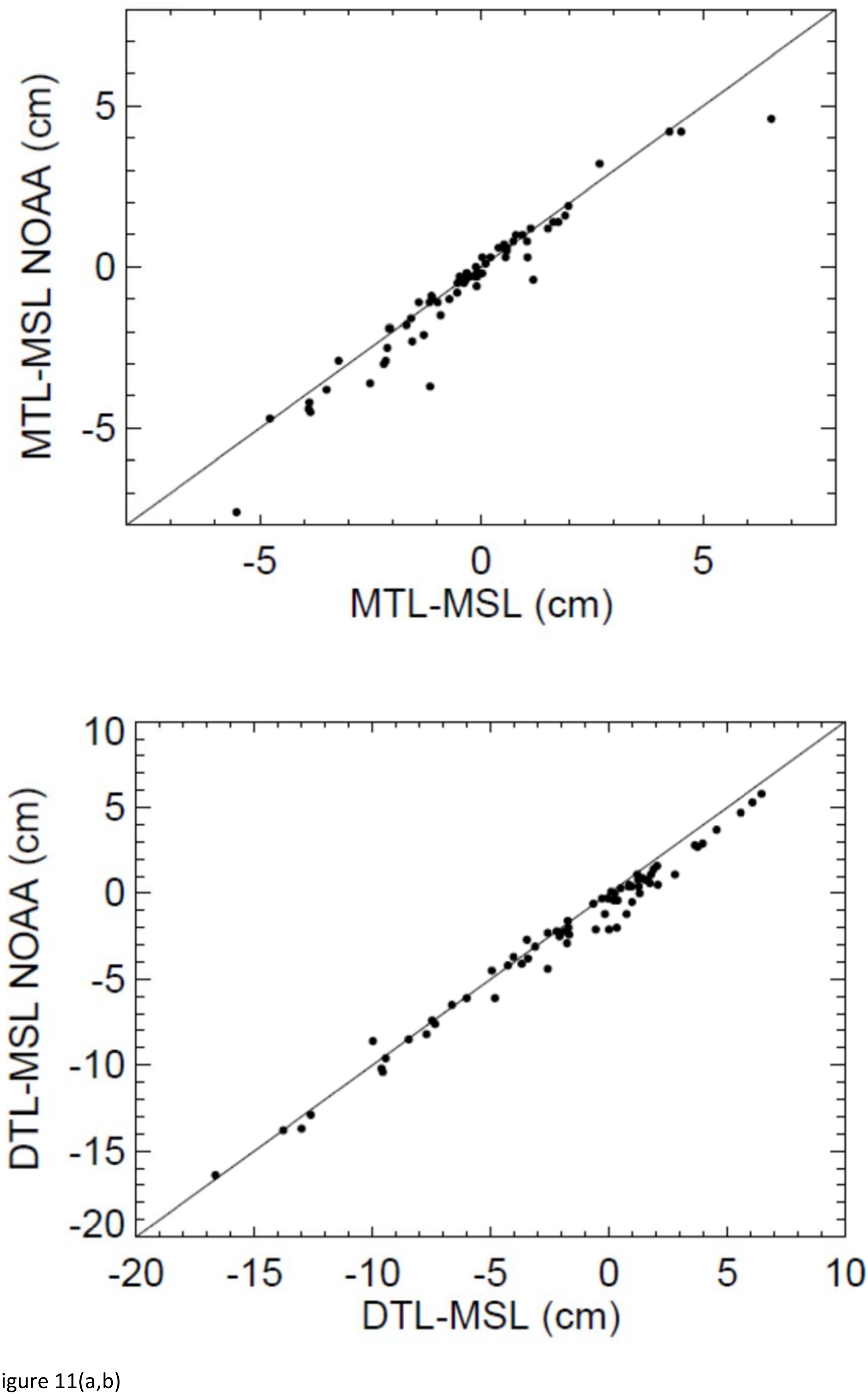
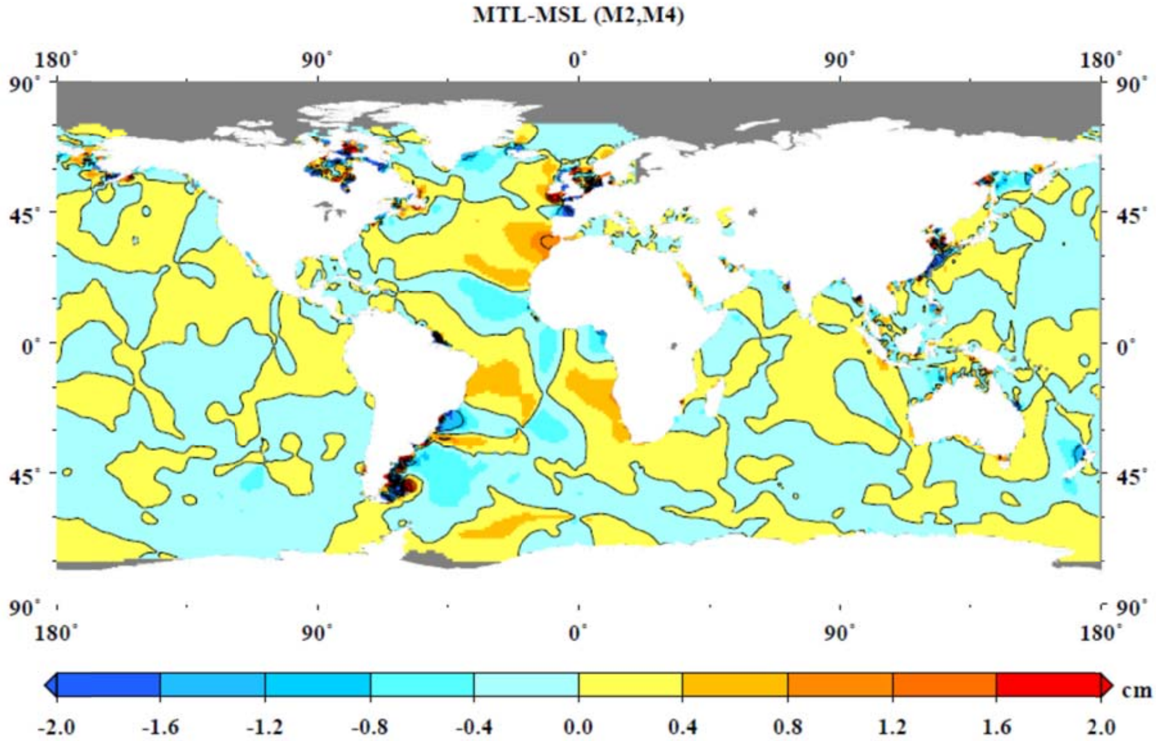

M2 + K1 + O1 (Phase Set 2)
MTL-MSL (cm)
0.10
0.05
0.00
-0.05
-0.10
1850
1900
1950
2000
2050

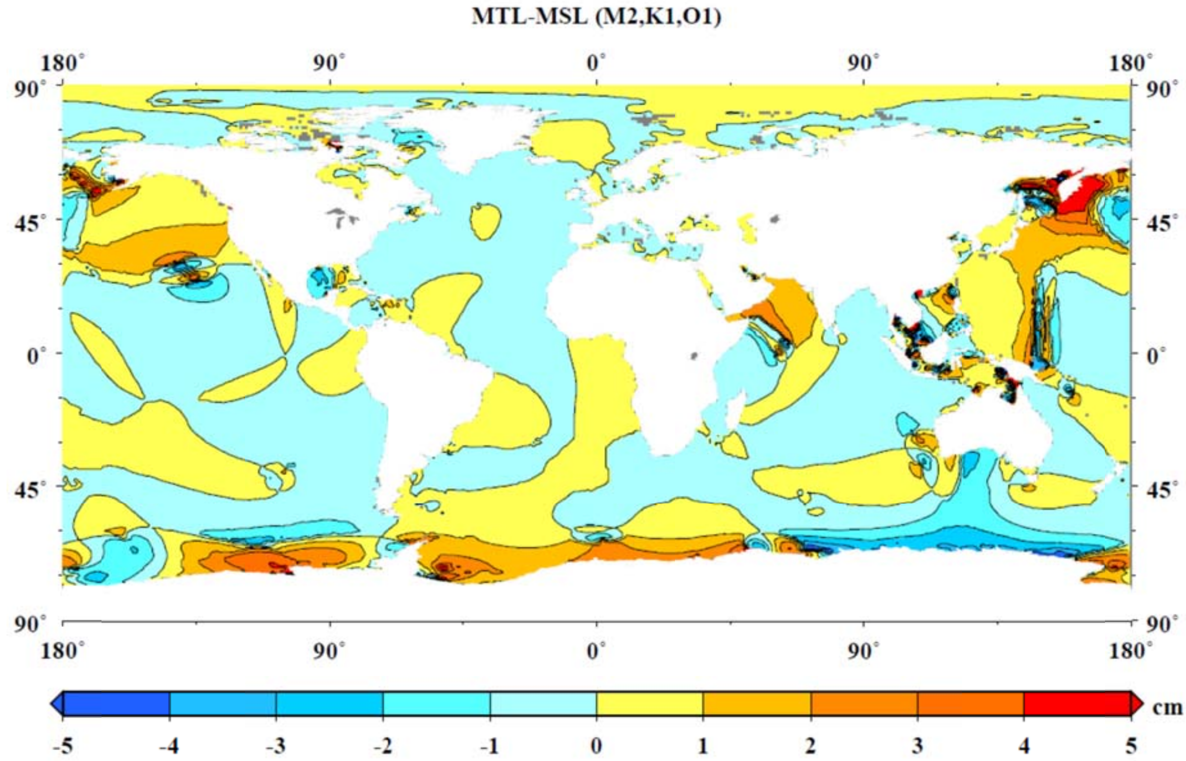
Figure 10

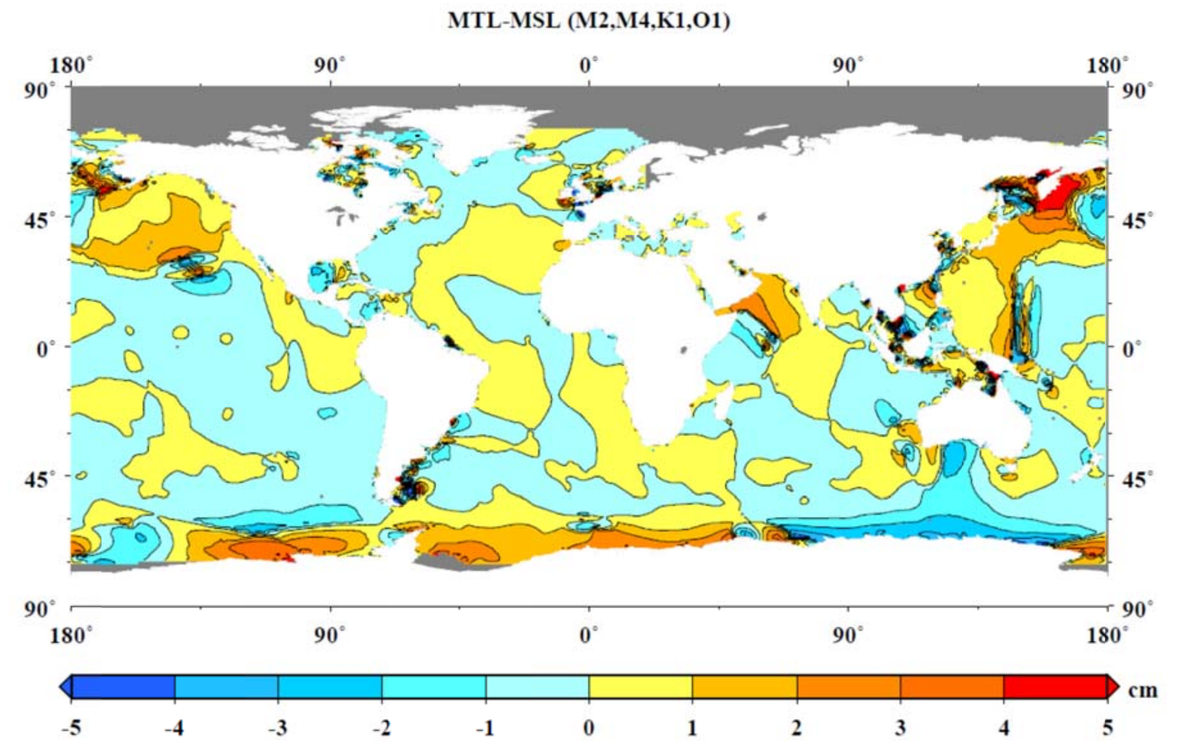


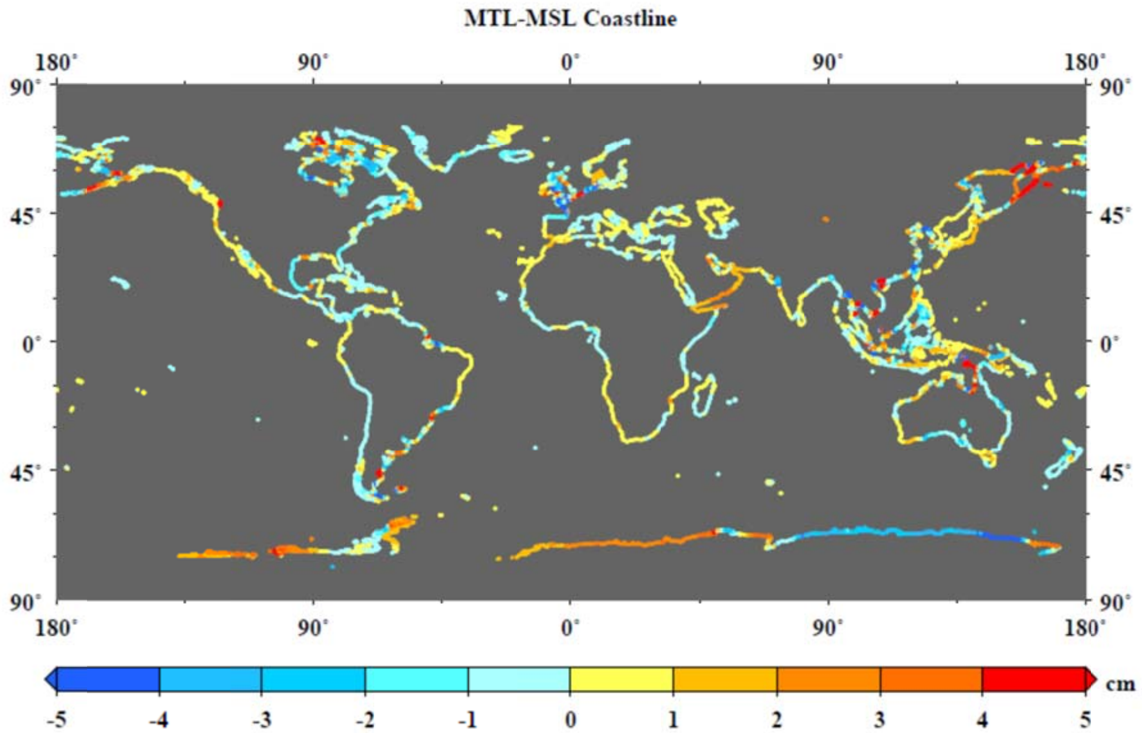
MTL-MSL NOAA (cm)
5
0
-5
-5
0
5
MTL-MSL (cm)
DTL-MSL NOAA (cm)
10
5
0
-5
-10
-15
-20
-20
-15
-10
-5
0
5
10
DTL-MSL (cm)

Figure 11(a,b)


MTL-MSL (M2,M4)
180°
90°
0°
90°
180°
90°
45°
0°
45°
90°
-2.0
-1.6
-1.2
-0.8
-0.4
0.0
0.4
0.8
1.2
1.6
2.0
cm


MTL-MSL (M2,K1,O1)
180°
90°
0°
90°
180°
90°
45°
0°
45°
90°
cm
-5
-4
-3
-2
-1
0
1
2
3
4
5


MTL-MSL (M2,M4,K1,O1)
180°
90°
0°
90°
180°
90°
45°
0°
45°
90°
cm
-5
-4
-3
-2
-1
0
1
2
3
4
5


MTL-MSL Coastline
180°
90°
0°
90°
180°
90°
45°
0°
45°
90°
cm
-5
-4
-3
-2
-1
0
1
2
3
4
5

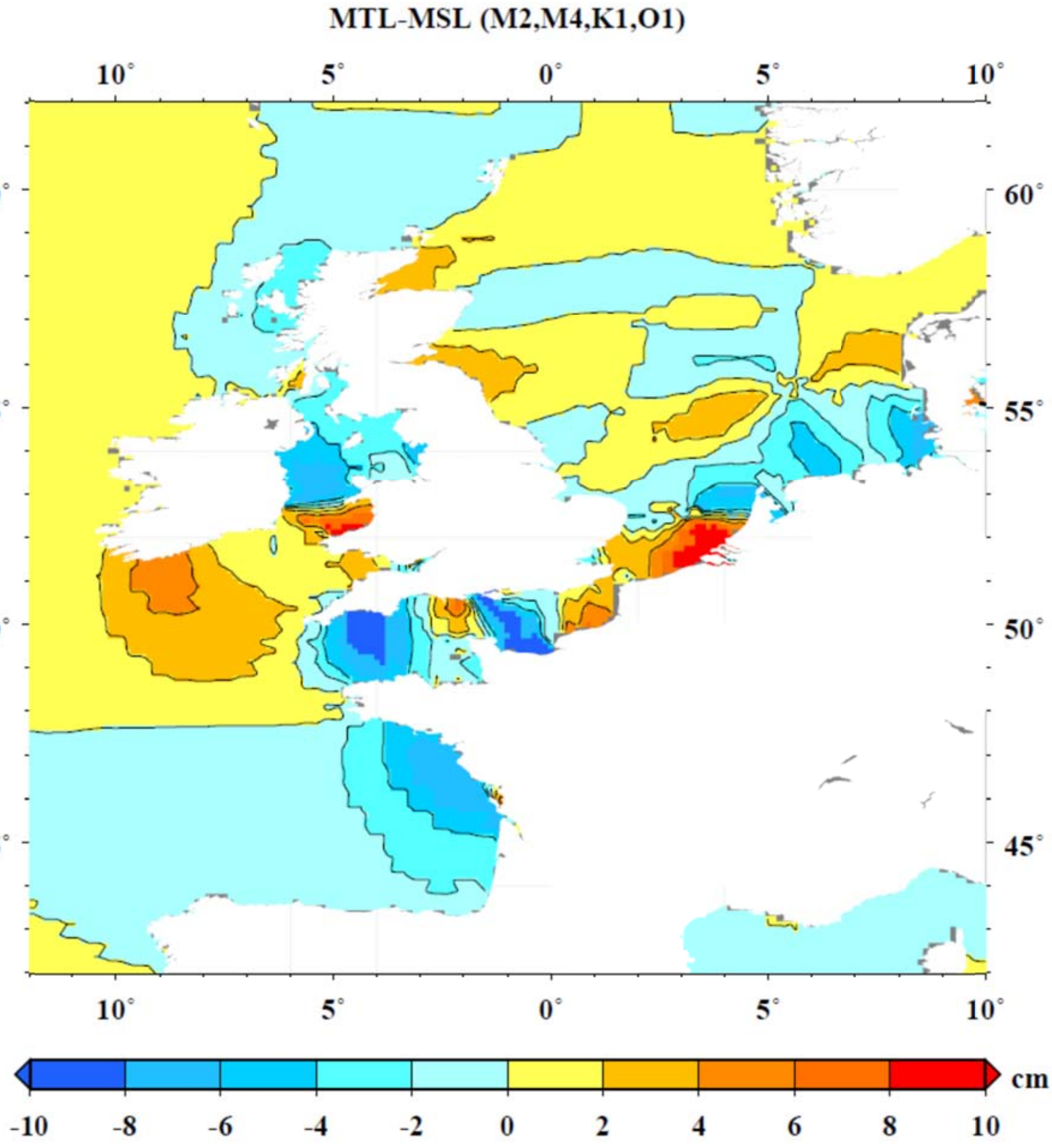
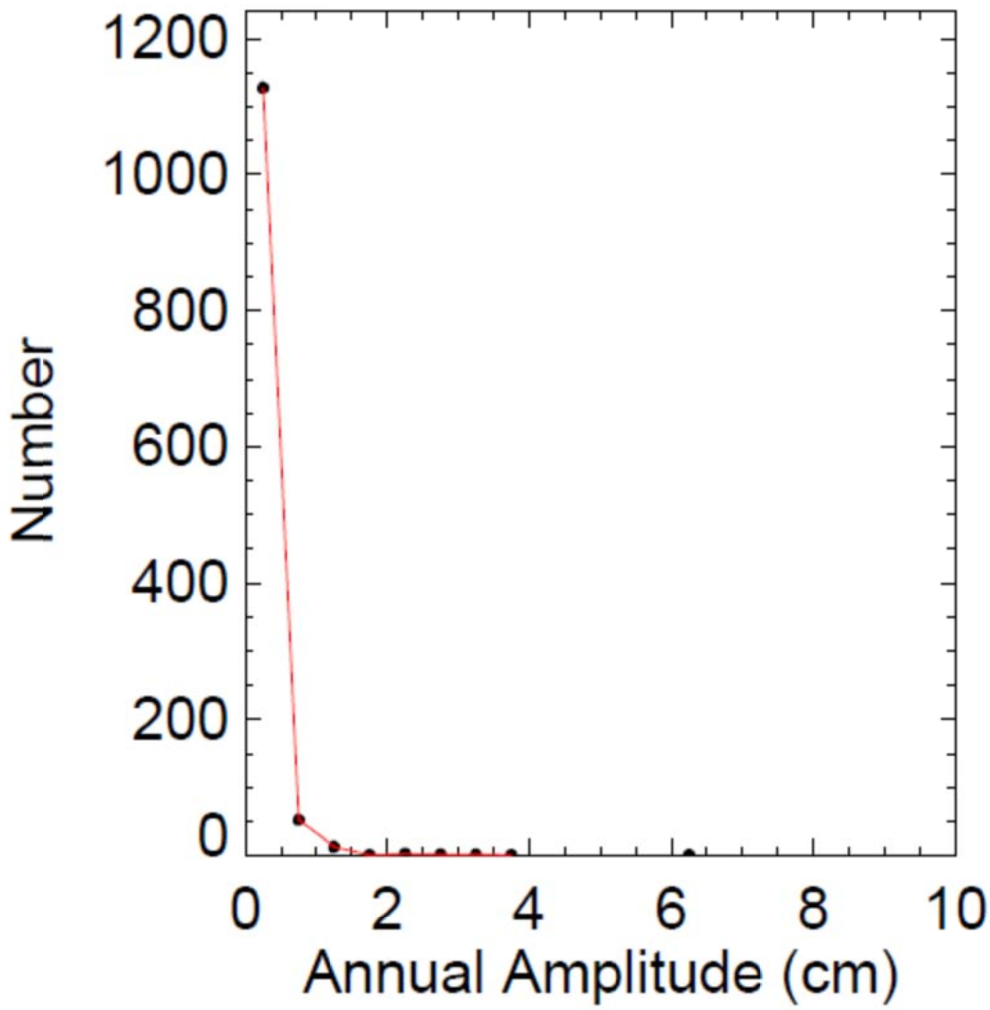
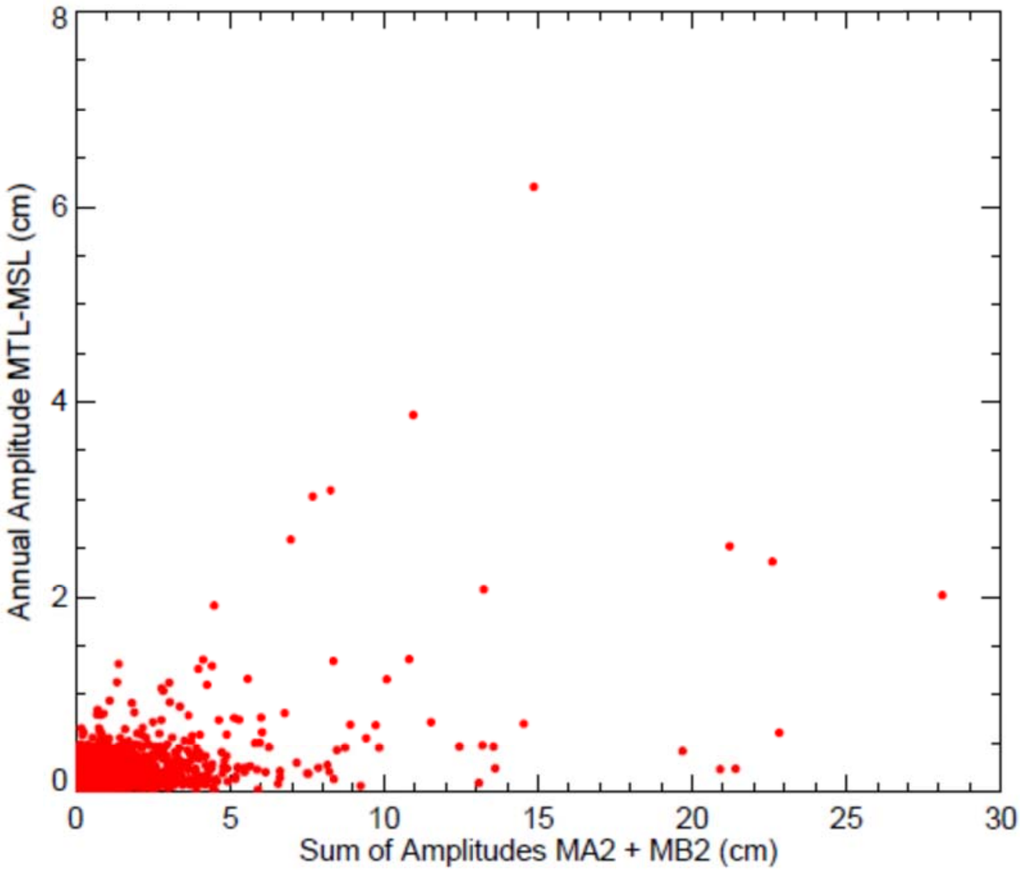


Figure 12 (a-e)


1200
1000
800
600
400
200
0
Number
0
2
4
6
8
10
Annual Amplitude (cm)



Annual Amplitude MTL-MSL (cm)
Sum of Amplitudes MA2 + MB2 (cm)
0
2
4
6
8
0
5
10
15
20
25
30

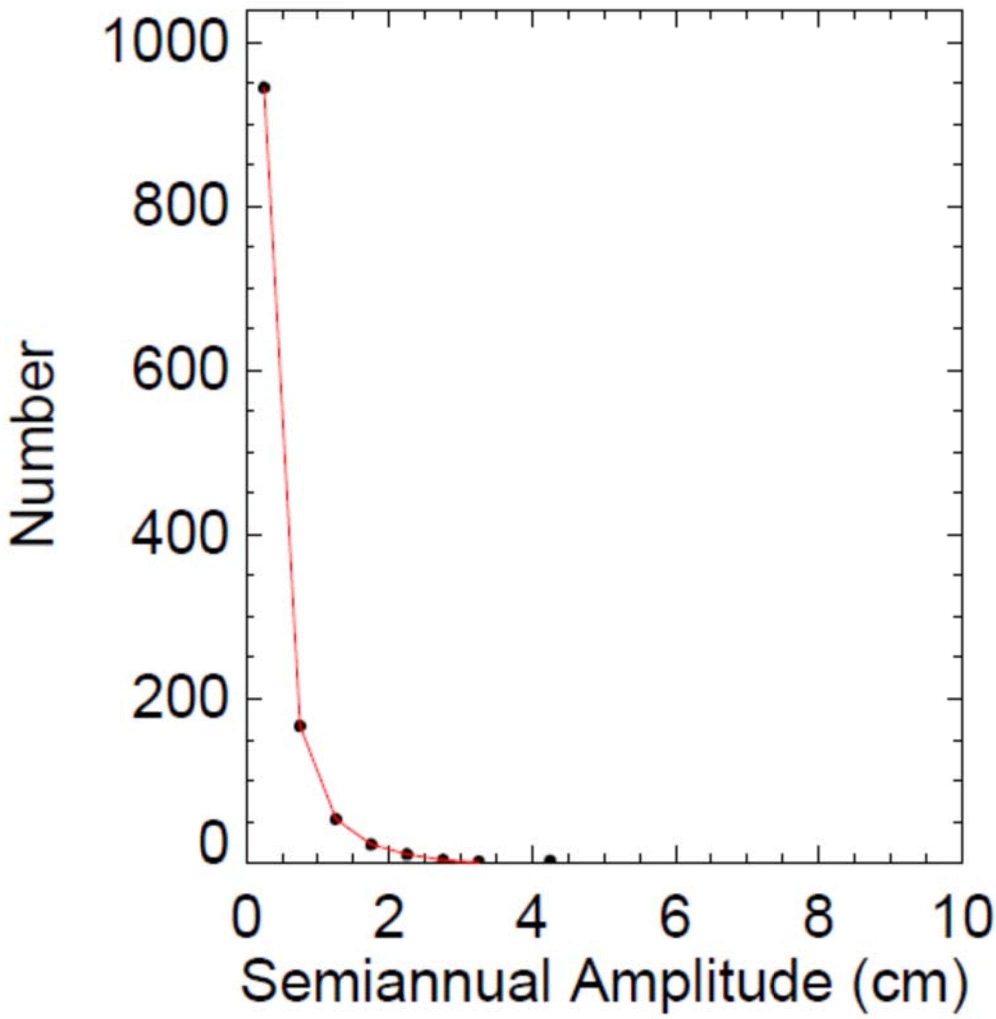


Figure 13a,b,c

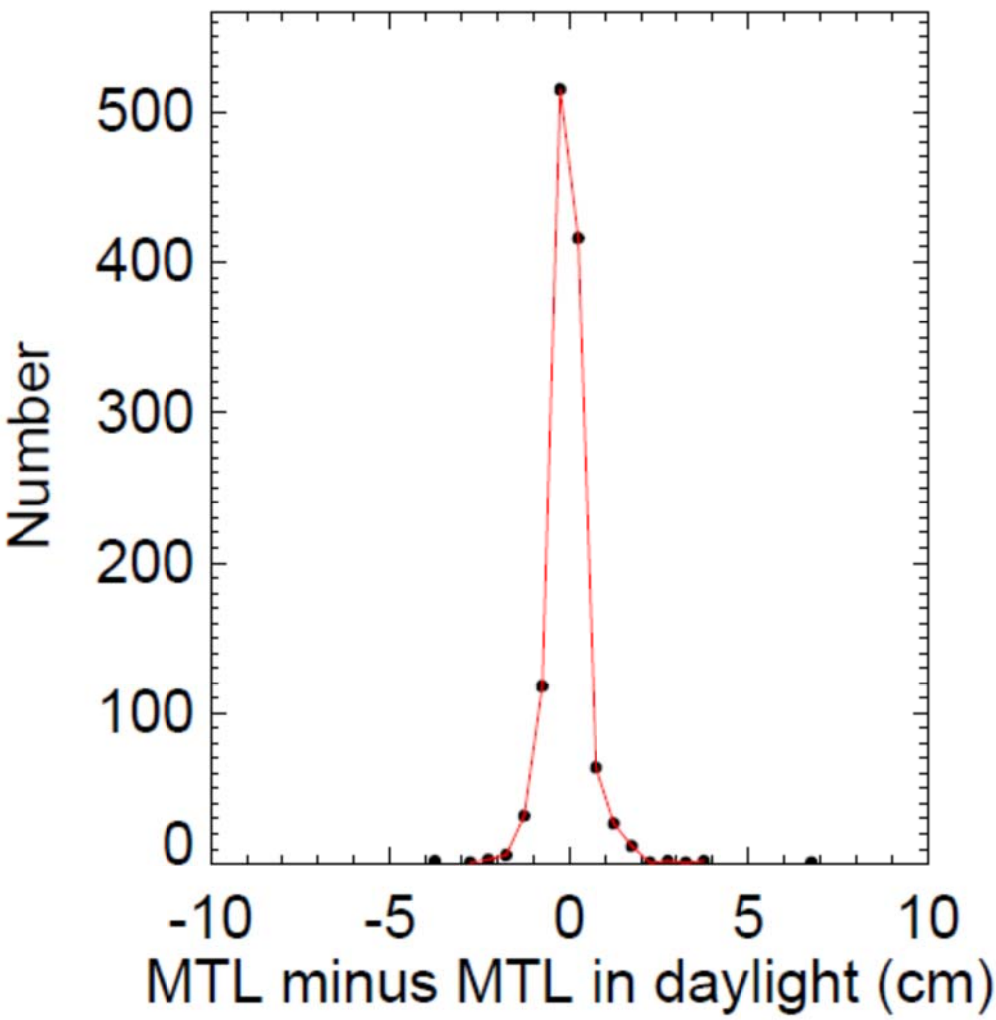

Number
500
400
300
200
100
0
-10
-5
0
5
10
MTL minus MTL in daylight (cm)

Figure 14

Supplementary Material for:

Differences between Mean Tide Level and Mean Sea Level

Journal of Geodesy

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Figure SM1 shows estimates of MTL-MSL obtained from tidal constants in the databank of the International Hydrographic Organization. (a) Global distribution. (b) North America. (c) North West Europe. (d) Asia and Australasia. Note that the colour scale in (c) differs from that in (a,b,d).

Figure SM2(a) shows a schematic example of time series of K1 (green) and S1 (blue), sampled every hour, with the two components given equal amplitudes, and phase lags chosen such that the two are in approximate anti-phase and so approximately cancel in the middle of the year. The black line shows the two added together, while the sections of data shown in red refer to those hourly values in black that happen to occur in daylight. (Time in this schematic simulation refers to local time and daylight is defined as between 6-18 hours local time.) The red values at this time of year are close to zero, which is as positive as they get during the 2 year envelope. Figure SM2(b) shows the corresponding plot at the start of the year. At this time, the K1 and S1 time series are approximately in phase and their addition has twice their individual amplitudes. The red sections can be seen to be more negative on average than in Figure SM2(a). Finally, Figure SM2(c) shows the same information but for the whole 2 years. The K1 and S1 time series overlap when plotted at this scale, while the amplitude of their addition varies over the 2 years. The important point is that there is a net (negative in this case) bias in MSL averaged over the daylight hours of the year, because of S1, which will bias any other sea level measurements made at these times, such as high and low waters and hence MTL. In addition, there is a seasonal dependence, because of K1. If a phase lag for S1 is chosen that is 180° different from that used above, then an average positive bias results.

This study of daylight bias is admittedly a rather esoteric one, but it does provide a further demonstration of how constituents can combine to produce significant contributions to MTL-MSL.

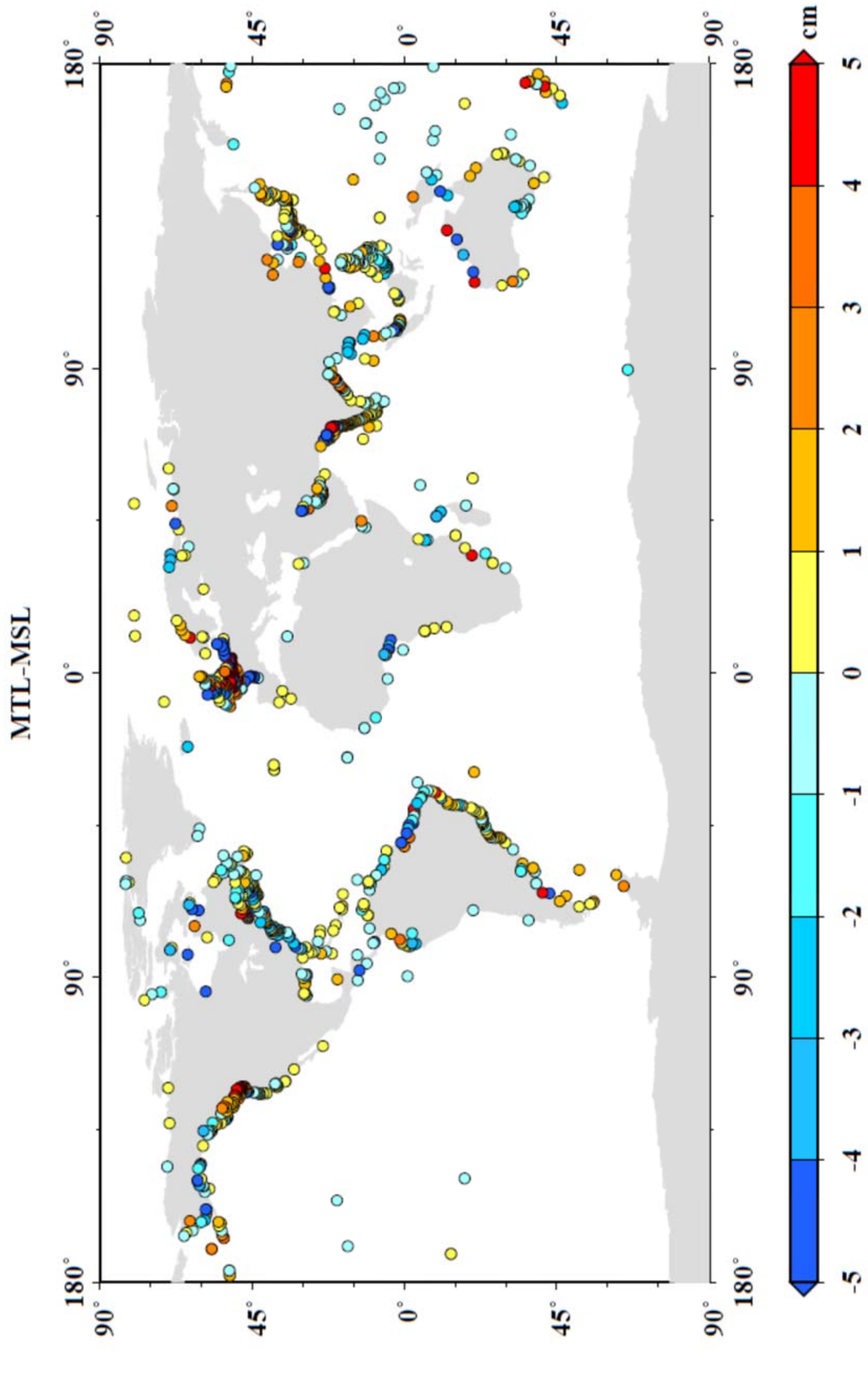

Figure SM1(a)

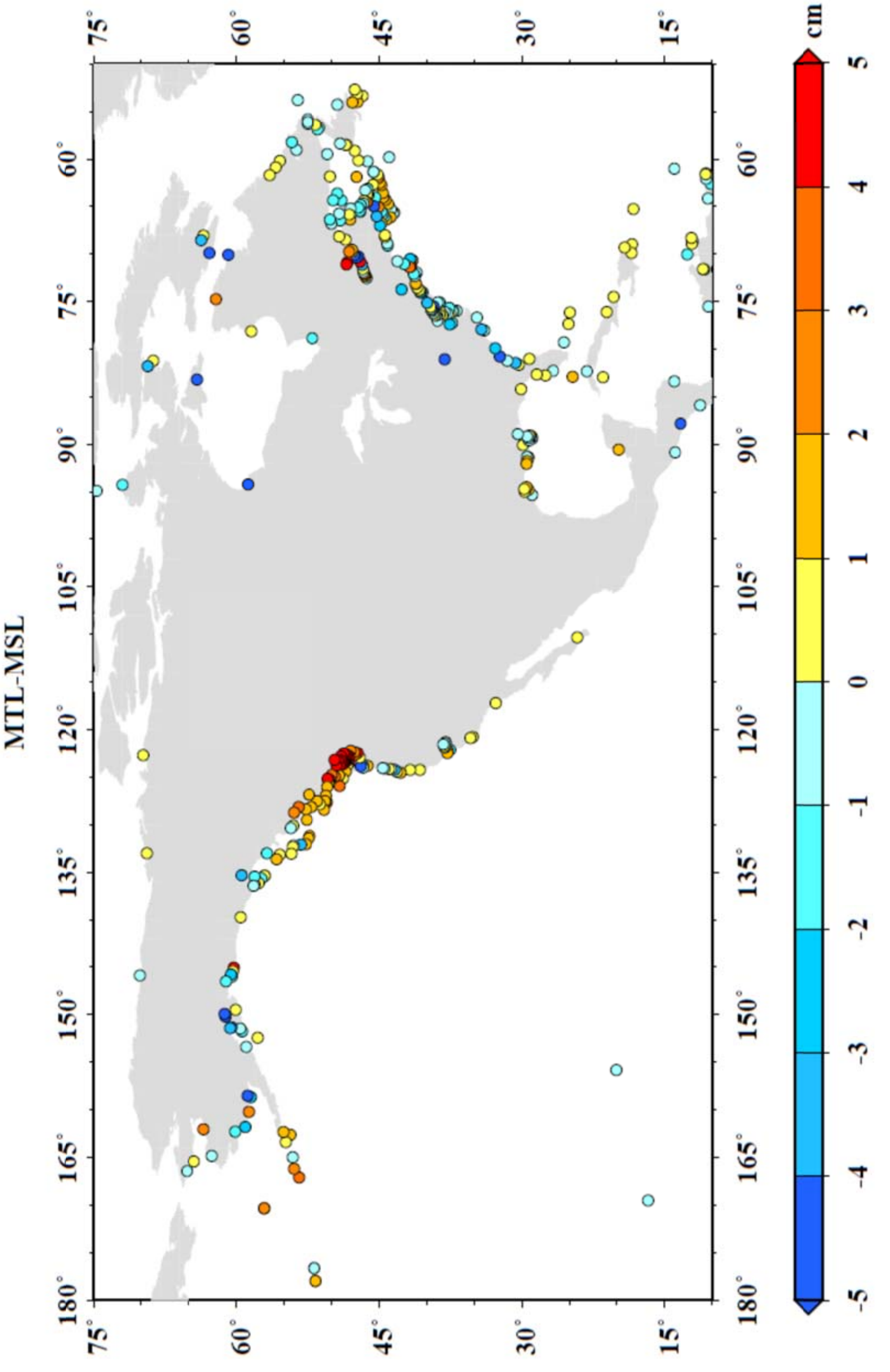

Fig. SM1(b)

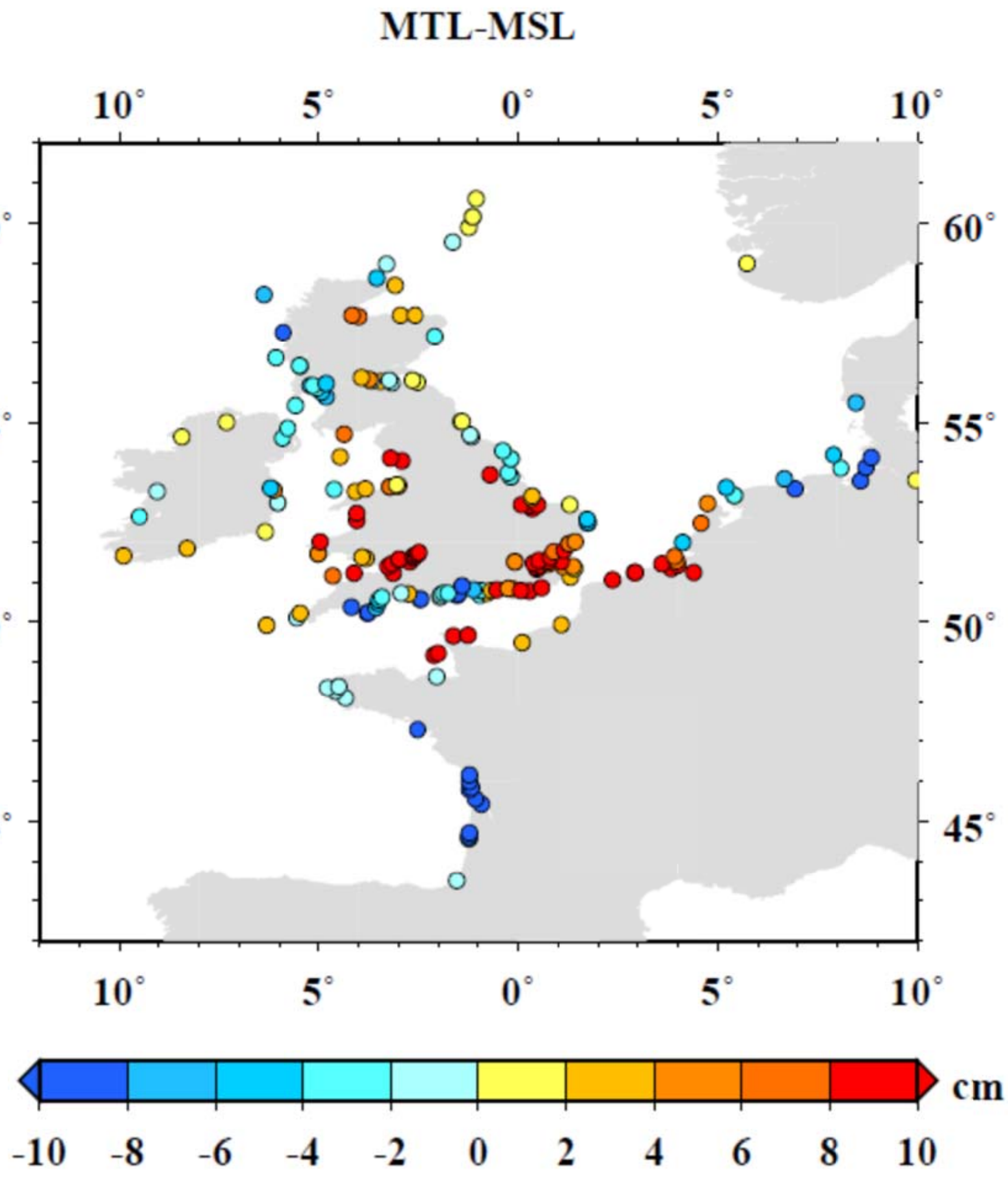

Figure SM1(c)

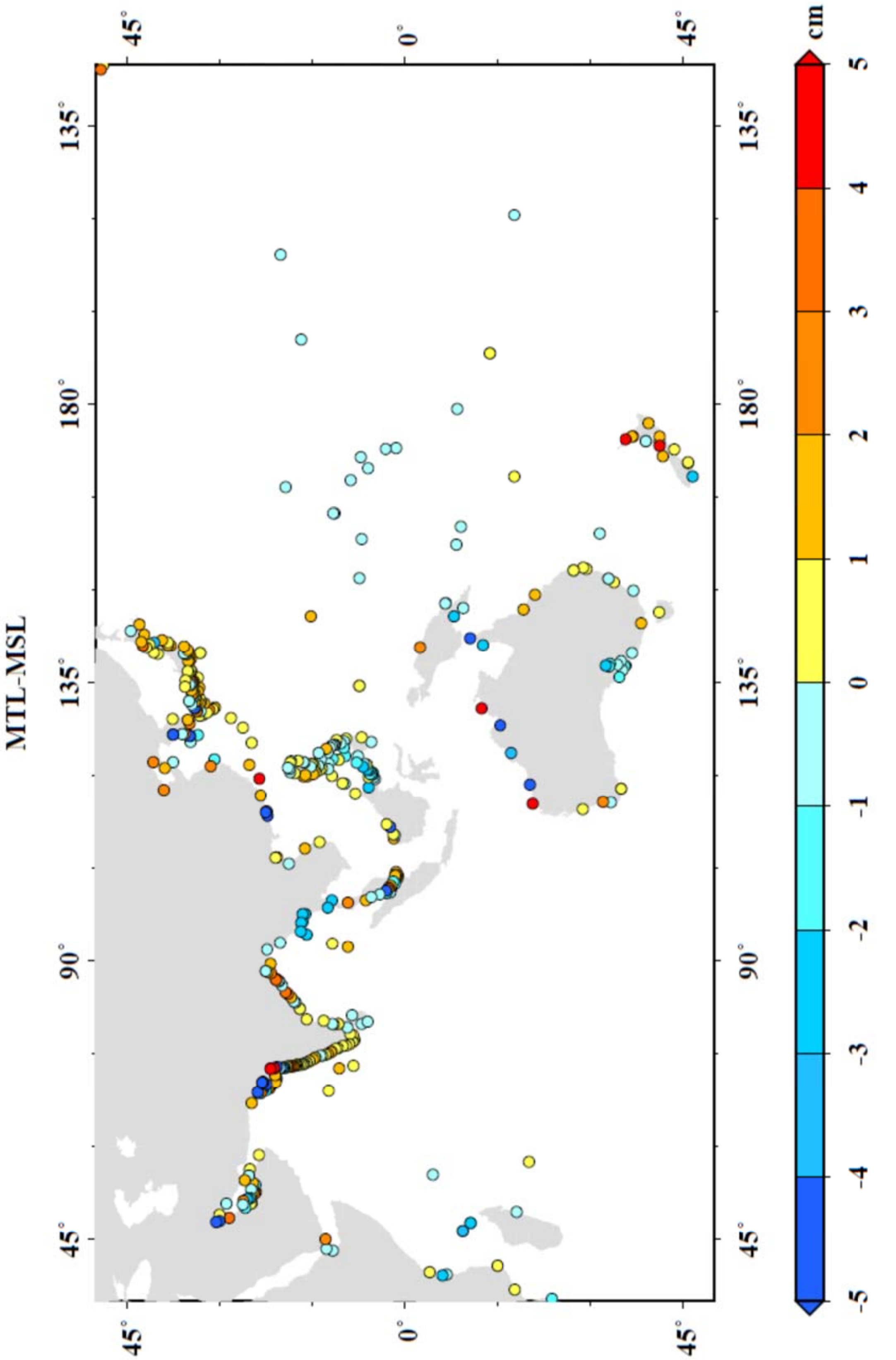

Figure SM1(d)

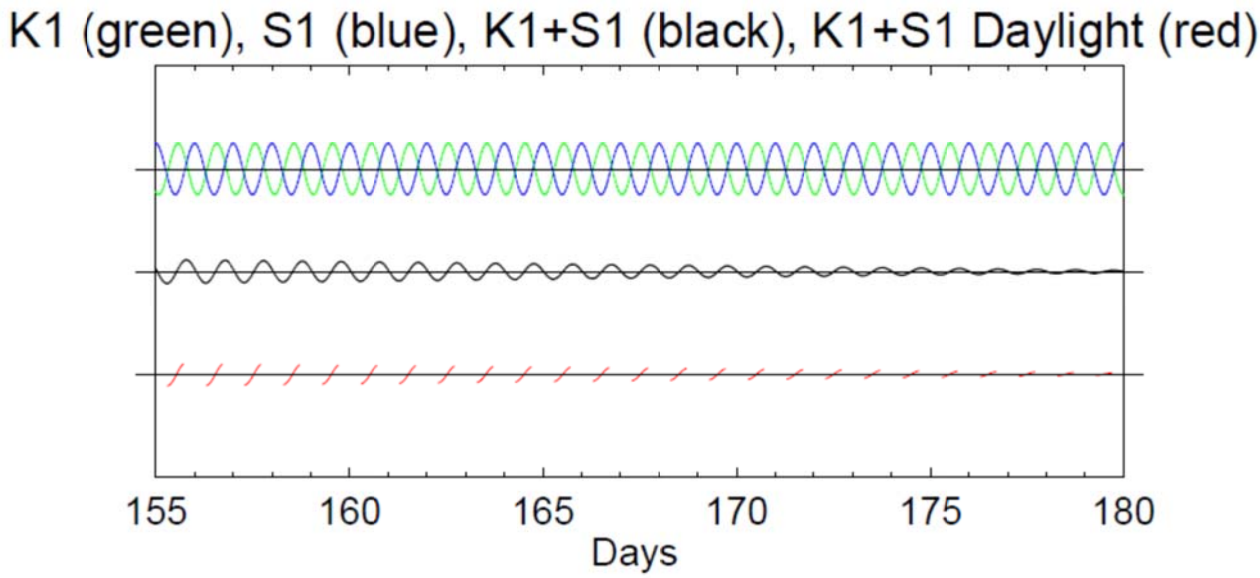

Figure SM2(a)

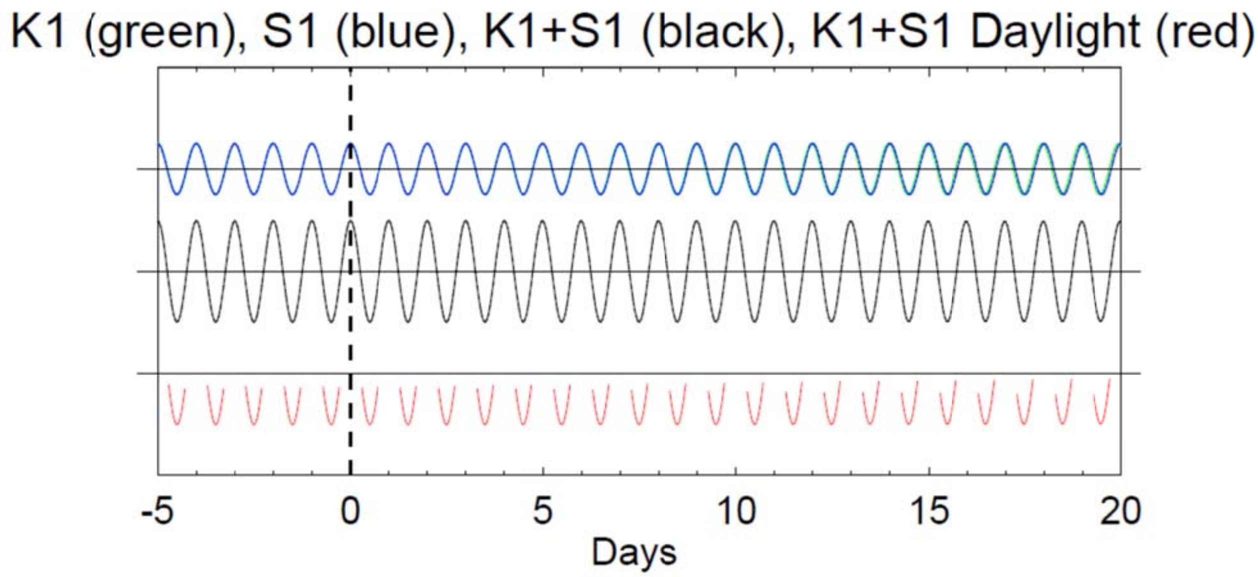

Figure SM2(b)

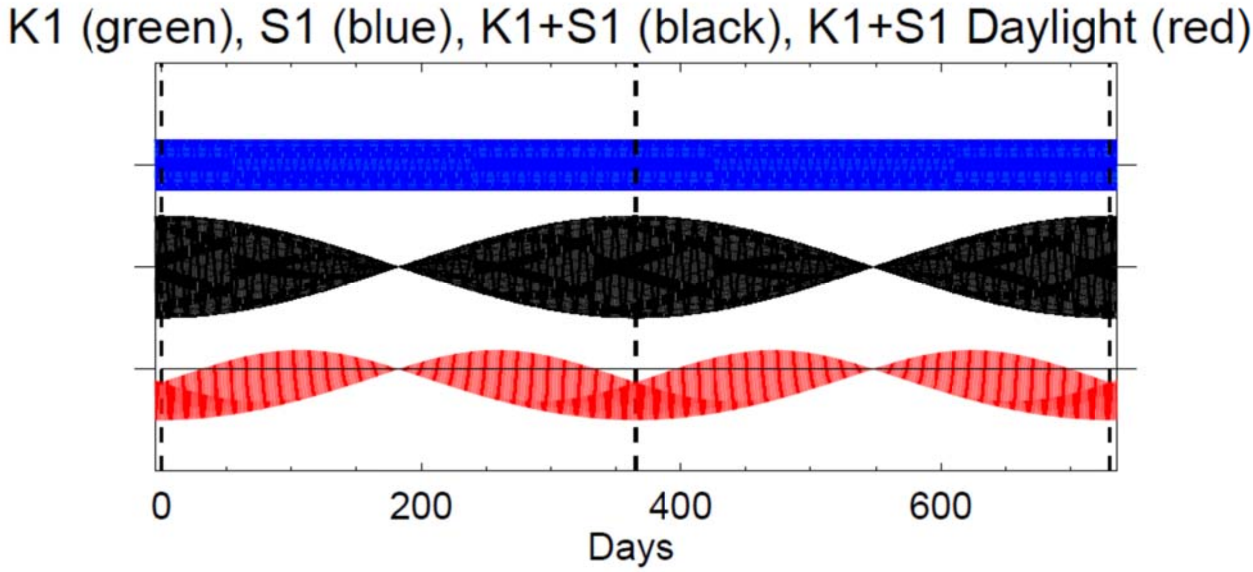
K1 (green), S1 (blue), K1+S1 (black), K1+S1 Daylight (red)
0
200
400
600
Days

Figure SM2(c)