A thin-sheet model of electromagnetic induction in northern England and southern Scotland

Jozwiak, W. & Beamish, D., 1986. Geophysical J. R. astr. Soc., 85, 629-643.

DOI: 10.1111/j.1365-246X.1986.tb04537.x

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Summary. Electric currents induced in the seas surrounding the British Isles influence the electromagnetic fields observed on land. Observational data suggest that, at certain periods, anomalous currents concentrate in a thin-sheet comprising the shallow seas and onshore sedimentary sequences. The block and basin structure of northern England and southern Scotland provides a physical basis for the implementation of a thin-sheet approximation in quantitative electromagnetic modelling studies of the region. The model comprises a non-uniform thin-sheet at the surface of a layered half-space. The solution to this problem, as formulated by Vasseur & Weidelt, has been applied to a regional thin-sheet model of northern England and southern Scotland.

A regional observational data set, comprising the anomalous vertical field at a period of 750 s, is used to constrain possible models. Geological and geophysical observations are used to provide a realistic model for the major sedimentary basin of the region and the results obtained are compared with observations. The results suggest that such a basin is capable of establishing *a substantial onshore perturbation in the anomalous vertical field*, of similar geometry to that observed. Although the present study is necessarily limited in its depth resolution, it is suggested that other models presented go some way to defining the lateral extent of the conductive configuration for the region.

Key words: electromagnetic induction, models, vertical fields, Britain

Introduction

It is evident from the results of geomagnetic deep sounding experiments (Edwards, Law & White 1971; Bailey & Edwards 1976; Hutton *et al.* 1981) that the deep ocean and shelf seas surrounding the British Isles strongly influence the electromagnetic fields observed on land.

The effects are frequency dependent. From a study of local and regional induction in the British Isles, Banks & Beamish (1984) identify three modes of induction in northern England and southern Scotland. At periods greater than 2000 s, geomagnetic transfer functions are determined largely by electric current flowing to the west and south-west in the Atlantic Ocean. In the period range 400–2000 s, the transfer functions are determined by currents induced in a thin-sheet of laterally varying conductance, comprising the shallow seas and their underlying sediments, together with the onshore extensions of the sedimentary basins. At periods less than 200 s, the transfer functions are compatible with an induction process which is controlled by the local geological structure. Thus, transfer functions calculated for the intermediate range of periods are not only determined by the local conductivity structure but also by the regional role of the conductive geometry in the surface sheet. This study addresses the problem of regional induction in northern England and southern Scotland.

The block and basin structure of the region is shown schematically in the north–south profile of Fig. 1. Two Carboniferous basins, the Northumberland and Stainmore troughs, are separated by the Southern Uplands, Alston and Askrigg blocks. Rocks of Ordovician and Silurian age outcrop to the north, in the Southern Uplands and around the Cheviot Volcanic Centre. The Alston Block, Cheviots and Southern Uplands were all intruded by granite batholiths towards the end of the Caledonian orogeny and have formed relatively buoyant blocks of crust.

The Caledonian metamorphic basement, including granite batholiths, can be expected to form a high resistivity basement over which there is a thin-sheet of relatively high, but laterally variable conductance. In the first instance, the present study considers the regional influence exerted by the Northumberland Basin. This superficial conductor is not isolated but is linked to the west and east with more extensive bodies with greater conductivity–thickness products: the Irish and North Seas and the underlying sedimentary basins of Permo–Triassic to Tertiary ages.

The suggested configuration of conductive material in a surface layer permits the use of a thin-sheet approximation in quantitative electromagnetic modelling studies for the region. The thin-sheet approximation produces an algorithm which is more effective than a complete 3-D model since the solution only requires the calculation of two horizontal electric field components across a 2-D surface grid, thereby drastically reducing the number of unknowns. Such a model was first considered by Price (1949). A solution for the global

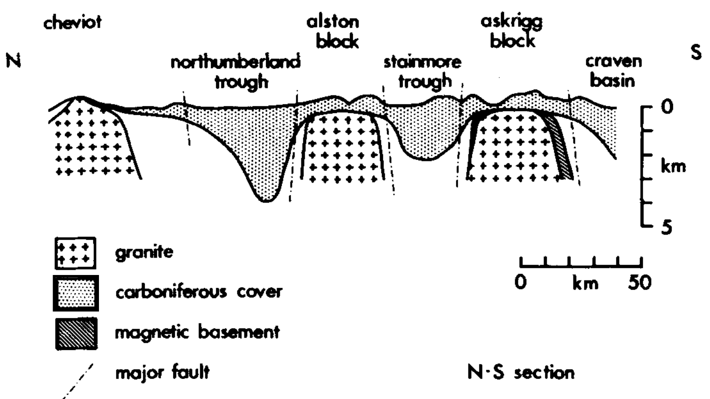

Figure 1. Schematic north–south profile of basement-cover relationships through northern England. Vertical exaggeration × 10.

problem was developed in terms of a scalar integro-differential equation. The work has subsequently been extended and modified by Vasseur & Weidelt (1977) and Dawson & Weaver (1979) in the form of an integral equation approach to the problem. An important feature of such models is the fact that the thin-sheet is in electrical contact with the substratum and this therefore gives rise to both toroidal and poloidal induced current systems or bimodal induction. Although the substratum may be modelled as containing one or more deep conductive layers, we wish to confine our attention to conductivity variations in the surface sheet and hence the underlying substratum is constructed in the form of a resistive half-space.

Models appropriate to such 'disturbed skin-effect' problems in three dimensions have been considered by a number of authors. Idealized structures have been considered by Weidelt (1977), Dawson & Weaver (1979) and McKirkdy & Weaver (1983). More realistic configurations have been considered by Vasseur & Weidelt (1977) and by Weaver (1982) who undertook a regional induction study of the entire Scottish mainland. The performance of the two construction algorithms (i.e. Vasseur & Weidelt 1977; Dawson & Weaver 1979) has recently been critically examined by Mareschal & Vasseur (1984). The present modelling study can be considered particularly well-posed in view of the number of observational data that can be applied in order to constrain otherwise free parameters.

The regional observational data set of Banks, Beamish & Geake (1983) consists of vertical field geomagnetic transfer functions presented in the form of a contour map derived from hypothetical event analysis, at a period of 750 s. This data set, which covers a major part of northern England and southern Scotland is used to constrain both the free conductances modelled within the onshore region and their spatial extent. Independent geological and geophysical information is then used to provide a realistic model for the Northumberland Basin and the results obtained are compared with the observations. The results suggest that such a basin is capable of establishing a substantial onshore perturbation in the anomalous vertical field, of similar geometry to that observed. A particular feature of the region considered is that the two major linear anomalies observed are likely to be due to the separate or combined effects of both near-surface and deep lateral conductivity gradients. The technique used is not capable of resolving or indeed separating such features, which both have limited lateral extent. It is argued, however, that the models developed go some way to defining the lateral extent of conductive material(s) beneath northern England and southern Scotland.

Observational data

Single-station, vertical field transfer functions for northern England and southern Scotland have been compiled into a uniform data set by Banks *et al.* (1983). The largest spatial data set (94 sites) was generated at a period of 750 s. Banks & Beamish (1984) suggest that at this period the regional context of local conductivity structures should be taken into account. The regional data set was presented using hypothetical event analysis. In this approach the single-station transfer functions are used to predict the vertical field of anomalous currents which would be produced when the horizontal component of the magnetic field across the region is spatially uniform and has a specified polarization and intensity. The inherent, spatially variable, bias in such a map due to the anomalous horizontal field can be a problem if quantitative inferences are required from the results in isolation. When comparing such results with theoretical models, however, it is a straightforward matter to provide results equivalent to those observed.

The direction of the horizontal magnetic field for which the anomalous vertical field was

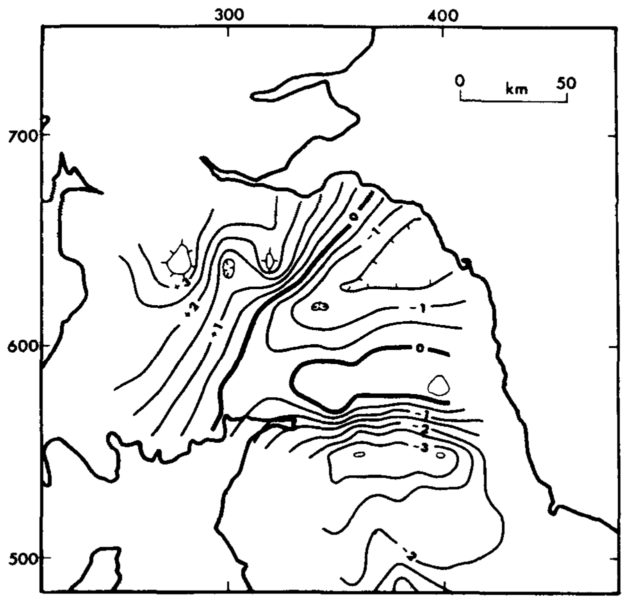

Figure 2. Map of the in-phase part of the vertical magnetic field produced when the horizontal field is directed towards magnetic north (9°W), and has unit amplitude. After Banks *et al.* (1983). Coordinate positions are National Grid. Contour interval is 0.5 with contour values × 10.

found to be greatest coincides with magnetic north–south and the anomalies are most pronounced in that part of the vertical field which is in phase with the horizontal. Fig. 2 reproduces the contours of the in-phase part of the vertical field related to a unit horizontal field in the north direction. For this direction of the unit field, the regional current flow is from east to west. If this current were only a function of depth, the vertical component of the magnetic field would be zero. Thus, the variations observed result from horizontal perturbations in current density, i.e. horizontal variations in the conductivity distribution. The regions with the largest spatial gradients in the anomalous vertical field correspond to areas with the largest conductivity contrasts.

The results indicate two principal linear features which can be used in the construction of numerical models. The two features have been commented upon by Banks *et al.* (1983). The first feature, which we will term anomaly 1, runs from NE to SW across the Southern Uplands with its axis to the SE of the Southern Uplands Fault. This feature does not coincide with any known conductive rock sequence close to the surface. The second feature, which we will term anomaly 2, runs from east to west through the Northumberland Basin between the North Sea and Solway Firth, and, has been studied in greater detail by Banks & Beamish (1984). The axis of the feature coincides with the southern margin of the basin where the sediment thickness is greatest.

A numerical regional model

The numerical analysis is based on a thin-sheet model proposed by Vasseur & Weidelt (1977). Using this formulation a computer program was developed by W. Józwiak and has been used to carry out the calculations described in this study. A detailed description of the model, together with the solution algorithm, can be found in the original paper by Vasseur & Weidelt (1977).

The study necessarily considers a simplified model of the onshore geological structure which is represented by a poorly conductive crystalline basement over which a thin-sheet of variable conductance is modelled. The observational data are used initially to constrain the range of conductance values and spatial distribution of the sediments. The free parameters are then adjusted to bring the theoretical and experimental results into agreement.

The mathematical method requires that the region considered, with anomalous conductance $\tau_a(\mathbf{r}) \neq 0$, should be surrounded by a normal medium for which $\tau_a(\mathbf{r}) = \tau_n$ = a constant. For the region considered, the normal medium is physically provided by the inner shelf areas with a depth of 100 m. This corresponds in the east to the North Sea and in the west to the Irish Sea. To the north and south, however, this assumption establishes an artificial 'cutting-off' of the land and close to the north and south boundaries the results have no physical significance. At the appropriate depth of 100 m, the conductance for the sea will be taken as 360 S. The inner shelf seas, to the west and east, are underlain by substantial thicknesses of post-Carboniferous sediments which would provide the basis for a much larger value for the offshore conductance. Numerical experiments indicate that if a larger value is used, it is necessary to increase the conductance value of the onshore sediments in order to maintain agreement between the theoretical and observed results. In this initial study, this additional physical complexity is ignored and the values of conductance obtained for the offshore sediments can be regarded as minimum values.

The resistivity of the underlying half-space is taken as 1000 Ωm (Hutton *et al.* 1981; Weaver 1982). In the absence of *a priori* information a value of 20 S was chosen to represent the conductance of the onshore resistive non-sediments. The above parameters are then regarded as non-free for the model under consideration. The region under study is covered by a square grid of 25 x 25 squares with distances of 15 km between the nodes as shown in Fig. 3. The inner square defines the region across which the observations are defined in Fig. 2 and over which the results of various models are presented. Calculations have been made using a number of models. Four models, shown in Fig. 4, have been chosen for presentation.

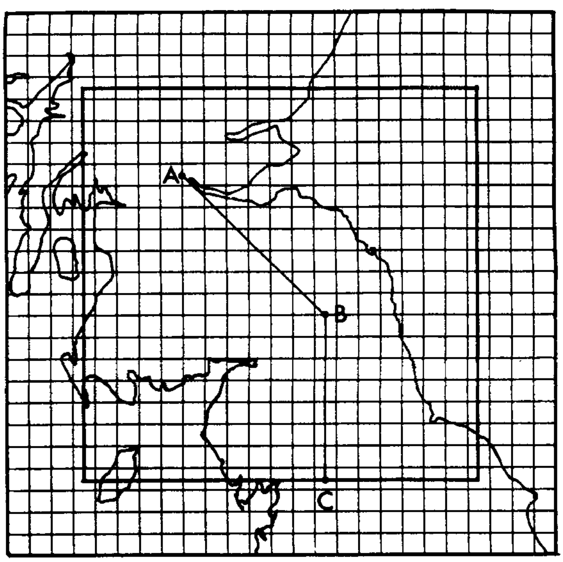

Figure 3. Square grid defining the regional model. The grid comprises 25 X 25 squares with distances of 15 km between the nodes. The inner square defines the region over which the results are presented. The profile ABC is used for the comparisons of Fig. 7.

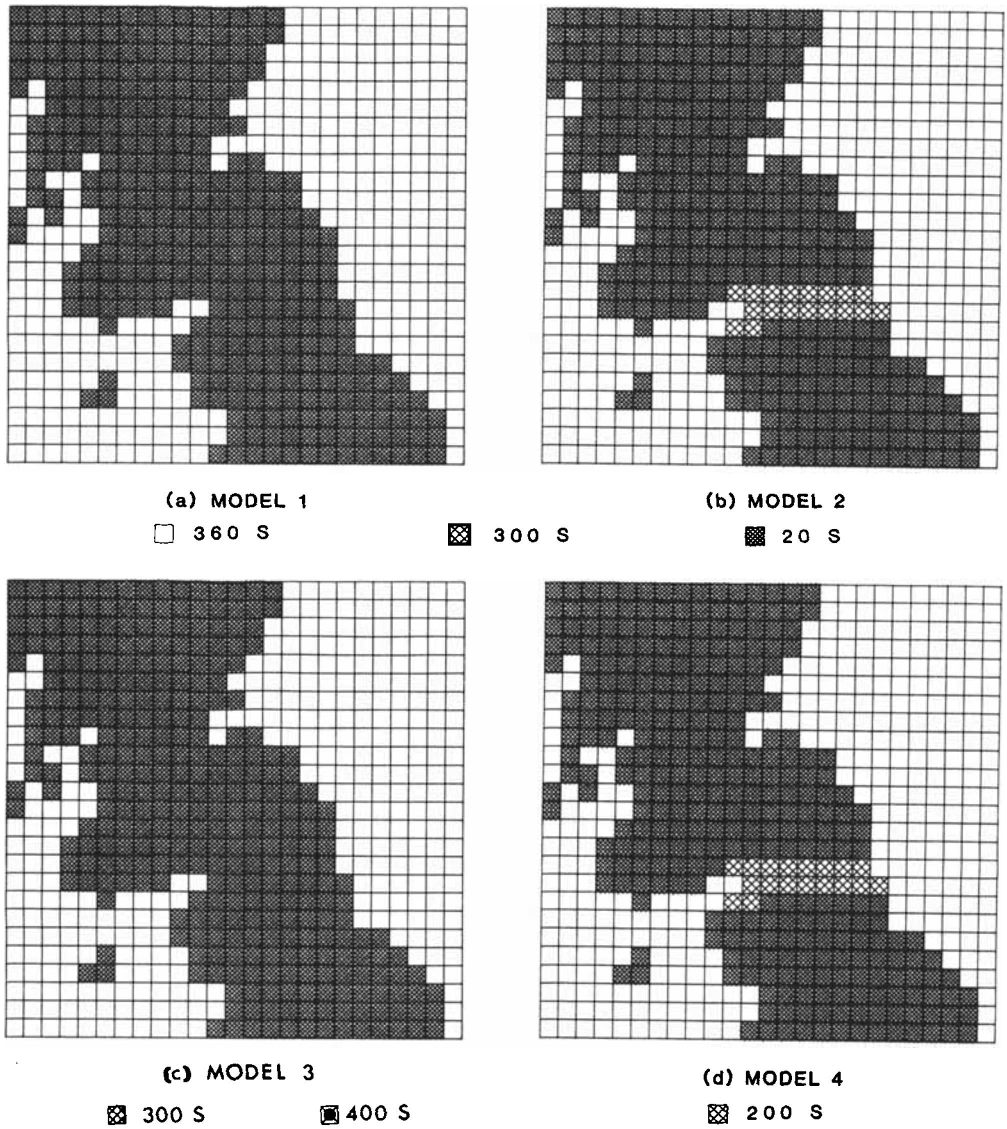

Figure 4. Four models, with associated values of conductance, used in the present study. (a) Model 1, (b) model 2, (c) model 3, (d) model 4.

The first three models (models 1 to 3) increase in order of complexity and differ in the distribution of the onshore conductive material. Model 4 is used to illustrate the effect of a change in value of the conductance of the onshore conductive material. The modelling exercise should be regarded, in the first instance, as an attempt to model the observational data by a single sedimentary basin, i.e. the Northumberland Basin, linking the North and Irish Seas. A detailed spatial model is provided by a seismic study of cover-basement relationships across the region presented by Bott, Swinburn & Long (1984). Although well-established variations in cover thickness exist both north–south and east–west, a first-order spatial model can be established by taking a mean depth for the sedimentary basin of 2.5 km and including only that part of the Northumberland Basin where the depth is greater than

1750 m. This is model 2 of Fig. 4(b). The problem of the remaining free parameter, i.e. the conductance of the sedimentary material, was approached using the constraints provided by the observational data. The value of the basin conductance was increased until the magnitude of the model results were in close agreement, but slightly less than, those observed. The value of 300 S obtained therefore represents a minimum value if the observational data are to be adequately modelled. This value represents material of conductivity 0.12 S m^{-1} if we assume a thickness of 2.5 km.

Before applying the model computations, it is necessary to re-examine the conditions under which the thin-sheet approximation is valid. There are two such conditions. The thickness of the surface layer should be small, to first order, compared with the skin-depth in the resistive substratum and the same thickness should also be small, to second order, compared with the skin-depth in the material defining the surface layer. In our case, for a period of 750 s, the respective skin-depths are 433 km for the resistive substratum, 7 km for the seawater and 40 km for the conductive material defining the sedimentary basin. Thus the thin-sheet conditions are satisfied for the models under consideration. For each model, values of the three magnetic components H_x, H_y and H_z are calculated for two polarizations of the normal horizontal field. It is then possible to determine the transfer functions (A, B) at all points across the surface grid such that

$$H_z = A \cdot H_x + B \cdot H_y.$$

Since in our models a geographical coordinate system was used, the normal horizontal field vector was rotated by 9°W to allow direct comparison with the observational results. The real part of the transfer function A then corresponds to the observed data set of Fig. 2.

For the relatively long period of 750 s, the electromagnetic penetration depth, in the underlying medium, is of the order of 400 km. It is advisable, therefore, to estimate and remove the artificial effects of the north and south model termination from the results obtained using the model grid (Fig. 3) having side dimensions of 375 x 375 km. We have calculated the transfer functions (A, B) for the whole British Isles represented by a grid of 25 x 25 squares with 37.5 km between nodes as indicated in Fig. 5(a). We have then calculated the transfer function for the 'artificial' model shown in Fig. 5(b), having a north–south land dimension of 375 km and corresponding apart from grid size, to the detailed regional model of Fig. 3. The effect of the artificial north–south termination of the landmass, in our detailed regional model, is then estimated as the difference in transfer functions generated by the models depicted in Fig. 5. The estimated values, allowing for the approximation due to the reduced grid size of Fig. 3, have been subtracted from all the model results presented.

Results

The results of the modelling procedure are presented in two ways. In the first instance a contour map across the inner region depicted in Fig. 3 is presented for comparison with the equivalent set of observations shown in Fig. 2. In the second instance, model results and observations are compared along the profile (ABC), again depicted in Fig. 3. Observational and modelling results are presented and compared using only the in-phase part of the vertical field transfer function. As noted previously, the observed anomalies are most pronounced in this component of the complex response. At a period of 750 s, anomaly 2 possesses a negligible in-quadrature response.

Fig. 6(a) shows the results obtained from a base model (model 1, Fig. 4a) with no

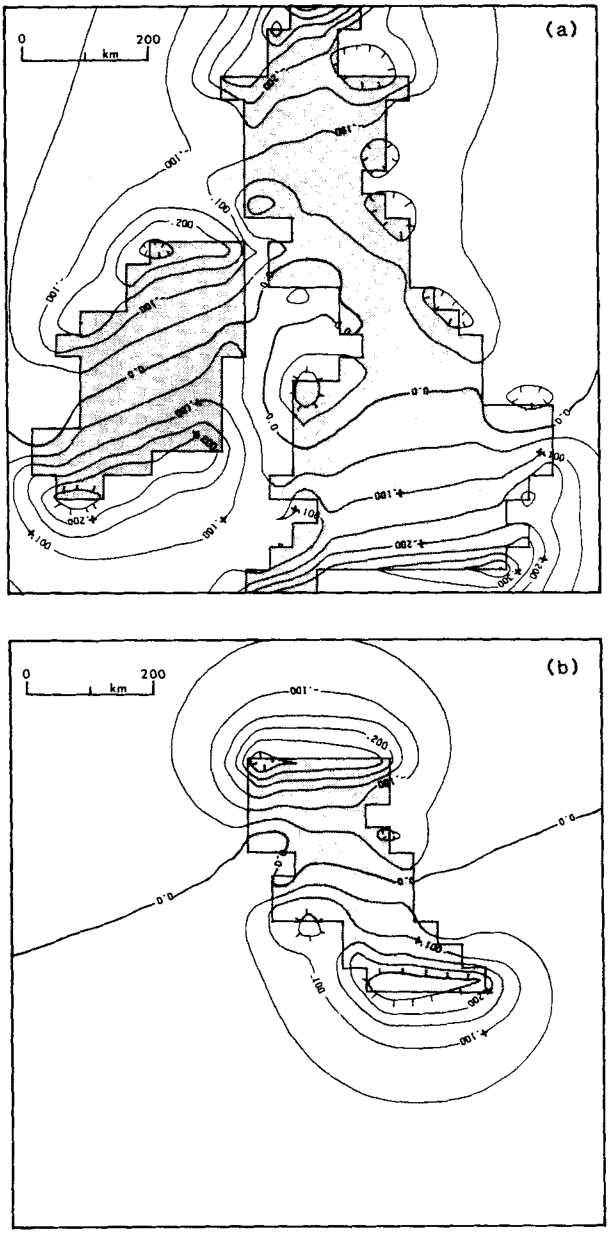

Figure 5. Two models used to estimate the effect of the artificial termination of the land-mass in the regional model of Fig. 3. The results contoured are the in-phase part of the vertical magnetic field produced when the horizontal field is directed towards magnetic north and has unit amplitude. Contour interval is 0.05. Conductance values are 360 S (offshore) and 20 S (onshore). The grid comprises 25 × 25 squares with distances of 37.5 km between the nodes.

onshore conductivity anomalies. In the distribution of values of the anomalous vertical field we observe only a coastline boundary effect.

Fig. 6(b) shows the results obtained from the simplified model of the Northumberland Basin described by model 2 (Fig. 4b). Apart from the coastline boundary effects, there is a distinct anomaly provided by the distortion of the contours which correlate with the boundaries of the onshore basin. A comparison between these results and the observational data of Fig. 2 reveals a correspondence only in the second linear feature (anomaly 2) running from east to west across the Northumberland Basin. This spatial comparison, together with

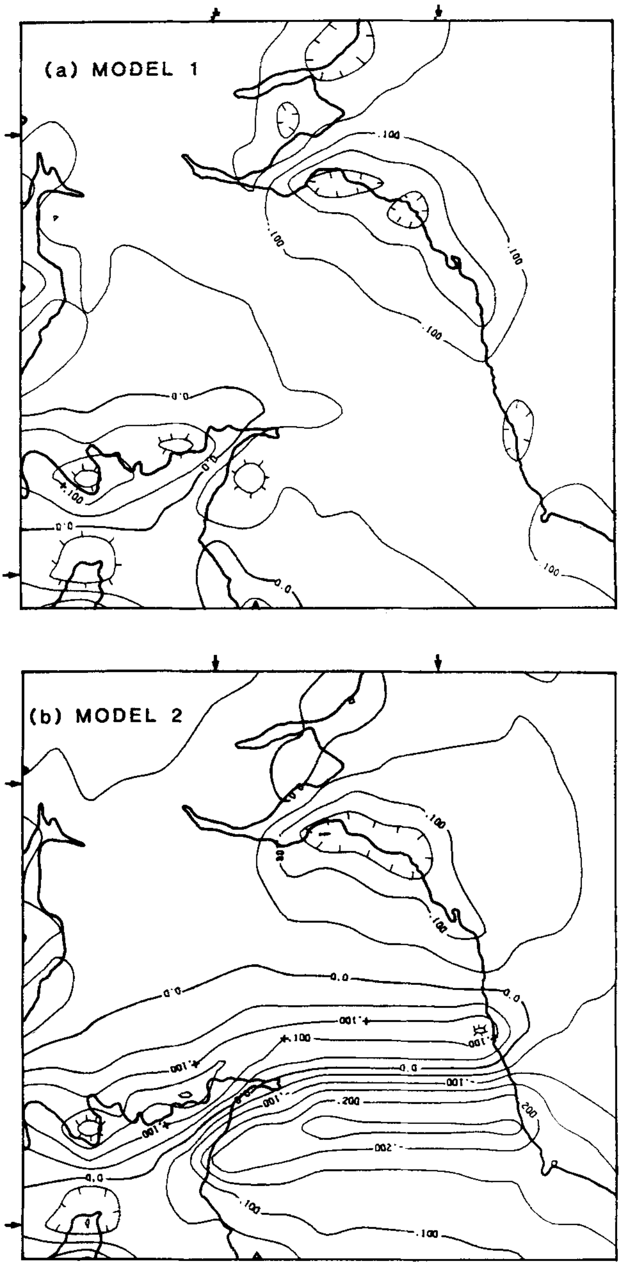

Figure 6. Maps of the in-phase part of the vertical magnetic field produced when the horizontal field is directed towards magnetic north, and has unit amplitude, produced by four models. Contour interval is 0.05 and all positive contours are provided with a plus sign. (a) Model 1, (b) model 2, (c) model 3, (d) model 4.

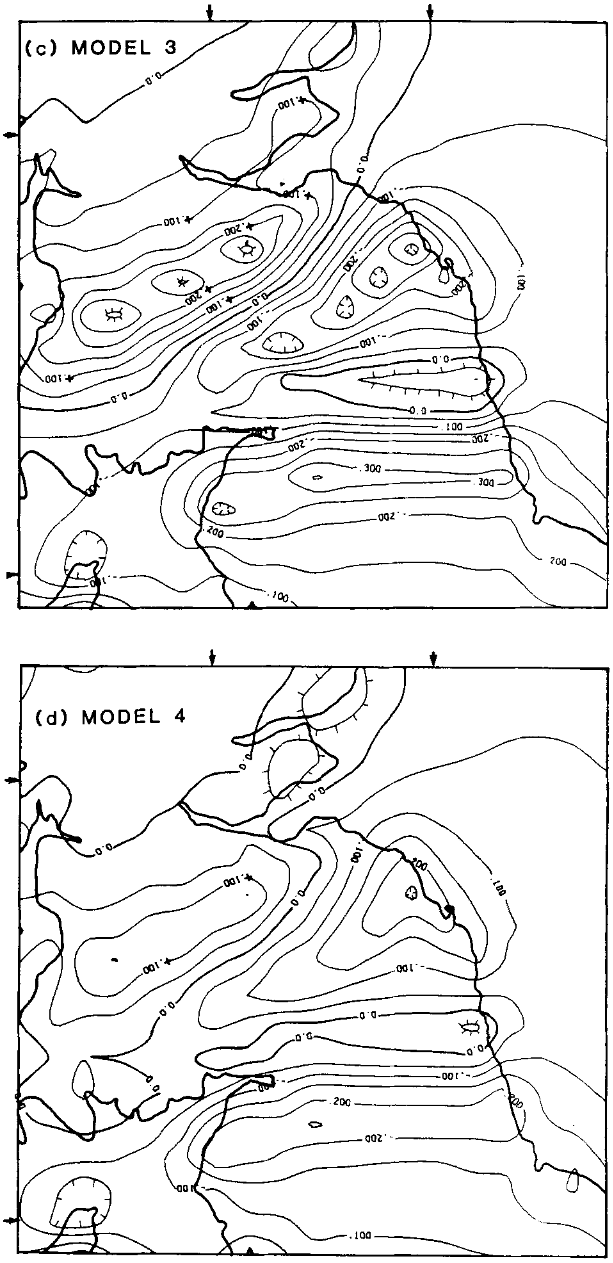

Figure 6 – *continued*

the profile comparison of Fig. 7 reveals that in the northern section (AB) there is considerable disagreement and thus the single basin model cannot fully account for the entire regional data set. It is clear that while model 2 can account for a substantial portion of the observed features of anomaly 2, a model with conductive material extending northwards from that of model 2 is required in order to generate the principal linear feature of anomaly 1.

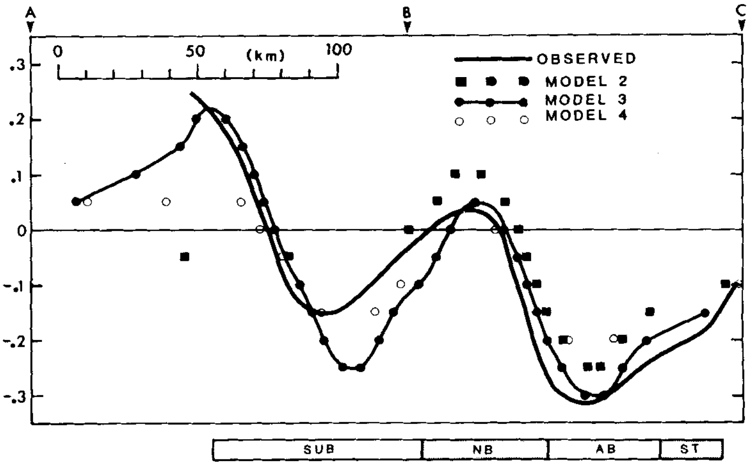

Figure 7. Profile comparison of observed and model results along profile ABC shown in Fig. 3. SUB: Southern Uplands Block, NB Northumberland Basin, AB: Alston Block, ST: Stainmore Trough.

A number of other models have been constructed and the results compared with the observations. Model 3 (Fig. 4c) is one of the simplest and includes a second conductive corridor along the southern margin of the Southern Uplands Fault. Fig. 6(c) shows the contoured results obtained using this model while the profile comparison is again displayed in Fig. 7. In general terms it can be noted that the model results are close to the observational data. The regions with maximum and minimum values essentially coincide and develop similar amplitudes. Perhaps the strongest observational constraint is provided by the half-widths of the two anomalies which restrict both the location and spatial width of the conductive corridors of model 3.

A further model (model 4, Fig. 4d) was developed from model 3 to illustrate the effect of the conductance values chosen for the onshore conductive material. Model 4 is identical to model 3 apart from the conductance values of the onshore conductive material. The results shown in Figs 6(c, d) and 7 clearly establish minimum conductance values for the two conductive corridors of order 400 S (anomaly 1) and 300 S (anomaly 2). These figures correspond to conductance ratios between onshore materials of order 20 (anomaly 1) and 15 (anomaly 2).

Taking into account that the geometry of the thin-sheet was modelled by surface elements with dimensions 15 x 15 km, we can regard model 2 as the simplest thin-sheet regional model of a single sedimentary basin than can be provided by the constraints of well established geological and geophysical parameters. It is recognised that this model, together with model 3 (discussed later), can only account for the first-order anomaly features, i.e. the spatial gradients established by the maximum and minimum values of the in-phase, anomalous vertical field.

Comparison of 3-D and equivalent 2-D models for the Northumberland Basin

The simplified thin-sheet model of the Northumberland Basin (model 2, Fig. 4b) consists of an east–west conducting corridor having a conductance of 300 S. The corridor is of width 30 km and is connected to resistive material of conductance 20 S, to the north and south. The length of the corridor is some 120 km and it is connected to the shelf-seas of conductance 360 S. The complete thin-sheet is underlain by material of conductivity 0.001 S m^{-1}. It is of interest to compare the results obtained from this model with those obtained from an equivalent 2-D model, in which the dimensions in one horizontal direction are assumed infinite.

An equivalent 2-D model was constructed along a north–south profile through model 2. The 2-D model consists of a half-space containing a surface layer, whose depth is taken as 2.5 km, underlain by material of conductivity 0.001 S m^{-1}. The surface layer contains the Northumberland Basin, of width 30 km and conductivity 0.12 S m^{-1}. To the north and south the conductivity of the surface layer is taken as 0.008 S m^{-1}. The values chosen generate the conductance values used in the corresponding thin-sheet model. The 2-D model necessarily assumes that the length of the conducting basin is infinite. The 2-D representation cannot therefore model the effect of the electrical connection of basin and shelf-seas or the geometrical effect of the coastline.

A numerical finite-difference solution to the 2-D problem, as formulated by Brewitt-Taylor & Weaver (1976), was used to solve the equations involved in the model defined above. Clearly for comparisons involving only the vertical magnetic field, only E-polarization calculations are required. The results obtained along profile B–C (Fig. 3) are shown in Fig. 8, for both model calculations. In Fig. 8(a), the real part of the transfer function A is shown alongside the observed results. In Fig. 8(b), the imaginary part of A is shown for the two model calculations. The observed imaginary part of A is less than 0.05 in magnitude across the profile B–C.

The comparison of results in Fig. 8 reveals that distinct differences exist in the maximum value of the vertical field generated by the modelling techniques together with differences in the spatial wavelengths obtained. Similar gradients are, however, observed across the central portion of the conducting region. The 2-D model, because of the symmetry involved in the model under consideration, generates results which are symmetrical about the centre of the conducting region. The 3-D model results appear much closer to the observed values in both real and imaginary parts of the transfer function. The results obtained indicate the extent to which 3-D formulations of the electromagnetic induction problem must be considered when vertical field data sets are to be modelled at intermediate and long periods.

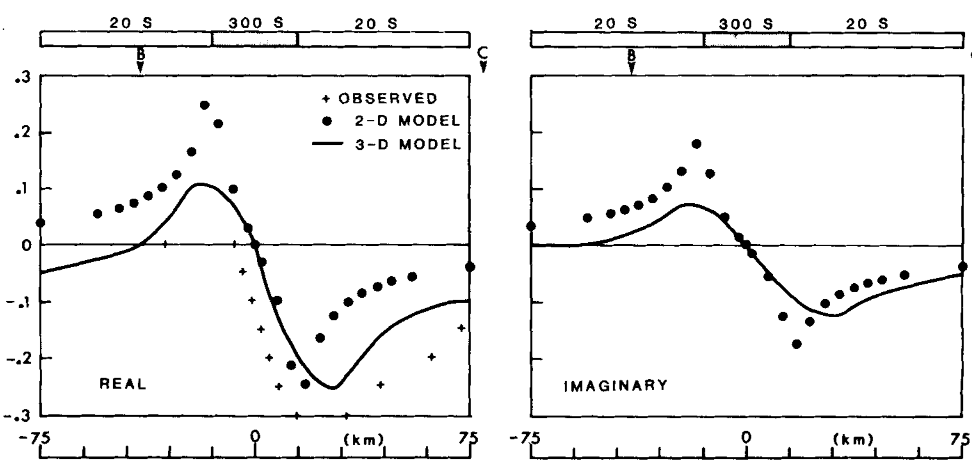

Figure 8. Comparison of 3-D (model 2) and equivalent 2-D model results along profile BC, across the Northumberland Basin.

Conclusions

One can adopt the view that the role of the thin-sheet algorithm is to model the likely influence of well-defined, near-surface conductivity contrasts within the context of the regional induction problem. This has been achieved, at least in part, for model 2 comprising a representation of the Northumberland Basin. From the results represented it can be recognized that model 2 can account for the belt of gradients running east–west across the southern margin of the Northumberland Basin. The spatial position and extent of the conductive basin used in this model can be considered well-constrained by geological and geophysical considerations. From the results presented we conclude that such a basin plays a substantial role in terms of the regional induction problem. At a period of 750 s, such a feature establishes a substantial onshore perturbation in the anomalous vertical field of similar geometry to that observed.

The magnitude of the onshore perturbation level produced by the model sedimentary basin can be considered in two ways. If the conductance of the onshore resistive material is considered as non-free, then the minimum conductance of 300 S obtained for the conductive material requires a mean longitudinal conductivity of the whole sedimentary complex to lie in the range 0.15 $\mathrm{S\,m^{-1}}$ (basin depth 2 km) to 0.1 $\mathrm{S\,m^{-1}}$ (basin depth 3 km). It is arguable whether such bulk conductivities are available over the whole sedimentary sequence despite the favourable presence of substantial thicknesses of water-saturated sandstone sequences producing artesian conditions (Hodgson & Gardiner 1971; Craddock–Hartopp & Holliday 1984). If, however, both onshore conductances are regarded as free, we require only the minimum conductance ratio 300/20 or 15 to be physically realizable. If we assume a thin-sheet of uniform depth (2.5 km) underlain by a uniform half-space, we can invoke Archie's Law (Archie 1942) to estimate conductivity from other measured rock parameters. We take Archie's Law in its simplest form.

$$\sigma_r = \sigma_f\, a\Phi^m$$

with σ_r, σ_f as the conductivities of the fluid saturated rock and electrolyte respectively, Φ the porosity expressed as vol. per cent and a and m as empirical constants. Typical value ranges are $a = 0.6$–3.5 and $m = 1.4$–2.0 (Keller 1967); these parameters then include the permeability and crack density of the rock. Since σ_f, a and m are largely unknown across a wide spatial scale we make the simplifying assumption that they do not differ between the two rock formations considered. The problem is then reduced to estimating the ratio of the bulk porosity of the conductive and non-conductive formations. To make the problem tractable we consider only the higher porosity sequences within the Carboniferous succession as defining the conductive material; these include the sandstone and some limestone sequences. Laboratory (Phemister 1982) and geothermal potential studies (e.g. Gale *et al.* 1984) would suggest a realistic figure of 10–15 vol. per cent for the porosity of such formations. If the resistive material is taken to include pre-Devonian (mainly Ordovician and Silurian) formations together with major granite intrusions, a realistic figure for their porosity of 1 vol. per cent can be suggested (Phemister 1982; Lee 1984). The ratio of the two suggested figures provides a conductivity ratio which ranges from 25 ($m = 1.4$, i.e. spherical particle texture and a porosity of 10 vol. per cent for the conductive formation) to 225 ($m = 2.0$, i.e. particles of decreasing sphericity and a porosity of 15 vol. per cent for the conductive formation). The minimum conductance ratio of 15 used in the model calculations can however be achieved using a porosity of 7 vol. per cent and $m = 1.4$. Although such values may be unrealistic, this brief discussion demonstrates the difficulties in obtaining useful bounds from two variables related by a power law. From the results presented it appears we can neither accept nor reject the hypothesis that the basin is *entirely* responsible for the observed anomalous vertical field at a period of 750 s.

In model 3, which was developed from model 2, it was assumed that in addition to the conductive region (30 km width) corresponding to the Northumberland Basin there is a second conductive corridor (40–50 km width) located on the southern side of the Southern Uplands Fault. A value of 400 S was chosen to represent the conductance of this corridor. Such a feature is clearly required if both linear features of the observational data set (anomalies 1 and 2) are to be adequately modelled. There is, to our knowledge, no known belt of conductive rock(s) close to the surface, equivalent to that modelled within the region. Beneath the Southern Uplands block, however, there undoubtedly exists a substantial conductive 'layer' whose upper surface lies within the lower crust and whose conductance, although spatially variable, lies in the range 250–750 S (Larsen 1981; Fischer & Le Quang 1981; Parker 1983; Beamish 1986). The model considered is necessarily restricted to an electrically thin-sheet. Although all the free conductivity variations take place in the surface sheet, for the model parameters considered and at the period of interest (750 s), the entire crustal depth may be included in the thin-sheet representation. On this basis it would be possible to construct an equivalent thin-sheet model at a depth of 15 km (say) and obtain a set of model results equivalent to those presented. In fact, the present observational data are too limited to determine the depth of the conductive anomalies correctly.

Model 3 should thus be viewed as modelling the *lateral* effects of two regions of major conductivity gradients within the crust under the assumption that all the regions are simply connected electrically. Given this view, model 3 defines a likely configuration of conductive material at some unspecified depth underlying northern England and southern Scotland. We therefore believe that such a model goes some way to defining the lateral extent of the conductivity variations responsible for the observed distribution of the regional anomalous vertical field. It is highly probable that the observed response of Fig. 2 is derived from the combined effects of both deep and shallow conductivity gradients, each of limited lateral extent. The thin-sheet models considered, and the currently available disturbed skin-effect algorithms, cannot incorporate such complexity. If a full 3-D modelling approach is to be adopted for the region considered, the present study points to the necessity of incorporating adequate regional boundary conditions, in the form of the shelf seas, for the modelling of results at intermediate and long periods.

Acknowledgments

One of the authors, WJ, is very grateful to Dr V. R. S. Hutton for many helpful discussions during his stay at the Department of Geophysics, University of Edinburgh, where part of this work was done. The authors also thank Professor J. Jankowski for valuable suggestions. This paper is published with the approval of the Director, British Geological Survey (NERC).

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