Induction in arbitrarily shaped oceans – VI. Oceans of variable depth

Geophysical Journal of the Royal Astronomical Society, 1983. **75**, 387-396.
DOI: 10.1111/j.1365-246X.1983.tb01931.x

D. Beamish *Geomagnetism Unit, Institute of Geological Sciences, Murchison House, West Mains Road, Edinburgh EH9 3LA*

R. C. Hewson-Browne *Department of Applied and Computational Mathematics, The University of Sheffield, Sheffield S10 2TN*

P. C. Kendall *Department of Mathematics, University of Keele, Keele, Staffordshire ST5 5BG*

S. R. C. Malin *National Maritime Museum, London SE10 9NF*

D. A. Quinney *Department of Mathematics, University of Keele, Keele, Staffordshire ST5 5BG*

Summary. We present calculations of the electric currents induced in a model ocean by the 24 hr S_q variation. The model has realistic bathymetry and represents a further step of a consistent approach to oceanic induction in which we have introduced various complexities in order of priority. The solution obtained is compared with our previously published results for the case of an ocean of realistic shape and uniform depth. It is found that the major shelf-seas considerably modify the pattern of electric current circulation in the oceans of the southern hemisphere. The consistent nature of the method of matched outer and inner expansions which we have used is also discussed. It seems probable that other methods, not limited to finding an outer solution and consequently more complicated, will at best only produce an outer solution since, in practice, the relatively coarse mesh necessarily used in the computation of realistic global problems precludes a fine resolution of the coastal bathymetry. Finally, the meaning of the internal and external Legendre coefficients and their relationship to S_q over the oceans is briefly discussed.

1 Introduction

In a series of papers (Hewson-Browne & Kendall 1978a, b; Hewson-Browne 1978; Beamish *et al.* 1980a, b), the currents induced in the Earth and oceans by the S_q variation fields have been calculated in order to assess the relative importance of various features. We have increased the complexity of the model by stages and undertaken validation at each stage. In all the models the spherically symmetrical conducting earth is simulated by a perfectly conducting sphere at a depth of 500 km. When the overall model becomes sufficiently refined there will be no difficulty in principle in replacing this perfectly conducting sphere by a spherically symmetric finite conductor. So far our refinement of the model has been confined to a thin shell representing the ocean. The first two studies (Hewson-Browne & Kendall 1978a, b – papers I and II in this series) concerned a perfectly conducting hemispherical shell and its edge corrections. The main purpose of these studies was to establish the validity of the method of matched asymptotic expansions and to check the results against analytic solutions. Having done so, the next model considered was a more realistic ocean of finite conductivity in paper III (Hewson-Browne 1978). In papers IV and V (Beamish *et al.* 1980a, b) we applied the method to two model oceans of finite conductivity using observationally obtained S_q coefficients. Both of these models were based on a $2° \times 2°$ representation of the continents and oceans with the ocean assumed to be of constant depth (4 km) and of uniform conductivity (3.3 S m^{-1}). All continental regions were assumed to be insulators so that currents induced in the thin shell are restricted to the oceans. In paper IV all continent/ocean boundaries were assumed to be at the same potential so that no current could circulate around the continents. The results obtained were shown to reproduce the results of Bullard & Parker (1970) when applied to the same S_q coefficients. In paper V the unrealistic constraint of no circulation of currents was removed and it was found that large induced currents circulated in the southern hemisphere around Antarctica and Australia. Where these circulatory currents were constrained to pass through narrow oceanic channels such as the Scotia Sea between South America and Antarctica, or the straits between Australia and South-east Asia, the current density became particularly high. While this may be realistic for the relatively deep Scotia Sea, it seems less plausible in the shallower, continental shelf-seas between Australia and South-east Asia. Because the model assumed a uniform depth for the oceans, no distinction was made between these two channels.

In the present paper we move a further step nearer to reality by taking account of the variable depth of the ocean using the 100 m bathymetric contour as its edge. This involves non-trivial modifications to the mathematical formulation and to the computer program. As in all previous stages, the modified version was validated by confirming that it reproduced the earlier results before proceeding to calculate new ones. In the present case, the variable depth program was applied to an ocean of constant depth for comparison with the results of paper V. Again we have used the outer expansion from the method of matched asymptotic expansions. This approach is defended in some depth in the conclusion where it is pointed out that all methods used to date can produce only the outer expansion. We therefore suggest that one might as well obtain it directly, thereby adopting a uniform approach to the problem. The significance of the calculations for S_q over the land and over the ocean is also elucidated.

2 Theory

Following the analysis given in paper V, it is sufficient to consider a single harmonic for the inducing S_q field with external potential $(r/a)^n S_n^m$, where (r, θ, ϕ) are spherical polar coordinates, a is the Earth's mean radius and

$$S_n^m = \Big\{[\gamma_n^m(a_p) - i\gamma_n^m(b_p)] \cos m\phi + [\sigma_n^m(a_p) - i\sigma_n^m(b_p)] \sin m\phi\Big\}\, aP_n^m(\cos\theta) \exp(i\omega t). \tag{1}$$

In terms of universal time t^*, $\omega t = pt^*$ and in the subsequent computations, an S_q period of 24 hr corresponds to $p = 1$. The oceanic electric current may be expressed in terms of a current function ψ so that

$$\mathbf{J} = -a^{-1}\hat{\mathbf{r}} \times \nabla_s(\psi),$$

where ∇_s is the dimensionless surface operator

$$\nabla_s = [0, \partial/\partial\theta, (\sin\theta)^{-1}\partial/\partial\phi].$$

In paper III it was shown that ψ approximately satisfies the differential equation

$$\nabla_s \cdot (\rho + i\gamma)\nabla_s\psi = -i\mu_0^{-1}\gamma n(2n+1)S_n^m, \tag{2}$$

where $\rho(\theta, \phi) = \kappa_0/\kappa$, $\gamma = (1-k)\Gamma$, $\Gamma = \omega\kappa_0\mu_0 a$ and $k = b/a$. Here κ is the integrated surface conductivity of the ocean with a convenient constant scale value κ_0 and b denotes the radius of the inner perfectly conducting sphere. It was suggested at that time, that replacing γ by $\gamma_n = (1-k^{2n+1})\Gamma/(2n+1)$ might be beneficial; however γ_n is dependent on n, and the present computation is much simplified by retaining γ. The equation is to be solved under the boundary condition ψ = constant on each coastline. However, since electric current is able to flow between separated land masses, the constant value of ψ may differ for different coastlines. We suppose that there are M separate land masses with coastlines $C_\upsilon (1 \leqslant \upsilon \leqslant M)$ and for convenience take $\psi = 0$ on the 'mainland' C_M (say). The remaining $M-1$ constants may then be evaluated by applying the conditions

$$\oint_{C_\upsilon} \left\{(\rho + i\gamma)\frac{\partial\psi}{\partial N} - \frac{i\gamma}{\mu_0}\left(\frac{2n+1}{n+1}\right)\frac{\partial S_n^m}{\partial N}\right\} ds = 0, \quad 1 \leqslant \upsilon \leqslant M-1, \tag{3}$$

where $\partial/\partial N$ denotes differentiation along the surface in a direction normal to C_υ.

In practice, we put

$$\psi = g_c\psi_c + g_s\psi_s, \tag{4}$$

where

$$g_c = -i\mu_0^{-1}\gamma n(2n+1)\,a\left\{\gamma_n^m(a_p) - i\gamma_n^m(b_p)\right\}\exp(ipt^*),$$
$$g_s = -i\mu_0^{-1}\gamma n(2n+1)\,a\left\{\sigma_n^m(a_p) - i\sigma_n^m(b_p)\right\}\exp(ipt^*). \tag{5}$$

The functions ψ_c and ψ_s then satisfy

$$\nabla_s \cdot (\rho + i\gamma)\nabla_s\psi_c = P_n^m(\cos\theta)\cos m\phi,$$
$$\nabla_s \cdot (\rho + i\gamma)\nabla_s\psi_s = P_n^m(\cos\theta)\sin m\phi, \tag{6}$$

together with the boundary conditions

$$\psi_s = \psi_c = 0 \text{ on } C_M,$$

and

$$f_\upsilon(\psi_c) = -I_\upsilon^c,$$
$$f_\upsilon(\psi_s) = -I_\upsilon^s, \tag{7}$$

for $1 \leqslant \upsilon \leqslant M-1$, where we have defined

$$f_\upsilon(\alpha) = \oint (\rho + i\gamma)\frac{\partial\alpha}{\partial N} ds \tag{8}$$

and

$$I_v^c = \frac{1}{n(2n+1)} \oint_{C_v} \frac{\partial}{\partial N} \left\{ P_n^m(\cos\theta) \cos m\phi \right\} ds,$$
$$I_v^s = \frac{1}{n(2n+1)} \oint_{C_v} \frac{\partial}{\partial N} \left\{ P_n^m(\cos\theta) \sin m\phi \right\} ds. \tag{9}$$

This formulation may be further simplified for computation by solving for particular integrals ψ_c^M, ψ_s^M satisfying

$$\psi_c^M = \psi_s^M = 0 \text{ on } \textit{all } C_v. \tag{10}$$

For the complementary functions, we compute $H_q(1 \leqslant q \leqslant M-1)$ satisfying

$$\nabla_s \cdot (\rho + i\gamma) \nabla_s H_q = 0 \tag{11}$$

with

$$H_q = \begin{cases} 1, & v = q, \\ 0, & v \neq q, \end{cases} \tag{12}$$

on $C_v(1 \leqslant v \leqslant M-1)$. Putting

$$\psi_c = \psi_c^M - \sum_{q=1}^{M-1} \lambda_q H_q,$$
$$\psi_s = \psi_s^M - \sum_{q=1}^{M-1} \mu_q H_q, \tag{13}$$

equations (7) then reduce to

$$\sum_{q=1}^{M-1} \lambda_q f_v(H_q) = f_v(\psi_c^M) + I_u^c,$$
$$\sum_{q=1}^{M-1} \mu_q f_v(H_q) = f_v(\psi_s^M) + I_v^s, \tag{14}$$

for $1 \leqslant v \leqslant M-1$. These provide $2M-2$ linear algebraic equations for the constants λ_q, $\mu_q(1 \leqslant q \leqslant M-1)$.

Finally for the computation, we take the real part of ψ in (4) and then sum over the appropriate values of m and n for a given value of p.

3 Practical details

In the case of the present model, separate calculations are required for the four S_q periods of 24, 12, 8 and 6 hr. We here restrict ourselves to the dominant S_q period of 24 hr. The external spherical harmonic coefficients obtained by Malin & Gupta (1977) for a period of 24 hr are used. Fifteen complex terms up to P_4^1 are included in the representation. The results of paper V indicated that the island of Spitzbergen had negligible effect on the results obtained, consequently the present model of the continental land-mass contains four islands (the mainland, Antarctica, Australia and New Zealand).

The model ocean bathymetry on a $2° \times 2°$ grid is obtained by smoothing the $1° \times 1°$ model of Gates & Nelson (1975). The $2° \times 2°$ bathymetry grid values have been contoured and are shown in Fig. 1. The contour interval is 1 km. We particularly wish to emphasize that

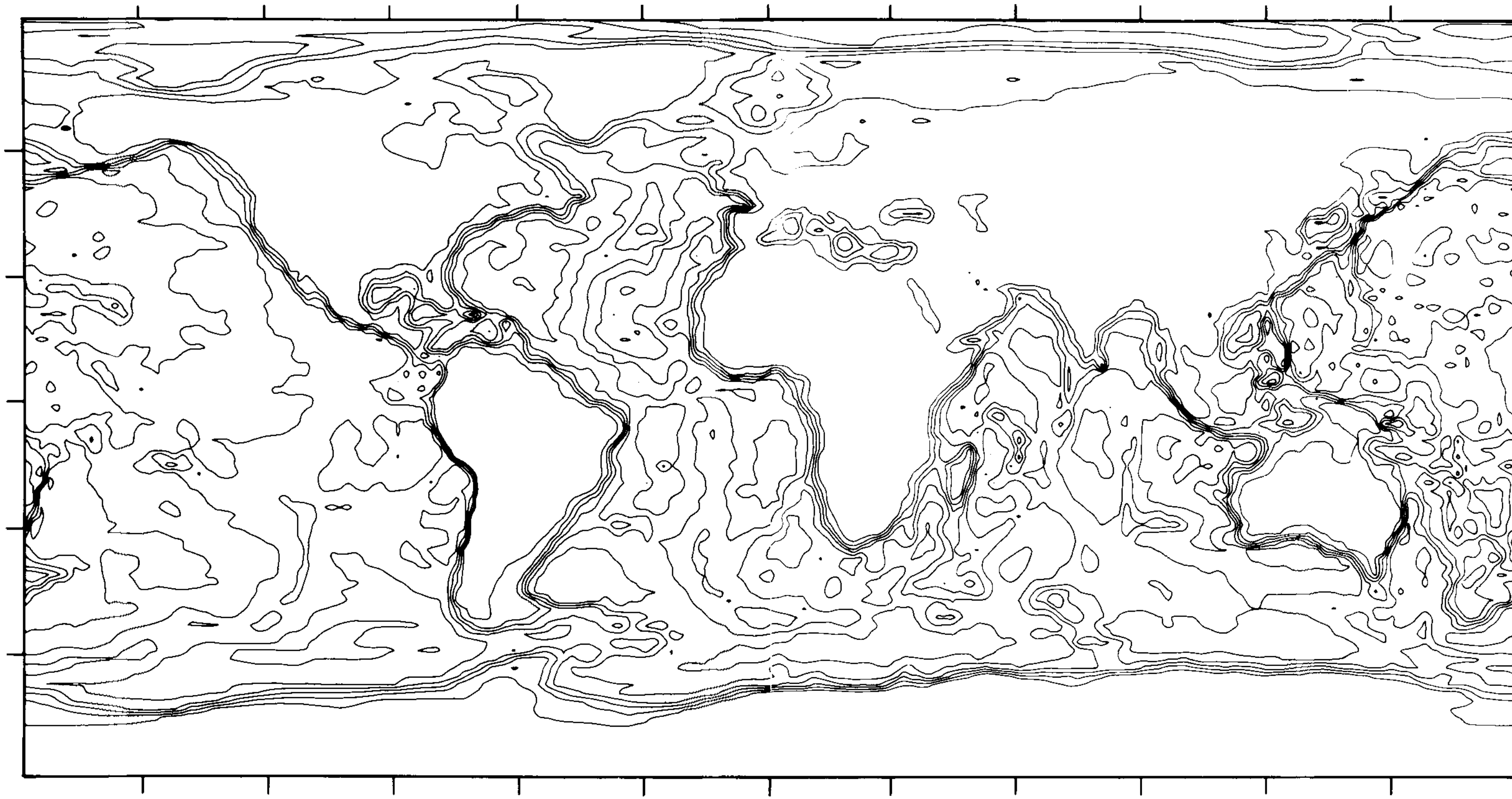

Figure 1. Ocean bathymetry of variable depth model defined by a 2° × 2° grid. The contour interval is 1 km.

although the mean meridional (50°N–70°S) depth of ocean, when zonally averaged, is 4 km, major shelf seas such as those around Indonesia and Madagascar are well-defined by the $2^\circ \times 2^\circ$ representation. Several of the world's mid-ocean ridge systems appear as smoothed representations.

4 Results

The results of our analysis are displayed in Fig. 2. Fig. 2(a) shows the in-phase part and Fig. 2(b) the in-quadrature part of the electric current system induced in the oceans by the S_q variation field of period 24 hr. The contour interval is 2500 A. These results may be directly compared with fig. 1(a, b) of papers IV and V obtained for an ocean of uniform depth.

As in most thin-sheet formulations of the induction problem (Price 1949) it is the variation in depth-integrated conductivity, or conductance, that determines the geometry of the induced current flow. In our model, electric currents are restricted to the oceans and, due to the dominance of the $P_2^1 S_q$ harmonic, are constrained to be of ocean-wide extent. Consequently it is only in those oceanic areas where the conductance differs on a regional scale between the two models that we expect significant differences. In the present case the conductivity has not been altered and the above argument applies simply to the extent to which our $2^\circ \times 2^\circ$ representation of ocean depths differs substantially from the uniform depth of 4 km in the previous model.

Considered globally we note that the positions of the main induced current foci have not altered. The introduction of variable depth has therefore not significantly affected the phase of the induced S_q current system. In both models, of uniform and variable depth, the major circulatory current system is maintained in the three oceans of the southern hemisphere. It is in this hemisphere that differences between the two models are most pronounced. Consider first the in-phase results. In our previous uniform depth model the current density in the shelf seas between Australia and South-east Asia was particularly high. The high current density circulating through these straits enabled a large current focus of 32 000 A to be maintained to the west of Australia. In our present model, the realistic ocean depths used (Fig. 1) result in a large regional decrease in conductance and consequently the current density that can be maintained across these shelf seas is much reduced. This constraint also reduces the amount of current that can circulate between the Indian and Pacific Oceans, resulting in a much reduced current focus to the west of Australia. Overall, the current density circulating in the Indian ocean has been reduced by a factor of 2 in the case of the present model. These modifications should be contrasted with the results obtained for the Scotia Sea separating South America and Antarctica. In this region, the deep ocean is continuous between the South Pacific and South Atlantic oceans and very little modification to the regional current density is obtained in the case of the present model.

We next consider the in-quadrature results for the same two regions. Again only a minor modification in current density is observed across the Scotia Sea so that the main current foci obtained in the South Pacific and South Atlantic do not differ substantially between the two models. In the case of the shelf-seas of Indonesia we note that the main current focus, for both models, lies in the centre of the Indian ocean and consequently there is far less current circulating into the Pacific Ocean through the shelf-seas of Indonesia than is the case for the in-phase current system. The current density obtained for the present model in this region has, however, been reduced. The modification to current circulation in the southern hemisphere introduced by the ocean of variable depth clearly depends on the spatial geometry of the inducing field.

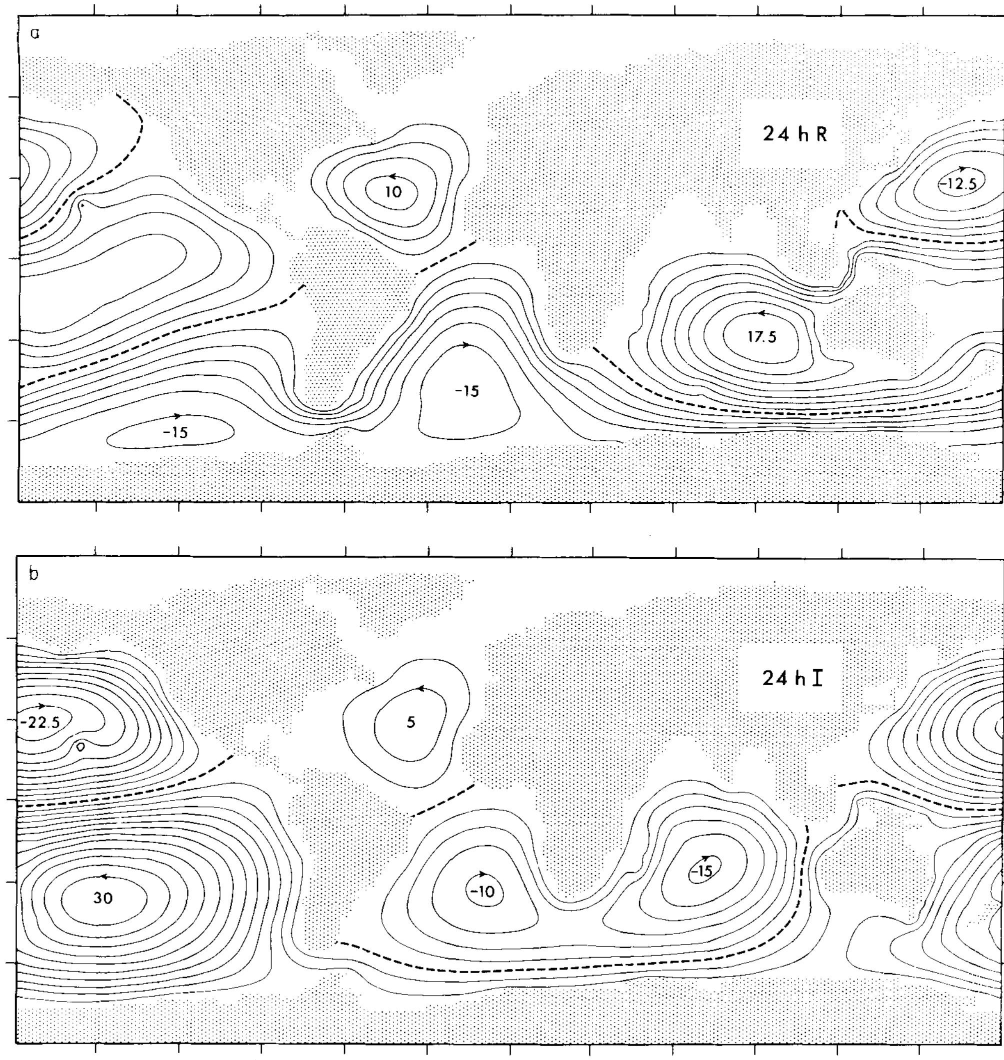

Figure 2. (a) World curves of the in-phase part of the electric current induced in the variable depth ocean for 24 hr period. Circulation conditions and four islands are taken into account as described in the text. 2500 A intervals between contours. Zero value contours are shown as broken lines. (b) The in-quadrature part of induced currents for 24 hr period.

There are other modifications worthy of note. The continental margin off the east coast of Madagascar produces a clear perturbation of the current system circulating in the Indian ocean in both the in-phase and in-quadrature parts. Generally the mid-ocean ridge systems are not well-resolved on the $2^\circ \times 2^\circ$ model of ocean bathymetry. The Hawaiian ridge striking NW–SE in the central Pacific is observed to introduce a substantial perturbation to the circulating current system in the northern Pacific ocean.

So far we have considered only the case of the induced ocean currents due to the S_q variation field of period 24 hr. The induced current systems for periods of 24, 12, 8 and 6 hr obtained in the case of the uniform depth ocean have been published in paper V. The external field variations at periods of 12, 8 and 6 hr possess progressively higher order spherical harmonic terms giving rise to induced current systems as obtained in paper V. We suggest that the modifications introduced in the case of the variable depth ocean at these higher frequencies may be readily deduced from the modifications observed in the present analysis.

5 Discussion

The present analysis has considered a model ocean containing realistic depths and the procedure has been validated by confirming that it reproduces the results of previous models. The analysis has been applied using an observationally derived model of S_q to obtain the induced electric current system in the ocean at a period of 24 hr. The results obtained have been compared with those obtained using a uniform depth ocean. The main finding to be emphasized is that it is the distribution of constraints to current flow that introduce the main modification to current flow in the oceanic thin shell. Thus for the case of our $2^\circ \times 2^\circ$ model it is the major shelf-seas of Indonesia in relation to the geometry of the induced current system (and hence the structure of the inducing field) that are capable of considerable modification to the current systems of the Indian and northern Pacific oceans. A large reduction in current density occurs when the seas between Australia and South-east Asia are given realistic depths. Elsewhere the definition of major restrictions such as mid-ocean ridges and sea-mounts is necessarily smoothed or omitted by our $2^\circ \times 2^\circ$ model. It should be noted however that such a model is adequate when consideration is given to the spatial extent of the S_q variation field.

We obtained these results by using the *outer* solution of the method of matched asymptotic expansions. This is not the only way of solving these problems, but it is arguably the best in that it permits greater resolution and a more extensive Legendre expansion. In addition it constitutes a uniform approach to the problem; all other methods of solution to date appear to be feasible only if their resolution is poor – in that case these other methods can produce at best the outer solution and at worst a truly inaccurate solution. The arguments are as follows. The main competitors are: (1) the method of shifting the spectrum (Hutson, Kendall & Malin 1972, 1973; Kendall 1978); (2) the method of the biharmonic Green's function (Fainberg & Zinger 1980, 1981; Zinger & Fainberg 1980, 1981); (3) the multiple integral equation method (Hewson-Browne & Kendall 1981). These have been described in a recent review by Kendall & Quinney (1983).

The method of shifting the spectrum appears to have lived up to expectations. However, in practice its use has been limited to cases with three harmonics, an ocean of uniform depth but realistic shape and a $5^\circ \times 5^\circ$ mesh. This method has now been overtaken in speed and convenience by the method we are currently using. The biharmonic Green's function method seems to be promising, but its present stage of development lies at the level of one harmonic with a more sophisticated model of the oceans incorporating sedimentary deposits, and a $5^\circ \times 5^\circ$ mesh. Lastly, the multiple integral equation method was certainly not designed to increase the speed and resolution of the computations. Indeed, the last two methods named are heading towards better overall modelling.

Let us now consider the question of resolution. At worst, poor resolution will merely produce an inaccurate solution, and the accuracy may be irregular and unknown. At best, a solution with poor resolution will reproduce only our outer solution. Some insight into the physical reasons why this is so may be obtained by analogy with the problem of incompressible viscous flow at high Reynold's number past an aerofoil. Suppose that the boundary layer on the aerofoil, governed by the inner solution, is of a thickness equivalent to 1°, and we choose a 5° mesh on which to discretize the problem. Away from the aerofoil all that will be seen at best is the inviscid flow pattern governed by the outer solution. At worst the solution may be quite inaccurate. In the case of magnetic induction at high magnetic

Reynolds number similar arguments apply. The relevant dimensions for the present configuration are the skin depth of the mantle (corresponding to about 5°, although its value is unknown, almost to an order of magnitude) and the length $1/(\omega\mu_0\kappa)$ which at 14 hr periods also corresponds to 5°. These correspond to a magnetic Reynolds number of unity. Apart from papers I and II only one attempt has been made to link together an inner and outer solution; for the well known problem of the uniform hemispherical cap. Hobbs & Dawes (1980) achieved this by means of a refined mesh near the ocean's circular edge. However, this aspect appears to be missing from Hobb's later computations (1981) for realistically shaped oceans, moreover the magnetic field calculations used in his fig. 7 appear to contain multiple singularities along the coast. We take the view that local bathymetry needs to be taken into account in any comparison between observed and calculated magnetic fields.

Finally, we consider briefly the meaning and necessity of calculations of the electric current induced in the oceans by S_q. The matter seems to have become confused with the question of observationally based Legendre expansions of the S_q and their separation into internal and external parts. Hobbs (1981) has used the S_q data of Malin & Gupta (1977) and Winch (1981) to calculate equivalent currents over the ocean. This procedure seems to have inverted the argument which ought to follow once it is conceded that the oceanic magnetic field penetrates at most only about 500 km inland or perhaps a little more, and is hardly present except in the vertical components of those stations on the coastline. The correct calculation of the S_q induced magnetic field over the ocean should probably be made by using the calculated induced electric currents combined with the internal coefficients. This question is important from the point of view of the experimenters and we hope to report on it at a later date. Eventually S_q will be measured by satellite above both the oceans and land. The results will have to be treated specially in view of the non-uniqueness of Legendre coefficients in any domain covering only part of the Earth's surface. The Legendre coefficients for the internal part of S_q correspond at present only to the land. The external S_q coefficients should be the same over both sea and land. But the internal S_q coefficients may be *chosen* to be different in different physical regions comprising large parts of the surface of the globe. The method of data analysis when applied to statistics from one region only, such as the land, seems designed to produce the best set of coefficients, probably the most convergent set for that group of statistics. However, Legendre and Fourier series are not unique over a partial domain. In paper I we showed how to re-expand the Legendre series of a decaying function in such a way that its expansion also became clearly decaying. This question is being further analysed.

We note, therefore, that calculations of this kind will continue to be necessary. It is hoped to continue with a study of the ocean tidal effect (which Hewson-Browne 1973 has shown to be an essentially similar problem) once the tidal velocity of the oceans is readily available.

Acknowledgments

This paper is published with the approval of the Director, Institute of Geological Sciences (NERC). Part of the work was carried out while one of us (PCK) was on leave at the Geomagnetism Unit, IGS, Edinburgh.

References

Beamish, D., Hewson-Browne, R. C., Kendall, P. C., Malin, S. R. C. & Quinney, D. A., 1980a. Induction in arbitrarily shaped oceans IV: *Sq* for a simple case, *Geophys. J. R. astr. Soc.*, **60**, 435.

Beamish, D., Hewson-Browne, R. C., Kendall, P. C., Malin, S. R. C. & Quinney, D. A., 1980b. Induction in arbitrarily shaped oceans V: the circulation of *Sq* induced currents around land masses, *Geophys. J. R. astr. Soc.*, **61**, 479. [See also erratum for improved diagrams, *Geophys. J. R. astr. Soc.*, **63**, 573.]

Bullard, E. C. & Parker, R. L., 1970. Electromagnetic induction in the oceans, *The Sea*, vol. IV, part 1, pp. 695–730, eds Bullard, Sir Edward & Worzel, J. L., Wiley, New York.

Fainberg, E. B. & Zinger, B. Sh., 1980. Electromagnetic induction in a non-uniform spherical model of the Earth, *Annls Géophys.*, **36**, 127.

Fainberg, E. B. & Zinger, B. Sh., 1981. Electromagnetic induction in a spherical model of the earth with a real distribution of near-surface conductivity, *Phys. Earth planet. Int.*, **25**, 52.

Gates, W. L. & Nelson, A. B., 1975. *A new (revised) tabulation of the Scripps topography on a 1° global grid, Part II: ocean depths*, The Rand Corporation, R-1277-1-ARPA, September.

Hewson-Browne, R. C., 1973. Magnetic effects of sea tides, *Phys. Earth planet. Int.*, **7**, 161.

Hewson-Browne, R. C., 1978. Induction in arbitrarily shaped oceans III: oceans of finite conductivity, *Geophys. J. R. astr. Soc.*, **55**, 645.

Hewson-Browne, R. C. & Kendall, P. C., 1978a. Some new ideas on induction in infinitely-conducting oceans of arbitrary shapes I, *Geophys. J. R. astr. Soc.*, **53**, 431.

Hewson-Browne, R. C. & Kendall, P. C., 1978b. Induction in arbitrarily shaped oceans II: edge correction for the case of infinite conductivity, *J. Geomagn. Geoelect., Kyoto*, **32**, Suppl. I, SI51.

Hewson-Browne, R. C. & Kendall, P. C., 1981. Electromagnetic induction in the Earth in electrical contact with the oceans, *Geophys. J. R. astr. Soc.*, **66**, 333.

Hobbs, B., 1981. A comparison of *Sq* analyses with model calculations, *Geophys. J. R. astr. Soc.*, **66**, 435.

Hobbs, B. & Dawes, G. J. K., 1980. The effect of a simple model of the Pacific Ocean on *Sq* variations, *J. Geomagn. Geoelect., Kyoto*, **32**, Suppl. I, SI59.

Hutson, V. C. L., Kendall, P. C. & Malin, S. R. C., 1972. Computation of the solution of geomagnetic induction problems: a general method, with applications, *Geophys. J. R. astr. Soc.*, **28**, 489.

Hutson, V. C. L., Kendall, P. C. & Malin, S. R. C., 1973. The modelling of oceans by spherical caps, *Geophys. J. R. astr. Soc.*, **33**, 377.

Kendall, P. C., 1978. Oceanic induction and shifting the spectrum, *Geophys. J. R. astr. Soc.*, **52**, 201.

Kendall, P. C. & Quinney, D. A., 1983. Induction in the oceans, *Geophys. J. R. astr. Soc.*, **74**, 239–255.

Malin, S. R. C. & Gupta, J., 1977. The *Sq* current system during the International Geophysical Year, *Geophys. J. R. astr. Soc.*, **49**, 515.

Price, A. T., 1949. The induction of electric currents in non-uniform thin-sheets and shells, *Q. Jl Mech. appl. Math.*, **2**, 283.

Winch, D. E., 1981. Spherical harmonic analyses of geomagnetic tides 1964–1965, *Phil. Trans. R. Soc. A*, **303**, 1.

Zinger, B.Sh. & Fainberg, E. B., 1980. Method for computing electromagnetic fields in the world ocean, *Geomagn. Aeron.*, **20**, 67.

Zinger, B. Sh. & Fainberg, E. B., 1981. Electromagnetic induction in a heterogeneous spherical model of the Earth with a realistic distribution of near surface conductivity, *Izv. Earth Phys.*, **17**, 435.