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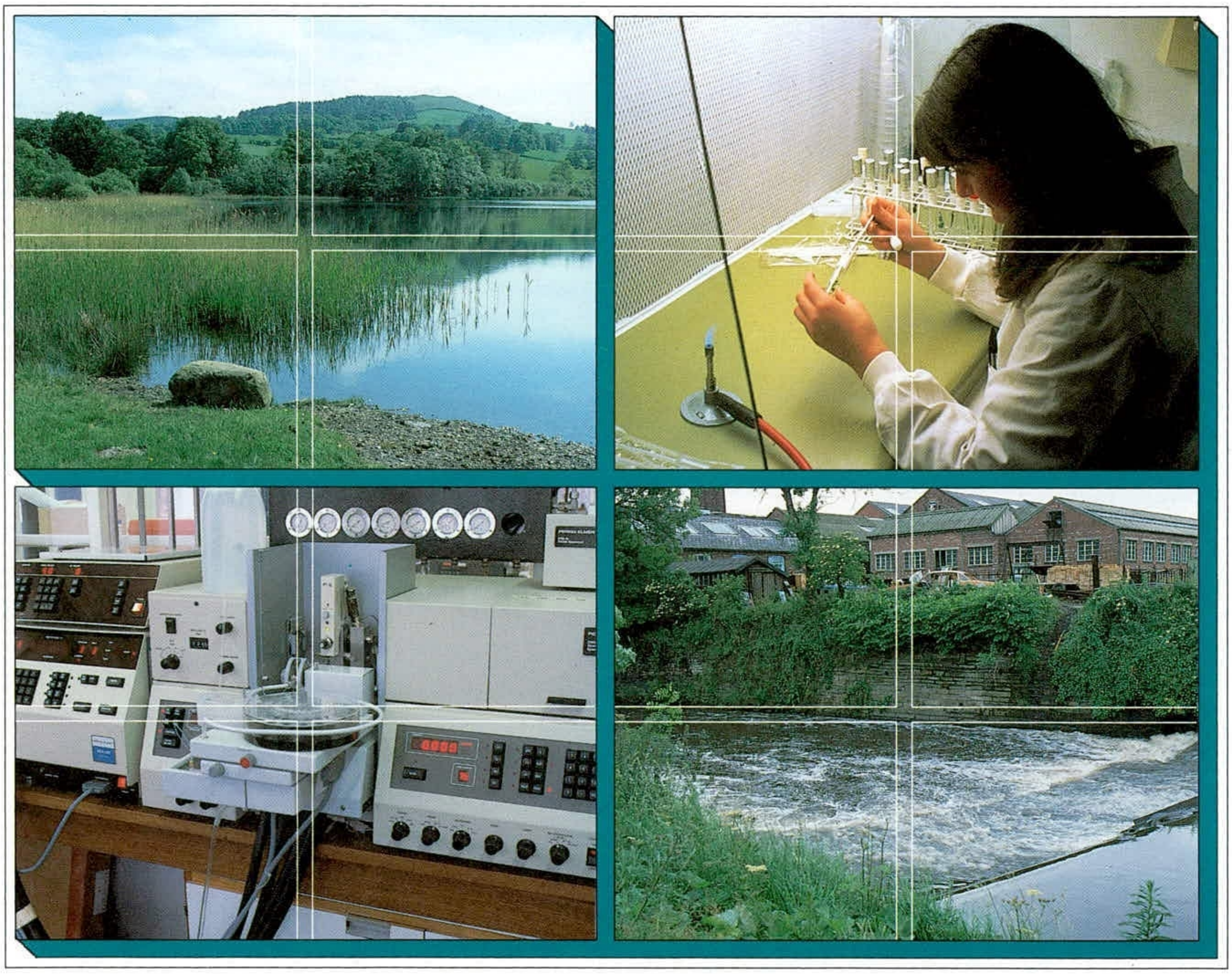

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Advanced Numerical Modelling of Fluvial Dunes: A Review

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Executive Summary

Deterministic approaches to the calculation of duneform are reviewed for aeolian and fluvial dunes. Based on this review, recommendations for an approach to the deterministic calculation of duneforms in the River Rhine are made.

Modelling of duneforms is a three stage procedure, requiring calculation of flow, sediment transport rate and the effects of sediment transport on duneform. The most important components are the prediction of bed shear stress in the flow solution and the expression for sediment transport rate.

The recommended model of flow, described in Johns (1991), calculates flow over the dune using a finite difference calculation. The results obtained from this model can be compared with measurements made on the River Rhine to check the accuracy of solution, in particular the ability to predict bed shear stress. The calculations of sediment transport rate and the effects of sediment transport can then be made.

The combination of flow model and sediment transport rate calculation will allow the calculation of duneform development. The use of existing data for comparison and the evaluation of the effects of variation in the calculations will allow the uncertainty in these calculations to be evaluated.

Introduction

Review of the literature concerning modelling dunes revealed two types of descriptions of dune behaviour, namely their occurrence and form. For both of these types of information a number of different approaches have been taken. For predicting the occurrence of dunes, empirical analysis has led to the compilation of bedform occurrence charts, for example Southard & Boguchwal (1990). Deterministic consideration of dune occurrence also has been calculated using stability analysis, the results of this work having been reviewed in Kennedy (1969), Reynolds (1976) and Engelund & Fredsoe (1982).

A number of approaches to the calculation of duneform have also been used. These can be loosely categorised as empirical, stochastic and deterministic.

Empirical models of duneform make no attempt to describe the detailed mechanisms and processes involved in dune formation and in the preservation of duneform. The models are based on simple physical reasoning or on analysis of the interaction of variables. The parameters used to define the duneform are based on the results of observations with the model calibrated to give the best fit to these observations. A model produced in this way may well be specific to the site and even the series of data used in its construction. The model of duneform described by O'Connor (1992) is of this type.

At the other extreme of model complexity a deterministic model of duneform evolution can be developed. This is based on physical descriptions of the processes involved in the formation of dunes. For the calculation of the form of dunes such a model would consist of a component representing the action of the flow on the surface of the dune and a description of the resulting sediment transport and its effects on the duneform. The aim of developing a physically-based description of the process of dune formation is at least two-fold. The model firstly will attempt to improve understanding of the process of dune formation, the mechanisms involved and their interactions. The other aim is to produce a model which, rather than being site and data specific, can be applied to any chosen site with little or no modification.

The first of these objectives can be said to have been achieved, the second is more difficult to realize. The complexity of the processes involved and the range of scales which influence these processes is such that not all processes can be represented directly, coefficients or other simplified representations, must be used to represent sub-scale processes to allow calculations to be performed. Thus though the results of calculations performed using a physically-based model give a qualitative insight into the process of duneform development the quantitative results obtained may still be limited to the sites and conditions for which the model has been tested. The calculated duneform will be a mean form for the flow.

The third approach to the calculation of duneform is based on stochastic analysis of observed duneforms. This approach has been used because observations show a range of duneforms produced by a set of conditions, rather than a single, mean, equilibrium form. One result of this, is that elements of stochastic analysis have to be used with both empirical and deterministic models of duneform. These elements are used to calculate the range of duneforms from the mean value of the duneform derived using the model; this can be seen in the maximum dune height calculation in O'Connor (1992). The aims of the stochastic analyses are to build descriptions of the behaviour of dunes, in time and space, these then allow the derivation of quantities which can usefully describe the duneform and ultimately its variation in time and space linked to the unsteadiness of the flow forming the dunes. This work is reviewed by Schilperoort (1984) and Moll *et al* (1987).

All three approaches rely on empirical data to some extent, in two of the approaches, empirical and stochastic, this is the basis for the model, for deterministic models some of the required information must be supplied from this source.

The approach to the calculation of duneform considered here is the deterministic approach. The calculation of duneforms within both fluvial and aeolian environments will be reviewed, in two sections, these are: a description of the flow causing the sediment transport and; a description of sediment transport and its effects on duneform. The calculations that can be performed and hence the type of information that can be obtained about such systems will then be described, with reference to the data available from measurements made on the River Rhine.

Deterministic models of duneform

Deterministic models to calculate equilibrium duneforms, and the response of these forms to changing conditions, have been developed for the aeolian, fluvial and marine environments. The development of the aeolian models has been independent of the fluvial and marine models. The most obvious difference between the models developed for the different environments is that the aeolian models are three-dimensional while those for the fluvial and marine environment are two-dimensional. There are a number of reasons for this, related both to the nature of the flow and the type of dune studied. The duneforms studied in aeolian models are barchan dunes, a three-dimensional duneform, initially chosen for study due to its simplicity of form and ease with which measurements could be made (Howard *et al*, 1978). The wind direction involved in the formation of aeolian dunes is also more likely to be variable than aqueous flows, though it is the predominant wind which ultimately determines duneform. Though the models of flow over fluvial and marine dunes are two-dimensional some of these models can easily be extended to three dimensions. This could be done either, if solutions obtained did not match observations, or to check whether the solution would be influenced by three-dimensional effects. The flow models described in Dawson *et al* (1983) and Johns *et al* (1993) have already being used to perform three-dimensional flow calculations in freshwater and marine systems.

The aeolian models of duneform will be reviewed first, followed by the models of duneform for fluvial and marine forms. For both types of model the flow calculation will be described first then the calculation of sediment transport and its effects.

Development of aeolian barchan dunes

Flow and sediment transport have been measured over three-dimensional aeolian barchan dunes in the field and in wind tunnel experiments (Howard *et al*, 1978, Weng *et al*, 1991). Data obtained from such experiments has been used to compare predicted flows and in some cases predicted sediment transport and the development of duneform.

The first attempt to predict sediment transport and hence duneform development due to flow over aeolian barchan dunes was made by Howard *et al* (1978). In this study the flow used to calculate shear stress and hence sediment transport over the surface of the dune was based on measurements, in subsequent studies (Howard & Walmsley, 1985, Wippermann & Gross, 1986, Weng *et al*, 1991) the flow field has been calculated from theory.

The numerical simulation of duneform development due to flow is performed in three steps:

(1) Calculation of flow over existing dune form.

(2) Calculation of sediment transport due to calculated flow field.

(3) Calculation of change in dune form due to sediment transport.

In all the numerical simulations, the solutions of flow field and duneform development were uncoupled, with flow solutions calculated over shorter time intervals than changes in dune height.

Flow over dunes

Howard *et al* (1978) used a combination of field and wind tunnel measurements to describe the flow field at a fixed height above the surface of a dune. The measured data were used to produce a contour map of velocity speed over the surface of the dune at this height compared to an upstream reference velocity at the same height,

$$u = U_R(z)\,U_0(z) \tag{1}$$

where **u** is the velocity at height **z** above a point on the surface of the dune, $U_0(z)$ is the undisturbed velocity upstream of the dune at height **z** and $U_R(z)$ is the ratio of the velocity at a point at a height **z** above the surface of the dune to the undisturbed upstream velocity at height **z**. The shear velocities required to calculate the rate of sediment transport were then calculated assuming a constant shear velocity below the height at which measurements were taken. The logarithmic velocity profile, modified by sand movement can be written

$$u(z) = \frac{u_*}{\kappa}\ln\left(\frac{z}{k}\right) + U_c \tag{2}$$

where **κ** is von Karman's constant, ***k*** is a roughness length scale and U_c is the flow velocity at the threshold of motion of the particles forming the dune at a height $z = k$. With the assumption of constant shear velocity up to a measurement height, ***H***, then;

$$u_* = \frac{\kappa\,(U_R(H)\,U_0(H) - U_c)}{\ln\left(\frac{H}{k}\right)} \tag{3}$$

The data of Howard *et al* (1978) has also been used to compare measured results with calculated flow fields (Walmsley & Howard, 1985).

The calculation of the flow field in Walmsley & Howard (1985) and Weng *et al* (1991) are both based on analytical studies of air flow over low hills by Jackson & Hunt (1975). The flow model used by Walmsley & Howard (1985) was MS3DJH/1.5; that by Weng *et al* (1991) FLOWSTAR - a later development in which an inconsistency between inner and outer regions was resolved. The advantage of using these models was that the flow solution was analytical rather than iterative giving a fast solution to the flow. The disadvantage is that these models are incapable of predicting separation and therefore cannot successfully model the wake region downstream of the brinkline. The flow model used by Wippermann & Gross (1986), FITNAH, was a non-hydrostatic model for which a solution was calculated using finite differences. The fact that the model was non-hydrostatic allows the possibility of flow separation downstream of the brinkline. The flow solution obtained was therefore a more accurate representation of the flow field at the expense of the use of an iterative flow solution leading to an increased computation time. All of the numerical flow solutions used the assumption of the logarithmic velocity profile of the type described in Howard *et al* (1978) to calculate bed shear velocity based on velocities calculated for a height above the bed.

Sediment transport

All the models of duneform development described here used the transport formula of Lettau & Lettau (1978)

$$q = C \frac{\rho}{g} u_*^2 (u_* - u_{*_c}) \qquad u_* > u_{*_c} \qquad \textbf{(4)}$$

$$q = 0 \qquad u_* \leq u_{*_c} \qquad \textbf{(5)}$$

where C is a constant, ρ is the fluid density, g the acceleration due to gravity, $\mathbf{u_*}$ the shear velocity and $\mathbf{u_{*_c}}$ the critical shear velocity for movement of particles. The transport equation was assumed to be applicable for three-dimensional transport, in which case it became

$$q = C \frac{\rho}{g} \tilde{u}_* |\tilde{u}_*| (|\tilde{u}_*| - |\tilde{u}_{*_c}|) \qquad |\tilde{u}_*| \geq |\tilde{u}_{*_c} \qquad \textbf{(6)}$$

$$q = 0 \qquad |\tilde{u}_*| < |\tilde{u}_{*_c}| \qquad \textbf{(7)}$$

where the ~ represent a vector quantity. Howard *et al* (1978) made calculations including the effects of slope on the rate of transport but found calculations that did not include this effect gave a slightly better fit to observations. They also investigated the effects of lag on transport rate, that is whether the rate of transport lags behind spatial variation in shear stress. Again this was found to reduce the accuracy of the model, especially when large lags were used. As a result of these experiments Howard & Walmsley (1985), Wippermann & Gross (1986) and Weng *et al* (1991) did not include either of these effects in their simulations.

Calculation of erosion and deposition

Erosion and deposition on the stoss side in the model of Howard *et al* (1978) were calculated along streamlines, giving an expression for the rate of change of height over an interval, $\boldsymbol{\delta x}$

$$\frac{\delta h}{\delta t} = \frac{1}{\rho_s}\,\frac{2}{l_1+l_2}\,\frac{1}{\delta x}\,(q_1 l_1 - q_2 l_2) \qquad \textbf{(8)}$$

where $\boldsymbol{h}$ is the height of the dune, $\boldsymbol{t}$ is time, $\boldsymbol{\rho_s}$ is the bulk density of the sand, $\boldsymbol{l}$ is the distance between streamlines, $\boldsymbol{q}$ is the rate of transport and subscripts 1 and 2 refer to the upstream and downstream boundaries of a cell of length $\boldsymbol{\delta x}$. This equation can also be written in differential form

$$\frac{\delta h}{\delta t} = \frac{1}{\rho_s}\left(\frac{q}{l}\frac{\delta l}{\delta x} + \frac{\delta q}{\delta x}\right) \qquad \textbf{(9)}$$

To calculate the change in height over the surface of the dune, streamlines were fitted over the surface of the dune then broken down into cells $\boldsymbol{\delta x}$ long. The quantities necessary to calculate the divergence, $\boldsymbol{\delta q} / \boldsymbol{\delta l}$, can be calculated from the geometry of the flow and the cell, while the change in transport rate can be calculated from the transport rate at the upstream and downstream boundaries of a cell, as determined by the local flow conditions at that position. Since the sides of the cell were defined by streamlines no transport occurs across these. Two sets of streamlines were fitted to the surface of the dune, based on flow directions from measurements over the dune and based on the direction of ripples observed on the surface of the dune. The results obtained from calculations using both these sets of streamlines were similar, the streamlines based on flow direction gave a slightly better fit at the brinkline of the dune. The change in height on the slip side of the dune was calculated based on the rate of transport over the brinkline of the dune.

The models in which flow and sediment transport were calculated, Howard & Walmsley (1985), Wippermann & Gross (1986) and Weng *et al* (1991) used a more general form of sediment continuity, defined as

$$\rho_s \frac{\partial h}{\partial t} = \mathrm{div}\tilde{q} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \qquad \textbf{(10)}$$

a form more suitable for Cartesian coordinates. This expression could only be used for small surface slope and not on the slip-face; Weng *et al* (1991) do not describe how the slip-face is treated. Wippermann & Gross (1986) force the angle between each grid point and the foot of the dune to be equal to the slope for which the slope is stable, redistributing material in the cells downslope.

The transport conditions upstream of the dune can affect the form of the dune, the values associated with this condition differ with the model considered. Howard *et al* (1978) calculated the upstream transport rate with the requirement of a zero average bias in transport rate at the upstream boundary. That is the upstream transport rate was adjusted until the difference between observed and calculated rates, summed over the cells at the upstream boundary was zero. This gave a transport rate of 80-85% of the calculated equilibrium transport rate. Wippermann & Gross (1986) allowed upstream transport of sediment to vanish, while Weng *et al* (1991) acknowledge that the upstream transport may not be saturated but do not evaluate the upstream transport rate.

Equilibrium duneform

The equilibrium condition of the dune requires that the rate of translation of the dune in the direction of the wind must be constant. That is the value can vary with time but not in space. The model of Howard *et al* (1978) shows that the calculated advance of the brinkline does appear to occur in equilibrium (though there was a large amount of scatter attributed to data error and incorrect assumptions). The model of Wippermann & Gross (1986) successfully predicted the development of a barchan duneform with slip face from initial conditions of a cone of sand. For equilibrium

$$\frac{\partial h_E}{\partial t} = U_D \frac{\partial h_E}{\partial x} \tag{11}$$

where $\boldsymbol{h_E}$ is the equilibrium height and $\boldsymbol{U_D}$ is the translation rate of the dune. Comparing this rate with the calculated value of change in dune height they found that their calculated duneform appeared to be slowly approaching equilibrium. In Weng *et al* (1991) contours of the rate of change of height with time and distance were plotted, these show a similar though not identical form to the results of Wippermann & Gross (1986).

In all the calculations of duneform the results seem to show a form close to but not in equilibrium, this may be due to limitations of the simulations or the fact that barchan dunes are never completely in equilibrium with their surroundings.

Fluvial dunes

The flow over fluvial dunes is assumed to be two-dimensional incompressible flow. For steady-state flow the equation of continuity can be written:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \qquad \textbf{(12)}$$

and that for momentum:

$$\frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial z}(uw) = -g\frac{\partial \zeta}{\partial x} - \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} \qquad \textbf{(13)}$$

where **u** and **w** are the velocities in the horizontal, **x** and vertical, **z** directions respectively; **ζ** is the water surface height, **g** is the acceleration due to gravity, **ρ** is the density of the fluid, **P** is the pressure and τ_{zx} is the shear stress. The momentum equation is in a Reynolds-average form; hence the shear stress term. This introduces the closure problem and an expression to determine the shear stress must be supplied to enable the flow over dunes to be solved.
The two-dimensional equation of sediment continuity can be written:

$$(1 - p)\frac{\partial h}{\partial t} = \frac{\partial q_v}{\partial x} \qquad \textbf{(14)}$$

where **t** is time, **p** is the porosity, **h** is the bed height and q_v is the volumetric transport rate.

The calculation of flow over dunes where the ultimate aim is to calculate duneform, and duneform modification, due to the flow is performed to obtain the values of bed shear stress over the dune. The values of bed shear stress are then used to calculate the rate of transport of sediment which is used in the equation of sediment continuity to determine the modification of the bedform.

The flow calculations leading to the distribution of bed shear stress are reviewed first, followed by approaches to the calculation of sediment transport. There are more descriptions of flow calculations over duneforms in the literature than of combined flow and sediment transport calculations. This is true even where the stated aim of developing models of flow over dunes was for the prediction of sediment transport. Possible reasons for this discrepancy will be considered in **Discussion.**

Flow over dunes

The model of Fredsoe (1982) calculated shear stress distribution over a dune based on the variation of shear stress measured downstream of the reattachment point of flow over a negative step. Comparison with measurements made downstream of the reattachment points of dunes showed a similar variation.

A number of alternative approaches to flow over dunes have been used. The different approaches are (1) to break down the flow field into components, describing the regions of different behaviour of the flow, or (2) to use a single turbulence model throughout the flow.

The models of flow described in McLean & Smith (1986), Nelson & Smith (1989) and Wiberg & Nelson (1992) are of the first type. The model described in Nelson & Smith (1989) is a development of that described in McLean & Smith (1986), while that in Wiberg & Nelson (1992) is a modification of this model to calculate flow over ripples rather than dunes.

In McLean & Smith (1986) the flow is considered to form three regions:

(1) An inner boundary layer region, on the stoss slope of the dune, growing from the point of reattachment.

(2) A wake region, from the point of flow separation on the dune crest to a limit height.

(3) A quasi-inviscid region above the wake region.

The reason for using this approach to the calculation of flow is that grid scales and turbulence closures, where the whole flow was treated as one, were thought to be to crude to resolve the behaviour in the different flow regions. The solutions of the flow in the different regions were performed as follows:

(1) Inner boundary layer; a regular perturbation about a zero-order logarithmic profile, using a mixing length closure.

(2) Wake region; far-field flow solution using far-field wake theory (Schlicting, 1979).

(3) Quasi-inviscid region; potential flow solution, weighted by a spatially averaged logarithmic function.

The velocity and shear stress were matched between 1 and 2 and the velocities between 2 and 3. The height of the inner boundary layer and the mean flow velocity at this height was determined by the interaction of the wake and the boundary layer and requires the discharge to be conserved at all streamwise points. Flow in the recirculating region downstream of the crest of the dune was not modelled, the downstream position of the reattachment point was one of the variables in the model that had to be set for a calculation.

In Nelson & Smith (1989) the model of McLean & Smith (1986) is modified to model the outer region as a region of wake interaction, to the surface of the flow, and to include a calculation of flow in the separation region downstream of the crest of the dune. The modelling of wake interactions removes a semi-empirical component from the previous model and allows the flow velocity and shear stress to be continuously defined throughout the depth of the flow. This is done by matching velocity and shear stress between the wakes as well as between the boundary layer and wake. A flow field in the region of flow separation was calculated by using the same expression as for the boundary layer of the stoss slope to calculate a boundary layer, but calculating back from the point of reattachment to the separation point. The recirculating velocity is matched to the wake velocity at the separation surface.

The model was altered to calculate flow over ripples not dunes, as described in Wiberg & Nelson (1992). The major difference was the introduction of a drag term due to the ripples acting on the outer flow. The use of a far field wake assumes that the wake is far downstream of the cause of disturbance. The results of calculations showed this to be a reasonable assumption for dunes but, for ripples, the wake interacted with the bedform and this assumption did not hold. The introduction of a drag term connecting the flow regions allowed the calculation to fit observations.
The information about flow over dunes derived from all these models was the mean flow velocity and the bed shear stress.

In Termes (1988) the results of predictions of flow over a dune from two different flow models are compared with measurements made over a series of fixed dunes in a flume. The models used to calculate the flows were WABED, based on boundary integral calculations, and ODYSSEE, a finite difference solution using a k-ϵ turbulence closure. The flow solution from WABED is a component of a model of bedform development, DUGRO.

The variation through the boundary layer in WABED is calculated by solving the von Karman integral momentum balance. Outside the boundary layer is an outer region with a constant value of streamwise velocity at each streamwise position. One of the conditions of the flow solution is that continuity is obeyed. Conditions for the streamwise velocity variation and a function describing entrainment of flow from the outer region into the boundary layer, with consequent boundary layer growth, allow a flow solution to be calculated. The flow is calculated over a periodic boundary, with the conditions at the downstream boundary matched to those at the upstream boundary.

The other flow model described, ODYSSEE, solved the hydrodynamic flow momentum and continuity equations using a finite difference solution and a k-ϵ turbulence model. The flow conditions at the surface were set to be a rigid, friction free, lid. This is an acceptable assumption for low Reynolds number flows. The first vertical grid point was set away from the bed, the velocity at this point was first set using the law of the wall but errors in the solution led to an alternative description of flow near the wall being used; the Reichardt bed boundary condition.

Models of shallow-water flow over topography, again with the aim of using the results from such models to examine bedform development are described in Dawson *et al* (1983), Johns *et al* (1990), Johns (1991) and Johns *et al* (1993).

The initial model described in Dawson *et al* (1983) was for hydrostatic flow over topography. A mixing length closure was used, based on a similarity hypothesis, and the equations were solved using a finite difference method. The use of this model to calculate bed shear stresses over dunes, to calculate sediment transport and hence bedform development is described in Johns *et al* (1990). A hydrostatic model cannot calculate flow separation, so any occurrence and any associated effects on the transport of sediment cannot be included in the calculations of duneform development. The description of mixing length used in this model was also found to over-estimate vertical mixing away from the bed.

A hydrodynamic version of this model is described in Johns (1991) and can calculate flow separation. An alternative description of the mixing length is also used, such that the mixing length decreases approaching the surface as well as the bed. A comparison of the flow calculated with this model for the conditions described in Termes (1988) is found in Johns *et al* (1993).

Another finite difference solution of the hydrodynamic momentum and continuity equations for flow over dunes is described in Mendoza & Shen (1990). The turbulence closure model used is an algebraic stress model. In this approach a **k-ϵ** model is used to supply a solution which is used as the starting condition for the algebraic stress model solution. As with the model ODYSSEE described in Termes (1988) a friction free, rigid lid is used at the surface of the flow.

Sediment transport

The reason given for the development of all the flow models was to calculate bed shear stress and hence the sediment transport rate and its effects on dune form. However the calculation of sediment transport rate and its effects are only described in Fredsoe (1982), Termes (1988) and Johns *et al* (1990). The calculations of sediment transport rate and the calculation of bedform modification due to this transport differ in these models.

Fredsoe (1982) only calculates the effects of bedload transport, the rate of transport is calculated using the Meyer-Peter formula (Meyer-Peter & Muller, 1948). The formula relates the non-dimensional rate of transport to the non-dimensional bed shear stress to the power 3/2. In this application the non-dimensional bed shear stress is modified to account for the effects of gravity due to the transport occurring on the slope of a dune.

The model of sediment transport described in Termes (1988) is part of the DUGRO model of sediment transport and uses the shear stress distribution calculated using the flow model, WABED, which has already being described. As with the model described in Fredsoe (1982) only bedload transport is considered. Rather than a direct calculation of sediment transport rate, a pickup rate, $\boldsymbol{n_p}$, and a deposition rate, $\boldsymbol{n_d}$, are calculated. The sediment continuity equation can then be written

$$\frac{\partial q_p}{\partial x} = n_p - n_d \qquad \textbf{(15)}$$

where $\boldsymbol{q_p}$ is the transport rate of particles. The pickup rate of sediment includes the effects of turbulence on the shear stress (represented by a Gaussian distribution of shear stress) and also includes the effects of the slope of the dune on transport rate. The deposition is calculated by the use of a mean step length, $\boldsymbol{\lambda}$, which also includes the effects of a Gaussian distribution of shear stress due to turbulence. The deposition rate, $\boldsymbol{n_d}$, is assumed to be given by the expression

$$n_d = q_p/\lambda \qquad \textbf{(16)}$$

If this expression for $\boldsymbol{n_d}$ is substituted into equation 15, the sediment continuity equation, this can then be solved to give the rate of sediment transport. The rate of transport of sediment is only calculated along the stoss slope of the dune, from the point of reattachment of the flow downstream of the previous dune crest to the crest of the dune. In the region of recirculating flow, between the crest of the dune and the point of reattachment, bedload transport will not be important.

In Johns *et al* (1990) the suspended as well as the bedload rate of transport are calculated, allowing for the possibility of the prediction of transition to upper-stage plane bed to be calculated. The suspended sediment concentration at the bed is calculated using a pickup function due to van Rijn (1984a) along with a value for critical shear stress from van Rijn (1984b). The distribution of suspended sediment through the depth of the flow is then calculated using an advection-dispersion equation. The effects of the suspended sediment concentration on the flow are modelled by using a modified fluid density. The rate of transport of suspended sediment at a streamwise position is calculated by integrating the sediment concentration through the water depth. The bedload transport is calculated using an expression for rate of transport due to Bagnold (1956). The form of the equation used here is one that includes the effects of the slope of the dune on transport rate described in Richards (1980).

Duneform

The modification of the bedform due to sediment transport is always treated using sediment continuity over the stoss slope of dunes but the treatment of the slip face varies.

In Fredsoe (1982) the form of a dune is calculated for the shear stress distribution downstream of a negative step. The predictions made using this model predict dune steepness increasing to a limit value with increasing shear stress. This is not in agreement with observations which show a reduction in dune steepness with increasing bed shear stress. The inclusion of a suspended sediment transport term alters the predicted variation in dune steepness so that the transition to upper-stage plane bed is included.

Termes (1988) calculated the effects of sediment transport on the stoss slope of the dune using the sediment continuity equation. In the region of flow separation the mass balance between transport of sediment over the crest of the dune and sediment transport at the point of reattachment is calculated. The slip face of the dune is assumed to remain at a constant angle; the angle of repose. This stricture along with other assumptions about the bed between the foot of the dune and the point of reattachment allow the change in duneform and the rate of migration to be calculated.

Finally in Johns (1990) the effects of the transport of sediment on the duneform are calculated using the sediment continuity equation.

Discussion

The models of aeolian dune development were examined to study whether an equilibrium form had been reached, as the emphasis in those studies was placed on distinguishing whether the dune is in equilibrium. In Howard *et al* (1978), they look for constant brinkline advance, while Weng *et al* (1991) sought the correspondence:

$$\frac{\partial h}{\partial t} = -U_D \frac{\partial h}{\partial x} \tag{17}$$

where h is the height of the dune and U_D is the speed of dune advance; that is, the form of the dune is preserved during translation of the dune. In contrast to this the fluvial literature puts much more emphasis on the response to a flow, rather than the formation of an equilibrium form. Observations in the fluvial environment show duneforms surviving after the flow which caused them is no longer present, Levin *et al* (1992).

The calculation of duneforms is a three stage process, flow; transport; and form. The purpose of calculating the flow over dunes is to determine the shear stress distribution over the dune which is then used to determine the rate of sediment transport. The equations describing the flow are based on the Navier-Stokes equations, simplified to an appropriate form for the assumptions made in these calculations. The Navier-Stokes equations themselves are a good description of the flow. To describe a turbulent flow a Reynolds-averaged form of these equations is used. To allow a solution of these equations a turbulence model must be used to close the equations. This set of equations must then be solved. All these stages introduce sources of uncertainty and error into the calculations. The solution of the flow calculation is used to calculate the bed shear stress which will therefore also contain errors and uncertainty in its values.

While the Navier-Stokes equations describe fluid flow there is no such basic system of equations to describe sediment transport. Instead a variety of equations are used to describe the rate of transport based on dimensional and physical reasoning. Empirically determined parameters are used to calibrate the calculated transport rate. One result of this is that equations describing the rate of transport of sediment can be limited in their application, being constrained within the conditions for which they were originally developed or even by data with which they were originally calibrated. For example a slope effect was included in the calculations of sediment transport rate in Howard *et al* (1977), because the entrainment and transport of sediment were occurring on the stoss slope of a dune. However the fit of the results to observations were found to be better without including this term. Thus in attempting to calculate sediment transport and its effects on duneform the equation used to calculate the rate of sediment transport can affect the resulting duneform. All the fluvial models of duneform development include a slope term in the expression for sediment transport. The term, discussed in Fredsoe (1982), is included because of observations of its effects. The three different models in which sediment transport is calculated use three different equations to determine the transport rate, with resulting differences in the calculated quantity in motion. However, though the exact terms and parameters used in a calculation vary, the relation between bed shear and rate of transport is always some power law function. One effect of this is that any error in the calculation of bed shear stress is magnified in the calculation of rate of transport.

The calculation of modification of dune form in all these models is an uncoupled solution, due to the difference in response times between flow and sediment transport. This is a reasonable approximation, though the time interval used to calculate bedform must be such that the change in duneform and hence flow pattern is not to large at each step.

The combination of errors due to inaccuracies in the calculation of bed shear stress and uncertainty in the values of parameters used in the calculation of sediment transport rate mean that the calculated range of behaviour in the duneform due to a flow can vary to a large degree. The effects of varying parameters within observed ranges can be examined by the performance of multiple calculations.

Recommendations

The models of flow described are capable of calculating the velocity and shear stress distributions over bedforms, therefore they could all be used in the calculation of transport and shear stress distribution over bedforms. The models can be considered in two groups, those where the flow field is built-up from components and those where it is treated as a whole. The argument for treating the flow in sections is the presence of very different scales within the boundary layer over the dune stoss slope, wake and outer layer (or wakes summed through depth). McLean & Smith (1986) argued that treatment of the whole flow using a single turbulent closure, along with the scale of the grid on which results were calculated would not be capable of resolving the detail necessary to model the behaviour of flow over dunes.

Comparison of the published results of calculations made using the different models described is complicated by the fact that the different data sets have been used to compare observed and calculated results. Comparing the results from the model of Nelson & Smith (1989), built up from components, with those of Johns *et al* (1993), a finite difference solution, the calculated velocities along the stoss slope of the dune show a similar fit. The region of flow separation is the region of worst fit in the results of Johns *et al* (1993). Nelson & Smith (1989) show no results for this region so no comparison can be made, however their solution in this region is ad hoc and there is no reason to think that they would obtain better results than those of Johns *et al* (1993).

Initial calculation comparing calculated flows using the hydrostatic and non-hydrostatic versions of the model described in Johns (1991) show no separation downstream of the crest of the dune and very little difference between the velocity profiles calculated with the hydrodynamic pressure term. The model of Nelson & Smith (1989) assumes that flow separation does occur and may therefore not be suitable for calculations of flow over dunes on the River Rhine. Further the models described in Johns (1991), or any other of the finite difference flow solutions, can be modified to calculate a three-dimensional flow field, whilst the model described in Nelson & Smith (1989) would be much more difficult, if not impossible to modify. The use of a finite difference solution would therefore seem to be a reasonable approach to calculation of the flow over dunes observed in the River Rhine.

The finite difference solutions vary in the exact solution scheme used and the turbulence closure model used. These affect the accuracy of the solution and the computation time required to produce a solution. The solution method chosen should be capable of reproducing the observed flows without imposing an unreasonable computational load. In order of increasing complexity the finite difference solutions described used the following turbulence closures: mixing length model (Johns *et al*, 1993), k-ϵ model (Termes, 1988) and algebraic stress model (Mendoza & Shen, 1990). The results of the calculations described in Johns *et al* (1993) and Termes

(1988) can be compared directly since the calculations were made for the same observations. In particular the calculated bed shear stress in Johns *et al* (1993) was a better fit to observations than that shown in Termes (1988). Comparison of the results of Johns *et al* (1993) and Mendoza & Shen (1990) is more difficult since different observations are being reproduced. The models appear to give similar fits for velocity but that of Mendoza & Shen (1990) appears to give a better fit for the other quantities. However the model of Mendoza & Shen (1990) was run on a supercomputer, so while the possibility of a better model fit exists it is at the expense of a considerable increase in the computing power required. The model of flow described in Johns (1991) and applied in Johns *et al* (1993) gives a reasonable balance between the results obtained and computation time with the computing facilities that are immediately available. In addition the solution can be extended to three dimensions if required. This model would therefore seem to be the most sensible choice of flow model for the calculation of duneform development.

Flow calculations

The first calculations to be performed with any model are to be compared with measured flows. These calculations can be performed for each set of flow measurements, comparing measured and calculated velocities and bed shear stress calculated from the velocity curve. For the field measurements made in September 1993 comparisons can also be made of calculated and measured turbulent kinetic energy and the directly measured values of bed shear stress. For calculations with the model of Johns (1991) the calculations can be performed to give hydrostatic and non-hydrostatic flow solutions, both the results are a steady-state flow solution. The non-hydrostatic solution uses the hydrostatic flow solution for its initial conditions. The non-hydrostatic approach is iterative and since any non-hydrostatic solution requires the calculation of a hydrostatic solution it will always be more computationally intensive. If future calculations show that a hydrostatic solution can be used without affecting the end result then this will be used.

For comparisons with measurements, the upstream boundary conditions of flow depth and surface velocity can be set from observations. When a calculation for which no measurements exist needs to be performed a rating curve such as that presented in O'Connor (1992) can be used to find the values for flow depth and discharge. The initial upstream boundary conditions can then be set to match these conditions.

Sediment transport

Measurements exist for bedload and suspended load transport. Consequently, a check can be made between calculations and observed transport as well as the effect of the calculated sediment transport on duneform. Calculations can be performed both for suspended and bedload transport of sediment, though bedload is more likely to be important in this application. Since the calculated rate of sediment transport will determine the calculated duneform, the effect of using different expressions for the transport rate should be examined to determine possible ranges of behaviour. The effects of using a calculation based on pickup rate and step-length could also be examined.

Duneform

The calculation of duneform is based on considerations of sediment continuity, the calculation must always obey continuity. The calculation of flow and sediment transport is uncoupled, with the duneform being updated less frequently than the flow. The update of duneform must be such that the flow is only slightly changed by the change in duneform, otherwise the assumption of an uncoupled solution is not obeyed.

Complete calculation

Due to the number of separate components in the calculation, each with its own errors and uncertainties, there is a large scope for uncertainty in the calculation of duneform change. In many ways the results are liable to be more qualitative than quantitative. The results of varying parameters can be evaluated by performing multiple calculations, and analyzed to give indications of the range of behaviour.

Possible alterations to existing model

A number of alterations to the existing model could be made, one of these, the effects of using different expressions for sediment transport rate, has already been mentioned. The introduction of variable element size in the downstream flow direction of the grid would enable a higher resolution in the region downstream of the crest of the dune; the region of most interest. The use of a variable roughness length scale would make the model a better representation of the system being studied. Finally the effects of calculating a three-dimensional flow solution could be examined.

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