

A Decadal-Scale Eigen-Analysis of Terrestrial Magnetic Data

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Motivation and aims

In the depth range 1,200 to 2,000 km, electric fields in the Earth's mantle are induced by magnetospheric and ionospheric magnetic fields with long periods (of several months to years). The induced electric fields in the mantle cause magnetic fields which add to measurements of the magnetic field made at and above the Earth's surface. If the inducing source is known, studies of the measured magnetic field can provide information on the conductivity structure of the Earth's mantle, as well as leading to accuracy improvements in magnetic field modelling in general. Unfortunately, the long period inducing signals overlap with the periods on which the core field changes significantly, hence the geometry and temporal behaviour of the long-period inducing fields are poorly resolved. It is common to use spherical harmonic analysis (SHA), choosing *a priori* a morphology for the inducing source (typically the P10 term, intended to represent the zonally-invariant (symmetric) magnetospheric ring current), then solving for the behaviour of this structure through time. This approach leads to large error bars. In this study, we use a method (called Empirical Orthogonal Functions – defined below) which allows the temporal and spatial behaviour of the inducing source to be solved for in the same analysis. We resolve the external field inducing source on annual to decadal periods, encompassing a full 11-year solar cycle. Particular attention is paid to the deviation from zonal symmetry (the P10 term) in local time.

Method

Empirical Orthogonal Functions (EOFs) are commonly used in climatological statistics. They are a series of basis vectors which project onto the maximum variance in the input data, allowing a decomposition of the full data set into a smaller number of orthogonal 'modes' (patterns of oscillation), the temporal variations of which are uncorrelated. The spatial patterns we seek are not necessarily orthogonal, but are likely to be temporally uncorrelated, hence the decomposition provides useful information about the magnetic field. If the original data are spatially arranged, maps of the EOF vectors are spatial patterns which successively maximise variance. By applying EOFs to magnetic data in the local time frame, we will produce a set of vectors which describe the change in the external fields over the input timespan. Across a full solar cycle, this can identify and quantify the degree of asymmetry in these fields.

EOF Calculation Process:

1: Arrange data matrix X (n observations at p observatories)

$$X = \begin{bmatrix} x_{11} & \dots & x_{1p} \\ \dots & \dots & \dots \\ x_{n1} & \dots & x_{np} \end{bmatrix}$$

Single time series at each observatory Single map of hourly mean magnetic data.

2: form covariance matrix F and solve for its eigenvectors C (the EOF vectors) and eigenvalues L

$$F = X^T X \quad FC = CL$$

3: Project EOF vectors C onto original data X to obtain A , a series of p 1-dimensional time series (each of length n) describing the temporal behaviour of each EOF pattern. Can also reconstruct X for one or more modes

$$A = XC \quad X = AC^T$$

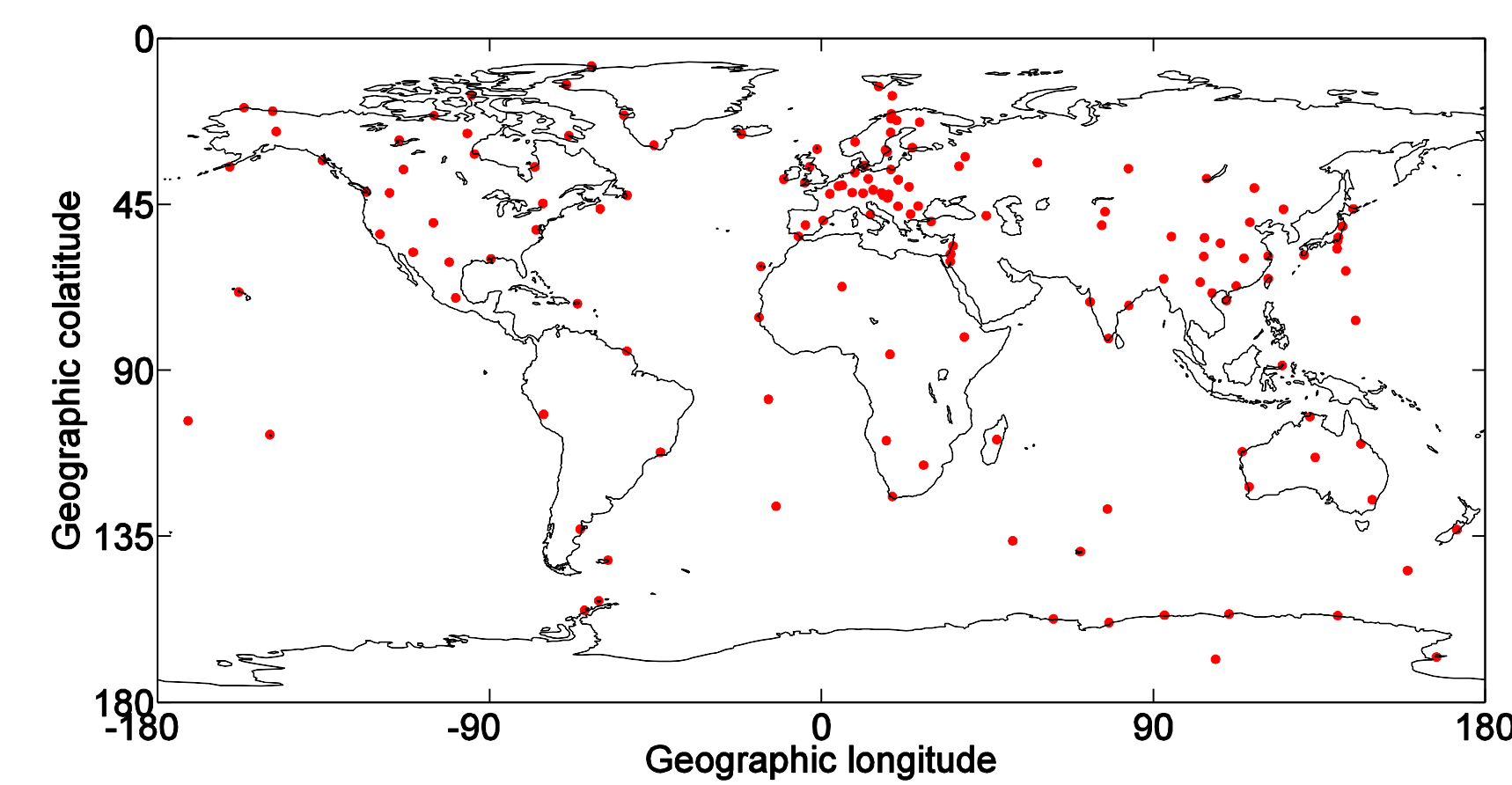


Fig 1: Geographic locations of magnetic observatories which contribute hourly mean vector data over the full time span of the data set (start-1997 to end-2010).

Synthetic data tests

To demonstrate how the EOF analysis responds to a known input, ionospheric (and induced) θ -component (south-pointing) magnetic values were predicted using the CM4 magnetic model to the nodes of an equal-area grid, for each hour in the period 1997.0-1998.0. As can be seen from the solar flux values (shown in Fig 2), this corresponds to the early part of the solar cycle we are studying. These hourly mean values (shown in Fig 3) were analysed with the EOF method, using only one hour of data per day to keep the observation locations static in local time (LT).

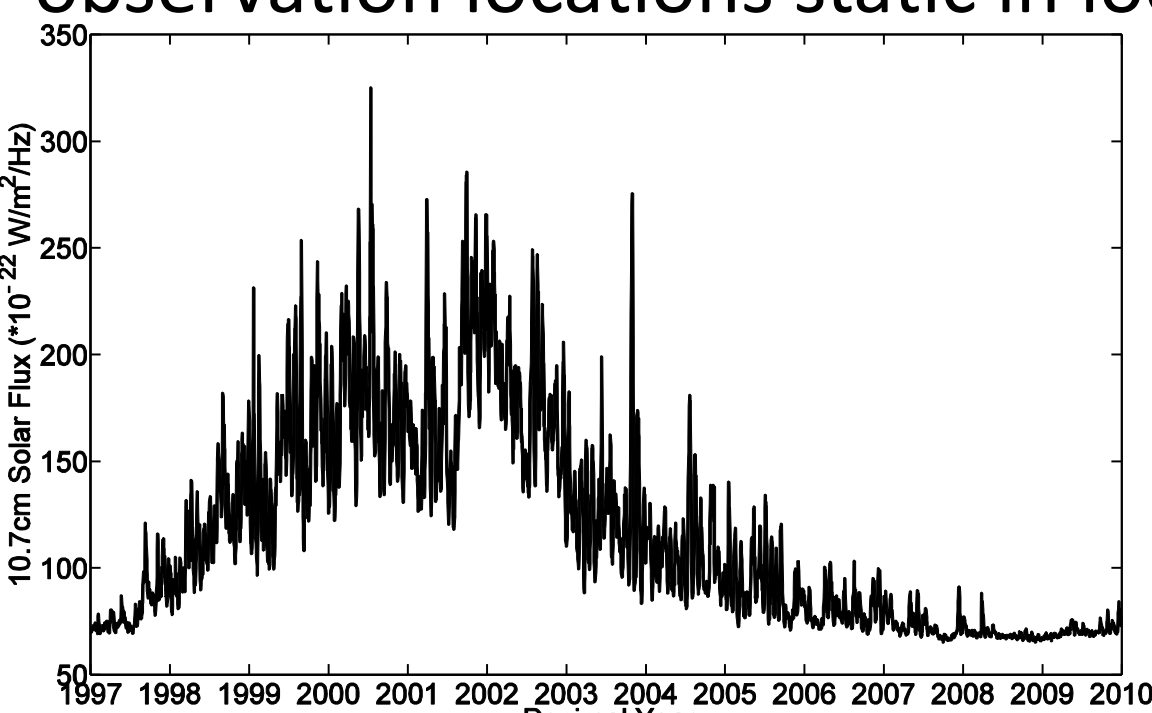


Fig 2: F10.7 index: Daily values of solar flux density at the 10.7 cm wavelength. A measure of solar activity.

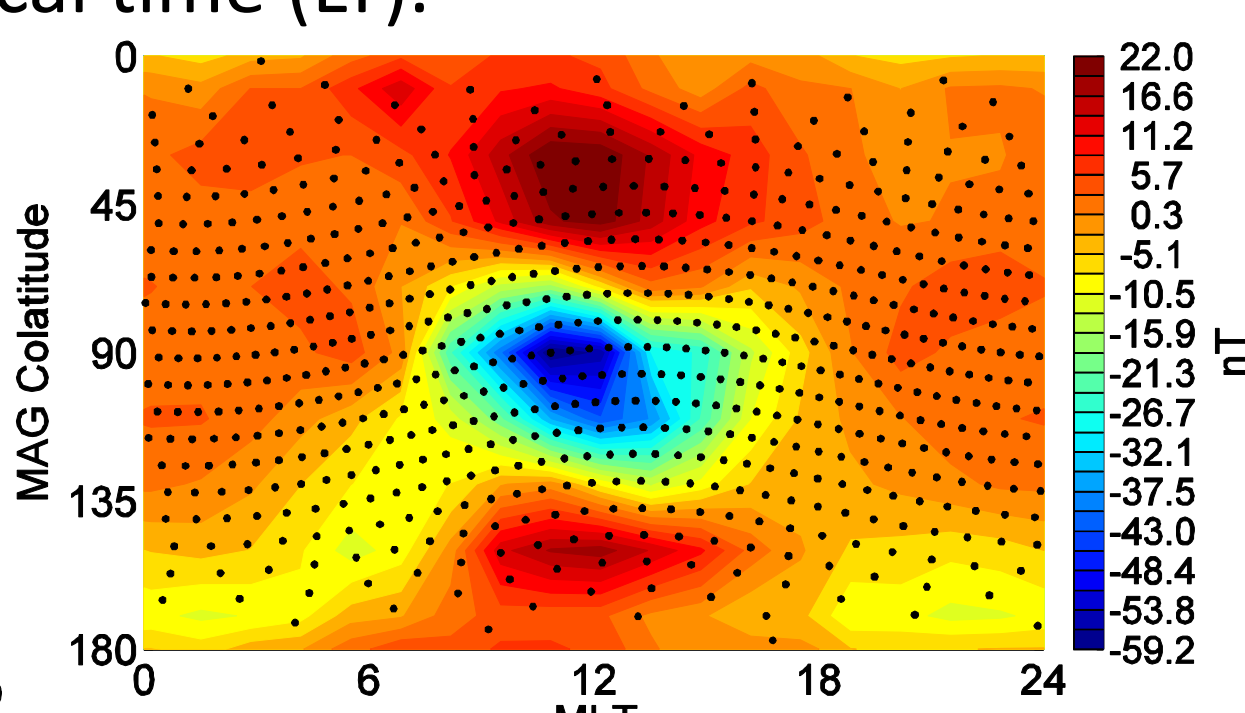


Fig 3: θ -component magnetic hourly mean values for a single epoch in northern hemisphere winter. Predicted using the CM4 model for ionospheric and induced fields only.

The data matrix reconstructions (at a single epoch) for modes 1 and 2 are shown in Fig 4, along with their temporal oscillations. The first mode represents the geometry of annual oscillation (the seasonal signal). Note that it is zero at the equator. The second mode represents the day-to-day variation in the input signal itself – it is maximal at the equator, and has a biannual pattern. The biannual signal is formed in response to the zero-crossings of the annual signal, and because the ionospheric magnetic field has greater strength at the equinoxes. The diminished magnitude of the northern hemisphere signal in mode 2 is due to the tilt from geographic coordinates to the dipole coordinates used in the EOF analysis.

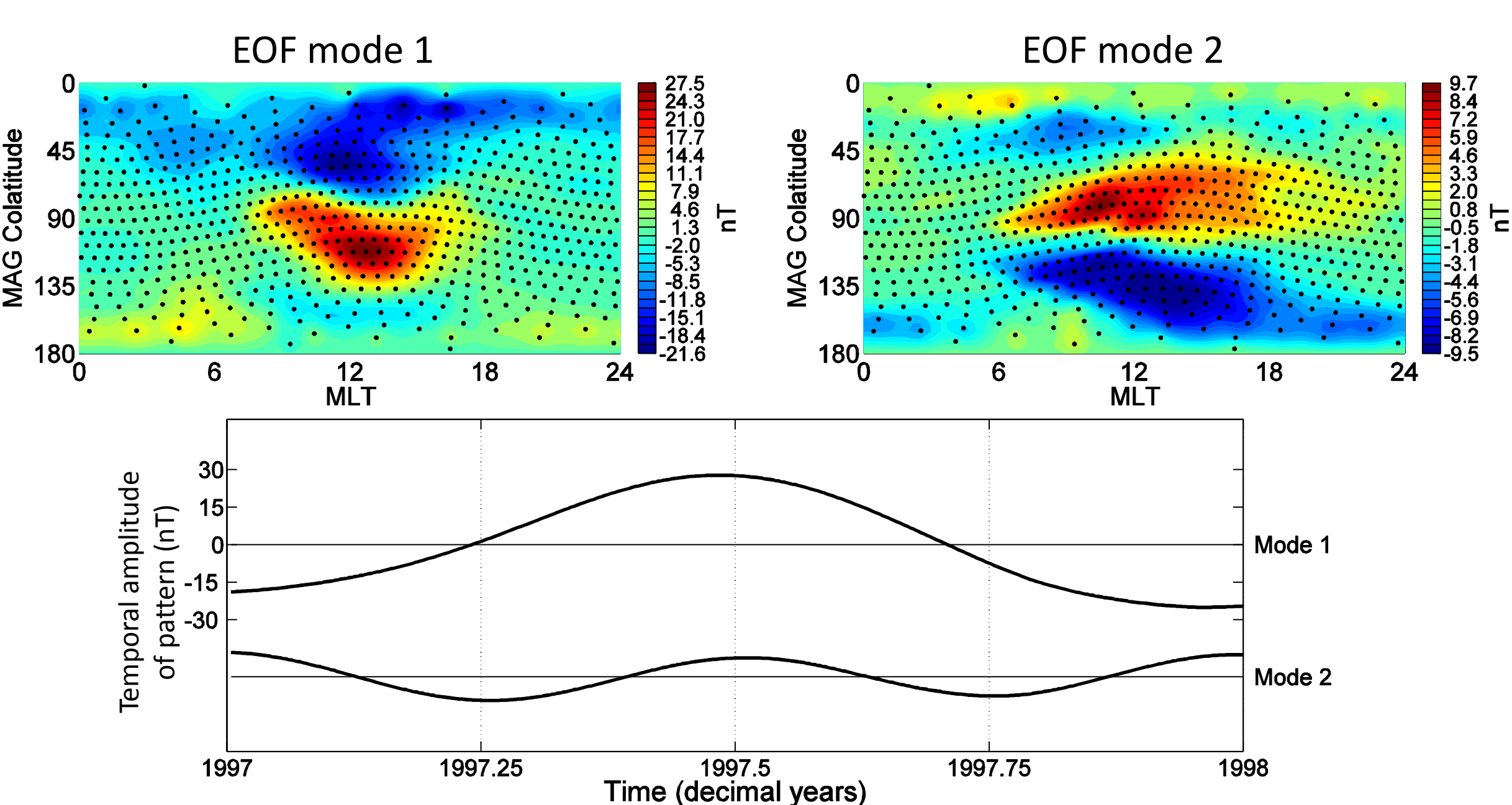


Fig 4: Spatial patterns and temporal oscillations for EOF modes 1 and 2 of the synthetic data analysis, collectively describing the majority of the ionospheric signal.

Real-data results

Hourly mean θ -component values from all stations in Fig 1 were analysed using the EOF method for the years 1997-2010 covering the solar cycle shown in Fig 2). To keep the station locations fixed in LT, 24 separate EOF analyses were performed, each covering the same span of time, but with a different UT used for each day. The outputs of the 24 EOF analyses were then combined using a degree 5, order 5 SHA. This had the side-effect of lowering the amplitudes of the spatial patterns – refer to the temporal oscillations for true values ranges. The combined results are shown in Fig 5. Mode 1 is the pattern of the symmetric ring current (with a region of lower magnitude on the dayside, likely due to the magnetopause current). Its temporal oscillation has a Pearson's correlation of $r=0.85$ with the Dst index. The residual of the mode 1 time series and Dst (not shown) has an amplitude of 15 nT, and appears broadly similar to the F10.7 index (when this is smoothed with a 2-month moving window (not shown)). Hence, the ring current appears to have a decadal-scale modulation in accordance with the solar cycle.

Mode 2 (shown in Fig 5) is the annual (seasonal) oscillation. It is anticipated that this pattern will be important to mantle induction studies due to its long period. The dayside signal has a strong effect in mode 2, but we focus here on the dawn-dusk asymmetry on the nightside. Note that the (temporal) amplitude of the annual signal (mode 2) is broadly similar to that of the symmetric ring current (mode 1). Note also that the biannual signal has a similar dawn-dusk asymmetry to the annual signal, implying that they are two parts of a linked process, or stem from the same current systems.

Our results indicate the annual and biannual patterns are important to the description of the combined long-period inducing source. We infer that a P10 term for the long-period inducing source is likely a significant misrepresentation on annual and semi-annual periods.

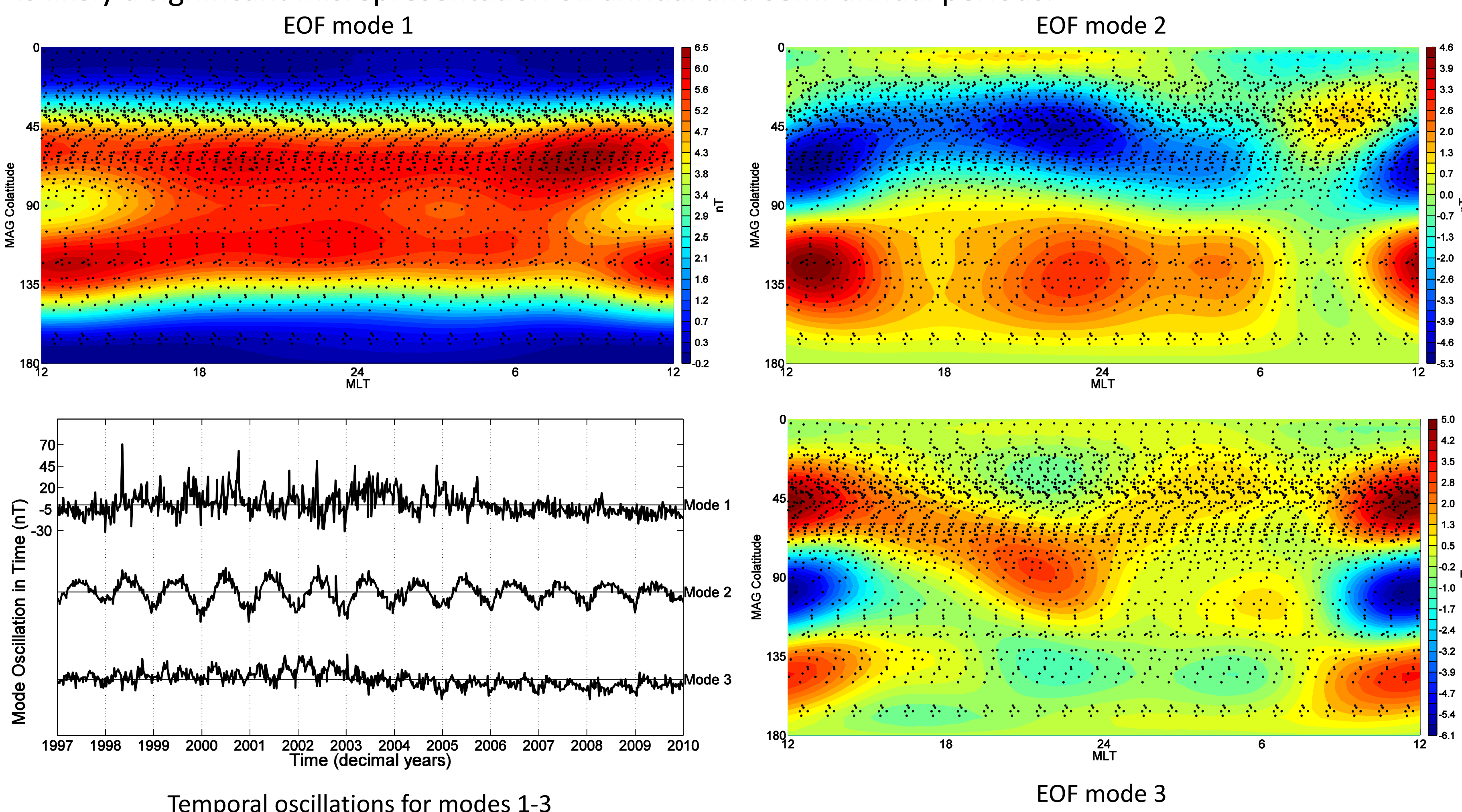


Fig 5: Results for 24 combined θ -component EOF analyses for data from 1997-2010. Clockwise from top left are the spatial patterns for the first three EOF modes (which collectively account for most of the variance of the input data), followed by a chart of their temporal oscillations for the analysis time span.