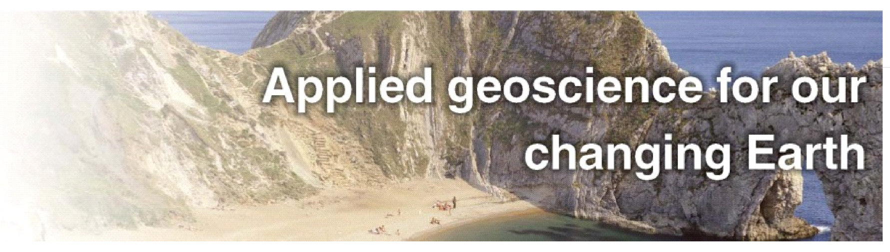

British Geological Survey
NATURAL ENVIRONMENT RESEARCH COUNCIL

INTERPOLATION OF ELECTRIC AND MAGNETIC FIELDS USING SPHERICAL ELEMENTARY CURRENT SYSTEMS

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Introduction

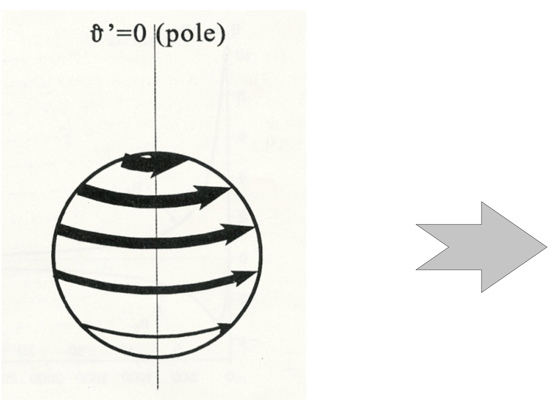
Estimation of the external (to the Earth's surface) electric or magnetic field disturbances at a point remote from an observatory can be difficult. We show how interpolation of electric field disturbances in the ionosphere can be achieved with the Spherical Elementary Current System (SECS) method using data from a number of ground-based magnetic observatories. The SECS method represents the complex electrical currents in the ionosphere as a simple set of equivalent currents placed at a specific altitude. The resultant magnetic field recorded at observatories can be used to invert for the electrical currents across a large area. We can also use this to interpolate the magnetic field.

Here we apply the SECS method to auroral zone latitudes using a series of observatories to test how well the external field can be interpolated over large distances. We demonstrate that relatively few observatories are required to produce an estimate which is better than the null hypothesis (i.e. assuming no change).

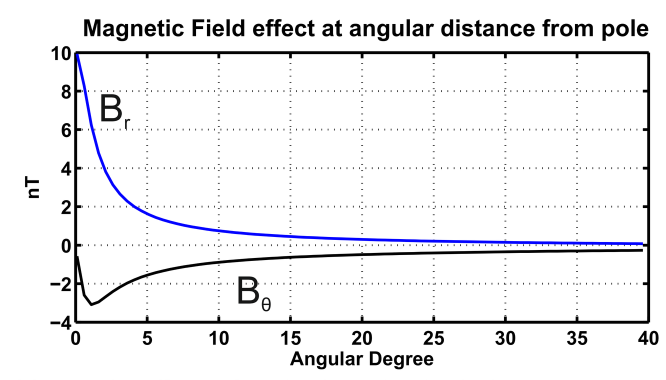
Spherical Elementary Current Systems

The basic concept of SECS is to construct model current systems using a linear superposition of divergence-free elementary current systems, all of which are placed freely within the current plane (e.g. at 110km altitude). Amm and Viljanen (1999) introduced this method for continuation of the magnetic field disturbance from the ground to the ionosphere after Pirjola and Viljanen (1998) showed that the electric field could be calculated from superposition of the measured magnetic effect of horizontal current layers composed of divergence-free elementary current systems emanating away from a pole (see image below, left).

The magnetic field (**B**) induced by a system of external current sheets (**I**) can be defined, for the ionosphere, as a function of linear equations which relate current and magnetic fields in a spherical reference frame (r, θ, φ). Pictorially, this can be imagined as the diminishing influence of a current system with distance (as below, right). By placing many such systems together, we can reproduce the magnetic field as measured on the ground.



Single spherical elementary current system



Magnetic effect of a single spherical elementary current system

For practical implementation, we assume that the magnetic field **B** measured at a network of observatories has been created by multiple current systems and construct a linear set of equations relating the measured magnetic field to (a) the geometry of the region (usually a grid of lat/lon positions) in a matrix (**T**) and (b) the scaling factors (**I**) of the external current systems. The scaling factors which best fit the observations of the ground magnetic field disturbances are determined by solving the inverse problem, using Singular Value Decomposition. In matrix term this is: $\mathbf{I} = \mathbf{B}^{-1}\,\mathbf{T}$.

The field over the entire area of interest or at a specific point can then be solved for.

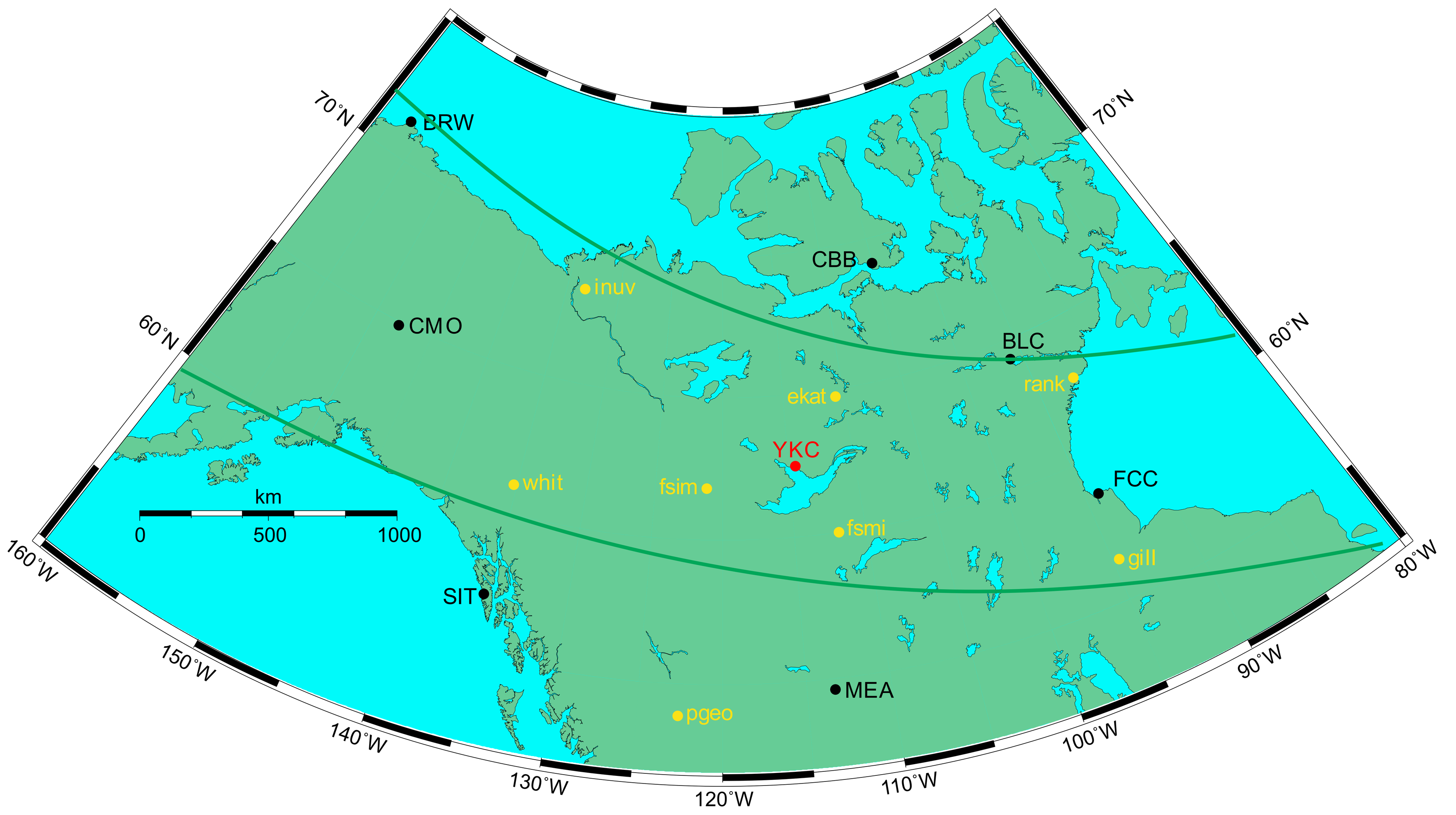


Figure 1: Locations of the observatories operated by the United States Geological Survey [BRW; CMO; SIT] and observatories operated by the Geological Survey of Canada [BLC; CBB; FCC; MEA] (black) and the eight variometers from the THEMIS ground network (yellow). The position of Yellowknife observatory [YKC] (red) operated by the Geological Survey of Canada at the centre is also shown.
Note most variometers and observatories are within the auroral zone (green lines: ~60-70° Geomagnetic North).

An example: testing SECS in North America

Figure 1 shows the network of observatories and variometers in North America. The minute-mean magnetic field values (with a daily average constant subtracted off) from each observatory were used to solve for the current systems over the entire area and then to estimate the magnetic field disturbance each minute at Yellowknife (YKC). The long term comparison (approximately four months) of the estimate for YKC was computed using the following sets of observatories and variometers:

Case 1: Seven observatories only
Case 2: Seven observatories and three close variometers *(ekat, fsim, fsmi)*
Case 3: Seven observatories and five distant variometers *(gill, rank, pgeo, whit, inuv)*
Case 4: Seven observatories and all eight variometers

A rectangular grid of current systems evenly-spaced in latitude and longitude was constructed with a grid spacing of 2°. The estimates of the magnetic field have been shown to be relatively insensitive to varying the grid spacing between 0.5 and 2.0°. The scaling factors (**I**) for the current systems across the grid of points were solved for every minute of every day. The magnetic field at YKC was then estimated and the root-mean-square difference between the recorded data and the interpolated value at the observatory for each day was computed using the RMS difference (in units of nanoTesla). The results of the RMS comparison of 110 days of minute-mean data are shown in Figure 2.

In almost all cases the SECS method proves to be better than assuming no change of the magnetic field (i.e. RMS difference is smaller than the power) and thus it is worthwhile using the technique to correct for external field disturbances even during magnetically quiet conditions or to estimate the electrical field strength within the ionosphere.

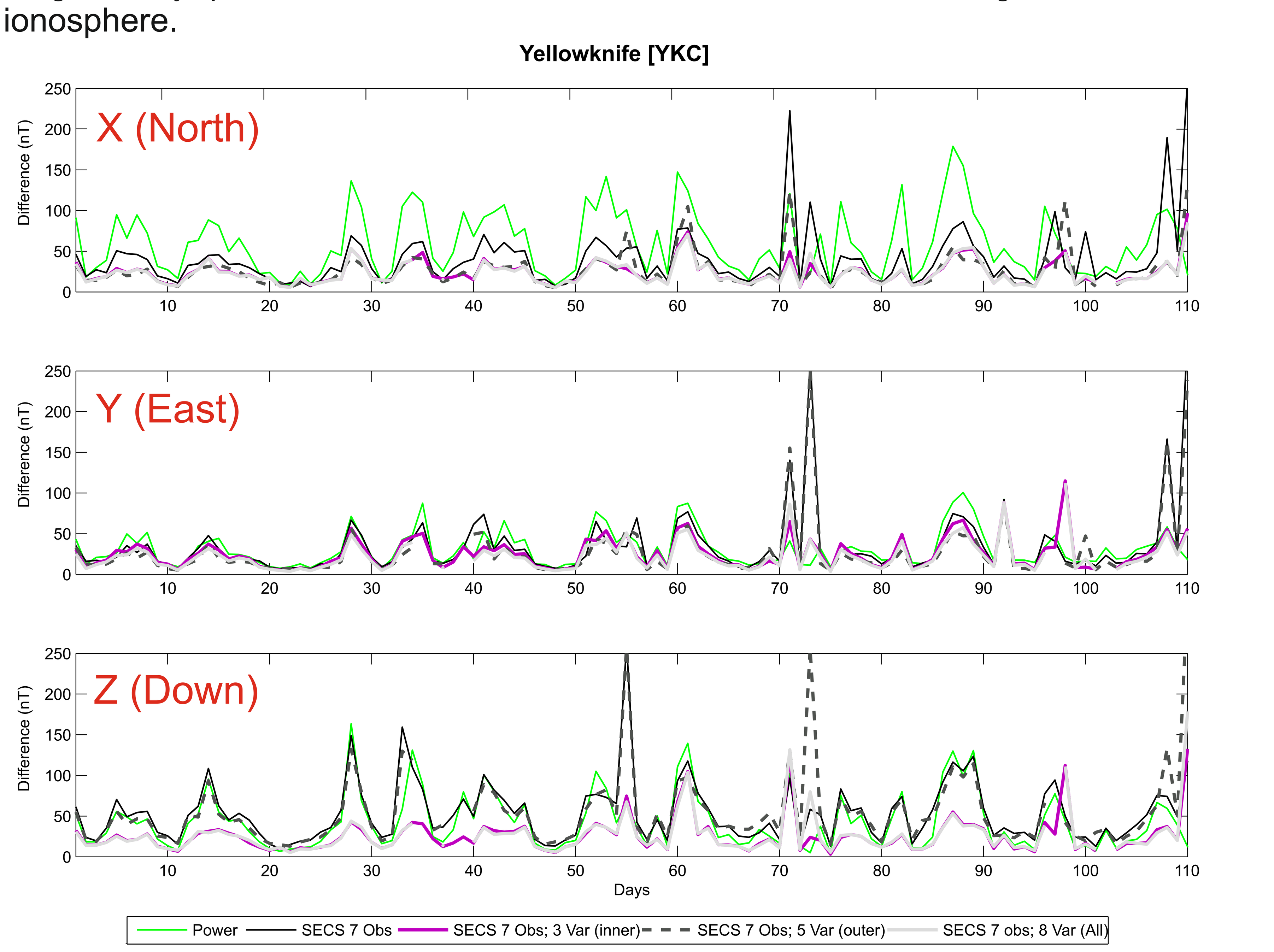


Figure 2: Comparison between Root Mean Square difference (in nanoTesla) in the three orthogonal components of the magnetic field for the (a) daily power of measured external disturbances (green) with (b) the estimate from the SECS using seven observatories (black solid), SECS using seven observatories and the three closest variometers (purple solid), SECS using seven observatories and the five distant variometers (gray dash) and SECS using seven observatories and the all variometers (light grey solid) over a four month period (February-May 2007) at Yellowknife (YKC).

Application to E-field interpolation

Near-realtime estimates of the ionospheric field could be made using calibrated variometer and observatory data. Figure 3 shows a snapshot of the modelled electric field. This could be used in addition to other method of estimating the auroral oval.

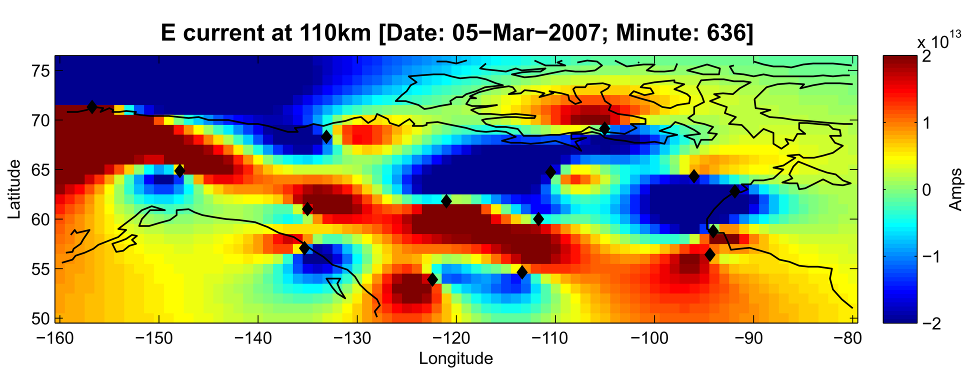


Figure 3: Snapshot of the ionospheric electric field on 5th May 2007 at 10.36UT modelled at 110km altitude using data from 15 variometers and observatories (diamonds). Note the strong red band of the auroral electojet trending across the region. Further observatories or variometers would help fill in the 'gaps'.

Acknowledgments

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