COMPUTER ANALYSIS OF ORIENTATION DATA IN STRUCTURAL GEOLOGY

by

T. Victor Loudon

TECHNICAL REPORT NO. 13

of

ONR Task No. 389-135

Contract Nonr 1228(26)

Office of Naval Research

Geography Branch

Northwestern University

Evanston, Illinois

November, 1964

COMPUTER ANALYSIS OF ORIENTATION DATA IN STRUCTURAL GEOLOGY

by

T. Victor Loudon

TECHNICAL REPORT NO. 13

of

ONR Task No. 389-135

Contract Nonr 1228(26)

Office of Naval Research

Geography Branch

Northwestern University

Evanston, Illinois

This report has been made possible through support and sponsorship by the United States Department of the Navy, Office of Naval Research, under ONR Task Number 389-135, Contract Nonr 1228(26).

Prefatory Remarks

This report is the sixth in a series of computer manuals arising from a continuing study of computer application in the earth and environmental sciences, including geology, geography, geophysics, geochemistry, and environmental engineering. In some of these subjects, notably oceanography, atmospheric science, and solid-earth geophysics, computer capability has advanced far beyond that in the more classical fields of geology and geography, as well as in such aspects of environmental science as soil mechanics and sanitary engineering. Our study is concerned mainly with these more classical fields in which computer utilization is less far advanced.

Our project has as its purposes the evaluation of developments in computer capability in our fields through assessment of present activities; and an obligation to make available in the public domain a series of computer programs especially adapted to the needs of workers in our fields. The first purpose is being met through conferences and literature search; and the second is being met by reports such as this, which will include programs arising from our own studies, as well as program reports contributed by others active in the field.

This report is an extension of work begun by Dr. T. Victor Loudon at the University of Edinburgh in connection with his Ph.D. dissertation. Dr. Loudon came to Northwestern University as a Research Associate in Geology, for the specific purpose of preparing this computer program and report. Although the examples used in the report are concerned with structural geology, the underlying methodology is applicable to a wide variety of problems that include directional data.

W. L. Garrison

W. C. Krumbein

CONTENTS

COMPUTER ANALYSIS OF ORIENTATION DATA IN STRUCTURAL GEOLOGY

by

T. Victor Loudon

ABSTRACT

Three-dimensional orientation data can be simplified by transforming them to refer to axes related to the symmetry of the distribution. Three uncorrelated scalar variates are generated by the transformation. Each of the scalar variates generally has direct geological significance. Each variate can be described individually by the usual statistical parameters. Thus, the orientation, tightness, asymmetry, shape and size of folds can be measured statistically; first, in the direction of greatest buckling; second, parallel to the fold axis. The degree to which a fold is cylindrical or conical in form can be estimated by a least-squares method. A Fortran IV program has been written to perform the computations.

A consideration of the spatial variation of the parameters allows various types of folding to be discriminated quantitatively, and permits the testing of hypotheses of the origin of folding.

INTRODUCTION

Purpose of this report:

Statistical methods are most frequently used with scalar variables, that is, with quantities each of which has a magnitude that can be described by a single number. Standard statistical methods are less readily applied to problems in the earth sciences which deal with orientation or location of objects in three-dimensional space. As has been pointed out by Paterson and Weiss (1961), the investigator frequently wishes to consider three-dimensional phenomena in terms of their symmetry. Symmetry arguments are well known in structural geology. For instance, a geologist may consider structure in terms of folding parallel to the fold axis and folding normal to the fold axis, in the belief that these directions are related to the stress pattern which produced the folding. But Paterson and Weiss show that the symmetry argument is implicit in the conclusions of many other geologists, who may relate, for example, the symmetry of a sedimentary deposit to the symmetry of the current movement which produced it. Paterson and Weiss quote Curie (1894) as stating: "When certain causes produce certain effects, the elements of symmetry of the causes must be found again in the effects produced."

It is the purpose of this report to show how a computer may be used to analyze a distribution of three-dimensional vectors into components which have meaning in terms of the symmetry of the distribution, and how the components may be analyzed subsequently by standard statistical methods. The breakdown of a complex distribution into components allows quantitative methods to be applied in fields where such an approach had not previously been widely used.

Scope of the method:

Structural geology is the primary concern of the present study, but the methods are also applicable in other fields. Structural data may be in the form of measurements of attitudes of planar or linear features, or of elevations of a number of points on a geological horizon. The results of the computations describe and analyze the form of a surface or series of surfaces. The methods are therefore directly applicable to geographical investigations which involve the form of land surfaces, with data in the form either of elevations or of slopes, (see, for example, Snell, 1961).

Many other kinds of mappable observations can be treated as surfaces which can be analyzed by the procedures described below. Geological structure maps and isopach maps, for instance, can be regarded as representations of surfaces in three dimensions. Such features as the geometry of sand bodies can be described quantitatively, by methods applied here to structural geology.

A problem similar to that of map projection arises in the construction of stereograms, which are commonly used in structural geology for the two-dimensional representation of points distributed on the surface of a sphere. A subroutine of the program handles the automatic plotting of a distribution of points on a Wulff net. This subroutine may have application in other fields such as crystallography and petrofabrics.

Geophysics, meteorology, and oceanography are other fields in which the approach taken in this study would appear to have some application. In general, the approach is likely to be of value in investigating any three-dimensional distribution in which directional properties or symmetry properties are important.

Background of the present study:

An excellent review of the procedures which geologists have used to analyze vectorial data was provided by Steinmetz (1962). Steinmetz advocated the use of three-dimensional methods based on a study by Fisher (1953) of probability density on the surface of a sphere. Fisher showed how estimates of the mean and dispersion (that is, variability about the mean) could be derived from a sample of three-dimensional vectors. The mean is itself a three-dimensional vector, but the dispersion is a scalar quantity which can be described by one number.

Fisher's approach is appropriate in many geological problems where the dispersion of vectors about the mean is not related to direction. Where the dispersion is greater in one direction than in another, and where the relationship between dispersion and direction is a fundamental part of the problem, Fisher's method is less suitable. An alternative approach is suggested by the techniques used in factor analysis, (see Harman, 1960, or Kendall, 1961). The method of factor analysis has been used in geology by Imbrie (1963). Matrix algebra procedures are used by these authors to simplify complex sets of measurements. The original data may consist of a group of variables measured on each of a number of items. It is generally found by factor analysis that most of the variability of the data can be accounted for by a smaller number of idealized variables, each obtained by combining some of the original variables in different proportions.

Each measurement of orientation can be expressed by three numbers: the cosines of the angles between the measurement and each of three axes at right angles, such as axes directed south, east and vertically upwards. Similarly, a measurement of location can be expressed by three numbers, representing distances from a selected origin parallel to each of three axes. If the numbers are regarded as the original variables, techniques similar to those of factor analysis can be used to compute new variables, which have real physical significance. The new variables are the same measurements referred to new axes - three mutually perpendicular lines in space, referred to as the principal axes. The geological significance of the principal axes and the new variables is considered in section 1 below.

Acknowledgments:

The interest taken in this work by E. K. Walton, F. H. Stewart and M. R. W. Johnson of the University of Edinburgh; and the help, encouragement and advice received at Northwestern University, particularly from W. C. Krumbein and E. H. T. Whitten, is gratefully acknowledged.

Financial support was received from the British Department of Scientific and Industrial Research during the early stages of this project, and from the Geography Branch of the Office of Naval Research at a later stage, during which the computer program described here was developed.

SECTION 1 : QUANTITATIVE FOLD DESCRIPTION

A large terminology is used in describing folds, (see, for instance, chapter 13 of Turner and Weiss, 1963); but little apparent progress has been made in quantifying fold description. However, a computer program has now been prepared which can be used to calculate quantitative measures for summarizing field data. It is hoped that quantitative methods will make it possible to describe folds more objectively and precisely. Standard methods of statistics and solid geometry are used in the computations, with a minimum of constraining assumptions.

The data collected by the field geologist usually are in numerical form, and are likely to consist of measurements of attitudes of planar and linear features at given geographic points. Planar features may include foliation planes, bedding planes, planes of cross-stratification, cleavage planes, fold axial planes, joints, and faults. Linear features may include fold and other tectonic axes, cleavage-bedding intersections, mineral, fossil or pebble elongations, and other sedimentary and tectonic lineations. The geographic point at which each measurement is made can be recorded as three coordinates, giving the distances south, east and vertically above some selected origin.

Vector representation:

For purposes of analysis, the measurements of each planar or linear feature can be regarded as a set of three-dimensional vectors. The attitude of a planar feature can be represented by a vector of unit length perpendicular to the plane, and the attitude of a lineation can be represented by a unit vector parallel to it. The sense of each vector can be assigned in some consistent, even though arbitrary, manner. For instance, vectors referring to bedding planes can be regarded as directed away from the younger side of the bed.

Vectors are commonly and conveniently represented by direction cosines (see, for example, Cohn, 1961, or Harman, 1960). The direction cosines of a vector are the cosines of the angles which the vector makes with three orthogonal axes, for example, axes directed south, east and vertically upwards. It can be proved by simple trigonometry that, with these axes, the direction cosines (p,q,r) of the pole to a plane dipping a v degrees in a direction u degrees east of north are: p = (-cos u) x (sin v); q = (sin u) x (sin v); r = cos v, (see figure 1).

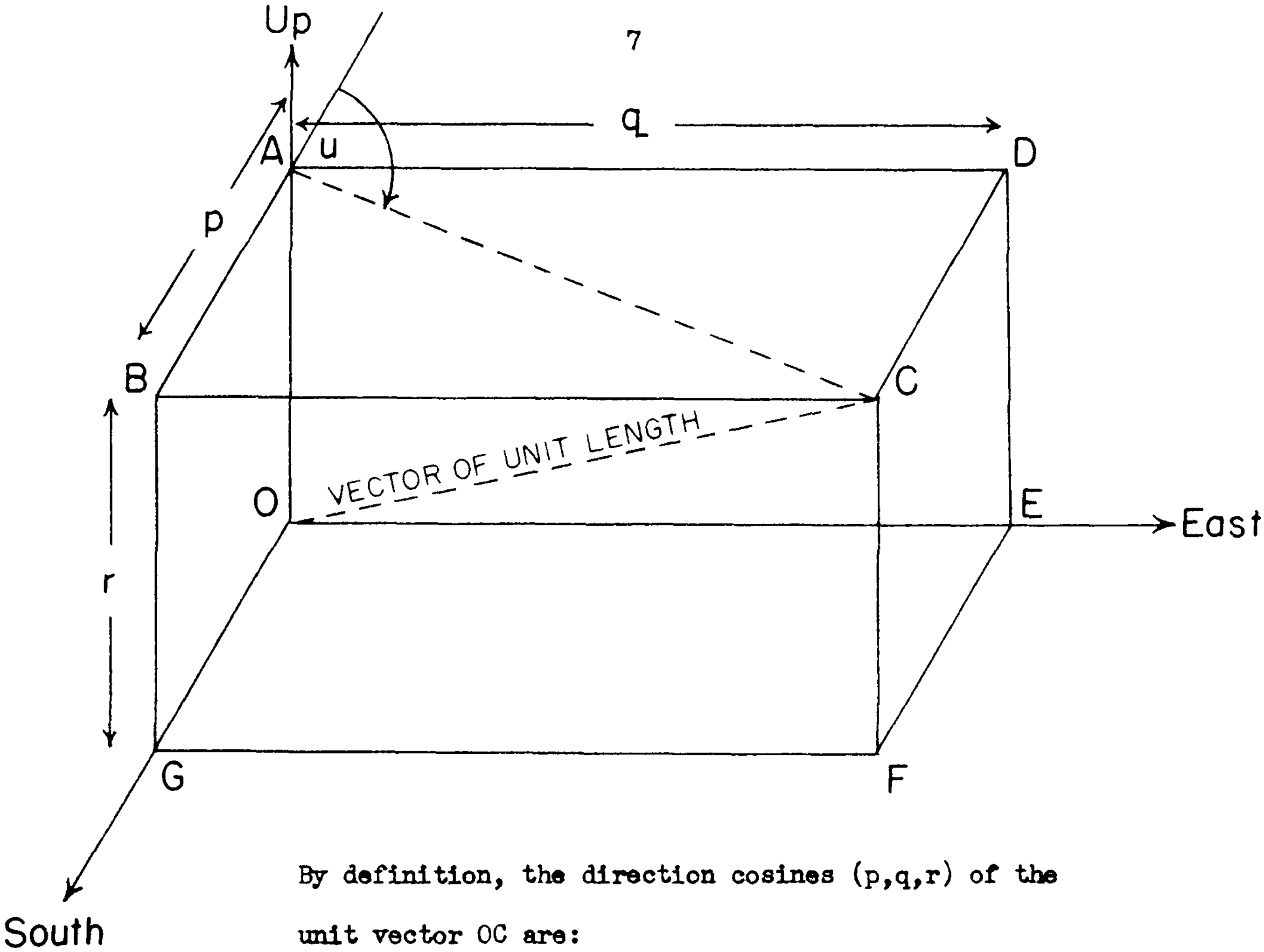
Direction cosines of a vector perpendicular to a bedding plane can be used to calculate the thickness of strata traversed in moving from one point to another. In figure 2, for instance, it can be seen that in moving from O to C, the thickness of strata traversed is (p x OG) + (q x GF) + (r x FC). Referring to figure 1, this is seen to equal $p^2 + q^2 + r^2$, which is equal to one, since it follows from the theorem of Pythagoras that the sum of squares of direction cosines is unity, (see Cohn, 1961, page 16). That is, unit thickness of strata is traversed in moving from O to C.

<u>The mean vector</u>:

The mean of a set of vectors is itself a vector, (see Fisher, 1953). The direction cosines of the mean vector are found as follows:

$$\bar{p} = \Sigma p/\sqrt{(\Sigma p)^2 + (\Sigma q)^2 + (\Sigma r)^2}$$
$$\bar{q} = \Sigma q/\sqrt{(\Sigma p)^2 + (\Sigma q)^2 + (\Sigma r)^2}$$
$$\bar{r} = \Sigma r/\sqrt{(\Sigma p)^2 + (\Sigma q)^2 + (\Sigma r)^2}$$

where the mean vector has direction cosines $(\bar{p},\bar{q},\bar{r})$, and Σp, Σq, and Σr denote the sums of individual values of p, q, and r, for the set of vectors. The direction cosines of the mean vector are calculated in this way to ensure that $\bar{p}^2 + \bar{q}^2 + \bar{r}^2 = 1$, an essential feature of direction cosines,



By definition, the direction cosines (p,q,r) of the unit vector OC are:

p = cosGOC = OG = DC; q = cosEOC = OE = AD; r = cos AOC = OA.

If the unit vector OC is the pole to a plane which dips v degrees in a direction u degrees east of north, then, DAC = u - 90^{o}, and AOC = v.

AC = sin AOC = sin v

p / sin v = DC / AC = sin DAC = sin (u - 90^{o}) = -cos u

Hence, p = -cos u . sin v

q / sin v = AD / AC = cos DAC = cos (u - 90^{o}) = sin u

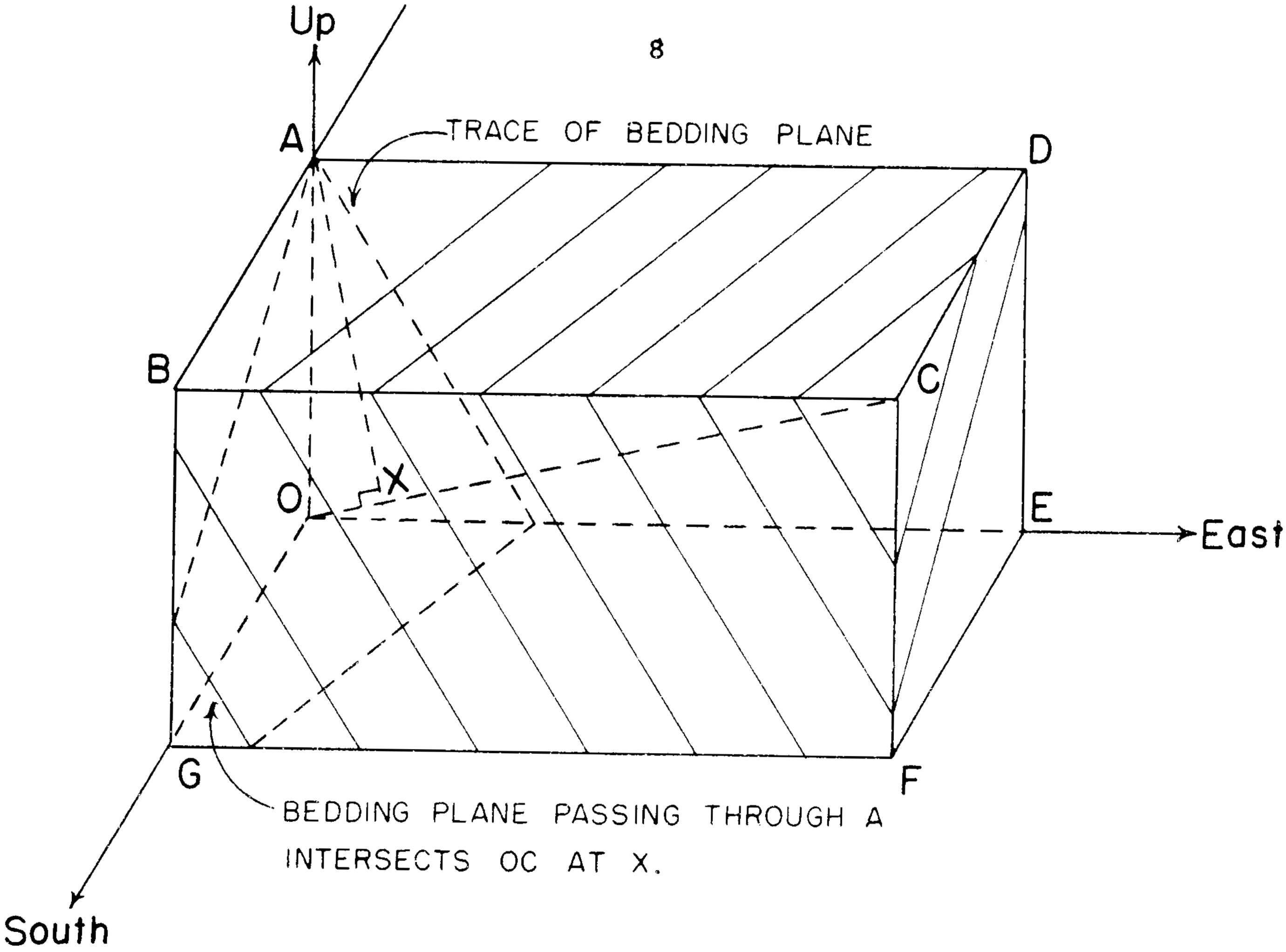
Hence, q = sin u . sin v

r = cos AOC = cos v

Summarizing: p = -cos u . sin v
q = sin u . sin v
r = cos v

where u and v are the direction and amount of dip of a plane with pole OC.

Figure 1 : Direction cosines (p,q,r) of a vector



OC is a vector of unit length normal to the bedding. Distances along OC represent thicknesses of strata measured normal to the bedding. It is possible to calculate thicknesses of strata traversed in moving between two points, say O and A, from the direction cosines (p,q,r) of the unit vector OC.

OX / OA = cos AOC = r (see figure 1)

Therefore, OX = r . OA

That is, the thickness of strata traversed in moving a distance OA vertically is r times OA. Similarly it can be shown that the thickness of strata traversed in moving east for a given distance, say OE, is q times OE. And the thickness traversed in moving OG southwards is p times OG.

Figure 2 : Calculation of thickness of strata from direction cosines

(see Cohn, 1961, page 16).

Estimates of thicknesses of strata can be calculated from direction cosines of the mean vector, thus allowing estimates to be made of the differences of stratigraphic level at various points. This may have value in stratigraphic correlation or in examining the tectonic attenuation of fold limbs. An example may make the procedure clearer.

Computation of mean vector:

Example 1 : The fictitious measurements recorded on the map in figure 3, are supposedly representative of the attitude of bedding within a sandstone unit. A band of limestone outcropping at AB is known to correlate with limestone outcropping at CD.

Two alternative hypotheses are considered.

(1) A fault may exist near FF', in which case the movement along the fault is to be estimated.

(2) The second hypothesis is that the fault does not exist, but that horizontal compression caused unequal attenuation of the fold limbs. In this case the amount of compression is to be estimated.

The first step in the computation is to calculate the mean value of the bedding attitude within the area ABCD. This will indicate the difference in stratigraphic level between AB and CD on the assumption that neither faulting nor attenuation occurred. The calculation proceeds as shown on table 1.

The value of the grand mean, (.1089, -.0996, .9891) indicates that on average one moves .1089 miles higher in the section for every mile south, and .0996 miles lower for every mile moved east. C is 2.6 miles north and 3 miles east of A. One might therefore expect to be (2.6 x .1089) + (3.0 x .0996) miles, that is about 3000 feet, lower in the section

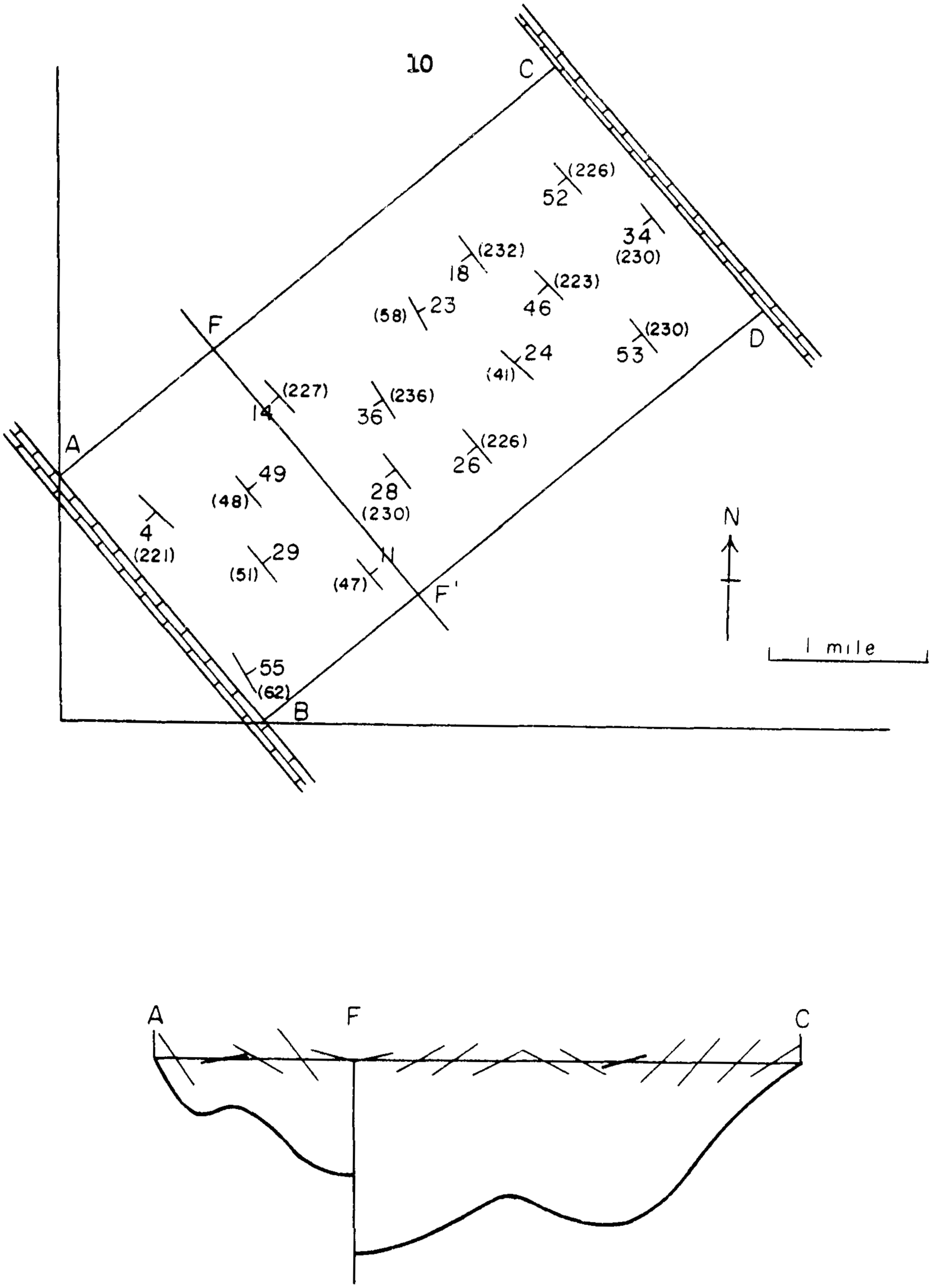


Figure 3 : Map and cross-section of hypothetical fold to illustrate example 1. The azimuth of dip is shown in brackets at each measurement.

TABLE 1

Dip (in degrees) Direction u	Amount v	sin u	sin v	cos u	cos v	p	q	r	Limb of fold	Side of fault
62	55	.8830	.8192	.4695	.5736	-.3846	.7234	.5736	E	SW
221	4	-.6561	.0698	-.7547	.9976	.0527	-.0458	.9976	W	SW
51	29	.7772	.4848	.6293	.8746	-.3051	.3768	.8746	E	SW
47	11	.7314	.1908	.6820	.9848	-.1301	.1396	.9848	E	SW
48	49	.7431	.7547	.6691	.6561	-.5050	.5608	.6561	E	SW
227	14	-.7314	.2419	-.6820	.9703	.1650	-.1769	.9703	W	NE
230	28	-.7660	.4695	-.6428	.8830	.3018	-.3596	.8830	W	NE
236	36	-.8290	.5878	-.5592	.8090	.3287	-.4873	.8090	W	NE
226	26	-.7193	.4384	-.6947	.8988	.3046	-.3153	.8988	W	NE
58	23	.8481	.3907	.5299	.9205	-.2070	.3314	.9205	E	NE
41	24	.6561	.4067	.7547	.9136	-.3069	.2668	.9136	E	NE
223	46	-.6820	.7193	-.7314	.6945	.5261	-.4906	.6945	W	NE
232	18	-.7880	.3090	-.6157	.9511	.1903	-.2435	.9511	W	NE
230	53	-.7660	.7986	-.6428	.6081	.5133	-.6117	.6081	W	NE
226	52	-.7193	.7880	-.6945	.6157	.5473	-.5668	.6157	W	NE
230	34	-.7660	.5592	-.6428	.8290	.3595	-.4283	.8290	W	NE

	Totals			Direction cosines of mean vectors		
East limb	-1.8387	2.3988	4.9232	-.3182	.4151	.8519
West limb	3.2893	-3.7258	8.2571	.3413	-.3866	.8568
NE of fault	2.7227	-3.0818	9.0936	.2728	-.3088	.9112
SW of fault	-1.2721	1.7548	4.0867	-.2750	.3794	.8835
Grand total	1.4506	-1.3207	13.1803	.1089	-.0996	.9891

Calculation for example 1, illustrating the use of direction cosines of the mean.

at C than at A. Since the stratigraphic level is in fact the same, this suggests that the northeast side of the fault FF' is downthrown about 3000 feet. On the southwest side of the fault, the mean vector has direction cosines (-.2750, .3794, .8835). F lies -0.8 miles south and 0.9 miles east of A. Therefore, the stratigraphic level of the sandstone at the southwest side of the fault FF' is expected to be (0.8 x .2750) + (0.9 x .3794) miles, that is about 3000 feet above the limestone band. On the northeast side of the fault, the mean vector has direction cosines (.2788, -.3088, .9122), and F lies 1.8 miles south and -2.1 miles east of C. The stratigraphic level at the surface of the northeast side of the fault is expected to be (1.8 x .2728) + (2.1 x .3088) miles or about 6000 feet above the stratigraphic level of the limestone. Again, this indicates about 3000 feet of movement on the fault.

If the discrepancy in stratigraphic level were a consequence of limb attenuation due to horizontal compression, and not a result of faulting, then the attenuation would be related to the angle which the limbs make with a horizontal axis. The average slope of the east limbs in direction AC is [(2.6 x .3182) + (3.0 x .4151)]/4.0, that is, 0.52. The average slope of the west limbs is [(2.6 x -.3413) + (3.0 x -.3866)]/4.0, that is, -0.52. Since the slope of the two limbs is about the same, there is no indication that horizontal compression would cause unequal limb attenuation.

The variance of a set of vectors:

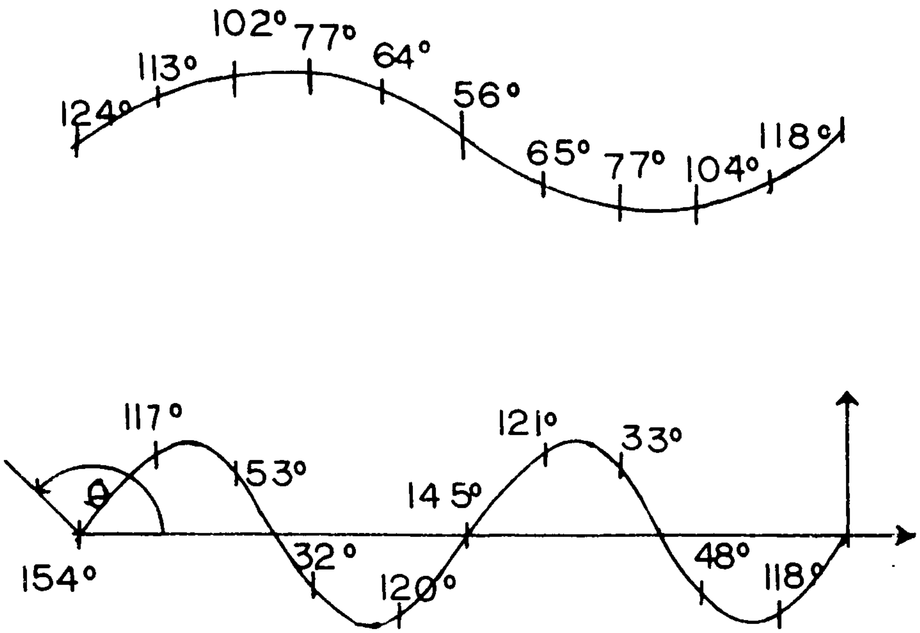
Having found a mean value for the measurements, the next descriptive statistics to be considered are the second moment and the variance, (see Yule and Kendall, 1958, or Dixon and Massey, 1957, page 18). The variance measures the variation, or dispersion, of values about the mean. The second moment measures the dispersion of values about any origin, not necessarily the mean. Any axis could be chosen in a set of three-dimensional vectors, and the second moment of direction cosines measured about that axis could be found by summing the squares of cosines of the angles between individual vectors and the axis.

Computation of variance:

Example 2 : The second moments of the two-dimensional distribution of direction cosines shown for each of the two folds in figure 4 are to be calculated.

The direction cosines (p,q) of poles to the foliation planes shown in figure 4 are, by definition, the cosines of the angles which the poles make with two orthogonal reference axes. Possible reference axes are the mean pole, which is directed vertically upwards, and a horizontal axis, directed to the right. The second moments can be calculated by finding the arithmetic means of p^2 and q^2 terms (denoted $\overline{p^2}$ and $\overline{q^2}$). From the theorem of Pythagoras, $p^2 + q^2 = 1$ for each individual term and hence $\overline{p^2} + \overline{q^2} = 1$. It is therefore unnecessary to calculate two second moments, since one can be found directly from the other. At first sight, it appears logical to use $\overline{p^2}$ to measure the dispersion of the vectors. But it has an unfortunate property. Because the cosine of 0 degrees is 1, and the cosine of 90 degrees is 0, the second moment measured about the vector mean varies from 1, for a flat plane, to nearly 0 for a fold with parallel limbs. A more natural measure of dispersion is $\overline{q^2}$, since it increases, from zero to one, as the variation increases. Referring to figure 4, $\overline{q^2}$ is found to be .16 for the first fold and .47 for the second fold. The second moment is an indication of the tightness of the folding.

The above example brings out two points. First, that the second moment is a suitable measure of tightness of folding. Second, that the



Fold 1:

Cosines of θ — Totals/10

-5592	-3907	-2079	2250	4384	5592	4226	2250	-2419	-4695	0.0001

$\cos^2 \theta$

3127	1526	0432	0506	1922	3127	1786	0506	0508	2204	0.1571

Fold 2:

Cosines of θ

-8988	-4540	6081	8480	-5000	-8192	-5150	8387	6691	-4695	-0.0699

$\cos^2 \theta$

8078	2061	3622	7191	2500	6711	2652	7034	4477	3204	0.4653

Figure 4 : Calculation of the second moment of direction cosines

value of the second moment depends on the choice of reference axes. The mean pole and a line at right angles to it are a suitable choice of axes in two dimensions.

Other measures of tightness of folding could easily be developed. One could, for instance, estimate the total cumulated thickness of strata traversed (up and down) in moving unit distance parallel to each of the reference axes. This can be done by calculating the average absolute value (that is, all values taken as positive) of direction cosines about each axis. Alternatively, one could determine the average slope of the fold limbs by calculating the average value of the sine of the angles which the poles make with a horizontal axis. These alternative measures may have their uses in certain cases, but the mathematical properties of the second moment make it a most valuable measure of dispersion, and give it a central position in the analytical scheme advocated here.

When the variance is considered in three dimensions, $\overline{p^2} + \overline{q^2} + \overline{r^2} = 1$, where $\overline{p^2}$ is the arithmetic mean of p^2 terms, $\overline{q^2}$ of q^2 terms and $\overline{r^2}$ of r^2 terms. It remains true in three dimensions that the value of the second moments depends on the orientation of the reference axes relative to the distribution.

Choice of reference axes:

Fisher (1953), in a study of probability density on the surface of a sphere, estimated the dispersion of the distribution about the mean by one number - a scalar quantity which has no directional significance. The use of Fisher's technique in geology has been described by Steinmetz (1962). The method does not appear to be altogether appropriate in structural geology, for the relationship between dispersion and direction is a fundamental part of most structural problems. The dispersion can instead be considered in terms of components in different directions. It is possible to consider dispersion separately in each of the p, q, and r components. The second moments are calculated by squaring each item and finding the average value of terms of the type p^2, q^2, and r^2. This still does not fully specify the relationship between dispersion and direction, for it gives no indication of correlations between the p, q, and r terms. Covariance terms of the type pq, qr, and pr, can also be computed for each measurement and the average values of the covariance terms found. The results can be arranged in rows and columns to form the following matrix:

$$\begin{bmatrix} \overline{p^2} & \overline{pq} & \overline{pr} \\ \overline{qp} & \overline{q^2} & \overline{qr} \\ \overline{rp} & \overline{rq} & \overline{r^2} \end{bmatrix} \qquad \text{where } \overline{pq} = \overline{qp}, \quad \overline{pr} = \overline{rp} \quad \text{and } \overline{qr} = \overline{rq} \qquad (1)$$

This is a dispersion matrix.

The dispersion or covariance matrix does not at first appear to give the geologist any information of value. Matrices which refer to different folds cannot be used to compare the properties of the folds, unless the folds happen to be similarly oriented in space. The reason for this is that the reference axes are arbitrary - namely, lines directed in south, east and vertical directions - and the axes are not related to the particular fold under consideration. The values of the variance and covariance terms in the matrix reflect the amount of folding in different directions, but the directions do not have any apparent geological significance. However, directions which do have geological significance can be found. They relate to the symmetry of the fold or folds. The technique of finding such axes is similar to the technique used in factor analysis for finding principal axes, and is described in Harman (1960, chapter 9), or Kendall (1961, chapter 2).

Principal axes:

Three orthogonal axes are found such that the dispersion matrix of direction cosines about the axes has every covariance term equal to zero. The axes are termed the principal axes of the distribution. If p', q', and r' are the direction cosines of the vectors referred to the principal axes, the dispersion matrix

$$\begin{bmatrix} \overline{p'^2} & \overline{p'q'} & \overline{p'r'} \\ \overline{q'p'} & \overline{q'^2} & \overline{q'r'} \\ \overline{r'p'} & \overline{r'q'} & \overline{r'^2} \end{bmatrix} \quad \text{is} \quad \begin{bmatrix} \overline{p'^2} & 0 & 0 \\ 0 & \overline{q'^2} & 0 \\ 0 & 0 & \overline{r'^2} \end{bmatrix} \quad . \quad . \quad (2)$$

This is the dispersion matrix referred to the principal axes. It is convenient to arrange it in such a way that $\overline{p'^2} \geq \overline{q'^2} \geq \overline{r'^2}$. Since the covariance terms are equal to zero, the variates p', q', and r' are said to be uncorrelated, (see Yule and Kendall, 1958).

The values $\overline{p'^2}$, $\overline{q'^2}$, and $\overline{r'^2}$ of matrix (2) are the second moments of the distribution referred to the principal axes. The relative values of the three second moments give an indication of the relationship of dispersion and direction. The second moments about the principal axes have important properties:

(1) The second moment $\overline{p'^2}$ has the largest value of any second moment of the distribution, measured about any axis that can be chosen. In other words, the dispersion of the distribution is largest when measured about the principal axis corresponding to $\overline{p'^2}$.

(2) The second moment $\overline{q'^2}$ has the largest value of any second moment measured about any axis lying in a plane normal to the p' axis.

(3) The third principal axis is at right angles to the other two, and the corresponding second moment $\overline{r'^2}$ has the smallest value of any second moment measured about any axis of the distribution.

(4) One of the axes, usually the one corresponding to $\overline{p'^2}$, is the vector mean of the distribution. In areas of very tight folding, where the fold limbs usually slope at more than 45 degrees, the mean pole is the axis corresponding to $\overline{q'^2}$.

Geological significance of principal axes:

The geological significance of the mean vector has already been considered. Since the vectors are poles to foliation planes, and the cosine of 90 degrees is zero, the $\overline{r'^2}$ axis is the line which, when moved parallel to itself, most nearly generates the folded surface. It is the fold axis (Clark and McIntyre, 1951). The axis about which the second moment is intermediate is at right angles to the mean pole and the fold axis. It is the horizontal axis of example 2. As in that example, the second moment measured about it is a measure of the tightness of folding measured in the plane normal to the fold axis. The folding parallel to the fold axis is measured separately by the minimum second moment, $\overline{r'^2}$.

Subscript notation:

At this stage, it is convenient to adopt a more compact notation, similar to that used in solid geometry (see Cohn, 1961, page 6 and page 34), and factor analysis (see Harman, 1960, chapter 2), and related to the notation of a computer program (see section 2 of this report). The three coordinate axes to which the measurements are referred can be denoted: 1-axis, 2-axis, and 3-axis. The original measurements might be in terms of axes directed south (1-axis), east (2-axis), and upwards (3-axis). The direction cosines of the jth measurement can be denoted $[d_{j1}, d_{j2}, d_{j3}]$, where the first subscript denotes the number of the measurement, and the second subscript denotes the axis from which the cosine is measured. The vector $[d_{j1}, d_{j2}, d_{j3}]$ can be more compactly denoted $\underline{d}_j$. The entire array of n such measurements, arranged in n rows and three columns, comprises the data matrix D. Underlining is used to indicate a vector, a capital letter to indicate a matrix.

The location of each measurement can be recorded as coordinates giving the distance south, east, and above an arbitrary origin. These distances together constitute a vector which for the jth measurement, can be denoted $\underline{g}_j$ or $[g_{j1}, g_{j2}, g_{j3}]$. The array of n geographical coordinates contains n rows and three columns, and can be denoted G, another data matrix.

As was mentioned above, the distribution of direction cosines, D, and the distribution of geographical coordinates, G, can be transformed to refer to the principal axes of the distribution. The principal axes would then be the coordinate axes to which the distributions were referred. The principal 1-axis, 2-axis, and 3-axis can be arranged in sequence so that the variance of the distribution measured about the 1-axis is the maximum variance, the variance measured about the 2-axis is intermediate, and the variance measured about the 3-axis is the minimum possible for the distribution measured about any axis, (see section on principal axes, above).

The transformed direction cosines for the jth measurement can be denoted $\underline{t}_j$ or $[t_{j1}, t_{j2}, t_{j3}]$. The corresponding geographical coordinates $\underline{g}_j$ are transformed to coordinates which have structural significance, denoted $\underline{s}_j$ or $[s_{j1}, s_{j2}, s_{j3}]$. The coordinates s_{j1}, s_{j2}, and s_{j3} are distances from the selected origin to the point of measurement, measured parallel to the principal 1-axis, 2-axis, and 3-axis respectively. The array of n transformed orientation measurements can be denoted T, and the array of structural coordinates can be denoted S. Both T and S are matrices with n rows and three columns.

Computation of principal axes:

Harman (1960, page 179), discussed the computation of principal axes at considerable length. The algebraic computation has a geometrical counterpart in the operations of rotation which are performed on a stereogram. This is illustrated below in example 3. The calculation of principal axes for structural data proceeds as follows.

(1) The dispersion or covariance matrix (1) mentioned above, is computed. It will be denoted C. It is found by summing squares and cross-products of the direction cosines. Using the symbol $\sum_j$ to indicate summation of the expression which follows it, for all values of j from 1 to n.

$$\underline{C} = \begin{bmatrix} \sum_j(d_{j1}\ d_{j1}) & \sum_j(d_{j1}\ d_{j2}) & \sum_j(d_{j1}\ d_{j3}) \\ \sum_j(d_{j2}\ d_{j1}) & \sum_j(d_{j2}\ d_{j2}) & \sum_j(d_{j2}\ d_{j3}) \\ \sum_j(d_{j3}\ d_{j1}) & \sum_j(d_{j3}\ d_{j2}) & \sum_j(d_{j3}\ d_{j3}) \end{bmatrix}$$

(2) The eigenvectors and eigenvalues of C are computed, (see Harman, 1960, page 157). The three eigenvectors can be arranged as the columns of a matrix with three rows and three columns, denoted E.

$$\underline{E} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}$$

The first eigenvector $[e_{11}, e_{21}, e_{31}]$ can be denoted $\underline{e}_1^T$, where the T indicates that it is a column vector. The eigenvector $\underline{e}_1^T$ is associated with the largest of the three eigenvalues, $\underline{e}_2^T$ with the intermediate eigenvalue, and $\underline{e}_3^T$ with the smallest eigenvalue.

The principal axes of the distribution are given by the eigenvectors, $\underline{e}_1^T$, $\underline{e}_2^T$, $\underline{e}_3^T$, in terms of direction cosines referring to the original coordinate axes. The eigenvalue associated with each eigenvector is equal to the second moment measured about the axis indicated by the eigenvector.

(3) Transformation from vectors referred to the original coordinate axes to vectors referred to the principal axes is achieved by postmultiplying the original vector by $\underline{E}$, the matrix of eigenvectors.

Thus: $\underline{d}_j \cdot \underline{E} = \underline{t}_j$

and $\underline{g}_j \cdot \underline{E} = \underline{s}_j$

As mentioned by Harman, the operations of matrix multiplication have exact geometrical analogs. The original population of vectors, $\underline{D}$, could be represented geometrically by lines of unit length radiating from the center of a sphere. The transformed vectors, $\underline{T}$, could be obtained by rotating the sphere until the principal axes lay in the south, east, and vertical positions. Rotation of the sphere is analogous to multiplication by the matrix $\underline{E}$. The Jacobi method of computing eigenvectors, also described by Harman, page 179, proceeds by a series of matrix multiplications, each analogous to a rotation about one axis of the distribution. The matrices are chosen to reduce each of the covariance terms of $\underline{C}$ successively to zero. Finally the various matrices used in the computation are combined (by multiplication) to yield one matrix, $\underline{E}$, which represents the total rotation. The analogous geometrical procedure could be used to find approximate principal axes. The original vectors could be plotted as points on a stereogram. The stereogram could be rotated about each axis in turn, until the points were as nearly possible symmetrically distributed about the stereographic axes. The axes of the stereogram would then be approximately parallel to the principal axes of the distribution.

<u>Transformation of data to refer to principal axes</u>:

Example 3 : The principal axes of the distribution of poles to foliation shown in stereogram 1, figure 5, are to be found using the Jacobi method. The effect of each rotation is to be illustrated on a stereogram. Each vector is to be transformed to refer to the new axes.

Direction cosines of the poles to bedding are calculated from the strike and dip as in example 1, with the results shown in table 2.

TABLE 2

j =	1	2	3	4	5	6	7	Totals
$\underline{D}^T$ =	.8743	.8079	.6973	.4255	.1114	-.0611	-.2276	
	-.1229	-.0423	.0363	.1984	.3234	.3859	.4280	
	.4514	.5878	.7193	.8829	.9397	.9205	.8746	
$(d_{1j})^2$ =	.76440	.65270	.48122	.18105	.01241	.00373	.05180	2.1473
$d_{1j} \cdot d_{2j}$ =	-.10745	-.03417	.02518	.08442	.03603	-.02358	-.09741	-0.1170
$d_{1j} \cdot d_{3j}$ =	.39466	.47488	.49898	.37567	.10468	-.05624	-.19906	1.5936
$(d_{2j})^2$ =	.01510	.00179	.00132	.03936	.10459	.14892	.18318	0.4943
$d_{2j} \cdot d_{3j}$ =	-.05548	-.02468	.02611	.17517	.30390	.35522	.37433	1.1544
$(d_{3j})^2$ =	.20376	.34551	.51739	.77951	.88304	.84732	.76493	4.3415

The totals divided by the number of items (seven), can be arranged as the covariance matrix, $\underline{C}$.

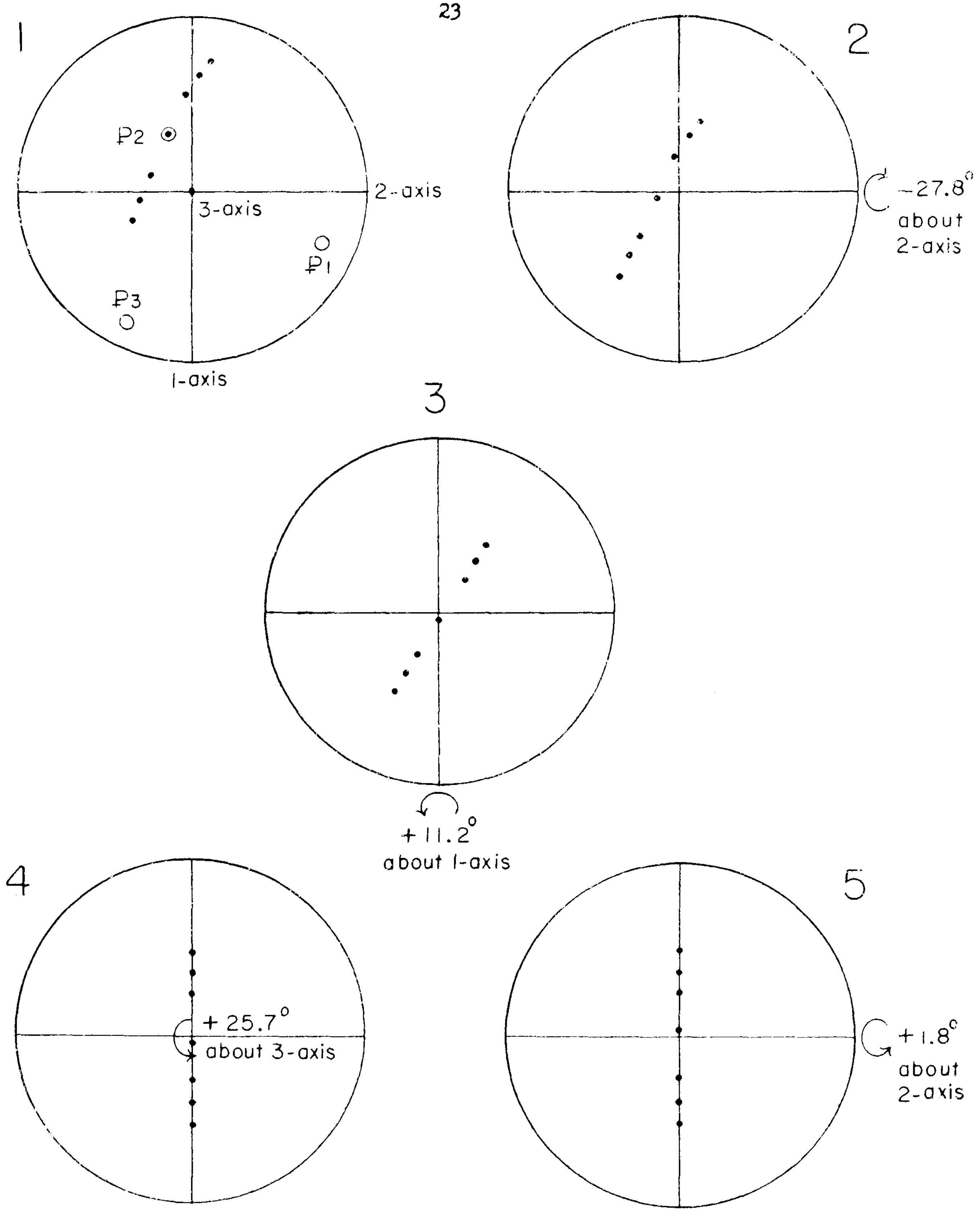


Figure 5 : Illustration of the rotations used in finding the principal axes of the vector distribution of example 3.

$$\underline{C} = \begin{bmatrix} .30676 & -.01671 & .22765 \\ -.01671 & .07060 & .16491 \\ .22765 & .16491 & .62021 \end{bmatrix}$$

The terms in $\underline{C}$ in the positions c_{11}, c_{22}, and c_{33} measure the variability of the distribution about the 1-axis, 2-axis, and 3-axis respectively. Each of the other terms is a measure of the association of values between the appropriate pair of axes. By rotating the distribution about one of the axes, say the i-axis, the orientation of the distribution relative to the other two axes, say the j-axis and k-axis, is changed. Hence the association between values measured about the j- and k-axes is changed. A position can be found such that there is no correlation between cosines measured from the j- and k-axes, and the c_{jk} term is reduced to zero.

Algebraically, the operation equivalent to rotating the distribution represented by $\underline{C}$ through θ degrees about the 3-axis, is to calculate the product $\underline{E}_1 . \underline{C} . \underline{E}_1^T$, where

$$\underline{E}_1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $\underline{E}_1^T$ is the transpose of $\underline{E}_1$, formed by interchanging the rows and columns.

$$\underline{E}_1^T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A similar algebraic operation, with the e_{ii} term equal to 1, and terms in the same row and column as e_{ii} equal to zero, would represent a rotation of θ degrees about the i-axis. The covariance term c_{jk} is reduced to zero by a rotation of θ degrees about the i-axis, where θ is given by

$$\tan 2\theta = \frac{2\,c_{jk}}{c_{jj} - c_{kk}}$$ (see Harman, 1960, page 181).

Rotations are performed to reduce the magnitude of each of the covariance terms, in decreasing order of size, until all are negligibly small.

Returning to example 3, the largest covariance term in C is c_{13} = 0.22765.

$$\tan 2\theta = \frac{2c_{jk}}{c_{jj} - c_{kk}} = \frac{2\ (0.22765)}{.30676 - .62021} = -1.4525$$

Hence, $\theta = -27.8$ degrees, and $\cos\theta = .8846$ and $\sin\theta = -.4664$.

The rotation matrix $\underline{E}_1$ is therefore

$$\begin{bmatrix} .8846 & 0 & -.4664 \\ 0 & 1 & 0 \\ .4664 & 0 & .8846 \end{bmatrix} \quad \text{and } \underline{E}_1.C = \begin{bmatrix} .16518 & .09170 & -.08789 \\ -.01671 & .07060 & .16491 \\ .34445 & .13890 & .65481 \end{bmatrix}$$

this is now multiplied by the transpose of E_1, namely E_1^T

$$\begin{bmatrix} .16518 & .09170 & -.08789 \\ -.01671 & .07060 & .16491 \\ .34445 & .13890 & .65481 \end{bmatrix} \times \begin{bmatrix} .8846 & 0 & .4664 \\ 0 & 1 & 0 \\ -.4664 & 0 & .8846 \end{bmatrix}$$

with the result

$$\begin{bmatrix} .18711 & .09170 & \underline{-.00070} \\ .09170 & .07060 & .13809 \\ -.00070 & .13809 & .73990 \end{bmatrix}$$

The term c_{13} has been reduced to a negligible quantity. The effect of this rotation is shown in stereogram 2, figure 5. The same procedure is used to reduce the c_{32} term from 0.13809 to nearly zero, and the other covariance terms are subsequently reduced until all covariance terms are negligible. The successive rotations are:

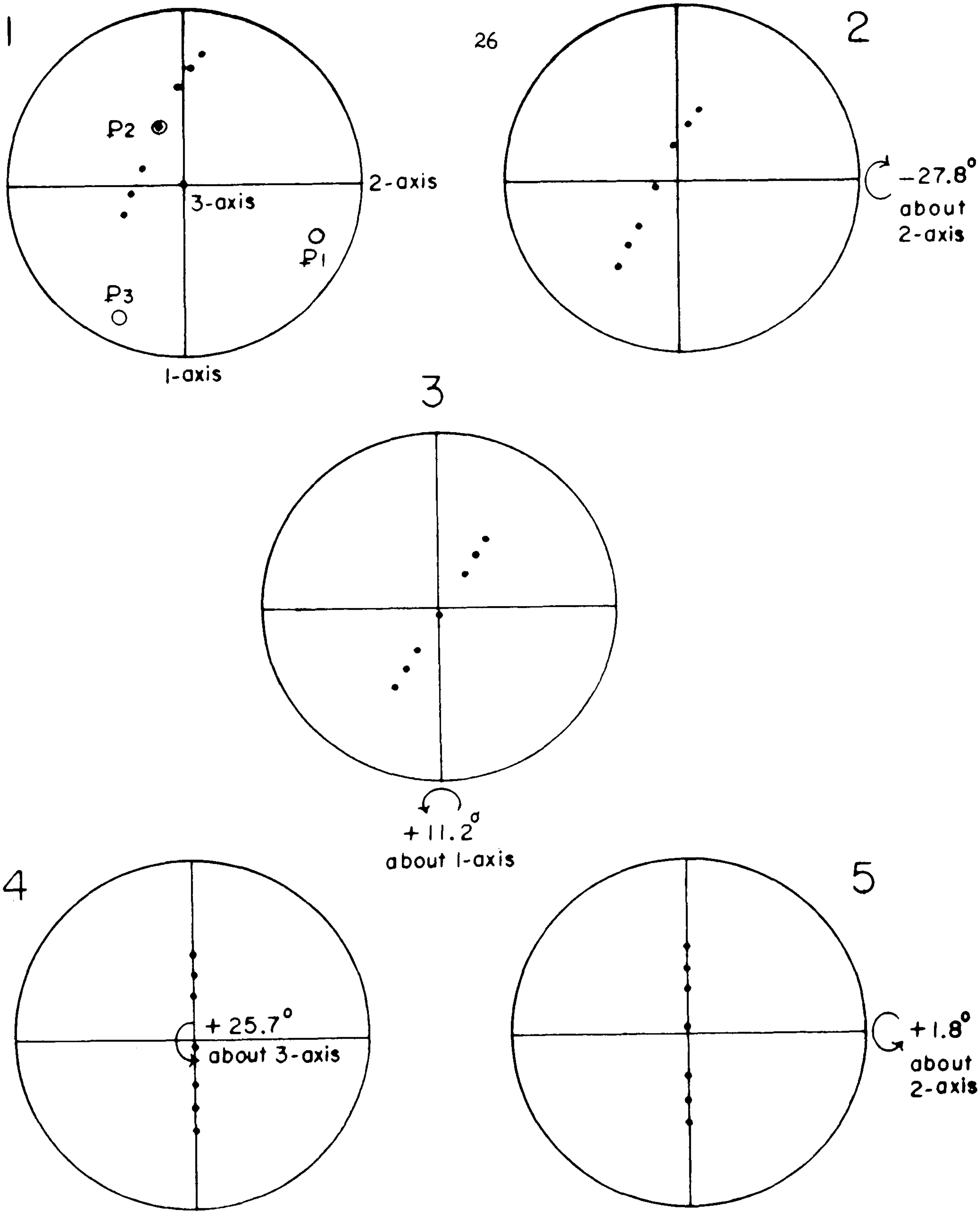


Figure 5 : Illustration of the rotations used in finding the principal axes of the vector distribution of example 3.

$\underline{E}_2$ about the 1-axis, 11.2 degrees, see stereogram 3.

$\underline{E}_3$ about the 3-axis, 25.7 degrees, see stereogram 4.

$\underline{E}_4$ about the 2-axis, 1.8 degrees, see stereogram 5.

After these rotations the matrix has reached the form:

$$\begin{bmatrix} .22975 & .00006 & .00014 \\ .00006 & .00009 & .00786 \\ .00014 & .00786 & .76784 \end{bmatrix}$$

The terms of the diagonal are the eigenvalues: 0.7678, 0.22975 and 0.00009.

The eigenvectors are found by multiplication of successive rotation matrices.

$$\underline{E} = \underline{E}_1 \cdot \underline{E}_2 \cdot \underline{E}_3 \cdot \underline{E}_4$$

$$\underline{E} = \begin{bmatrix} .8503 & .3020 & .4310 \\ -.4192 & .8840 & .2075 \\ -.3182 & -.3571 & .8782 \end{bmatrix}$$

Rearranging the columns to correspond with the eigenvectors in decreasing order of magnitude.

$$\underline{E} = \begin{bmatrix} .4310 & .8503 & .3020 \\ .2075 & -.4192 & .8840 \\ .8782 & -.3182 & -.3571 \end{bmatrix}$$

The principal axes, $\underline{e}_1^T = [.4310, .2075, .8782]$

$\underline{e}_2^T = [.8503, -.4192, -.3182]$

$\underline{e}_3^T = [.3020, .8840, -.3571]$

are plotted on stereogram 1, figure 5.

The original direction cosines $\underline{d}_j$ can now be transformed to refer to the principal axes by multiplying each one by $\underline{E}$. The first vector $\underline{d}_1$ had the value (.8743, -.1229, .4514).

t_{11} = (.8743 x .4310) + (-.1229 x .2075) + (.4514 x .8782) = .7477

t_{12} = (.8743 x .8503) + (-.1229 x -.4192) + (.4514 x -.3182) = .6513

t_{13} = (.8743 x .3020) + (-.1229 x .8840) + (.4514 x -.3517) = -.0034

$\underline{t}_1$ = [.7477, .6513, -.0034]

The other values of $\underline{t}_j$ are found in the same way, yielding the matrix

$$\underline{T}^T = \begin{bmatrix} .7477 & .8556 & .9382 & .9999 & .9409 & .8621 & .7588 \\ .6513 & .5176 & .3456 & -.0023 & -.3399 & -.5066 & -.6512 \\ -.0034 & -.0033 & -.0153 & -.0114 & -.0160 & -.0061 & -.0027 \end{bmatrix}$$

The use of transformed direction cosines:

The transformation of direction cosines referred to the original coordinate axes, to direction cosines referred to the principal axes has an important function in simplifying subsequent statistical treatment of the data. The original vector distribution contains three correlated variates, d_{j1}, d_{j2}, and d_{j3}. None of these variates can be treated independently, and none of the variates has any apparent geological meaning when considered alone. By referring the vectors to the principalaxes, three uncorrelated variates t_{j1}, t_{j2}, and t_{j3} are obtained. Each uncorrelated variate has direct geological significance and can be treated independently of the other two. One variate measures folding in the direction of greatest buckling; another measures folding parallel to the main fold axis; the third measures change in attitude in successive layers in a direction normal to the form surface or mean foliation plane.

The original three-dimensional vector distribution can be regarded as broken down into three uncorrelated scalar distributions, by rotation to a position where covariance terms vanish. Statistical methods can be applied much more readily to the uncorrelated components than to the original vectors. For instance higher moments can be calculated for each of the transformed

variates. Of the two moments calculated so far, the first moment, or mean, measures the position of the fold in space. The second moments about the principal axes measure the tightness of folding in different directions. The third and fourth moments measured about the principal axes measure the relative slope of fold limbs and the relative abundance of unusually steep slopes on the fold limbs.

Skewness and kurtosis:

The second moment about the i-th principal axis was calculated as $m_{2i} = \sum_{j=1}^{n} (t_{ji}^2)/n$, where i = 1, 2, or 3. The third and fourth moments are similarly defined.

The third moment = $m_{3i} = \sum_{j=1}^{n} (t_{ji}^3)/n$

The fourth moment = $m_{4i} = \sum_{j=1}^{n} (t_{ji}^4)/n$

There are advantages, (see Yule and Kendall, 1958) in using the skewness, $\left(\frac{m_{3i}}{m_{2i}^{3/2}}\right)$ and the kurtosis $\left(\frac{m_{4i}}{m_{2i}^{2}}\right)$ in place of the third and fourth moments. The skewness and kurtosis are both pure numbers, unaffected by the unit of measurement. Skewness and kurtosis were devised by statisticians for the description of unimodal frequency distributions. The justification for using them with frequency distributions of direction cosines, which for structural data are likely to be strongly dimodal, is simply that they appear to measure properties of interest to the structural geologist.

The skewness measures theasymmetry of a frequency distribution. If the frequency distribution is that of cosines measured about the principal

2-axis, it is clear that where the angle between the 2-axis and the vector is less than 90 degrees, the cosine lies between 0 and 1. Where the angle is more than 90 degrees the cosine lies between 0 and -1. Since the 2-axis is at right angles to the mean, the first moment about the 2-axis is zero. If the limbs facing towards and away from the 2-axis have the same slope, then the third moment m_{31}, is also zero, (see figure 7). But if the limbs facing the 2-axis have the steeper slope, the positive cosines, cubed, will be larger than the negative cosines, cubed. The third moment and the skewness will therefore be positive. On the other hand, if the steeper limb faces away from the 2-axis, the third moment and the skewness will be negative.

Computation of skewness:

Example 4 : The skewness of fold 1 in figure 4 is to be estimated.

The cosines have the following values:

											Totals
$\cos\theta$	-5592	-3907	-2079	2250	4384	5592	4226	2250	-2419	-4695	0.0001
$\cos^3\theta$	-1749	-0596	-0090	0114	0843	1749	0755	0114	-0142	-1035	-0.0004

The second moment m_2 was calculated to be 0.1571, therefore

$$\text{the skewness} = \frac{-0.0004}{(.1571)^{3/2}} = 0.0000$$

In contrast to this, consider the two folds illustrated in figure 6. The second and third moments are calculated as above. The second moments are 0.1513 and 0.1586. The skewness is calculated to be 0.6407 for the fold with the steep limb facing p_2 and -0.8411 for the fold with the steep limb facing away from p_2. In this sense, the skewness measures the asymmetry of the folding, but the word "asymmetry" can be ambiguous in structural geology (see Turner and Weiss, 1963, page 122). The asymmetry of folding may be related to the fold having an oblique axial plane.

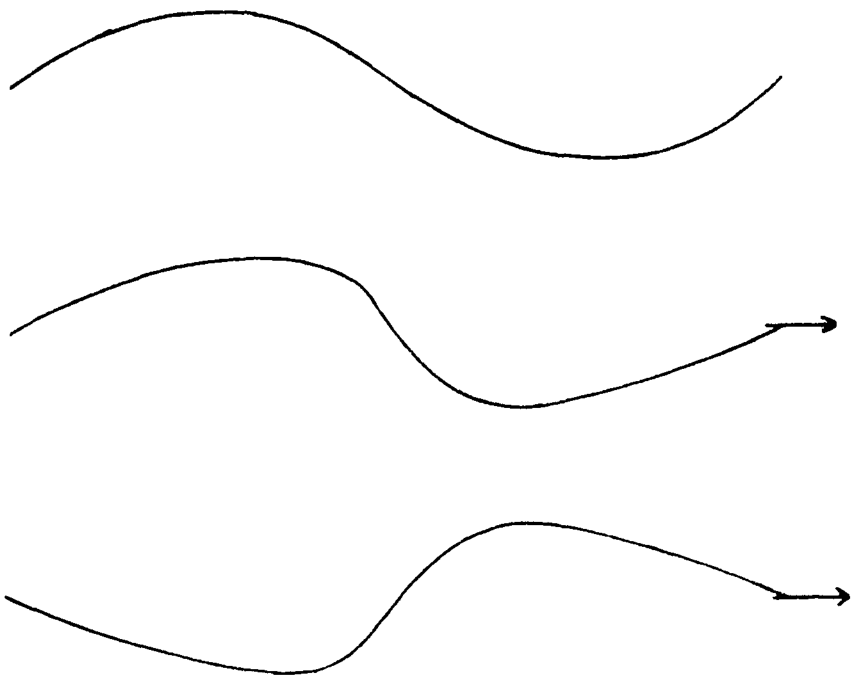


Figure 6 : Cross-sections of three folds. The upper fold has a skewness of 0.0000, the center fold has a skewness of 0.6407, the lowest fold has a skewness of -0.8411. The direction cosines are measured about the principal 2-axis, directed horizontally toward the right of the page.

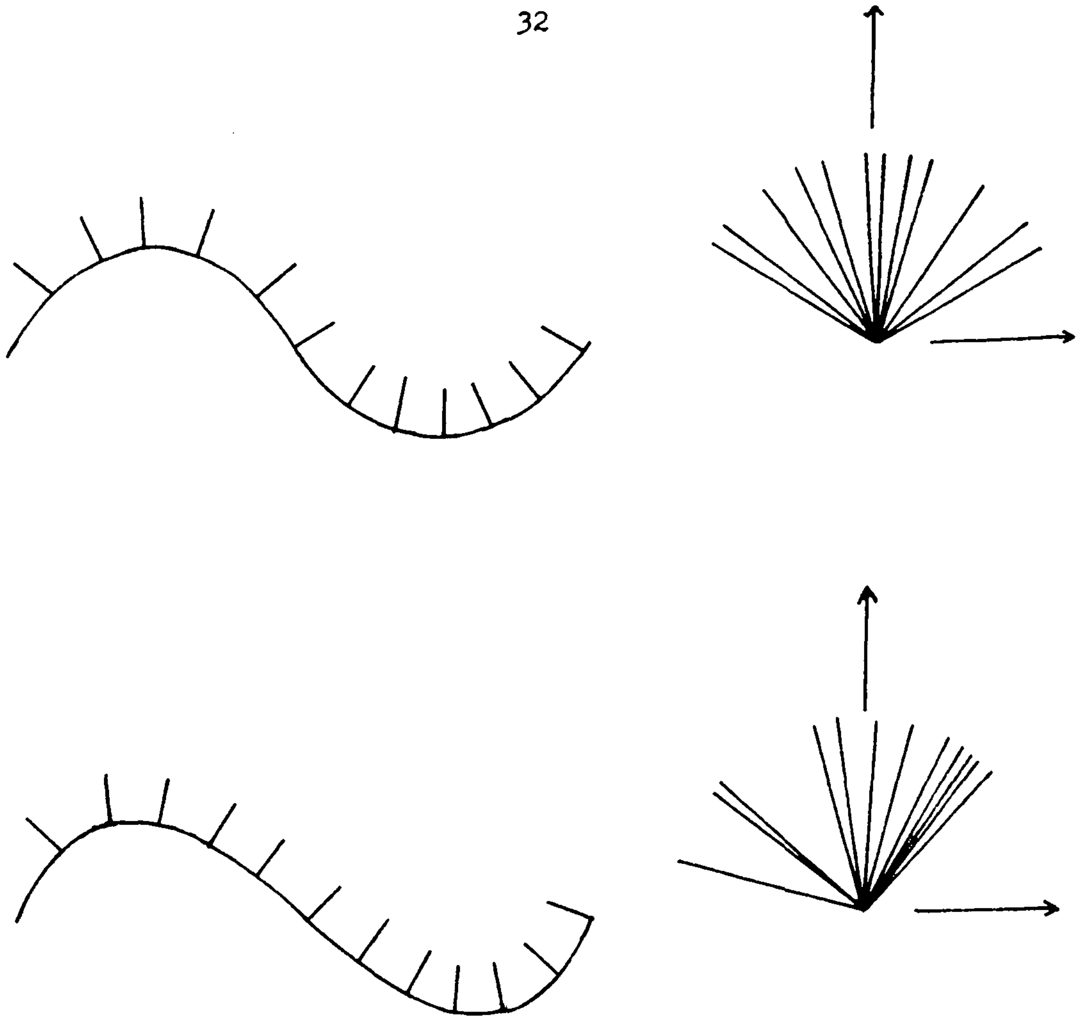


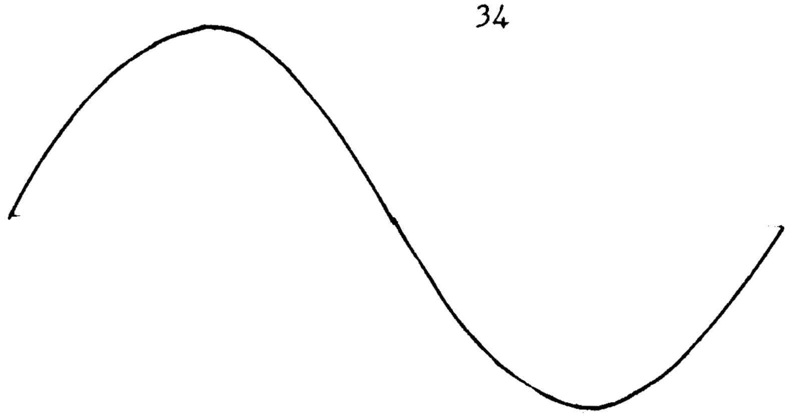
Figure 7 : Cross-sections of two folds (on right), the upper diagram illustrating a symmetrical fold, the lower diagram an asymmetrical fold. The diagrams on the left show distributions of vectors normal to the bedding planes. The mean vector is vertical in both folds, the principal 2-axis is horizontal and directed to the right of the page.

The kurtosis of a normal frequency distribution has the value 3. The fourth moment is more affected by extreme values than is the second moment. As a result the kurtosis is larger where there are more extreme values. A fold with rounded profile has a higher proportion of extreme values than does a fold with angular crests and flat limbs, and the kurtosis is correspondingly higher. Kurtosis can thus be used to measure the shape of a fold. The procedure for computing the kurtosis is similar to that for calculating the skewness, and an example of the computation does not appear to be necessary. However, it may be useful to illustrate moments for a few folds of different shapes. This is done in figure 8.

<u>Matrix methods used to describe operations of stretching and rotation</u>:

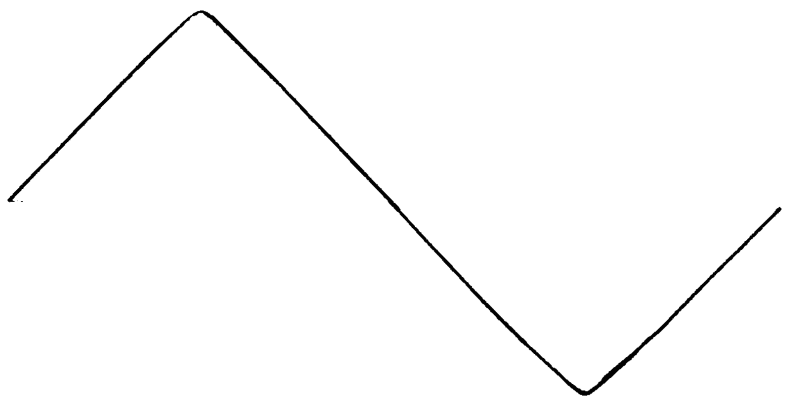
In example 3 above, the use of matrix multiplication as an algebraic representation of rotation was considered at some length, for the usefulness of this procedure in structural geology goes far beyond finding principal axes. Its importance is enhanced by the fact that stretching and reflection, see Cohn, 1961, page 36, are two other geometrical operations which can be represented by matrix multiplication. Matrices representing different operations performed in succession can be combined into one matrix representing the total effect of the sequence of operations, by multiplying the several matrices in their correct order, (see Harman, 1960, page 53).

Example 5 : A bedding plane $\underline{d}_1$ with direction cosines (.8743, -.1229, .4514) is situated 400 yards south, 50 yards east, and 20 feet above a reference point in the core of a fold. The matrix $\underline{E}$ for the fold is as in example 3. The dimensions of a number of green spots in the red shale were measured, and the conclusion was reached that the spots were



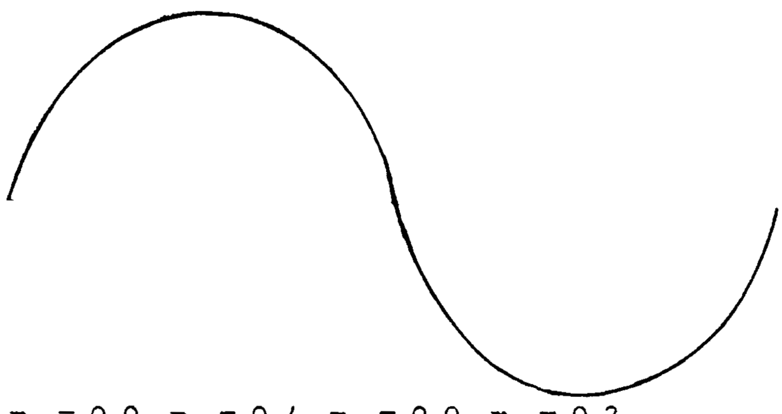
$m_1 = 0.0$, $m_2 = 0.4$, $m_3 = 0.0$, $m_4 = 0.3$,

skewness = 0.0, kurtosis = 1.3.



$m_1 = 0.0$, $m_2 = 0.5$, $m_3 = 0.0$, $m_4 = 0.3$,

skewness = 0.0, kurtosis = 1.0.



$m_1 = 0.0$, $m_2 = 0.4$, $m_3 = 0.0$, $m_4 = 0.3$,

skewness = 0.0, kurtosis = 1.7.

Figure 8: First four moments, skewness and kurtosis of a number of folds of different tightness, asymmetry, and shape.

$m_1 = 0.0$, $m_2 = 0.2$, $m_3 = 0.0$, $m_4 = 0.0$,

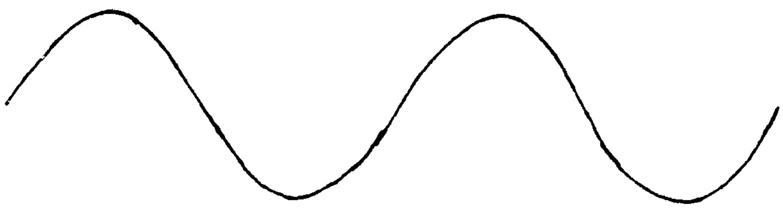
skewness = 0.0, kurtosis = 1.4.

$m_1 = 0.0$, $m_2 = 0.2$, $m_3 = 0.1$, $m_4 = 0.1$,

skewness = 0.7, kurtosis = 2.4.

$m_1 = 0.0$, $m_2 = 0.2$, $m_3 = -0.1$, $m_4 = 0.1$,

skewness = -0.8 kurtosis = 2.3.



$m_1 = 0.0$, $m_2 = 0.5$, $m_3 = 0.0$, $m_4 = 0.3$,

skewness = 0.0, kurtosis = 1.2.

elongated in the plane defined by the 1- and 2-axes. On average, the dimensions of the spots parallel to the 1-, 2-, and 3-axes were in the ratio 0.7 : 1 : 1.5. On the hypothesis that these dimensions reflect compression of spherical spots after buckling was complete, find the attitude and location of the bedding plane after buckling but before compression.

From the above information, $\underline{d}_1 = [.8743, -.1229, .4514]$, $\underline{g}_1 = [1200, 150, 20]$ and from example 3,

$$\underline{E} = \begin{bmatrix} .4310 & .8503 & .3020 \\ .2075 & -.4193 & .8840 \\ .8782 & -.3182 & -.3571 \end{bmatrix}$$

Hence, $\underline{t}_1 = \underline{d}_1.\underline{E} = (.7477, .6513, -.0058)$,

and $\underline{s}_1 = \underline{g}_1.\underline{E} = (566, 951, 488)$.

The dimensions of the reduction spots parallel to the 1, 2, and 3-axes have the ratio 0.7 : 1 : 1.5. On the assumption that they were originally spherical, the ratios before stretching were 1 : 1 : 1. The stretching apparently had the effect $x_{j1} = 0.7(y_{j1})$; $x_{j2} = y_{j2}$; and $x_{j3} = 1.5(y_{j3})$, where $\underline{x}_j$ gives the ratio of the diameters of the spot after stretching, and $\underline{y}_j$ gives the ratios of the diameters before stretching. The effect of the stretching can thus be described by the equation:

$\underline{x}_j = \underline{y}_j.\underline{E}_1$, where $\underline{E}_1$ is the matrix $\begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$

and $\underline{y}_j = \underline{x}_j.\underline{E}_1^{-1}$, where $\underline{E}_1^{-1}$ is the inverse matrix of E_1, (see Harman, 1960, chapter 3).

Since $\underline{E}_1$ is a diagonal matrix,

$$\underline{E}_1^{-1} = \begin{bmatrix} 1/0.7 & 0 & 0 \\ 0 & 1/1 & 0 \\ 0 & 0 & 1/1/5 \end{bmatrix} = \begin{bmatrix} 1.4286 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.6666 \end{bmatrix}$$

The ratios of the terms on the diagonal of $\underline{E}_1^{-1}$ are fixed, but their absolute values are not. The movement, relative to the reference point in the fold core, of the location $\underline{s}_1$ due to stretching and compression can be examined on the assumption that there has been no volume change in the rock due to compression. With this assumption, it follows that the product of the diagonal terms of $\underline{E}_1^{-1}$ is unity. Dividing each term by (1.4286 x 1 x 0.6666),

$$\underline{E}_1^{-1} = \begin{bmatrix} 0.7000 & 0 & 0 \\ 0 & 1.0501 & 0 \\ 0 & 0 & 1.4983 \end{bmatrix}$$

and $\underline{s}_1.\underline{E}_1^{-1}$ = [396, 999, 731], giving the distance along the principal axes from the origin to the subject location, after buckling but before compression. To express the coordinates of this location in terms of the original coordinate axes, the rotation $\underline{E}$ must be reversed. The inverse of the matrix $\underline{E}$ is therefore required, (see Harman, 1960). A rotation matrix such as $\underline{E}$ has the special property that the inverse is equal to the transpose, that is, $E^{-1} = E^T$, (see Harman, 1960). The inverse can thus be found by interchanging the rows and columns of $\underline{E}$.

$$\underline{E}^{-1} = \begin{bmatrix} .4310 & .2075 & .8782 \\ .8503 & -.4192 & -.3182 \\ .3020 & .8840 & -.3571 \end{bmatrix}$$

and (396, 999, 731).$\underline{E}^{-1}$ = (1241, 310, -231). That is, after buckling but before compression, the measured point was located 414 yards south, 103 yards west and 231 feet below the reference point in the core of the fold.

The procedure for finding the direction cosines of the bedding plane after buckling but before compression is similar to that outlined above. $\underline{t}_1 . \underline{E}_1^{-1}$ = (.5234, .6839, -.0087). The sum of squares of these numbers does not equal one, and they are not they are not therefore direction cosines. When converted to direction cosines the numbers become (.6077, .7941, -.0101). If a large number of direction cosines were being calculated a scalar multiple of the matrix $\underline{E}_1^{-1}$ could be used such that the sum of the diagonal terms would be unity. Multiplication of direction cosines by this matrix would yield direction cosines directly.

The direction cosines (.6077, .7941, -.0101) can be multiplied by $\underline{E}^{-1}$ to obtain direction cosines in terms of the original coordinate axes. (.6077, .7941, -.0101).$\underline{E}^{-1}$ = (.9341, -.2157, .2774).

While the above example is simplified for illustrative purposes, it is clear that by applying this procedure a number of times to a number of measurements, one could attempt to reconstruct the shape of the folded surface at various moments during the folding.

Statistical symmetry planes:

There are three principal planes in each distribution, namely the 12-plane, the 13-plane, and the 23-plane. Each is defined by the pair of principal axes which are parallel to it. Because the principal axes are chosen in such a way that there is no association between the distribution measured about pairs of axes, it follows that the distribution is as nearly possible symmetrically distributed about the principal planes. The principal planes may therefore be statistical symmetry planes, of the kind that would be found from a stereogram rather than from an individual fold, (see Turner and Weiss, page 122). The skewness of the distribution about each of the axes is a guide to whether or not the distribution is symmetrical about the principal plane. If symmetry is perfect, the skewness is zero.

A set of structural elements may have more than three planes of symmetry. The elements may for instance be randomly distributed on a uniform pattern over the surface of a sphere. The population would then have a center of symmetry. This would be clearly indicated by each of the three second moments having approximately the same value. Similarly, an axis of symmetry would be indicated by two of the variances having about the same value. An axis of symmetry might be present, for instance, in a set of bedding planes where these were folded into a series of nearly circular domes and basins. Two of the principal axes would then be indeterminate, since no definite fold axis would exist.

A conical fold would have less than three planes of symmetry as defined by Turner and Weiss (1963). It may therefore be advisable to examine the possibility that the distribution of vectors under consideration might be arranged as a cone rather than as a cylinder. This can be done using a least-squares method, (see Yule and Kendall, 1958, or Dixon and Massey, 1957, page 193). The axis of the cone and the apical angle of the cone are calculated. If the distribution is in fact cylindrical, the axis of the cone will coincide with one of the principal axes, and the apical angle will be zero.

<u>Cylindrical folds</u>:

The method of computing the axis of a conical fold depends on the fact that the cosine of the angle between two vectors with direction cosines $[d_{11}, d_{12}, d_{13}]$ and $[d_{21}, d_{22}, d_{23}]$ is equal to $d_{11} \cdot d_{21} + d_{22} \cdot d_{22} + d_{13} \cdot d_{23}$ (see Cohn, 1961, page 22). If a number of vectors $\underline{d}_j$ are arranged on the surface of a cone with an unknown axis $\underline{a}$, the cosine of the angle between the cone axis and the j-th vector is $a_1.d_{j1} + a_2.d_{j2} + a_3.d_{j3}$, a scalar quantity which can be denoted k_j. The quantity k_j is equal to a constant, k, for all the vectors of the distribution, apart from residual deviations of the form $(k_j - k)$. The best fit value of $\underline{a}$, namely the value such that the sum of squares of deviations, $\sum_{j=1}^{n} (k_j - k)^2$, is a minimum, can be found by the method of least squares.

A parallel approach to the problem of finding a cone axis can be taken. Geometrically, a distribution of vectors can be represented by lines of unit length radiating from the origin, 0. The ends of the lines are points which are arranged on the surface of a sphere, and which can be plotted on a stereogram. The position of each point in space can be described in terms of conventional coordinate axes, by the direction cosines of the vector, (see figure 1). The points are likely to lie on or near a plane. If the vectors are radii of a cylinder, the points lie on a plane passing through the origin, and would plot near a great circle on a stereogram. If the vectors are arranged in a conical pattern, the points lie on a plane which does not pass through the origin, and would plot near a small circle on a stereogram.

The plane on which the points lie can be represented by the equation: $a_1.d_{j1} + a_2.d_{j2} + a_3.d_{j3} - k = 0$, where k is the distance from the plane to the origin. The plane passes through the origin, and k is zero, if the distribution is cylindrical. If the distribution is conical, k is equal to the cosine of the apical angle of the cone. Three quantities in the above equation are measured, namely d_{j1}, d_{j2}, and d_{j3}. The equation can be written in three different ways, each with a different observation on the left-hand side:

$$d_{j1} = -\frac{a_2}{a_1} \cdot d_{j2} - \frac{a_3}{a_1} d_{j3} + \frac{k}{a_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

$$d_{j2} = -\frac{a_1}{a_2} \cdot d_{j1} - \frac{a_3}{a_2} d_{j3} + \frac{k}{a_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

$$d_{j3} = -\frac{a_1}{a_3} \cdot d_{j1} - \frac{a_2}{a_3} d_{j2} + \frac{k}{a_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

There are thus three different ways in which one can minimize the sum of squares of deviations between observations (on the left of the equation) and the calculated values on the right of the equation. Three best-fit planes can be found for the points. Each of the best-fit planes has a minimum value for the sum of squares of distances from observed points to the plane. But, for one plane the distances are measured parallel to the 1-axis; for another parallel to the 2-axis; for the third parallel to the 3-axis. It is convenient computationally to calculate all three planes, and it is of some interest to see how closely the planes coincide. The average of the three planes can then be used to derive an estimate of the orientation of the cone axis.

A better procedure, particularly if the distribution is referred to the principal axis, is to use the plane for which the sum of squares of deviations is a minimum. This indicates that the axis along which the deviations are measured is the axis most nearly perpendicular to the best-fit plane. Aberrant values will have least effect on the position of the best-fit plane, if it is the plane for which the sum of squares of deviations is a minimum.

Example 6 : Find the axis and apical angle of the best-fit cone for the foliation planes plotted as poles of stereogram 1 of figure 9.

$$\underline{D}^T = \begin{bmatrix} -.6100 & -.5534 & -.4807 & -.3338 & -.1761 & -.1069 & -.0502 \\ -.3812 & -.2699 & -.1378 & .1701 & .4839 & .6063 & .7176 \\ .6947 & .7880 & .8660 & .9272 & .8572 & .7880 & .6947 \end{bmatrix}$$

The best-fit cone can be calculated directly from these measurements, and the direction cosines of the axis found in terms of the original reference axes. But it is of interest to see how far the axis of the best-fit cone deviates from the principal axis. The data matrix is therefore transformed to refer to the principal axes, and the best-fit cone calculated in terms of the transformed data matrix $\underline{T}$.

The covariance matrix derived from $\underline{D}$ is found to be:

$$\underline{C} = \begin{bmatrix} .1523 & .0293 & -.2651 \\ .0293 & .1976 & .1360 \\ -.2651 & .1360 & .6502 \end{bmatrix}$$

The eigenvalues of $\underline{C}$ are: 0.7866; 0.2119; and 0.0009.

The corresponding eigenvectors, arranged as columns of a matrix, are

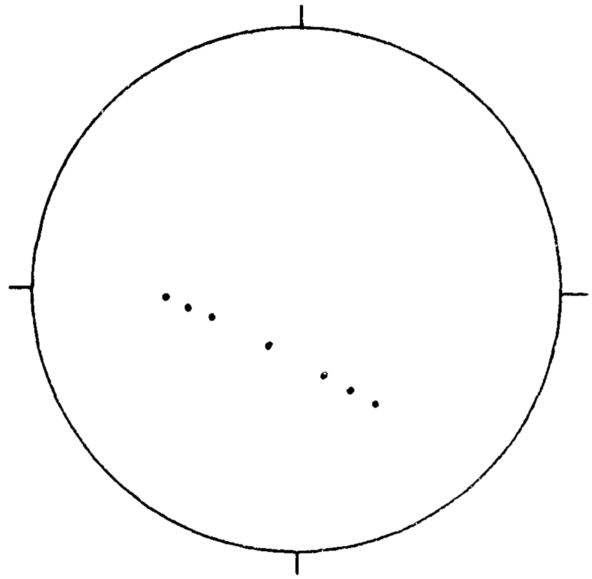
$$\underline{E} = \begin{bmatrix} -.3900 & .4490 & -.8041 \\ .1891 & .8936 & .4072 \\ .9013 & .0067 & -.4334 \end{bmatrix}$$

Hence,

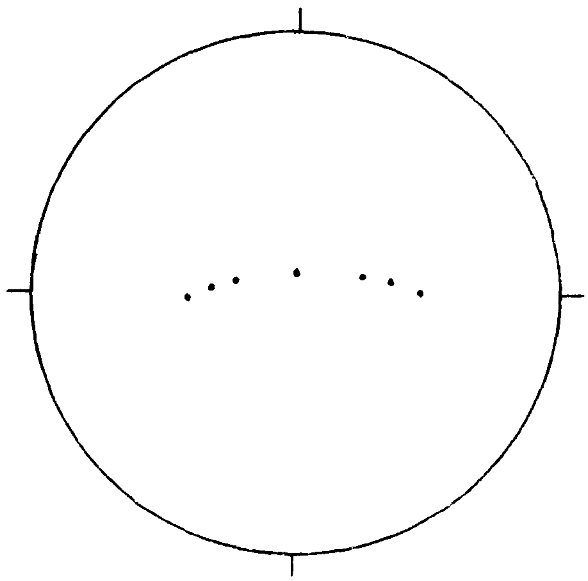
$$\underline{T}^T = \begin{bmatrix} .7919 & .8750 & .9419 & .9980 & .9328 & .8666 & .7814 \\ -.6099 & -.4844 & -.3332 & .0083 & .3591 & .4991 & .6234 \\ .0342 & -.0113 & -.0449 & -.0642 & -.0329 & -.0087 & .0315 \end{bmatrix}$$

The above points are plotted on stereogram 2, figure 9.

The sum of squares of deviations between the observed t_{j1}, and t_{j1} calculated from equation (3) above, is



Stereogram 1, showing poles to seven foliation planes.



Stereogram 2, showing poles to the same foliation planes, rotated to refer to the principal axis.

Figure 9: A best-fit cone is computed, in example 6, using a least-squares method.

$$\sum_{j=1}^{n}[t_{j1} - (-\frac{a_2}{a_1} t_{j2} - \frac{a_3}{a_1} t_{j3} + \frac{k}{a_1})]^2 = G$$

Putting $b_1 = -\frac{a_2}{a_1}$; $b_2 = -\frac{a_3}{a_1}$; and $b_0 = \frac{k}{a_1}$, the expression becomes

$$\sum_{j=1}^{n}(t_{j1} - b_0 - b_1.t_{j2} - b_2.t_{j3})^2 = G$$

These equations are similar to those which arise in trend surface analysis, (see Whitten, 1963). The unknowns can be found in a similar manner. The minimum value of the variable G is required. It is a condition of G being a minimum that $\frac{\partial G}{\partial b_0} = \frac{\partial G}{\partial b_1} = \frac{\partial G}{\partial b_2} = 0$.

$$\frac{\partial G}{\partial b_0} = \Sigma 2\ (t_{j1} - b_0 - b_1 t_{j2} - b_2 t_{j3})\ (-1) = 0$$

$$\frac{\partial G}{\partial b_1} = \Sigma 2\ (t_{j1} - b_0 - b_1 t_{j2} - b_2 t_{j3})\ (-t_{j2}) = 0$$

$$\frac{\partial G}{\partial b_2} = \Sigma 2\ (t_{j1} - b_0 - b_1 t_{j2} - b_2 t_{j3})\ (-t_{j3}) = 0$$

These expressions simplify to the normal equations:

$$b_0.N + b_1.\Sigma t_{j2} + b_2.\Sigma t_{j3} = \Sigma t_{j1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$b_0.\Sigma t_{j2} + b_1.\Sigma(t_{j2})^2 + b_2.\Sigma(t_{j2}.t_{j3}) = \Sigma(t_{j2}.t_{j1}) \quad . \quad . \quad . \quad . \quad (7)$$

$$b_0.\Sigma t_{j3} + b_1.\Sigma(t_{j2}.t_{j3}) + b_2.\Sigma(t_{j3})^2 = \Sigma(t_{j3}.t_{j1}) \quad . \quad . \quad . \quad . \quad (8)$$

where N is the number of measurements in the data.

The following values are calculated from the T matrix of example 6:

$N = 7$; $\Sigma t_{j1} = 6.1876$; $\Sigma t_{j2} = 0.0624$; $\Sigma t_{j3} = -0.0963$

$\Sigma(t_{j1})^2 = 5.50761$; $\Sigma(t_{j2})^2 = 1.48439$; $\Sigma(t_{j3})^2 = 0.00959$

$\Sigma(t_{j1}.t_{j2}) = 0.04223$; $\Sigma(t_{j1}.t_{j3}) = -0.10278$; $\Sigma(t_{j2}.t_{j3}) = 0.00252$

These values can be substituted in normal equations (6) to (8), giving

$$7.0000b_0 + 0.0624b_1 - 0.0963b_2 = 6.1876 \quad \ldots \quad (9)$$

$$0.0624b_0 + 1.4844b_1 + 0.0025b_2 = 0.0422 \quad \ldots \quad (10)$$

$$-0.0963b_0 + 0.0025b_1 + 0.0096b_2 = -0.1028 \quad \ldots \quad (11)$$

Multiply (10) by (7/.0624)

$$7.0000\, b_0 + 166.5185b_1 + 0.2804b_2 = 4.7340 \quad \ldots \quad (12)$$

Multiply (11) by (7/.0963)

$$-7.0000b_0 + .1817b_1 + 0.6978b_2 = -7.4725 \quad \ldots \quad (13)$$

Add (12) and (13)

$$166.7002b_1 + 0.9782b_2 = -2.7385 \quad \ldots \quad (14)$$

Subtract (9) from (12)

$$166.4561b_1 + 0.3767b_2 = -1.4536 \quad \ldots \quad (15)$$

Multiply (15) by 1.00147

$$166.7008b_1 + 0.3773b_2 = -1.4557 \quad \ldots \quad (16)$$

Subtract (14) from (16)

$$0.6009b_2 = -1.2828$$

$$b_2 = -2.1348$$

Substitute in (16)

$$b_1 = -.00136$$

Substitute in (1)

$$b_0 = 0.85458$$

$$b_1 = -\frac{a_2}{a_1} = -.0014; \qquad b_2 = -\frac{a_3}{a_1} = -2.1348$$

Hence $a_1 : a_2 : a_3 = 1.0000 : 0.0014 : 2.1348$

Dividing by the square root of the sum of squares to express $\underline{a}$ as direction cosines, $a_1 = 0.4242$; $a_2 = -0.0006$; $a_3 = -0.9056$. The angles of which these are cosines are 65°, 90°, and 166°. The constant $k = b_0 \times a_1 = .3625$, which is the cosine of 68°. Due to the small number of decimal places retained in the calculation, the results are only approximate.

An analogous procedure to that described above can be used to determine the coefficients when the sum of squares of deviations parallel to the 2- and 3-axes is minimized. Again using desk calculator accuracy only, the results are

$\underline{a} = (0.4208, 0.0016, -0.9071)$ and $k = 0.3350$

$\underline{a} = (0.4208, 0.0240, -0.9068)$ and $k = 0.3593$

Taking the average values of the three sets of results:

$\underline{a} = (0.4258, 0.0262, -0.9065)$ and $k = 0.3523$. The angles of which these are cosines are (64°, 91°, 155°) and 69° for the apical angle. These differ by up to 2° from the angles calculated on a stereogram, because of rounding errors in the calculation.

Computations for subsurface data:

Structural information is frequently obtained in the form of elevations of a particular horizon, rather than in the form of orientation of structural features measured at a number of points. It is possible, from the location and elevation of measured points on the given horizon, to calculate the orientations of a set of lines which join each pair of measured points. The set of lines can be treated as before, although they refer to lines parallel to the surface rather than to poles to the surface. The geological significance of the principal axes computed in this manner is somewhat different from the significance of principal axes calculated from poles.

Variance of measurements about principal axis	Vectors parallel to a surface	Vectors normal to planar features
Maximum	Mean vector	Mean pole
Minimum	Fold axis	Fold axis

The calculation of direction cosines for subsurface data is illustrated in the following example.

Example 7 : The elevation above sea level of four points on a marker horizon at locations measured south and east of an origin (0,0) are:

	South of 0	East of 0	Elevation
1	0 feet	0 feet	1230 feet
2	-70 feet	3462 feet	1608 feet
3	2560 feet	410 feet	1534 feet
4	1892 feet	2807 feet	1527 feet

The direction cosines of the six lines which join pairs of the points are to be calculated.

The distances between points in a south, an east, and a vertical direction determine the direction ratios of the lines joining the points. The direction cosines can be found directly from the direction ratios, by dividing each of the direction ratios by the root of the sum of squares, (see Cohn, 1961, or Harman, 1960). The calculations are summarized in table 3.

<u>Quantitative description of minor folds</u>:

It may be possible to make large numbers of measurements of bedding plane attitudes on folds that are small enough to be completely visible within an outcrop. From the measurements, estimates of orientation, tightness, asymmetry, and shape of the minor folds could be derived by

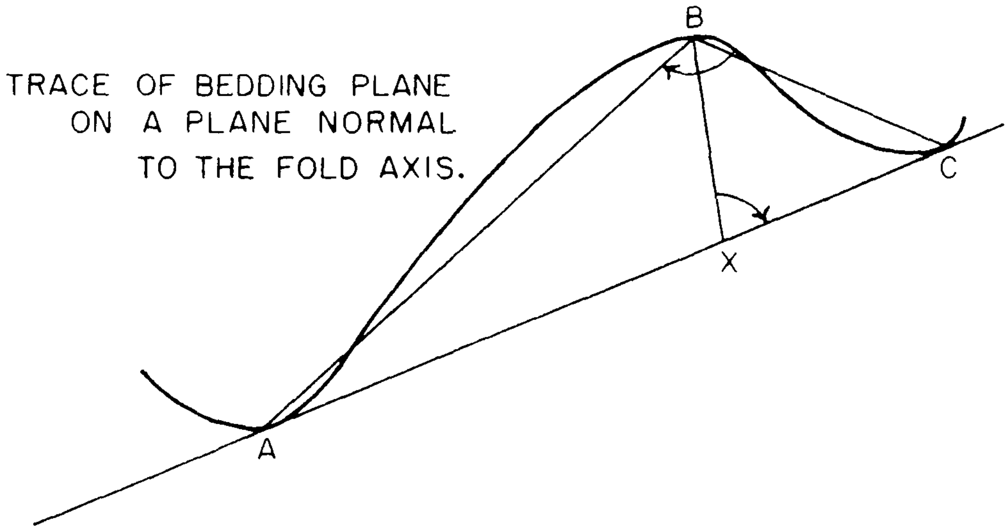
TABLE 3

Points joined by line	Distance between points South	East	Vertical	Sum of squares (S.Sq.)	Reciprocal of root of S.Sq.	Direction cosines of line		
1 and 2	-70	3462	378	12133200	.0002871	-.0201	.9940	.1085
1 and 3	2560	410	304	6814100	.000383	.9805	.1570	.1164
1 and 4	1892	2807	297	11547100	.000294	.5563	.8261	.0874
2 and 3	2630	3052	-74	16237100	.000248	.6528	.7575	-.0184
2 and 4	1962	-655	-81	4285000	.000483	.9476	-.3164	-.0391
3 and 4	-668	2397	-7	6191900	.000402	-.2685	.9634	-.0028

Calculation of direction cosines describing the orientation of a series of lines joining pairs of points, the elevation and location of which is known. The direction cosines can be used in the same way as direction cosines calculated from orientation measurements.

the procedures described above.

If profiles of the folded surfaces can be seen, however, it is much more convenient to make direct measurements of the features of the fold. The convenience may outweigh the fact that the measures established for the minor folds cannot be compared directly with those calculated for the major folds by the computer program. Figure 10 shows one manner in which estimates of the properties of the fold could be made directly from the fold profile.



For the folded bedding plane between A and C:

AC is the mean attitude of the bedding;

the angle ABC between the mean attitudes of the limbs can be used as a measure of the tightness of folding;

the angle BXC between the bisector of angle ABC, namely BX, and the mean attitude, AC, can be used as a measure of asymmetry of the fold;

the ratio of the length of the bedding plane between A and C to the length of the straight lines AB + BC can be used as a measure of the shape of the fold.

Figure 10 : Illustrating possible quantitative measures for minor folds

SECTION 2 : THE COMPUTER PROGRAM

Notation:

A listing of the program, written in Fortran IV for the IBM 709, is given in the appendix. An attempt has been made to use a notation in the program similar to that used in the body of this report. A list of some of the symbols follows:

Variable referred to:	Algebraic symbol:	Fortran name:
Covariance matrix	$\underline{C}$	COVMAT
Element of covariance matrix in i-th row and j-th column	c_{ij}	COVMAT(I,J)
Direction cosine of j-th measurement about the i-axis	d_{ji}	DIRCOS(J,I)
Rotation matrix of eigenvectors	$\underline{E}$	EIGMAT
Element of above matrix in i-th row and j-th column	e_{ij}	EIGMAT(I,J)
I-th eigenvalue, arranged in order of decreasing magnitude		EIGVAL(I)
Geographical coordinate of j-th measurement, measured parallel to i-th axis	g_{ji}	GCOORD(J,I)
I-th moment measured about the j-th axis	m_{ij}	
Code number (from 1 to 9) indicating the category of the i-th factor to which the j-th measurement belongs		LIST(J,I)
Name, in alphameric characters, of the k-th category of the i-th factor		NAME(K,I)

Coordinate of j-th measurement, measured parallel to i-th principal axis	s_{ji}	SCOORD(J,I)
Direction cosine of j-th measurement transformed to refer to the i-th principal axis	t_{ji}	TRACOS(J,I)

<u>The output from the subroutines:</u>

Because the user must first decide which program to use, the output of the program is described before the arrangement of data input. To provide flexibility, the program has been written as a number of subroutines, which can be combined in different ways, according to the complexity of the geological problem. A choice of five main programs is provided in the program listing in the appendix. The user may choose whichever of the programs is felt to be most appropriate to the data, and the main program automatically calls the required subroutines, see table 4. As far as possible, the subroutines have been written in such a way that each one represents a step in the geological investigation. The user should thus be able to write a new main program with little difficulty, if none of the five is appropriate.

The subroutines which provide printed output are:

ANOV Computes and prints the mean and variance of the distribution of direction cosines about any one of the principal axes, and the mean and variances of subsets of the distribution, the subsets being the categories of the various factors specified by the user in the array LIST. The purpose of the subroutine is to indicate whether significant differences in the orientation or amount of folding occur between different categories, and to indicate the factors with which the greatest degree of variation is associated.

TABLE 4

List of subroutines required with each main program, and list of subroutines which provide the output

Main program	Subroutines required	Subroutines contributing printed output
PROGV1	AXES, CONFIT, ENISOC, EQUATE, SOLSP, WULFF	AXES, CONFIT, WULFF
PROGV2	As above, and MOMENT, SPIN	AXES, CONFIT, MOMENT, SPIN, WULFF
PROGV3	As above, and HOWBIG, SEKSHN	AXES, CONFIT, HOWBIG, MOMENT, SEKSHN, SPIN, WULFF
PROGV4	As above, and ANOV, READIN, SELECT	ANOV, AXES, CONFIT, HOWBIG, MOMENT, SEKSHN, SPIN, WULFF
PROGS5	As above, and SCALAR. SEKSHN is not required.	ANOV, AXES, CONFIT, HOWBIG, MOMENT, SPIN, WULFF

AXES Computes principal axes of the distribution of direction cosines. The subroutine is based on a library program HDIAG, written by Corbato and Merwin. The principal axes are printed out as a matrix of eigenvectors which has as its columns the direction cosines of the principal axes. With each eigenvector is printed the corresponding eigenvalue, which is equal to the variance of the distribution about that axis. The trend and plunge of each of the principal axes is also printed out, the transformation from direction cosines to trend and plunge being performed by subroutine ENISOC.

CONFIT Computes and prints the axes and apical angle of the conical surface on which the measured vectors most nearly lie. The root-mean-square deviation from the conical surface is printed as an indication of how well the conical surface fits the data. If instructed by the main program, CONFIT prints out each of the measurements which deviate from the conical surface by more than 1.5 root-mean-square deviations. The deviant measurements may indicate the presence of errors in the data, or simply unusual orientations which may require further investigation. Again ENISOC is used to convert from direction cosines to trend and plunge. Subroutine SOLSP (a Northwestern University library program, written by I. Waye) is used to solve the simultaneous equations which arise in the computation.

HOWBIG Prints a histogram showing the relative frequency of folds of different sizes, parallel to any one of the principal axes.

MOMENT Prints a histogram of frequency distribution of direction cosines measured about any one of the principal axes. It also computes and prints the values of the first four moments, and the skewness and kurtosis of the distribution about that axis.

SEKSHN Supplies computed values of the slope of intersection of each measurement with each of the principal planes, and the coordinates of each measurement in terms of the principal planes. The output from this subroutine can be used directly to draw cross-sections parallel to any two of the principal axes.

SPIN If so instructed by the main program, prints out each of the measurements which deviates considerably from the mean value. The deviant measurements may indicate the presence of errors in the data, or of unusual orientations which deserve further investigation.

WULFF Prints a scatter diagram showing the distribution of vectors plotted on a Wulff stereogram. Each number printed on the stereogram indicates the number of vectors falling within the rectangle on which the number is printed. The vectors represent either the orientation of lineation or the orientation of poles to planar features. The output can be used as an overlay on a standard 20-centimetre Wulff net.

Choice of main program:

The programs handle two main types of measurement. The data may be in the form of orientations measured at a number of points on a series of surfaces, such as dip and strike measurements collected by the field geologist. Alternatively, the data may be in the form of measurements of elevations of points of known location on one surface, or series of surfaces, such as elevations of marker horizons obtained in subsurface geology. All the subroutines can be used with data in either form, except for subroutine SEKSHN, which is limited to orientation data.

The main programs PROGV1 to PROGV4, listed in the appendix, handle progressively more complex orientation data. PROGS5 can be used with scalar data, such as elevations. See table 4. PROGV1 uses azimuth and dip measurements only. If the data are expressed as dip and strike, a correction card can be inserted as mentioned on a comments card within the program. This automatically converts from strike to azimuth, provided that the strike is measured in degrees east of north, and is counter-clockwise from the direction of dip. The program prints out any identification in columns 1 to 12 of the data cards; the azimuth and amount of dip as it is read in; and the computed direction cosines for each measurement. Subroutines AXES, CONFIT, and WULFF (see section above on "The output of the subroutines") provide the remainder of the output from this program.

PROGV2 is similar to PROGV1, except that it also calls subroutines SPIN and MOMENT, the latter for each of the three principal axes in turn. There is considerably more output than for PROGV1.

PROGV3 requires geographical coordinates for each measurement as well as azimuth and dip. The distances south, east, and above some selected origin must be recorded, with each distance measured in the same units. Direction cosines transformed to refer to the principal axes can be obtained as output by inserting cards as indicated on comments cards within the program. Subroutines HOWBIG and SEKSHN are called in addition to those used in PROGV2.

PROGV4 is more flexible in the arrangement of data which it accepts (see following section on "The preparation of data input"). Along with each measurement of orientation and location, the field geologist may have recorded a number of factors such as lithology, stratigraphic horizon, type of structural feature, metamorphic grade, confidence in the accuracy of measurement, type of sample, density of sampling, etc. The categories of each factor can be coded as integer numbers from 1 to 9 for factors 1 to 6, and the code numbers can be punched on the data cards, and read in by this program. Subroutine READIN controls the input of data to the computer, where the code numbers are stored as an array LIST. In addition to the subroutines used in PROGV3, ANOV is called by this program. Control cards following the data specify which subsets of the data are to be used with each subroutine. The subsets are specified by referring to the code numbers of LIST. Different subsets may be used on different runs of the program. Thus the first control card could specify that only measurements made in sandstone should be used in the computation of principal axes. The second control card could indicate that on the second run only measurements of cleavage should be used, etc.

PROGS5 is similar to PROGV4 except that it handles elevation measurements rather than orientation measurements. The height coordinate is used to record the elevation of a marker horizon. If a number of horizons have been measured they may be distinguished by assigning them different code numbers in the array LIST. It is possible to record a number (such as KB elevation of a well) on each card. The number is automatically subtracted from the height coordinate, before the elevation is stored in the computer.

The preparation of data input:

One data card is used for each measurement. All of the programs, as at present dimensioned, are limited to groups of 300 cards or less. The data should be arranged on the cards as follows:

PROGV1

Columns 1 - 6	Identification, such as project number.
Columns 6 - 12	Identification, such as item number.

The above columns may be left blank if desired.

Columns 31 - 36	Azimuth of dip, in degrees. A decimal point is assumed to follow column 33 unless punched elsewhere.
Columns 37 - 42	Amount of dip, in degrees. A decimal point is assumed to follow column 39 unless it is explicitly punched.

Any other information on the card is ignored by this program. Each group of data should be terminated by a card with 999000 punched in columns 31 - 36. The final group of data should be terminated by a card punched 999000 in columns 31 - 36 and 999000 in columns 37 - 42.

PROGV2

Arrangement of data as for PROGV1.

PROGV3

Arrangement of data as for PROGV1. In addition, the geographical coordinates of the point of measurement are recorded in columns 13 - 30. The distances south, east, and above a selected origin are recorded in columns 13 - 18, 19 - 24 and 25 - 30 respectively. A decimal point is assumed to follow the sixth digit in each of these six-digit fields, unless one is explicitly punched in a different position.

PROGV4

The data cards may be arranged as for PROGV3, or any other format may be used, provided that the format is described on the format card, and that the data is arranged in the same sequence as described for PROGV3, namely: identification, geographical coordinates (south, east, and vertical), azimuth and amount of dip. In addition, integer code numbers from 1 to 9 may be punched to indicate the category of each factor (1 to 6) into which each measurement falls. The code numbers are read into the array LIST.

Column 51	Category code number for factor 1.
Column 54	Category code number for factor 2.
Column 57	Category code number for factor 3.
Column 60	Category code number for factor 4.
Column 63	Category code number for factor 5.
Column 66	Category code number for factor 6.

Again, the listing may be in different columns if so specified on the format card. Table 5 and figure 11 illustrate the arrangement of data on cards for this program.

TABLE 5

Data abstracted from a geological map, arranged as in a field notebook:

Measurements from USGS sheet GQ 130, Wildwood Quadrangle, Tennessee, April, 1964.

Origin for geographical coordinates is northwest corner of map. Distances are measured in inches, height in feet above sea level.

Project	Item	Geographical coordinates			Measurement of dip		Factor 1	2	3	4	5	6
		South	East	Height	Azimuth (-90°)	Amount	Category code number					
GQ130	1	017.	013.	940'	59°	40°	1	3	7	1		
"	2	012.	043.	1000'	56°	85°	1	4	7	1		
"	3	015.	082.	1000'	44°	50°	1	4	4	1		
"	4	014.	103.	1000'	64°	15°	1	3	5	1		
"	.	.	.	.	.	.	.	.	.	.		

Explanation of category code numbers:

Category code number	Factor 1 Sample Type	Factor 2 Lithology	Factor 3 Stratigraphic horizon	Factor 4 Feature measured	Factor 5	Factor 6
1	Grid sample (3 inch squares)	Sandstone	Precambrian	Bedding plane		
2	Unusual measurements	Sandstone and shale	Lower Cambrian	Fault plane		
3		Shale	Lower Cambrian	Joint plane		
4		Carbonate	Middle Cambrian			
5		Evaporite	Upper Cambrian			
6			Lower Ordovician			
7			Middle Ordovician			
8						
9						

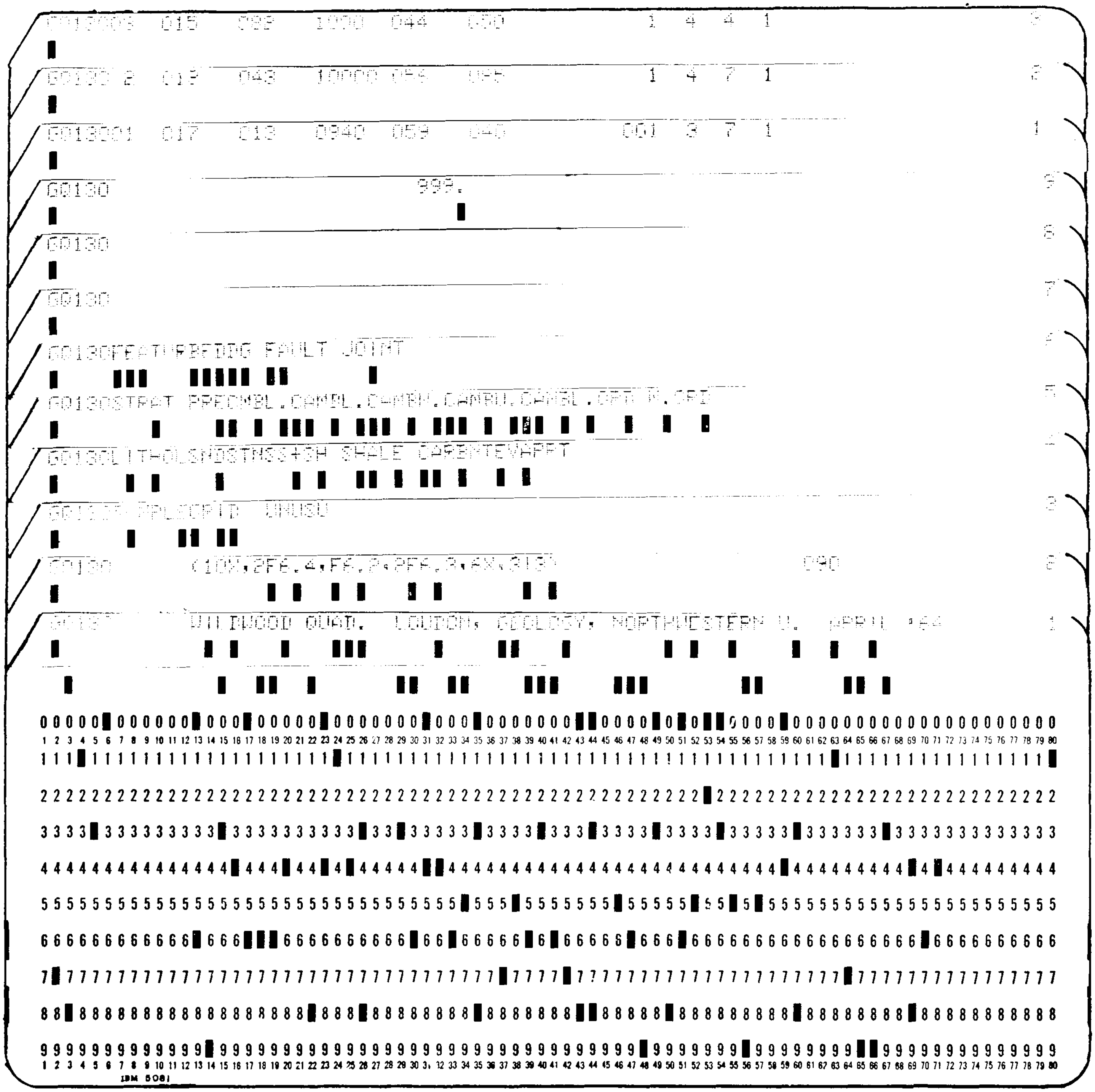


Figure 11: Arrangement of data for PROGV4. The data of table 5 are punched as an example.

The numerical data for PROGV4 must be preceded by nine cards. Any or all of the first eight may be left blank, or they can be punched as follows:

Card (1) Title card: Columns 13 - 72 may be punched with the date, project name and user's name. This information will be printed at the head of the output.

Card (2) Format and correction factor card: Columns 13 - 48 should contain a format statement, if the format suggested above has not been used.

Columns 49 - 54 : If a correction factor is punched here, it will be used to multiply the south and east geographical coordinates as they are read in. A decimal point is assumed to follow the sixth digit unless punched elsewhere.

Columns 55 - 60 : If a correction factor is punched here, it will be used to multiply the height coordinates as they are read in. A decimal point is assumed to follow the sixth digit unless punched elsewhere.

The two correction factors mentioned above can be used to correct for the scale of the map and to bring horizontal and vertical coordinates to the same scale.

Columns 61 - 66 : If a correction factor is punched here, it will be added to each of the measurements of azimuth of dip as they are read in. It can be used to correct a magnetic deviation from true north, or to convert from a strike measurement to an azimuth measurement. A decimal point is assumed to follow the sixth digit, unless punched elsewhere.

Card (3) NAME card for factor 1, (see PROGV4 in section on "Choice of main program" above).

Card (4) NAME card for factor 2.

Card (5) NAME card for factor 3.

Card (6) NAME card for factor 4.

Card (7) NAME card for factor 5.

Card (8) NAME card for factor 6.

The above NAME cards may contain the names, in alphameric characters, of the categories of the appropriate factor.

They are stored in the computer as the array NAME.

Columns 13 - 18	Name of category with category code number 1.
Columns 19 - 24	Name of category with category code number 2.
Columns 25 - 30	Name of category with category code number 3.
Columns 31 - 36	Name of category with category code number 4.
Columns 37 - 42	Name of category with category code number 5.
Columns 43 - 48	Name of category with category code number 6.
Columns 49 - 54	Name of category with category code number 7.
Columns 55 - 60	Name of category with category code number 8.
Columns 61 - 66	Name of category with category code number 9.

Card (9) Nines card indicating beginning of numerical measurement data.
Columns 31 - 36 : Should contain 999.0. If the azimuth of dip is punched elsewhere, according to another format, the 999.0 should be punched in the columns which would normally contain the azimuth of dip.

Change of format card: A card containing a new format, or new correction factors, similar to card (2) above, can be inserted anywhere in the numerical data. It should be immediately preceded and followed by a nines card similar to card (9). The cards that follow it will be read in according to the new format, and the new correction factors will be applied.

Nines card terminating data: The numerical measurement data should be terminated by a card punched 999000999000 in the columns used for azimuth and amount of dip (normally columns 31 - 42).

Control cards: The nines card terminating the measurement data should be immediately followed by one or more control cards.

Columns 13 - 72 : Code numbers are punched in successive three-digit fields. They are read into the array KONTRL, which can hold up to twenty code numbers. Each control card specifies the subsets of measurements to be used:

(1) For determination of the cone axis and apical angle.

(2) For determination of the principal axes.

(3) For calculation of the moments and cross-section data.

(4) to (9) Up to six subsets within which an analysis of variance is to be performed (subroutine ANOV).

The subsets are specified on the control card in sequence from (1) up to a maximum of (9). More than one three-digit word can apply to each subset. The beginning of each new subset is indicated by a non-zero digit in column 1 of the three-digit word. The category to be included in each subset is indicated by the number of the factor in column 2 of the three-digit word, and the category code number in column 3. The factor number and category code number correspond to those recorded for each measurement in the array LIST. The specified category, and only that category, is retained in the subset. If a minus sign is punched in column 1 of the three-digit word, the specified category is removed from the subset, and all other measurements are retained.

As an example of the arrangement of a control card, suppose that the following subsets are required for the data of table 5.

(1) For CONFIT, bedding plane measurements only, for all rocks of Cambrian age. Punch 141-31-36-37 in columns 13 - 24.

(2) For AXES, grid sample measurements only, for bedding plane measurements in rocks of Cambrian age. Punch 241011-31-36-37 in columns 25 - 39.

(3) For MOMENT and SEKSHN, all measurements of bedding planes. Punch 341 in columns 40 - 42.

(4) For ANOV, all bedding plane measurements of the grid sample. Punch 441011 in columns 43 - 48.

(5) For ANOV, all measurements. Punch 500 in columns 49 - 51.

(6) For ANOV, all grid measurements of bedding planes in the Cambrian. Punch 641011-31-36-37 in columns 52 - 66. If instead 641011032033034035 had been punched, it would not have been successful, since 032 indicates that only measurements of category 2 of factor 3 are to be retained in the subset. The following word 033 would then have retained only the members of the subset which belong to category 3 of factor 3. As this category has been removed by the preceding instruction, no measurements would remain in the subset.

After the computations are completed for the subsets specified on the first control card, the second control card is read, and the computations are repeated with the subsets that it specifies. The procedure is repeated for successive control cards. There must be at least one control card, but as many can be included as desired.

A blank card following the control cards indicates the end of the program. A card punched -99-99-99 in columns 13 to 21 indicates that another batch of data follows, beginning as before with card (1) containing data, project name and user's name.

Data for this program may be in the form of direction cosines rather than azimuth and amount of dip.

Columns 31 - 36 Direction cosines about the 1-axis.

Columns 37 - 42 Direction cosines about the 2-axis.

Columns 43 - 48 Direction cosines about the 3-axis.

Columns 43 - 48 are left blank when the measurements are azimuth and amount of dip, and this indicates to the computer that direction cosines have to be computed.

PROGV5

Arrangement of data as for PROGV4. The data must be in the form of elevation measurements (scalar quantities) not orientation measurements (vectorial quantities). The height coordinate in columns 25 - 30 indicates the elevation of a particular horizon. If measurements have been made on a number of horizons they can be distinguished by different category code numbers in array LIST. Since the program is designed for use with subsurface data, the azimuth columns (normally 31 - 36) can be used for a quantity, such as KB elevation of a well, that is to be subtracted from the height coordinate on the same card, before the measurement is stored in the computer. The columns used for the amount of dip, normally 37 - 42, should be left blank, or the computer will treat the measurements as orientation data.

List of subroutines:

A complete list of the subroutines of the program follows. The names in parentheses following the subroutine name are dummy variables which can be given values or names each time the subroutine is called.

ANOV (IAXIS)   IAXIS can be put equal to 1, 2, or 3. It refers to the axis about which the cosines are measured.

AXES

CONFIT (KOOKS)   KOOKS is put equal to 1 if a print-out of extreme deviations from the cone surface is required. Otherwise it is put equal to zero.

ENISOC (LINFOL)   LINFOL is put equal to 1 if the direction cosines refer to a lineation of which the trend and plunge is required, 2 if the direction cosines refer to a pole to a plane for which the azimuth and amount of dip are required, 3 if the direction cosines refer to a pole to a plane for which the strike and dip are required. The direction cosines should enter this subroutine named D(1), D(2), and D(3).

EQUATE (DIRCOS,T,KOUNT,KOUNT2,COVMAT,D)   The array T is put equal to the array DIRCOS. KOUNT2 is put equal to KOUNT - the number of items in DIRCOS. The covariance matrix COVMAT is calculated for T. The mean values of the columns of DIRCOS are stored in D. Since these are dummy variables, the subroutine may be called, for example, with SCOORD substituted for DIRCOS, and S substituted for T.

HOWBIG (IAXIS)   IAXIS has the same meaning as before.

MOMENT (IAXIS)

READIN

SCALAR   This subroutine computes a covariance matrix from scalar data. If the data are scalar this subroutine must be called immediately before subroutines AXES or CONFIT is called.

SEKSHN

SELECT (NUMB)   NUMB indicates the number of the subset by its position on the control card.

SOLSP (A,X,NC,NV,ZERO,IERR)   This subroutine is called only by subroutine CONFIT, where the dummy variables are named automatically.

SPIN (KOSCOR,KOOKS)   This subroutine uses the variables GCOORD and TRACOS. If KOSCOR equals 1, only GCOORD are transformed. If KOSCOR equals 2, both GCOORD and DIRCOS are transformed. If KOOKS is put equal to 1, a list of values which deviate considerably from the mean is printed out. If KOOKS is put equal to zero, the listing is supressed.

WULFF

Program organization:

To economize in storage space within the computer, the subroutines all operate on sets of the data held in common storage. Direction cosines are stored in T, and coordinates are stored in S. The actual values of the numbers held in T and S may change at various stages of the running of the program. The geographical coordinates and direction cosines of each measurement are stored in GCOORD and DIRCOS, where they remain unchanged throughout the running of the program. Subroutine EQUATE can be used at any time to transfer the arrays GCOORD and DIRCOS into S and T, (see figure 12). After transfer to S and T, the original data can be used by any of the other subroutines. If a subset of the original data was required, the subset would be retained in S and T by subroutine SELECT, and the other data in S and T would be discarded. The subset would be chosen according to numbers on a control card (see above).

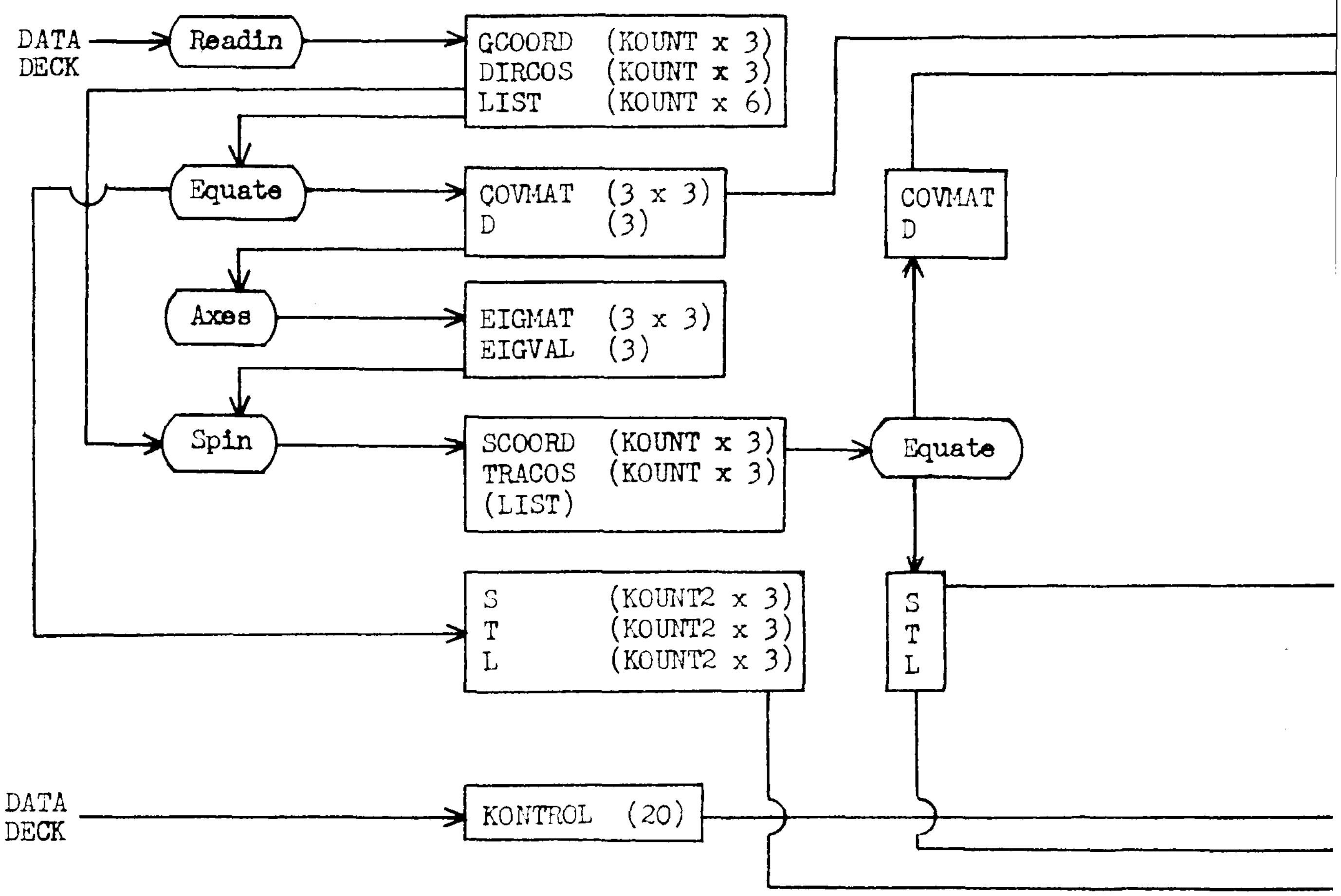
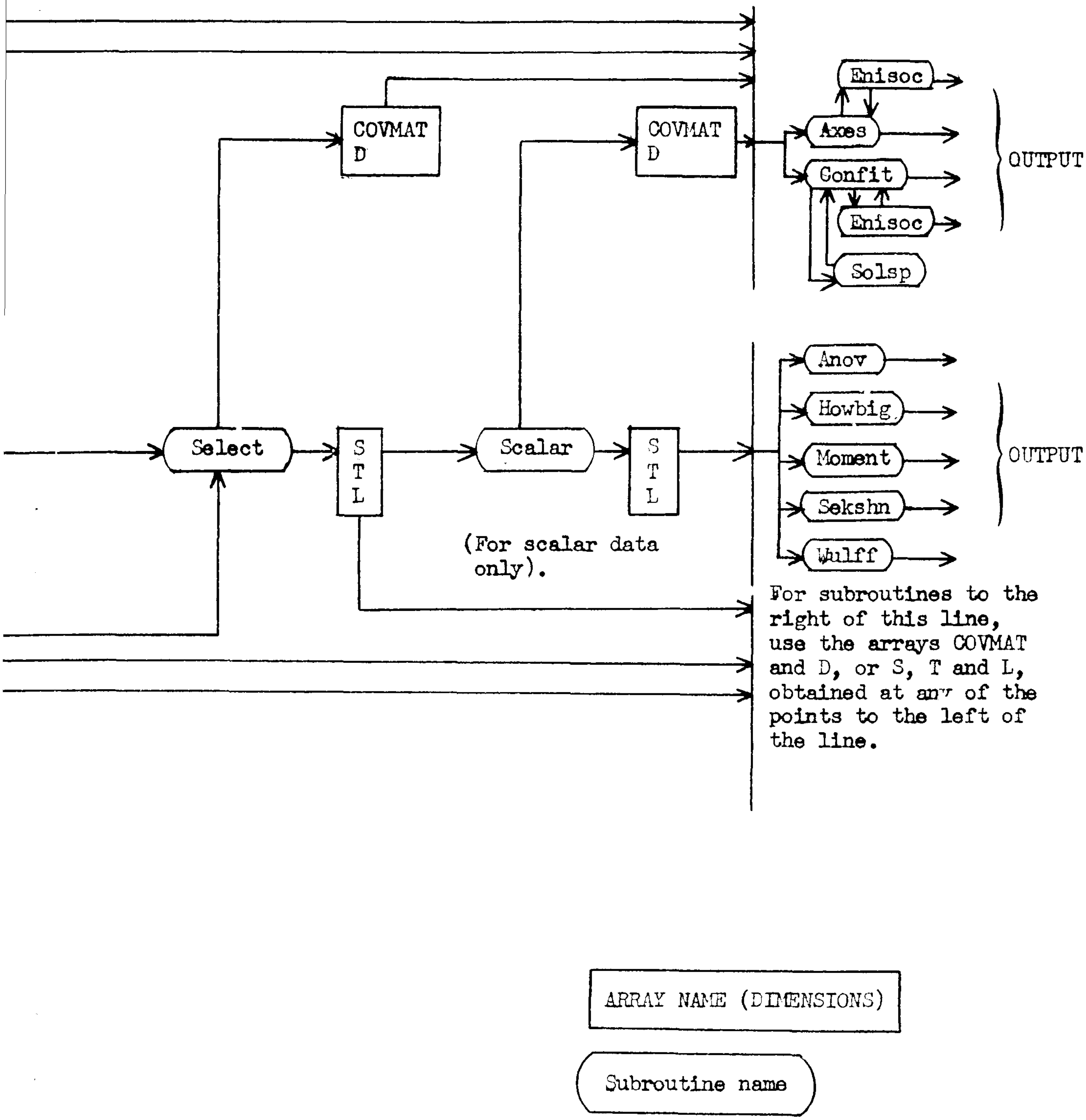


Figure 12 : This is not a flow chart of the usual type, but is a diagram showing the movement of data within the storage area of the computer. All the arrays in the diagram are in common storage, and available to most of the subroutines at any time during the running of the program. The lines with arrows show which arrays are used by each subroutine, and which arrays are generated by each subroutine. The arrays S, T and L, and the arrays COVMAT and D can be generated in several different ways. They may contain different subsets of the data at different times during the running of the program.


COVMAT
D
COVMAT
D
Enisoc
Axes
Confit
Enisoc
Solsp
OUTPUT
Anov
Howbig
Moment
Sekshn
Wulff
OUTPUT
Select
S
T
L
Scalar
S
T
L
(For scalar data only).
For subroutines to the right of this line, use the arrays COVMAT and D, or S, T and L, obtained at any of the points to the left of the line.
ARRAY NAME (DIMENSIONS)
Subroutine name

After the matrix of eigenvectors has been calculated by subroutine AXES, the eigenvectors can be used by subroutine SPIN as a rotation matrix, to transform the original coordinates and direction cosines and refer them to the principal axes. The transformed coordinates are stored in SCOORD and the transformed direction cosines are stored in TRACOS. Subroutine EQUATE can be used to transfer the array SCOORD to S, and the array TRACOS to T. A subset can then be chosen by subroutine SELECT, or any of the other subroutines can be used with the transformed data.

Subroutine WULFF:

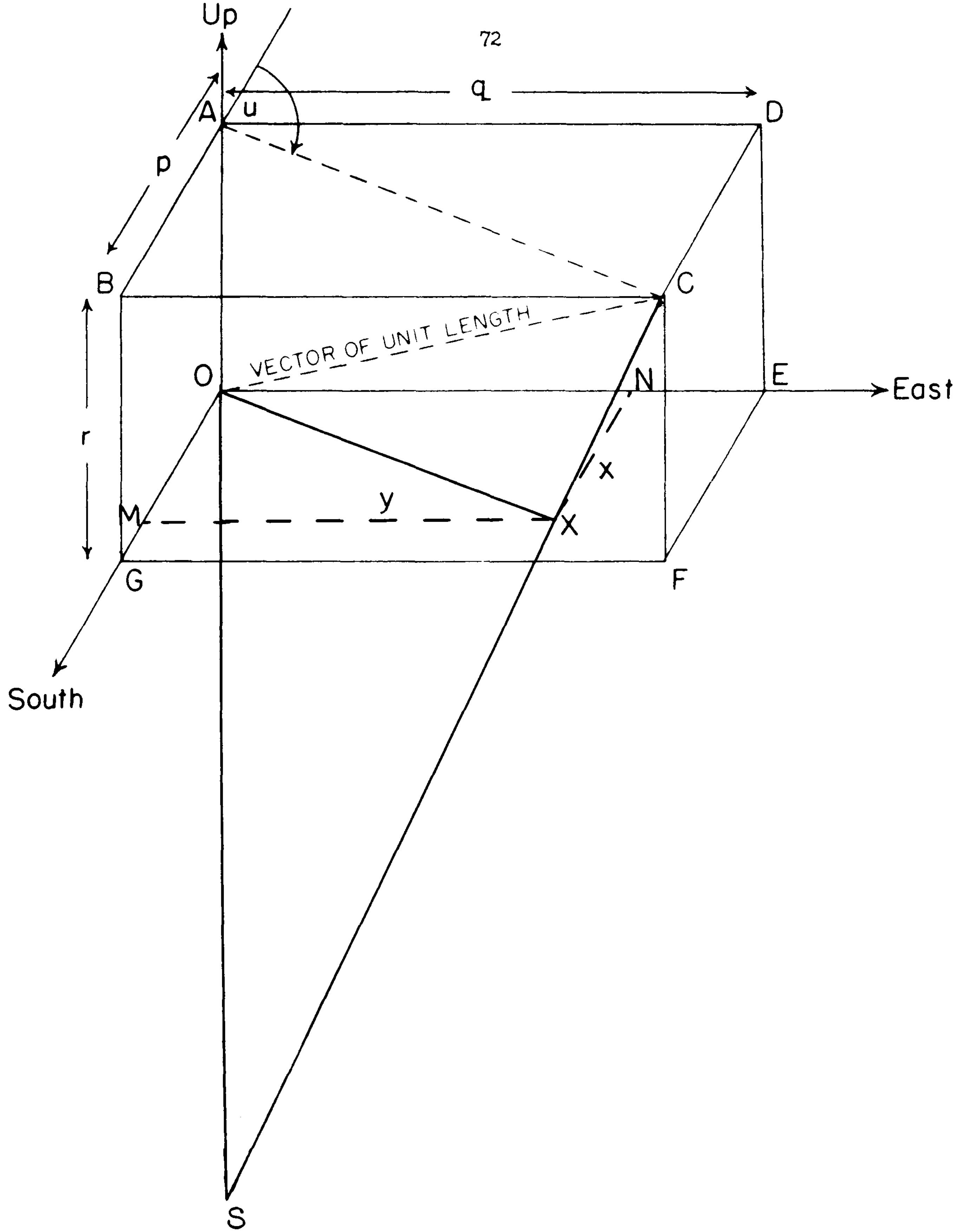
The mathematical backgound of the other subroutines has been explained in section 1 of the report. An explanation of subroutine WULFF will now be given. The purpose of the subroutine is to illustrate how a distribution of vectors - either lineations or poles to foliation - is distributed on a Wulff stereogram. Standard line-printer output is used.

The line-printer prints 10 characters per inch and 6 lines per inch. Hence, it can be calculated that an array of two-digit characters of 47 rows and 40 columns would produce a square with sides about 20 centimetres in length. This array is stored in the computer as IROW. The elements of IROW which lie outside a circle of 20 cm diameter are initially set at -9, the other elements are initially set equal to zero. The portion of each row lying beyond the circle can be calculated from the formula: $x^2 + y^2 = 1$, which gives the position of the circumference of the circle in terms of x, the north-south distance from the center of the circle, and y, the east-west distance from the center of the circle.

The orientations of the vectors are stored within the computer as direction cosines. As shown in figure 13, the direction cosines are related to the position of the plot of the vector on the stereogram. The row and column for each vector is calculated from the direction cosines, and one is added to the number stored in the corresponding element of IROW. When all the orientation measurements have been included in IROW, the matrix is printed out, row by row. If the data are in the form of elevations rather than orientations, subroutine WULFF computes the direction cosines of the vectors joining each pair of elevations, and plots the corresponding points on the stereogram.

Output for further analysis:

Since the direction cosines measured about each of the principal axes are uncorrelated variables, many statistical procedures can be used with them, treating the cosines about each axis as a scalar variable. Many programs, such as those listed in the BIMD computer programs manual (1964) are available for this purpose. The present programs can readily be used to obtain an output of punched cards containing the measurements and coordinates transformed to refer to the principal axes (see comments cards within the programs). The transformed measurements may be usable directly as input to BIMD or similar programs. Suggestions are made in table 6 about the geological information which such an analysis might yield.


Up
A
u
q
D
p
B
C
VECTOR OF UNIT LENGTH
O
N
E
East
r
x
y
M
X
G
F
South
S

O is the center of a sphere of unit radius. OEFG is a horizontal plane through the center. OS is normal to OEFG and intersects the sphere at S. Since OC is a vector of unit length (compare with figure 1), C also lies on the sphere. The vector OC plots at the point X with coordinates (x,y) when plotted on the upper hemisphere by Wulff projection.

In triangles ACS and OXS,

angle CAS = angle XOS = 90°

Tan OSX = AC/AS = OX/OS

Hence AC/OX = AS/OS

In triangles ADC and ONX, the vertical plane CAS is normal to the horizontal planes ABCD and OGFE,

therefore, angle DAC = angle NOX = $u-90^{\circ}$

cos $(u-90^{\circ})$ = AD/AC = ON/OX

Therefore, AD/ON = AC/OX = AS/OS

But AD = q (see figure 1); OS = 1 (sphere is of unit radius); and AS = r + 1.

Therefore AD/ON = (r + 1)

$y = ON = q/(r+1)$

Similarly,

$x = OM = p/(r+1)$

If the vector is plotted on the lower hemisphere,

$y = -q/(r-1)$, and $x = -p/(r-1)$.

See Phillips (1960), chapter 1.

Figure 13 : The coordinates (x,y) of the point at which a unit vector with direction cosines (p,q,r) is plotted on a circle of unit radius by Wulff projection on the lower hemisphere.

Qualitative descriptive terms	Quantitative relationships
	Variation, parallel to the 1-axis, of successive layers
Vertical extent of folds - similar, concentric, disharmonic folding.	Variability of variance in successive strata
Curvature of axial planes.	Variability of skewness in successive strata
Conjugate folding.	Variability of kurtosis in successive strata
	Variation parallel to the 2-axis or 3-axis
Parasitic folding.	Correlation of variance of minor fold with mean value of minor fold (that is, with position on major fold)
Parasitic folding.	Skewness and mean value of minor folds negatively correlated
Refolded folds.	Skewness and mean of minor folds uncorrelated
	Correlations of measurements about different axes
Canoe-shaped folds.	Mean of minor folds measured parallel to the 3-axis correlated with the variance of the folds measured parallel to the 2-axis
Minor folds refolded by superposed folds at right angles.	Mean of minor folds measured parallel to the 3-axis uncorrelated with the variance of the folds measured parallel to the 2-axis
	Relationships between different structural features
Axial plane cleavage.	Cleavage correlated with axial plane, lineation correlated with fold axis, uncorrelated with bedding orientation

Axial plane cleavage with slight fanning.	As above, but slightly correlated with bedding plane orientation
Refolded cleavage or lineation.	Cleavage uncorrelated with axial plane, lineation uncorrelated with fold axis, but correlated with bedding plane orientation
Cross-cutting cleavage.	Cleavage uncorrelated with axial plane, lineation uncorrelated with fold axis, both uncorrelated with bedding orientation
Small circle distribution of early lineation on stereogram. Late folding concentric (see Ramsay, 1960).	Early lineation folded to conical form, with axis parallel to later fold axis.
Great circle distribution on stereogram of early lineation. Later folding is similar, intersection of great circle and axial plane indicates tectonic "a" direction.	Early lineation folded to cylindrical form.
Compressional buckling (see Ramberg, 1963).	Size of fold related to thickness of competent unit. Variance of each unit related to viscosity contrast between competent unit and surrounding medium.

Table 6 : Some properties of folding which involve relative location as well as orientation. The axes referred to are the principal axes of the distribution. Punched card output of measurements and locations referred to the principal axes can be used as input for programs used to investigate the above correlations.

BIBLIOGRAPHY

BIMD, 1964. Computer programs manual. University of California, Los Angeles.

Clark, R.H. and McIntyre, D.B., 1951. The use of the terms pitch and plunge. Amer. Jour. Sci., v. 249, pp. 591 - 599.

Cohn, P.M., 1961. Solid geometry. Routledge and Kegan Paul, London.

Dixon, W.J. and Massey, F.J., 1957. Introduction to statistical analysis. McGraw-Hill Book Co., Inc., New York.

Fisher, R.A., 1953. Dispersion on a sphere. Proc. Roy. Soc., London, Series A, v. 217, pp. 295 - 306.

Harman, H.H., 1960. Modern factor analysis. University of Chicago Press.

Imbrie, J., 1963. Factor and vector analysis programs for analyzing geologic data. Technical Report No. 6 of ONR Task No. 389-135, contract Nonr 1228(26).

Kendall, M.G., 1961. A course in multivariate analysis. Charles Griffin and Company, Ltd., London.

Noble, D.C., 1964. Mathematical rotation of orientation data. Geol. Soc. Amer. Bull., v. 75, pp. 247 - 248.

Paterson, M.S. and Weiss, L.E., 1961. Symmetry concepts in the structural analysis of deformed rocks. Geol. Soc. Amer. Bull., v. 72, pp. 841 - 882.

Phillips, F.C., 1960. The use of stereographic projection in structural geology. Edward Arnold Ltd., London.

Ramberg, H., 1963. Fluid dynamics of viscous buckling applicable to folding of layered rocks. Bull. Amer. Assoc. Petrol. Geol., v. 47, pp. 484 - 505.

Ramsay, J.G., 1960. The deformation of early linear structures in areas of repeated folding. Jour. Geol., v. 68, pp. 75 - 93.

Steinmetz, R., 1962. Analysis of vectorial data. Jour. Sed. Pet., v. 32, pp. 801 - 812.

Snell, J.B., 1961. Method for predicting line-of-sight capabilities: preliminary investigations. Geol. Soc. Amer. Bull., v. 72, pp. 479 - 484.

Turner, F.J. and Weiss, L.E., 1963. Structural analysis of metamorphic tectonites. McGraw-Hill Book Co., Inc., New York.

Yule, G.U. and Kendall, M.G., 1958. An introduction to the theory of statistics. Charles Griffin and Company, Ltd., London.

Examples

Example 1 uses the data of Example 3, section 1

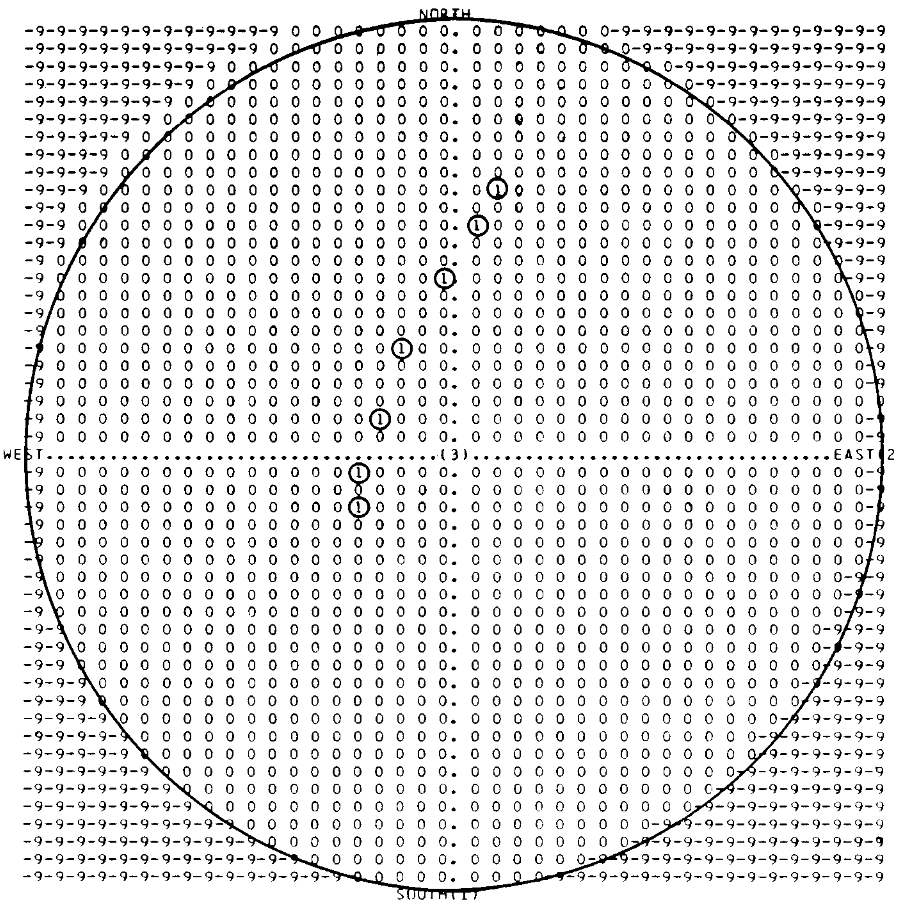
Example 2 uses the data of Example 6, section 1

Example 3 refers to fold 4 of figure 8, page 35.

Example 4 uses a number of measurements abstracted from the United States Geological Survey map of the Wildwood Quadrangle, Tennessee. It will be noted that while the conical surface on which the bedding planes of Example 4 were found to lie is mathematically plausible, it is geological ly improbable. It can be attributed to the sample being unduly small.

Example 1

ITEM	AZIMUTH AND AMOUNT OF DIP		COMPUTED DIRECTION COSINES
1	188.000	62.000	0.87435-0.12288 0.46947
2	183.000	54.000	0.80791-0.04234 0.58779
3	177.000	44.000	0.69371 0.03636 0.71934
4	155.000	28.000	0.42548 0.19841 0.88295
5	109.000	20.000	0.11135 0.32339 0.93969
6	81.000	23.000	-0.06112 0.38592 0.92050
7	62.000	29.000	-0.22760 0.42806 0.87462



7 MEASUREMENTS

STEREOGRAM SHOWING DISTRIBUTION OF POINTS PROJECTED ON LOWER HEMISPHERE OF 20-CENTIMETRE WULFF NET

COVARIANCE MATRIX
0.30677-0.01672 0.22991
-0.01672 0.07062 0.16461
0.22991 0.16461 0.62261

EIGENVECTORS AND EIGENVALUES IN COLUMNS
0.42919-0.84865-0.30917
0.19685 0.42196-0.88499
0.88150 0.31897 0.34816
0.77131 0.22867 0.00002

PRINCIPAL AXES AND ASSOCIATED VARIANCE
TREND AND PLUNGE (LINEAR STRUCTURE) OF 1-AXIS
155.4 / 61.8 NORTHWEST
0.77131
TREND AND PLUNGE (LINEAR STRUCTURE) OF 2-AXIS
26.4 / 18.6 SOUTHWEST
0.22867
TREND AND PLUNGE (LINEAR STRUCTURE) OF 3-AXIS
109.3 / 20.4 SOUTHEAST
0.00002

NOTE: This and following stereograms are for rapid visual scanning, inasmuch as points can be positioned by the machine only at locations specified by the spacing of lines and zeroes. Hence some rounding is inevitable, and smooth great circles or other projections may appear to have jogs in them. The exact position of the points is given in a table at the top of each circle printout.

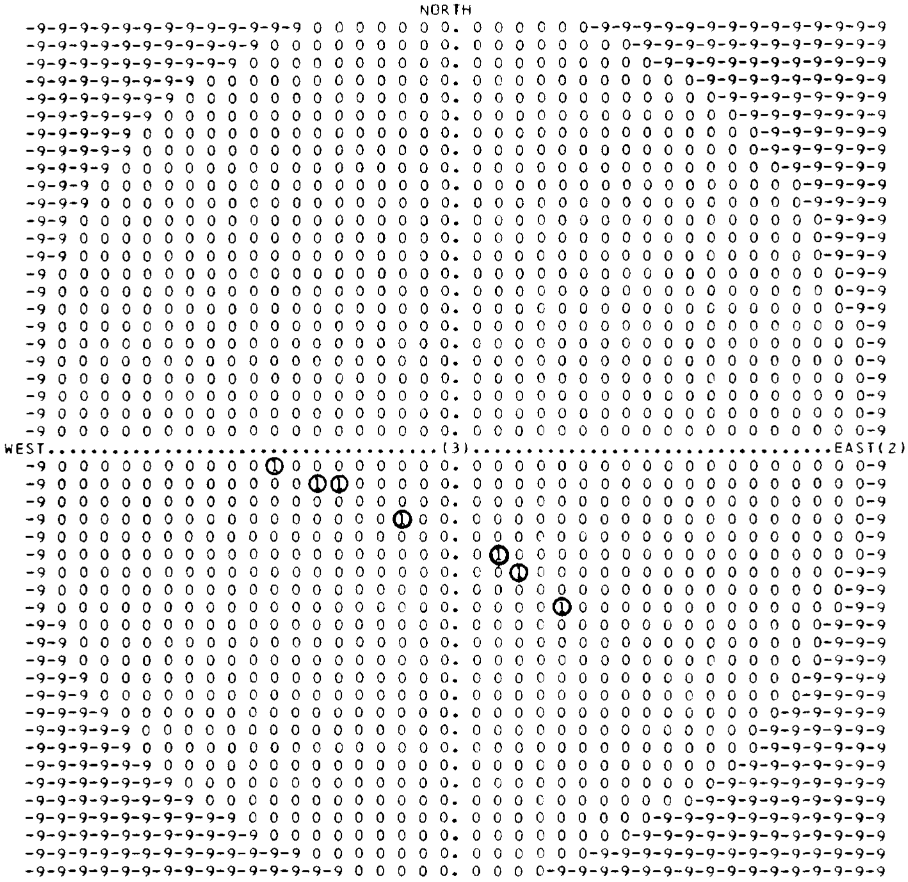
Example 1 (con't.)

```
COEFFICIENTS OF L.S. EQUATIONS
  1.00000  0.37487  0.17242  0.77062
  0.37487  0.30677 -0.01672  0.22991
  0.17242 -0.01672  0.07062  0.16461
  0.77062  0.22991  0.16461  0.62261

CONSTANTS MINIMIZING SQUARES IN SOUTH, EAST, AND VERTICAL DIRECTIONS IN TURN,
COS APEX . DIRECTION COSINES OF AXIS . RMS DEVIATION
  0.04914  0.70103  0.        -0.71313      0.51780
  0.02140  0.33751  0.94132  0.             0.27385
  0.01445  0.31616  0.88806 -0.33374       0.00407

DIRECTION COSINES OF CONE AXIS WITH LOWEST RMS DEVIATION
  0.31616  0.88806 -0.33374

TREND AND PLUNGE OF CONE AXIS
TREND AND PLUNGE (LINEAR STRUCTURE)
 109.6 /   19.5 SOUTHEAST

APICAL ANGLE OF CONE IS  89.17 DEGREES

              --------------------------------------------

                     THE MEASUREMENTS BELOW ARE IN TERMS OF THE PRINCIPAL AXES

LIST OF COORDINATES AND DIRECTION COSINES OF MEASUREMENTS TRANSFORMED TO REFER TO PRINCIPAL AXES
  1   -0.000E-19   -0.000E-19   -0.000E-19           0.7649 -0.6441  0.0019
  2   -0.000E-19   -0.000E-19   -0.000E-19           0.8565 -0.5160 -0.0077
  3   -0.000E-19   -0.000E-19   -0.000E-19           0.9390 -0.3439  0.0038
  4   -0.000E-19   -0.000E-19   -0.000E-19           1.0000  0.0043  0.0003
  5   -0.000E-19   -0.000E-19   -0.000E-19           0.9398  0.3417  0.0065
  6   -0.000E-19   -0.000E-19   -0.000E-19           0.8612  0.5083 -0.0022
  7   -0.000E-19   -0.000E-19   -0.000E-19           0.7576  0.6528 -0.0040

                                   NORTH
  -9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 1. 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9-9 0 0 1 0 0 0 0 0 0. 0 0 0 0 0 0 1 0-9-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 1-9-9-9-9-9-9-9
  -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
  -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
  -9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9
  -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
  -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
  -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
WEST.....................................(3)......................................EAST(2)
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
  -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
  -9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9
  -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
  -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
  -9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 1-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9 0 1 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 1-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0. 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0. 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9-9-9
                                  SOUTH(1)

 7 MEASUREMENTS

STEREOGRAM SHOWING DISTRIBUTION OF POINTS PROJECTED ON LOWER HEMISPHERE OF 20-CENTIMETRE WULFF NET
```

Example 2

--- NEXT BATCH OF DATA ---

ITEM	AZIMUTH AND AMOUNT OF DIP		COMPUTED DIRECTION COSINES
1	328.000	46.000	-0.61003-0.38119 0.69466
2	334.000	38.000	-0.55335-0.26989 0.78801
3	344.000	30.000	-0.48063-0.13782 0.86603
4	387.000	22.000	-0.33378 0.17007 0.92718
5	430.00C	31.000	-0.17616 0.48398 0.85717
6	440.000	38.000	-0.10691 0.60631 0.78801
7	446.000	46.000	-0.05018 0.71759 0.69466



7 MEASUREMENTS

STEREOGRAM SHOWING DISTRIBUTION OF POINTS PROJECTED ON LOWER HEMISPHERE OF 20-CENTIMETRE WULFF NET

COVARIANCE MATRIX
 0.15225 0.02932-0.26509
 0.02932 0.19755 0.13599
-0.26509 0.13599 0.65020

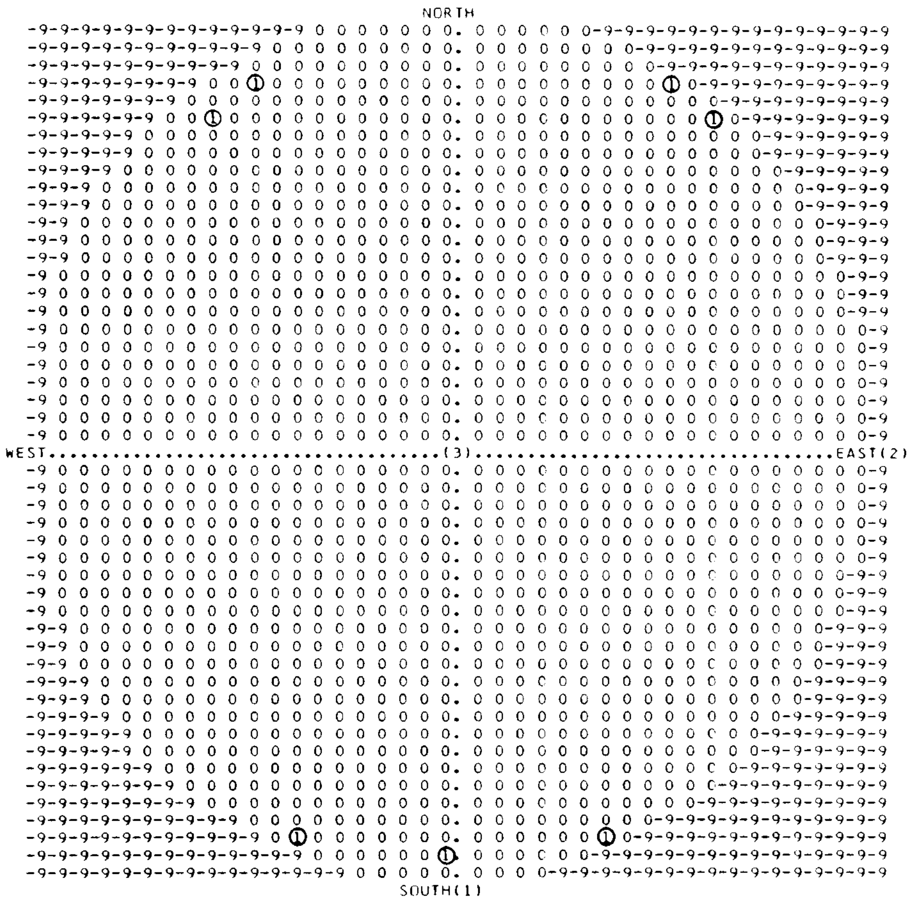
EIGENVECTORS AND EIGENVALUES IN COLUMNS
 0.37074 0.45228 0.81117
-0.19125 0.89187-0.40987
-0.90883-0.00318 0.41715
 0.78696 0.21194 0.00110

PRINCIPAL AXES AND ASSOCIATED VARIANCE
TREND AND PLUNGE (LINEAR STRUCTURE) OF 1-AXIS
 27.3 / 65.3 SOUTHWEST
 0.78696
TREND AND PLUNGE (LINEAR STRUCTURE) OF 2-AXIS
 116.9 / 0.2 SOUTHEAST
 0.21194
TREND AND PLUNGE (LINEAR STRUCTURE) OF 3-AXIS
 26.8 / 24.7 NORTHEAST
 0.00110

Example 2 (con't.)

```
COEFFICIENTS OF L.S. EQUATIONS
  1.00000 -0.33015  0.16986  0.80225
 -0.33015  0.15225  0.02932 -0.26509
  0.16986  0.02932  0.19755  0.13599
  0.80225 -0.26509  0.13599  0.65020

CONSTANTS MINIMIZING SQUARES IN SOUTH, EAST, AND VERTICAL DIRECTIONS IN TURN,
COS APEX . DIRECTION COSINES OF AXIS . RMS DEVIATION
  0.40535 -0.99991  0.        -0.01341      0.22501
  0.36168 -0.89215  0.45175 -0.             0.01006
  0.27626 -0.88630  0.44851 -0.11535        0.00888

DIRECTION COSINES OF CONE AXIS WITH LOWEST RMS DEVIATION
 -0.88630  0.44851 -0.11535

TREND AND PLUNGE OF CONE AXIS
TREND AND PLUNGE (LINEAR STRUCTURE)
  26.8 /     6.6 NORTHEAST

APICAL ANGLE OF CONE IS  73.96 DEGREES
```

```
                         THE MEASUREMENTS BELOW ARE IN TERMS OF THE PRINCIPAL AXES

LIST OF COORDINATES AND DIRECTION COSINES OF MEASUREMENTS TRANSFORMED TO REFER TO PRINCIPAL AXES
  1   -0.000E-19   -0.000E-19   -0.000E-19              -0.7846 -0.6181 -0.0488
  2   -0.000E-19   -0.000E-19   -0.000E-19              -0.8697 -0.4935 -0.0095
  3   -0.000E-19   -0.000E-19   -0.000E-19              -0.9389 -0.3431  0.0279
  4   -0.000E-19   -0.000E-19   -0.000E-19              -0.9989 -0.0022  0.0463
  5   -0.000E-19   -0.000E-19   -0.000E-19              -0.9369  0.3492  0.0163
  6   -0.000E-19   -0.000E-19   -0.000E-19              -0.8718  0.4899 -0.0065
  7   -0.000E-19   -0.000E-19   -0.000E-19              -0.7872  0.6151 -0.0450
```

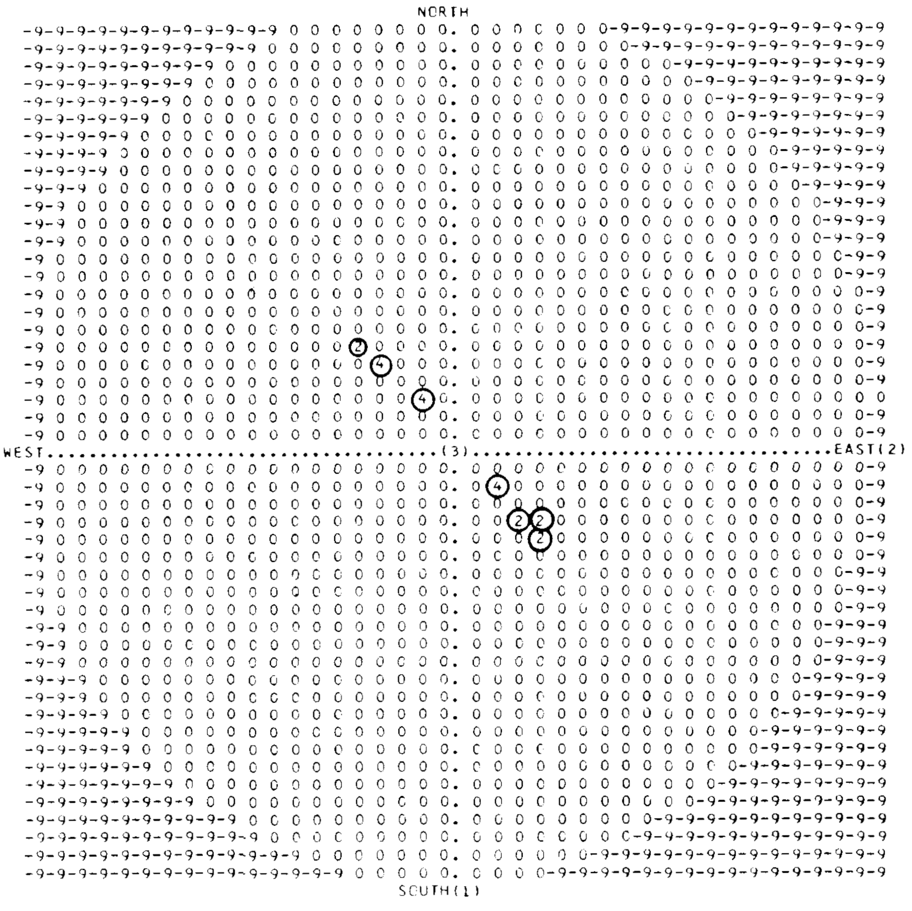


7 MEASUREMENTS

STEREOGRAM SHOWING DISTRIBUTION OF POINTS PROJECTED ON LOWER HEMISPHERE OF 20-CENTIMETRE WULFF NET

Example 3

ITEM	GEOGRAPHICAL COORDINATES			AZIMUTH AND AMOUNT OF DIP		COMPUTED DIRECTION COSINES		
1	10.000	10.000	0.	315.000	34.000	-0.39541	-0.39541	0.82904
2	20.000	20.000	0.	315.000	23.000	-0.27629	-0.27629	0.92050
3	30.000	30.000	0.	315.000	12.000	-0.14701	-0.14702	0.97815
4	40.000	40.000	0.	135.000	13.000	0.15906	0.15906	0.97437
5	50.000	50.000	0.	135.000	26.000	0.30997	0.30998	0.89879
6	60.000	60.000	0.	135.000	34.000	0.39541	0.39541	0.82904
7	70.000	70.000	0.	135.000	25.000	0.29884	0.29884	0.90631
8	80.000	80.000	0.	135.000	13.000	0.15906	0.15906	0.97437
9	90.000	90.000	0.	315.000	14.000	-0.17106	-0.17107	0.97030
10	100.000	100.000	0.	315.000	28.000	-0.33196	-0.33197	0.88295
11	110.000	110.000	0.	315.000	34.000	-0.39541	-0.39541	0.82904
12	120.000	120.000	0.	315.000	23.000	-0.27629	-0.27629	0.92050
13	130.000	130.000	0.	315.000	12.000	-0.14701	-0.14702	0.97815
14	140.000	140.000	0.	135.000	13.000	0.15906	0.15906	0.97437
15	150.000	150.000	0.	135.000	26.000	0.30997	0.30998	0.89879
16	160.000	160.000	0.	135.000	34.000	0.39541	0.39541	0.82904
17	170.000	170.000	0.	135.000	25.000	0.29884	0.29884	0.90631
18	180.000	180.000	0.	135.000	13.000	0.15906	0.15906	0.97437
19	190.000	190.000	0.	315.000	14.000	-0.17106	-0.17107	0.97030
20	200.000	200.000	0.	315.000	28.000	-0.33196	-0.33197	0.88295



20 MEASUREMENTS
STEREOGRAM SHOWING DISTRIBUTION OF POINTS PROJECTED ON LOWER HEMISPHERE OF 20-CENTIMETRE WULFF NET

COVARIANCE MATRIX

0.07861	0.07861	0.00022
0.07861	0.07861	0.00022
0.00022	0.00022	0.84278

EIGENVECTORS AND EIGENVALUES IN COLUMNS

0.00032	0.70710	-0.70711
0.00032	0.70711	0.70710
1.00000	-0.00045	0.00000
0.84278	0.15722	0.00000

PRINCIPAL AXES AND ASSOCIATED VARIANCE
TREND AND PLUNGE (LINEAR STRUCTURE) OF 1-AXIS
135.1 / 90.0 NORTHWEST
0.84278
TREND AND PLUNGE (LINEAR STRUCTURE) OF 2-AXIS
135.0 / 0.0 SOUTHEAST
0.15722
TREND AND PLUNGE (LINEAR STRUCTURE) OF 3-AXIS
45.0 / 0.0 SOUTHWEST
0.00000

Example 3 (con't.)

THE FOLLOWING MEASUREMENTS ARE IN TERMS OF THE PRINCIPAL AXES

LIST OF COORDINATES AND DIRECTION COSINES OF MEASUREMENTS TRANSFORMED TO REFER TO PRINCIPAL AXES

1	0.641E-02	0.141E 02	-0.474E-04	0.8288	-0.5596	-0.0000
2	0.128E-01	0.283E 02	-0.948E-04	0.9203	-0.3911	-0.0000
3	0.192E-01	0.424E 02	-0.142E-03	0.9781	-0.2084	0.0000
4	0.256E-01	0.566E 02	-0.190E-03	0.9745	0.2245	0.0000
5	0.320E-01	0.707E 02	-0.237E-03	0.8990	0.4380	-0.0000
6	0.384E-01	0.849E 02	-0.284E-03	0.8293	0.5588	-0.0000
7	0.448E-01	0.990E 02	-0.332E-03	0.9065	0.4222	-0.0000
8	0.513E-01	0.113E 03	-0.379E-03	0.9745	0.2245	0.0000
9	0.577E-01	0.127E 03	-0.426E-03	0.9702	-0.2424	0.0000
10	0.641E-01	0.141E 03	-0.474E-03	0.8827	-0.4699	-0.0000
11	0.705E-01	0.156E 03	-0.521E-03	0.8288	-0.5596	-0.0000
12	0.769E-01	0.170E 03	-0.568E-03	0.9203	-0.3911	-0.0000
13	0.833E-01	0.184E 03	-0.616E-03	0.9781	-0.2084	0.0000
14	0.897E-01	0.198E 03	-0.664E-03	0.9745	0.2245	0.0000
15	0.961E-01	0.212E 03	-0.710E-03	0.8990	0.4380	-0.0000
16	0.103E-00	0.226E 03	-0.758E-03	0.8293	0.5588	-0.0000
17	0.109E-00	0.240E 03	-0.806E-03	0.9065	0.4222	-0.0000
18	0.115E-00	0.255E 03	-0.853E-03	0.9745	0.2245	0.0000
19	0.122E-00	0.269E 03	-0.900E-03	0.9702	-0.2424	0.0000
20	0.128E-00	0.283E 03	-0.948E-03	0.8827	-0.4699	-0.0000

THE FOLLOWING MEASUREMENTS DEVIATE CONSIDERABLY FROM THE MEAN,
ITEM LOCATION COORDS DIRECTION COSINES FACTORS

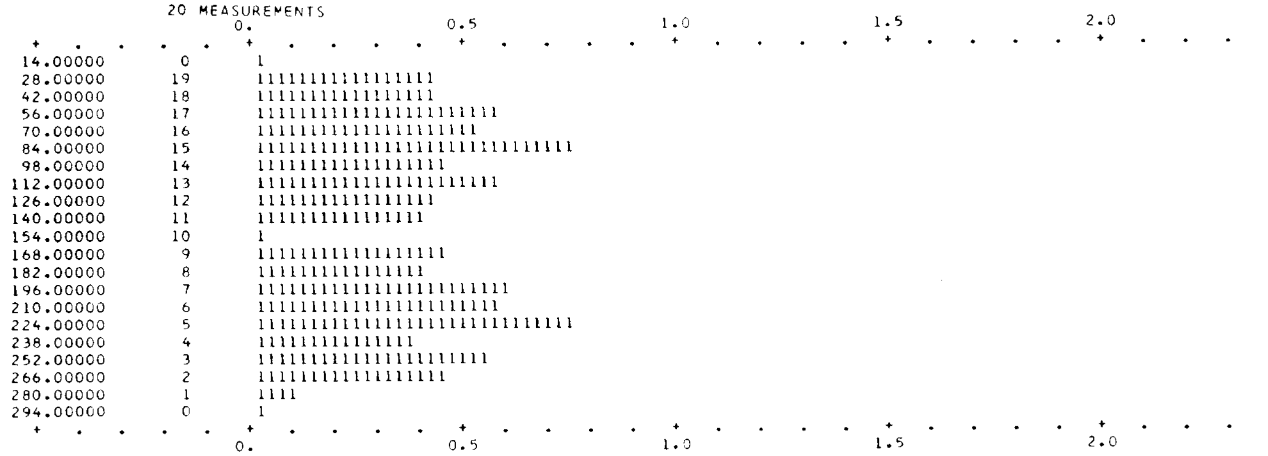
```
                                    NORTH
   -9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 4 0 0 0 0. 0 0 0 2 2 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9+9-9-9 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9
   -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
   -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
   -9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9
   -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
   -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
   -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
   -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
   -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
WEST.....................................(3).....................................EAST(2)
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
   -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
   -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
   -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
   -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
   -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
   -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
   -9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9
   -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
   -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
   -9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9-9 2 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 2-9-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9-9-9-9 2 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 2-9-9-9-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9-9-9-9-9 2 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 2-9-9-9-9-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0. 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9
   -9-9-9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0. 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9-9-9
                                    SOUTH(1)
```

20 MEASUREMENTS
STEREOGRAM SHOWING DISTRIBUTION OF POINTS PROJECTED ON LOWER HEMISPHERE OF 20-CENTIMETRE WULFF NET

```
COEFFICIENTS OF L.S. EQUATIONS
  1.00000   0.06727148.49240 -0.00050
  0.06727   0.00589 13.00259 -0.00004
148.49240  13.00259699.99219 -0.09616
 -0.00050  -0.00004 -0.09616  0.00000

CONSTANTS MINIMIZING SQUARES IN SOUTH, EAST, AND VERTICAL DIRECTIONS IN TURN,
COS APEX . DIRECTION COSINES OF AXIS . RMS DEVIATION
  0.00000  0.12742  0.00000  0.99185      0.11697
  0.00000 -1.00000  0.00047  0.00000      0.91803
  0.00000  0.00447  0.00000  0.99999      0.00410
DIRECTION COSINES OF CONE AXIS WITH LOWEST RMS DEVIATION
  0.00447  0.00000  0.99999

TREND AND PLUNGE OF CONE AXIS
TREND AND PLUNGE (LINEAR STRUCTURE)
 180.0 /    89.7 NORTHWEST

APICAL ANGLE OF CONE IS  90.00 DEGREES

THE FOLLOWING MEASUREMENTS DEVIATE CONSIDERABLY FROM THE CONE SURFACE,
ITEM        COS DEVN  DIRECTION COSINES      LOCATION COORDS        FACTORS
```

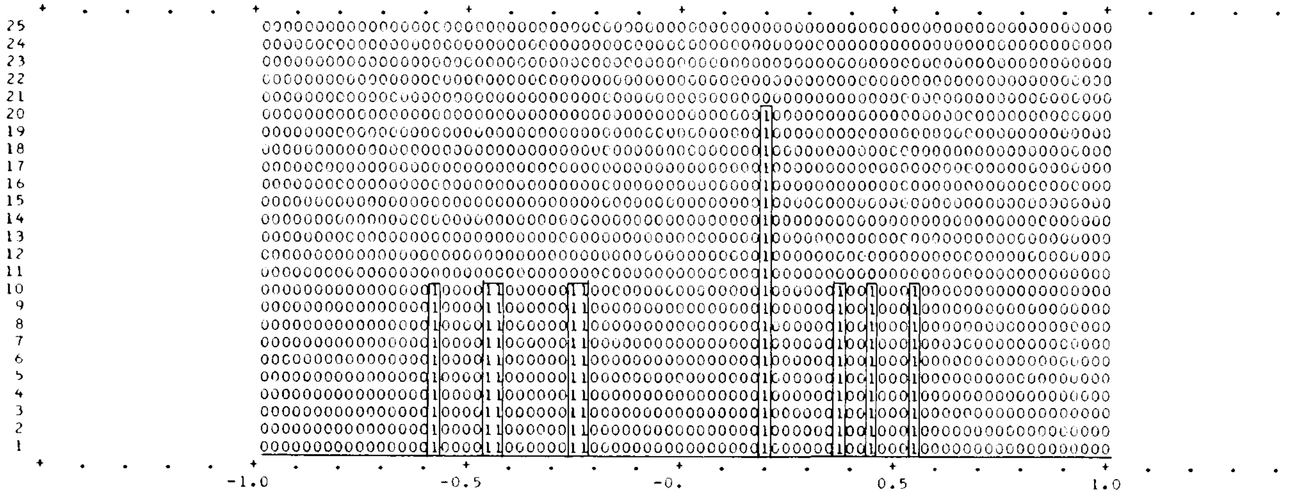
Example 3 (con't.)

HISTOGRAM OF MEAN ATTITUDE DEVIATION MEASURED PARALLEL TO THE 2-AXIS
20 MEASUREMENTS



MEAN DIFFERENCE BETWEEN DIRECTION COSINES AT GIVEN DISTANCE APART
DISTANCE BETWEEN MEASUREMENTS IS PLOTTED VERTICALLY,THE SCALE IS ON THE LEFT
THE SECOND COLUMN SHOWS THE NUMBER OF VALUES REPRESENTED IN EACH ROW
A LOW VALUE IN THE HISTOGRAM MAY INDICATE THE PRESENCE OF FOLDS OF THAT WAVELENGTH

DISTRIBUTION OF DIRECTION COSINES ABOUT THE 2-AXIS
20 MEASUREMENTS



PERCENTAGE FREQUENCY PLOTTED VERTICALLY

MOMENTS OF THE ABOVE DISTRIBUTION ARE -
1 -0.0000
2 0.1572
3 0.0004
4 0.0347

SKEWNESS AND KURTOSIS
0.0070 1.4026

DATA FOR DRAWING CROSS-SECTIONS PARALLEL TO THE PRINCIPAL AXES

SLOPE (TANGENT OF ANGLE) OF INTERSECTION OF BED AND			DISTANCE FROM ORIGIN PARALLEL TO		
1-2 PLANE	1-3 PLANE	2-3 PLANE	1-AXIS	2-AXIS	3-AXIS
-1.4811	2698275.8750	-0.0000	0.0064	14.1421	-0.0000
-2.3529	26055223.2500	-0.0000	0.0128	28.2843	-0.0001
-4.6942	4074153.2813	-0.0000	0.0192	42.4264	-0.0001
4.3405	4497966.3125	0.0000	0.0256	56.5685	-0.0002
2.0527	8600166.7500	0.0000	0.0320	70.7107	-0.0002
1.4840	2733073.4375	0.0000	0.0384	84.8528	-0.0003
2.1470	11063403.7500	0.0000	0.0448	98.9949	-0.0003
4.3405	4497966.3125	0.0000	0.0513	113.1371	-0.0004
-4.0031	5149113.0625	-0.0000	0.0577	127.2792	-0.0004
-1.8787	5535486.8125	-0.0000	0.0641	141.4213	-0.0005
-1.4811	2698275.8750	-0.0000	0.0705	155.5635	-0.0005
-2.3529	26055223.2500	-0.0000	0.0769	169.7056	-0.0006
-4.6942	4074153.2813	-0.0000	0.0833	183.8477	-0.0006
4.3405	4497966.3125	0.0000	0.0897	197.9899	-0.0007
2.0527	8600166.7500	0.0000	0.0961	212.1320	-0.0007
1.4840	2733073.4375	0.0000	0.1025	226.2741	-0.0008
2.1470	11063403.7500	0.0000	0.1089	240.4163	-0.0008
4.3405	4497966.3125	0.0000	0.1153	254.5584	-0.0009
-4.0031	5149113.0625	-0.0000	0.1217	268.7005	-0.0009
-1.8787	5535486.8125	-0.0000	0.1281	282.8427	-0.0009

Example 4

WILDWOOD QUAD, TENN. LOUDON, GEOL, NU. APR,64.

```
PRINT-OUT OF MASTER CARDS AND DATA CARDS
GQ130      (X,A5,A4,2F6.4,F6.2,3F6.3,6I3)       2000.   -0.   90.
 GQ130    SAMPLE    GRID      UNUSU
 GQ130    LITHOL    SNDSTN    SS+SH     SHALE     CARBNT    EVAPRT
 GQ130    STRAT     PRECMB    L.CAMB    L.CAMB    M.CAMB    U.CAMB    L.ORD     M.ORD
 GQ130
 GQ130
 GQ130
```

GQ130	01	1.7000	1.3000	940.0000	59.0000	40.0000	-0.	1	3	7	-0	-0	-0
GQ13	02	1.2000	4.3000	1000.0000	56.0000	85.0000	-0.	1	4	7	-0	-0	-0
GQ130	03	1.5000	8.2000	1000.0000	44.0000	50.0000	-0.	1	4	4	-0	-0	-0
GQ13	04	1.4000	10.3000	1000.0000	64.0000	15.0000	-0.	1	3	5	-0	-0	-0
GQ130	05	1.8000	12.9000	1120.0000	49.0000	35.0000	-0.	1	4	5	-0	-0	-0
GQ130	06	1.8000	16.8000	1060.0000	53.0000	55.0000	-0.	1	4	6	-0	-0	-0
GQ130	07	5.0000	1.5000	1040.0000	64.0000	15.0000	-0.	1	2	3	-0	-0	-0
GQ130	08	4.3000	4.4000	1120.0000	32.0000	45.0000	-0.	1	3	4	-0	-0	-0
GQ130	09	4.5000	6.9000	1080.0000	38.0000	65.0000	-0.	1	4	5	-0	-0	-0
GQ130	10	4.2000	11.4000	1040.0000	36.0000	35.0000	-0.	1	4	6	-0	-0	-0
GQ130	12	3.2000	17.5000	1045.0000	50.0000	15.0000	-0.	1	4	6	-0	-0	-0
GQ130	13	7.7000	2.8000	900.0000	52.0000	75.0000	-0.	1	4	4	-0	-0	-0
GQ130	14	7.2000	4.7000	1060.0000	49.0000	70.0000	-0.	1	4	5	-0	-0	-0
GQ130	15	7.5000	7.5000	1020.0000	50.0000	30.0000	-0.	1	4	6	-0	-0	-0
GQ130	16	7.6000	10.5000	1000.0000	47.0000	25.0000	-0.	1	4	6	-0	-0	-0
GQ130	17	8.9000	13.1000	1010.0000	259.0000	40.0000	-0.	1	4	7	-0	-0	-0
GQ130	18	8.4000	15.9000	1020.0000	239.0000	20.0000	-0.	1	4	7	-0	-0	-0
GQ130	19	10.8000	2.2000	920.0000	54.0000	80.0000	-0.	1	4	6	-0	-0	-0
GQ130	20	9.2000	4.3000	1060.0000	39.0000	75.0000	-0.	1	4	6	-0	-0	-0
GQ130	21	10.7000	8.6000	1000.0000	230.0000	35.0000	-0.	1	4	7	-0	-0	-0
GQ130	22	10.5000	10.5000	1040.0000	250.0000	65.0000	-0.	1	4	6	-0	-0	-0
GQ130	23	9.4000	14.5000	1040.0000	240.0000	25.0000	-0.	1	4	6	-0	-0	-0
GQ130	24	10.4000	16.1000	1040.0000	60.0000	45.0000	-0.	1	4	6	-0	-0	-0
GQ130	25	13.1000	1.8000	960.0000	46.0000	50.0000	-0.	1	4	5	-0	-0	-0
GQ130	26	12.4000	4.1000	960.0000	43.0000	55.0000	-0.	1	4	6	-0	-0	-0
GQ130	28	12.6000	10.8000	980.0000	73.0000	30.0000	-0.	1	4	7	-0	-0	-0
GQ130	29	12.6000	14.2000	1000.0000	68.0000	35.0000	-0.	1	2	7	-0	-0	-0
GQ130	30	13.5000	16.6000	1040.0000	61.0000	45.0000	-0.	1	2	7	-0	-0	-0
GQ130	31	16.8000	2.5000	880.0000	227.0000	60.0000	-0.	1	4	6	-0	-0	-0
GQ130	32	17.1000	3.9000	940.0000	58.0000	25.0000	-0.	1	4	6	-0	-0	-0
GQ130	33	15.3000	7.5000	1040.0000	43.0000	45.0000	-0.	1	4	6	-0	-0	-0
GQ130	34	16.7000	10.9000	920.0000	52.0000	45.0000	-0.	1	2	7	-0	-0	-0
GQ130	35	15.9000	12.9000	1000.0000	65.0000	50.0000	-0.	1	2	7	-0	-0	-0
GQ130	36	16.0000	15.9000	1040.0000	67.0000	45.0000	-0.	1	2	7	-0	-0	-0
GQ130	37	17.4000	2.8000	900.0000	52.0000	40.0000	-0.	1	4	5	-0	-0	-0
GQ130	38	18.7000	4.6000	880.0000	47.0000	45.0000	-0.	1	4	7	-0	-0	-0
GQ130	40	18.3000	10.8000	1100.0000	39.0000	45.0000	-0.	1	2	7	-0	-0	-0
GQ130	41	18.5000	17.4000	1000.0000	86.0000	45.0000	-0.	1	3	7	-0	-0	-0
GQ130	42	18.7000	16.2000	1600.0000	62.0000	30.0000	-0.	1	2	1	-0	-0	-0
GQ130	43	21.3000	2.8000	820.0000	47.0000	75.0000	-0.	1	3	7	-0	-0	-0
GQ130	44	20.8000	5.7000	1000.0000	38.0000	45.0000	-0.	1	2	7	-0	-0	-0
GQ130	45	21.7000	8.1000	1120.0000	55.0000	20.0000	-0.	1	1	7	-0	-0	-0
GQ130	46	21.2000	10.8000	1360.0000	81.0000	50.0000	-0.	1	2	7	-0	-0	-0
GQ130	47	21.2000	13.8000	1740.0000	79.0000	35.0000	-0.	1	1	2	-0	-0	-0
GQ130	48	21.4000	16.9000	2300.0000	54.0000	25.0000	-0.	1	1	2	-0	-0	-0
GQ130	49	1.3000	0.7000	920.0000	59.0000	25.0000	-0.	2	3	7	-0	-0	-0
GQ130	50	4.8000	0.4000	1140.0000	156.0000	15.0000	-0.	2	2	3	-0	-0	-0
GQ130	51	5.8000	1.3000	1060.0000	5.0000	50.0000	-0.	2	2	3	-0	-0	-0
GQ130	52	8.4000	10.5000	1000.0000	146.0000	35.0000	-0.	2	4	6	-0	-0	-0
GQ130	53	12.9000	0.8000	920.0000	11.0000	75.0000	-0.	2	3	7	-0	-0	-0
GQ130	54	1.4000	10.3000	1000.0000	335.0000	15.0000	-0.	2	3	5	-0	-0	-0
GQ130	55	2.6000	17.6000	980.0000	7.0000	10.0000	-0.	2	4	6	-0	-0	-0
GQ130	56	2.6000	17.8000	980.0000	276.0000	5.0000	-0.	2	4	6	-0	-0	-0
GQ130	57	8.2000	17.5000	980.0000	272.0000	15.0000	-0.	2	4	6	-0	-0	-0
GQ130	58	8.4000	17.5000	960.0000	0.5000	10.0000	-0.	2	4	6	-0	-0	-0
GQ130	59	8.6000	17.4000	960.0000	293.0000	10.0000	-0.	2	4	6	-0	-0	-0
GQ130	60	9.9000	15.7000	940.0000	107.0000	15.0000	-0.	2	4	6	-0	-0	-0
GQ130	61	8.9000	14.2000	980.0000	33.0000	60.0000	-0.	2	4	7	-0	-0	-0
GQ130	62	11.5000	10.1000	940.0000	305.0000	10.0000	-0.	2	4	6	-0	-0	-0
GQ130	63	11.5000	10.5000	930.0000	181.0000	15.0000	-0.	2	4	6	-0	-0	-0
GQ130	64	11.7000	10.7000	940.0000	202.0000	15.0000	-0.	2	4	6	-0	-0	-0
GQ130	65	11.6000	10.9000	930.0000	318.0000	15.0000	-0.	2	4	6	-0	-0	-0
GQ130	66	13.4000	18.4000	1100.0000	5.0000	35.0000	-0.	2	2	7	-0	-0	-0
GQ130	67	23.5000	8.4000	1060.0000	5.0000	40.0000	-0.	2	2	7	-0	-0	-0
GQ130	68	23.5000	16.8000	1500.0000	183.0000	10.0000	-0.	2	1	2	-0	-0	-0
		-0.	-0.	-0.	999.0000	999.0000	-0.	-0	-0	-0	-0	-0	-0

65 MEASUREMENTS

CONTROL CARD 100 211 300 411 512 611 -31 -36 -37 -0 -0 -0 -0 -0 -0 -0 -0

Example 4 (con't.)

```
                                        NORTH
  -9-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9 0 0 0 1 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9
  -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
  -9-9-9-9 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9
  -9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9
  -9-9-9 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
  -9-9 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 1 2 1 2 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 1 0 0 1 1 2 0 1 1 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 2 0 0 0 0 1 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 1 1 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 3 0 0. 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 2 0. 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  -9 0 0 0 0 0 0 0 0 0 1 0 1 1 0 0 0 0 0 0. 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 0 0. 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
WEST.....................................(3)......................................EAST(2)
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1. 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
  -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
  -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
  -9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
  -9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9
  -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
  -9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
  -9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0. 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9
  -9-9-9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0. 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9-9-9
                                       SOUTH(1)
```

65 MEASUREMENTS

STEREOGRAM SHOWING DISTRIBUTION OF POINTS PROJECTED ON LOWER HEMISPHERE OF 20-CENTIMETRE WULFF NET

IN COMPUTING THE NEXT SET OF RESULTS, ONLY MEASUREMENTS IN THE FOLLOWING CATEGORIES WERE USED -

```
INCLUDED                               EXCLUDED

ALL MEASUREMENTS

COEFFICIENTS OF L.S. EQUATIONS
  1.00000  0.27571  0.25820  0.74311
  0.27571  0.21936  0.15109  0.16733
  0.25820  0.15109  0.17536  0.15035
  0.74311  0.16733  0.15035  0.60528

CONSTANTS MINIMIZING SQUARES IN SOUTH, EAST, AND VERTICAL DIRECTIONS IN TURN,
COS APEX . DIRECTION COSINES OF AXIS . RMS DEVIATION
  0.24066  0.98262  0.       0.18562     0.39348
  0.45237 -0.39709  0.91778  0.          0.40183
  0.80575  0.07884  0.30464  0.94920     0.18163

DIRECTION COSINES OF CONE AXIS WITH LOWEST RMS DEVIATION
  0.07884  0.30464  0.94920

TREND AND PLUNGE OF CONE AXIS
TREND AND PLUNGE (LINEAR STRUCTURE)
 104.5 /    71.7 NORTHWEST

APICAL ANGLE OF CONE IS  36.32 DEGREES

THE FOLLOWING MEASUREMENTS DEVIATE CONSIDERABLY FROM THE CONE SURFACE,
ITEM       COS DEVN   DIRECTION COSINES    LOCATION COORDS       FACTORS
   2         0.4882    0.826 0.557 0.087   -0.000-0.000-0.000    GRID    CARBNT  M.ORD   Q00000  Q00000  Q00000
  18         0.4018    0.797 0.579 0.174   -0.000-0.000-0.000    GRID    CARBNT  L.ORD   Q00000  Q00000  Q00000
  21         0.5662   -0.852-0.310 0.423   -0.000-0.000-0.000    GRID    CARBNT  L.ORD   Q00000  Q00000  Q00000
  29         0.5610   -0.633-0.591 0.500   -0.000-0.000-0.000    GRID    CARBNT  L.ORD   Q00000  Q00000  Q00000
```

Example 4 (con't.)

THE MEASUREMENTS BELOW ARE IN TERMS OF THE PRINCIPAL AXES

LIST OF COORDINATES AND DIRECTION COSINES OF MEASUREMENTS TRANSFORMED TO REFER TO PRINCIPAL AXES

1	0.355E 04	0.257E 04	0.518E 02	0.9973	-0.0333	-0.0659
2	0.546E 04	0.459E 04	0.546E 04	0.7279	0.6841	-0.0467
3	0.890E 04	0.841E 04	0.114E 05	0.9833	0.1352	0.1222
4	0.105E 05	0.102E 05	0.148E 05	0.8896	-0.4538	-0.0509
5	0.131E 05	0.129E 05	0.185E 05	0.9920	-0.1194	0.0407
6	0.161E 05	0.164E 05	0.248E 05	0.9735	0.2287	0.0029
7	0.730E 04	0.663E 04	-0.357E 04	0.8896	-0.4538	-0.0509
8	0.894E 04	0.831E 04	0.192E 04	0.9668	0.0195	0.2550
9	0.111E 05	0.108E 05	0.569E 04	0.8981	0.3697	0.2383
10	0.144E 05	0.145E 05	0.133E 05	0.9761	-0.1378	0.1683
11	0.182E 05	0.187E 05	0.242E 05	0.8925	-0.4509	0.0120
12	0.111E 05	0.111E 05	-0.471E 04	0.8365	0.5476	0.0216
13	0.122E 05	0.121E 05	-0.106E 04	0.8795	0.4707	0.0698
14	0.148E 05	0.150E 05	0.307E 04	0.9784	-0.2048	0.0263
15	0.172E 05	0.178E 05	0.776E 04	0.9558	-0.2907	0.0437
16	0.207E 05	0.216E 05	0.104E 05	0.1859	-0.9430	0.2761
17	0.224E 05	0.235E 05	0.155E 05	0.4740	-0.8800	0.0309
18	0.140E 05	0.143E 05	-0.937E 04	0.7856	0.6187	-0.0121
19	0.140E 05	0.141E 05	-0.409E 04	0.8171	0.5250	0.2382
20	0.190E 05	0.198E 05	0.101E 04	0.2293	-0.9727	-0.0359
21	0.204E 05	0.212E 05	0.430E 04	-0.2627	-0.9296	0.2587
22	0.224E 05	0.234E 05	0.120E 05	0.3963	-0.9170	0.0463
23	0.247E 05	0.261E 05	0.134E 05	0.9950	0.0528	-0.0844
24	0.161E 05	0.167E 05	-0.128E 05	0.9858	0.1381	0.0957
25	0.172E 05	0.179E 05	-0.824E 04	0.9650	0.2185	0.1451
26	0.228E 05	0.240E 05	0.227E 04	0.9591	-0.2256	-0.1710
27	0.255E 05	0.270E 05	0.773E 04	0.9802	-0.1317	-0.1478
28	0.284E 05	0.302E 05	0.105E 05	0.9940	0.0516	-0.0966
29	0.206E 05	0.218E 05	-0.161E 05	-0.2009	-0.9748	-0.0970
30	0.220E 05	0.233E 05	-0.142E 05	0.9564	-0.2897	-0.0373
31	0.231E 05	0.243E 05	-0.625E 04	0.9911	0.0474	0.1246
32	0.272E 05	0.291E 05	-0.247E 04	0.9983	0.0560	0.0142
33	0.280E 05	0.298E 05	0.170E 04	0.9788	0.1315	-0.1570
34	0.305E 05	0.326E 05	0.639E 04	0.9847	0.0416	-0.1693
35	0.215E 05	0.227E 05	-0.163E 05	0.9994	-0.0312	0.0125
36	0.243E 05	0.259E 05	-0.150E 05	0.9957	0.0528	0.0757
37	0.290E 05	0.308E 05	-0.454E 04	0.9842	0.0394	0.1728
38	0.344E 05	0.370E 05	0.581E 04	0.9230	-0.0265	-0.3839
39	0.341E 05	0.357E 05	0.365E 04	0.9750	-0.2083	-0.0781
40	0.256E 05	0.275E 05	-0.210E 05	0.8329	0.5432	0.1057
41	0.275E 05	0.293E 05	-0.157E 05	0.9821	0.0370	0.1847
42	0.304E 05	0.325E 05	-0.129E 05	0.9285	-0.3711	-0.0129
43	0.322E 05	0.341E 05	-0.800E 04	0.9305	0.0784	-0.3578
44	0.349E 05	0.365E 05	-0.319E 04	0.9547	-0.1598	-0.2510
45	0.380E 05	0.391E 05	0.154E 04	0.9574	-0.2887	-0.0078
46	0.263E 04	0.157E 04	-0.433E 03	0.9559	-0.2902	-0.0446
47	0.629E 04	0.535E 04	-0.510E 04	0.6819	-0.6854	-0.2555
48	0.801E 04	0.740E 04	-0.485E 04	0.8200	-0.0497	0.5702
49	0.181E 05	0.187E 05	0.681E 04	0.5917	-0.5645	-0.5755
50	0.151E 05	0.156E 05	-0.141E 05	0.6703	0.3587	0.6496
51	0.105E 05	0.102E 05	0.148E 05	0.7558	-0.6051	0.2504
52	0.176E 05	0.181E 05	0.251E 05	0.8146	-0.5670	0.1225
53	0.177E 05	0.183E 05	0.254E 05	0.7003	-0.7116	0.0560
54	0.235E 05	0.247E 05	0.183E 05	0.5860	-0.7946	0.1587
55	0.237E 05	0.250E 05	0.180E 05	0.8046	-0.5782	0.1353
56	0.238E 05	0.251E 05	0.176E 05	0.6763	-0.7218	0.1468
57	0.238E 05	0.252E 05	0.134E 05	0.8220	-0.5288	-0.2115
58	0.216E 05	0.226E 05	0.121E 05	0.9146	0.2718	0.2993
59	0.210E 05	0.221E 05	0.246E 04	0.6984	-0.6972	0.1618
60	0.213E 05	0.225E 05	0.310E 04	0.6144	-0.7611	-0.2079
61	0.217E 05	0.229E 05	0.319E 04	0.5727	-0.8082	-0.1376
62	0.218E 05	0.230E 05	0.363E 04	0.7049	-0.6620	0.2547
63	0.298E 05	0.317E 05	0.135E 05	0.8659	-0.2623	0.4259
64	0.326E 05	0.349E 05	-0.146E 05	0.8571	-0.1927	0.4777
65	0.396E 05	0.421E 05	-0.113E 04	0.6602	-0.7385	-0.1367

THE FOLLOWING MEASUREMENTS DEVIATE CONSIDERABLY FROM THE MEAN,
ITEM LOCATION COORDS DIRECTION COSINES FACTORS
IN COMPUTING THE NEXT SET OF RESULTS, ONLY MEASUREMENTS IN THE FOLLOWING CATEGORIES WERE USED -
INCLUDED EXCLUDED
GRID
COVARIANCE MATRIX

```
 0.30030 0.20806 0.23390
 0.20806 0.17771 0.17612
 0.23390 0.17612 0.52200
```

EIGENVECTORS AND EIGENVALUES IN COLUMNS

```
 0.53300 0.59935-0.59723
 0.39979 0.44369 0.80206
 0.74570-0.66627-0.00313
 0.78360 0.19430 0.02209
```

PRINCIPAL AXES AND ASSOCIATED VARIANCE
TREND AND PLUNGE (LINEAR STRUCTURE) OF 1-AXIS
143.1 / 48.2 NORTHWEST
0.78360
TREND AND PLUNGE (LINEAR STRUCTURE) OF 2-AXIS
143.5 / 41.8 SOUTHEAST
0.19430
TREND AND PLUNGE (LINEAR STRUCTURE) OF 3-AXIS
53.3 / 0.2 NORTHEAST
0.02209

Example 4 (con't.)

```
                                              NORTH
-9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 1 0 0 1 0. 1 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9
-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 1 0. 0 1 0 1 0 1 0 0-9-9-9-9-9-9-9-9-9-9-9-9
-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 2 1 0. 0 0 0 0 0 0 0 0 1-9-9-9-9-9-9-9-9-9-9-9
-9-9-9-9-9-9-9-9 0 0 0 0 0 0 0 1 0 0 0 1. 0 0 1 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9
-9-9-9-9-9-9-9 0 0 1 0 1 0 0 0 0 0 0 0 1. 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9
-9-9-9-9-9-9 0 0 0 0 1 0 0 0 0 0 0 0 0 1. 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9
-9-9-9-9-9 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0. 0 0 0 0 0 0 0 0 0 2 0 0 0 0-9-9-9-9-9-9
-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 1 0-9-9-9-9-9-9
-9-9-9-9 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9
-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 1 1 0 0 0 0 0 1 0 1 1 0 0 0-9-9-9-9
-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0-9-9-9-9
-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0-9-9-9
-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-9-9-9
-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0-9-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
WEST.....................................(3)..................................EAST(2)
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
-9 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9
-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
-9-9 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9
-9-9-9 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
-9-9-9 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9
-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9
-9-9-9-9-9 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1. 1 0 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9
-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0 1 0-9-9-9-9-9-9
-9-9-9-9-9-9 0 0 0 0 0 0 0 0 0 0 0 1 0 0. 0 0 0 0 0 0 0 0 0 0 0 1 0-9-9-9-9-9-9-9
-9-9-9-9-9-9-9 0 0 0 0 0 0 0 0 1 0 0 0 0. 0 0 0 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9
-9-9-9-9-9-9-9-9 0 0 0 2 0 0 0 0 0 1 0 0. 1 1 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9
-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0 1 0 0 0 0. 0 0 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9
-9-9-9-9-9-9-9-9-9-9-9 0 1 0 2 0 0 0 0 1. 2 0 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9
-9-9-9-9-9-9-9-9-9-9-9-9-9 0 1 0 0 0 0 0. 0 0 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9
-9-9-9-9-9-9-9-9-9-9-9-9-9-9-9 0 0 0 0 0. 0 0 0 0-9-9-9-9-9-9-9-9-9-9-9-9-9-9-9-9
                                         SOUTH(1)
```

65 MEASUREMENTS

STEREOGRAM SHOWING DISTRIBUTION OF POINTS PROJECTED ON LOWER HEMISPHERE OF 20-CENTIMETRE WULFF NET

IN COMPUTING THE NEXT SET OF RESULTS, ONLY MEASUREMENTS IN THE FOLLOWING CATEGORIES WERE USED -

INCLUDED EXCLUDED

ALL MEASUREMENTS

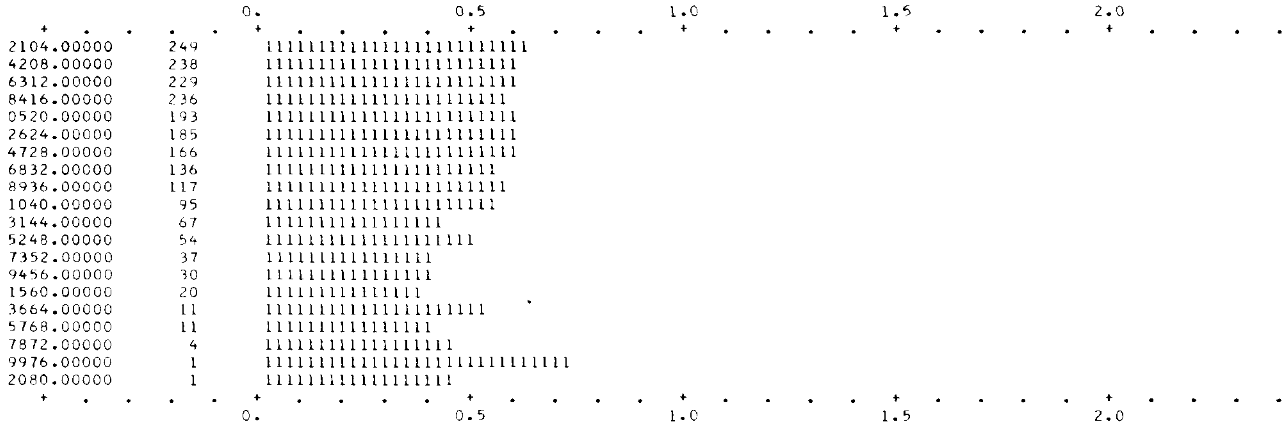
Example 4 (con't.)

DATA FOR DRAWING CROSS-SECTIONS PARALLEL TO THE PRINCIPAL AXES

SLOPE (TANGENT OF ANGLE) OF INTERSECTION OF BED AND			DISTANCE FROM ORIGIN PARALLEL TO		
1-2 PLANE	1-3 PLANE	2-3 PLANE	1-AXIS	2-AXIS	3-AXIS
-29.9695	15.1281	-1.9810	3552.6305	2565.0977	51.8434
1.0640	15.5830	0.0683	5463.1091	4587.9179	5461.2673
7.2747	8.0492	0.9038	8901.2745	8408.3206	11359.0240
-1.9604	17.4619	-0.1123	10473.7921	10151.9537	14847.1367
-8.3106	24.3581	-0.3412	13068.5886	12858.6770	18539.7073
4.2572	335.8955	0.0127	16142.2094	16359.4452	24795.9900
-1.9604	17.4619	-0.1123	7304.9454	6631.6659	-3569.3653
49.5221	3.7916	13.0609	8937.1782	8312.6816	1918.4757
2.4291	3.7694	0.6444	11119.5026	10797.5295	5690.0258
-7.0819	5.8004	-1.2209	14367.9827	14457.7910	13267.0604
-1.9792	74.3336	-0.0266	18183.1421	18668.7908	24246.6807
1.5275	38.7624	0.0394	11118.2245	11115.0380	-4708.6075
1.8685	12.5920	0.1484	12223.7352	12095.1102	-1064.0358
-4.7765	37.1747	-0.1285	14752.5343	14966.0431	3069.3065
-3.2878	21.8476	-0.1505	17242.9617	17761.3865	7762.3038
-0.1971	0.6734	-0.2928	20715.1384	21620.2314	10380.2032
-0.5387	15.3591	-0.0351	22428.4158	23498.8889	15468.9581
1.2698	64.9715	0.0195	13958.0171	14285.2610	-9373.9753
1.5566	3.4299	0.4538	14035.9203	14137.5624	-4094.6087
-0.2357	6.3931	-0.0369	19028.3857	19791.3374	1011.6332
0.2826	-1.0156	-0.2783	20364.2148	21210.9731	4298.2415
-0.4322	8.5650	-0.0505	22389.9263	23441.9304	12028.6569
18.8561	11.7946	1.5987	24735.2632	26060.4451	13400.7988
7.1379	10.3007	0.6930	16119.8330	16660.6733	-12763.0116
4.4167	6.6520	0.6640	17212.6614	17862.5613	-8237.3969
-4.2520	5.6080	-0.7582	22797.9648	24034.4397	2271.2991
-7.4426	6.6303	-1.1225	25531.4517	27038.2148	7725.2683
19.2458	10.2888	1.8706	28439.6802	30220.1145	10500.0326
0.2061	-2.0703	-0.0995	20564.1145	21770.3420	-16059.3785
-3.3017	25.6722	-0.1286	22048.0715	23332.3123	-14172.1265
20.9255	7.9565	2.6300	23082.3154	24302.5979	-6247.5521
17.8292	70.4087	0.2532	27203.8159	29077.8342	-2465.3909
7.4443	6.2351	1.1939	28009.8257	29840.3359	1698.1815
23.6832	5.8148	4.0729	30544.9956	32595.7034	6390.9913
-32.0111	79.9284	-0.4005	21458.5078	22742.4529	-16294.8795
18.8604	13.1487	1.4344	24268.6494	25911.3804	-14960.1879
24.9766	5.6961	4.3848	28963.6985	30787.0920	-4537.5038
-34.8746	2.4041	-14.5064	34379.5591	36950.1841	5811.1539
-4.6814	12.4843	-0.3750	34080.6870	35725.3032	3645.4345
1.5332	7.8831	0.1945	25556.2852	27470.6948	-20953.0276
26.5261	5.3170	4.9890	27476.2903	29324.8245	-15704.3916
-2.5018	71.8894	-0.0348	30444.1753	32453.4224	-12929.8770
11.8723	2.6009	4.5647	32249.0063	34090.0986	-8002.2536
-5.9759	3.8030	-1.5714	34931.1143	36499.0640	-3191.0604
-3.3161	122.9256	-0.0270	38040.6089	39116.5786	1541.0907
-3.2946	21.4335	-0.1537	2631.5649	1566.5126	-432.7858
-0.9949	2.6685	-0.3728	6286.7758	5349.1774	-5095.3251
-16.5118	1.4381	-11.4816	8012.7504	7399.8259	-4845.8220
-1.0481	1.0281	-1.0195	18095.7686	18720.3484	6806.7350
1.8686	1.0319	1.8109	15077.2230	15560.2009	-14128.1213
-1.2491	3.0177	-0.4139	10473.7921	10151.9537	14847.1367
-1.4366	6.6483	-0.2161	17575.0239	18081.6147	25123.9727
-0.9841	12.5144	-0.0786	17734.9399	18259.0913	25444.7981
-0.7375	3.6926	-0.1997	23464.7144	24705.6113	18274.5784
-1.3916	5.9473	-0.2340	23663.0022	24958.6772	18035.7488
-0.9369	4.6068	-0.2034	23796.2456	25109.6794	17636.4434
-1.5544	3.8858	-0.4000	23807.8564	25172.7678	13356.6912
3.3648	3.0560	1.1011	21572.3054	22616.3406	12144.8368
-1.0017	4.3174	-0.2320	21035.8210	22121.3496	2462.4421
-0.8073	2.9557	-0.2731	21348.1963	22482.9656	3104.1243
-0.7086	4.1623	-0.1703	21728.7708	22893.5195	3186.0261
-1.0647	2.7677	-0.3847	21774.6289	22957.7886	3626.3290
-3.3014	2.0333	-1.6237	29817.0659	31657.5566	13506.7198
-4.4478	1.7941	-2.4792	32558.1228	34917.2783	-14598.4816
-0.8939	4.8283	-0.1851	39602.7061	42078.1323	-1125.1902

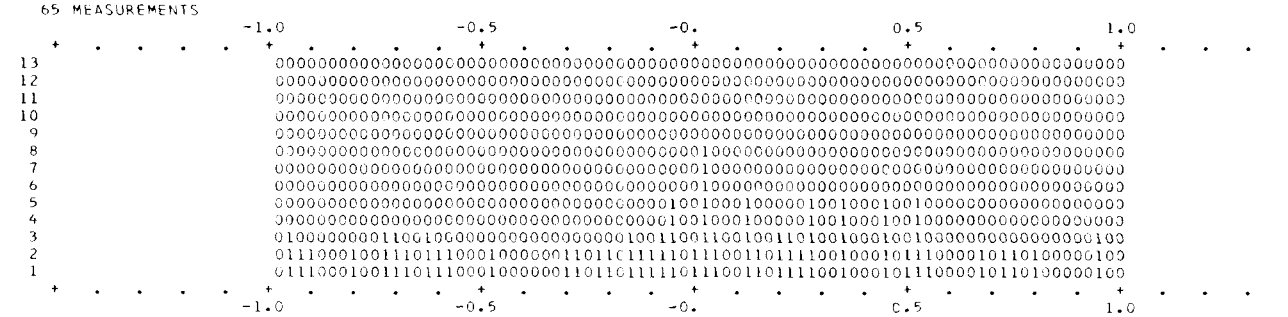
Example 4 (con't.)

```
HISTOGRAM OF MEAN ATTITUDE DEVIATION MEASURED PARALLEL TO THE 2-AXIS
  65 MEASUREMENTS
```




```
MEAN DIFFERENCE BETWEEN DIRECTION COSINES AT GIVEN DISTANCE APART

DISTANCE BETWEEN MEASUREMENTS IS PLOTTED VERTICALLY,THE SCALE IS ON THE LEFT

THE SECOND COLUMN SHOWS THE NUMBER OF VALUES REPRESENTED IN EACH ROW

     A LOW VALUE IN THE HISTOGRAM MAY INDICATE THE PRESENCE OF FOLDS OF THAT WAVELENGTH
```

```
DISTRIBUTION OF DIRECTION COSINES ABOUT THE 2-AXIS

  65 MEASUREMENTS
```




```
                              VALUE OF DIRECTION COSINES

PERCENTAGE FREQUENCY PLOTTED VERTICALLY

MOMENTS OF THE ABOVE DISTRIBUTION ARE -
1  0.0095
2  0.2398
3 -0.0208
4  0.1340

SKEWNESS AND KURTOSIS
 -0.1767       2.3288
```

Example 4 (con't.)

```
HISTOGRAM OF MEAN ATTITUDE DEVIATION MEASURED PARALLEL TO THE 3-AXIS
  65 MEASUREMENTS
                              0.                  0.5                 1.0                 1.5                 2.0
   +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .
1272.00000      126       1111111111
2544.00000      120       1111111111
3816.00000      121       111111111
5088.00000      111       1111111111
6360.00000      104       1111111111
7632.00000      113       1111111111
8904.00000      115       1111111111
0176.00000      105       11111111111
1448.00000      119       11111111111
2720.00000      114       1111111111
3992.00000       82       111111111
5264.00000       82       111111111
6536.00000       85       1111111111
7808.00000       80       1111111111
9080.00000       81       11111111111
0352.00000       67       1111111111
1624.00000       57       111111111111
2896.00000       67       1111111111
4168.00000       43       11111111
5440.00000       30       1111111
6712.00000       40       1111111111
   +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .
                              0.                  0.5                 1.0                 1.5                 2.0
```

MEAN DIFFERENCE BETWEEN DIRECTION COSINES AT GIVEN DISTANCE APART

DISTANCE BETWEEN MEASUREMENTS IS PLOTTED VERTICALLY,THE SCALE IS ON THE LEFT

THE SECOND COLUMN SHOWS THE NUMBER OF VALUES REPRESENTED IN EACH ROW

A LOW VALUE IN THE HISTOGRAM MAY INDICATE THE PRESENCE OF FOLDS OF THAT WAVELENGTH

```
DISTRIBUTION OF DIRECTION COSINES ABOUT THE 3-AXIS

  65 MEASUREMENTS
                         -1.0                -0.5                -0.                 0.5                 1.0
   +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .
13                          00000000000000000000000000000000000000000000000000000000000000000000000000000000
12                          00000000000000000000000000000000000000000000000000000000000000000000000000000000
11                          00000000000000000000000000000000000000000000000000000000000000000000000000000000
10                          00000000000000000000000000000000000000000000000000000000000000000000000000000000
9                           00000000000000000000000000000000000000000000000000000000000000000000000000000000
8                           00000000000000000000000000000000000000011000000000000000000000000000000000000000
7                           00000000000000000000000000000000000000011000000000000000000000000000000000000000
6                           00000000000000000000000000000000000101011001010001000000000000000000000000000000
5                           00000000000000000000000000000000110111111001110001000000000000000000000000000000
4                           00000000000000000000000000000000110111111001110001000000000000000000000000000000
3                           00000000000000000000000000001010110111111111110011100000000000000000000000000000
2                           00000000000000010000000110001010110111111111111011100000101001001000000000000000
1                           00000000000000010000000110001010110111111111111011100000101001001000000000000000
   +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .    +    .    .    .    .
                         -1.0                -0.5                -0.                 0.5                 1.0

                                                       VALUE OF DIRECTION COSINES
```

PERCENTAGE FREQUENCY PLOTTED VERTICALLY

MOMENTS OF THE ABOVE DISTRIBUTION ARE -

1 0.0401
2 0.0462
3 0.0071
4 0.0089

SKEWNESS AND KURTOSIS

0.7117 4.1555

Example 4 (con't.)

```
IN COMPUTING THE NEXT SET OF RESULTS, ONLY MEASUREMENTS IN THE FOLLOWING CATEGORIES WERE USED -
INCLUDED                            EXCLUDED
GRID



          ANALYSIS OF VARIANCE ABOUT 2-AXIS

NUMBER IN SAMPLE, MEAN AND VARIANCE FOR SUBGROUP.
           45.    -0.0979    0.1943   GRID

GRAND MEAN AND TOTAL VARIANCE
           45.    -0.0979    0.1943

NUMBER IN SAMPLE, MEAN AND VARIANCE FOR SUBGROUP.
            3.    -0.2732    0.0822   SNDSTN
           10.    -0.0358    0.0301   SS+SH
            5.     0.0098    0.1006   SHALE
           27.    -0.1214    0.2849   CARBNT

GRAND MEAN AND TOTAL VARIANCE
           45.    -0.0979    0.1943

NUMBER IN SAMPLE, MEAN AND VARIANCE FOR SUBGROUP.
            1.    -0.2083    0.0434   PRECMB
            2.    -0.2242    0.0544   L.CAMB
            1.    -0.4538    0.2059   L.CAMB
            3.     0.2341    0.1062   M.CAMB
            6.     0.0624    0.0997   U.CAMB
           14.    -0.1789    0.2751   L.ORD
           18.    -0.1038    0.2010   M.ORD

GRAND MEAN AND TOTAL VARIANCE
           45.    -0.0979    0.1943

          ANALYSIS OF VARIANCE ABOUT 3-AXIS

NUMBER IN SAMPLE, MEAN AND VARIANCE FOR SUBGROUP.
           45.     0.0104    0.0221   GRID

GRAND MEAN AND TOTAL VARIANCE
           45.     0.0104    0.0221

NUMBER IN SAMPLE, MEAN AND VARIANCE FOR SUBGROUP.
            3.    -0.0906    0.0211   SNDSTN
           10.    -0.0686    0.0285   SS+SH
            5.    -0.0280    0.0461   SHALE
           27.     0.0580    0.0154   CARBNT

GRAND MEAN AND TOTAL VARIANCE
           45.     0.0104    0.0221

NUMBER IN SAMPLE, MEAN AND VARIANCE FOR SUBGROUP.
            1.    -0.0781    0.0061   PRECMB
            2.    -0.1294    0.0315   L.CAMB
            1.    -0.0509    0.0026   L.CAMB
            3.     0.1329    0.0268   M.CAMB



            6.     0.0677    0.0125   U.CAMB
           14.     0.0597    0.0151   L.ORD
           18.    -0.0436    0.0309   M.ORD

GRAND MEAN AND TOTAL VARIANCE
           45.     0.0104    0.0221

IN COMPUTING THE NEXT SET OF RESULTS, ONLY MEASUREMENTS IN THE FOLLOWING CATEGORIES WERE USED -
INCLUDED                            EXCLUDED
UNUSU

SELECTED SUBGROUP HAD ONLY 20 DATA ITEMS.  COMPUTATIONS ON SUBGROUP WERE OMITTED.

IN COMPUTING THE NEXT SET OF RESULTS, ONLY MEASUREMENTS IN THE FOLLOWING CATEGORIES WERE USED -
INCLUDED                            EXCLUDED
GRID
                                     PRECMB
                                     L.ORD
                                     M.ORD

SELECTED SUBGROUP HAD ONLY 12 DATA ITEMS.  COMPUTATIONS ON SUBGROUP WERE OMITTED.

CONTROL CARD  -0   -0   -0   -0   -0   -0   -0   -0   -0   -0   -0   -0   -0   -0   -0   -0   -0   -0
```

- APPENDIX -

LISTING OF FORTRAN IV PROGRAM

THE MAIN PROGRAMS ARE -
PROGV1
PROGV2
PROGV3
PROGV4
PROGS5

THE SUBROUTINES ARE -
ANOV (ANALYSIS OF VARIANCE)
AXES (PRINCIPAL AXES)
CONFIT (BEST-FIT CONICAL SURFACE)
ENISOC (STRIKE AND DIP FROM DIRECTION COSINES)
EQUATE (READS ARRAYS INTO S OR T)
HOWBIG (FOLD SIZE)
MOMENT (MOMENTS OF DISTRIBUTION ABOUT PRINCIPAL AXES)
READIN (READS IN DATA FROM DATA CARDS)
SCALAR (COVARIANCE MATRIX FOR SCALAR DATA)
SEKSHN (COMPUTES DATA FOR DRAWING CROSS-SECTIONS)
SELECT (RETAINS SPECIFIED SUBSETS OF THE DATA IN S AND T)
SOLSP (SOLVES SIMULTANEOUS EQUATIONS)
SPIN (TRANSFORMS MEASUREMENTS TO REFER TO PRINCIPAL AXES)
WULFF (PLOTS VECTORS ON STEREOGRAM)

ASSISTANCE IN THE WRITING OF THESE PROGRAMS WAS GIVEN BY
MRS BENSON AND OTHER MEMBERS OF THE STAFF OF THE
NORTHWESTERN UNIVERSITY COMPUTING CENTER.

```
$IBJOB
$IBFTC PROGV1
C - - - - - - -
C      THIS IS THE MAIN PROGRAM.   USE ONE 'PROG' DECK ONLY.
C      SUBROUTINES REQUIRED. AXES,CONFIT,ENISOC,EQUATE,SOLSP,SPIN,WULFF
C
C      DATA CARDS SHOULD CONTAIN AZIMUTH OF DIP (IN DEGREES), IN
C         COLUMNS 31 TO 33, AND AMOUNT OF DIP IN COLUMNS 37 TO 39.
C         DECIMAL FRACTIONS OF A DEGREE CAN BE INSERTED IN COLUMNS 34
C         TO 36 AND 40 TO 42.
C      EACH GROUP SHOULD CONTAIN LESS THAN  300 ITEMS AND SHOULD BE
C         TERMINATED BY A CARD WITH 999000 IN COLUMNS 31 TO 36.
C         INSERT 999000 IN COLUMNS 37 TO 42 OF THE FINAL CARD OF THE LAST
C         GROUP.
C
C      FOR LINEAR STRUCTURES NOTE COMMENT CARD BELOW
```

```
C
      COMMON    COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
C          IN PROGV1 OR PROGV2, THE DIMENSIONS CAN BE INCREASED FROM 300
C     TO 500 ITEMS PER BATCH OF DATA.   TO DO SO, REPLACE 300 BY 500
C     WHEREVER 300 IS PUNCHED IN EACH OF THE THREE DIMENSION CARDS AT
C     THE HEAD OF THE MAIN PROGRAM AND THE SUBROUTINES.
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
C     PUT NOVEC = 0, TO INDICATE THAT THE DATA ARE VECTORS.
      NOVEC = 0
      I = 1
 6    WRITE (6,102)
 3    READ (5,100)   DIRCOS(I,1), DIRCOS(I,2)
      IF (DIRCOS(I,1) - 999.0)   1,2,1
 1    CONTINUE
C     A CORRECTION CARD (E.G. DIRCOS(I,1)=DIRCOS(I,1)+90.0) MAY BE USED.
      DIRCOS(I,1) = DIRCOS(I,1) + 90.0
      WRITE (6,103)  I, DIRCOS(I,1), DIRCOS(I,2)
C
C     FOR LINEAR STRUCTURES, INSERT A CARD  DIRCOS(I,2)=DIRCOS(I,2)+90.0
C
      A = DIRCOS(I,1) * 3.14159 / 180.0
      B = DIRCOS(I,2) * 3.14159 / 180.0
      DIRCOS(I,1) = COS(A) * SIN(B) * (-1.0)
      DIRCOS(I,2) = SIN(A) * SIN(B)
      DIRCOS(I,3) = COS(B)
      WRITE (6,104)   (DIRCOS(I,J),J=1,3)
C
      I = I + 1
      GO TO 3
C
 2    KOUNT = I - 1
      CALL EQUATE (DIRCOS,T,KOUNT,KOUNT2,COVMAT,D)
C     OMIT NEXT CARD IF STEREOGRAM IS NOT REQUIRED.
      CALL WULFF
C     OMIT NEXT CARD IF BEST-FIT CONE IS NOT REQUIRED.
      CALL CONFIT(0)
C     OMIT NEXT SEVEN CARDS IF ONLY A BEST-FIT CONE IS REQUIRED.
      CALL AXES
C
C     OMIT NEXT FOUR CARDS IF ROTATED VECTORS ARE NOT REQUIRED.
      WRITE (6,105)
      CALL SPIN (0,0)
      CALL EQUATE (TRACOS,T,KOUNT,KOUNT2,COVMAT,D)
      CALL WULFF
C
      IF (DIRCOS(KOUNT+1, 2) - 999.0)   4,5,4
 4    I = 1
      WRITE (6,101)
      GO TO 6
 5    CONTINUE
C
 100  FORMAT (30X, 2F6.3)
 101  FORMAT (1H1,50X,37H---      NEXT  BATCH  OF  DATA      ---)
 102  FORMAT (1HO,4HITEM,29X,25HAZIMUTH AND AMOUNT OF DIP,4X,
```

```
     1   26HCOMPUTED DIRECTION COSINES)
 103  FORMAT (1H+, I4,29X, 3F8.3)
 104  FORMAT (X, 62X, 3F8.5)
 105  FORMAT (1HO, 30X, 57HTHE MEASUREMENTS BELOW ARE IN TERMS OF THE PR
     1INCIPAL AXES)
      STOP
       END
```

```
$IBJOB
$IBFTC PROGV2
C - - - - - - -
C      THIS IS THE MAIN PROGRAM.   USE ONE 'PROG' DECK ONLY.
C
C      SUBROUTINES REQUIRED. AXES,CONFIT,ENISOC,EQUATE,MOMENT,SOLSP,SPIN.
C      SUBROUTINES REQUIRED, WULFF
C      DATA CARD LAYOUT AS IN PROG1.
C
       COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
C         IN PROGV1 OR PROGV2, THE DIMENSIONS CAN BE INCREASED FROM 300
C      TO 500 ITEMS PER BATCH OF DATA.   TO DO SO, REPLACE 300 BY 500
C      WHEREVER 300 IS PUNCHED IN EACH OF THE THREE DIMENSION CARDS AT
C      THE HEAD OF THE MAIN PROGRAM AND THE SUBROUTINES.
       DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
C      PUT NOVEC = 0, TO INDICATE THAT THE DATA ARE VECTORS.
       NOVEC = 0
       I = 1
 7     WRITE (6,102)
 3     READ (5,100)   DIRCOS(I,1), DIRCOS(I,2)
       IF (DIRCOS(I,1) - 999.0)    1,2,1
 1     CONTINUE
C      A CORRECTION CARD (E.G. DIRCOS(I,1)=DIRCOS(I,1)+90.0) MAY BE USED.
       DIRCOS(I,1) = DIRCOS(I,1) + 90.0
       WRITE (6,103)  I, DIRCOS(I,1), DIRCOS(I,2)
C
C      FOR LINEAR STRUCTURES, INSERT A CARD  DIRCOS(I,2)=DIRCOS(I,2)+90.0
C
       A = DIRCOS(I,1) * 3.14159 / 180.0
       B = DIRCOS(I,2) * 3.14159 / 180.0
       DIRCOS(I,1) = COS(A) * SIN(B) * (-1.0)
       DIRCOS(I,2) = SIN(A) * SIN(B)
       DIRCOS(I,3) = COS(B)
       WRITE (6,104)   (DIRCOS(I,J),J=1,3)
       I = I + 1
       GO TO 3
C
 2     KOUNT = I - 1
       CALL EQUATE (DIRCOS,T,KOUNT,KOUNT2,COVMAT,D)
       CALL WULFF
       CALL AXES
       WRITE (6,105)
```

```
      CALL SPIN (0,1)
      CALL EQUATE (TRACOS,T,KOUNT,KOUNT2,COVMAT,D)
      CALL WULFF
C     OMIT NEXT CARD IF BEST-FIT CONE IS NOT REQUIRED.
      CALL CONFIT (1)
C     OMIT NEXT TWO CARDS IF MOMENTS OF THE DISTRIBUTION ARE NOT NEEDED.
      DO 6   IAXIS=1,3
 6    CALL MOMENT (IAXIS)
C
      IF (DIRCOS(KOUNT+1, 2) - 999.0)   4,5,4
 4    I = 1
      WRITE (6,101)
      GO TO 7
 5    CONTINUE
C
 100  FORMAT (30X, 2F6.3)
 101  FORMAT (1H1,50X,37H---      NEXT  BATCH  OF  DATA      ---)
 102  FORMAT (1H0,4HITEM,29X,25HAZIMUTH AND AMOUNT OF DIP,4X,
     1   26HCOMPUTED DIRECTION COSINES)
 103  FORMAT (1H+, I4,29X, 3F8.3)
 104  FORMAT (1HX, 62X, 3F8.5)
 105  FORMAT (1H0, 25X, 57HTHE MEASUREMENTS BELOW ARE IN TERMS OF THE PR
     1INCIPAL AXES)
      STOP
       END




$IBJOB
$IBFTC PROGV3
C - - - - - -
C     THIS IS THE MAIN PROGRAM.   USE ONE 'PROG' DECK ONLY.
C
C     SUBROUTINES REQUIRED ARE AXES, CONFIT, ENISOC, EQUATE, HOWBIG,
C        MOMENT,SEKSHN,SOLSP,SPIN,WULFF
C     DATA CARD LAYOUT AS IN PROG1,  TOGETHER WITH GEOGRAPHICAL
C        COORDINATES IN COLUMNS 13 TO 18 AND 19 TO 24, AND HEIGHT
C        COORDINATE IN COLUMNS 25 TO 30.   A DECIMAL POINT IS ASSUMED
C        AFTER THE SIXTH DIGIT, UNLESS PUNCHED ELSEWHERE.
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
C     PUT NOVEC = 0, TO INDICATE THAT THE DATA ARE VECTORS.
      NOVEC = 0
      I = 1
      WRITE (6,102)
 3    READ(5,100)   (GCOORD(I,J),J=1,3),DIRCOS(I,1),DIRCOS(I,2)
      IF (DIRCOS(I,1) - 999.0)   1,2,1
 1    CONTINUE
C     A CORRECTION CARD (E.G. DIRCOS(I,1)=DIRCOS(I,1)+90.0) MAY BE USED.
      DIRCOS(I,1) = DIRCOS(I,1) + 90.0
      WRITE (6,103) I,(GCOORD(I,J),J=1,3),DIRCOS(I,1),DIRCOS(I,2)
```

```
C      FOR LINEAR STRUCTURES, INSERT A CARD  DIRCOS(I,2)=DIRCOS(I,2)+90.0
       A = DIRCOS(I,1) * 3.14159 / 180.0
       B = DIRCOS(I,2) * 3.14159 / 180.0
       DIRCOS(I,1) = COS(A) * SIN(B) * (-1.0)
       DIRCOS(I,2) = SIN(A) * SIN(B)
       DIRCOS(I,3) = COS(B)
       WRITE (6,104)   (DIRCOS(I,J),J=1,3)
C
       I = I + 1
       GO TO 3
C
 2     KOUNT = I - 1
       CALL EQUATE (DIRCOS,T,KOUNT,KOUNT2,COVMAT,D)
       CALL WULFF
       CALL AXES
       WRITE (6,107)
       CALL SPIN (2,1)
       CALL EQUATE (TRACOS,T,KOUNT,KOUNT2,COVMAT,D)
       CALL EQUATE (SCOORD,S,KOUNT,KOUNT2,COVMAT,D)
C      IF OUTPUT OF PUNCHED CARDS CONTAINING TRANSFORMED DATA IS WANTED,
C      INSERT TWO CARDS AS FOLLOWS,
C      DO 7   I=1,KOUNT
C7     WRITE (6,101)   (SCOORD(I,J),J=1,3),(TRACOS(I,J),J=1,3)
       CALL WULFF
C      OMIT NEXT CARD IF BEST-FIT CONE IS NOT REQUIRED.
       CALL CONFIT (1)
       DO 6   IAXIS=1,3
       CALL HOWBIG (IAXIS)
 6     CALL MOMENT (IAXIS)
       CALL SEKSHN
       IF (DIRCOS(KOUNT+1, 2) - 999.0)   4,5,4
 4     I = 1
       WRITE (6,106)
       WRITE (6,102)
       GO TO 3
 5     CONTINUE
C
 100   FORMAT (12X, 5F6.3)
 101   FORMAT (1HP, 12X, 3F6.0, 3F6.3)
 102   FORMAT (1H0,4HITEM,X,24HGEOGRAPHICAL COORDINATES,4X,
      1 25HAZIMUTH AND AMOUNT OF DIP,4X,26HCOMPUTED DIRECTION COSINES)
 103   FORMAT (1H+, I4, 3F8.3, 4X, 2F8.3)
 104   FORMAT (X, 62X, 3F8.5)
 106   FORMAT (1H1,50X,37H---     NEXT  BATCH  OF  DATA     ---)
 107   FORMAT (1H0,25X,61HTHE FOLLOWING MEASUREMENTS ARE IN TERMS OF THE
      1PRINCIPAL AXES)
       STOP
        END




$IBJOB
$IBFTC PROGV4
C - - - - - - -
C      THIS IS THE MAIN PROGRAM.   USE ONE 'PROG' DECK ONLY.
C
```

```
C       SUBROUTINES REQUIRED ARE ANOV, AXES, CONFIT, ENISOC, EQUATE,
C          HOWBIG, MOMENT, READIN, SEKSHN, SELECT, SOLSP, SPIN, WULFF
C       LAYOUT OF NUMERICAL DATA AS IN PROG3, TOGETHER WITH FACTORS
C         WHICH MAY BE LISTED IN THREE-DIGIT FIELDS IN COLUMNS 49 TO 66.
C       THE CATEGORY , OR LEVEL, OF EACH FACTOR IS REPRESENTED BY AN
C         INTEGER CODE NUMBER FROM 1 TO 9.
C       THIS PROGRAM ALSO ACCEPTS ATTITUDE MEASUREMENTS IN THE FORM OF
C          DIRECTION COSINES PUNCHED IN COLUMNS 31-36, 37-42, 43-48.
C
C       NUMERICAL DATA SHOULD BE PRECEDED BY CARDS 1) TO 9), CONTAINING
C         1) DATE, PROJECT NAME, AND USER NAME WITHIN COLUMNS 13 TO 72.
C         2) FORMAT STATEMENT (IF NON-STANDARD) IN COLUMNS 13 TO 48, AND
C          CORRECTION FACTORS TO MULTIPLY THE GEOGRAPHICAL COORDINATES,
C          (49 TO 54), AND HEIGHT COORDINATES(55 TO 60), AND TO ADD TO THE
C          AZIMUTH MEASUREMENTS (61 TO 66).
C         3) TO 8) NAMES OF THE CATEGORIES REPRESENTED BY INTEGER CODE
C          NUMBERS 1 TO 9 ON THE DATA CARDS, LISTED IN SEQUENCE IN
C          SIX-COLUMN ALPHAMERIC FIELDS BETWEEN COLUMNS 13 AND 66.
C          ONE CARD IS USED FOR EACH OF THE FACTORS, WHICH ARE IN THE SAME
C          SEQUENCE AS ON THE DATA CARDS.
C         ANY OR ALL OF THE CARDS (1) TO (8) MAY BE LEFT BLANK.
C         9) A CARD PUNCHED 999. IN THE AZIMUTH COLUMNS (31 TO 36).
C
C       NUMERICAL DATA BEGIN AT CARD 10 AND END AT A CARD PUNCHED
C          999000999000 IN THE AZIMUTH AND DIP COLUMNS (USUALLY 31 TO 42).
C          IF A CHANGE OF FORMAT OR OF CORRECTION FACTORS IS REQUIRED
C          WITHIN THE DATA DECK, INSERT A NEW FORMAT CARD
C          AT THE APPROPRIATE POINT, PRECEDED AND FOLLOWED BY CARDS
C          PUNCHED 999. IN THE AZIMUTH COLUMNS.
C
C       AFTER THE NINES CARD TERMINATING THE DATA THERE SHOULD BE ONE OR
C         MORE CONTROL CARDS, EACH SPECIFYING THE SUBSETS OF MEASUREMENTS
C         TO BE USED FOR DETERMINATION OF (1) THE CONE AXIS, (2) THE
C         PRINCIPAL AXES, (3)STATISTICAL MOMENTS AND CROSS-SECTION DATA
C         (4) UP TO 6 SUBSETS WITHIN WHICH AN ANALYSIS OF
C         VARIANCE IS TO BE PERFORMED.   THE SAME PROCEDURES WILL THEN
C         BE FOLLOWED USING NEW SUBSETS SPECIFIED ON SUBSEQUENT CONTROL
C         CARDS.   A BLANK CARD INDICATES THE END OF THE PROGRAM, AND A
C         CARD WITH -99-99-99 IN COLUMNS 13 TO 21 INDICATES THAT ANOTHER
C         BATCH OF DATA, PRECEDED AS BEFORE BY NINE MASTER CARDS,
C         FOLLOWS.   SUBSETS ARE CODED AS SUCCESSIVE THREE-DIGIT WORDS IN
C         COLUMNS 13 TO 72 OF THE CONTROL CARD.   WITHIN EACH THREE-DIGIT
C         WORD, THE FACTOR IS INDICATED BY THE CODE NUMBER IN COLUMN 2,
C         THE CATEGORY BY THE NUMBER IN COLUMN 3.   A NEGATIVE SIGN IN
C         COLUMN 1 CAUSES MEASUREMENTS BELONGING TO INDICATED CATEGORY TO
C         BE REMOVED BY SUBROUTINE SELECT, OTHERWISE THE CATEGORY IS
C         RETAINED AND EVERYTHING ELSE DISCARDED.   A NON-ZERO NUMBER IN
C         COLUMN 1 INDICATES THE BEGINNING OF A NEW SUBSET.   IF COLUMNS
C         2 AND 3 ARE ZERO, S,T AND L ARE LEFT UNCHANGED.  IF COLUMN 1 IS
C         ALSO ZERO,NUMB IS PUT EQUAL TO 9 AND CONTROL IS RETURNED TO THE
C         MAIN PROGRAM.
C
       COMMON    COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1    L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
       DIMENSICN    COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
```

```
C     PUT NOVEC = 0, TO INDICATE THAT THE DATA ARE VECTORS.
      NOVEC = 0
 4    CALL READIN
C     READ CONTROL CARD.
 3    READ (5,100)   (KONTRL(I), I=1,20)
      WRITE (6,110)   (KONTRL(I),I=1,20)
      IF (KONTRL(1) + KONTRL(2) + KONTRL(3))  10,6,5
 10   IF (KONTRL(1) + KONTRL(2) + KONTRL(3) + 3*99)   5,4,5
 5    CONTINUE
      CALL EQUATE (DIRCOS,T,KOUNT,KOUNT2,COVMAT,D)
C     OMIT NEXT CARD IF STEREOGRAM IS NOT REQUIRED.
      CALL WULFF
C     OMIT NEXT TWO CARDS IF BEST-FIT CONE IS NOT REQUIRED.
      CALL SELECT (1)
      IF (KOUNT2 - 20)   20,20,21
 20   WRITE (6,109)   KOUNT2
      GO TO 22
 21   CALL CONFIT (1)
 22   CALL EQUATE (DIRCOS,T,KOUNT,KOUNT2,COVMAT,D)
      CALL EQUATE (GCOORD,S,KOUNT,KOUNT2,COVMAT, D)
      CALL SELECT (2)
      IF (KOUNT2 - 20)   23,23,24
 23   WRITE (6,109)   KOUNT2
      GO TO 6
 24   CALL AXES
C
C     OMIT NEXT 34 CARDS IF BEST-FIT CONE AND PRINCIPAL AXES ONLY ARE
C        REQUIRED.
      WRITE (6,105)
      CALL SPIN (2,1)
      CALL EQUATE (TRACOS,T,KOUNT,KOUNT2,COVMAT,D)
      CALL EQUATE (SCOORD,S,KOUNT,KOUNT2,COVMAT,D)
      CALL WULFF
C
C     IF OUTPUT OF PUNCHED CARDS CONTAINING TRANSFORMED DATA IS WANTED,
C     INSERT THREE CARDS AS FOLLOWS,
C     DO 8   I=1,KOUNT
C8    WRITE(6,101) (SCOORD(I,J),J=1,3),(TRACOS(I,J),J=1,3),(LIST(I,J),
C    1  J=1,6)
C
C     IF CROSS-SECTION DATA, HISTOGRAM OF FOLD SIZES AND MOMENTS OF
C        THE DISTRIBUTION ARE NOT REQUIRED, OMIT NEXT NINE CARDS.
      CALL SELECT(3)
      IF (KOUNT2 - 30)   25,25,26
 25   WRITE (6,109)   KOUNT2
      GO TO 27
 26   CALL SEKSHN
      DO 9   IAXIS=1,3
      CALL HOWBIG (IAXIS)
 9    CALL MOMENT (IAXIS)
 27   CONTINUE
C     OMIT NEXT TEN CARDS IF ANALYSIS OF VARIANCE IS NOT REQUIRED.
      NUMB = 4
 31   CALL EQUATE (TRACOS,T,KOUNT,KOUNT2,COVMAT,D)
      CALL SELECT (NUMB)
      IF (KOUNT2 - 30)   28,28,29
 28   WRITE (6,109)   KOUNT2
      NUMB = NUMB + 1
```

```
      GO TO 2
 29   DO 7    IAXIS=1,3
 7    CALL ANOV (IAXIS)
      NUMB = NUMB + 1
 2    IF (NUMB - 9)    31,31,30
C                         DO NOT REMOVE CARDS BELOW THIS POINT.
C
 30   GO TO 3
 6    CONTINUE
 100  FORMAT (12X,20I3)
 101  FORMAT (1HP,12X,3F6.0,3F6.3,6I3)
 105  FORMAT (1H0, 25X, 57HTHE MEASUREMENTS BELOW ARE IN TERMS OF THE PR
     1INCIPAL AXES)
 109  FORMAT (1H0, 26HSELECTED SUBGROUP HAD ONLY, I3, 52H DATA ITEMS.  C
     1OMPUTATIONS ON SUBGROUP WERE OMITTED.)
 110  FORMAT (1H0, 12HCONTROL CARD, I5,19I6)
      STOP
       END
```

```
$IBJOB
$IBFTC PROGS5
C - - - - - - -
C     THIS IS THE MAIN PROGRAM.   USE ONE 'PROG' DECK ONLY.
C
C     THE PROGRAM REQUIRES SCALAR DATA, NOT STRIKE AND DIP.
C
C     SUBROUTINES REQUIRED ARE ANOV, AXES, CONFIT, ENISOC, EQUATE,
C        HOWBIG, MOMENT, READIN, SCALAR, SELECT, SOLSP, SPIN, WULFF
C     DATA LAYOUT AS IN PROG4, WITH GCOORD(I,3), LISTED IN COLUMNS 25
C        TO 30, BEING THE ELEVATION OF A PARTICULAR HORIZON.
C        A CORRECTION FACTOR, SUCH AS GROUND ELEVATION, MAY BE RECORDED
C        IN COLUMNS 31 TO 36.   IT WILL BE SUBTRACTED FROM GCOORD(I,3).
C        OTHERWISE, AZIMUTH AND DIP COLUMNS SHOULD BE LEFT BLANK.
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
C     PUT NOVEC = 1 TO INDICATE THAT THE DATA ARE SCALAR MEASUREMENTS.
      NOVEC = 1
 4    CALL READIN
C     READ CONTROL CARD.
 3    READ (5,100)    (KONTRL(I), I=1,20)
      IF (KONTRL(1) + KONTRL(2) + KONTRL(3))  10,6,5
 10   IF (KONTRL(1) + KONTRL(2) + KONTRL(3) + 3*99)    5,4,5
 5    CONTINUE
      CALL EQUATE (GCOORD,S,KOUNT,KOUNT2,COVMAT,D)
      CALL WULFF
C     OMIT NEXT SIX  CARDS IF BEST-FIT CONE IS NOT REQUIRED.
      CALL SELECT (1)
      IF (KOUNT2 - 20)    20,20,21
 20   WRITE (6,109)    KOUNT2
```

```
      GO TO 22
 21   CALL SCALAR
      CALL CONFIT (0)
 22   CALL EQUATE (GCOORD,S,KOUNT,KOUNT2,COVMAT,D)
      CALL SELECT (2)
      IF (KOUNT2 - 20)   23,23,24
 23   WRITE (6,109)   KOUNT2
      GO TO 6
 24   CALL SCALAR
      CALL AXES
C
C     OMIT NEXT 31 CARDS IF ONLY BEST-FIT CONE AND PRINCIPAL AXES ARE
C        REQUIRED.
      WRITE (6,105)
      CALL SPIN (1,0)
C     IF OUTPUT OF PUNCHED CARDS CONTAINING TRANSFORMED DATA IS WANTED,
C     INSERT TWO CARDS AS FOLLOWS,
C     DO 8   I=1,KOUNT
C8    WRITE (6,101)   (SCOORD(I,J),J=1,3), (LIST(I,J),J=1,6)
C
C     OMIT NEXT TEN CARDS IF MOMENTS AND HISTOGRAM OF FOLD SIZES ARE
C        NOT REQUIRED.
      CALL EQUATE (SCOORD,S,KOUNT,KOUNT2,COVMAT,D)
      CALL SELECT(3)
      IF (KOUNT2 - 20)   25,25,26
 25   WRITE (6,109)   KOUNT2
      GO TO 27
 26   DO 9   IAXIS=1,3
      CALL HOWBIG (IAXIS)
 9    CALL MOMENT (IAXIS)
 27   CONTINUE
C     OMIT NEXT CARD IF STEREOGRAM OF ROTATED VECTORS IS NOT REQUIRED.
      CALL WULFF
C     OMIT NEXT TEN CARDS IF ANALYSIS OF VARIANCE IS NOT REQUIRED.
      NUMB = 4
   31 CALL EQUATE (SCOORD,S,KOUNT,KOUNT2,COVMAT,D)
      CALL SELECT (NUMB)
      IF (KOUNT2 - 30)   28,28,29
 28   WRITE (6,109)   KOUNT2
      NUMB = NUMB + 1
      GO TO 2
 29   DO 7   IAXIS=1,3
 7    CALL ANOV (IAXIS)
      NUMB = NUMB + 1
 2    IF (NUMB - 9)   31,31,30
C                DO NOT REMOVE CARDS BELOW THIS POINT.
 30   GO TO 3
 6    CONTINUE
 100  FORMAT (12X,20I3)
 101  FORMAT (1HP,12X,3F6.0,18X,6I3)
 105  FORMAT (1H0, 25X, 57HTHE MEASUREMENTS BELOW ARE IN TERMS OF THE PR
     1INCIPAL AXES)
 109  FORMAT (1H0, 26HSELECTED SUBGROUP HAD ONLY, I3, 52H DATA ITEMS.  C
     1OMPUTATIONS ON SUBGROUP WERE OMITTED.)
 110  FORMAT (1H0, 12HCONTROL CARD, I5,19I6)
      STOP
       END
```

```
$IBFTC ANOV
C
      SUBROUTINE ANOV (IAXIS)
C     - - - - - - - - - - - - -
C     THE MEANS AND VARIANCES ARE COMPUTED AND PRINTED FOR INDIVIDUAL
C        SUBGROUPS OF THE DATA.   ONE FACTOR IS CONSIDERED AT A TIME.
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
      DIMENSION SUM0(9), SUM1(9), SUM2(9)
C
      WRITE (6,100)   IAXIS
      DO 10   J=1,6
C     INITIALIZE
      ITOTAL = 0
      GSUM0 =0.0
      GSUM1 = 0.0
      GSUM2 = 0.0
      DO 1   M=1,9
      ITOTAL = ITOTAL + L(M,J)
      SUM0(M) = 0.0
      SUM1(M) = 0.0
 1    SUM2(M) = 0.0
C
C     CHECK THAT FACTOR HAS BEEN CODED.
      IF (ITOTAL)   19,10,19
 19   IF (NOVEC - 1)   6,7,6
C     SPLIT INTO CATEGORIES, COUNT, FIND SUMS AND SUMS OF SQUARES.
C
C     THE NEXT SECTION IS FOR VECTORIAL DATA ONLY.
 6    DO 2   I=1,KOUNT2
      M = L(I,J)
      SUM0(M) = SUM0(M) + 1.0
      SUM1(M) = SUM1(M) + T(I, IAXIS)
 2    SUM2(M) = SUM2(M) + T(I,IAXIS)**2
      GO TO 8
C
C     THE NEXT SECTION IS FOR SCALAR DATA ONLY.
C     COMPUTE DIRECTION COSINES FROM ELEVATIONS.
 7    DO 11  I=1,KOUNT2
      M = L(I,J)
      DO 11   K=1,KOUNT2
      IF (I-K)   18,11,18
 18   IF (L(K,J) - M)   11,13,11
 13   D(1) = S(I,1) - S(K,1)
      D(2) = S(I,2) - S(K,2)
      D(3) = S(I,3) - S(K,3)
      P = D(1)**2 + D(2)**2 + D(3)**2
      P = SQRT(1.0/P)
      D(1) = D(1)*P
      D(2) = D(2) * P
```

```
      D(3) = D(3)*P
      SUMO(M) = SUMO(M) + 1.0
      SUM1(M) = SUM1(M) + D(IAXIS)
 9    SUM2(M) = SUM2(M) + D(IAXIS)**2
C
C
 11   CONTINUE
C     FIND MEAN AND VARIANCES
 8    WRITE (6,101)
      DO 5   M=1,9
      IF (SUMO(M) - 0.9)   5,5,4
 4    CONTINUE
      GSUMO = GSUMO + SUMO(M)
      GSUM1 = GSUM1 + SUM1(M)
      GSUM2 = GSUM2 + SUM2(M)
      SUM1(M) = SUM1(M) / SUMO(M)
      SUM2(M) = SUM2(M)/SUMO(M)
      WRITE (6,103) SUMO(M), SUM1(M), SUM2(M), NAME(J,M)
C
 5    CONTINUE
C
      IF (GSUMO - 0.9)   10,10,15
 15   WRITE (6,104)
      GSUM1 = GSUM1 /GSUMO
      GSUM2 = GSUM2 / GSUMO
      WRITE (6,103)   GSUMO, GSUM1, GSUM2
 10   CONTINUE
C
 100  FORMAT (1H0, 10X, 26HANALYSIS OF VARIANCE ABOUT, I2, 5H-AXIS)
 101  FORMAT(1H0,49HNUMBER IN SAMPLE, MEAN AND VARIANCE FOR SUBGROUP.)
 103  FORMAT (12X, F5.0, 2F10.4, 3X, A6)
 104  FORMAT (1H0, 29HGRAND MEAN AND TOTAL VARIANCE)
      RETURN
       END
```

```
$IBFTC AXES
      SUBROUTINE AXES
C     - - - - - - - -
C     COMPUTE PRINCIPAL AXES OF COVMAT, A 3X3 COVARIANCE MATRIX, SUCH AS
C     THAT PRODUCED BY SUBROUTINE EQUATE OR SELECT.   IF THE DATA ARE
C     SCALAR, THE MATRIX PRODUCED BY SUBROUTINE SCALAR SHOULD BE USED.
C     THE DIRECTION COSINES OF THE PRINCIPAL AXES ARE COMPUTED AND
C     STORED IN THE COLUMNS OF EIGMAT.   THE CORRESPONDING VARIANCES ARE
C     STORED IN EIGVAL.   PRINCIPAL AXES AND VARIANCES ARE PRINTED OUT.
C     SUBROUTINE ENISCC IS REQUIRED WITH THIS SUBROUTINE.
C     BASED ON LIBRARY SUBROUTINE HDIAG WRITTEN BY CORBATO AND MERWIN.
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
      DIMENSION H(3,3), U(3,3),X(3),IQ(3)
```

```
C
      WRITE(6,103)
      DO 6    J=1,3
      WRITE (6,102)    (COVMAT(J,K), K=1,3)
      DO 6    K=1,3
 6    H(J,K) = COVMAT(J,K)
      N = 3
      IEGEN = 0
C
C
C4F05.0EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC MATRIX BY THE
C      JACOBI METHOD. (HDIAG).  PROGRAMMED BY F. J. CORBATO AND M. MERWIN
C      OF THE M. I. T. COMPUTATION CENTER.
C      SUBROUTINE HDIAG.
C
C           THIS SUBROUTINE COMPUTES THE EIGENVALUES AND EIGENVECTORS
C      OF A REAL SYMMETRIC MATRIX, H, OF ORDER N ( WHERE N MUST BE LESS
C      THAN 51), AND PLACES THE EIGENVALUES IN THE DIAGONAL ELEMENTS OF
C      THE MATRIX H, AND PLACES THE EIGENVECTORS (NORMALIZED) IN THE
C      COLUMNS OF THE MATRIX U.  IEGEN IS SET AS 1 IF ONLY EIGENVALUES
C      ARE DESIRED, AND IS SET TO 0 WHEN VECTORS ARE REQUIRED.  NR CON-
C      TAINS THE NUMBER OF ROTATIONS DONE.
      IF (IEGEN) 15,10,15
 10   DO 14 I=1,N
      DO 14 J=1,N
      IF(I-J)12,11,12
 11   U(I,J)=1.0
      GO TO 14
 12   U(I,J)=0.0
 14   CONTINUE
C
 15   NR = 0
      IF (N-1) 1000,1000,17
C
C      SCAN FOR LARGEST OFF DIAGONAL ELEMENT IN EACH ROW
C      X(I) CONTAINS LARGEST ELEMENT IN ITH ROW
C      IQ(I) HOLDS SECOND SUBSCRIPT DEFINING POSITION OF ELEMENT
C
 17   NMI1=N-1
      DO 30 I=1,NMI1
      X(I) = 0.0
      IPL1=I+1
      DO 30 J=IPL1,N
      IF ( X(I) - ABS( H(I,J))) 20,20,30
 20   X(I)=ABS(H(I,J))
      IQ(I)=J
 30   CONTINUE
C
C      SET INDICATOR FOR SHUT-OFF.RAP=2**-27,NR=NO. OF ROTATIONS
      RAP=.745058059E-08
      HDTEST=1.0E38
C
C      FIND MAXIMUM OF X(I) S FOR PIVOT ELEMENT AND
C      TEST FOR END OF PROBLEM
C
 40   DO  70  I=1,NMI1
      IF (I-1) 60,60,45
 45   IF ( XMAX- X(I)) 60,70,70
```

```
 60    XMAX=X(I)
       IPIV=I
       JPIV=IQ(I)
 70    CONTINUE
C
C      IS MAX. X(I) EQUAL TO ZERO, IF LESS THAN HDTEST, REVISE HDTEST
       IF ( XMAX) 1000,1000,80
 80    IF (HDTEST) 90,90,85
 85    IF (XMAX - HDTEST) 90,90,148
 90    HDIMIN = ABS( H(1,1) )
       DO 110  I= 2,N
       IF (HDIMIN- ABS( H(I,I))) 110,110,100
 100   HDIMIN=ABS(H(I,I))
 110   CONTINUE
C
       HDTEST=HDIMIN*RAP
C
C      RETURN IF MAX.H(I,J)LESS THAN(2**-27)ABSF(H(K,K)-MIN)
       IF (HDTEST- XMAX) 148,1000,1000
 148   NR = NR+1
C
C      COMPUTE TANGENT, SINE AND COSINE,H(I,I),H(J,J)
 150   TANG=SIGN(2.0,(H(IPIV,IPIV)-H(JPIV,JPIV)))*H(IPIV,JPIV)/(ABS(H(IPI
      1V,IPIV)-H(JPIV,JPIV))+SQRT((H(IPIV,IPIV)-H(JPIV,JPIV))**2+4.0*H(IP
      2IV,JPIV)**2))
       COSINE=1.0/SQRT(1.0+TANG**2)
       SINE=TANG*COSINE
       HII=H(IPIV,IPIV)
       H(IPIV,IPIV)=COSINE**2*(HII+TANG*(2.0*H(IPIV,JPIV)+TANG*H(JPIV,JPI
      1V)))
       H(JPIV,JPIV)=COSINE**2*(H(JPIV,JPIV)-TANG*(2.0*H(IPIV,JPIV)-TANG*H
      1 II))
       H(IPIV,JPIV)=0.0
C
C       PSEUDO RANK THE EIGENVALUES
C       ADJUST SINE AND COS FOR COMPUTATION OF H(IK) AND U(IK)
       IF ( H(IPIV,IPIV) -  H(JPIV,JPIV)) 152,153,153
 152   HTEMP = H(IPIV,IPIV)
       H(IPIV,IPIV) = H(JPIV,JPIV)
       H(JPIV,JPIV) = HTEMP
       HTEMP = SIGN(1.0, -SINE) * COSINE
C        RECOMPUTE SINE AND COS
       COSINE = ABS(SINE)
       SINE = HTEMP
 153   CONTINUE
C
C      INSPECT THE IQS BETWEEN I+1 AND N-1 TO DETERMINE
C      WHETHER A NEW MAXIMUM VALUE SHOULD BE COMPUTED SINCE
C      THE PRESENT MAXIMUM IS IN THE I OR J ROW.
C
       DO 350 I=1,NMI1
       IF(I-IPIV)210,350,200
 200   IF(I-JPIV)210,350,210
 210   IF(IQ(I)-IPIV)230,240,230
 230   IF(IQ(I)-JPIV)350,240,350
 240   K=IQ(I)
 250   HTEMP=H(I,K)
       H(I,K)=0.0
```

```
      IPL1=I+1
      X(I) =0.0
C
C     SEARCH IN DEPLETED ROW FOR NEW MAXIMUM
C
      DO 320 J=IPL1,N
      IF ( X(I)- ABS( H(I,J)) ) 300,300,320
 300  X(I) = ABS(H(I,J))
      IQ(I)=J
 320  CONTINUE
      H(I,K)=HTEMP
 350  CONTINUE
C
      X(IPIV) =0.0
      X(JPIV) =0.0
C
C     CHANGE THE OTHER ELEMENTS OF H
C
      DO 530 I=1,N
C
      IF(I-IPIV)370,530,420
 370  HTEMP = H(I,IPIV)
      H(I,IPIV) = COSINE*HTEMP + SINE*H(I,JPIV)
      IF ( X(I) -  ABS( H(I,IPIV)) )380,390,390
 380  X(I) = ABS(H(I,IPIV))
      IQ(I) = IPIV
 390  H(I,JPIV) = -SINE*HTEMP + COSINE*H(I,JPIV)
      IF ( X(I) -  ABS( H(I,JPIV)) ) 400,530,530
 400  X(I) = ABS(H(I,JPIV))
      IQ(I) = JPIV
      GO TO 530
C
 420  IF(I-JPIV)430,530,480
 430  HTEMP = H(IPIV,I)
      H(IPIV,I) = COSINE*HTEMP + SINE*H(I,JPIV)
      IF ( X(IPIV) -  ABS( H(IPIV,I)) ) 440,450,450
 440  X(IPIV) = ABS(H(IPIV,I))
      IQ(IPIV) = I
 450  H(I,JPIV) = -SINE*HTEMP + COSINE*H(I,JPIV)
      IF ( X(I) -  ABS( H(I,JPIV)) ) 400,530,530
C
 480  HTEMP = H(IPIV,I)
      H(IPIV,I) = COSINE*HTEMP + SINE*H(JPIV,I)
      IF ( X(IPIV) -  ABS( H(IPIV,I)) ) 490,500,500
 490  X(IPIV) = ABS(H(IPIV,I))
      IQ(IPIV) = I
 500  H(JPIV,I) = -SINE*HTEMP + COSINE*H(JPIV,I)
      IF ( X(JPIV) -  ABS( H(JPIV,I)) ) 510,530,530
 510  X(JPIV) = ABS(H(JPIV,I))
      IQ(JPIV) = I
 530  CONTINUE
C
C     TEST FOR COMPUTATION OF EIGENVECTORS
C
      IF(IEGEN)40,540,40
 540  DO 550 I=1,N
      HTEMP=U(I,IPIV)
      U(I,IPIV)=COSINE*HTEMP+SINE*U(I,JPIV)
```

```
 550   U(I,JPIV)=-SINE*HTEMP+COSINE*U(I,JPIV)
       GO TO 40
 1000  CONTINUE
C
C      END OF LIBRARY SUBROUTINE HDIAG.
C
C      PRINT OUT RESULTS AS DIRECTION COSINES AND AS TREND AND PLUNGE.
       WRITE (6,104)
       DO 8  J = 1,3
       DO 7   K = 1,3
 7     EIGMAT(J,K) = U(J,K)
       EIGVAL(J) = H(J,J)
 8     WRITE(6,102)   (EIGMAT(J,K), K = 1,3)
       WRITE (6,102)   (EIGVAL(K), K = 1,3)
       WRITE (6,105)
       DO 1011   J = 1,3
       WRITE (6,107)   J
       DO 1010   I = 1,3
 1010   D(I) = EIGMAT(I,J)
       CALL ENISOC(1)
 1011  WRITE(6,102)   EIGVAL(J)
       WRITE (6,106)
C
 102    FORMAT(X,3F8.5)
 103   FORMAT(X, 17HCOVARIANCE MATRIX)
 104   FORMAT (1H0,40H EIGENVECTORS AND EIGENVALUES IN COLUMNS)
 105   FORMAT (1H0, 38HPRINCIPAL AXES AND ASSOCIATED VARIANCE)
 106   FORMAT (1H0,25X,45H---------------------------------------------)
 107   FORMAT (1H+, 36X, 3HOF , I1, 5H-AXIS)
       RETURN
        END
```

```
$IBFTC CONFIT
       SUBROUTINE CONFIT (KOOKS)
C      - - - - - - - - - - - - - -
C      SUBROUTINE TO FIND THE AXIS AND APICAL ANGLE OF A BEST-FIT CONE.
C
C      SUBROUTINES ENISOC AND SOLSP ARE REQUIRED WITH THIS SUBROUTINE.
C      CALLS N.U. LIBRARY SUBROUTINE SOLSP, WRITTEN BY I. WAYE.
C      IF KOOKS = 1, A LIST OF ITEMS WHICH DEVIATE CONSIDERABLY FROM THE
C      CONE SURFACE IS PRINTED OUT.   IF KOOKS = 0, THE LISTING IS
C      SUPPRESSED.
C      'CONFIT' SHOULD NOT FOLLOW 'AXES' DIRECTLY AS D WOULD THEN CONTAIN
C      IRRELEVANT VALUES.
C
       COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
      1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
       DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
      1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
      2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
       DOUBLE PRECISION   A
       DIMENSION COEFF(4,4), AXICOS(4,4), X(3,1), A(3,4)
C
```

```
      DO 1    J=1,4
         DO 1    I= 1,4
 1    COEFF(I,J) = 0.0
C     COMPUTE AND PRINT COEFFICIENTS OF LEAST-SQUARES EQUATIONS
      COEFF(1,1) = 1.0
      DO 3    J=2,4
      COEFF(1,J) = D(J-1)
      DO 4    K=2,4
 4    COEFF(J,K) = COVMAT(J-1,K-1)
 3    COEFF(J,1) = COEFF(1,J)
      WRITE (6,102)
      DO 2    J=1,4
 2    WRITE (6,101)    (COEFF(J,K),K=1,4)
C     THREE SETS OF THREE EQUATIONS (ABSTRACTED FROM COEFF) ARE SOLVED
C     IN TURN, EACH MINIMIZING SUM OF SQUARES PARALLEL TO A DIFFERENT
C     AXIS.
C     THE KTH ROW OF COEFF IS NOT USED, THE KTH COLUMN CONTAINS THE
C     INDEPENDENT VARIABLES.    THE REQUIRED COEFFICIENTS ARE READ INTO
C     A, AND THE INDEPENDENT VARIABLES INTO X(I,1).
      WRITE(6,108)
      WRITE (6,112)
      DO 5    K = 2,4
      N = 0
      DO 6    I = 1,3
 7    N = N+1
      IF (N-K)    11,7,11
 11   M = 0
      X(I,1) = COEFF(N,K)
      DO 6    J = 1,3
 8    M = M+1
 9    IF (M-K)    6,8,6
 6    A(I,J) = COEFF(N,M)
C     THE MATRIX 'A' CONTAINS THE COEFFICIENTS OF THE L.S. EQUATIONS.
      CALL SOLSP(A,X,3,1,0.0,IERR)
C     THE VECTOR 'X' IS RETURNED WITH THE ROOTS OF THE EQUATION.
C     X CONTAINS THE COSINE OF THE APICAL ANGLE, AND DIRECTION COSINES
C     OF THE CONE AXIS (OTHER THAN THE K-1TH), ALL DIVIDED BY THE
C     (K-1)TH DIRECTION COSINE.
      DO 15    J=1,3
      N = J
      IF (J-K)    16,15,17
 17   N = J+1
 16   AXICOS(K-1,N) = X(J,1)
 15   CONTINUE
      AXICOS(K-1,K) =-1.0
C     DIRECTION RATIOS ARE CONVERTED TO DIRECTION COSINES.
      SUM = 0.0
      DO 18    J = 2,4
 18   SUM = SUM + (AXICOS(K-1,J))**2
      SUM = SQRT(SUM)
      DO 19    J=2,4
 19   AXICOS(K-1,J) = AXICOS(K-1,J)/SUM
C     APICAL ANGLE IS CONVERTED FROM A RATIO TO AN ABSOLUTE VALUE.
      AXICOS(K-1,1) =AXICOS(K-1,1) * AXICOS(K-1,K)
      IF (AXICOS(K-1,1) ) 30,31,31
 30   DO 201    J=1,4
 201  AXICOS(K-1,J) = AXICOS(K-1,J) * (-1.0)
 31   CONTINUE
```

```
C     CALCULATE ROOT-MEAN-SQUARE DEVIATION OF MEASUREMENTS FROM CONE.
      TOTAL = 0.0
      DO 222   I=1,KOUNT2
      RMS = 0.0
      DO 212  J = 1, 3
 212  RMS = RMS + T(I,J) * AXICOS(K-1,J+1)
      RMS = RMS - AXICOS(K-1,1)
 222  TOTAL = TOTAL + RMS ** 2
      COUNT = KOUNT2
      RMS = SQRT(TOTAL/COUNT)
      D(K-1) = RMS
 5    WRITE(6,101)   (AXICOS(K-1,J), J=1,4), RMS
      A = D(1)
      B = D(2)
      C = D(3)
      RMS = AMIN1(A,B,C)
      DO 203   I=1,3
      IF (D(I) - RMS)   203,204,203
 203  CONTINUE
 204  DO 205   J=1,4
 205  AXICOS(4,J) = AXICOS(I,J)
      WRITE(6, 103)
      WRITE (6,101)   (AXICOS(4,J), J=2,4)
      WRITE (6,104)
      DO 213   J=2,4
 213  D(J-1) = AXICOS(4,J)
      CALL ENISOC(1)
C     COMPUTE APICAL ANGLE OF CONE (COSINE = AXICOS(4,1)).
      APICON = ABS((1.0/AXICOS(4,1))**2 - 1.0)
      APICON = SQRT(APICON)
      APICON = (ATAN(APICON)) * (180.0/3.141593)
      WRITE (6,113)   APICON
      IF (KOOKS)   23,23,21
 21   WRITE (6,109)
      WRITE (6,110)
      DO 20   I=1,KOUNT2
      R = 0.0
      DO 24   J=1,3
 24   R=R +T(I,J)*AXICOS(4,J+1)
      R = ABS(R-AXICOS(4,1))
      IF (R - 2.0*RMS)   20,22,22
 22   I1 = L(I,1)
      I2 = L(I,2)
      I3 = L(I,3)
      I4 = L(I,4)
      I5 = L(I,5)
      I6 = L(I,6)
      WRITE (6,111)   I,R,(T(I,J),J=1,3),(S(I,J),J=1,3),NAME(1,I1),
     1 NAME(2,I2),NAME(3,I3),NAME(4,I4),NAME(5,I5),NAME(6,I6)
 20   CONTINUE
 23   CONTINUE
      WRITE (6,107)
C
 101  FORMAT (X, 4F9.5, 3X, F9.5)
 102  FORMAT (1H0, 30HCOEFFICIENTS OF L.S. EQUATIONS)
 103  FORMAT (1H0, 56HDIRECTION COSINES OF CONE AXIS WITH LOWEST RMS DEV
     1IATION)
 104  FORMAT(1H0, 29HTREND AND PLUNGE OF CONE AXIS)
```

```
106   FORMAT (1H0,50HROOT-MEAN-SQUARE COSINE OF DEVIATIONS FROM SURFACE)
107   FORMAT(1H0,25X,45H---------------------------------------------)
108   FORMAT(1H0, 77HCONSTANTS MINIMIZING SQUARES IN SOUTH, EAST, AND VE
     1RTICAL DIRECTIONS IN TURN,)
109   FORMAT (1H0,70HTHE FOLLOWING MEASUREMENTS DEVIATE CONSIDERABLY FRO
     1M THE CONE SURFACE,)
110   FORMAT (X,4HITEM,7X,8HCOS DEVN,2X,17HDIRECTION COSINES,5X,
     115HLOCATION COORDS,7X,7HFACTORS)
111   FORMAT (X,I4,6X,F8.4,2X,3F6.3,3X,3F6.3,4X,6(2X,A6))
112   FORMAT   (X, 52HCOS APEX . DIRECTION COSINES OF AXIS . RMS DEVIATI
     1ON)
113   FORMAT (1H0, 23HAPICAL ANGLE OF CONE IS, F7.2, 8H DEGREES)
      RETURN
       END



$IBFTC ENISOC
C
      SUBROUTINE ENISOC (LINFOL)
C     - - - - - - - - - - - - - -
C     THIS SUBROUTINE CONVERTS DIRECTION COSINES TO RADIAL COORDINATES.
C     D IS DIMENSIONED FOR THREE, THEREFORE CALL ONLY SINGLE SETS
C     OF THREE DIRECTION COSINES, NAMELY D(1), D(2), AND D(3).
C
C     PUT 'LINFOL' = 1 FOR LINEATION, 2 FOR AZIMUTH AND AMOUNT OF DIP
C        OF FOLIATION, 3 FOR STRIKE AND DIP OF FOLIATION.
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
      DIMENSION MARK(5)
      DATA (MARK(J),J=1,5) /5HSOUTH,5HNORTH,5HWEST ,5HEAST ,5H     /
C
C     CAPTION THE OUTPUT
      IF (LINFOL - 2)   1,2,3
 1    WRITE(6, 100)
      GO TO 4
 2    WRITE(6, 101)
      GO TO 20
 3    WRITE(6,102)
C     FIND OUT WHICH WAY IT DIPS
 20   D(3) = -D(3)
 4    IF (D(1) * D(3))   12,10,11
 10   I = 5
      GO TO 17
 11   I = 2
      GO TO 17
 12   I = 1
 17    IF (D(2) * D(3))   14,15,13
 13   J = 3
      GO TO 16
 14   J = 4
```

```
      GO TO 16
 15   J = 5
 16   CONTINUE
C
C     CALCULATE THE TREND AND PLUNGE
      PLUNGE = ABS((1.0/D(3))**2 - 1.0)
      PLUNGE = SQRT(PLUNGE)
      PLUNGE = 90.0 - (ATAN(PLUNGE))*(180.0/3.141593)
      TREND = ABS(D(2)/D(1))
      TREND = (ATAN(TREND)) * (180.0/3.141593)
      IF (D(1) * D(2))   22,22,21
 21   TREND = 180.0 - TREND
 22   IF (LINFOL - 2)   5,6,7
 7    TREND = 90.0 + TREND
 6    PLUNGE = 90.0 - PLUNGE
      IF (TREND)   8,5,5
 8    TREND = TREND + 180.0
 5    WRITE(6, 103) TREND, PLUNGE, MARK(I), MARK(J)
C
 100  FORMAT (X,   35HTREND AND PLUNGE (LINEAR STRUCTURE))
 101  FORMAT (X,   46HDIRECTION AND AMOUNT OF DIP (PLANAR STRUCTURE))
 102  FORMAT (X,   33HSTRIKE AND DIP (PLANAR STRUCTURE))
 103  FORMAT (X, F6.1, 3H / , F6.1, X, 2A5)
      RETURN
       END
```

```
$IBFTC EQUATE
C
      SUBROUTINE EQUATE (DIRCOS,T, KOUNT, KOUNT2, COVMAT, D)
C     - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C     THIS SUBROUTINE PUTS THE MATRIX T EQUAL TO THE MATRIX DIRCOS,
C     PUTS KOUNT2 = KOUNT, AND COMPUTES THE COVARIANCE MATRIX, COVMAT.
C     MEAN VALUES OF THE COLUMNS OF 'DIRCOS' ARE STORED IN D(1) TO D(3).
C
      DIMENSION DIRCOS(300,3),T(300,3),COVMAT(3,3),D(3)
C
      KOUNT2 = KOUNT
      COUNT = KOUNT
C     INITIALIZE
      DO 1 J = 1,3
      D(J) = 0.0
      DO 1 K = 1,3
 1    COVMAT (J,K) = 0.0
C
C     EQUATE T AND DIRCOS, ACCUMULATE SUMS AND CROSS-PRODUCTS.
      DO 4   I = 1, KOUNT
      DO 5   J=1,3
      T(I,J) = DIRCOS(I,J)
 5    D(J) = D(J) + T(I,J)
      DO 4   J=1,3
      DO 4  K = 1,3
 4    COVMAT(J,K) = COVMAT(J,K) + T(I,J) * T(I,K)
C
```

```
C       DIVIDE BY 'COUNT' TO OBTAIN MEAN VALUES.
        DO 3    J=1,3
        D(J) = D(J)/COUNT
        DO 3    K=1,3
 3      COVMAT(J,K) = COVMAT(J,K) / COUNT
C
        RETURN
         END
```

```
$IBFTC HOWBIG
C
        SUBROUTINE HOWBIG (IAXIS)
C       - - - - - - - - - - - - - - -
C       THIS SUBROUTINE PLOTS A HISTOGRAM SHOWING RELATIVE FREQUENCY OF
C          FOLDS OF DIFFERENT SIZES.
C
        COMMON    COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
       1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
        DIMENSION    COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
       1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
       2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
        DIMENSION  AVE(50), NUM(50), A(5), MARK(6,5), NUM1(80)
C
C       INITIALIZE
        DO 8    I=1,80
 8      NUM1(I) = 1
C       CAPTION THE OUTPUT
        IF (NOVEC - 1)    18,19,18
 18     WRITE (6,101)    IAXIS
        GO TO 20
 19     WRITE (6,111)    IAXIS
 20     WRITE (6,102)    KOUNT2
        DATA (MARK(1,J), J=1,5) /4H    +,4*4H    ./
        DO 10   I=1,5
        DO 10    J=1,5
   10 MARK(I+1,J)=MARK(I,J)
        A(1) = 0.0
        DO 11   I=1,4
   11 A(I+1) = A(I) + 0.5
        IF (NOVEC - 1)    28,29,28
 28     WRITE (6,104)     A
 29     WRITE (6,105)    ((MARK(I,J), J=1,5),I=1,6)
C
C       COMPUTE A SCALE WHICH WILL MAKE THE HISTOGRAM A CONVENIENT SIZE.
        Y = S(1,IAXIS)
        DO 1    I=1,KOUNT2
        F = S(I,IAXIS)
 1      Y = AMAX1(F,Y)
        Z = Y
C       K IS A FIRST ESTIMATE OF THE NUMBER OF ROWS IN THE HISTOGRAM.
        K = KOUNT2/20
        IF (K-20)    2,2,3
 2      K=20
```

```
 3     B = K
       Z = Z/B
C      Z IS THE INTERVAL DISTANCE REPRESENTED BY ONE ROW.
C      ROUND Z SOMEWHAT.
       DO 5   I=1,5
       IF (Z*10.0**I - 10.0)   5,5,4
 5     CONTINUE
C      JMAX IS NOW THE NUMBER OF ROWS IN THE HISTOGRAM.
 4     K =(Z* 10.0**I) + 0.99
       Z = K / 10 **I
       JMAX = (Y/Z) +1.0
       DO 7   J=1,JMAX
C      INITIALIZE
       AVE(J) = 0.0
 7     NUM(J) = 0
C
C
C      DETERMINE THE VALUE OF EACH ROW OF THE HISTOGRAM.
C
       IF (NOVEC - 1)   12,13,12
C      THE NEXT SECTION IS FOR VECTORIAL DATA ONLY.
 12    K = KOUNT2 - 1
       DO 6   M=1,K
       I = M+1
       DO 6   N = I,KOUNT2
       J = ABS((S(M,IAXIS) - S(N,IAXIS))/Z) + 1.0
       AVE(J) = AVE(J) + ABS(T(M,IAXIS) - T(N,IAXIS))
 6     NUM(J) = NUM(J) +1
       GO TO 14
C
C      THE NEXT SECTION IS FOR SCALAR DATA ONLY.
 13    K = KOUNT2 - 1
       DO 21   M=1,K
       I = M+1
       DO 21   N = I,KOUNT2
       J = ABS((S(M,IAXIS) - S(N,IAXIS))/Z) + 1.0
C      S(I,1) MEASURES THE STRUCTURAL ELEVATION OF THE ITH POINT.
       AVE(J) = AVE(J) + ABS(S(M,1) - S(N,1))
 21    NUM(J) = NUM(J) + 1
C
C      COMPUTE AVERAGE VALUES FROM TOTALS.
C      COMPUTE A CONVENIENT HORIZONTAL SCALE FACTOR.
 14    DO 26   J=1,JMAX
       F = NUM(J)
 26    AVE(J) = AVE(J)/F
       IF (NOVEC - 1)   23,22,23
 23    SCALE = 40.0
       ASMALL = 0.0
       GO TO 25
 22    ABIG = S(1,IAXIS)
       ASMALL = S(1,IAXIS)
       DO 24   I=1,KOUNT2
       F = S(I,IAXIS)
       ASMALL = AMIN1 (F,ASMALL)
 24    ABIG = AMAX1(F,ABIG)
       ARANGE = ABIG - ASMALL
       SCALE = ARANGE/80.0
       DO 27   I=1,5
```

```
      F = 20 • (I-1)
 27   A(I) = ASMALL + F*SCALE
      SCALE = 80.0 / ARANGE
 25   CONTINUE
C     PLOT THE RESULTS.
      DO 9    J=1,JMAX
      WOO = J
      W = WOO*Z
      MEAN = (AVE(J) - ASMALL)*SCALE + 1.0
 9    WRITE (6,106)   W, NUM(J), (NUM1(I), I=1,MEAN)
C     CAPTION THE BASE OF THE HISTOGRAM
      WRITE(6,105) ((MARK(I,J), J=1,5), I=1,6)
      IF (NOVEC - 1)   15,16,15
 15   WRITE (6,104)   A
      WRITE(6,107)
      GO TO 17
 16   WRITE (6,103)   A
      WRITE (6,117)
 17   WRITE (6,108)
      WRITE (6,109)
      WRITE (6,118)
C
 101  FORMAT (1H1, 61HHISTOGRAM OF MEAN ATTITUDE DEVIATION MEASURED PARA
     1LLEL TO THE, I2, 5H-AXIS)
 102  FORMAT (X,I4,13H MEASUREMENTS)
 103  FORMAT (X, 10X, 5F20.6)
 104  FORMAT (X, 5X, 5F20.1)
 105  FORMAT (X, 30A4)
 106  FORMAT (X, F10.5, 5X, I3, 6X, 80I1)
 107  FORMAT (1H0,65HMEAN DIFFERENCE BETWEEN DIRECTION COSINES AT GIVEN
     1DISTANCE APART)
 108  FORMAT(1H0,76HDISTANCE BETWEEN MEASUREMENTS IS PLOTTED VERTICALLY,
     1THE SCALE IS ON THE LEFT)
 109  FORMAT (1H0, 68HTHE SECOND COLUMN SHOWS THE NUMBER OF VALUES REPRE
     1SENTED IN EACH ROW)
 111  FORMAT (1H1,63HHISTOGRAM OF MEAN ELEVATION DIFFERENCE MEASURED PAR
     1ALLEL TO THE, I2, 5H-AXIS)
 117  FORMAT (1H0,69HMEAN DIFFERENCE BETWEEN STRUCTURAL ELEVATIONS AT GI
     1VEN DISTANCE APART)
 118  FORMAT (1H0, 5X, 82HA LOW VALUE IN THE HISTOGRAM MAY INDICATE THE
     1PRESENCE OF FOLDS OF THAT WAVELENGTH)
      RETURN
       END
```

```
$IBFTC MOMENT  DECK
C
      SUBROUTINE MOMENT (IAXIS)
C     - - - - - - - - - - - - - -
C     A HISTOGRAM AND DESCRIPTIVE STATISTICS ARE CALCULATED FOR A
C     DISTRIBUTION OF DIRECTION COSINES. MEASURED ABOUT THE IAXIS AXIS.
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
```

```
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
      DIMENSION MARK(6,5), POWER(4), NUMBER(80), A(5), IROW(80)
      DATA(MARK(1,J),J=1,5)/4H    +,4*4H    ./
      DATA(MARK(2,J),J=1,5)/4H    +,4*4H    ./
      DATA(MARK(3,J),J=1,5)/4H    +,4*4H    ./
      DATA(MARK(4,J),J=1,5)/4H    +,4*4H    ./
      DATA(MARK(5,J),J=1,5)/4H    +,4*4H    ./
      DATA(MARK(6,J),J=1,5)/4H    +,4*4H    ./
C
C     WRITE TITLES FOR HISTOGRAM
      WRITE (6,101)   IAXIS
      WRITE (6,105)    KOUNT2
      A(1) = -1.0
      DO 2    I=1,4
 2    A(I+1) = A(I) + 0.5
      WRITE (6,102) A
      WRITE(6,103) ((MARK(I,J), J=1,5), I=1,6)
C     INITIALIZE
      COUNT = KOUNT2
      DO 3    J=1,4
 3    POWER(J) = 0.0
      DO 4    M=1,80
 4    NUMBER(M) = 0
C
C     CALCULATE SUMS OF FIRST TO FOURTH POWERS OF DATA
      IF (NOVEC - 1)    31,30,31
C     SCALAR DATA HAVE TO AVOID THE NEXT SECTION.
 31   CONTINUE
      DO 6    I=1, KOUNT2
      DO 5    J=1,4
 5    POWER(J) = POWER(J) + (T(I,IAXIS))**J
C     COUNT FREQUENCY OF MEASUREMENTS IN EACH COLUMN OF HISTOGRAM
      M = (T(I,IAXIS) + 1.0)/0.025
 6    NUMBER(M) = NUMBER(M) + 1
      GO TO 32
 30   CONTINUE
C
C     VECTORIAL DATA HAVE TO AVOID THE NEXT SECTION.
C     COMPUTE DIRECTION COSINES FROM ELEVATIONS.
      DO 21    I=1,KOUNT2
      DO 21    J=1,KOUNT2
C     TAKE ONLY LINES WITH A POSITIVE DIRECTION, TO AVOID DUPLICATION.
      IF (S(I,2) - S(J,2))    21,22,23
 22   IF (I-J)    28,21,28
 28   IF (S(I,1) - S(J,1))    21,24,23
 24   IF (S(I,3) - S(J,3))    21,23,23
 23   D(1) = S(I,1) - S(J,1)
      D(2) = S(I,2) - S(J,2)
      D(3) = S(I,3) - S(J,3)
      P = D(1)**2 + D(2)**2 + D(3)**2
      P = SQRT(1.0/P)
      D(1) = D(1)*P
      D(2) = D(2)*P
      D(3) = D(3)*P
C
      DO 25    K=1,4
```

```
 25    POWER(K) = POWER(K) + (D(IAXIS))**K
       M = (D(IAXIS) + 1.0)/0.025
       NUMBER(M) = NUMBER(M) + 1
 21    CONTINUE
C
       ITOTAL = 0
       DO 33   M=1,80
 33    ITOTAL = ITOTAL + NUMBER(M)
       COUNT = ITOTAL
 32    CONTINUE
C      CONVERT FREQUENCIES TO PERCENTAGES
       DO 7    M = 1, 80
       P = NUMBER(M)
 7     NUMBER(M) =(100.0*P/COUNT) + 0.5
C      FIND LARGEST PERCENTAGE
       M = 0
       DO 8    I = 1, 80
       INUM = NUMBER(I)
 8     M = MAX0(M,INUM)
C
C      DRAW HISTOGRAM
       M = M+5
C      FOR ONE ROW AT A TIME, FIND THE SYMBOL REQUIRED IN EACH COLUMN
 13    DO 12   I=1,80
       IF (NUMBER(I) - M)   10,9,9
 10    IROW(I) = 0
       GO TO 12
 9     IROW(I) = 1
 12    CONTINUE
C      FILL IN EACH ROW OF THE HISTOGRAM, AND REPEAT THE TITLES AT BASE
       WRITE (6,108)   M,IROW
       M = M-1
       IF (M)   14,14,13
 14    WRITE ( 6,103)((MARK(I,J),J=1,5),I=1,6)
       WRITE (6,102) A
       WRITE(6,109)
       WRITE (6,114)
C
C      COMPUTE AND PRINT THE MOMENTS, SKEWNESS AND KURTOSIS
       WRITE(6,110)
       DO 15   J = 1,4
       POWER(J) = POWER(J)/ COUNT
 15    WRITE (6,111)J, POWER(J)
       WRITE (6, 112)
       A(1) = POWER(3)/(SQRT(POWER(2) ** 3))
       A(2) = POWER(4) / (POWER(2)**2)
       WRITE (6,113)   A(1), A(2)
C
 101   FORMAT(1H1,43HDISTRIBUTION OF DIRECTION COSINES ABOUT THE, I2,
      15H-AXIS)
 102   FORMAT (X,5X,5F20.1)
 103   FORMAT(X, 30A4)
 105   FORMAT (1H0, I4, 13H MEASUREMENTS)
 107   FORMAT (X,I2)
 108   FORMAT (X,I2,22X,80I1)
 109   FORMAT (1H0, 51X, 26HVALUE OF DIRECTION COSINES)
 110    FORMAT (1H0, 39HMOMENTS OF THE ABOVE DISTRIBUTION ARE -)
 111   FORMAT (X, I1, F8.4)
```

```
 112  FORMAT (1H0, 21HSKEWNESS AND KURTOSIS)
 113  FORMAT (X,F8.4, 4X, F8.4)
 114  FORMAT (1H0, 39HPERCENTAGE FREQUENCY PLOTTED VERTICALLY)
      RETURN
       END
```

```
$IBFTC READIN  DECK                                                     READ 000
C
      SUBROUTINE READIN
C     - - - - - - - - -
C     THE SUBROUTINE READS IN INFORMATION FROM THE DATA DECK.
C     LAYOUT OF DATA AS FOR PROG4.
C     THE SUBROUTINE COMPUTES DIRECTION COSINES FOR EACH MEASUREMENT
C        AND STORES THE RESULTS IN DIRCOS, (DIMENSIONED KOUNT, 3).
C        THE NUMBER OF MEASUREMENTS IS COUNTED AND STORED IN KOUNT.
C     IF STRUCTURES ARE LINEATIONS, NOTE COMMENTS CARD BELOW.
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
      DIMENSION FMT(6), AMT(6)
      DATA  BLANK/6H      /
      DATA (AMT(I),I=1,6)/36H  (A6,A6,3F6.0,3F6.3,6I3)
C
C     READ IN ALPHAMERIC INFORMATION. PRINT OUT MAIN HEADINGS.
      READ (5,100)   (NAME(1,I), I=1,9), NAME(2,1), NAME(2,2)
      WRITE (6,110) (NAME(1,I), I=1,9) , NAME(2,1), NAME(2,2)
      WRITE (6,103)
      READ (5,101)   ID1, ID2, (FMT(I),I=1,6),COR12,COR3,CORAZI
      WRITE (6,101)  ID1, ID2, (FMT(I),I=1,6),COR12,COR3,CORAZI
 6    DO 7   I=1,6
      READ (5,102)   ID1,ID2, (NAME(I,J), J=1,9)
 7    WRITE(6,112)   ID1,ID2, (NAME(I,J), J=1,9)
      NEXT = 1
      I = 1
C
C     USE STANDARD FORMAT AND NO CORRECTIONS IF NONE SPECIFIED ON CARD 2
 32   IF (COR12)  2,1,2
 1    COR12 = 1.0
 2    IF (COR3 )   4,3,4
 3    COR3 = 1.0
 4    IF (FMT(1) - BLANK)   20,5,20
 5    DO 30   J=1,6
 30   FMT(J) = AMT(J)
 20   READ (5,FMT)   ID1,ID2,A1,A2,A3,A4,A5
      IF (A4-999.0)    22,21,22
 21   IF (A5-999.0)    8,22,8
 22   WRITE (6,105)
      STOP
C
 8    CONTINUE
```

```
 25    SUM=0.0
       SUM2 = 0.0
C
C      READ IN NUMERICAL INFORMATION FROM DATA CARDS.
 10    READ(5,FMT)   ID1,ID2,(GCOORD(I,J),J=1,3), (DIRCOS(I,J),J=1,3),
      1   (LIST(I,J), J= 1,6)
       WRITE(6,113)  ID1,ID2,(GCOORD(I,J),J=1,3), (DIRCOS(I,J),J=1,3),
      1   (LIST(I,J), J= 1,6)
       IF (DIRCOS(I,1) - 999.0)   11,12,11
 11    GCOORD(I,1) = GCOORD(I,1) * COR12
       GCOORD(I,2) = GCOORD(I,2) * COR12
       GCOORD(I,3) = GCOORD(I,3) * COR3
       DIRCOS(I,1) = DIRCOS(I,1) + CORAZI
       SUM = SUM + DIRCOS(I,3)**2
       SUM2 = SUM2 + DIRCOS(I,2)**2
       I = I+1
       GO TO 10
C
 12    KOUNT = I - 1
       WRITE(6,104)   KOUNT
C
C      COMPUTE DIRECTION COSINES FROM AZIMUTH AND AMOUNT OF DIP (DEGREES)
C      IF SOME NUMBERS HAVE BEEN INSERTED IN THE DIRCOS(I,3) COLUMN,
C         ASSUME THAT THE MEASUREMENTS ARE ALREADY DIRECTION COSINES.
       IF (SUM - 0.05)   31,31,16
C
C      IF THERE ARE NO NUMBERS IN THE DIRCOS(I,2) COLUMN, ASSUME THAT
C      THE MEASUREMENTS ARE SCALAR.
 31    IF (SUM2 - 0.005)   19,19,17
 19    NOVEC = 1
       DO 14   I=NEXT,KOUNT
 14    GCOORD(I,3) = GCOORD(I,3) - DIRCOS(I,1)
       GO TO 16
C
 17    DO 18   I=NEXT,KOUNT
       A = DIRCOS(I,1) * 3.14159 / 180.0
C
C      FOR LINEAR STRUCTURES, INSERT A CARD  DIRCOS(I,2)=DIRCOS(I,2)+90.0
C
       B = DIRCOS(I,2) * 3.14159 / 180.0
       DIRCOS(I,1) = COS(A) * SIN(B) * (-1.0)
       DIRCOS(I,2) = SIN(A) * SIN(B)
 18    DIRCOS(I,3)= COS(B)
C
 16    IF (DIRCOS(KOUNT + 1, 2) - 999.0)   23,24,23
 23    READ (5,101)   ID1,ID2,(FMT(I),I=1,6),COR12,COR3,CORAZI
       WRITE(6,101)   ID1,ID2,(FMT(I),I=1,6),COR12,COR3,CORAZI
       I = KOUNT + 1
       NEXT = I
       GO TO 32
 24    CONTINUE
C
 100   FORMAT (12X, 11A6)
 101   FORMAT (X,A5,7A6,3F6.0)
 102   FORMAT (12A6)
 103   FORMAT (1H0, 40HPRINT-OUT OF MASTER CARDS AND DATA CARDS)
 104   FORMAT (X,I6,X,12HMEASUREMENTS)
 105   FORMAT (1H0, 39HFAULTY READ IN, PLEASE CHECK DATA DECK.)
```

```
 110   FORMAT (1H1, 20X, 11A6)
 112   FORMAT (X,12(A6,3X))
 113   FORMAT (X,2(A6,3X),6F12.4,6I3)
       RETURN
        END
```

```
$IBFTC SCALAR
C
       SUBROUTINE SCALAR
C      - - - - - - - - -
C      THIS SUBROUTINE COMPUTES  A COVARIANCE MATRIX OF DIRECTION COSINES
C        (COVMAT), FROM A GROUP OF ELEVATIONS (S(I,3)) OF A GIVEN HORIZON
C
       COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
      1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
       DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
      1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
      2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
C      INITIALIZE
       COUNT = KOUNT2*(KOUNT2 - 1 )
       DO 5    J=1,3
       EIGVAL(J) = 0.0
       DO 5    K=1,3
 5     COVMAT(J,K) = 0.0
C
C      COMPUTE DIRECTION COSINES FROM ELEVATIONS.
       DO 1    I=1,KOUNT2
       DO 1    J=1,KOUNT2
       IF (I-J)    3,1,3
 3     D(1) = S(I,1) - S(J,1)
       D(2) = S(I,2) - S(J,2)
       D(3) = S(I,3) - S(J,3)
       P = D(1)**2 + D(2)**2 + D(3)**2
       P = SQRT (1.0/P)
       D(1) = D(1)*P
       D(2) = D(2)*P
       D(3) = D(3)*P
C
C      COMPUTE COVARIANCE MATRIX.
       DO 6    K=1,3
       EIGVAL(K) = EIGVAL(K) + D(K)
       DO 6    M=1,3
 6     COVMAT(K,M) = COVMAT(K,M) + D(K)*D(M)
 1     CONTINUE
C
       DO 7    K=1,3
       D(K) = EIGVAL(K) / COUNT
       DO 7    M=1,3
 7     COVMAT(K,M) = COVMAT(K,M)/COUNT
C
       RETURN
        END
```

```
$IBFTC SEKSHN
      SUBROUTINE SEKSHN
C     - - - - - - - - - -
C     COMPUTES SLOPE AND LOCATION OF INTERSECTION OF MEASURED PLANES
C       WITH PRINCIPAL PLANES.
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
      WRITE (6,103)
      WRITE(6,100)
      WRITE (6, 101)
C
      DO 1    I = 1,KOUNT2
      A = T(I,2) / T(I,1)
      B = T(I,3) / T(I,1)
      C = T(I,3) / T(I,2)
 1    WRITE(6,102)    A,B,C,(S(I,J), J=1,3)
C
 100  FORMAT ( 1H0, 51HSLOPE (TANGENT OF ANGLE) OF INTERSECTION OF BED A
     1ND, 19X, 33HDISTANCE FROM ORIGIN PARALLEL TO )
 101  FORMAT (X,5X, 9H1-2 PLANE, 5X, 9H1-3 PLANE, 5X, 9H2-3 PLANE,
     1 28X, 6H1-AXIS, 7X, 6H2-AXIS, 7X, 6H3-AXIS)
 102  FORMAT (X, 3F14.4, 23X, 3F13.4)
 103  FORMAT (1H1, 62HDATA FOR DRAWING CROSS-SECTIONS PARALLEL TO THE PR
     1INCIPAL AXES)
      RETURN
       END


$IBFTC SELECT  DECK
C
      SUBROUTINE SELECT (NUMB)
C     - - - - - - - - - - - - - -
C     THIS SUBROUTINE SELECTS SUBSETS OF DATA FROM T, S, AND L (OR LIST)
C     ACCORDING TO THE CODE ON THE CONTROL CARD (SEE PROG4).
C     NUMB INDICATES THE CONTROL CARD POSITION OF THE CHOSEN GROUPS.
C     KOUNT2 IS THE NUMBER OF ITEMS IN THE SUBSET.   A COVARIANCE MATRIX
C     (COVMAT) IS COMPUTED FOR T.  THE MEAN VALUES OF THE COLUMNS OF T
C     ARE STORED IN D(1) TO D(3).
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
      WRITE (6,100)
```

```
      WRITE (6,101)
      KOUNT2 = KOUNT
C     PUT L = LIST, AND INITIALIZE COVMAT.
      DO 26   J=1,6
      DO 26   I=1,KOUNT
 26   L(I,J) = LIST(I,J)
      DO 27   J=1,3
      D(J) = 0.0
      DO 27   K=1,3
 27   COVMAT(J,K) = 0.0
      KNUMB = NUMB
C
C     'KONTRL' IS A THREE-DIGIT INTEGER,
C        THE FIRST DIGIT INDICATES THE BEGINNING OF A NEW SUBSET,
C        THE SECOND DIGIT IS THE FACTOR CODE NUMBER (KLASS),
C        THE THIRD DIGIT IS THE CATEGORY CODE NUMBER (KAT).
C     FIND 'NUMB'TH SUBSET AND COMPUTE KLASS AND KAT.
      DO 13   I=1,20
 48   N=KONTRL(I)/100
      IF (N)    12,13,12
 12   KNUMB = KNUMB - 1
      IF (KNUMB)    8,8,13
 13   CONTINUE
 37   WRITE (6,106)
      KOUNT2 = 0
      GO TO 20
 8    M = I
 14   K = IABS(KONTRL(M))
      IF (K)    20,20,38
 38   N = K/100
      K = K - N*100
      KLASS = K/10
      IF (KLASS)    32,33,32
 33   WRITE (6,105)
      GO TO 20
 32   KAT = K - KLASS*10
      IF (KAT)    33,33,34
C
 34   K = 1
      IF (KLASS-6)    30,30,31
 31   WRITE (6,104)
      STOP
C
 30   IF (KONTRL(M))    23,20,24
 23   WRITE(6,102)    NAME(KLASS,KAT)
      GO TO 25
 24   WRITE (6,103)    NAME(KLASS,KAT)
C
 25   IF (KONTRL(M))    6,20,1
C
C
C     'KONTRL' WAS POSITIVE, SPECIFIED CATEGORY ONLY IS TO BE RETAINED.
 1    DO 2    J=1,KOUNT2
      IF (L(J,KLASS) - KAT)    2,4,2
 4    DO 5    I=1,3
      T(K,I) = T(J,I)
      S(K,I) = S(J,I)
      L(K,I) = L(J,I)
```

```
 5     L(K,I+3) = L(J,I+3)
       K = K + 1
 2     CONTINUE
       KOUNT2 = K-1
       GO TO 22
C
C      'KONTRL' WAS NEGATIVE, SPECIFIED CATEGORY IS TO BE OMITTED.
 6     DO 7   J=1,KOUNT2
       IF (L(J,KLASS) - KAT)   9,7,9
 9     DO 3   I = 1,3
       T(K,I) = T(J,I)
       S(K,I) = S(J,I)
       L(K,I) = L(J,I)
 3     L(K,I+3) = L(J,I+3)
       K=K+1
 7     CONTINUE
       KOUNT2 = K-1
 22    M = M+1
C
       N = KONTRL(M)/100
       IF (N)   35,14,35
C
C      'KONTRL' WAS ZERO, OR END OF SUBSET HAS BEEN REACHED,
C         END SELECTION, COMPUTE COVMAT AND D.
 20    CONTINUE
       IF (KONTRL(M))   35,36,35
 36    NUMB = 9
 35    DO 28   I = 1,KOUNT2
       DO 28   J=1,3
       D(J) = D(J) + T(I,J)
       DO 28   K=1,3
 28    COVMAT(J,K) = COVMAT(J,K) + T(I,J)*T(I,K)
C
       COUNT = KOUNT2
       DO 29   J=1,3
       D(J) = D(J) / COUNT
       DO 29   K=1,3
 29    COVMAT(J,K) = COVMAT(J,K)/COUNT
C
 100   FORMAT(1H0,95HIN COMPUTING THE NEXT SET OF RESULTS, ONLY MEASUREME
      1NTS IN THE FOLLOWING CATEGORIES WERE USED -)
 101   FORMAT(X,8HINCLUDED, 30X, 8HEXCLUDED)
 102   FORMAT (40X,A6)
 103   FORMAT (X,A6)
 104   FORMAT (1H1,63HSELECTION ERROR. PLEASE CHECK CONTROL CARDS AT END
      1OF DATA DECK)
 105   FORMAT (1H0, 16HALL MEASUREMENTS)
 106   FORMAT (1H0,  67HSELECTION UNSPECIFIED ON CONTROL CARD. NO ITEMS R
      2ETAINED IN SUBSET.)
       RETURN
        END



$IBFTC SOLSP
C
```

```
      SUBROUTINE SOLSP (A,X,NC,NV,ZERO,IERR)
C     - - - - - - - - - - - - - - - - - - - - -
C     THIS IS N.U. LIBRARY SUBROUTINE 4F01.1 (REDIMENSIONED).
C
      DOUBLE PRECISION   A, TEMP
      DIMENSION A(3,4), X(3,1), NJ(3)
C
      IERR=0
      NM1=NC-1
      NP1=NC+1
      NCV=NC+NV
      DO 10 I=1,NC
   10 NJ(I)=I
      DO 15 J=1,NV
      L=NC+J
      DO 15 I=1,NC
   15 A(I,L)=X(I,J)
      DO 100 K=1,NM1
      TEMP=0.0
      DO 30 I=K,NC
      DO 30 J=K,NC
      IF (ABS(A(I,J))-TEMP) 30,30,32
   32 TEMP=ABS(A(I,J))
      II=I
      JJ=J
   30 CONTINUE
      IF (TEMP-ZERO) 22,22,21
   22 IERR=K
      GO TO 111
   21 IF (II-K) 72,71,72
   72 DO 75 J=K,NCV
      TEMP=A(II,J)
      A(II,J)=A(K,J)
   75 A(K,J)=TEMP
   71 IF (JJ-K) 42,41,42
   42 DO 45 I=1,NC
      TEMP=A(I,JJ)
      A(I,JJ)=A(I,K)
   45 A(I,K)=TEMP
      NTEMP=NJ(JJ)
      NJ(JJ)=NJ(K)
      NJ(K)=NTEMP
   41 L=K+1
      DO 55 J=L,NCV
      A(K,J)=A(K,J)/A(K,K)
      DO 55 I=L,NC
   55 A(I,J)=A(I,J)-A(I,K)*A(K,J)
  100 CONTINUE
      IF (ABS(A(NC,NC))-ZERO ) 302,302,301
  302 IERR=NC
      GO TO 111
  301 DO 310 J=NP1,NCV
      A(NC,J)=A(NC,J)/A(NC,NC)
      K=NM1
  615 DO 320 I=K,NM1
  320 A(K,J)=A(K,J)-A(K,I+1)*A(I+1,J)
      K=K-1
      IF (K) 310,310,615
```

```
  310 CONTINUE
      DO 110 I=1,NC
      L=NJ(I)
      DO 110 J=1,NV
      M=J+NC
  110 X(L,J)=A(I,M)
  111 RETURN
       END
```

```
$IBFTC SPIN
C
      SUBROUTINE SPIN (KOSCOR, KOOKS)
C     - - - - - - - - - - - - - - - - - -
C     THE INPUT MAY BE A NUMBER (=KOUNT) OF DIRECTION COSINES (DIRCOS),
C     OR GEOGRAPHICAL COORDINATES (GCOORD) AND A ROTATION MATRIX(EIGMAT)
C     LIKE THAT OBTAINED FROM SUBROUTINE AXES.   TRANSFORMED COSINES OR
C     COORDINATES ARE STORED IN TRACOS OR SCOORD.   IF KOSCOR=0, ONLY
C     DIRCOS ARE TRANSFORMED.   IF KOSCOR = 1, GCOORD ARE TRANSFORMED.
C     IF KOSCOR = 2, BOTH DIRCOS AND GCOORD ARE ROTATED.
C
C     IF KOOKS = 1, A LIST OF ITEMS WHICH DEVIATE CONSIDERABLY FROM THE
C     AVERAGE IS PRINTED OUT.   IF KOOKS = 0, THE LISTING IS SUPPRESSED
C
      COMMON   COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
      IF (KOSCOR-1)    1,2,1
C     INITIALIZE
 1       DO 1100   J = 1,3
      DO 1200    I = 1, KOUNT
 1200 TRACOS(I,J) = 0.0
         DO 1100    I = 1,KOUNT
            DO 1100 K = 1,3
C
C     MUTIPLY EACH DIRCOS VECTOR BY THE MATRIX EIGMAT, TO OBTAIN TRACOS,
C        VECTORS IN TERMS OF DIRECTION COSINES ABOUT THE PRINCIPAL AXES.
 1100 TRACOS(I,J) = TRACOS(I,J) + DIRCOS(I,K) * EIGMAT(K,J)
      IF (KOSCOR-1)    5,2,2
C     TRANSFORM GCOORD TO SCOORD BY MULTIPLYING EACH VECTOR BY EIGMAT.
 2    DO 3    J=1,3
      DO 4    I=1,KOUNT
 4    SCOORD(I,J) =0.0
      DO 3    I=1,KOUNT
         DO 3    K=1,3
 3    SCOORD(I,J) = SCOORD(I,J) + GCOORD(I,K)*EIGMAT(K,J)
 5    CONTINUE
C
C     IF A LISTING OF TRANSFORMED MEASUREMENTS IS NOT DESIRED, OMIT NEXT
C     THREE CARDS.
      WRITE (6,106)
      DO 6    I=1,KOUNT
```

```
 6     WRITE (6,107)    I, (SCOORD(I,K),K=1,3),(TRACOS(I,K),K=1,3)
C
C      IF A DECK OF PUNCHED CARDS IS REQUIRED, CONTAINING TRANSFORMED
C      COORDINATES AND DIRECTION COSINES, INSERT TWO CARDS AS FOLLOWS,
C      DO 7    I=1,KOUNT
C7     WRITE (6,108)    I, (SCOORD(I,K),K=1,3),(TRACOS(I,K),K=1,3)
C
       IF (KOOKS)    23,23,21
C      CALCULATE THE ROOT-MEAN-SQUARE DEVIATION FROM THE MEAN VECTOR.
 21    WRITE (6,109)
       WRITE (6,110)
       E = SQRT((EIGVAL(1))**2+ (EIGVAL(2))**2 + (EIGVAL(3))**2)
       DO 20    I=1,KOUNT
       TSUM = SQRT((TRACOS(I,1))**2+ (TRACOS(I,2))**2 + (TRACOS(I,3))**2)
C      THE MULTIPLIER OF E ON THE NEXT CARD CAN BE ADJUSTED TO GIVE A
C         REASONABLE LENGTH OF LIST OF UNUSUAL MEASUREMENTS.
C
C      PRINT OUT LIST OF MEASUREMENTS DIFFERING FROM THE MEAN BY MORE
C         1.5 RMS DEVIATIONS.
       IF (TSUM - 1.5*E)    20,22,22
 22    I1 = LIST(I,1)
       I2 = LIST(I,2)
       I3 = LIST(I,3)
       I4 = LIST(I,4)
       I5 = LIST(I,5)
       I6 = LIST(I,6)
       WRITE (6,111)    I,(GCOORD(I,J),J=1,3),(DIRCOS(I,J),J=1,3),
      1NAME(1,I1),NAME(2,I2),NAME(3,I3),NAME(4,I4),NAME(5,I5),NAME(6,I6)
 20    CONTINUE
 23    CONTINUE
C
 106   FORMAT (1H0,96HLIST OF COORDINATES AND DIRECTION COSINES OF MEASUR
      1EMENTS TRANSFORMED TO REFER TO PRINCIPAL AXES)
 107   FORMAT (X,I4,3X,E10.3,3X,E10.3,3X,E10.3,10X,3F8.4)
 108   FORMAT (1HP,6X,I6,3F6.0,3F6.3)
 109   FORMAT (1H0, 62HTHE FOLLOWING MEASUREMENTS DEVIATE CONSIDERABLY FR
      1OM THE MEAN,)
 110   FORMAT (X,4HITEM,X,15HLOCATION COORDS,16X,17HDIRECTION COSINES,
      114X, 7HFACTORS)
 111   FORMAT (X,I4,X,3E10.3,3X,3F7.4,3X,6(A6,X))
       RETURN
        END
```

```
$IBFTC WULFF
C
       SUBROUTINE WULFF
C      - - - - - - - -
C         THIS SUBROUTINE PLOTS A SCATTER DIAGRAM OF
C      A GROUP OF DIRECTION COSINES (T(I,J).  THE DIAGRAMS MAY BE USED
C      AS OVERLAYS ON A 20-CENTIMETRE DIAMETER WULFF NET.
C      THE NUMBER IN EACH RECTANGLE INDICATES THE NUMBER OF POINTS
C      FALLING IN THAT RECTANGLE.
C
       COMMON    COVMAT,D,DIRCOS,EIGMAT,EIGVAL,GCOORD,KONTRL,KOUNT,KOUNT2,
```

```
     1   L,LIST,NAME,NOVEC,S,SCOORD,T,TRACOS
      DIMENSION   COVMAT(3,3),D(3),DIRCOS(300,3),EIGMAT(3,3),EIGVAL(3),
     1 GCOORD(300,3),KONTRL(20),L(300,6),LIST(300,6),NAME(6,9),S(300,3),
     2 SCOORD(300,3),T(300,3),TRACOS(300,3)
C
      DIMENSION IROW(48,40)
C
C     INITIALIZE, PUTTING -9 IN THE AREA OUTSIDE THE STEREOGRAM.
      WRITE (6,105)
C     SEE COMMENT ON CONSTANTS BELOW.
      P = 1.0 / (3.937*6.0)
      Q = 1.0 / (3.937*5.0)
C     FOR POINTS ON THE CIRCUMFERENCE OF THE PRIMITIVE CIRCLE,
C     (MP/2)**2 + (NQ/2)**2 = 1, BY THE THEOREM OF PYTHAGORAS.
      DO 6   M=1,48
      R = M
      R = ABS(R-24.0)
      DO 2   N=1,40
 2    IROW(M,N) = 0
      N2 = SQRT((1.05-(R*P)**2)/Q**2) + 0.5
      N3 = 20 - N2
      N2 = 20 + N2
      DO 7   N=1,N3
 7    IROW(M,N) = -9
      DO 6   N=N2,40
 6    IROW(M,N) = -9
C
C     COMPUTE THE NUMBER OF POINTS IN EACH RECTANGLE OF THE STEREOGRAM.
C
      IF (NOVEC-1)   24,25,24
C     THE NEXT SECTION IS FOR VECTORIAL DATA ONLY.
 24   DO 3   I=1,KOUNT2
      UP = T(I,3)
      IF (UP)   1,28,28
 1    T(I,1) = -T(I,1)
      T(I,2) = -T(I,2)
C     IN THE NEXT TWO EXPRESSIONS, 3.937 IS THE RADIUS OF THE NET, IN
C     INCHES, 6.0 IS THE NUMBER OF LINES OF OUTPUT PER INCH, 5.0 IS THE
C     NUMBER OF TWO-DIGIT CHARACTERS PER INCH, 0.5 IS A ROUNDING FACTOR.
 28   M = ((T(I,1)/(1.0+ABS(T(I,3))))*3.937*(-6.0))+1.0+3.937*6.0
      N = ((T(I,2)/(1.0+ABS(T(I,3))))*3.937*(-5.0))+1.0+3.937*5.0
 3    IROW(M,N) = IROW(M,N) + 1
      GO TO 26
C
C     THE NEXT SECTION IS FOR SCALAR DATA ONLY.
C     COMPUTE DIRECTION COSINES FROM ELEVATIONS.
 25   DO 11  I=1,KOUNT2
      DO 11  J=1,KOUNT2
C     PLOT EACH VECTOR ONCE ONLY - DIRECTED UPWARDS.
      IF (S(I,3) - S(J,3))    11,12,13
 12   IF (I-J)   18,11,18
 18   IF (S(I,1) - S(J,1))    11,14,13
 14   IF (S(I,2) - S(J,2))    11,13,13
 13   D(1) = S(I,1) - S(J,1)
      D(2) = S(I,2) - S(J,2)
      D(3) = S(I,3) - S(J,3)
      P = D(1)**2 + D(2)**2 + D(3)**2
      P = 1.0 / SQRT(P)
```

```
      D(1) = D(1) * P
      D(2) = D(2) * P
      D(3) = D(3) * P
C     SEE EXPLANATION OF THIS EXPRESSION ABOVE (PRECEDING STATEMENT 28).
 30   M = ((D(1)/(1.0+ABS(D(3))))*3.937*(-6.0)) + 1.0 + 3.937*6.0
      N = ((D(2)/(1.0+ABS(D(3))))*3.937*(-5.0)) + 1.0 + 3.937*5.0
      IROW(M,N) = IROW(M,N) + 1
 11   CONTINUE
 26   CONTINUE
      DO 4    M=1,24
C
C     PRINT THE RESULTS.
 4    WRITE (6,100)    (IROW(M,N),N=1,20), (IROW(M,N),N=21,40)
      WRITE (6,104)
      DO 5    M=25,48
 5    WRITE (6,100)    (IROW(M,N),N=1,20), (IROW(M,N),N=21,40)
      WRITE (6,106)
      WRITE (6,101)     KOUNT2
      WRITE (6,103)
      WRITE (6,107)
C
 100  FORMAT (X, 20X, 20I2, 1H., 20I2)
 101  FORMAT (1H0, I4, 13H MEASUREMENTS)
 103  FORMAT (1H0, 98HSTEREOGRAM SHOWING DISTRIBUTION OF POINTS PROJECTE
     1D ON LOWER HEMISPHERE OF 20-CENTIMETRE WULFF NET)
 104  FORMAT (19X,85HWEST.....................................(3).......
     1.........................EAST(2))
 105  FORMAT (1H1, 57X, 5HNORTH)
 106  FORMAT (56X, 8HSOUTH(1))
 107  FORMAT(1H0,38X,45H---------------------------------------------)
      RETURN
       END
```