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ROADFORD ENVIRONMENTAL
INVESTIGATION:
TAMAR RIVER QUALITY MODEL:
PHASE 1 REPORT TO WRc/SWWA
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Summary

A daily simulation model has been applied to the River Tamar system from the proposed site of the Roadford Reservoir to the tidal limit at Gunnislake. The model simulates flow, dissolved oxygen, biochemical oxygen demand, nitrate, ammonia and any conservative pollutant. Information on tributary inflows, effluent discharges and proposed reservoir discharges have been incorporated in order to assess the effects of these inputs on river quality. The fourteen reach model has been set up on a DEC mini-computer with extensive interactive graphics feature, so that the effect of input changes can be rapidly assessed from plots of flow and quality at key sites along the river system.

1. Introduction

This report describes research progress in the first phase of a water quality modelling study on the River Tamar. The objectives and schedule of the complete study are as follows.

- 1.1 To apply an existing river water quality model to non-tidal sections of the Tamar and those parts of its tributaries (Lyd, Thrushel and Wolf) leading to the site of the new Roadford Reservoir; the model to simulate dissolved oxygen concentrations in these rivers, corresponding to known or assumed steady-state loads of BOD and ammonia and to a specified regime of mean daily river discharge.
- 1.2 To extend the scope of the model so as to simulate the effects of algae upon oxygen balance and to calculate the river's pH.
- 1.3 To supply the fully calibrated models in magnetic tape form suitable for running on a DEC computer with FORTRAN 77 compiler and for interfacing with a tidal water quality model, to be developed by IMER.

The first phase of the study has involved an assessment of water quality on the Tamar and the tributaries leading to the site of the new Roadford Reservoir and the application of the IHQM, the Institute of Hydrology River Quality Model. This is a dynamic integrated flow and quality model that has been developed over many years and applied to many river systems including the Great Ouse, The Thames, and the River Tawe. Details of the model are presented and a preliminary application to the Tamar investigated.

2. Hydrological Data

Hydrological data forming the basis of the IH flow model consists of mean daily

flows from the hydrological record for 1982-1986. The gauging stations for which data is available are shown in Table 1 and Figure 1. This data base has been set up on a mini-computer and used by the model to simulate flow along the river system. Ungauged tributaries and inflows are computed on the same areal basis as the IH hydrological studies being undertaken by Dr M.J. Lowing.

3. Water Quality Data

The Water Quality Data base for the model consists of records of varying length from the period 1982-1986. Mean values of various water quality parameters are provided by the SWWA Water Archive System Statistical analysis. Table 2 and Figure 2 show the principal water quality monitoring sites in the system.

3.1 Water Chemistry of the Tamar and Tributaries

Tables 3 and 4 show the mean values of the water quality parameters used as inputs to the model for both the main river profile and the tributaries. Little seasonal variation has been observed in the four parameters of principal interest (D.O., BOD, Nitrate and Ammonia) except for in some cases high summer BOD values at Gunnislake possibly resulting from the decay of dead algae along the lower reaches of the river. Therefore no seasonal component has been included except in the case of Chlorophyll A. The data in Tables 3 and 4 shows the river system to be relatively pollution free at present although the input from the main Tamar tributary, monitored at Druxton Bridge, has a higher BOD than the remainder of the river system.

3.2 Algal growth in the Tamar

Table 5 shows chlorophyll A levels for the Tamar at Gunnislake and a marked seasonal variation is evident with increasingly high levels in recent years indicating the occurrence of algal blooms. There is not a facility for modelling algal growth at present although the contribution of algae to BOD and DO via decay and photosynthesis, respiration etc. is included in the form of the appropriate rate coefficients.

4. The Flow and Quality Model

The first stage of model development has been to divide the main stem into a series of reaches as indicated in Figures 3 and 4 and shown in Table 6. The reaches are selected on the basis of the location of tributaries, effluents, abstractions and weirs, since these all affect the mass balance and hence the quality of the river water. Table 6 gives information on the reach lengths. Considerable experience gained in previous modelling studies (Whitehead et al., 1979, 1981, 1982, 1984) has shown that such a reach structure is particularly

suitable for modelling major rivers such as the Tamar. Details of the flow and quality model are given in Appendix 1. As required in the first phase of the study (see preface) we have maintained a daily simulation time step. However, this time step needs to be reduced considerably to capture the true dynamic behaviour of the river system since recent tracer experiments suggest travel times of the order of 1-3 days along the river system. However the daily model provides an initial assessment of the steady state behaviour of the river system and a means of calibrating the model against observed water quality.

Tables 7 and 8 show the adopted rate coefficients for the model and observed and simulated water quality. In general the model performs well reproducing the main characteristics of stream quality. Figures 5A-G show a series of steady state runs. The model results can be presented as profiles along the river or as concentration curves against time at any site of interest. So Figure 5A shows the flow profile down the river on 20th June 1986 and Figures 5B and 5C show the flow variations at Combepark Farm and Gunnislake during June 1986. Figures 5D-G show the profiles of water quality along the river system.

The model has been set up to operate interactively so that it is relatively easy to select plots or to simulate the effects of a) effluent discharges at any site or b) discharges from Roadford Reservoir.

Figures 5H-K show the effect of a simulated sewage works breakdown at Launceston. The effect on BOD and Ammonia below the confluence of the Tamar is significant but these do not seem to affect DO significantly because of the short residence times in the river and high reaeration rates. This pollutant load may have a more significant effect however under extreme low flow conditions or in the tidal section of the river.

5. Computational Aspects

The flow and quality model has been set up initially in an interactive conversational mode on a micro computer, the DEC Micro PDP 11/23. The programs have been written in FORTRAN 77 under the DEC TSX-11 operating system. Some system-dependent software has been written in MACRO 11 assembler where FORTRAN would have been impractical. Extensive use of graphic facilities on a VT241 terminal are made so that the effects of system changes on flow and water quality can be rapidly assessed. Hard copy of plots are attainable via a HP7475 pen plotter and it is possible to dump screen plots to a DEC LA50 printer. A detailed users' guide for the Tamar River Management Modelling System will be prepared at a later date.

6. Conclusions and Recommendations

The initial five week study has shown that a model of the Tamar can be developed and used to simulate the river system. Its application in terms of predicting steady state behaviour is considered adequate although further

information on velocity flow relationships are required to further refine the model. Such data is being collected as part of the WRC tracer experiments.

A major requirement is to simulate the effects of discharges from Roadford Reservoir. These effects will inevitably be of short duration and therefore the time scale of the model should be altered to allow for dynamic changes in addition to simulating overall steady state behaviour.

Further data are required on chlorophyll A, pH and the extremes of the DO diurnal cycle (i.e. daily max and min DO) to assess oxygen production in the river during summer months.

A key variable of ecological significance in the Tamar is temperature and it might be appropriate to simulate this variable in place of the conservative pollutant or nitrate.

Table 1 Principal Gauging Stations in the Tamar River System

1. Model Profile

<u>River</u>	<u>Gauging Station</u>
Wolf	Roadford Dam
Thrushel	Tinhay
Lyd	Lifton Park
Tamar	Gunnislake

2. Tributaries

Tamar	Crowford Bridge
Inry	Beals Mill

Table 2 Water Quality Monitoring Sites

1. Main Profile

<u>River</u>	<u>Monitoring Site</u>
Wolf	Combepark Farm
Wolf	Upstream of R. Thrushel Confluence
Thrushel	Tinhay
Lyd	Lifton Park
Tamar	Greystones
Tamar	Horsebridge
Tamar	Gunnislake

2. Tributaries

Tamar	St Leonards Bridge Drupton Bridge
Thrushel	Stowford Bridge
Lew	u/s of R. Lyd confluence
Lowley Brook	Lowley Bridge
R. Lockett	Lockett Bridge
Inny	Beals Mill

Table 3 Existing Water Quality of River Profile

Site		BOD	DO(mg/l)	NO ₃	NH ₃
1. u/s Thrushel Confluence	Max	3.0	12.5	2.6	0.01
	Min	0.9	6.6	1.2	0.01
	Mean	1.9	10.6	1.9	0.01
	SD	0.71	1.78	0.45	0.00
2. Tinhay	Max	3.0	12.5	3.0	0.01
	Min	0.7	7.0	1.3	0.01
	Mean	1.97	10.26	2.27	0.01
	SD	0.76	1.88	0.56	0.00
3. Lifton	Max	3.1	12.5	3.0	0.01
	Min	0.2	10.0	1.6	0.01
	Mean	1.84	11.07	2.4	0.01
	SD	0.96	0.77	0.47	0.00
4. Greystones	Max	3.5	12.2	3.0	0.58
	Min	2.0	8.3	2.0	0.03
	Mean	2.76	10.13	2.61	0.2
	SD	0.51	1.26	0.34	0.17
5. Horsebridge	Max	4.6	12.1	3.2	0.6
	Min	1.8	8.7	2.2	0.02
	Mean	2.66	10.4	2.78	0.17
	SD	0.86	1.1	0.32	0.18
6. Gunnislake	Max	6.4	13.7	3.9	0.01
	Min	0.5	8.6	1.4	0.01
	Mean	2.46	10.57	2.94	0.01
	SD	1.26	1.27	0.59	0.00

All data refers to the period 1.1.1985 - 30.11.1986

Table 4 Existing Water Quality of Main Tributaries

Tributary	Monitoring Site		BOD	DO(mg/l)	NO ₃	NH ₃
1. Rexton 1982-1986	Rexton Bridge	Max	3.4	13.5	3.7	
		Min	0.4	6.6	0.7	-
		Mean	1.73	10.46	1.78	0.2
		SD	0.69	1.53	0.67	-
2. Thrushel	Stowford Bridge	Max	5.5	12.6	2.9	0.01
		Min	0.8	9.1	1.4	0.01
		Mean	2.29	10.36	2.17	0.01
		SD	1.49	1.16	0.46	0.00
3. Lew	u/s R. Lyd	Max	3.3	12.1	2.7	0.01
		Min	0.2	8.5	1.3	0.01
		Mean	1.44	10.51	2.09	0.01
		SD	0.90	1.38	0.51	0.00
4. Tamar	Druyton Bridge	Max	8.0	9.0	2.6	0.01
		Min	4.3	6.2	1.8	0.01
		Mean	6.15	7.6	2.2	0.01
		SD	2.62	1.98	0.57	0.00
5. Six (Synthetic data used for model. No data available)		Max				
		Min				
		Mean	2.0	10.0	2.0	0.2
		SD				
6. Lowley Brook	Lowley Bridge	Max	2.6	11.5	5.0	0.01
		Min	0.1	8.7	2.9	0.01
		Mean	1.70	10.54	4.26	0.01
		SD	0.87	0.96	0.55	0.00
7. Inny	Beals Bridge	Max	3.6	11.9	4.0	0.01
		Min	1.7	9.0	3.1	0.01
		Mean	2.44	10.81	3.36	0.01
		SD	0.65	1.06	0.36	0.00

All data refers to the period 1.1.1985-30.11.1986 unless otherwise indicated.

Table 5 Mean monthly chlorophyll A at Gunnislake

	April	May	June	July	August	September	August
1978		9.2		7.8	4.7		
1979	3.96	9.95	22.5	26.1	4.4	3.85	12.4
1980	8.05	9.5	12.5	9.5	10.0	5.0	3.0
1981	3.9	19.5		4.5	2.0		12.5
1982		12.0		7.0		7.6	3.7
1983	8.7	4.0	8.6	57.0	5.0	2.0	2.0
1984	4.0	13.5	54	156.0			

Table 6 Lengths of Reaches in River Tamar

Number	Distance (kms)	Start Point	End Point
1	1.55	Roadford Down	
2	2.94	Combepark Farm G.S.	
3	2.75	U/s of R. Thrushel Confluence	
4	2.06	Tinhay Gauging Station	
5	3.55	Lifton Park Gauging Station	
6	2.04	D/s of River Tamar Confluence	
-	1.99	Greystones Gauging Station	
8	2.71	Lowley Bridge	
9	2.0	U/s of River Inny Confluence	
10	3.0	D/s of Rivery Inny Confluence	
11	2.37	Tutwell	
12	3.0	Horsebridge	
13	2.0	Latchley	
14	2.37	Chilsworthy	Gunnislake

Table 7 Rate Coefficients for Tamar W.Q. Model

Rate of Denitrification	0.05
Rate of Decay of BOD	0.2
Ammonia Nitrification Rate	0.05
Reaeration Coefficient	Varies according to reach
Rate of oxygen uptake by sediment	0.1
Rate of BOD Addition by dead algae	0.01
Rate of O ₂ production by photosynthesis (up to 50 mg/l)	0.0027
Rate of O ₂ production by photosynthesis (above 50 mg/l)	0.001

Table 8 Comparison of observed and simulated mean values of principal water quality parameters

Site	BOD		DO		Nitrate		Ammonia	
	Observed	Simulated	Observed	Simulated	Observed	Simulated	Observed	Simulated
Gunnislake	2.46	2.457	10.57	10.2	2.94	2.38	0.11	0.17
Greystones	2.76	2.784	10.13	9.96	2.61	2.09	0.2	0.216
Tinhay	1.97	2.22	10.25	10.18	2.27	2.645	0.07	0.195

Observed values are 1985-1986 means

Simulated values are means calculated using data from a series of model runs throughout 1986

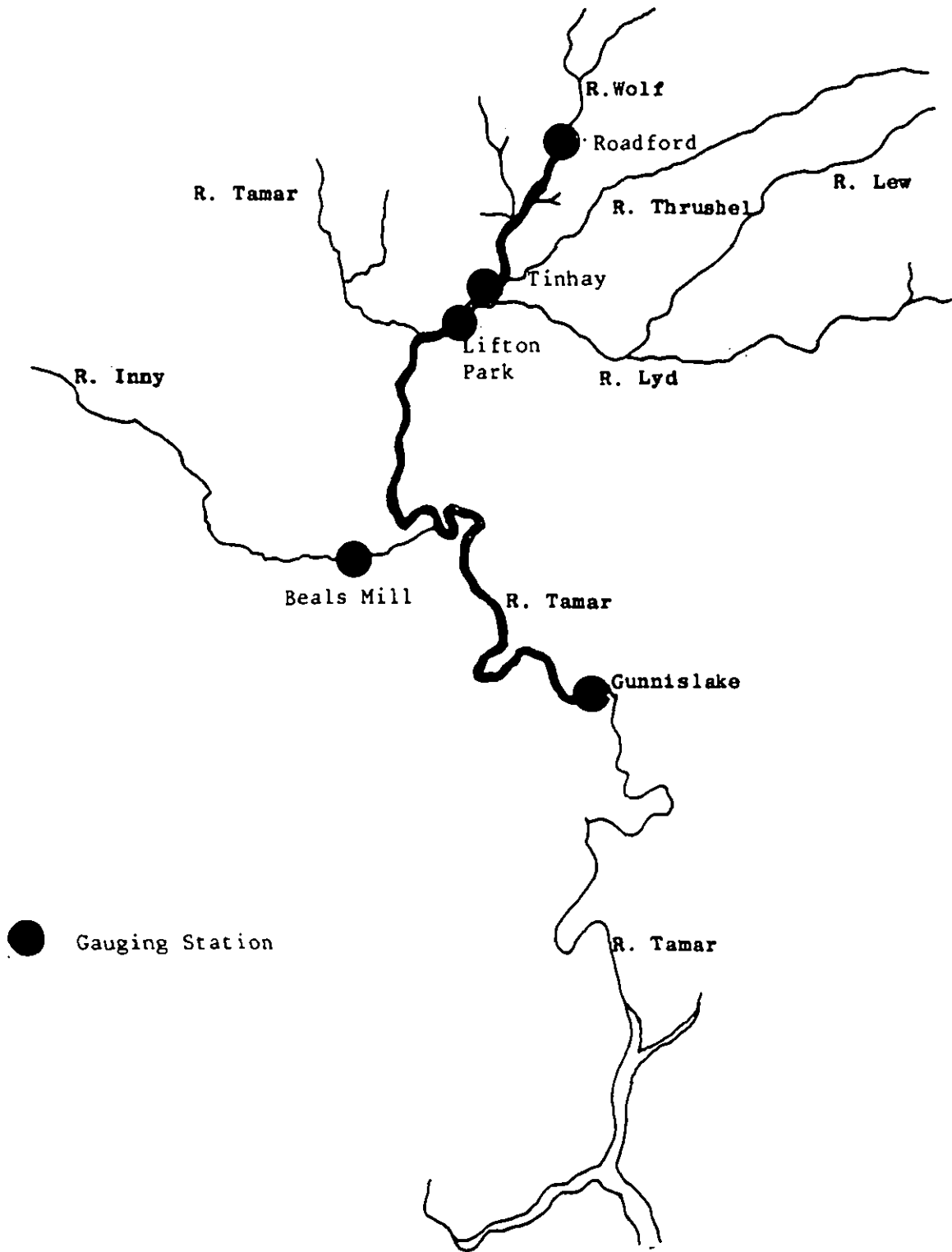


Figure 1 Map to show Tamar River System and gauging stations

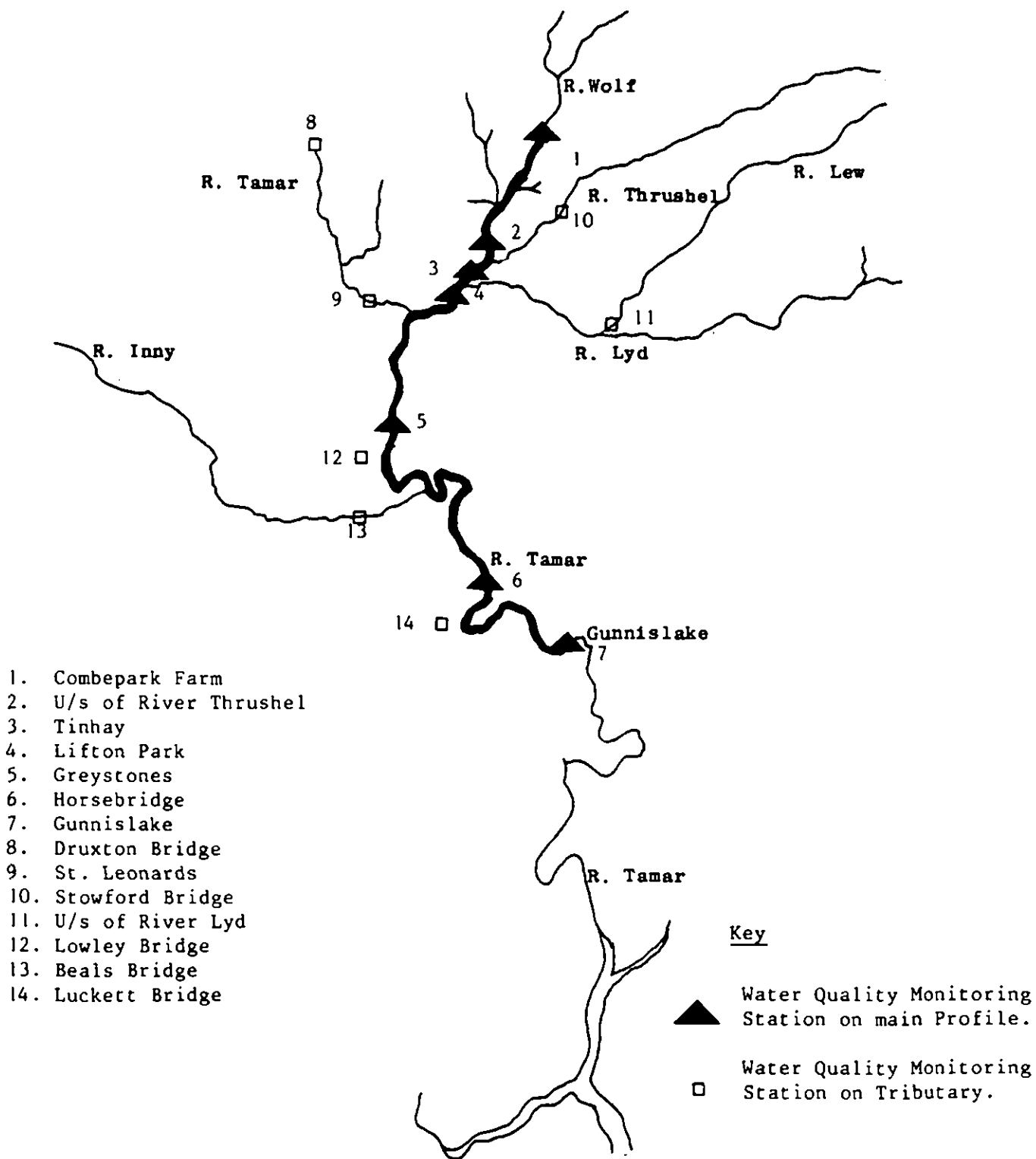


Figure 2 Map to show Principal Water Quality Monitoring Stations

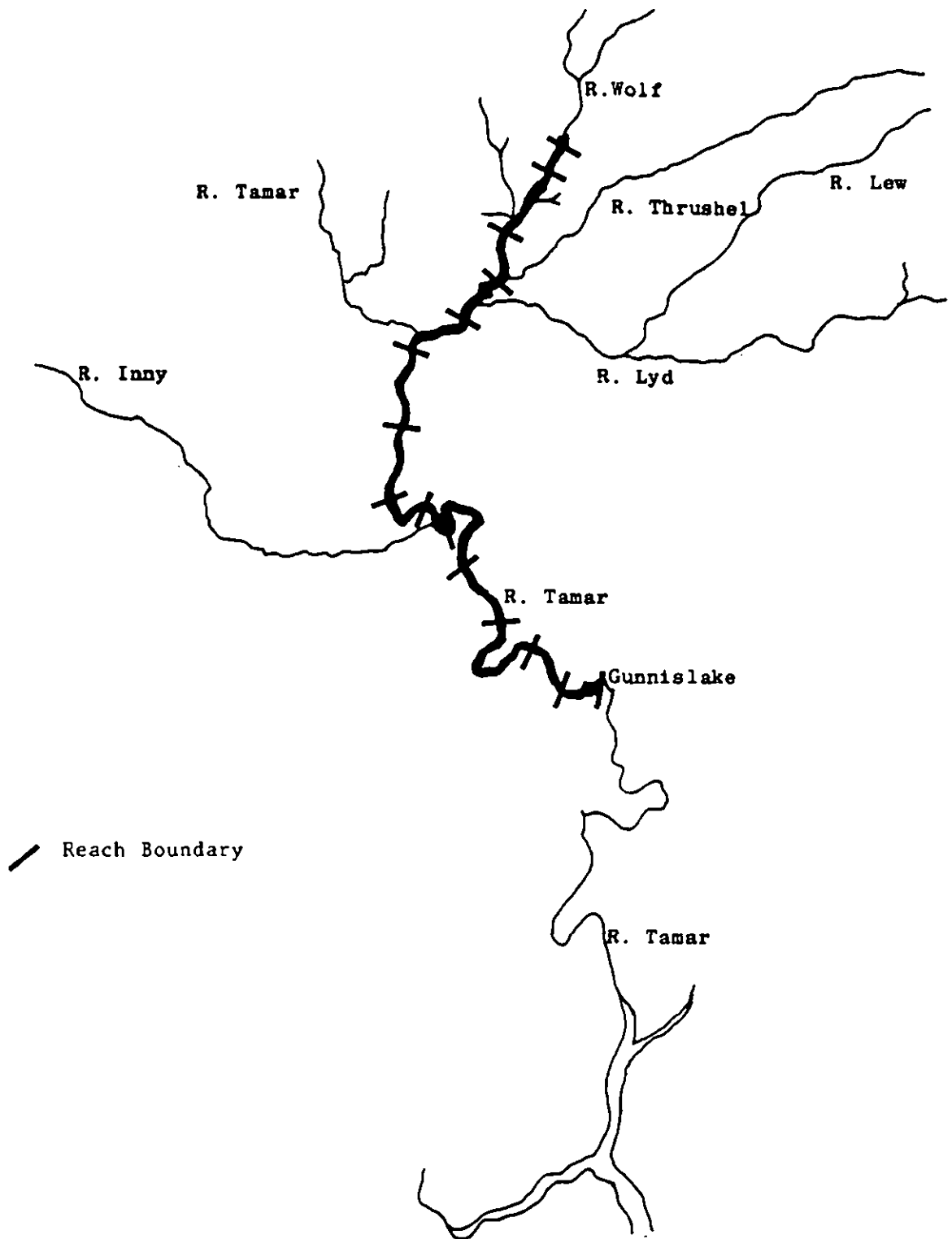


Figure 3 Map to show Profile Reach Structure

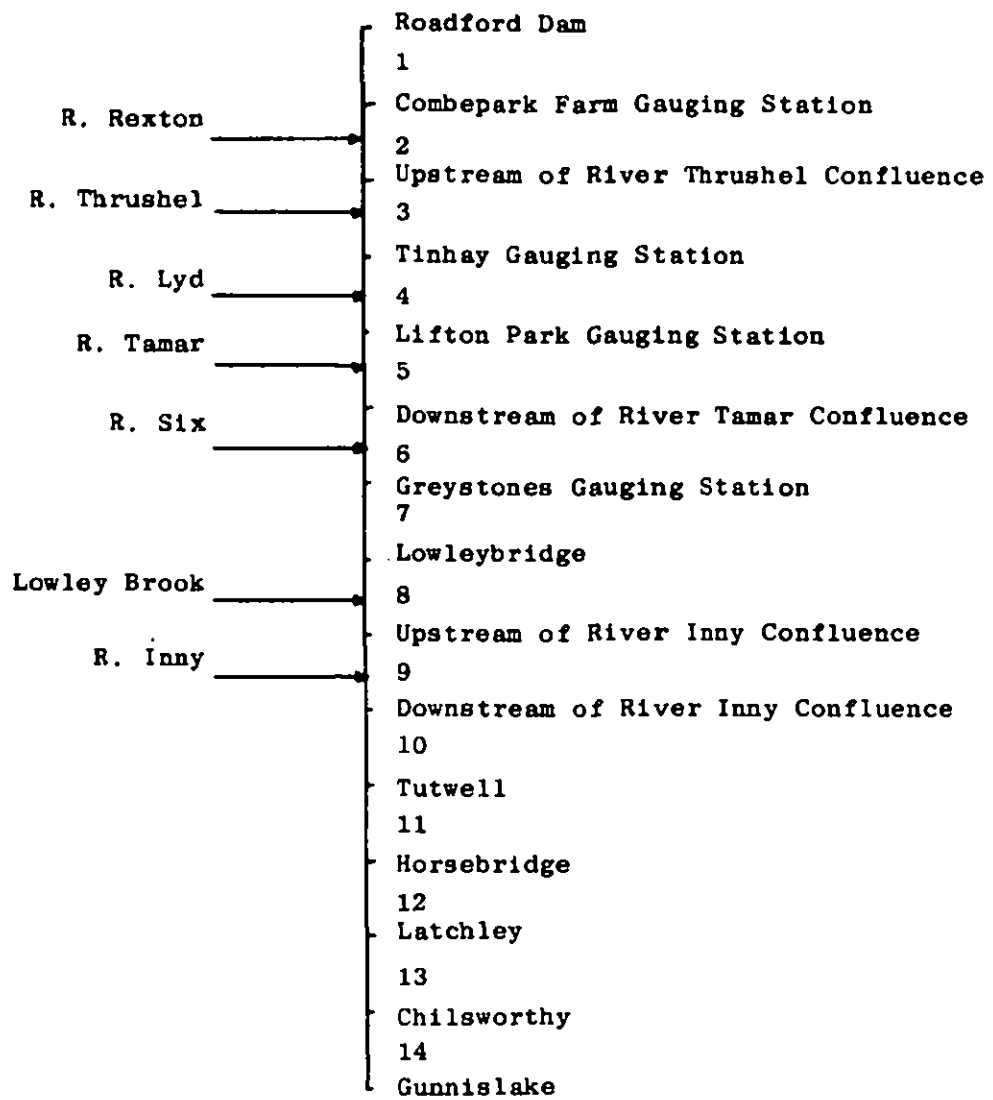
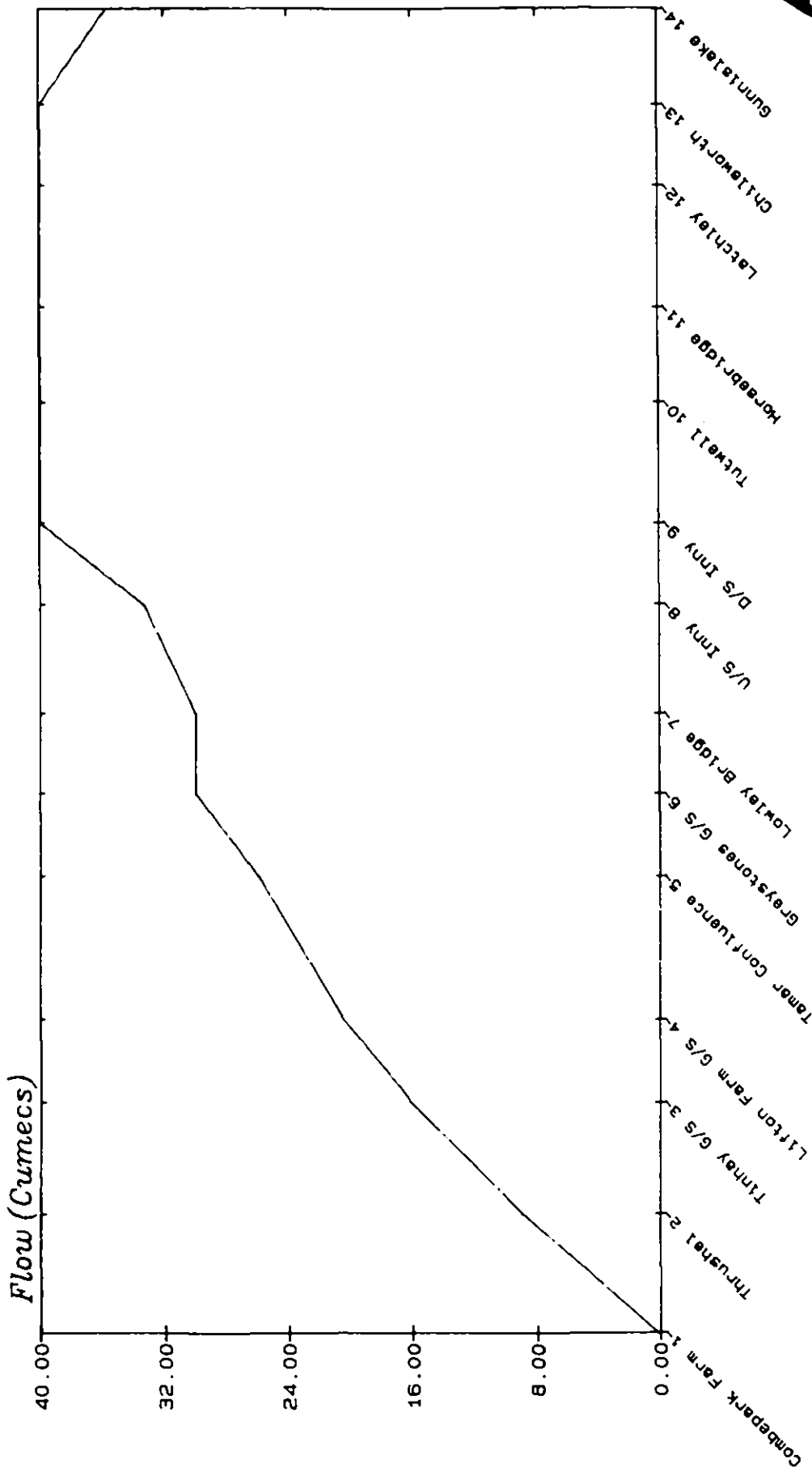


Fig. 4 Diagram to show Reach Structure of River Tamar

River profile for 20-JUN-1986



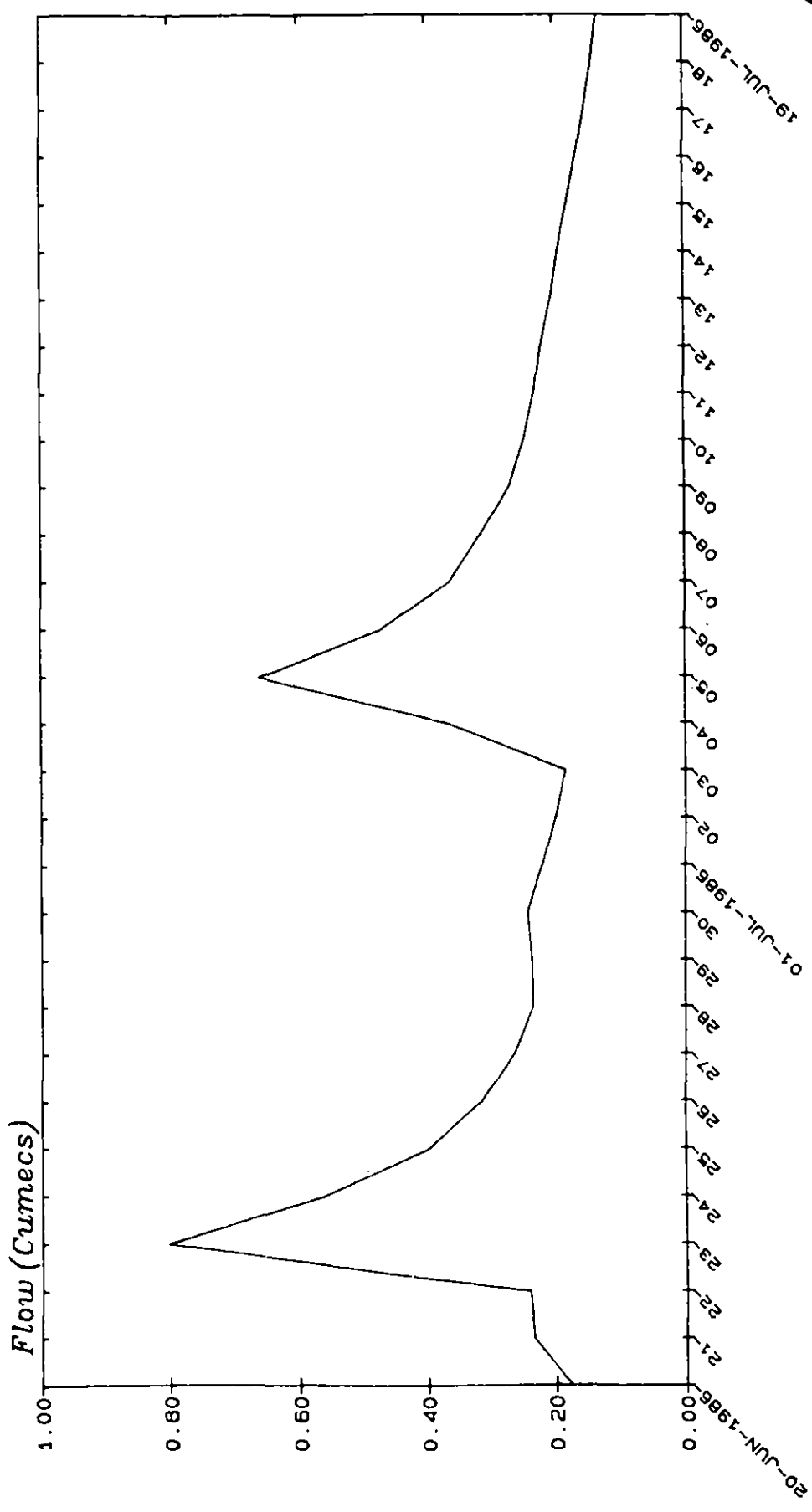
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Figure 5A

Profile in time for Combepark Farm G/S (1)



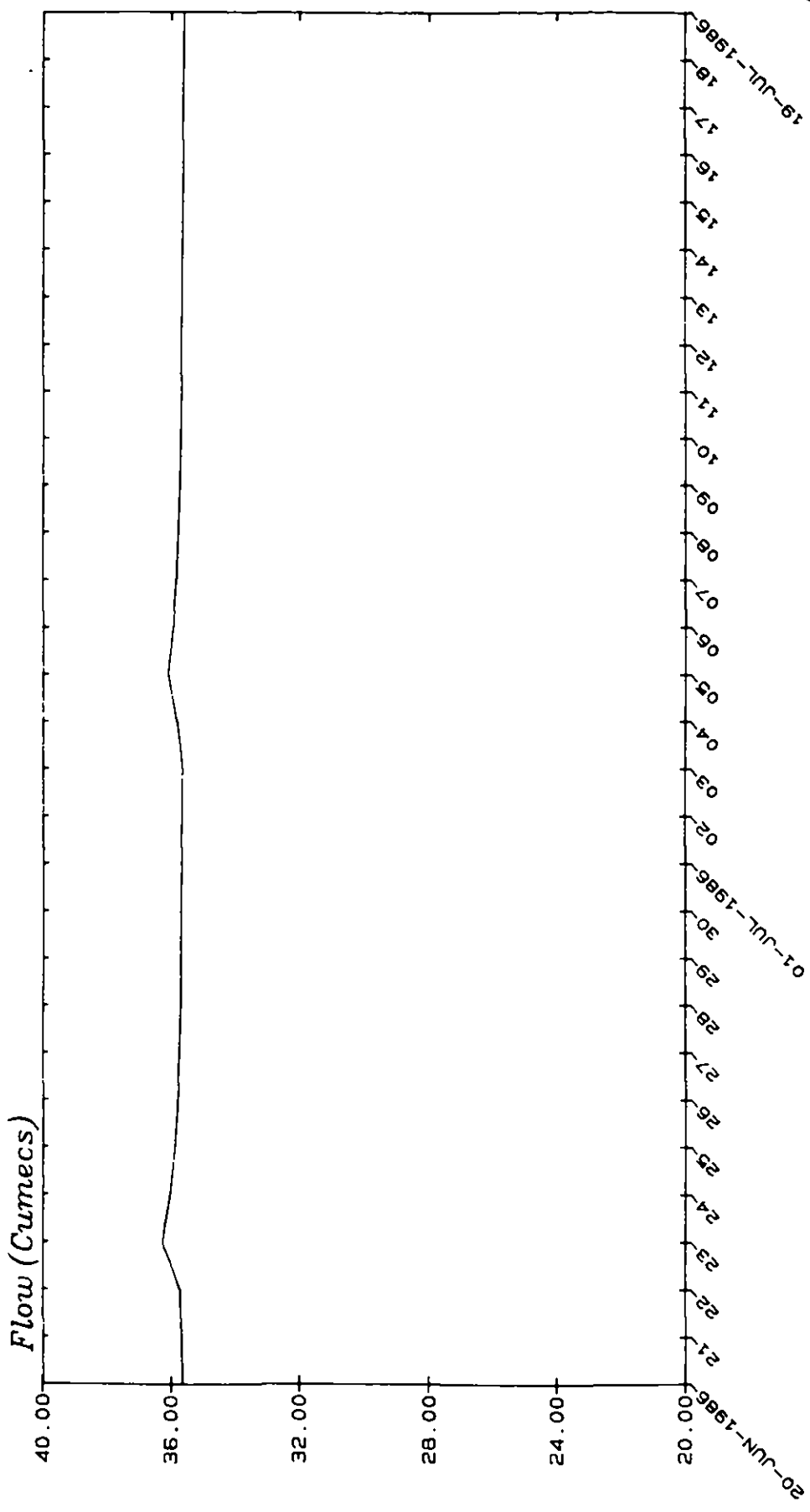
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Figure 5B

Profile in time for Gunnislake (14)



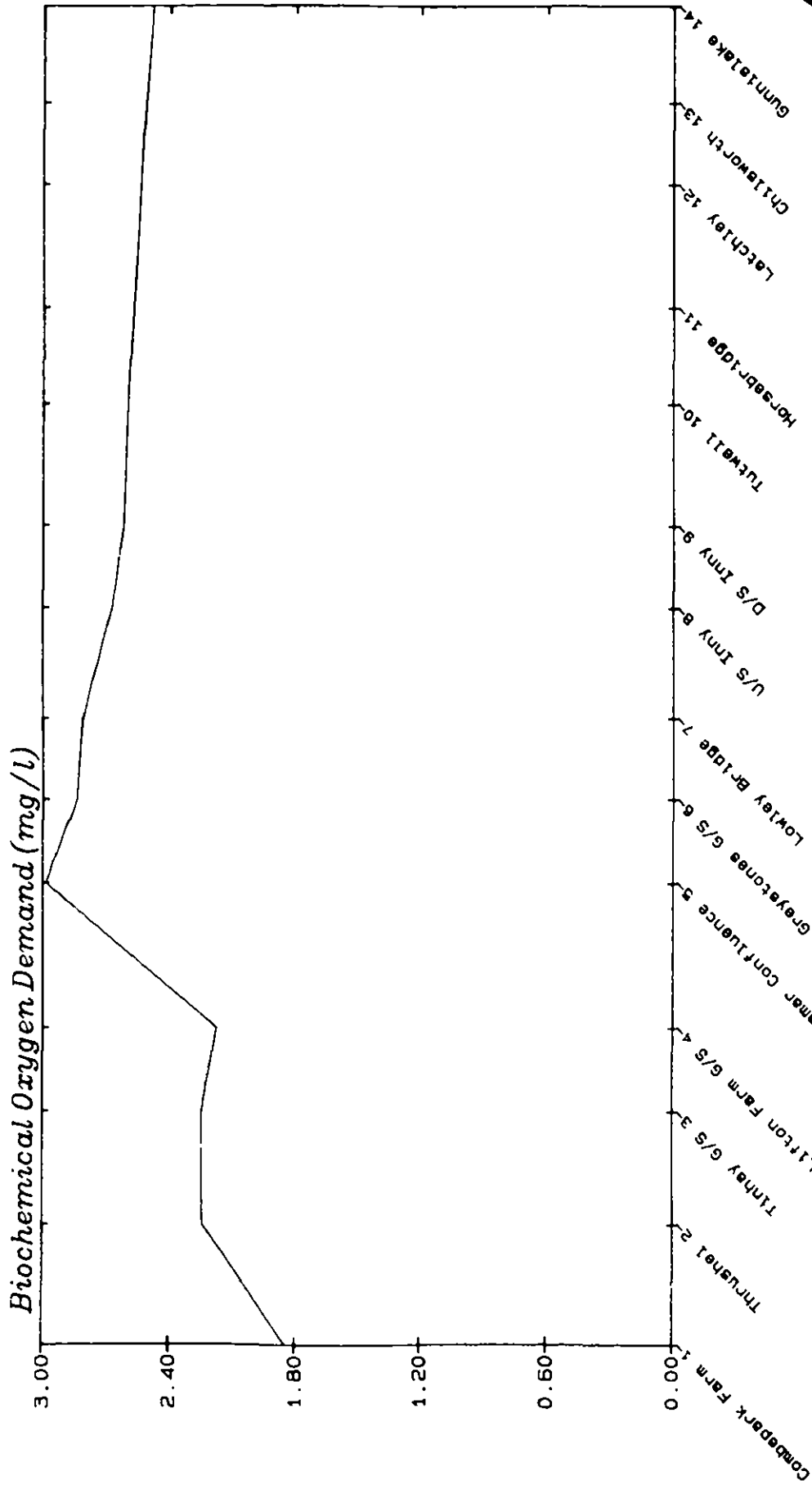
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Figure 5C

River profile for 20--JUN--1986



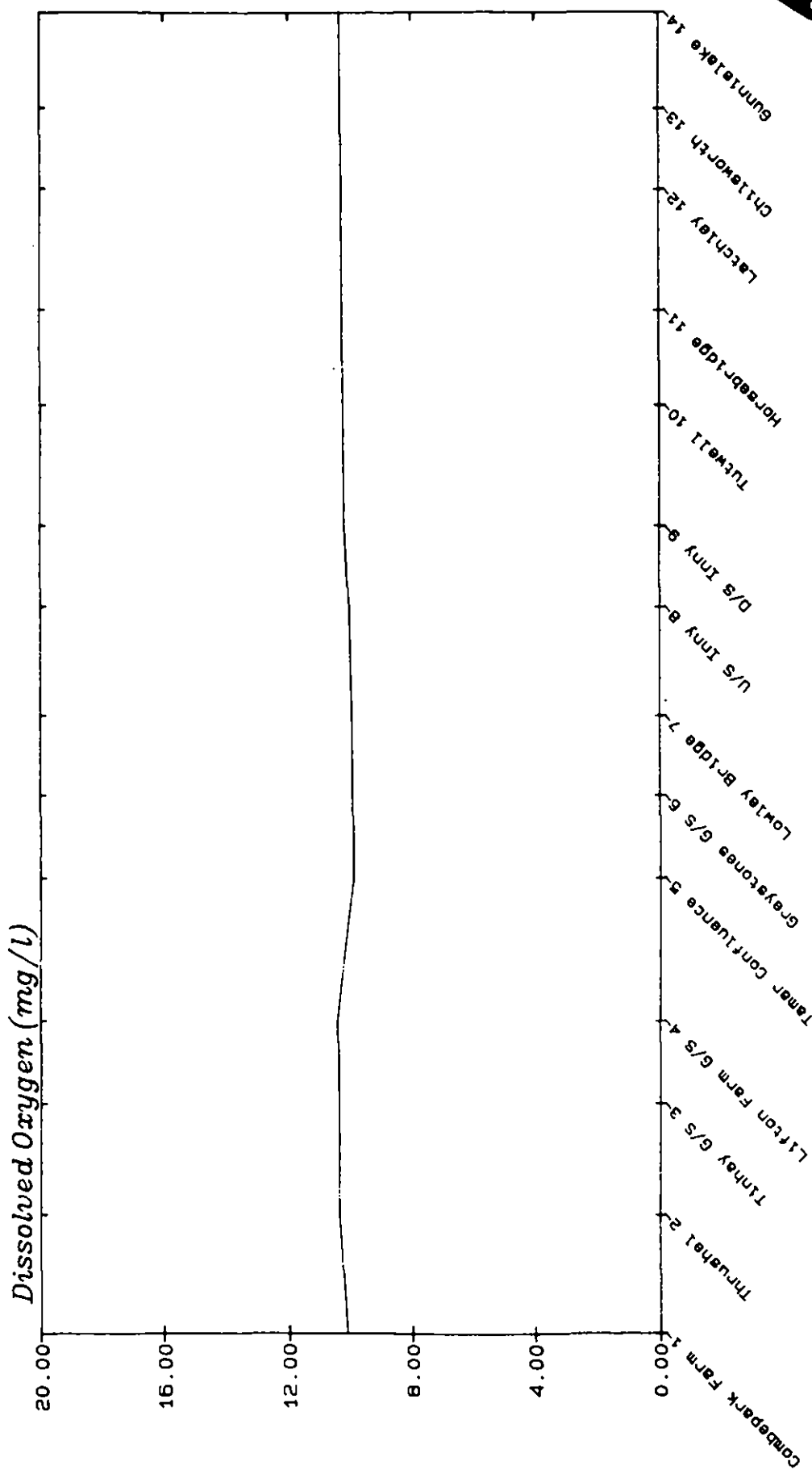
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Figure 6D

River profile for 20-JUN-1986



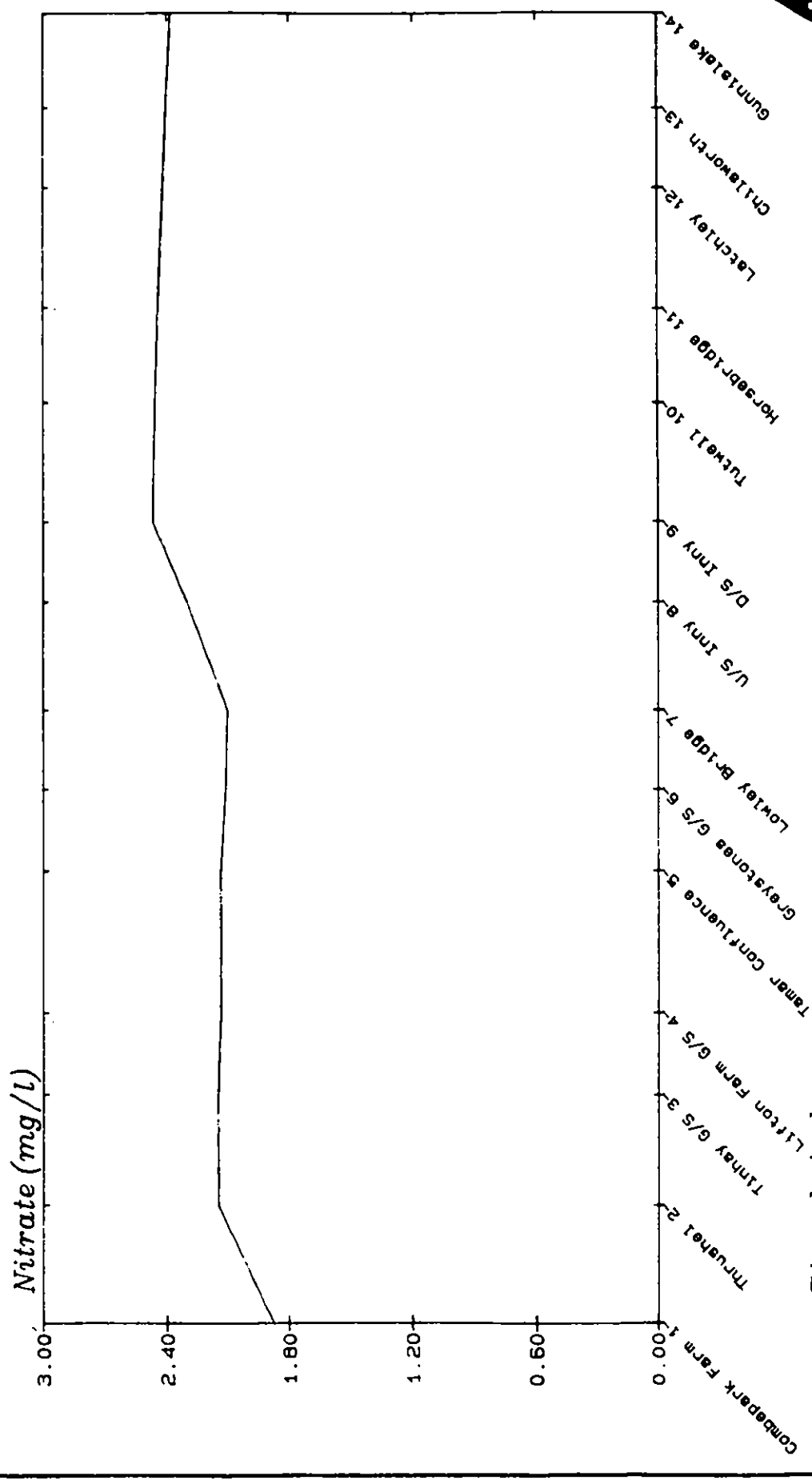
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Figure 5E

River profile for 20-JUN-1986



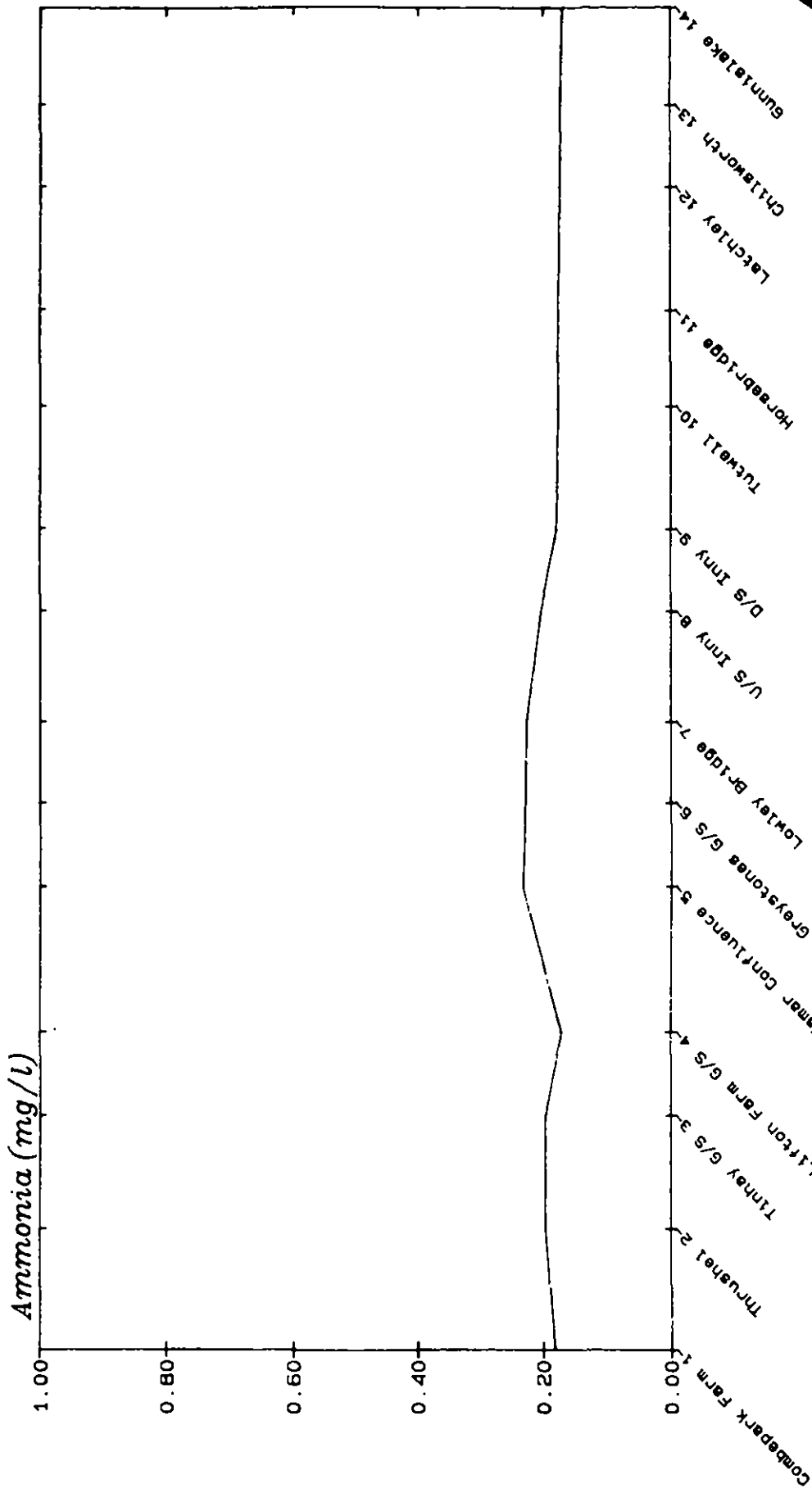
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Figure 5F

River profile for 20--JUN--1986



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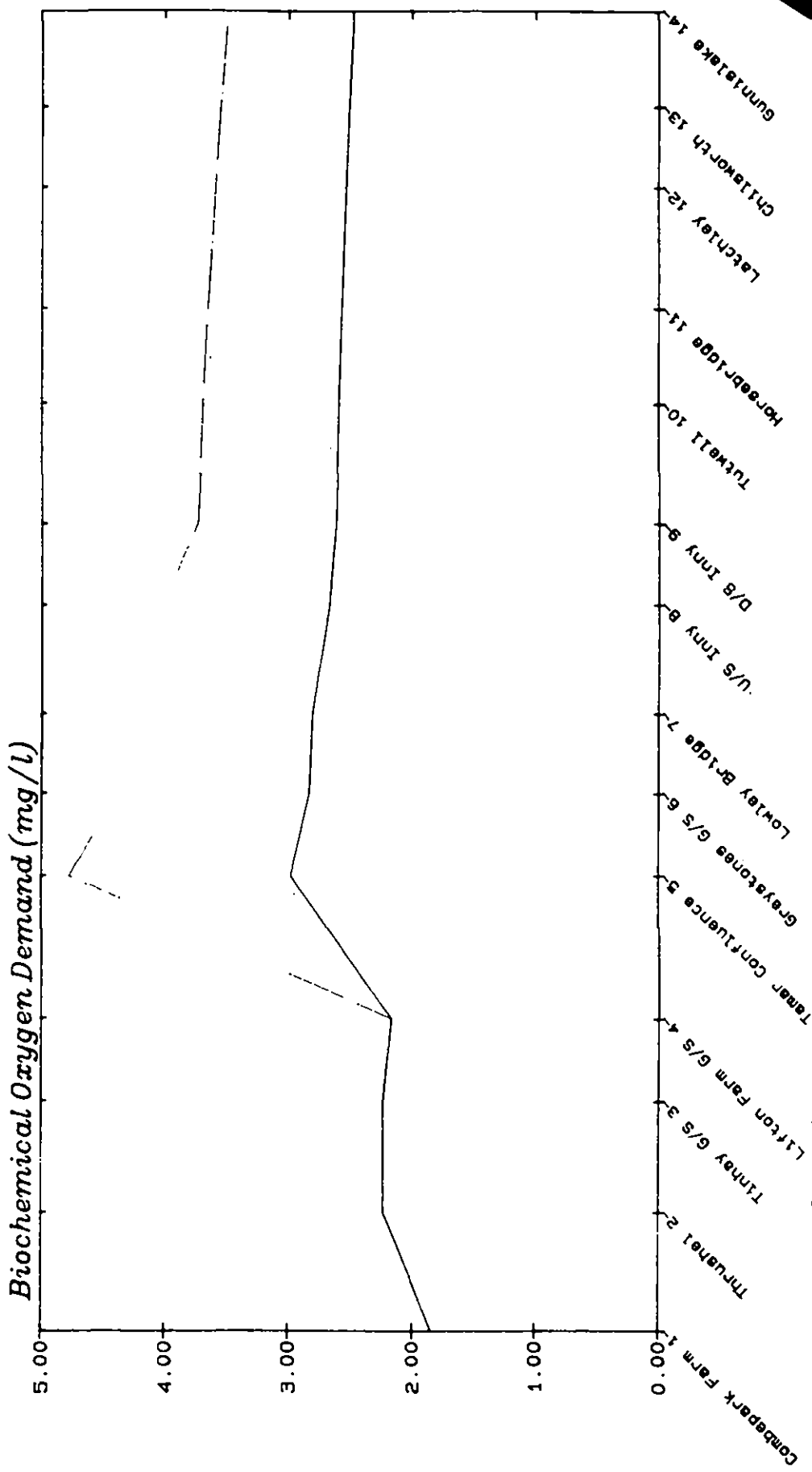
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Figure 5G

River profile for 20--JUN--1986

Biochemical Oxygen Demand (mg/l)



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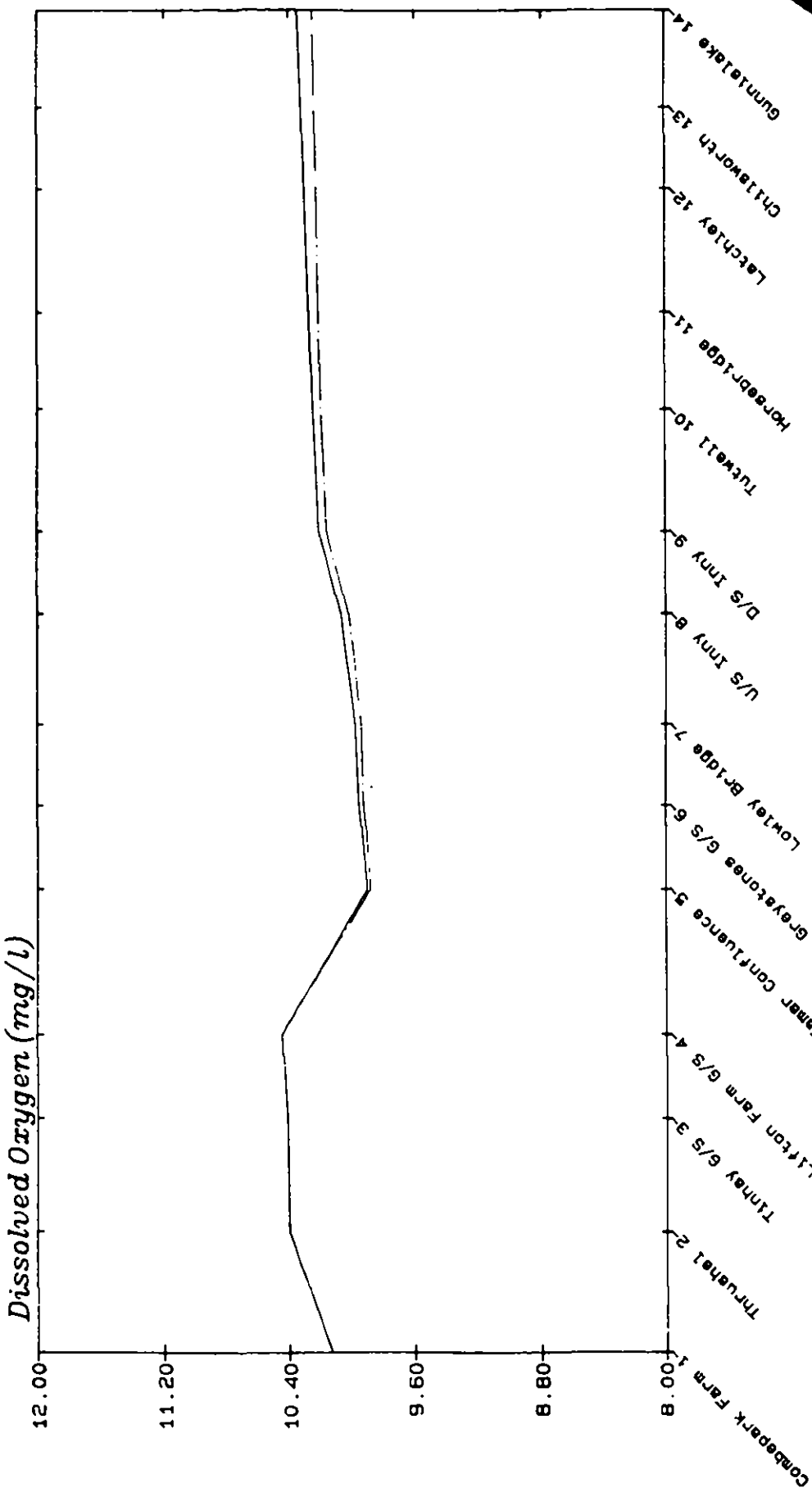


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Figure 6H

River profile for 20-JUN-1986



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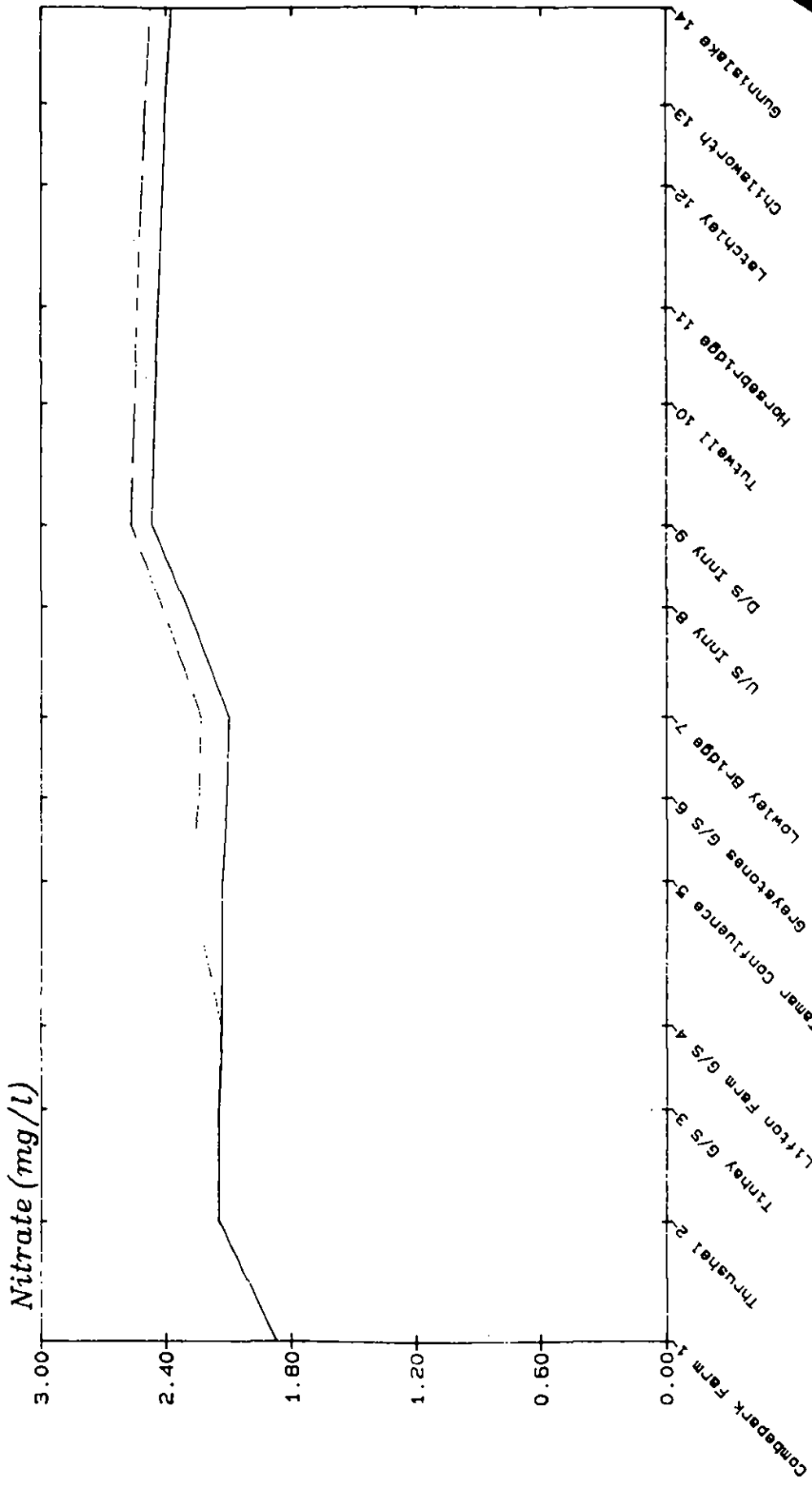


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Figure 61

River profile for 20-JUN-1986



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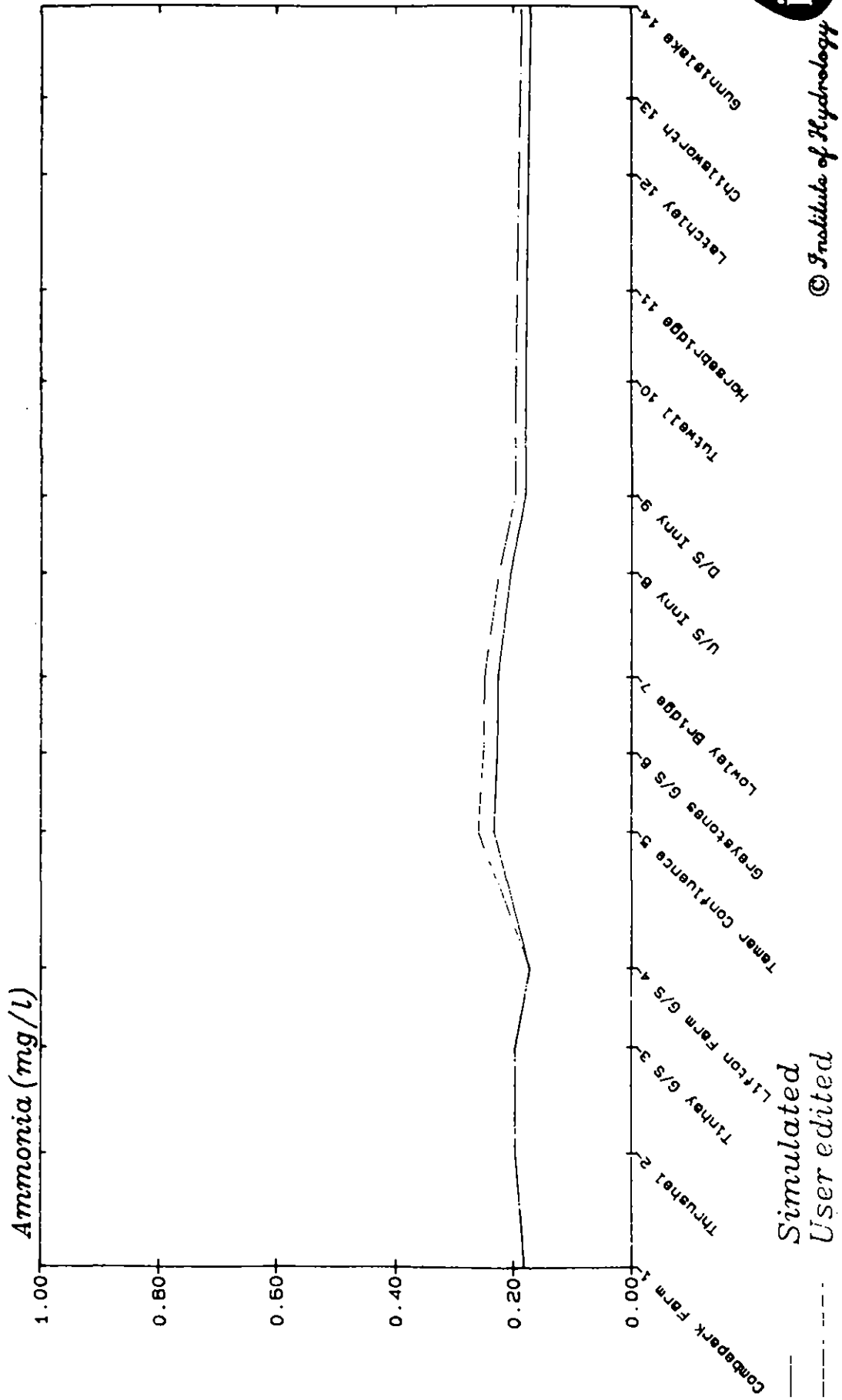


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Figure 5J

River profile for 20-JUN-1986



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Figure 6K

Appendix 1

The Institute of Hydrology River Quality Model (IHQM) Modelling Flow

In order to model water quality it is necessary to first simulate streamflow in all the reaches of the river. In the Tamar streamflow model each reach is characterised by a number of cells and the model for flow variations in each cell is based on an analogy with the lumped parameter equations for the variations in concentration of a conservative pollutant under the assumption of uniform mixing over the cell (Whitehead et al., 1979). The model may be viewed in hydrological flow routing terms as one in which the relationship between inflow I , outflow, Q , and storage, S , in each cell is represented by the continuity equation:

$$\frac{dS}{dt} = I - Q \quad (1)$$

with

$$S = \tau Q$$

where τ is a travel time parameter. In order to represent the variation in travel time with flow; τ is expressed as

$$\tau(Q) = \frac{L}{UN} \quad (2)$$

where N is the number of compartments in the reach, L is the reach length and U , the mean flow velocity in the reach, is related to discharge through

$$U = a Q_m^b \quad (3)$$

where Q_m is the mean flow in the reach and where a and b are coefficients to be estimated.

The value of N affects the relative importance of floodwave advection and dispersion in a reach; values of N , a and b can be determined by calibration on an observed record of downstream flow or from tracer experiments (see Whitehead et al., 1984).

Given information on upstream and tributary inputs, the flow routing model can be used to derive simulations of downstream flow by solving the differential equation (1). The equation is solved using a numerical integration technique which contains an automatic adjustment to the integration step length. This is particularly useful since under periods of low flow and high residence times, the integration step length can be increased thereby saving computer time. Under high flow conditions, however, residence times are reduced and in order to solve the equation to the same accuracy, it is necessary to reduce the integration step length. Since this is achieved automatically, there are relatively few numerical integration problems.

Modelling Water Quality

The water quality models for the River Tamar are based on a mass balance principle but include factors to allow for the non-conservative nature of water quality variables. For example dissolved oxygen in the river is a balance between the various sources and sinks of oxygen. On the one hand there is oxygen supplied by the reaeration from the atmosphere and photosynthetic oxygen produced by plants and algae and, on the other hand, oxygen is being consumed by respiration processes and the removal of oxygen during nitrification of ammonia or breakdown of organic material and effluents. The basic mass balance equations required to simulate water quality behaviour are as follows:

Chloride or any conservative determinand

$$\frac{dx_1(t)}{dt} = \frac{u_1(t)}{\tau(t)} - \frac{x_1(t)}{\tau(t)} \quad (4)$$

Nitrate

$$\frac{dx_2(t)}{dt} = \frac{u_2(t)}{\tau(t)} - \frac{x_2(t)}{\tau(t)} - k_1 x_2(t) \quad (5)$$

Ammonia

$$\frac{dx_3(t)}{dt} = \frac{u_3(t)}{\tau(t)} - \frac{x_3(t)}{\tau(t)} - k_2 \left(\frac{1}{Q(t)} \right) x_3(t) \quad (6)$$

Dissolved Oxygen (DO)

$$\begin{aligned} \frac{dx_4(t)}{dt} = & \frac{u_4(t)}{\tau(t)} - \frac{x_4(t)}{\tau(t)} - 4.33 k_2 \left(\frac{1}{Q_0(t)} \right) x_3(t) - k_3 x_5(t) \\ & + k_4 (C_s(t) - x_4(t)) + P(t) + R(t) + M(t) \end{aligned} \quad (7)$$

Biochemical Oxygen Demand (BOD)

$$\frac{dx_5(t)}{dt} = \frac{u_5(t)}{\tau(t)} - \frac{x_5(t)}{\tau(t)} - (k_3 + k_5) x_5(t) + A(t) \quad (8)$$

where x refers to the downstream (reach output) concentration mg l^{-1} ;
 u refers to the upstream (reach input) concentration mg l^{-1} ;
 Q is the flow rate (determined from the flow model $\text{m}^3\text{sec}^{-1}$);
 τ is the reach residence time (varying as a function of flow) days;
 P, R, M refer to the additional sources and sinks affecting dissolved oxygen such as photosynthesis, respiration and uptake by mud or the benthos.
 C_s is the saturation concentration of dissolved oxygen, and k_1, k_2, k_3, k_4 and k_5 are the rate coefficients of the various reactions.
 A refers to the addition BOD created by the death of algae in river systems.
 t is time.

The rate coefficients are not constants but generally vary as a function of temperature or other variables such as depth. For example, the denitrification rate k_1 is

$$k_1 = \frac{0.05}{d} 10^{0.0293\theta} \quad (9)$$

where d is river depth, m, and θ is water temperature in $^{\circ}\text{C}$. This nitrate relationship has been shown to provide a good representation of denitrification processes in rivers (see Whitehead and Williams, 1982).

The saturation concentration for DO is determined as,

$$C_s = 14.652 - 0.41022T + 0.0079910 T^2 - 0.000077774 T^3$$

where T is the stream temperature $^{\circ}\text{C}$.

A common problem with water quality models is to determine parameter values such as the BOD decay coefficient and reaeration rate coefficients. The standard approach is to select parameter values from the literature or from experimental measurements. Knowles and Wakeford (1978) describe a number of relationships and parameter values which can be used in situations where little information is available and this approach has been applied by Casapieri et al (1978) in a study of the Blackwater Catchment of the Thames.

A more sophisticated approach was developed by Beck and Young (1976) in which the parameters of a dynamic water quality model were estimated directly from field data using the extended Kalman filter (EKF). The EKF is essentially a statistical technique which accounts for measurement errors and system noise, both of which are highly significant in water quality studies. Whitehead (1978, 80, 81) applied the EKF technique and the instrumental variable (IV) technique to estimate water quality parameters in the dynamic models developed for the Bedford Ouse. However, a requirement of these techniques is that an extensive record of daily or continuous data is available. Since such a data set does not exist for the Tamar a set of standard relationships have been used to provide estimates of the various processes in the model. For example in the case of photosynthetic oxygen production in river systems, Owens et al (1969) developed a simplified model in which oxygen production is related to light intensity and plant biomass or algal levels. Whitehead et al (1981) used a modified version of the Owens model and estimated the relevant parameters for the Bedford Ouse. A similar approach was adopted for the Thames and the following relationship developed.

$$P = \frac{8.6}{10^5} Cl_a I^{0.79} 1.08^{(T-20)}$$

Here Cl_a is the chlorophyll-A concentration mg m^{-3}
 I is the solar radiation level watt hours m^{-2} per day. The coefficient 8.6 was determined from a linear regression analysis using as variates the observed oxygen production, obtained from continuous data recorded by TWA. This relationship has been employed for the Tamar although continuous DO data and Cl_a information in summer 1987 will be used to update this relationship.

R in the DO equation refers to the loss of oxygen via algal respiration.

Kowalczewski and Lack (1971) developed a relationship between algal concentration measured as chlorophyll A and respiration rate for the River Thames, where

$$R = (0.14 + 0.013 Cl_a) 1.08^{(T-20)}$$

and this relationship has been incorporated into the model. Again this can be updated for the Tamar given suitable records.

M in the DO equation refers to the respiration of the river bed or mud. There has been considerable research into this process (Edwards and Rolley, 1965) and the following equation has been used,

$$M = \frac{k_6}{d} x_4^{0.45} 1.08^{(T-20)}$$

where x_4 is the DO concentration $mg\ l^{-1}$, d is depth, m , and k_6 is a parameter to be determined. The original work of Edward and Rolley was conducted on the highly polluted muds of the River Ivel and later studies by Rolley and Edwards (1967) showed that the parameter k_6 varied considerably from river to river. In the Tamar study a value for k_6 of 0.15 days was found to provide the best fit to the observed DO data.

Finally A in the BOD equation refers to the conversion of algae to decaying organic matter. In previous algal modelling studies on the Thames (see Whitehead 1984) the concentration of dead algae is assumed proportional to the concentration of live algae. Thus A can be expressed

$$A = k_7 Cl_a .1.047^{(T-20)}$$

Where Cl_a is the chlorophyll a concentration $mg\ m^{-3}$ and k_7 is a parameter. From simulation studies on the Thames k_7 was found to be 0.01. This parameter has been included in the Tamar model.

The remaining rate coefficients in the model refer to the ammonia decay, k_2 , which is flow dependent (see Whitehead 1984), k_3 is the BOD decay coefficient, k_4 is the reaeration coefficient and k_5 is a BOD sedimentation coefficient. All the rate coefficients can be altered using an interactive feature in the model program.

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