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A Review of the Flood Studies Rainfall/Runoff Model Prediction Equations for North-West England

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for the North West Water Weather Radar Project -
Applications and Research Working Party.

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CONTENTS

1. INTRODUCTION

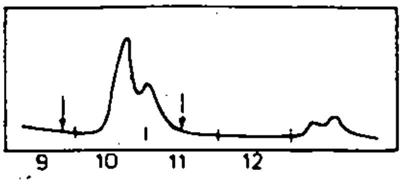
The Flood Studies Report (FSR) suggests two methods of estimating the magnitude of a flood at an ungauged site, one based on flow statistics and the other on rainfall statistics and a rainfall-runoff model. The advantages of the rainfall-runoff model approach are that it produces not just an estimate of the peak flow but of the whole flood hydrograph and that the model, once calibrated, can be used with any rainfall input to estimate the response runoff, for example as an estimate of an ungauged inflow in a flood forecasting scheme.

The purpose of this study was to calibrate the FSR rainfall-runoff model on catchments in north west England. The work was carried out as part of a Ministry of Agriculture, Fisheries and Food (MAFF) commissioned project to investigate catchment response to heavy rainfall.

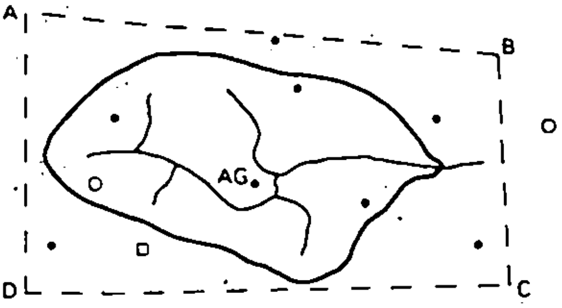
1.2 The FSR rainfall-runoff model

The rainfall-runoff model used in the FSR is the unit hydrograph/losses model. This is detailed in FSR I.6 but a brief description is included here. A graphical summary of the analysis procedure is shown in Fig. 1.1.

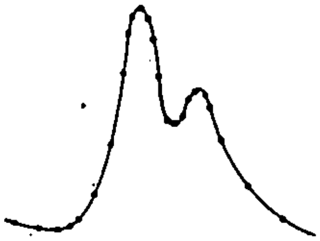
The model requires three types of data: flow data (usually obtained from stage charts and a rating equation), catchment average rainfall (at hourly intervals for the period of the storm and daily totals for the preceding five days) and soil moisture deficit data (taken from a nearby meteorological station). The flow hydrograph is separated into response runoff and baseflow and then, knowing the volume of response runoff the rainfall hyetograph is separated into excess rainfall and losses. In the analysis procedure (Fig. 1.1) the excess rainfall is determined using a loss rate curve based on a catchment wetness index (CWI). This index combines soil moisture deficit (SMD) information with an antecedent precipitation index (API). The main parameter of the rainfall separation model is the percentage runoff (PR) which is the volume of response runoff divided by that of the total rainfall. For each event a unit hydrograph is derived from the separated rainfall and flow data using a least squares analysis technique (FSR I. 6.4.6) and then approximated by three parameters Tp, Qp and W as shown in Fig. 1.2. Together with PR these form the four main parameters of the unit hydrograph/losses model.



Event specified (arrows). Preliminary examination of rain records shows that some rain occurred end of 9th.
Hydrograph digitised at as many points as necessary for adequate definition.



Rain recorder (○) charts collected for 9th, 10th, 11th. Hourly rainfalls extracted. Input together with definition of enclosing quadrilateral ABCD to program which obtains (from magnetic tape) daily falls for all gauges (●) inside ABCD, weights hourly patterns according to distance from centre of catchment, calculates average fall on each of the three days and distributes; also extracts 5 days of antecedent rain for gauge nearest centre (AG).



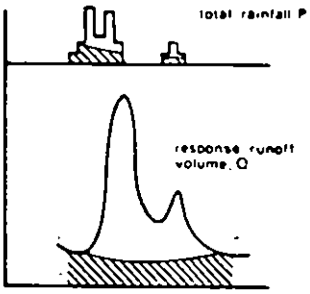
SMD data obtained (□)

Rain program produces catchment average pattern of hourly rainfall totals. Trivial amounts at either end of event are removed.

Paper tape to computer which produces flow hydrograph at specified intervals (usually hourly) and plots it.

Data types:

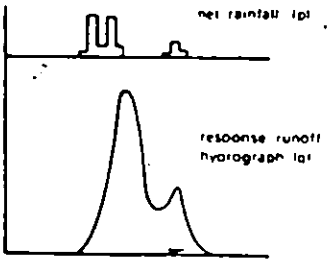
1. Flow hydrograph
2. Hourly rainfall
3. SMD
4. Antecedent rainfall
5. Comment.



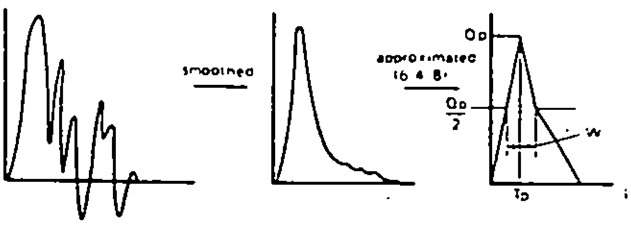
Plot checked for 'sense'. Edits made if necessary.

$\frac{Q}{P} \times 100$ = percentage runoff

Fig 1.1 Summary of hydrograph analysis



unit hydrograph derivation (6.4.6, 6.4.7)



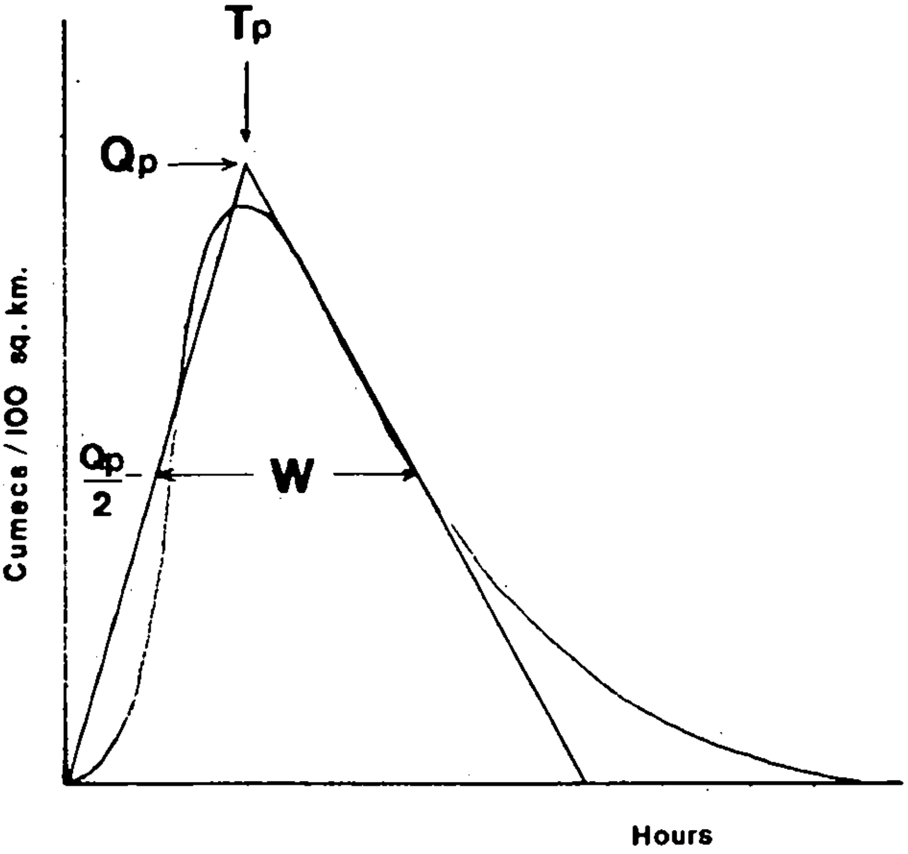


Fig 1.2 Straight line approximation to the unit hydrograph

In the original flood study the model parameters were thus derived from 1447 events on 130 catchments. Regression analyses were then performed to relate the parameters to catchment, climate and event characteristics, enabling a method to be developed for flood synthesis on ungauged catchments.

1.3 Recent enhancements of the FSR data base

One of the main aims of the MAFF catchment Response project being undertaken at the Institute is to enhance the flood event data base both in terms of quality and quantity. The additional period of record has enabled further events to be collated on previously studied catchments and for the inclusion of a number of new catchments where the quality of available data has improved sufficiently. A particular aim of this process is to seek out catchments having characteristics that were not well represented in the original database. At the same time the quality of data is being monitored to ensure that events accepted for analysis conform to certain minimum standards of hydrological acceptability. This data review has already led to the total rejection of some catchments while others have been assessed suitable for loss estimation purposes only. Generally, however, the assessment is made event by event. Where raingauge totals indicate an uncharacteristic distribution of rainfall across the catchment then the event is rejected from the unit hydrograph analysis but retained for loss estimation provided that the distribution is adequately defined. Apart from imposing these higher standards of quality control, the method of analysis being used is unchanged from that described in FSR 1.6.4.

1.4 Objectives of this study

During the course of the data extension and review the request was received to look specifically at catchments in north west England. The North West Water Authority (NWWA) is considering use of the unit hydrograph/losses model for flood forecasting on ungauged catchments as part of the weather radar based flood warning scheme being set up in the region.

The project therefore had two main objectives:

(1) To provide a procedure for estimating the unit hydrograph and loss parameters on ungauged catchments within the area of operation of

the weather radar being installed at Hameldon Hill near Blackburn.

(2) To gauge the likely effect of stricter quality control on the size of the new data bank and to examine whether this is likely to produce more significant regressions.

2. DATA

2.1 Selection of Catchments

Catchments were selected from the extended data base for inclusion in this study on the basis that they should be within 75 km of the weather radar site. Thirty two catchments fell into this category and are listed in Table 2.1 and shown mapped in Fig. 2.1. As can be seen from this map not all the catchments are from the NWWA area; some from the east of the Pennines are also included. The distribution of these catchments is fairly good although coverage is better in the area to the north of the radar site than in the southern part. A further six catchments were initially selected but were later rejected because of inadequate data. On the majority of catchments between ten and fifteen events were selected so it was initially hoped that about 400 events would be analysed. However only 257 events were found to be acceptable and of these only 162 were suitable for unit hydrograph analysis. Although well below expectations this still represents a considerable increase from the number of events used in the original flood study (40% of the total were from the original study).

The number of events from each catchment varies considerably as shown in Table 2.1. Where, for analyses in later sections, catchment average values were required, average parameter values were only considered adequate when they came from at least five events. Less than half of the catchments had at least five events usable for unit hydrograph analysis.

2.2 Catchment Characteristics

Table 2.2 gives various catchment characteristics for the selected catchments. A brief description of each characteristic together with the FSR reference for a fuller account are given below:

(a)	AREA catchment area in sq. km	
(b)	S1085 a measure of catchment slope in m/km	FSR 1.4.2, Fig. 1.4.3
(c)	SOIL a measure of winter rain acceptable potential obtained from map FSR V.4.18 (revised Feb 1978)	FSR 1.4.2.3.
(d)	STMFRQ a measure of drainage density, the number of stream junctions per sq. km	FSR 1.4.2.2

Catchment Number	Catchment Name	No. of Events	
		Total	Suitable for u.h.
27027	Wharfe at Ilkley	23	11
20731	Colne at Colne Bridge	2	2
27032	Hebden Beck at Hebden	6	
27034	Ure at Kilgram Bridge	9	
27035	Aire at Kildwick Bridge	11	9
27301	Crimple Beck at Burn Bridge	8	5
28023	Wye at Ashford	8	8
68006	Dane at Hulme Walfield	6	4
68014	Sanderson at Sanderson	6	6
69008	Dean at Stanneylands	2	2
69011	Mickle Brook at Cheadle	2	2
69012	Bollin at Wilmslow	3	3
69018	Newton Brook at Newton-le-Willows	5	2
69020	Medlock at Ardwick	7	
69027	Tame at Portwood	7	5
69031	Netherly Brook at Greens Bridge	9	7
69034	Musbury Brook at Helmshore Intake	6	5
69802	Etherow at Woodhead	2	2
70006	Tawd at Newburgh	8	8
71003	Croasdale Beck at Croasdale	24	17
71004	Calder at Whalley	6	
71008	Hodder at Hodder Place	10	4
71011	Ribble at Halton West	6	4
71804	Dunsop at Footholme	6	6
72002	Wyre at St. Michael's	14	14
72006	Lune at Kirkby Lonsdale	8	
72818	New Mill Brook at Carvers Bridge	9	9
73005	Kent at Sedgewick	12	10
73007	Troutbeck at Troutbeck Bridge	6	2
73008	Bela at Beetham	7	
73803	Winster at Lobby Bridge	4	1
76014	Eden at Kirkby Stephen	15	14

Table 2.1 Catchments used in this study and the number of events from each catchment

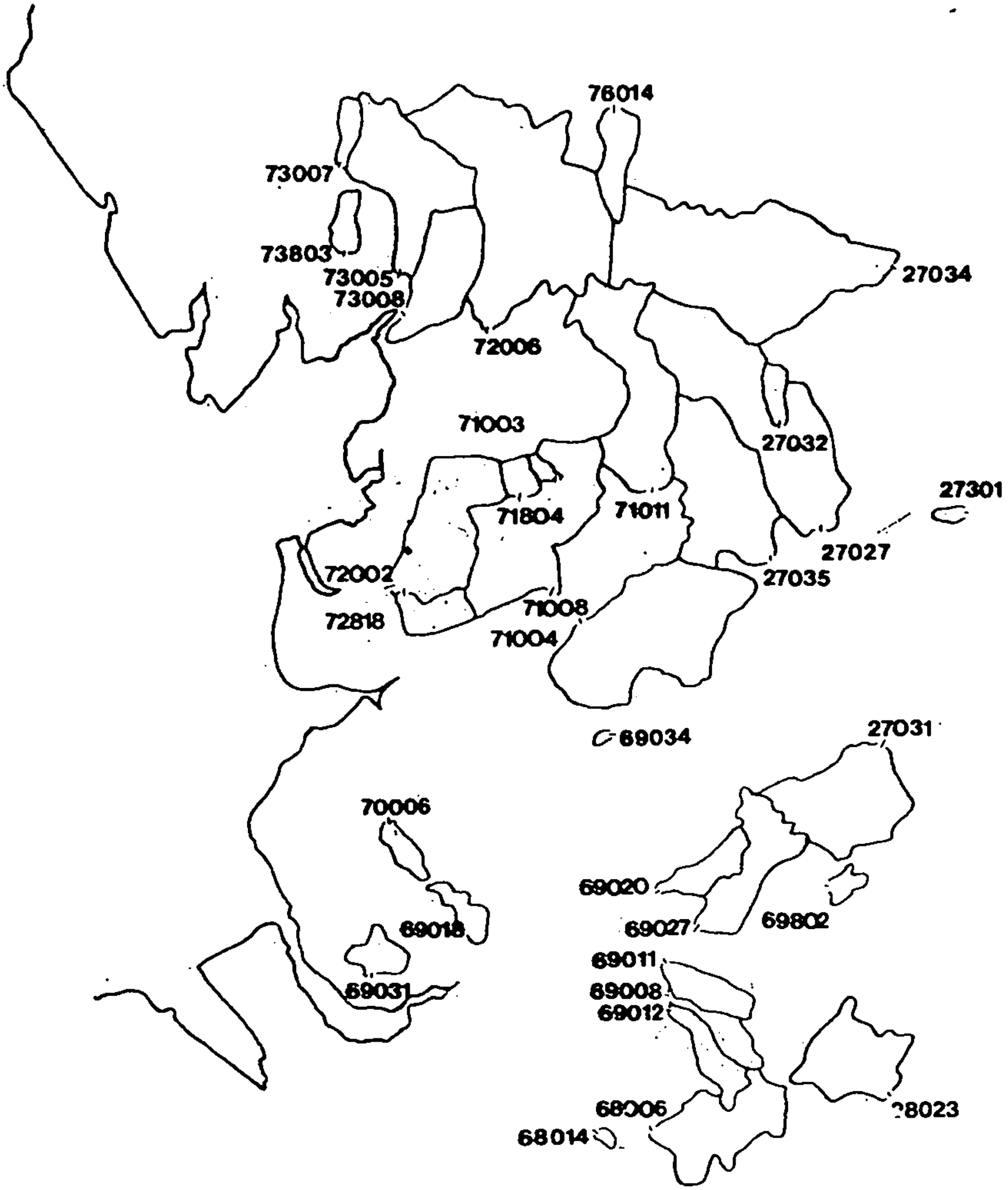


Fig 2.1 Locations of catchments used in this study

	CATCHMENT	SAAR	RSMD	AREA	SL1085	SOIL	STMFRQ	MSL	URBAN	OVF
1	27027	1382.000	51.400	443.000	4.460	.418	1.670	55.050	.000	.011
2	27031	1107.000	38.147	245.000	9.870	.393	1.410	23.680	.113	.005
3	27032	1429.000	53.340	6.800	25.310	.500	.520	11.330	.000	.050
4	27034	1429.000	53.850	510.200	4.100	.472	2.400	50.250	.000	.013
5	27035	1108.000	42.670	282.000	4.470	.460	1.800	31.660	.008	.082
6	27301	831.000	34.460	8.500	26.480	.450	1.530	3.590	.000	.276
7	28023	1145.000	45.240	152.800	10.040	.208	.260	26.410	.020	.009
8	68006	1034.000	36.013	151.500	9.410	.430	1.420	31.480	.021	.023
9	68802	749.000	28.570	5.400	3.430	.390	.190	4.610	.000	.245
10	69008	931.000	38.630	51.800	9.950	.430	.660	22.750	.012	.075
11	69011	873.000	36.890	67.300	9.070	.436	.460	17.540	.300	.045
12	69012	939.000	36.680	72.500	6.800	.290	.660	19.740	.103	.564
13	69018	825.000	30.870	32.800	5.740	.450	.490	13.550	.202	.129
14	69020	1064.000	39.550	57.300	8.730	.450	.840	19.730	.389	.030
15	69027	1179.000	41.250	150.000	6.900	.492	.720	38.200	.186	.007
16	69031	840.000	34.620	47.900	5.360	.444	.750	10.000	.306	.045
17	69034	1486.000	51.850	3.100	89.570	.500	2.580	2.630	.000	.206
18	69802	1530.000	60.625	13.000	35.850	.480	7.540	5.860	.000	.085
19	70006	965.000	33.840	28.900	5.850	.400	.710	9.060	.160	.127
20	71003	1839.000	64.724	10.360	35.600	.500	3.360	5.150	.000	.212
21	71004	1227.000	43.674	316.000	4.220	.464	.890	26.570	.110	.031
22	71008	1480.000	55.090	264.200	6.850	.477	3.350	28.880	.000	.065
23	71011	1529.000	52.680	206.700	6.770	.480	1.210	35.290	.006	.020
24	71804	1809.000	64.206	24.900	28.520	.500	2.810	7.520	.000	.228
25	72002	1257.000	44.529	274.500	7.380	.458	1.000	33.960	.000	.019
26	72006	1437.000	61.880	507.200	5.150	.462	2.750	47.540	.000	.026
27	72818	1024.000	36.630	64.500	6.940	.440	1.740	16.390	.000	.045
28	73005	1925.000	71.750	216.000	9.560	.420	1.750	28.180	.020	.220
29	73007	2199.000	73.700	23.600	19.040	.500	3.900	11.080	.000	.042
30	73008	1318.000	49.320	131.000	4.760	.292	1.690	53.790	.000	.005
31	73803	1509.000	50.780	20.700	8.590	.300	1.010	7.210	.000	.022
32	76014	1439.000	52.160	69.400	19.650	.460	4.030	20.130	.000	.029

Table 2.2 Catchment Characteristics

(e) URBAN the fraction of urban development FSR 1.4.2.3

(f) DVF dry valley factor - the distance from the end of the stream to the watershed divided by the main stream length FSR 1.4.2.2

(g) SAAR the standard annual average rainfall in mm

(h) RSMD a measure of flood producing rainfall FSR 1.4.2.4

The selected catchments show a wide range of values for most of these characteristics. For nearly all catchments, however, the SOIL index is in the range 0.4 to 0.5 (SOIL must be between 0.15 and 0.5) and there is no type 3 soil in any of the catchments. From looking at the geology of the region alone a greater spread of values could have been hoped for as there are sandstone aquifers underlying a large fraction of the area. However, the locations of the catchments selected and the drift cover have combined to remove much of the expected variation.

3. UNIT HYDROGRAPH PARAMETERS

3.1 FSR unit hydrograph internal relationships

As described in Section 1.2 three unit hydrograph parameters were extracted from each unit hydrograph derived: Qp, the peak flow in cumecs per 100 sq. km; Tp, the time to peak and W, the width at half the peak, both in hours. These three parameters were expected to be interdependent and so the unit hydrograph was transformed by multiplying the ordinate by Tp and dividing the abscissae by Tp to produce a semi-dimensionless unit hydrograph. Choosing Tp as the key parameter was partly due to the long tradition of using Tp in this type of investigation and partly because Tp was thought to be less influenced by the smoothing of the unit hydrograph and the method of separation.

Although regressions were made of QpTp and W/Tp on catchment and storm characteristics the equations used to estimate these parameters were based solely on Tp. The QpTp relationship, presented in the FSR, is

$$QpTp = 2.6Tp + 162 \qquad 3.1 \qquad \text{(FSR I.6.10)}$$

for which the multiple determination coefficient (R^2) was 0.241 and the standard error of estimate (s.e.e.) was 33.4. For W/Tp the relationship is

$$W/Tp = 1.40 - 0.008Tp \qquad 3.2 \qquad \text{(FSR I, Table 6.2)}$$

for which R^2 was 0.049 and the s.e.e. 0.267.

In fact both these relationships were adjusted to allow for the effects of smoothing of the uh and for the use, in application, of a different separation method (see 3.4 below).

3.2 Qp - Tp relationship for the north west

A QpTp against Tp regression was carried out on the north west data and yielded the following equation

$$QpTp = 7.00\,Tp + 137 \qquad 3.3$$

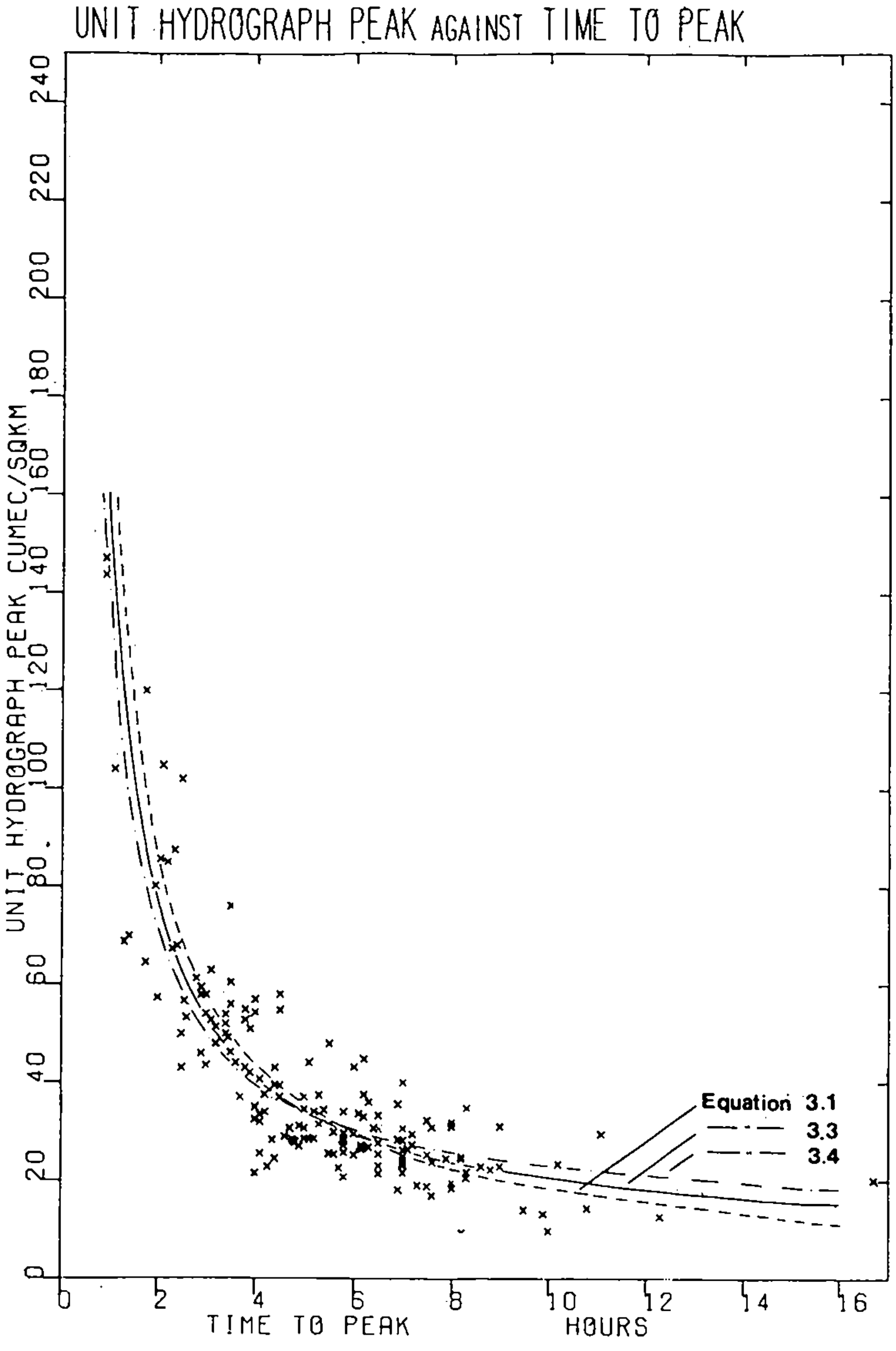
The associated R^2 value was 0.13 and s.e.e. 44.5. Both the constant and the Tp term were significant at the 1% level but the resulting regression is poor.

Since large Qp values are associated with small Tp values and vice versa it seems surprising that such a poor regression should result. For the FSR data set FSR 1 Fig. 6.17 shows QpTp versus Tp for catchment average values (both regressions, in the FSR and in this report, were on individual events); the figure shows a clear trend of increasing QpTp with Tp but the scatter around the line is great.

For all events used in this study Qp is plotted against Tp in Fig. 3.1. This shows the expected large Qp - small Tp, small Qp - large Tp relationship and suggests that this relationship would be well represented by an equation relating Qp to 1/Tp. Such a regression gives the equation

$$Qp = 10.78 + 120.2\,(1/Tp) \qquad 3.4$$

Again both terms are significant at the 1% level but $R^2 = 0.75$ and s.e.e is 11.3. Having found that the dimensionless product QpTp was significantly related only to Tp, a regression equation of the form of equation 3.4 might have been preferable to equation 3.1. In fact equations 3.3 and 3.4 differ very little over the range of Tp's used, and indeed equation 3.1 only diverges from the north west equations for larger Tp's. As very few Tp's greater than 10 hours were in the north west data set, whereas many were used in deriving equation 3.1, it would seem reasonable to retain equation 3.1 for the north west as it should be more reliable when large Tp's are encountered.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8
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14
16
TIME TO PEAK
HOURS
Equation 3.1
3.3
3.4

Fig 3.1 Qp v Tp

3.3 W-Tp relationship for the north west

The regression that produced equation 3.2 was repeated on the north west data and gave

$$W/Tp = 1.54 - 0.03Tp \qquad 3.5$$

where $R^2 = 0.035$ and s.e.e. = 0.36. The constant is significant at the 1% level but the Tp term only at the 5% level. Again having chosen a relationship relating W to Tp and not catchment or storm characteristics it would seem better to regress W directly on Tp; for the north west this yielded:

$$W = 0.83 + 1.20\ Tp \qquad 3.6$$

for which $R^2 = 0.63$ and s.e.e. = 2.2. However the constant in this relationship is barely significant (only at 10% level). Forcing the equation to pass through the origin gives

$$W = 1.33\ Tp \qquad 3.7$$

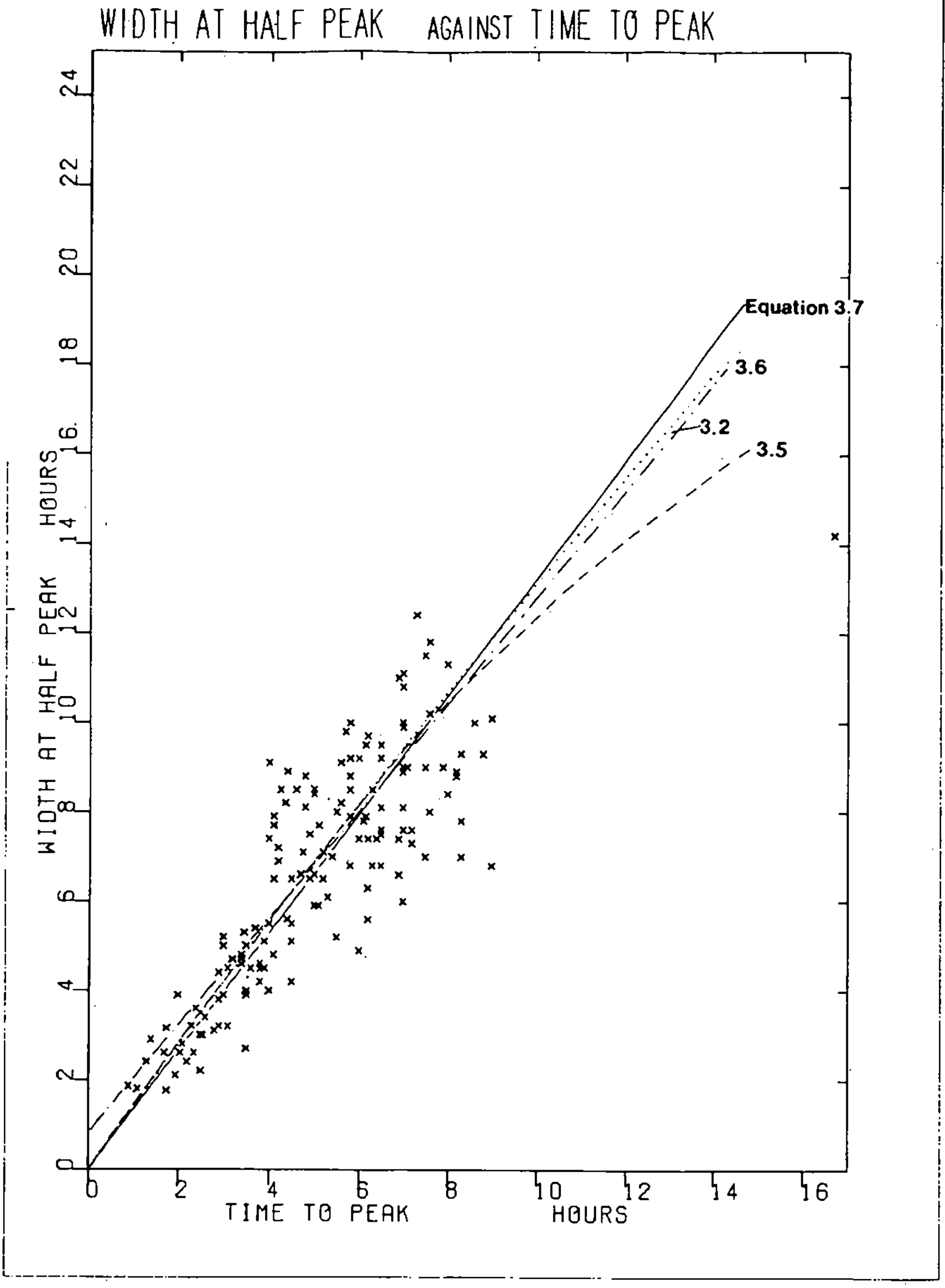
with s.e.e. only 1% greater than for equation 3.6.

W is plotted against Tp for each event in Fig.3.2 which also shows equations 3.2, 3.5, 3.6 and 3.7. As can be seen there is little difference between these equations over most of the range of observed values and so the FSR relation (equation 3.2) is preferred, again because it was calibrated on a more extensive data set.

3.4 Adjustment of Qp-Tp and W-Tp relationships

The conclusion arising from sections 3.2 and 3.3 is that the national FSR equations relating Qp-Tp and W-Tp are suitable for use in the north west. However it is not equations 3.1 and 3.2 that defined the unit hydrograph recommended for use in the FSR.

As all unit hydrographs evaluated in this study and the FSR were smoothed during derivation it was felt desirable to adjust the peak of the design unit hydrograph to compensate. In addition to this factor, while the rainfall data used here were separated using a loss rate method a further


WIDTH AT HALF PEAK AGAINST TIME TO PEAK
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24
Equation 3.7
3.6
3.2
3.5

Fig 3.2 W v Tp

correction is necessary because an increasing percentage runoff method is recommended for design use. This second correction works in the same direction as the first - unit hydrographs coming from data separated by the increasing percentage method tend to be peakier.

Thus equations 3.1 and 3.2 were replaced by

$$QpTp = 220 \qquad 3.8 \qquad \text{(FSR I.6.11)}$$

and

$$W = 1.2\,Tp \qquad 3.9 \qquad \text{(FSR I.6.12)}$$

The unit hydrograph resulting from these equations is so close to triangular that it was decided to make the recommended unit hydrograph triangular, replacing the two part recession by a single line. The simple triangle is achieved by adjusting equation 3.9 to

$$W = 1.26\,Tp \qquad 3.10 \qquad \text{(FSR I.6.13)}$$

As the original equations 3.1 and 3.2 were judged suitable, the transformed equations 3.8 and 3.10 must also be recommended for continued use in the north west.

3.5 Estimation of Tp

To estimate Tp on an ungauged catchment, the FSR recommends the use of the equation

$$Tp = 46.6\,S^{-0.33}\,(1+URBAN)^{-1.99}\,RSMD^{-0.4}\,L^{0.14} \qquad 3.11 \quad \text{(FSR I.6.18)}$$

where S is the slope of channel in m/km
URBAN is the urban fraction of the catchment
RSMD is a measure of flood producing rainfall in mm
and L is the length of the main channel in km.
(Full definitions can be found in FSR I.4.2).

This equation resulted from regressing mean observed Tp's for each catchment against climate and catchment characteristics. The reasons for using catchment average values are fully described in FSR.1.6.5.3. Briefly, it seemed impossible to relate variations in response times to the size of events; whilst some catchments showed Tp decreasing with storm size, others showed the opposite effect. It was considered better to use a good estimate of the mean Tp. In this study the estimate is accepted when obtained from at least five events.

As only fifteen catchments have sufficient suitable events it would be wrong to produce a new four variable equation for the north west, but it is possible to assess the performance of the FSR equation on these catchments.

Table 3.1 gives mean observed Tp and the predicted Tp using equation 3.11. Alongside the mean observed Tp is the standard deviation indicating the spread of observed Tp values. Fig. 3.3 shows mean observed Tp plotted against the estimated value. The error bars on this graph indicate one standard deviation either side of the observed mean. It should be noted that there is considerable error associated with the Tp estimate by the FSR equation. The standard factorial error for the equation was 1.4 (ie. 70% to 140% of the estimated value).

These data suggest a slight overestimation of Tp when using equation 3.11 but that the basic trend of the observed values is echoed by the predicted ones. Whereas a new equation based on so few catchments would not be justified it would be reasonable to adjust the multiplier (46.6). However, a new multiplier estimated from the north west data was not significantly different from 46.6 at the 5% level and so again it seems that the national FSR equation should be retained for use in the north west.

Finally, values of the ratio of mean observed Tp to FSR estimated Tp were mapped (Figure 3.4) to see if any local trends of under or over prediction emerged. Although several of the central catchments do show over-predictions by the FSR equation it would be with very low confidence that this effect could be isolated.

It is therefore recommended that the procedure described in the FSR for estimating the parameters of a design unit hydrograph remains unchanged for use in the north west.

Catchment Number	Estimated T_p from eqn 3.11	Observed Mean	T_p s.d.	$T_{p\,obs}/T_{p\,est}$
27027	9.57	7.9	1.05	0.82
27035	9.38	7.01	0.86	0.75
27301	3.89	3.32	0.25	0.85
28023	6.41	9.41	1.47	1.47
68802	9.51	4.85	1.00	0.51
69027	5.99	7.44	1.06	1.24
69031	4.84	5.19	0.61	1.07
69034	1.99	1.18	0.30	0.59
70006	4.06	4.11	0.24	1.01
71003	2.84	2.62	0.54	0.92
71804	3.27	2.07	0.30	0.63
72002	7.82	5.59	0.79	0.72
72818	7.82	6.73	0.60	0.86
73005	5.49	6.19	1.06	1.13
76014	4.70	3.85	0.64	0.82

Table 3.1 FSR estimated T_p and Mean observed T_p

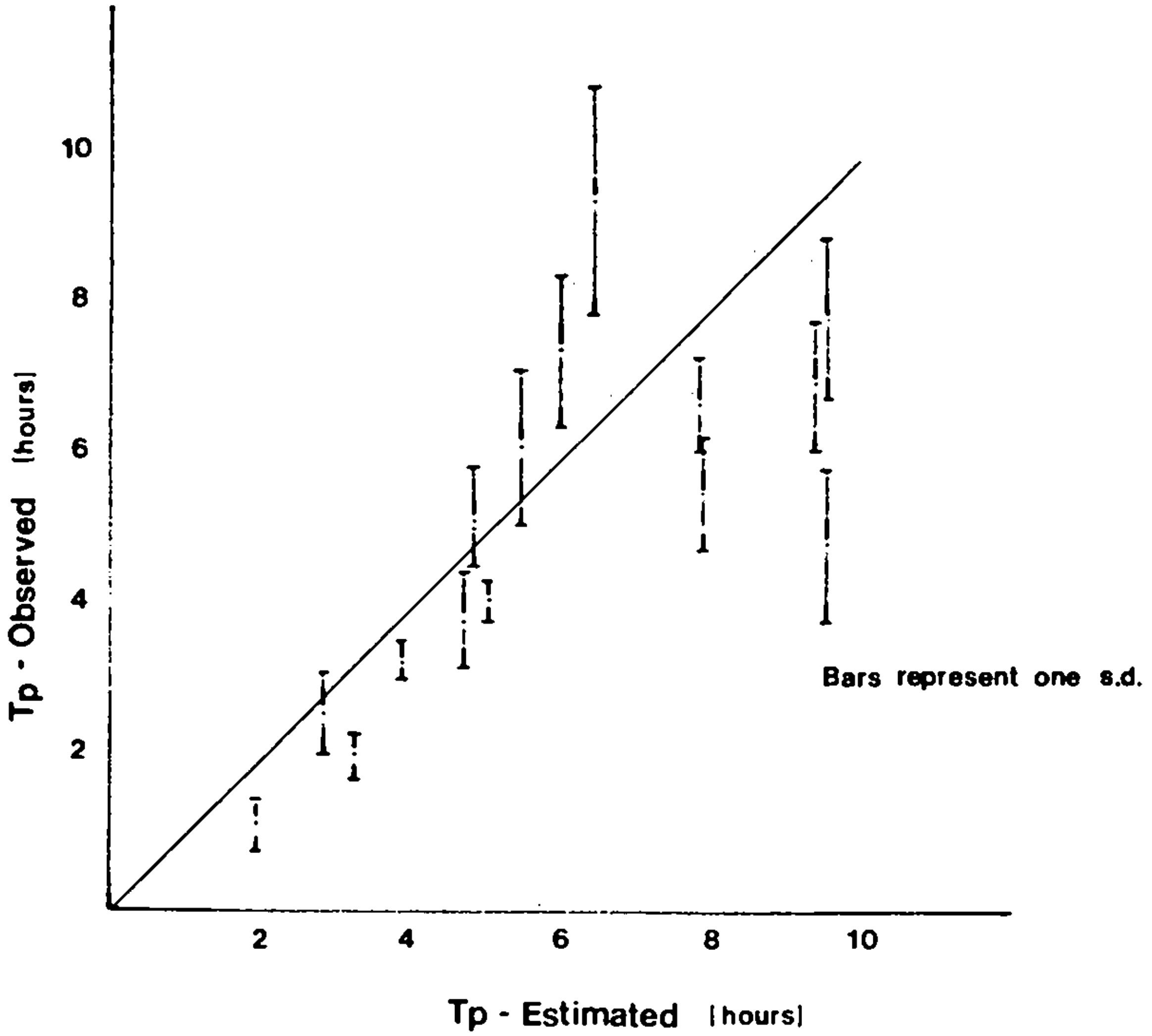

Fig3.3 Observed v FSR Estimated Tp

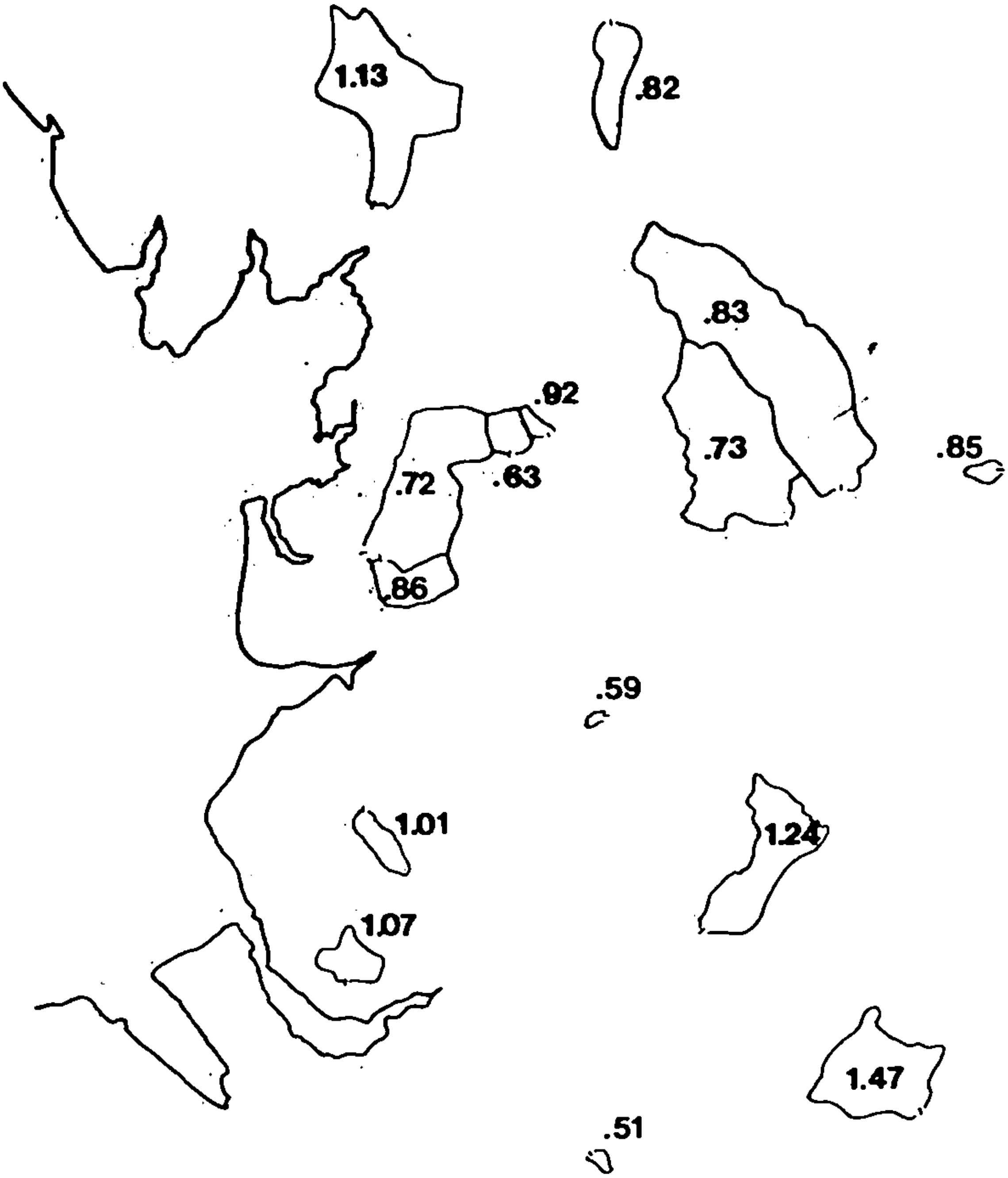

Fig 3.4 $\frac{T_p \text{ Observed}}{T_p \text{ Estimated}}$

4. Estimation of percentage runoff

4.1 The FSR method

For estimation of percentage runoff the FSR recommends the use of the following equation:

$$\frac{Q}{P} \times 100\% = 0.22\,(CWI-125) + 0.10(P-10) + 95.5\,SOIL + 12\,URBAN \qquad 4.1 \quad \text{(FSR I.6.4.0)}$$

where Q is the response runoff (mm)
P is the total rainfall (mm)
CWI is a catchment wetness index (see FSR I.6.4.4)
SOIL is the soil index obtained from the FSR winter rain acceptance potential map; and
URBAN is the urban fraction of the catchment.

The equation can be divided into two parts, the standard percentage runoff (SPR) given by

$$SPR = 95.5\,SOIL + 12\,URBAN \qquad 4.2$$

which is a constant for a catchment, and a dynamic term given by

$$0.22(CWI-125) + 0.10(P-10) \qquad 4.3$$

which represents the increase in percentage runoff that might be expected from a wetter catchment or a bigger storm. The dominant term in the whole equation is the soil term; the index SOIL can be between 0.15 and 0.50 thereby contributing between 14% and 48% to the total estimate of percentage runoff.

A full description of the process leading to the choice of this model is given in FSR I.6.5.5. to I.6.5.8.

4.2 Performance of the FSR equation in the north west

For each of the 257 events used in this study the estimated percentage runoff was calculated using equation 4.1 and plotted against the observed value; the resulting plot is shown as Fig. 4.1. At first sight the scatter of points is disappointing but analysis reveals that the means of the observed and estimated values agree well (45.8% and 43.8% respectively) and that the scatter about the 'observed equals estimated' line is only slightly greater than for the original regression equation (s.e.e. 18.3 for north west data, 15.1 for national data). Despite this reassurance it is easy to appreciate the anxiety of a user when presented with Fig.4.1 as evidence of how well the FSR equation performs. It is therefore worth considering the difficulties of determining percentage runoff and ways in which estimated values could be erroneous before proceeding to develop a new or modified equation.

4.3 Determining Percentage Runoff

Percentage runoff cannot be measured directly for an event; it is obtained by analysing rainfall and runoff data using a particular model. The value obtained will therefore be affected by errors in both types of data and by modelling decisions.

As rainfall is only measured at points within the catchment a method must be defined to extrapolate these measurements to the whole catchment. The FSR technique is based on the assumption that the percentage of the annual average occurring over the whole catchment is the average of percentage falls occurring at the individual gauges (FSR IV. 3.2). Where the percentages recorded at the gauges vary considerably then the method becomes suspect and the event must be rejected. Although the annual average for a gauge is known from past records, that for the catchment must be estimated from a map of annual average rainfall. In upland regions accurate data are more difficult to obtain and the reliability of the map suffers accordingly. This point is worth stressing as any error in the standard annual average rainfall for a catchment is passed directly to the estimate of storm rainfall and hence to that of percentage runoff.

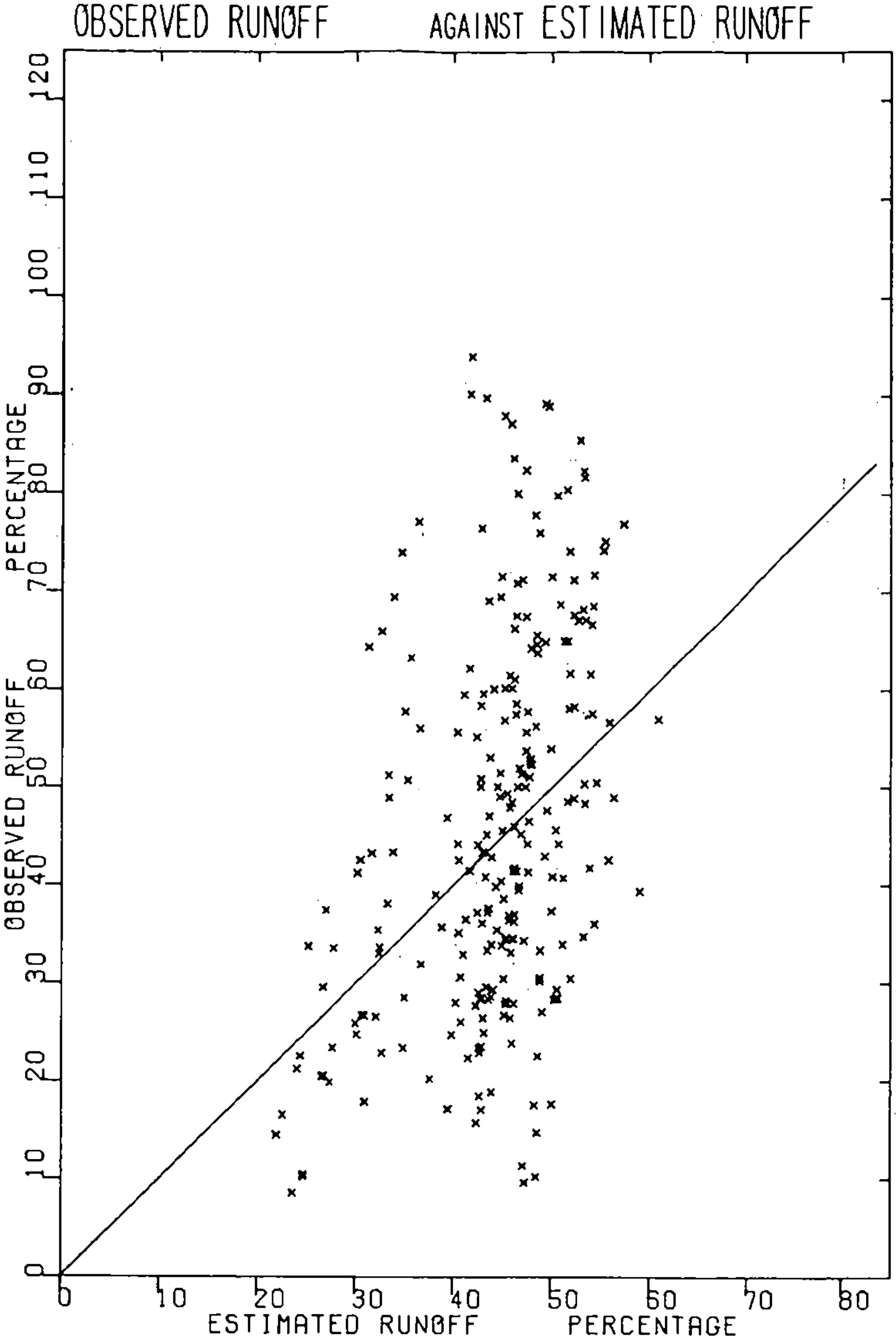

Fig 4.1 Observed v Estimated Percentage Runoff

With flow data the main problem is in defining a rating curve and, especially for natural sections, in monitoring changes in the rating due to shifts in the river bed. The only way of minimizing this source of error in the present study is by careful selection of catchments on the evidence of well defined ratings.

One of the first stages in the analysis of the data is to separate the flow into quick response runoff and baseflow. The FSR method of fixing the end point of quick response runoff at four times the lag after the final rainfall is quite arbitrary but works well; it seems to bias slightly towards longer lasting response runoff which introduces less variability with timing errors than would a trend in the opposite direction. Having decided which is the response runoff it may be clear that not all of the rainfall profile has contributed to the flow peak being analysed and that the profile should be truncated. In some cases this can be a very subjective decision, but one that will change percentage runoff for an event. What can also happen is that there is insufficient rainfall to cause the response runoff; in this case something is clearly wrong. Perhaps the data should be suspected, or there was some snow lying on the catchment at the start of the event or the flow separation is at fault. While is it clear that more than 100% runoff indicates such a fault, lesser values do not necessarily imply a trouble-free event.

4.4 Errors in estimating percentage runoff

As expected from a regression model the range of observed values of percentage runoff is considerably greater than that of the estimated values most of which are between 40% and 55%. This is a direct consequence of the soil index being almost entirely in the range 0.4 to 0.5 and a reminder of the fact that the north west catchments are not a representative subset of FSR catchments.

The use of incorrect values of the independent variables in equation 4.1 will add to the error of estimation; errors in SOIL or URBAN, introduced by poor mapping or abstraction of values, will of course effect all estimates for a catchment. However, for the catchments studied in the north west area the derived soil index was at, or near, its maximum value; errors in abstracting SOIL could, therefore, help to explain only positive errors in percentage runoff (ie. overestimation).

4.5 Investigation of anomalous estimations

Bearing in mind the difficulties associated with percentage runoff determination (4.3 and 4.4 above) it was decided to concentrate on catchments where all percentage runoff values were similarly over or underestimated. To facilitate this, events were grouped together for each catchment and the average observed value plotted against the average estimate. For catchments with at least five events the result is shown in Fig. 4.2 and reveals several outliers.

(i) 27032 Hebden Beck at Hebden

Percentage runoff is grossly overestimated by equation 4.1. However the catchment description held at the Institute indicates uncertainty about the soil type (mainly limestone which can fall into almost any SOIL class depending on its fracturing) and the catchment area (a road embankment has been constructed across the catchment). The description also indicates mining activities which in such an area can lead to considerable water loss from a catchment. As the problems can only be resolved by a detailed site visit it seems reasonable to reject this catchment from further consideration in this study.

(ii) 69020 Medlock at Ardwick

Again equation 4.1 overestimates percentage runoff. It has been suggested that the gauging station, which is in the heart of Manchester could well be by-passed by a large storm sewer. Since this would most influence high flows this catchment has been withheld from further analysis.

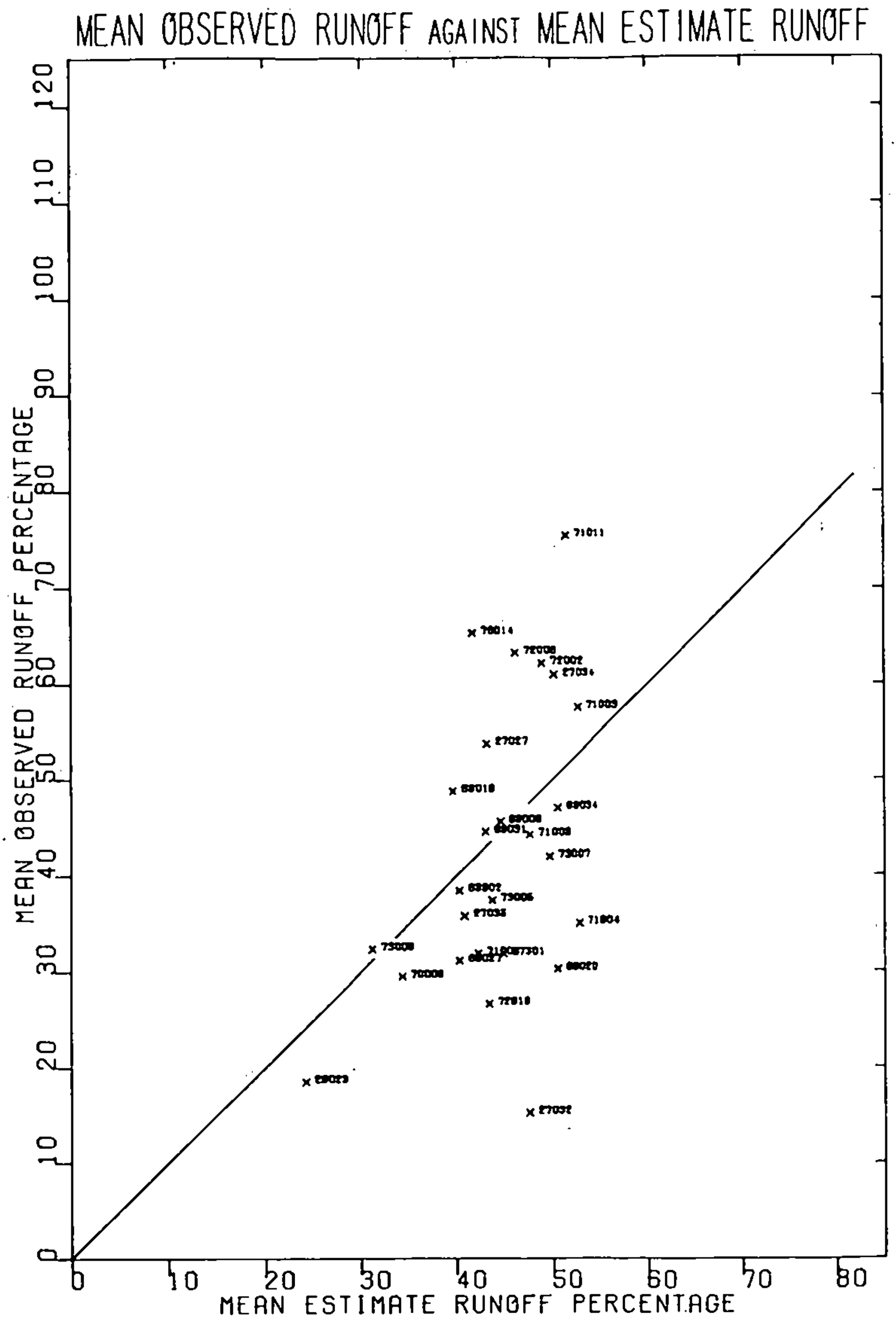

Fig 4.2 Mean Observed v Mean Estimated Percentage Runoff

(iii)	71011	Ribble at Hatton West
	76014	Eden at Kirkby Stephen
	72006	Lune at Kirkby Lonsdale

These three catchments, on which the estimated percentage runoff is markedly lower than observed, form a group at the north of the Pennines. Furthermore, on neighbouring catchments, there is also underestimation although it is not so marked. This suggests that there might be a common factor. The problem could be the one mentioned earlier of measuring rainfall in upland areas and placing too much importance on the standard annual average for a catchment. Another possibility is that the valley side slopes are important runoff generating areas in these steep Pennine catchments and that this characteristic is not represented by equation 4.1. However in the equation as it stands the weakest link is generally regarded as the SOIL index; should this index be revised so that its maximum value is increased? This question is considered in Section 4.8.

4.6 Using the FSR Percentage Runoff equation in the north west

In the complete absence of any local data the best estimate of percentage runoff for an ungauged catchment is the one obtained from equation 4.1. However, the FSR recommends that wherever local data from similar catchments are available (which is frequently the case in this country) they are used in preference to or to improve upon estimates from the regression equation. For percentage runoff estimation, values of SPR can be transferred between catchments in the same locality that have similar characteristics; the SPR is then augmented by the dynamic term to give an estimate of the percentage runoff for a particular storm.

To facilitate this transference of the SPR values a map (Fig. 4.3) has been produced that shows observed SPR for those catchments in the north west from which at least five events were analysed.

Extreme care should be exercised in using this map to ensure that the gauged and ungauged catchments are of a similar nature especially where high SPRs were found or where neighbouring catchments show markedly different values.

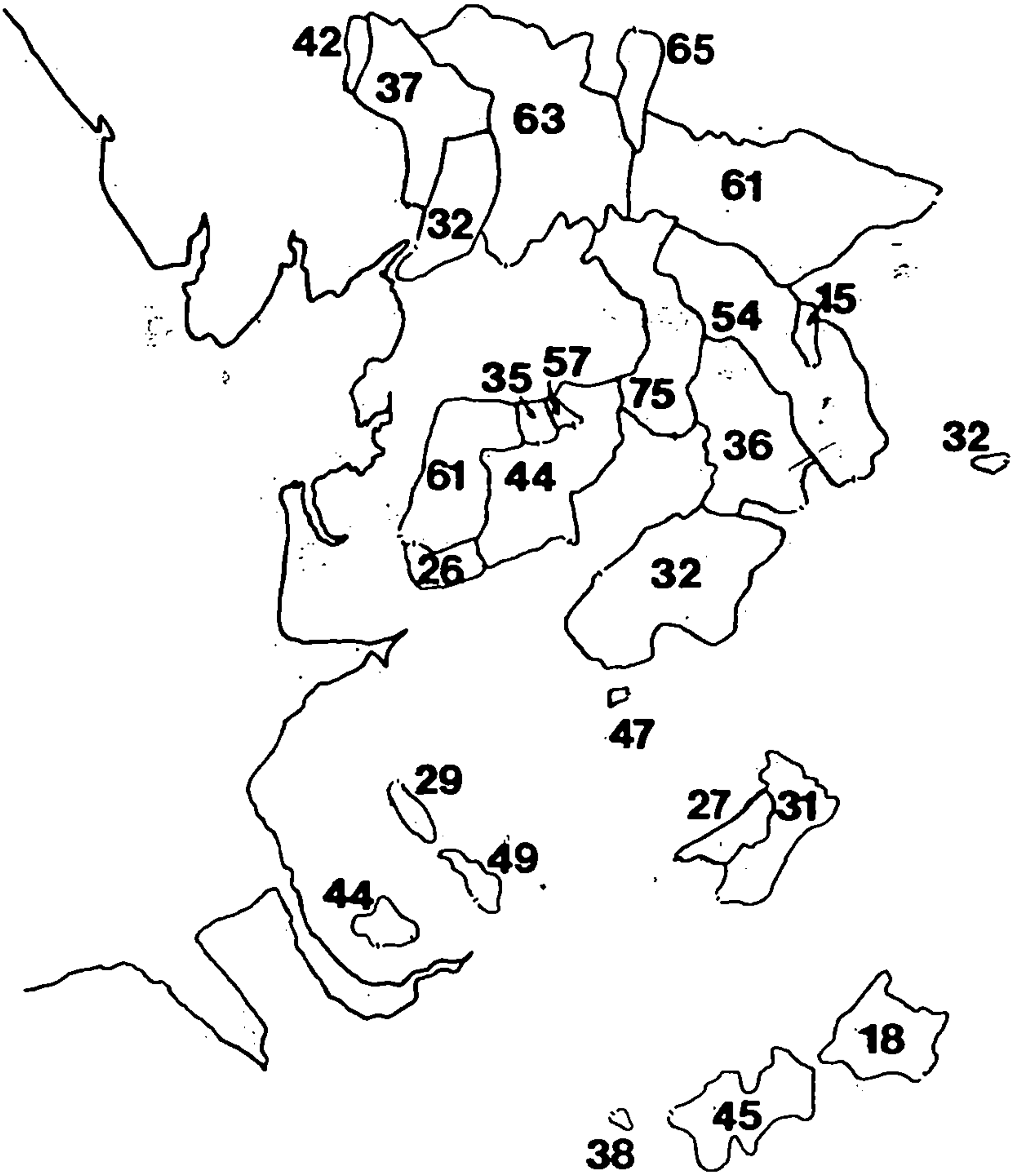

Fig 4.3 Observed Standard Percentage Runoff

4.7 A new percentage runoff equation for the north west?

With the events from the two catchments rejected in section 4.5 removed from the data set, the standard error of estimate in using the FSR equation in the north west drops slightly to 16.9; still just greater than from the FSR data.

A new regression equation, possibly based on other variables, would do better but how useful would the improvement be? Table 4.1 presents the results of four regressions using the most significant variables to estimate percentage runoff. The soil index again emerges as the most significant variable. Interestingly the four variables can be divided into pairs representing standard and dynamic components in the same way as for the FSR equation. The other variable in the fixed term is the dry valley factor, instead of URBAN which was unlikely to be significant on these catchments as there is hardly any urban development. The dynamic component comes from two variables the initial flow per square kilometre and SMD, which is the principal component of CWI. Although the four variable equation is a slight statistical improvement on equation 4.1 it is conceptually unsound as particular combinations of independent variables could yield values of percentage runoff either greater than 100% or less than zero. This type of problem is generally avoided when the independent variables cover as great a range as possible as would seem to be the case for the FSR data but not the north west data.

The FSR percentage runoff equation is therefore recommended for continued use in the region.

4.8 A review of the SOIL index

In Section 4.5 it was noted that if the soil index could have values greater than 0.5 then the estimations on several catchments would be improved. Modifying the index in this way could be achieved in two ways: by adjusting the coefficients used in obtaining the soil index (FSR I.6.5.7) or by introducing an additional soil category with greater runoff potential, an idea first mooted in the FSR (I.4.2.3). The latter approach is favoured as it could leave unaffected the soil index for catchments containing none of the new type 6 soil class. Other evidence from the Soil Survey and the Low Flows Project supports the observation made here that some soils have a much lower rain acceptance potential than 'average' type 5 soils and that these soils are found in the northern Pennines.

Dependent Variable PR

Variable:		Coefficient	Constant	R^2	s.e.e.
Best 1:	SOIL	113	3.7	.13	17.7
Best 2:	SOIL	116	-12.8	.26	16.5
	IFLOW	263			
Best 3:	SOIL	137.	-28.5	.31	15.9
	MSL	0.27			
	IFLOW	243			
Best 4:	SOIL	144.	-17.5	.35	15.4
	DVF	-50.5			
	IFLOW	226			
	SMD	-0.17			

IFLOW is the flow per square kilometre at the start of the hydrograph rise

DVF is the dry valley factor (see FSR I.4.2.2)

Table 4.1 <u>Regressions of PR on catchment and antecedent conditions for all events</u>

DB/JVP

A preliminary trial of a soil classification including a sixth type has been carried out on the north west catchments. Figure 4.4 shows how the type 5 soil has been sub-divided into two classes. This map was used to divide the class 5 soil present on the catchments into the two new classes and a regression carried out to determine the coefficients that would give the best form for the new index. Reassuringly it emerged that the original coefficients could remain unchanged and that the coefficient for the class 6 soil should be 0.6. Assessing the performance of the new index is difficult to isolate but a simple analysis indicates that the improvement it introduces is only to reduce the s.e.e. by a factor of 0.85. For such a small improvement proposing a revision to the recommended procedure would seem unwarranted.

Fig 4.4 **Subdivision of Soil type 5**

All stippled areas are currently classified as type 5.

Potential type 6 soils ▩
stagnohumic gleys
raw peats

Other type 5 soils ░

5. CONCLUSION

The FSR model prediction equations have been given a thorough test on data coming from the north west of England. The equations were found to give regionally unbiased estimates of unit hydrograph and losses parameters although on particular catchments large estimation errors were present. The outcome of the study is to recommend the continued use of the equations presented in the FSR but with the strong proviso that local data be incorporated into such estimates wherever possible. New prediction equations were not presented on the grounds that the characteristics of the catchments were not sufficiently diverse to give a representative sample of catchments and also because regressions indicated any improvement on existing equations would be slight.

The study did indicate that the soil index should be reviewed with the possibility of incorporating a sixth soil type in the winter rain acceptance potential map. Such an investigation will be carried out when the current enhancements of the FSR data base are completed. From the number of new events available in the north west it seems likely that the new data base will contain roughly double the number of events contained in the FSR data set but that the stricter quality control could reduce the number of events suitable for unit hydrograph analysis to roughly 60% of the total.

APPENDIX

A COMPARISON OF CHARACTERISTIC CATCHMENT RESPONSE TIMES

by S B Jones

Introduction

The aim of this study is to compare some of the various methods of estimating rainfall-runoff response times from catchment characteristics, and to assess how closely they agree with the observed times on a number of catchments in north-west England.

There are several hydrograph properties which have been used to characterise catchment response times, eg:

(i) T_p - the time to peak of a unit hydrograph (of specified unit time)

(ii) LAG - the time between (variously) defined features of the causative rainfall and the resulting runoff. One convenient measure, used in the UK Flood Studies, is the time between the centroid of total rainfall and the peak of the hydrograph (or some weighted location in time if there is more than one peak).

(iii) m_1 - the first moment about the origin of the instantaneous unit hydrograph, which can be shown (eg. by Nash 1960) to be equal to the lag between the centroids of effective rainfall and separated response runoff.

Bell and Om Kar (1969) also define 'critical lag' as the value of m_1 for 'extreme' floods, which they found to be 90% of the median value of m_1.

Catchment Characteristics

I Lengths (in km)

the length of the main channel specified as in the Flood Studies Report, i.e. measured in 0.1 km chords from a 1:25000 scale map.

L_b - the 'basin length', which approximates the main channel by two or three straight lines terminated at the point where 5% of the catchment by area lies further upstream. This is rather an unsatisfactory length, as it can be altered appreciably by different choices of the lines.

L_{ca} - the distance along the main channel from the outfall to the point nearest the centroid of the catchment. A weakness of this definition is shown up in one case (Wyre at St Michaels), where the main channel is a long way from the centroid and L_{ca} appears artificially small.

II Slopes (in m per km, or parts per 1000)

simply the difference in elevations of the watershed and outfall, divided by the total stream length L.

S1085 - the slope between points which are 0.1 L and 0.85 L along the main channel (as defined in the Flood Studies Report).

S095 - the slope associated with the basin length, i.e. the difference in elevations of the point on those lines with 5% of the area upstream and the outfall, divided by L_b.

EA - the 'equal area' slope, which is the uniform slope from the outfall to the watershed along the main channel so that the area below it is equal to that under the hypsometric curve.

III Area (in km^2)

AREA - the area of the whole catchment.

IV Urbanisation Factors

URBAN - the urban fraction of the catchment as defined in the Flood Studies Report.

- the population density (per km^2) in the catchment; this is used by Kennedy & Watt (1967), who suggest taking the population density for each town, and estimating the average for the basin by proportion of areas. They used the variable POP = p/1930, but the difficulty in evaluating this suggested that it would be convenient to suppose a density of 1930 per km^2 (or 5000 per sq mile) in towns, which is not unreasonable, and to assume that POP = URBAN.

V Climate

RSMD the 5-year effective rainfall (in mm) as defined in the Flood Studies Report.

VI Storage

LAKE* - the proportion of the upper 2/3 of the catchment (by area) which is lake, pond or marsh. This is converted into the factor $ST = 1 + 20\ LAKE^*$

Estimates of Times

Note that most of the constant multipliers in these estimates have been changed from the original papers in order to standardise the units.

In the following, the parameter $L/\sqrt{S}$ appears frequently. This is because both the Manning and Chézy equations for flow in an open channel give a velocity proportional to $\sqrt{S}$, and hence the time of travel along a channel of length L is proportional to $L/\sqrt{S}$.

Flood Studies Report (1974) (UK)

$$T_p = 46.6\ S1085^{-0.38}\ (1 + URBAN)^{-1.99}\ RSMD^{-0.4}\ L^{0.14}$$

Where T_p is the time to peak of the one-hour unit hydrograph.

Snyder (1938) (USA)

$$LAG = C_t\ (L\ L_{ca})^{0.3}$$

C_t is an unknown coefficient which is supposed to represent slope and storage. Its range is roughly $1.35 < C_t\ 1.65$, with a mean value of 1.50. Extreme values of 0.4 and 8.0 have been observed in the USA.

Method attributed to the US Soil Conservation Service

$$LAG = 1.76(L/\sqrt{S})^{0.77}$$

There is disagreement about the constant factor; Gray (1970) gives 0.56, but the above gives better results here, and comes from Bell and Om Kar (1969).

Bell and Om Kar (1969) (USA)

$$m_1 = M\ AREA^{0.4}$$

M is a constant which is related to the type of vegetation cover. These types are not well related to the UK, as they were defined for the USA. However, a value of 0.88 for 'crops and poor to fair pasture' seems appropriate to the North-West.

Nash (1960) (UK)

$$m_1 = 8.11\ L^{0.3}\ EA^{-0.33}$$

Gray (1964) (Canada)

$$T_p = b(L/\sqrt{S})^n$$

b and n are not given by Gray, but the Flood Studies Report found b = 2.8, n = 0.47 to be the best choice for the UK.

Kennedy and Watt (1967) (Canada)

$$m_1 = 3.80\ (L_b/\sqrt{S095})^{0.63}\ ST^{0.86}\ POP^{-0.63}$$

The analysis here used POP = 1 + URBAN. Kennedy and Watt produced three other relations, but this was claimed to be the best, and in fact was so on the North-West catchments.

Fok and Lau (1973) (Hawaii)

$$T_p(3) = 1.41\ (L\ L_{ca}/\sqrt{S})^{0.22}$$

where $T_p(3)$ is the time to peak of the three-hour unit hydrograph.

Other characteristic times and methods of estimation

The Flood Studies Report Vol I Table 6.4 gives a summary of a number of investigations which have not been considered here, mainly because they involve parameters which are not easily evaluated, and are not at present available. Three such cases are:-

Nash (1960): A second method of evaluation of m_1 requires knowledge of the "overland slope" - the mean value of the steepest gradients measured at grid points over the catchment.

Hickok, Keppel & Rafferty (1959): An estimate for LAG involving the "drainage density", which is defined as the total length of visible channels per unit area, and also the overland slope as above. They also recommend that the parameters should be evaluated only on the so-called "source area" - that half of the catchment with the highest overland slopes.

Mitchell (1972): The lag-time is defined as the time between the centre of mass of the rainfall, and the point of inflexion on the recession limb of the total flow hydrograph.

The Comparisons

Two sets of data were available: firstly, all events on 26 catchments (162 events in all), and secondly the catchment averages for the 15 catchments which had more than 4 events each. The characteristic times were tested on both sets of data using the ASCOP statistical analysis program. To explain the results given in the tables, suppose that the observed time is T_{obs}, and the estimated time is T_{est} using catchment characteristics. Constant factors in T_{est} were ignored, for example in the case of Nash's estimate,

$$T_{obs} = m_1 \qquad T_{est} = L^{0.3}\ EA^{-0.33}$$

As the estimate equations have been derived originally using logarithmic regressions, comparisons were made between $\mathrm{Log}(T_{obs})$ and $\mathrm{Log}(T_{est})$ as well. The comparison criteria used were as follows:-

(a) The correlation between Log (T_{obs}) and Log (T_{est}), which indicates whether any linear relationship between the two is a strong tendency, but does not give any indication as to whether the relation is the one claimed.

(b) A linear regression of Log (T_{obs}) on $\mathrm{Log}(T_{est})$ was performed using ASCOP. This reads to a relation of the form $T_{obs} = K(T_{est})^C$, where C and K are constants. If C is close to unity, this is an indication of the validity of the claimed relation, provided the correlation in (a) is also close to 1.

(c) If the constant C above is close to 1, attention should now turn to the multiplying factor in the estimate. This is tested by forcing a relation of the form Log $(T_{obs}) = A + \mathrm{Log}(T_{est})$ (See Fig.1), where the value of A which gives the least squares error can be shown to be the average value of $\text{Log}(T_{obs}) - \text{Log}(T_{est})$. The ratio of the antilogarithm of A to the claimed value of the constant should be close to 1, and it is this ratio which is tabulated.

For the Snyder and Bell and Om Kar results, where the estimates do not specify a constant, the values observed are given in a separate table.

The three criteria above should be viewed in the sequence given, answering in turn the questions: Is T_{obs} simply related to T_{est}? Is the relation simple proportionality? Is the constant of proportionality the one claimed?

Another, separate, criterion is provided by:-

(d) The root mean square value of the error in using the claimed estimate. In order to make comparisons with different types of T_{obs}, this should be 'normalised' by dividing by the standard deviation of T_{obs}, and for the estimate to be regarded as 'good', this ratio should be as small as possible.

These comparison criteria are tabulated in Tables 1-3.

Conclusions

Firstly, it should be noted that no estimate does better than the full Flood Studies Report equation, although the Gray/FSR version does almost as well. Nash's estimate also performs well, having the second lowest error ratios. It is noticeable that the estimates which are less satisfactory are based on North American data, and so close agreement might not be expected.

In this light, the Fok and Lau estimate is surprisingly good at predicting the form of the relation. It has been suggested that a relation $T_p = b(L\, L_{ca}/\sqrt{S})^n$ for the time to peak of the one-hour unit hydrograph should be used in the same way as Gray's formula, provided discrepancies in the defined values of L_{ca} could be removed. It appears that this produces satisfactory results with n = 0.23 and b = 2.3 for these catchments.

All the criteria used above have been objective ones, and perhaps further comparison should be made on more subjective matters such as the ease of evaluation (as in the case of Nash's EA) and the satisfactory definition (for L_{ca}) of the catchment characteristics involved.

It should be noted that the small number of catchments used does not allow for effects due to some individual parameters (such as URBAN and storage), which the simpler estimates do not take account of, to affect the overall pattern in any dramatic way.

SJ/JVP

	(a)	(b)	(c)	(d)	
	Correlation between logs	Index in logarithmic regression	Ratio of constants	RMS error	RMS error ÷ s.d. of time
Flood Studies Report	0.797	0.957	0.861	0.987	0.410
Fok and Lau	0.784	0.768	2.026	4.841	2.011
Gray/FSR	0.810	0.812	0.904	3.999	1.661
Snyder	0.606	0.656			
Soil Conservation Service	0.647	0.414	1.051	3.709	1.361
Nash	0.767	0.783	0.883	2.108	0.533
Bell and Om Kar	0.682	0.582			
Kennedy and Watt	0.773	0.519	1.004	3.455	0.874

Table 1: Comparison criteria evaluated on all events

	(a)	(b)	(c)	(d)	
	Correlation between logs	Index in logarithmic regression	Ratio of constants	RMS error	RMS error ÷ s.d. of time
Flood Studies Report	0.863	1.039	0.839	0.987	0.424
Fok and Lau	0.879	0.820	2.079	4.355	1.869
Gray/FSR	0.897	0.887	0.914	3.709	1.591
Snyder	0.755	0.652			
Soil Conservation Service	0.798	0.415	1.151	3.434	1.433
Nash	0.850	0.760	0.913	1.904	0.484
Bell and Om Kar	0.769	0.585			
Kennedy and Watt	0.866	0.535	1.078	3.045	0.774

Table 2: Comparison of criteria evaluated on catchment averages for catchments with more than 5 events.

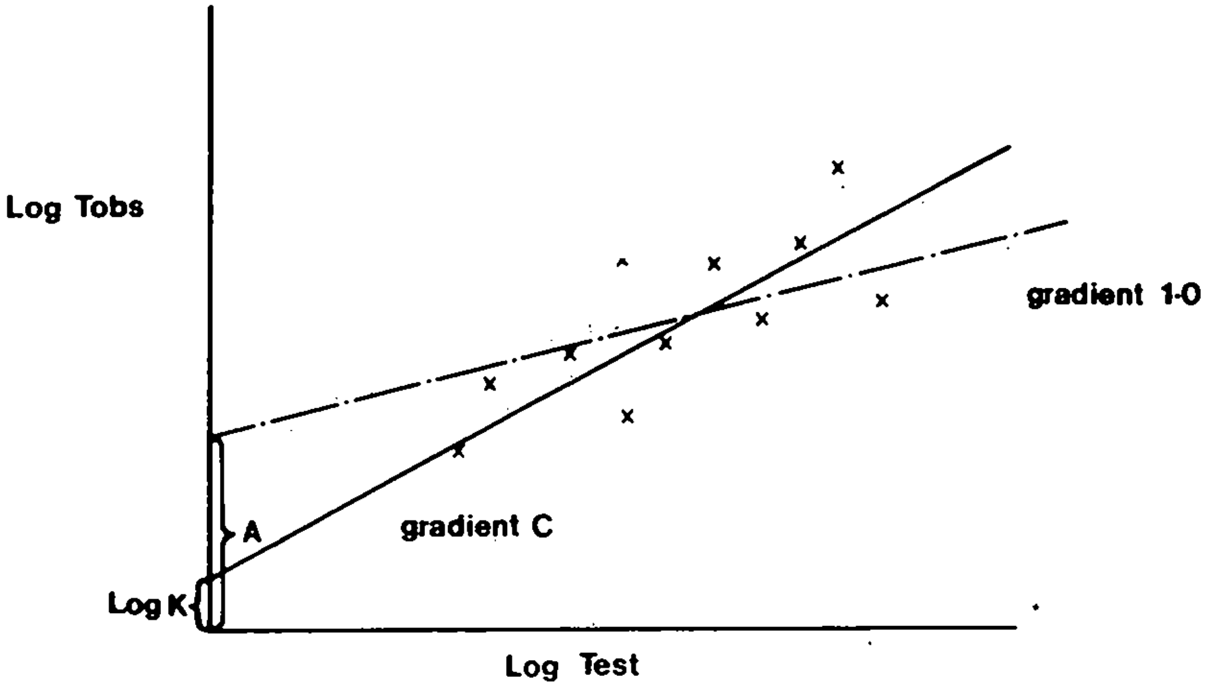

Fig 1. Logarithmic regression

—— Full linear regression of Log Tobs on Log Test (criterion (b))

—·— Regression of Log Tobs on Log Test with unit gradient (criterion (c))

	Claimed range	Observed on all events	Observed on catchment averages	Comments
Snyder	1.35 - 1.65	1.290	1.396	Lower values are expected for hilly regions
Bell and Om Kar	0.46 - 1.55	1.438	1.557	Suggests 'good pasture and forest'

Table 3: Constants found for Snyder and Bell and Om Kar estimates.

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