

Melting of floating ice and sea level rise

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[1] Contrary to popular belief, the melting of floating ice (in the form of ice shelves, icebergs and sea ice) may have a non-zero impact on sea level. This is because the melting process cools and dilutes the oceans on average, and unless these opposing effects exactly balance each other there will be a net change in the ocean density. We discuss how these subtle effects can be quantified and put bounds on the potential sea level rise associated with melting of the ice masses that are currently afloat in the world's oceans.

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1. Introduction

[2] Rising sea level is potentially one of the most disruptive consequences of climate change. Recent decades have seen both improvements in our ability to monitor sea level and acceleration in the rate at which it is observed to rise [Cazenave and Nerem, 2004]. However, attribution and prediction of variations in sea level remains problematic, and it is not yet possible to say whether the recent acceleration represents a change in the long-term signal or decadal-scale variability. In this paper we discuss and quantify one contribution to sea level change that is traditionally ignored. Its magnitude is indeed small and the critical reader might dismiss it as a curiosity. However, it would be impossible to close the ocean volume budget at the level of the current errors in observed sea level rise without consideration of this contribution. While uncertainties in other contributions remain as high as they currently are such an accurate closure of the budget is unachievable. However, as work proceeds to improve estimates of present and future contributions to sea level variability, estimation of the signal that can be attributed to the melting of the world's floating ice masses will become increasingly important.

2. Archimedes Principle

[3] As every schoolchild learns at an early stage of their science curriculum, a floating body displaces its own weight in water. This deceptively simple yet immensely powerful principle was first articulated by the Greek mathematician and philosopher Archimedes (287–212 BC), who, according to popular legend, leaped from his bath and ran naked through the streets of Syracuse shouting “Eureka!” (“I have

found it!”) upon its realisation. Because of Archimedes Principle, ice discharged from a grounded ice sheet into the ocean has an immediate impact on sea level, even though it remains part of the cryosphere until it subsequently melts. Indeed, once the ice is fully afloat its effective contribution to the total mass of the ocean has been made. Subsequent melting simply replaces the mass of seawater, which has already been displaced by the ice, with an identical mass of meltwater. Since the total mass of the ocean and ice remains unchanged during the melting process, this has often led to the erroneous assumption that sea level will be unaffected by the melting of floating ice.

3. Freshening of the Ocean

[4] In fact melting causes a change in the ocean density, and hence its volume, while leaving the total mass unaltered (Figure 1). Consider a mass of floating ice simply converted to freshwater without any mixing with the surrounding ocean. The freshwater displaces the same mass of seawater as the ice did, but since its density is lower than that of the ocean it still has a freeboard, which will now be seen by an observer as a rise in the mean sea level. Subsequent mixing of the seawater and freshwater will have only a small impact, which we will ignore for the moment, on their total volume, since the density of seawater is a nearly linear function of its salinity, and that function is approximately independent of temperature. Following complete mixing there will be a net freshening of the ocean and thus a steric sea level rise is associated with the conversion of floating ice to meltwater.

[5] This effect has recently been discussed by Noerdlinger and Brower [2007], who demonstrate the validity of the principles behind Figure 1 with a simple laboratory experiment. They also make estimates of the rise in mean sea level that would be associated with the melting of current ice shelf and sea ice cover. In Table 1 we present our own estimates based on the densities given in Figure 1 and a calculation of the change in ocean volume, ΔV , as:

$$\Delta V = V_{fw} \frac{(\rho - \rho_{fw})}{\rho} \quad (1)$$

where V_{fw} and ρ_{fw} are the volume and density of freshwater and ρ is the density of the displaced ocean water. The effects are quite small (the freeboard in Figure 1b is only 2–3% of the volume of the displaced ocean water), but they are not totally insignificant when compared with the magnitude of the quoted error on current estimates of sea level rise [Cazenave and Nerem, 2004], and could conceivably make a detectable contribution to the observed interannual variability in global mean sea level. Noerdlinger and

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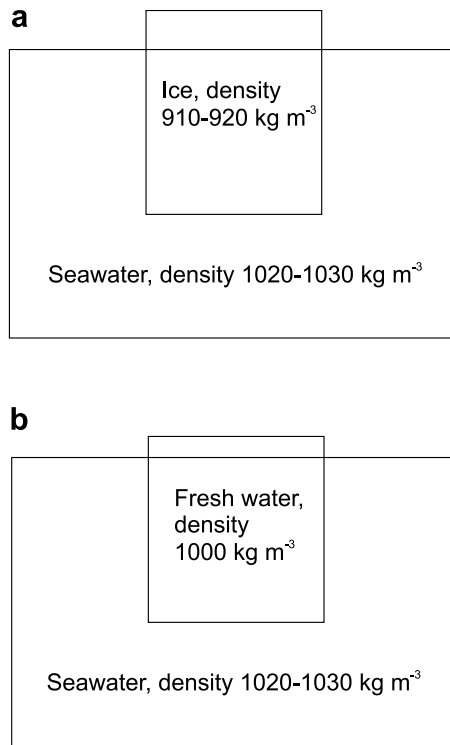


Figure 1. Schematic diagram illustrating how a mass of (a) ice or (b) freshwater floats in isostatic equilibrium in the ocean. Note that both have a freeboard.

Brower [2007] also point out that this subtle effect has been almost completely overlooked in the literature to date.

4. Cooling Associated With Freshening

[6] While it might appear that the simple principles outlined in Figure 1 and demonstrated by *Noerdlinger and Brower* [2007] represent the last word on the subject, we argue that the impact of melting ice on sea level is more complex and subtle still. The problem is that the situation in Figure 1b is unrealisable in nature. A considerable energy input is required to effect the melting of ice, and because its albedo is very high, most of that energy comes from the ocean itself. There is thus an oceanic cooling that accompanies the freshening discussed above, and this at least partially offsets the density decrease. The size of the offset depends on the ocean temperature and is a non-linear function of the cooling.

[7] Let us return to the situation shown in Figure 1, but consider how the ice is converted to water (Figure 2). Ocean and ice interact via a turbulent boundary layer, the motion of which is driven both by the density difference between the boundary layer and the surrounding ocean and by relative motion between the ice and surrounding ocean. Ocean waters are entrained into the turbulent boundary layer and give up some of their heat to effect melting. The freshwater produced by melting is mixed into the boundary layer and carried away from the ice, ultimately to mix with the far-field ocean. Thus our picture in Figure 1b should have had the ice replaced with a much larger volume of water having the temperature and salinity of the turbulent boundary layer. The freeboard of this water mass, which will be determined by the density difference between it and the ocean, is what determines the impact of melting ice on mean sea level. Thus our expression for the overall change in ocean volume becomes:

$$\Delta V = V_{bl} \frac{(\rho - \rho_{bl})}{\rho} \quad (2)$$

where V_{bl} and ρ_{bl} are the total volume and density of boundary layer water produced.

[8] With this added complexity, we need to consider the balance of both heat and salt within the boundary layer in order to estimate its volume and density. We make the assumptions that all the heat for melting the ice is extracted from the ocean, and that, as a first approximation, the boundary layer properties are steady in space and time. There must then be an exact balance between the rate at which heat is brought into the boundary layer by the incorporation of water from the surrounding ocean and the rate at which it is used to warm and melt the ice:

$$\rho \dot{e} c (T - T_{bl}) = \rho_{fw} \dot{m} c (T_{bl} - T_f) + \rho_{fw} \dot{m} L + \rho_{fw} \dot{m} c_i (T_f - T_i) \quad (3)$$

where \dot{e} and \dot{m} (dimensions of velocity) are the rate at which ocean water (at temperature, T) and meltwater, respectively, are entrained into the boundary layer, the subscripts bl , f and i refer to the boundary layer, freezing point and ice, respectively, c is specific heat capacity and L is latent heat of fusion. Similarly the salt balance must be:

$$\rho \dot{e} (S - S_{bl}) = \rho_{fw} \dot{m} (S_{bl} - S_i) \quad (4)$$

Table 1. Estimates of Sea Level Rise Caused by Melting of Floating Ice

Ice mass	Volume, 10^3 km ³	Equivalent Freshwater Volume, 10^3 km ³	Change in Ocean Volume, 10^3 km ³	Global Mean Sea Level Rise, mm
Antarctic ice shelves (total)	700	640	12–19	35–52
Icebergs formed by a major calving event	5	4.6	0.089–0.13	0.25–0.37
Thinning of all ice shelves by 1 m	1.5	1.4	0.027–0.040	0.074–0.11
Larsen B Ice Shelf	0.72	0.66	0.013–0.019	0.036–0.054
Antarctic sea ice (maximum)	28	26	0.50–0.75	1.4–2.1
Annual cycle (Antarctic)	10	9.2	0.18–0.27	0.50–0.74
Arctic sea ice (maximum)	42	38	0.75–1.1	2.1–3.1
Annual cycle (Arctic)	10	9.2	0.18–0.27	0.50–0.74

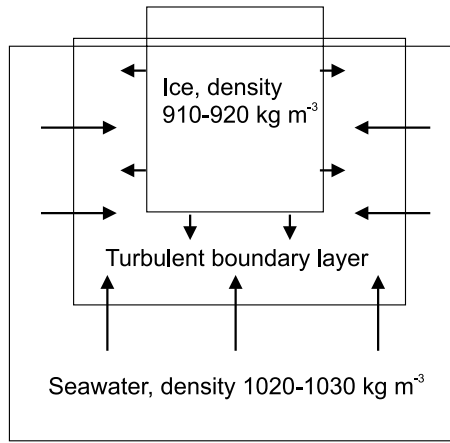


Figure 2. Schematic diagram illustrating how a mass of ice melts into the ocean. Meltwater and ocean water are mixed in a turbulent boundary layer that brings ocean heat to the ice and carries meltwater away. The relative density, and hence the freeboard, of the boundary layer waters is determined by both the cooling and freshening caused by the melting process.

where S is salinity, and we will generally assume that the salinity of the ice is zero. If we rewrite the left-hand side of equations (3) and (4) in terms of the total mass of water added to the boundary layer:

$$(\rho \dot{e} + \rho_{fw} \dot{m}) c(T - T_{bl}) = \rho_{fw} \dot{m} c(T - T_f) + \rho_{fw} \dot{m} L_i + \rho_{fw} \dot{m} c_i(T_f - T_i) \quad (5)$$

$$(\rho \dot{e} + \rho_{fw} \dot{m})(S - S_{bl}) = \rho_{fw} \dot{m}(S - S_i) \quad (6)$$

the right-hand sides are then in terms of known quantities. The cooling and freshening of the boundary layer, relative to the surrounding ocean, are thus closely coupled, such that as meltwater is added to the boundary layer, its properties evolve along a straight line in temperature/salinity space:

$$\frac{(T - T_{bl})}{(S - S_{bl})} = \frac{(T - T_f) + L_i/c + (c_i/c)(T_f - T_i)}{(S - S_i)} \quad (7)$$

This relationship, or similar, has been derived by a number of authors [Gade, 1979; Greisman, 1979; Moore and Wallace, 1988; Nøst and Foldvik, 1994], and its widespread validity has been demonstrated by numerous studies of ocean properties near melting ice. By comparing the slope of the line defined in equation (7) with the isopycnal (density contour) slope we can determine how evolution of the boundary layer properties along this line will impact the density (Figure 3). In the lower part of Figure 3, a water mass following one of the dashed, blue lines towards lower temperature and salinity becomes less dense. In the upper part of the diagram it becomes denser, at least initially,

although further progress along the line will eventually bring the density back to and below the original value.

5. Limits on Cooling and Freshening Caused by Melting Ice

[9] In order to make further progress we need to estimate how far along the dashed line the boundary layer properties are relative to the far-field ocean properties. Let us return to our assumption that the boundary layer properties are constant in space and time, embodied in equations (3) and (4). In addition, we know that at the interface between the solid ice and turbulent boundary layer the phase change is driven by the divergence of the heat flux:

$$\rho_{fw} \dot{m} L = \rho \gamma u_{bl} c(T_{bl} - T_f) - \rho_{fw} \dot{m} c_i(T_f - T_i) \quad (8)$$

where u_{bl} is the boundary layer velocity and γ is the heat transfer coefficient. The formulation of the turbulent heat

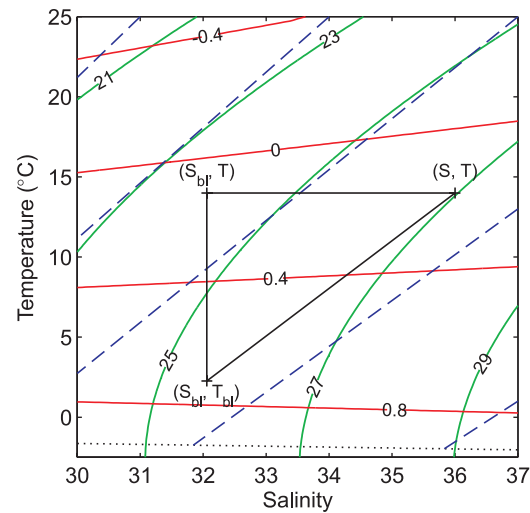


Figure 3. Temperature/salinity diagram with contours of seawater density (ρ -1000 kg m⁻³) at atmospheric pressure shown by the curved (green) lines. When ice melts into seawater the properties evolve towards the lower left corner of the diagram, following the dashed (blue) lines. The near-horizontal, solid (red) lines are contours of the relative slope of the dashed, meltwater evolution lines and the curved, density contours. The zero contour indicates where the slopes are equal, implying that the cooling and dilution associated with melting exactly balance each other, leaving the density unchanged. The near-horizontal, dotted (black) line near the bottom indicates the surface freezing point. The crosses joined into a (black) triangle illustrate for one particular case how the opposing effects of freshening and cooling combine to give the overall density change. In this case cooling offsets about half of the density change that would result from freshening alone, corresponding to a mean value of about 0.5 for the relative slope term contoured in red. Note that as the pressure increases, the curved density contours become flatter (temperature becomes more important in the equation of state), and the red contours move down the diagram towards lower temperatures.

transfer to the ice (first term on the right-hand side) is solidly grounded in observation [McPhee, 1992], and if we assume a typical drag coefficient of 2.5×10^{-3} for the ice-ocean interface, the heat transfer coefficient takes a value of 3×10^{-4} . Note that the rate of heat transfer to the ice is directly proportional to both the boundary layer velocity and the elevation of the boundary layer temperature above the freezing point (dotted line in Figure 3). Thus, if the turbulent boundary layer is at the freezing point there will be no transfer of heat and no melting, so the intersection with the freezing point line represents an absolute limit on the boundary layer properties. Immediately obvious from this is the fact that nothing resembling the freshwater shown schematically in Figure 1 will appear in the real ocean.

[10] Now combining equations (8) and (3) we obtain, after some rearrangement:

$$\rho \varepsilon u_{bl} c (T - T_{bl}) = \rho \gamma u_{bl} c (T_{bl} - T_f) \left[1 + \frac{T_{bl} - T_f}{L/c + (c_i/c)(T_f - T_i)} \right] \quad (9)$$

where ε is an entrainment coefficient, and we have invoked simple boundary layer theory to set the entrainment rate of equation (3) directly proportional to the water velocity. Note that the denominator of the last term on the right-hand side is typically $\sim 100^\circ\text{C}$, so we can generally assume that the term in parentheses is approximately equal to one. Rearranging equation (9), we can now write the temperature difference between the ocean and boundary layer as a fraction of the temperature difference between the ocean and the freezing point:

$$\frac{T - T_{bl}}{T - T_f} = \frac{\gamma}{\varepsilon + \gamma} \quad (10)$$

and this enables us to estimate how far the boundary layer properties will be along the dashed lines. In practice the entrainment coefficient is a function of the geometry of the ice-ocean boundary. For a vertical ice face the coefficient is ~ 0.1 [Ellison and Turner, 1959] and the ratio in equation (10) then has a value of typically less than 0.01, so that the boundary layer properties lie very close to the far-field ocean properties. For a gently sloping ice-ocean interface, such as that at the base of an ice shelf, the entrainment coefficient is much smaller (typically around 1×10^{-4}) and the ratio in equation (10) grows to around 0.75 or more. The black triangle in Figure 3 illustrates such a case schematically. With such large temperature and salinity differences, the boundary layer could be less dense than the far-field ocean, even if the far-field properties were at the upper extreme of the values shown in Figure 3.

6. Combined Effect of Cooling and Freshening on Steric Sea Level

[11] We are now in a position to quantify the change in ocean volume defined in equation (2). For the relatively small changes in temperature and salinity we are now concerned with we can linearise the equation of state by choosing appropriate mean values for the haline contraction

and thermal expansion coefficients, enabling us to rewrite equation (2) as:

$$\Delta V = \frac{V_{bl}}{\rho} \left((S - S_{bl}) \frac{\partial \rho}{\partial S} \Big|_T - (T - T_{bl}) \frac{\partial \rho}{\partial T} \Big|_S \right) \quad (11)$$

Since the relative size of the temperature and salinity differences is determined by the slope of the relevant dashed line plotted in Figure 3 and quantified in equation (7):

$$\frac{(T - T_{bl})}{(S - S_{bl})} = \frac{\partial T}{\partial S} \Big|_{bl} \quad (12)$$

and the relative size of the thermal expansion and haline contraction coefficients by the slope of the local isopycnal:

$$\frac{\partial \rho / \partial S \Big|_T}{\partial \rho / \partial T \Big|_S} = \frac{\partial T}{\partial S} \Big|_{\rho} \quad (13)$$

we can rewrite equation (11) as:

$$\Delta V = \frac{V_{bl}}{\rho} (S - S_{bl}) \frac{\partial \rho}{\partial S} \Big|_T \left(1 - \frac{\partial T / \partial S \Big|_{bl}}{\partial T / \partial S \Big|_{\rho}} \right) \quad (14)$$

Integrating equation (6) over time we obtain:

$$\rho_{bl} V_{bl} (S - S_{bl}) = \rho_{fw} V_{fw} (S - S_i) \quad (15)$$

and using equation (15) we can rewrite (14) in terms of the volume of meltwater in the boundary layer:

$$\Delta V = \frac{\rho_{fw} V_{fw}}{\rho_{bl} \rho} (S - S_i) \frac{\partial \rho}{\partial S} \Big|_T \left(1 - \frac{\partial T / \partial S \Big|_{bl}}{\partial T / \partial S \Big|_{\rho}} \right) \quad (16)$$

If we assume that the salinity of the ice is zero and that the haline contraction coefficient is constant over this wide range of salinities we arrive at an expression analogous to equation (1):

$$\Delta V = V_{fw} \frac{(\rho - \rho_{fw})}{\rho} \frac{\rho_{fw}}{\rho_{bl}} \left(1 - \frac{\partial T / \partial S \Big|_{bl}}{\partial T / \partial S \Big|_{\rho}} \right) \quad (17)$$

where we now formally require that the densities of the ocean water and freshwater are evaluated at the same temperature.

[12] There are two corrections to be applied to our earlier equation (1). The first (ρ_{fw}/ρ_{bl}) is the small correction for mixing the freshwater with the ocean that we ignored previously. It arises because it is the mass of salt that is conserved during the mixing process, rather than the salinity. With complete mixing the correction approaches a value of (ρ_{fw}/ρ) and reduces our earlier estimates of sea level rise by a few percent. The second correction, represented by the last term in parentheses on the right-hand side is much more significant. This correction quantifies our earlier qualitative discussion about the impact of melting on density. If the relevant dashed line in Figure 3 is steeper than the local isopycnal, the ratio term in the parentheses is greater than

one and the ocean volume change is negative, i.e. there is a fall in sea level. If the isopycnal is steeper than the dashed line the ratio is smaller than one and there is a positive impact on sea level. The full expression in the parentheses is the relative slope term contoured in Figure 3. Over the temperature and salinity ranges considered, the correction factor ranges from -0.45 to 0.9 . Note that we recover equation (1) from (17) only if the parenthetical correction factor is equal to one, which requires an assumption either that the relevant dashed line has zero slope (i.e. runs parallel to the salinity axis) or that the relevant isopycnal is vertical (i.e. is parallel to the temperature axis).

[13] While the former assumption is invalid at oceanic salinities, the latter is almost true in very cold waters (note the steepness of the isopycnals near the freezing point line in Figure 3 that makes the ratio in equation (17) about 0.9), where most floating ice is found. In this case equation (1) would be a good first approximation, but the low temperatures mean that the melt rates are also low. Large icebergs can survive for decades in the coastal waters of Antarctica, where their gradual melt will have a correspondingly gradual impact on steric sea level. Only when they drift into warmer waters do the melt rates rise. However, the potentially greater impact on the rate of steric sea level rise is then reduced because the cooling effect of the icebergs offsets the freshening to a greater degree.

7. Summary

[14] When floating ice melts in the ocean the effective mass of the ocean remains unchanged, but there is a change in its mean density. Both the change in temperature and the change in salinity shown in Figure 3 must be considered when calculating the steric sea level rise (or fall) associated with this density change. If the ocean is subsequently returned to its original temperature through heat exchange with the atmosphere, a further rise in sea level will accompany the warming, leaving the freshening as the only lasting impact of the ice. In this case our final estimate of the net contribution from melting ice would be that presented in equation (1) (with the ρ_{fw}/ρ correction applied) and

described by *Noerdlinger and Brower* [2007]. However, the subsequent warming of the ocean could be significantly delayed, particularly if the melting occurs in waters that are warm enough for the boundary layer water produced to be denser than its surroundings and to sink away from the sea surface. For this reason we suggest that it is more appropriate to calculate the steric change in sea level resulting from the melting of floating ice using equation (17) presented above, and regard any subsequent sea level rise associated with the recovery of the ocean to its initial temperature as a separate thermosteric effect.

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References

- Cazenave, A., and R. S. Nerem (2004), Present-day sea level change: Observations and causes, *Rev. Geophys.*, *42*, RG3001, doi:10.1029/2003RG000139.
- Ellison, T. H., and J. S. Turner (1959), Turbulent entrainment in stratified flows, *J. Fluid Mech.*, *6*(3), 423–448.
- Gade, H. G. (1979), Melting of ice in sea water: A primitive model with application to the Antarctic ice shelf and icebergs, *J. Phys. Oceanogr.*, *9*(1), 189–198.
- Greisman, P. (1979), On upwelling driven by the melt of ice shelves and tidewater glaciers, *Deep Sea Res., Part A*, *26*(9), 1051–1065.
- McPhee, M. G. (1992), Turbulent heat flux in the upper ocean under sea ice, *J. Geophys. Res.*, *97*(C4), 5365–5379.
- Moore, R. M., and D. W. R. Wallace (1988), A relationship between heat transfer to sea ice and temperature-salinity properties of Arctic Ocean waters, *J. Geophys. Res.*, *93*(C1), 565–571.
- Noerdlinger, P. D., and K. R. Brower (2007), The melting of floating ice raises the ocean level, *Geophys. J. Int.*, *170*(1), 145–150, doi:10.1111/j.1365-246X.2007.03472.x.
- Nøst, O. A., and A. Foldvik (1994), A model of ice shelf-ocean interaction with application to the Filchner-Ronne and Ross ice shelves, *J. Geophys. Res.*, *99*(C7), 14,243–14,254.

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