

I.O.S.

WAVES RECORDED WEST OF THE SCILLY ISLES

1979 - 1986

BY

S. BACON

REPORT NO. 246

1987

**INSTITUTE OF
OCEANOGRAPHIC SCIENCES
DEACON LABORATORY**

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<i>ABSTRACT</i>	<p>Measurements of surface waves were made routinely with a Waverider buoy off the Isles of Scilly over seven years from 1979 to 1986. This report gives details of the location, the instrumentation and the data return. It describes the method of data processing, to obtain in particular from the wave records estimates of significant wave height, H_s, and zero-up-crossing wave period, T_z. Histograms from these values are presented indicating the probability distributions of H_s and T_z throughout the year and during each season of the year. The observed distribution of H_s throughout the year and for each month and season of the year are fitted to various probability distribution functions which are extrapolated to obtain estimates of the fifty-year return value of H_s. The durations of storms, with H_s above a specified threshold, are investigated. There is some evidence that gaps in the data were preferentially associated with high wave heights.</p>		
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CONTENTS	Page
1. INTRODUCTION	5
2. LOCATION	5
3. INSTRUMENTATION	6
3.1 Description of Measuring and Recording Systems	6
3.2 Maintenance and Calibration	6
4. WAVE DATA COVERAGE	7
5. ANALYSIS AND PRESENTATION OF DATA	7
5.1 Definitions	7
5.2 Statistics of Significant Wave Height	8
5.2.1 Observed Distributions of H_s	8
5.2.2 50-Year Return Value of H_s	8
5.2.3 Persistence of High Values of H_s	9
5.2.4 Relationship Between H_s , Gaps in the Data and Wind Speed	11
5.3 Statistics of Wave Periods	11
5.4 Joint Probability Distribution of Wave Height and Period	11
5.4.1 Construction of the Scatterplot	11
5.4.2 Results from the Scilly Isles Waverider Data	12
6. ACKNOWLEDGEMENTS	12
7. REFERENCES	13
APPENDIX 1: FREQUENCY LOGGING OF WAVE DATA	14
APPENDIX 2: METHODS OF SPECTRAL ANALYSIS AND COMPUTATION OF WAVE PARAMETERS	15
APPENDIX 3: DETAILS OF METHODS USED FOR CALCULATING 50-YEAR RETURN VALUES BY FINDING THE LONG-TERM DISTRIBUTION OF H_s	18
APPENDIX 4: DEPLOYMENT HISTORY	21

1. INTRODUCTION

Wave measurements were recorded routinely off the Isles of Scilly using a Waverider buoy from 1979 to 1986 for the UK Department of Energy. This publication is a report on this wave measurement programme. It describes the site and the general topography of the location, then gives a description of the measuring and recording systems, and lists the data return achieved. It describes the method of data analysis, in particular the derivation of frequency spectra and, from these, estimates of significant wave height, H_S , and zero-up-crossing period, T_Z . The analysis and presentation of H_S and T_Z data includes discussion of storm duration (with H_S above a specified threshold), and of the presence of gaps in the data.

2. LOCATION

The site at which the wave measurements were taken is shown on the maps in figures 1(a) and (b). The buoy was moored in a water depth of 100m, initially at position $49^\circ 55.0N$, $6^\circ 40.0W$, later moved to $49^\circ 55.1N$, $6^\circ 36.6W$ (see Appendix 4 for deployment history). It was 15n.m. (29km) west of the Scilly Isles. The receiving site was at the Coastguard look-out tower near Hugh Town on St. Mary's, Isles of Scilly.

The sea floor in the area of the Waverider mooring is generally very nearly flat, with the Scilly Isles, the Seven Stones rocks and the southern Cornish coast forming very steep-sided projections from the floor, rising abruptly to the surface from (roughly) the 50m depth contour. The site is open to the Atlantic from directions NW through S to SE. There is a limited fetch of about 100 miles from NW to NE (SE. Ireland, Irish Sea, S. Wales) and from E to SE (English Channel, N. France). The narrow E to NE sector is sheltered by the Scilly Isles and Cornwall, 15 and 40 miles distant respectively.

3. INSTRUMENTATION

3.1 Description of Measuring and Recording Systems

Wave measurements were made using a standard Waverider buoy moored as described by Humphery (1982). This instrument senses the vertical acceleration of the buoy by means of a stabilized accelerometer and uses analogue double integrators to reconstitute the surface elevation. This information is transmitted ashore via a high-frequency radio link employing a frequency-modulated subcarrier to encode the wave height information.

On St. Mary's, the buoy's transmissions were received and a counting arrangement (described in Appendix 1) was used to decode the wave height information. In this way a measure of surface elevation was obtained at 0.5 second intervals in the form of a digital count, which was recorded on magnetic tape. The times between starts of successive records was 3 hours, later reduced to 1.5 hours, and the length of each record was initially 17 minutes, later increased to 34 minutes. A back-up cassette system was also employed to log the FM buoy-signal directly. These cassettes could be used to provide in-fill data in the event of a malfunction of the magnetic tape data logger. See Appendix 4 - Deployment history for further details.

3.2 Maintenance and Calibration

The buoy was maintained on site by Wimpey Laboratories Ltd. (now Wimpol Ltd.) under contract to IOS. Buoy replacement visits were made regularly; special visits were also made either when data reception stopped, or when data quality fell below acceptable standards. See Appendix 4 - Deployment history. A local coastguard was responsible for all tape and/or cartridge changes, and for reporting any malfunctions.

The buoys were calibrated before deployment and after recovery by Wimpey Laboratories Ltd. staff using the facilities at the National Maritime Institute (now British Maritime Technology) at Hythe. The buoys are subjected to circular motion in the vertical plane through attachment to a rotating arm of variable frequency (up to 0.4Hz) and amplitude 1.5m. Calibration is effected by comparing frequency response, and also comparing the amplitude of the record with 1.5m. Buoy sensitivity has been found to be within $\pm 2\%$, and the decoding technique (described in Appendix 1) has reduced variations in sensitivity due to the shore station equipment to a negligible value. The overall sensitivity of the system is therefore stable to within $\pm 2\%$.

4. WAVE DATA COVERAGE

The data cover the period 1 April 1979 to 31 March 1986. For the period 1 April 1979 to 31 March 1983 (3-hourly records), there were 8,147 valid records out of a possible 11,688; for the period 1 April 1983 to 31 March 1986 (1.5-hourly records), there were 12,477 valid records from 17,536 possible. The average resulting data return was 70.3%. Figures 2(a)-2(u) present all records of significant wave height, H_S , over the recording period as a time series, where each vertical bar represents a valid record whose height is proportional to the value of H_S for that record. Missing or invalid records are discerned as gaps. There was no attempt to fill gaps, with the single exception of gap-filling by simple linear interpolation for the purpose of persistence calculations only (see 5.2.3).

Table 1 presents the data return by month. The highest data return came from June with 91.6% average; the lowest was September, with 56.4%. There also appears to be a seasonal bias, with greater loss in the winter months. The possibility that loss of data is more likely to occur at times of large H_S is investigated in 5.2.4.

5. ANALYSIS AND PRESENTATION OF DATA

5.1 Definitions

The n^{th} moment of a continuous spectrum is

$$m_n = \int_0^{\infty} f^n E(f) df$$

where $E(f)$ is the spectral density at frequency f . Values of significant wave height, H_S , and zero-up-crossing wave period, T_Z , presented in this report have been derived from the spectral moments using the following definitions:

$$\begin{aligned} H_S &= 4\sqrt{m_0} \\ T_Z &= \sqrt{(m_0/m_2)} \end{aligned}$$

Significant steepness, S_S , is defined by

$$S_S = \frac{2\pi H_S}{g T_Z^2}$$

The fifty-year return value of H_S , $H_S(50)$, is defined as the value of H_S which is exceeded on average once in fifty years.

Where figures show seasonal data, the seasons are defined as follows

Spring - March to May

Summer - June to August

Autumn - September to November

Winter - December to February

5.2 Statistics of Significant Wave Height

5.2.1 OBSERVED DISTRIBUTIONS OF H_S

The maximum values of H_S recorded in these seven years of data occurred on the 7th March 1980 at 1200 hours with $H_S = 10.06\text{m}$ and associated $T_Z = 11.88\text{sec}$; and on the 11th December 1982 at 0300 hours with $H_S = 10.04\text{m}$ and associated $T_Z = 10.88\text{sec}$. But see the discussion on missing data in 5.2.4.

The mean and maximum values of H_S recorded during each individual month are given in Tables 2 and 3 respectively. Note: figures given in parentheses indicate values for a month with less than 50% valid records. Where no figure is given, no data were recorded in that month.

Estimates of the probability distributions of H_S are given in figures 3(a), (c), (e), (g), (i) which present histograms giving the percentage occurrence of H_S over all data and over each season, with the H_S grouped in 0.5 metre bins. These histograms are the marginal H_S distributions of the scatterplot which was constructed allowing for the variations in the number of records both throughout the years and between months; see 5.4. Modal values from these histograms are included in Table 5. Cumulative H_S probability distribution values can be obtained from figures 4,5,6.

5.2.2 50-YEAR RETURN VALUE OF H_S

Estimates of the fifty year return value of H_S , $H_S(50)$, were obtained by fitting various theoretical probability distributions to the observed distribution of H_S , and extrapolating to the required probability. Further details are given in Appendix 3. A summary of the results is given in Table 4. A drawback to this method is that there is no theoretical or physical justification for the choice of probability distribution. A method which avoids this problem is extreme value analysis; see for example Carter et al. (1986) for details. However, the Scilly Isles Waverider data cover too short a period and there are too many gaps to apply this method.

Fisher-Tippett Type 1 Distribution of H_S

The results of fitting a Fisher-Tippett Type 1 distribution to the H_S values are shown in figure 4. The parameters of the Fisher-Tippett 1 distribution estimated by the method of moments (see Appendix 3) are A (location parameter) = 1.77m and B (scale parameter) = 1.04m. This distribution is represented by the solid straight line in figure 4, extrapolation of which gives a 50-year return value of $H_S = 14.15m$.

Table 4 gives estimates of the distribution parameters and fifty year return values from H_S data from fitting individual years where a complete year's data was available (from April-March), and from fitting H_S data from individual months and seasons (over all years). The wide range of values for $H_S(50)$ from the four individual years (from 12.0m to 15.6m) indicates a considerable between-year variation in the distribution of H_S .

Weibull (2-parameter) distribution

The 2-parameter Weibull distribution fits all the data very poorly, so the usual practice of only fitting the upper tail was adopted. Fitting data of $H_S > 4m$ gave B (scale parameter) = 2.43m, C (shape parameter) = 1.53 and corresponding 50-year return value of $H_S = 12.27m$. See figure 5, in which all data above the vertical bar are fitted. The data cut-off was determined by subjectively balancing the need to maximise the quantity of data included in the fit against the severe worsening of the fit (judged by eye) as the cut-off is lowered below 4m.

Weibull (3-parameter) distribution

See figure 6. The parameters of the Weibull distribution which most closely fit the data are A (location) = 0.62m, B (scale) = 1.91m, C (shape) = 1.32. The corresponding 50-year return value of H_S is 13.14m.

Values of B and C were obtained by the method of moments for a range of values for the location parameter, A. See Appendix 3 for details. The value for A of 0.62m was chosen by minimising the χ^2 -distribution. See for example Gibbons (1971).

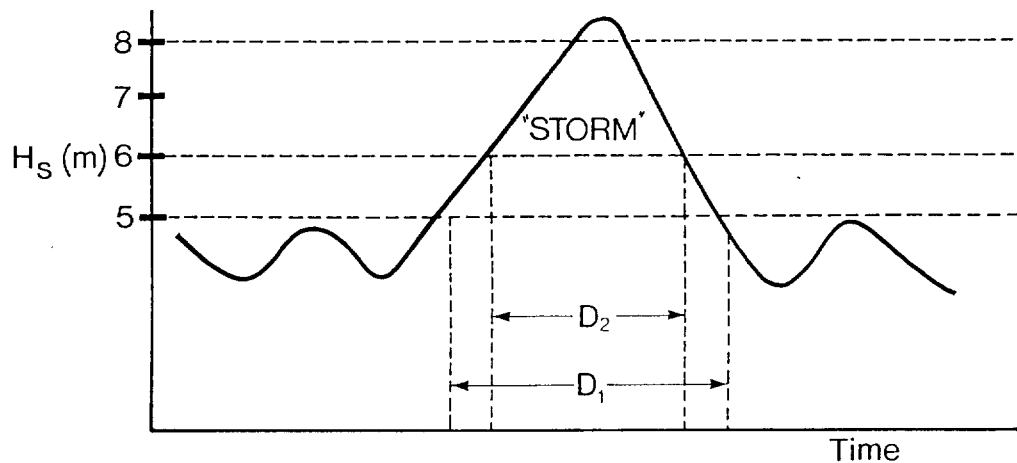
5.2.3 PERSISTENCE OF HIGH VALUES OF H_S

Definition of persistence

A storm of height H_S is said to occur when the significant wave height exceeds

a specified threshold. The storm duration or persistence is the time from the up-crossing of significant wave height through the threshold value to the time of the subsequent down-crossing.

One single climatic event can therefore be registered more than once by this method; if significant wave height reaches, say, 8m, then it will be counted as a 5m storm for some duration D_1 , a 6m storm for duration D_2 (less than D_1), etc. See sketch below by way of explanation.



Calculation of Persistence

The distribution of duration of storms of specified threshold levels can be estimated from discrete data by counting the number of consecutive values of H_s above the threshold.

For the purpose of these persistence calculations gaps in the data of duration <12 hours were filled by linear interpolation. See figures 12(a)-(f).

The presence of large unfilled gaps in the data introduced a difficulty to the computation, in that in the measurement of storm duration, no distinction was made between the recorded storm ends being either 'natural' or due to data gaps. One could only therefore state with confidence that measured storm durations represent minimum durations, the only effect of gaps being to truncate events. Therefore it was decided to present the persistence data in cumulative form; the histograms presented in figure 12 show the frequency of all events of specified duration D or longer.

Example Using figure 12(b) for storm $H_s > 6m$, consider the fifth bar which shows freq. = 3 events/year, minimum event duration = 15 hours. Assuming these

data represent average conditions, then one might expect values of H_S exceeding 6m persisting for 15 hours or more to occur three times in a year.

5.2.4 RELATIONSHIP BETWEEN H_S , GAPS IN THE DATA AND WIND SPEED

The winter months show poor data returns, as also does September, a month associated with equinoctial gales, and it seems unlikely that the distribution of gaps occurs by chance. This possible bias has been investigated using the technique employed by Stanton (1984). Figure 13 gives a histogram showing the frequency distribution of H_S records immediately prior to a gap, together with the histogram of all H_S records. It indicates the clear tendency of recording to be interrupted in high sea states. This may be due to breaks in the line of communication between the buoy's telemetered output and the shore receiving station.

There were also gaps of several months when the buoy went adrift or failed to function. For example, no wave data were recorded from 30 June to 28 August 1979 which covers the period of the Fastnet Race disaster (13th-14th August 1979).

Wind measurements were not made on the Isles of Scilly during much of the period covered by this report; but 10-minute mean wind velocities were recorded hourly at Gwennap Head on Land's End about 35 miles east of the Waverider, and a study of them indicates the following.

December 1979 was a month of severe gales with the highest wind speed recorded at Gwennap Head during the period covered by this report (60 knots from 270°): but no wave records were taken during this month. The largest December monthly mean wind speed was 22.4 knots in 1982 with particularly stormy periods around the 10th and the 20th. Fig. 2(1) shows a reasonable coverage of wave data throughout December 1982, but with gaps during these stormy periods.

The second highest wind speed recorded during the period was 55 knots at 2300 on 14th February 1985 with speeds greater than 40 knots persisting for 24 hours (1700 on the 14th to 1600 on the 15th). The wind direction was from 100° - 120° and seems to have raised a relatively low sea at the Waverider (see Fig. 2(p)), although most of the wave records for the 15th are missing.

The highest wind speed recorded during the month of January was 47 knots in 1980 and in 1984 and the largest January monthly mean was 1984; no wave data were obtained during these two Januaries.

The highest wind speed during March was 45 knots in 1986 which also had the largest March mean value (19.6 knots). The Waverider went adrift during a period of gales at the end of February 1986 and no data were obtained for March 1986. The next highest mean speed for March was 18.8 knots in 1981, and Fig.2(f) shows many gaps in the wave records during that month. The mean wind speed during this month at 3-hourly intervals with wave data was 16.9 knots, and without wave data was 20.4 knots. (Taking logs of the values to obtain approximately normal data, a Student's t-test shows differences at the 99% level.)

The highest monthly mean wind speed for November was 21.6 knots, recorded in 1982 which had a 96% wave data return; but the storm with the highest November wind speed, of 50 knots, occurred on 22-23 November 1984 when - as Fig.2(q) shows - there was a gap in the wave records.

Thus there is good evidence that monthly maximum wave heights were often missed. It also appears that many wave records are missing from some months with particularly high winds, suggesting that the results of wave height analysis in this report could underestimate the severity of the wave climate - although wind direction is an important factor in wave growth - but it is not feasible to quantify the error.

5.3 Statistics of Wave Periods

The maximum recorded value of zero-up-crossing wave period, T_2 , from the seven years' data occurred on the 26th December 1984 at 1500 hours with $T_2 = 12.74s$ and associated $H_5 = 8.41m$.

The percentage occurrence of T_2 for all data and for data in seasons is shown in figures 3(b), (d), (f), (h), (j) as histograms with 0.5 second bin size. Modal values are included in Table 5. These histograms are the marginal distributions of the scatterplots, the derivation of which is described below.

5.4 Joint Probability Distribution of Wave Height and Period

5.4.1 CONSTRUCTION OF THE SCATTERPLOT

The scatterplot shows the number of wave records in parts per thousand having particular combinations of H_5 and T_2 ; that is, it is a bivariate histogram representing the joint distribution of H_5 and T_2 . It is calculated by finding the number of records whose combination of (H_5, T_2) falls in each 0.5sec x 0.5m bin, dividing each final bin total by the total number of records,

then multiplying by 1000. In this case, however, the following adjustments have to be made for shortcomings in the data set (but it is not possible to correct for the probable bias of gaps towards high H_S , described in 5.2.4).

Allowance for change in recording interval

Up to 31st March 1983, records were taken once every 3 hours. After this date, records were taken every 1.5 hours, doubling the number of data records per day, and increasing the yearly average from $(365.25 \times 8) = 2922$ records to $(365.25 \times 16) = 5844$ records. If no allowance were made for this, any scatterplot would have twice as many points from a higher-frequency year than from a lower frequency one, unduly weighting the former. Allowance could be made for this either by 1. ignoring alternate 1.5 hourly records, effectively reducing the data to 3-hourly records, or by 2. multiplying each 3-hourly record by a weighting factor of 2.

Option 1 involves a loss of information and consequently was rejected. The scatterplot presented in figure 7 was therefore computed using option 2, whereby each 3-hourly record is accordingly treated as two equivalent 1.5 -hourly records.

Allowance for seasonal variation in the number of observations

Allowance was made for the variation in the number of records throughout the year by computing a cumulative scatterplot for each calendar month, then combining the twelve resulting scatterplots (suitably weighted for different numbers of days per month) into one representing all year. Seasonal plots were produced by combining the appropriate monthly scatterplots.

5.4.2 RESULTS FROM THE SCILLY ISLES WAVERIDER DATA

Figures 7-11 show the scatterplots for each season and for the entire year, for the observations off the Scilly Isles. Included on these plots are lines of significant steepness S_S which for much of the data are around $1/15$, with most of the steeper waves ($\sim 1/12$) recorded in the Spring.

Modal values of (H_S, T_Z) from the scatterplots are given in Table 5, along with the modal values of H_S and T_Z from the marginal histograms.

6. ACKNOWLEDGEMENTS

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APPENDIX 1

FREQUENCY LOGGING OF WAVE DATA

The Waverider buoy employs a 259Hz frequency modulated (FM) subcarrier to encode wave height information and has a nominal calibration so that an upward motion of the buoy of 1 metre displacement results in an increase in subcarrier frequency by 1.86Hz. In the standard receiving system supplied by Datawell the wave height information is extracted from the signal by monitoring the V.C.O. (voltage control oscillator) voltage in a phase locked loop demodulator. In the IOS system however a different approach is used. The 259Hz FM signal is applied to a phase locked loop which multiplies the subcarrier frequency by a factor of 128 and also serves to filter out any extraneous noise. The phase locked loop output is subtracted from a signal with a fixed frequency of 128 x 290Hz. This gives a signal whose frequency depends upon wave height and for which (290-259) x 128Hz corresponds to zero wave height. The frequency of this signal is counted over a period of one half second, so that for zero wave height a count of 1984 is obtained. In the presence of waves the count will change by -1.86 x 64 counts per metre of upward displacement.

The counting scheme described above determines the frequency response of the detector system which has the form:

$$\frac{\sin x}{x}$$

where $x = \pi ft$, t is the time period over which the frequency is counted (0.5 seconds); and f is the sea wave frequency. For high frequencies (above 1Hz) the response function should be slightly modified to take account of the frequency response inherent in the phase locked loop.

This system has two advantages over the standard analogue logging system: First, the receiver and demodulator do not require regular calibration. Second, the system frequency response serves as a precisely defined low pass anti-alias filter.

APPENDIX 2

METHODS OF SPECTRAL ANALYSIS AND COMPUTATION OF WAVE PARAMETERS

The wave data described in this report are derived from time-series containing 2048, or latterly 4096, values of sea-surface elevation sampled at 0.5 second intervals producing 17- or 34- minute records. Where two values are quoted below for any parameter, the former refers to the shorter record, the latter the longer.

The spectrum produced from the time series is considered to be representative of the 3-hourly or 1.5-hour period of which it is a sample.

An outline of the method of spectral analysis is given below.

THE FAST FOURIER TRANSFORM

Using the Fourier theorem, the elevation of the sea surface above its mean at time t is given by

$$h(t) = \sum_{i=1}^{\infty} \left\{ a_i \cos\left(\frac{2\pi i t}{T}\right) + b_i \sin\left(\frac{2\pi i t}{T}\right) \right\}$$

where

$$a_i = \frac{2}{T} \int_0^{\infty} h(t) \cos\left(\frac{2\pi i t}{T}\right) dt$$

$$b_i = \frac{2}{T} \int_0^T h(t) \sin\left(\frac{2\pi i t}{T}\right) dt$$

and T is the record length.

The Fast Fourier Transform, based on the above relationships, is used to compute the pairs of coefficients (a_i, b_i) at the fundamental (lowest measurable) frequency

$$f_0 = \frac{1}{T}$$

and at integral multiples of this frequency up to the maximum measurable Nyquist frequency

$$f_{\max} = \frac{1}{2\Delta T}$$

where ΔT is the sampling interval.

The sample estimate of the spectrum at the i^{th} frequency, Φ_i , is then computed as

$$\Phi_i = \frac{1}{2f_0} (a_i^2 + b_i^2)$$

TAPERING OF THE DATA

Variance of the wave record which is not located at one of the harmonic frequencies appears in the spectral estimates not only at the harmonics adjacent to the true frequency but in a band of harmonics. This leakage leads to biased estimates in that on balance a small proportion of the variance which should appear in the neighbourhood of the spectral peak leaks towards higher and lower frequencies. The effect can be reduced by tapering the ends of the time-series data smoothly to zero before performing the Fast Fourier Transform; a cosine taper applied to 12.5% of the record at each end has been used on the data described in this report. (This leads to a small increase in the sampling errors of the spectral estimates.)

SMOOTHING THE SPECTRAL ESTIMATES

The spectral estimates Φ_i have a standard error of 100%. This may be reduced by taking the average of consecutive spectral estimates, and assigning to it the mid-frequency of the band of estimates used. The smoothed spectral estimates, S_j have been averaged in blocks of 10 or 15:

$$S_j = \frac{1}{10} \sum_{i=10_j-9}^{10_j} \Phi_i \quad \text{or} \quad \frac{1}{15} \sum_{i=15_j-14}^{15_j} \Phi_i$$

and

$$f_j = (10_j - 4.5)f_0 \quad \text{or} \quad (15_j - 7.0)f_0$$

APPLICATION TO THE WAVE DATA

The fundamental frequencies used are

$$f_0 = \frac{1}{1024} = 0.9766 \times 10^{-3} \text{ Hz}$$

or

$$f_0 = \frac{1}{2048} = 0.4883 \times 10^{-3} \text{ Hz}$$

and the maximum frequency is

$$f_{\text{max}} = \frac{1}{2 \times 0.5} = 1.0 \text{ Hz}$$

smoothed estimates at the following frequencies

$$\begin{aligned} f_1 &= 5.37 & \text{or} & & 3.91 \times 10^{-3} & \text{Hz} \\ f_{\max} &= 0.992 & \text{or} & & 0.993 & \text{Hz} \\ \Delta f &= 9.766 & \text{or} & & 7.324 \times 10^{-3} & \text{Hz} \end{aligned}$$

The normalised standard error of the smoothed spectral estimates is 32% or 26%, although the tapering process increases these by a small amount.

COMPUTATION OF SPECTRAL MOMENTS

The n^{th} moment of a continuous spectrum is

$$m_n = \int_0^{\infty} f^n E(f) df$$

where $E(f)$ is the spectral density at frequency f . The unsmoothed spectral estimates Φ_i are used to compute the spectral moments;

$$m_n = \frac{1}{T} \sum_{i=i_L}^{i_U} f_i^n \Phi_i$$

where

$$\begin{aligned} i_L &= 42 \text{ or } 83 & f_{42} &= 0.0410 \text{ Hz} \\ & & f_{83} &= 0.0405 \text{ Hz} \end{aligned}$$

and

$$\begin{aligned} i_U &= 651 \text{ or } 1302 & f_{651} &= 0.6357 \text{ Hz} \\ & & f_{1302} &= 0.6357 \text{ Hz} \end{aligned}$$

APPENDIX 3

DETAILS OF METHODS USED FOR CALCULATING 50-YEAR RETURN VALUES BY FINDING THE LONG-TERM DISTRIBUTION OF H_S

H_S is used as a measure of the "sea-state", (i.e., the intensity of wave activity) and it is sampled either every 3 hours or every 1.5 hours. It is assumed that a set of H_S data for one year, or an integral number of years, is representative of the wave climate.

For each binned data value of H_S , the probability that this value will not be exceeded is calculated; this probability is then plotted against H_S . The axes are scaled according to an appropriate distribution, so that data with a perfect fit would appear as a straight line on the diagram. In practice, the class of functions known as extreme-value distributions are often found to give a close fit to the data. It should be noted that these functions are used only as 'templates' and not strictly as extreme-value distributions. These functions describe independent random data only, which climatic data are not, given 3-hourly data records and weather-system time-scales ranging from hours to years, etc.

FORMULAE

i) Weibull (3-parameter)

$$\text{Prob}(H_S \leq h) = \begin{cases} 1 - \exp\left\{-\left(\frac{h-A}{B}\right)^C\right\}, & \text{for } h > A \\ 0 & , \text{ for } h \leq A \end{cases}$$

ii) Weibull (2-parameter)

$$\text{Prob}(H_S \leq h) = \begin{cases} 1 - \exp\left\{-\left(\frac{h}{B}\right)^C\right\}, & \text{for } h > 0 \\ 0 & , \text{ for } h \leq 0 \end{cases}$$

iii) Fisher-Tippett I

$$\text{Prob}(H_S \leq h) = \exp\left[-\exp\left\{-\left(\frac{h-A}{B}\right)\right\}\right], \text{ where } B > 0$$

In each case, A is the location parameter, B is the scale parameter, C is the shape parameter. The Weibull 2-parameter distribution is the Weibull 3-parameter distribution with A=0.

For each distribution, the best fit straight line is drawn, then extrapolated to some desired probability and the corresponding value of H_S read off as the "design sea-state".

FITTING OF DISTRIBUTIONS BY THE METHOD OF MOMENTS

Fitting the Fisher-Tippett 1 Distribution

The mean and variance of this distribution are $A + \gamma B$ and $\pi^2 B^2 / 6$ respectively, where γ (Euler's constant) = 0.5772...; so the moments estimators given data $X_i, 1 \leq i \leq n$, are given by

$$\begin{aligned} \tilde{A} &= \bar{x} - \gamma \tilde{B} \\ \tilde{B} &= \sqrt{6} s / \pi \end{aligned}$$

where

$$\begin{aligned} \bar{x} &= \sum_i x_i / n \\ s^2 &= \sum_i (x_i - \bar{x})^2 / (n - 1) \end{aligned}$$

and values of \bar{x} and s^2 may be estimated from grouped data.

Fitting the Weibull (2- and 3-parameter) Distributions

The inverse form of the 3-parameter Weibull distribution is

$$H_s = A + B [\log_e \{1 - \text{Prob}(H_s < h)\}]^{1/C}$$

The three-parameter Weibull distribution can be 'converted' into the two-parameter by subtracting the location parameter A from each grouped H_s data value; the following fitting method was applied to both, with the proviso that the location parameter for the 3-parameter distribution be entered beforehand having been determined by prior attempts at fitting and judged by eye and minimum χ^2 distribution.

The probability distribution function for the 2-parameter Weibull distribution is

$$P_X(x) = \frac{C}{B} \left(\frac{x}{B}\right)^{C-1} \exp[-(x/B)^C] \quad (3.1)$$

Note that the distribution of $-\log_e(x)$ is FT-1 with location and scale parameters of $-\log_e(B)$ and $1/C$ respectively, so to fit a Weibull distribution to ungrouped data, it would be possible to take $-\log_e(\text{data})$ and fit an FT-1. However, as the two-parameter Weibull is usually only fitted to the upper tail of the data, it is easier to work with the Weibull distribution, it being integrable in terms of the incomplete gamma function.

Defining 'partial' moments about the origin of values above some specified level x_0 by

$$v_\gamma = \int_{x_0}^{\infty} x^\gamma P_X(x) dx \quad (3.2)$$

and substituting for $P_X(x)$ from 3.1 for $\gamma=1$ and 2 leads to

$$\begin{aligned} v_1 &= \frac{x_0}{Z^Y} \Gamma(1 + Y, Z) \\ v_2 &= \frac{x_0}{Z^Y} \Gamma(1 + 2Y, Z) \end{aligned} \quad (3.3)$$

where $Y = 1/C$ and $Z = (x_0/B)^C$.

and
$$\Gamma(p, D) = \int_D^{\infty} y^{p-1} e^{-y} dy$$

Therefore given estimates of v_1 and v_2 from data using equation 3.2 and a value for the lower limit of data to be fitted x_0 , then estimates of Y and Z , and hence of B and C can be obtained by numerical solution of 3.3.

CALCULATION OF 50-YEAR RETURN VALUE

The 50-year return value of H_S is defined as that value of H_S which is exceeded on average once in 50 years. In each case this has been determined by extrapolating the relevant distribution to the required probability of exceedance which is determined by assuming some frequency of observation (taken in this report to be 3-hourly), and by assuming all H_S observations to be independent.

The 50-year return value of H_S , $H_S(50)$ is then given by

$$\begin{aligned} \text{Prob}(H < H_S(50)) &= 1 - \frac{1}{50 \times 365.25 \times 8} \\ &= 0.99999316 \end{aligned}$$

Fitting to seasonal or monthly data reduces the number of days observation per year from 365.25 to 365.25/4 or 365.25/12 respectively, and reduces the relevant probabilities to 0.99997262 and 0.99991786 respectively.

APPENDIX 4
DEPLOYMENT HISTORY

Date	Buoy Number	Event
April 1979	67201	Deployed
10 Oct 79	67201	Recovered
	67326	Deployed
24 Jan 80	67326	Recovery by fishing boat after going adrift
20 Feb 80	67191	Deployed
20 Mar 80	-	Rapco logger changed to Microdata
24 Jun 80	67191	Recovered
	67326	Deployed
22 Jan 81	67326	Recovered
	67550	Deployed
1 Mar 81	-	Change to frequency logging double length records
2 Apr 81	67550	Recovered
	67552	Deployed
23 Oct 81	67552	Recovered
	67327	Deployed
18 Mar 82	67327	Recovered
	67407	Deployed
1 Jul 82		Buoy moved from 49°55.0N, 6°40.0W to 49°55.1N, 6°36.6W due to change in position of traffic separation zones
20 May 83	67407	Recovered
	67327	Deployed
? Jan 84		HP9915 microcomputer replaced
10 Feb 84	67327	Recovered
	67043	Deployed
21 Jul 84	67043	Recovered
	67214	Deployed
? Jan/Feb 85	67214	Recovered
	67043	Deployed
16 May 86		Buoy reported off station; never recovered; formally written off; site closed

Each buoy was recalibrated upon recovery. Changes in recording sensitivity during the on-site period were of order of 0.1%.

SCILLY ISLES WAVERIDER BUOY 1979-86

TABLE 1(a)

MONTHLY AND ANNUAL DATA RETURN

	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	TOTAL	% TOTAL	%(MOD) TOTAL*
1979-80	196	147	211	0	30	72	0	0	0	0	50	138	844	28.8	32.6
1980-81	104	214	167	88	231	145	211	232	237	146	124	117	2016	69.0	79.5
1981-82	218	190	230	215	213	129	213	227	236	243	219	241	2574	88.2	93.2
1982-83	233	246	235	226	231	225	220	230	214	205	214	234	2713	92.9	97.7
1983-84	473	491	469	435	423	251	412	447	457	0	215	424	4513	77.1	80.9
1984-85	455	476	450	394	399	183	128	213	385	385	342	370	4180	71.6	79.7
1985-86	316	407	473	346	242	319	345	309	289	397	341	0	3784	64.8	87.7
Average Monthly & Annual %ages	81.7	85.5	91.6	64.3	71.3	56.4	62.6	69.9	72.2	56.7	66.7	75.7		70.3	78.8

Average Monthly & Annual %ages

TABLE 1(b)

TOTAL DATA RETURN

RECORDING PERIOD (HRS)	TOTAL VALID RECORDS	TOTAL POSSIBLE RECORDS	VALID %AGE
Apr.1979-Mar.1983	8,147	11,688	69.7
Apr.1979-Mar.1983	12,477	17,536	71.2

*Modified total %ages - after filling gaps <12 hours duration for persistence diagrams ONLY.

SCILLY ISLES WAVERIDER BUOY 1979-86

TABLE 2

MONTHLY MEAN $\overline{H_s}$, H_s (metres)

	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR
1979-80	2.36	2.11	1.56	-	(1.74)	(1.39)	-	-	-	-	(2.42)	2.89
1980-81	1.41	1.35	1.52	(1.98)	1.70	2.46	2.87	2.88	3.49	2.36	2.77	3.26
1981-82	1.89	2.31	1.72	1.81	1.23	2.78	2.71	2.24	3.31	3.28	3.63	3.51
1982-83	1.70	1.93	1.94	1.10	2.06	2.13	3.57	3.47	3.84	3.71	2.84	2.73
1983-84	2.12	2.07	1.43	1.15	1.26	2.80	2.90	2.08	3.50	-	1.90	1.74
1984-85	1.55	1.54	1.49	1.24	1.33	(1.98)	(2.62)	2.86	3.17	2.58	2.59	2.58
1985-86	2.85	1.74	1.60	1.42	2.45	1.82	1.79	2.15	3.57	4.00	2.50	-

TABLE 3

MONTHLY AND ANNUAL MAXIMA OF H_s

	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	ANNUAL
1979-80	5.36	7.32	3.30	-	(2.19)	(3.40)	-	-	-	-	(5.08)	10.06	10.06
1980-81	2.92	3.43	4.21	(3.34)	4.54	6.08	7.06	6.48	8.91	4.86	5.22	6.89	8.91
1981-82	6.78	5.19	4.04	3.62	3.15	5.92	7.79	4.71	7.73	5.71	7.85	7.55	7.85
1982-83	4.46	4.26	5.18	2.42	4.81	5.46	9.34	6.74	10.04	9.18	7.55	6.68	10.04
1983-84	5.59	6.85	3.96	3.40	2.44	6.12	9.00	7.31	7.67	-	4.62	5.05	9.00
1984-85	3.81	5.52	3.77	3.54	3.35	(3.35)	(4.86)	5.58	8.59	5.88	5.79	7.21	8.59
1985-86	8.40	6.01	5.92	3.16	4.91	4.44	5.31	6.30	7.63	9.30	5.41	-	9.30
ALL YEARS	8.40	7.32	5.92	3.62	4.91	6.12	9.34	7.31	10.04	9.30	7.85	10.06	10.06

SCILLY ISLES WAVERIDER BUOY 1979-86

TABLE 4

50-YEAR RETURN VALUES AND FUNCTION PARAMETERS

FUNCTION TYPE	DATA PERIOD	Hs(50) (m)	A(location) (m)	B(scale) (m)	C(shape)
Weibull (2-parameter, 4.0m cutoff)	All	12.27	0.00	2.43	1.53
Weibull (3-parameter)	All	13.25	0.62	1.88	1.30
Fisher-Tippett 1	All	14.16	1.77	1.04	-
	Spring	11.72	1.65	0.96	-
	Summer	7.47	1.20	0.60	-
	Autumn	12.72	1.97	1.02	-
	Winter	13.94	2.26	1.08	-
	Jan	12.34	2.65	1.03	-
	Feb	11.41	2.23	0.98	-
	Mar	12.76	2.11	1.13	-
	Apr	9.24	1.51	0.82	-
	May	8.12	1.45	0.71	-
	Jun	7.33	1.24	0.65	-
	Jul	5.88	1.11	0.51	-
	Aug	7.02	1.28	0.61	-
	Sep	9.75	1.75	0.85	-
	Oct	12.83	2.15	1.14	-
	Nov	11.52	2.05	1.01	-
	Dec	13.73	2.81	1.16	-
	Apr.-80-Mar.81	12.98	1.80	0.94	-
	Apr.81-Mar.82	13.99	1.95	1.01	-
	Apr.82-Mar.83	15.60	1.93	1.15	-
	Apr.84-Mar.85	11.98	1.63	0.87	-

SCILLY ISLES WAVERIDER BUOY 1979-86

TABLE 5

MODAL VALUES of Hs,Tz
from Histograms and Scatterplots
Values quoted are bin-central

QUANTITY	TIME OF YEAR	VALUE	%AGE OCCURRENCE
HS	All	1.25m	19.1
HS	Spring	1.75m	23.1
HS	Summer	1.25m	29.8
HS	Autumn	1.75m	17.7
HS	Winter	3.25m	14.5
TZ	All	5.25s	13.1
TZ	Spring	5.25s	14.2
TZ	Summer	5.25s	19.3
TZ	Autumn	6.25s	15.6
TZ	Winter	7.25s	14.0
Joint (Hs,Tz)	All	1.75m,5.25s	4.6
Joint (Hs,Tz)	Spring	1.75m,5.25s	5.3
Joint (Hs,Tz)	Summer	1.25m,4.75s	7.1
Joint (Hs,Tz)	Autumn	1.75m,4.75s	4.1
Joint (Hs,Tz)	Winter	2.25m,5.75s	3.3

Fig.1

Fig.1, showing the locations of Gwennap Head wind recording station, the Seven Stones Shipborne Wave Recorder, and the earlier (1) and later (2) positions of the Scilly Isles Waverider Buoy.

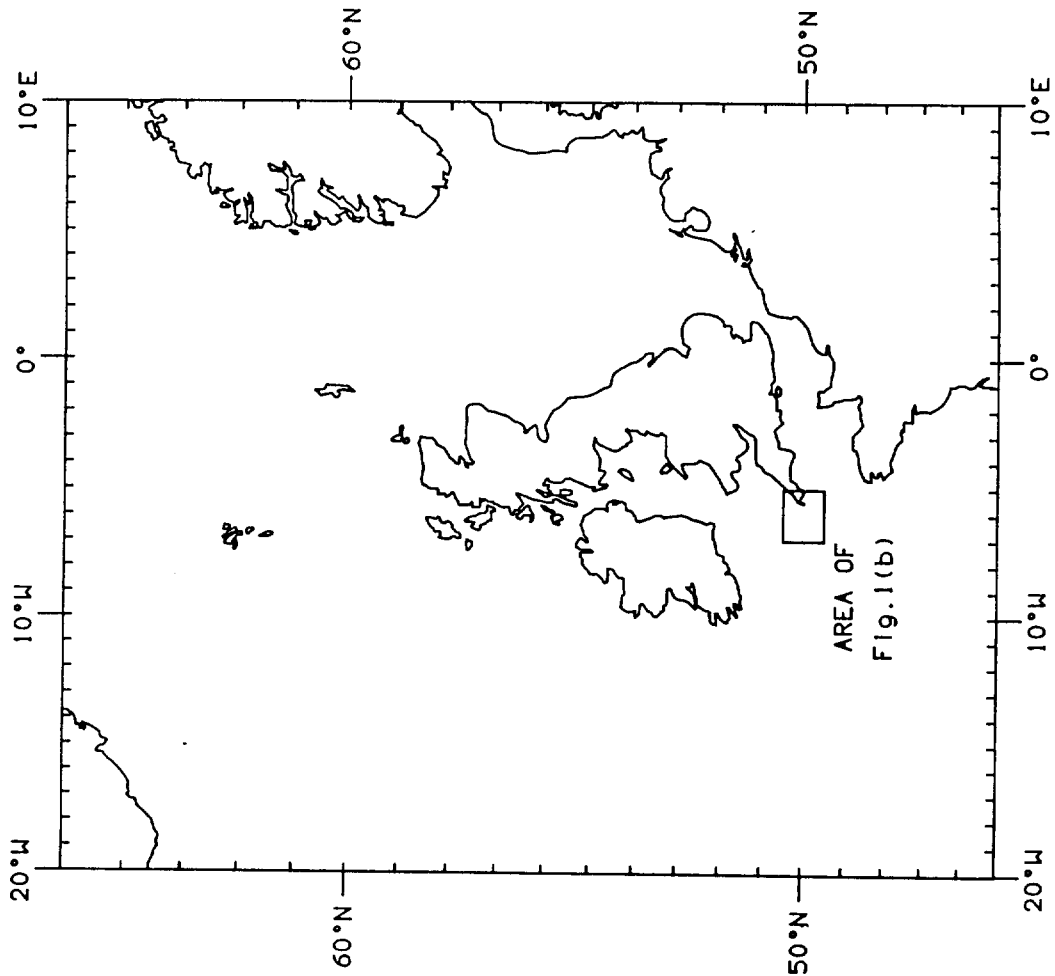


Fig.1(a)

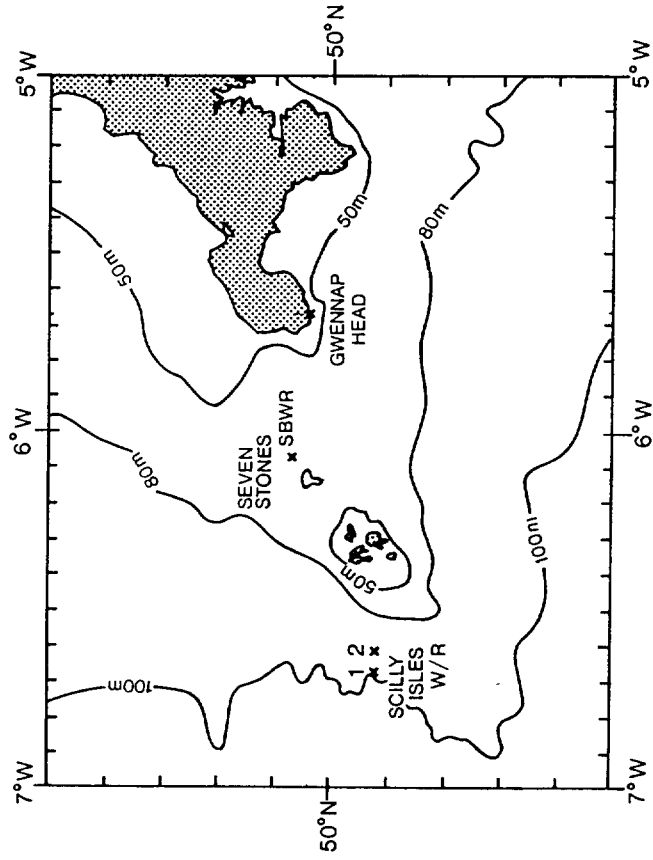


Fig.1(b)

Fig.2(a)

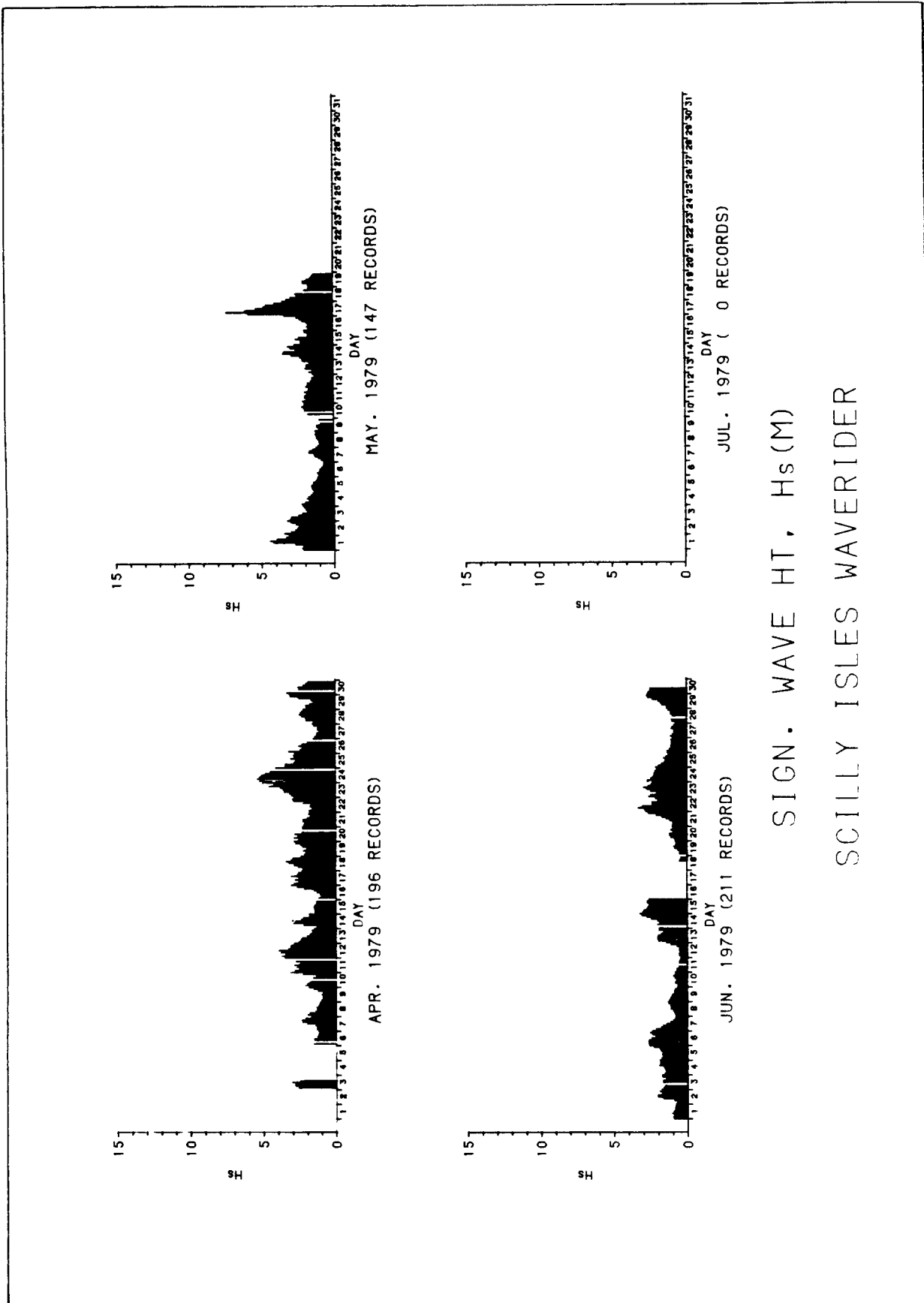


Fig.2(b)

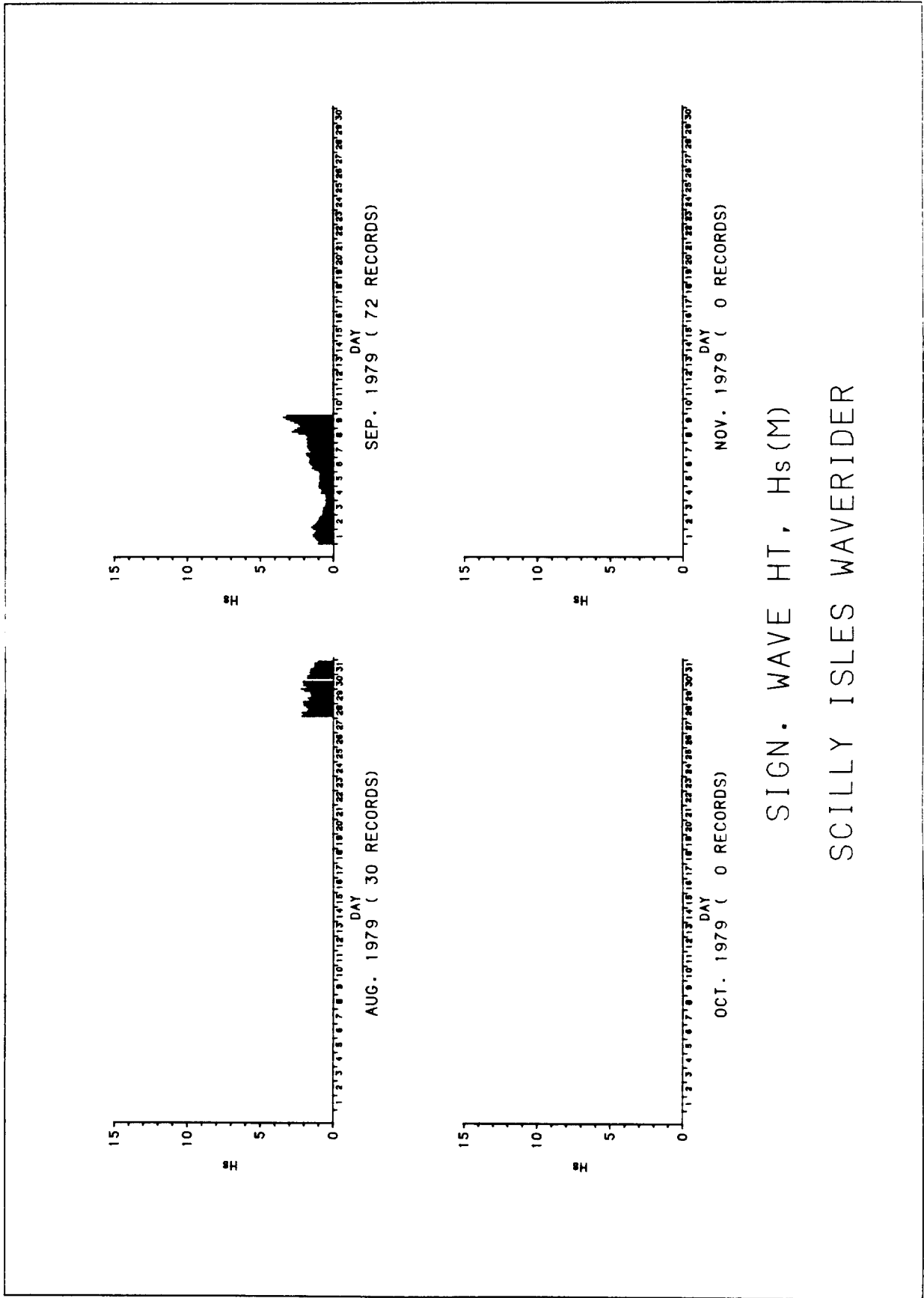


Fig.2(c)

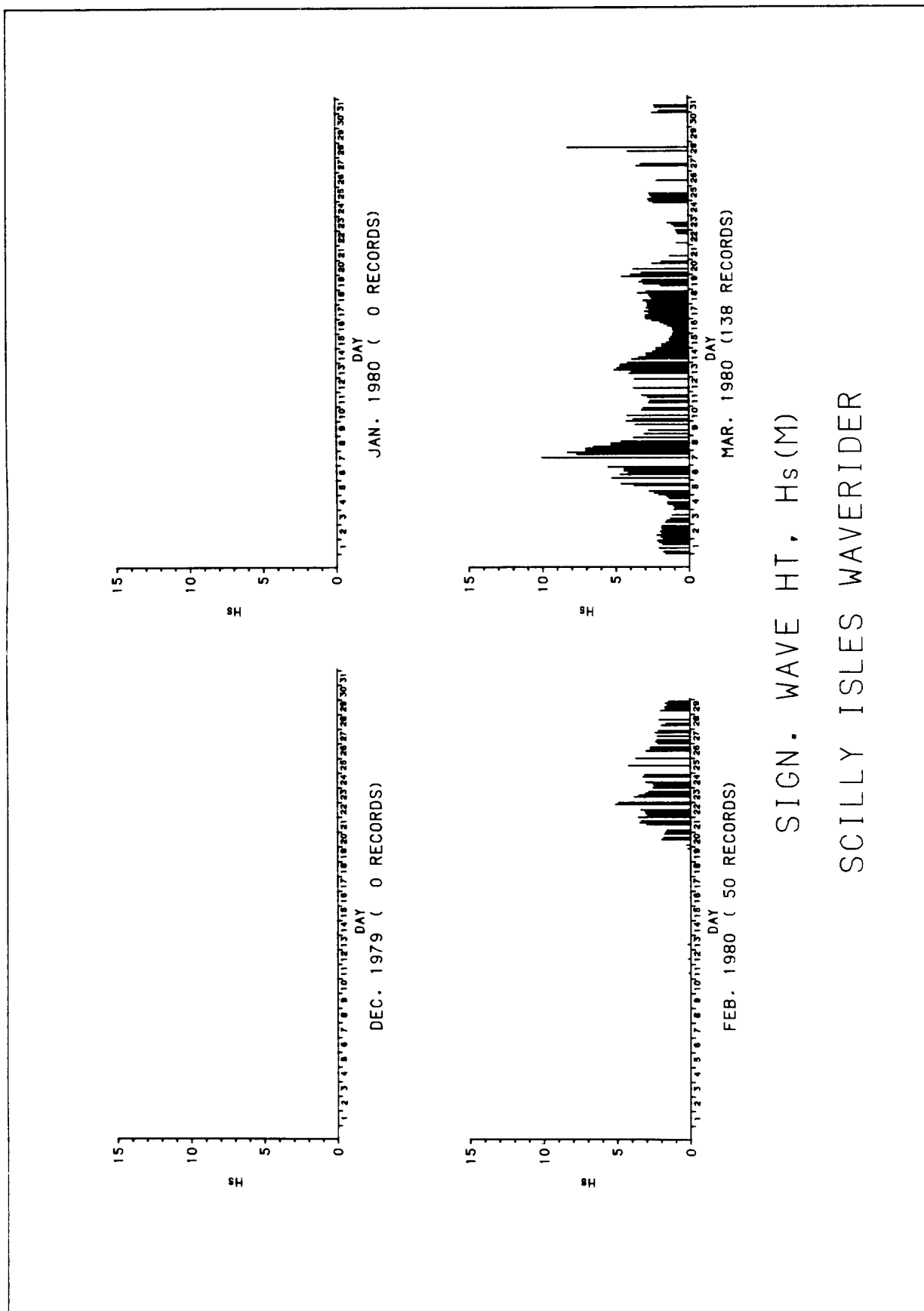


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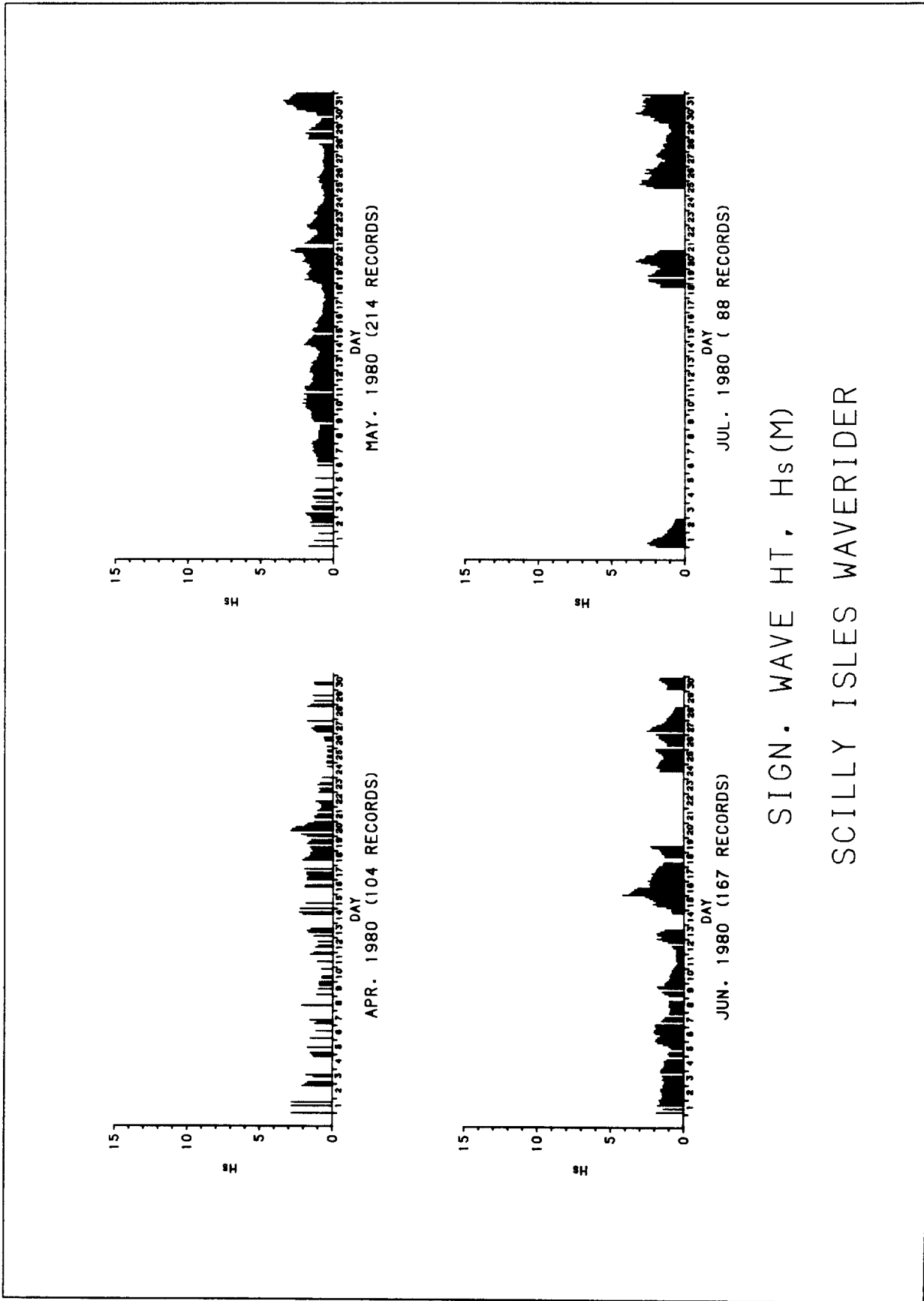


Fig.2(e)

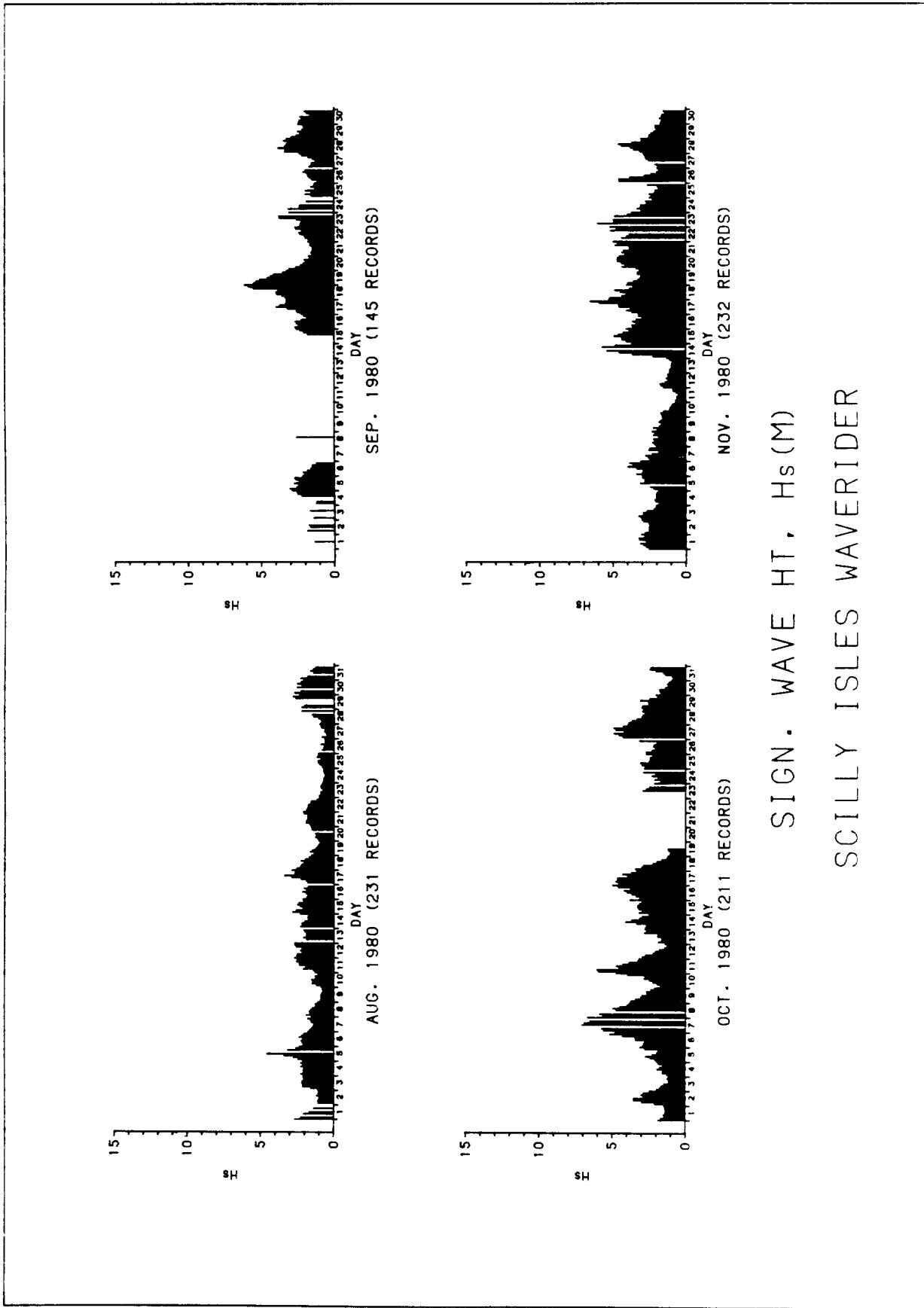
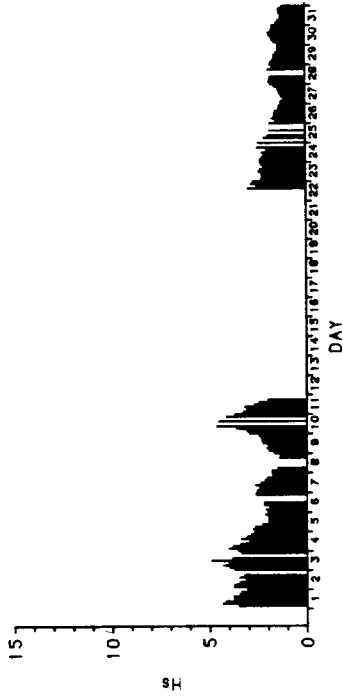
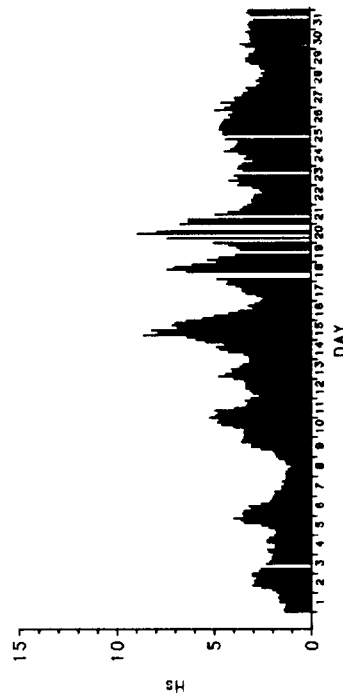


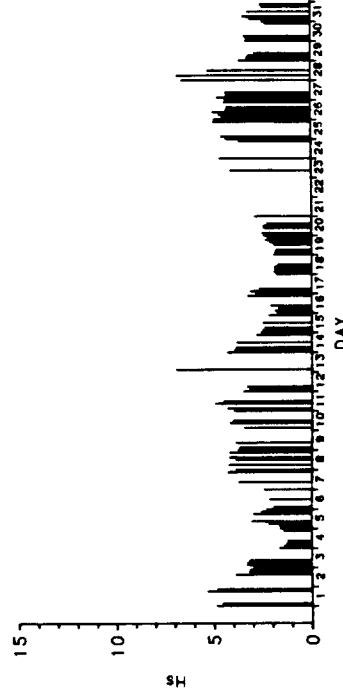
Fig.2(f)



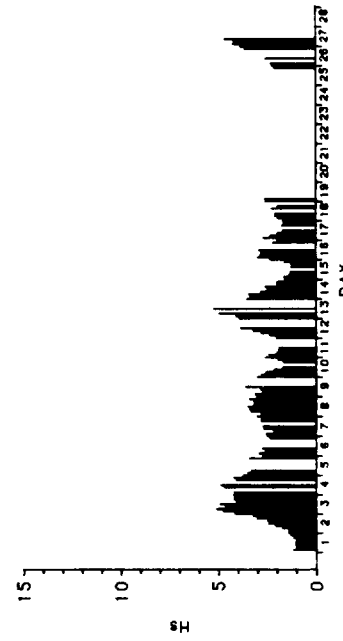
JAN. 1981 (146 RECORDS)



DEC. 1980 (237 RECORDS)



MAR. 1981 (117 RECORDS)



FEB. 1981 (124 RECORDS)

SIGN. WAVE HT, Hs(M)
SCILLY ISLES WAVERIDER

Fig.2(g)

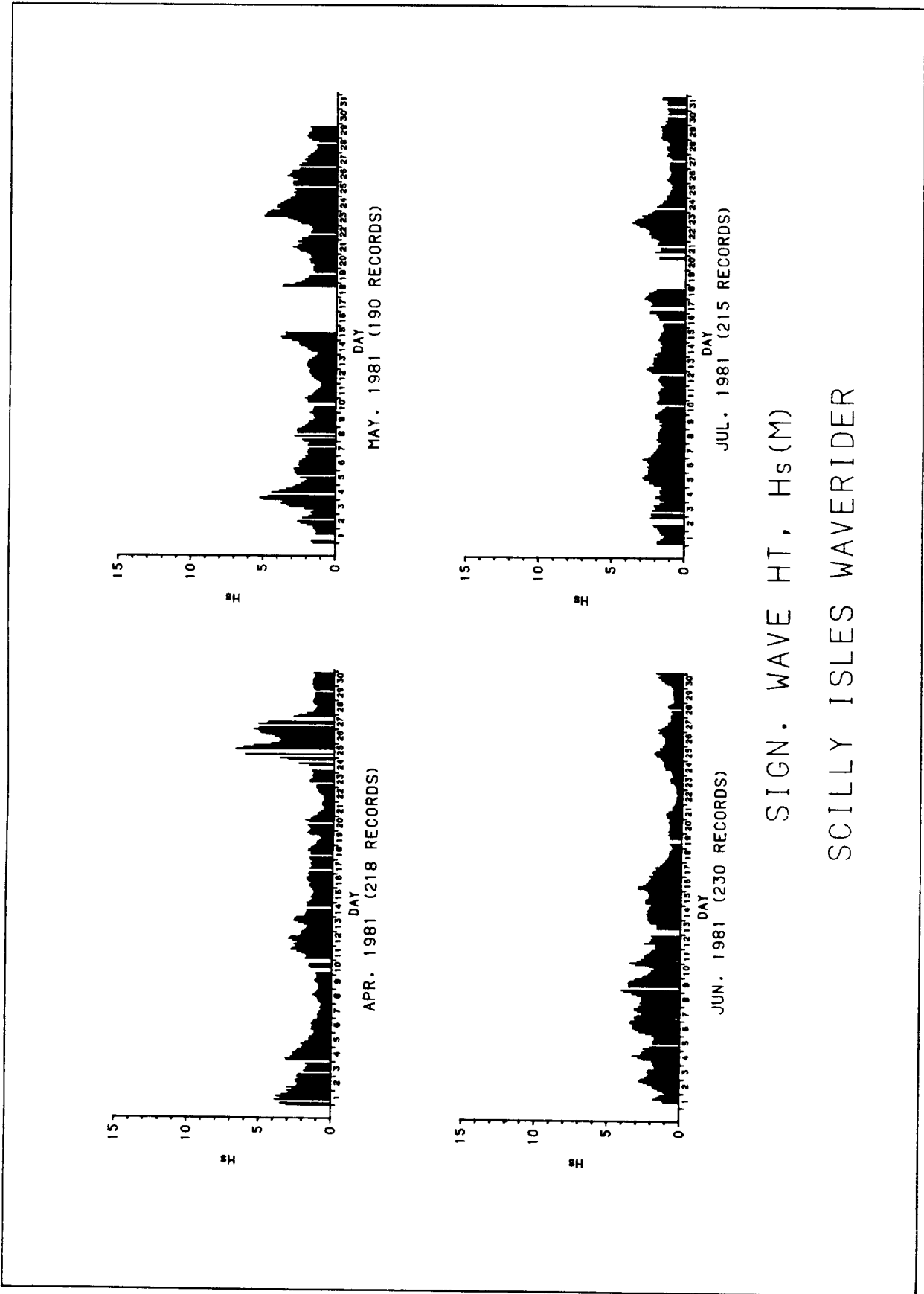
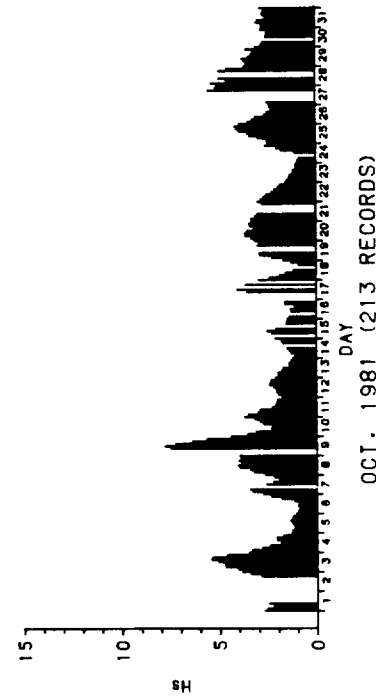
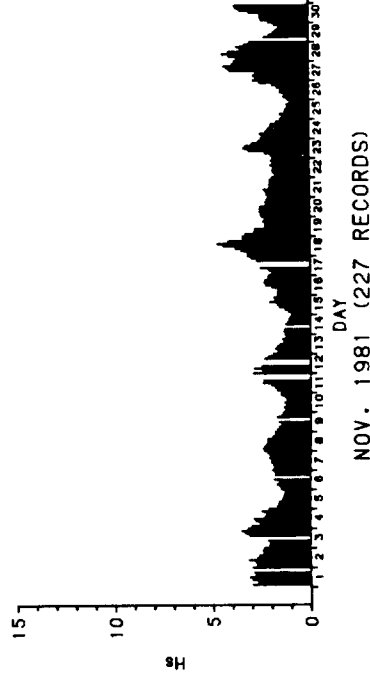
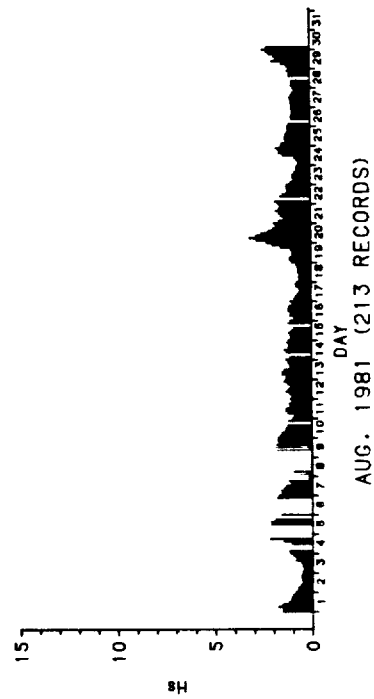
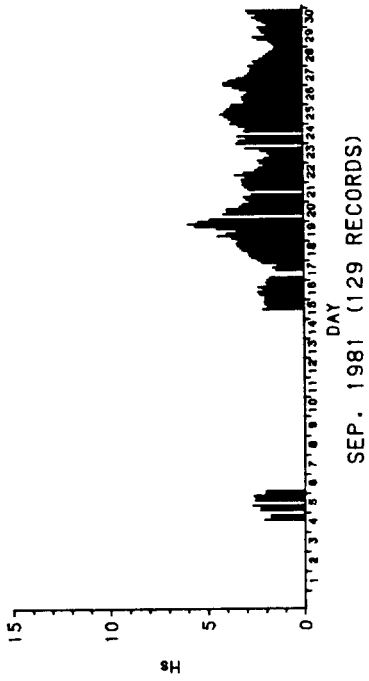


Fig.2(h)



SIGN. WAVE HT, Hs(M)
SCILLY ISLES WAVERIDER

Fig.2(i)

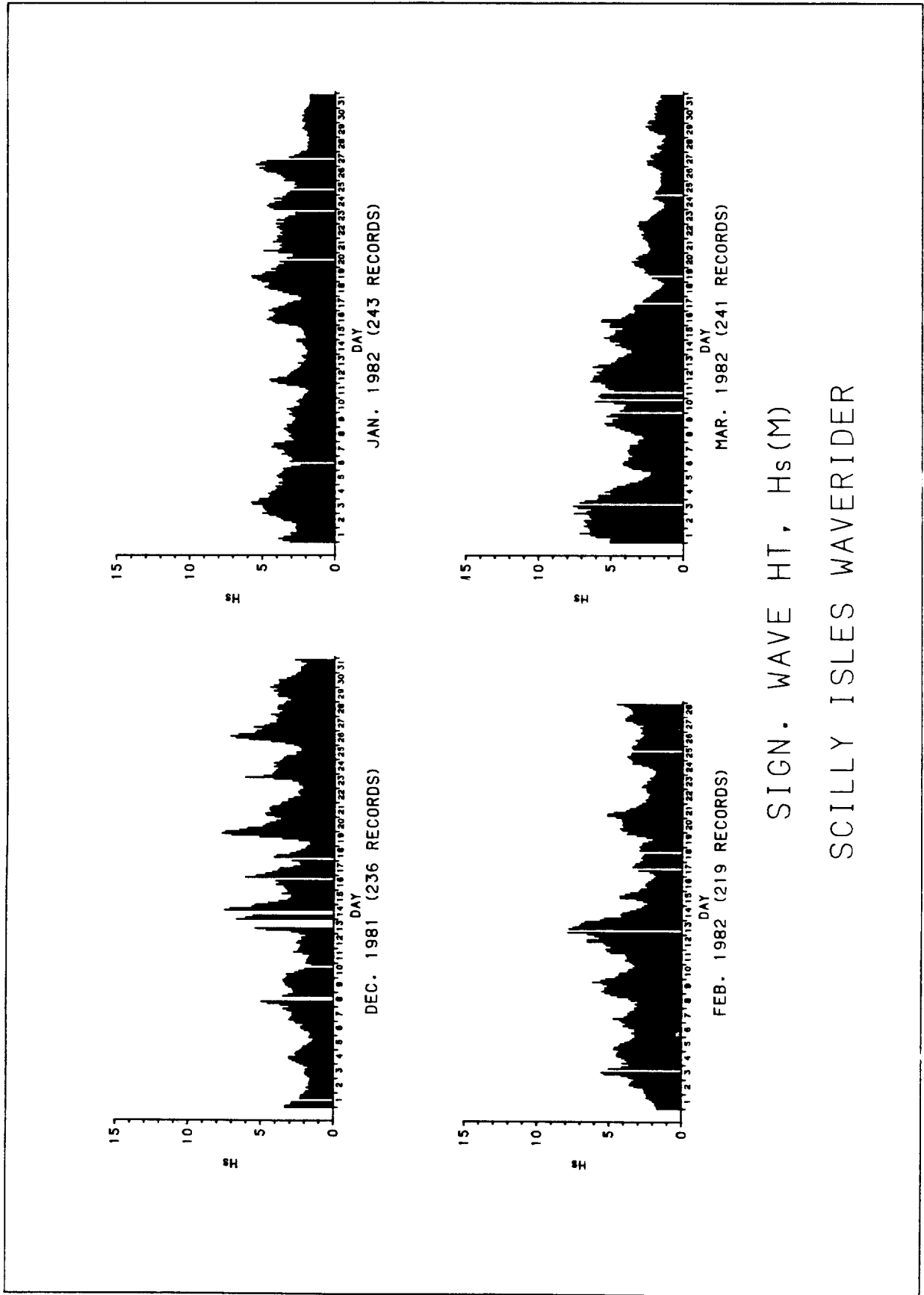
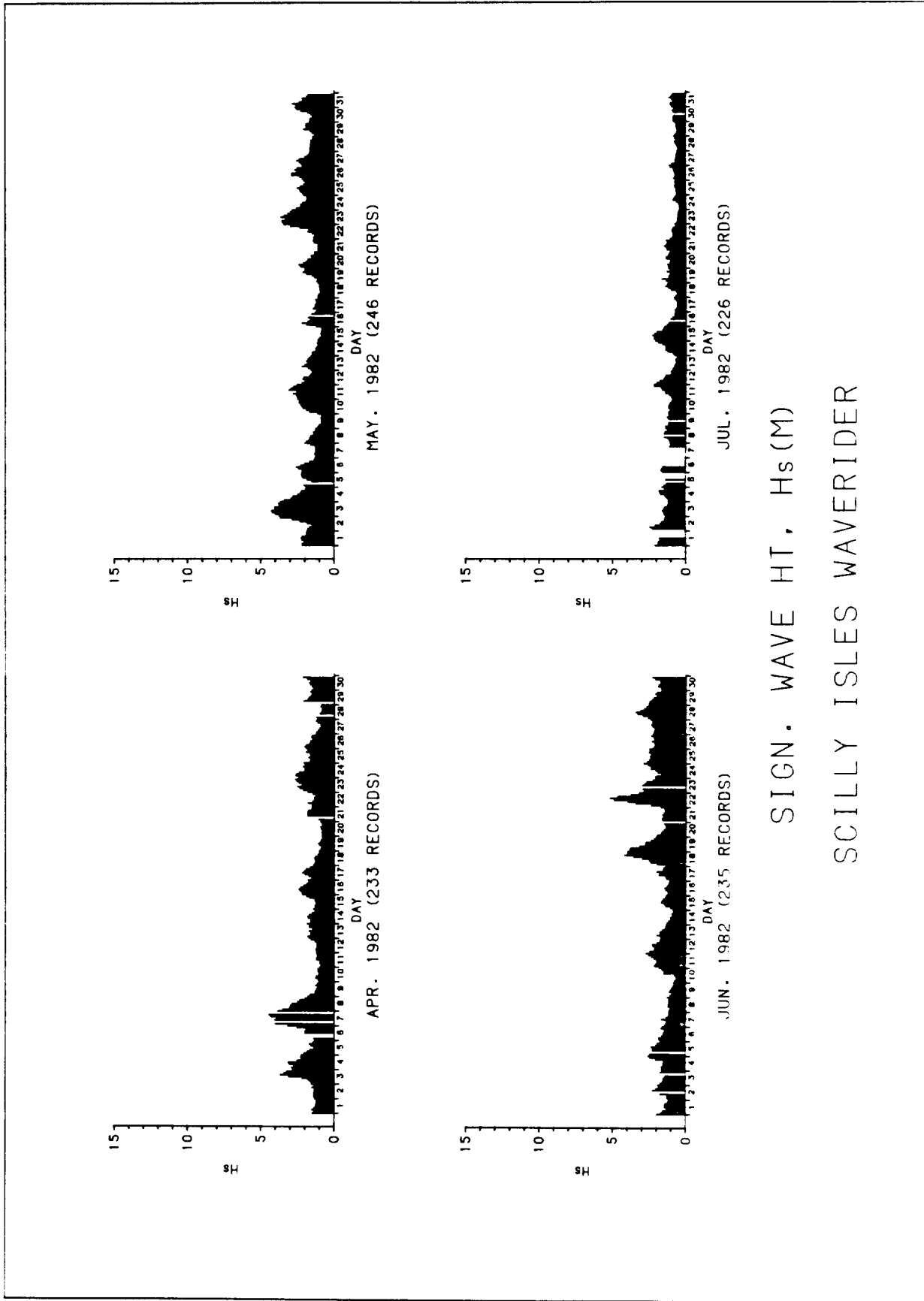
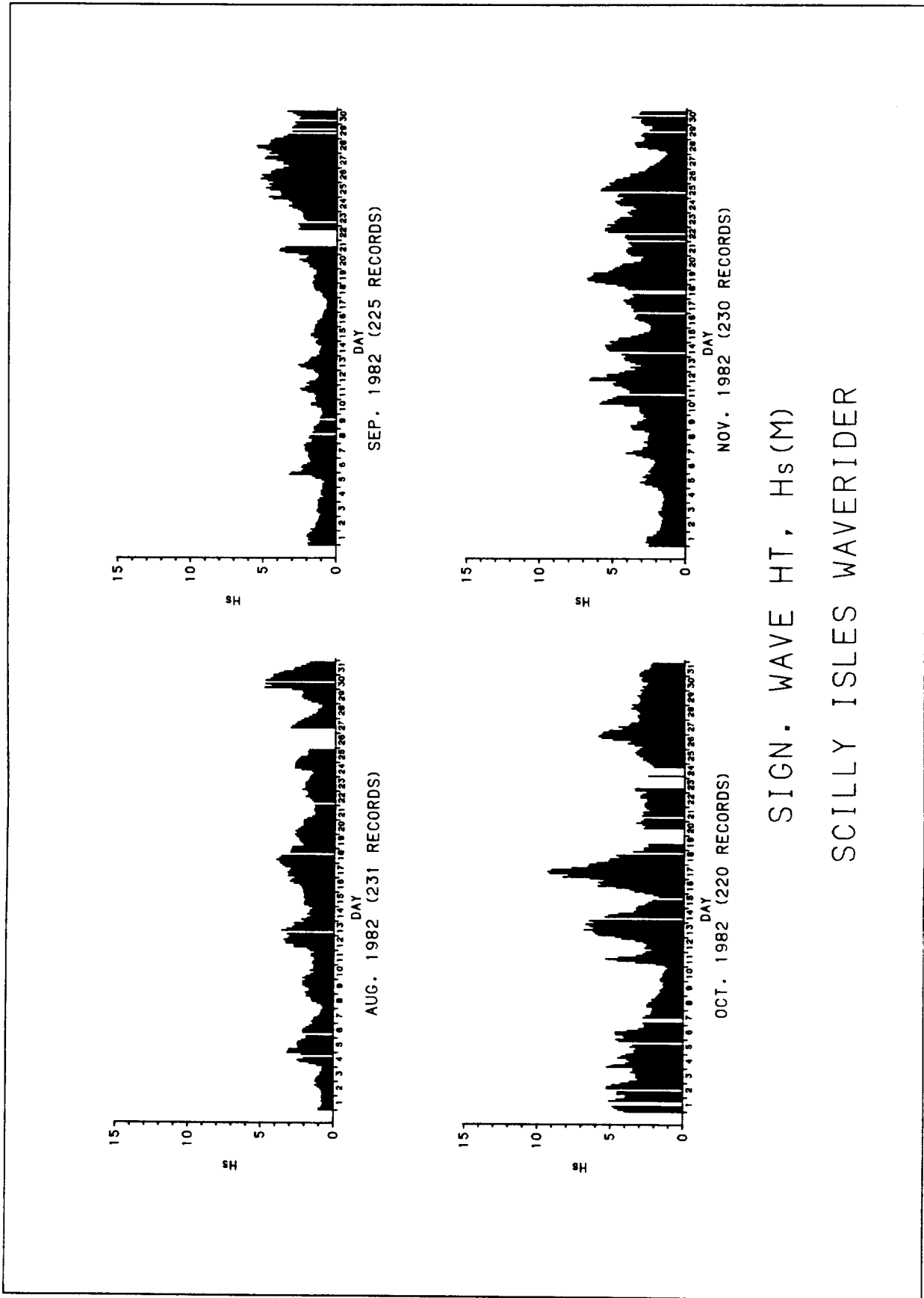


Fig.2(j)



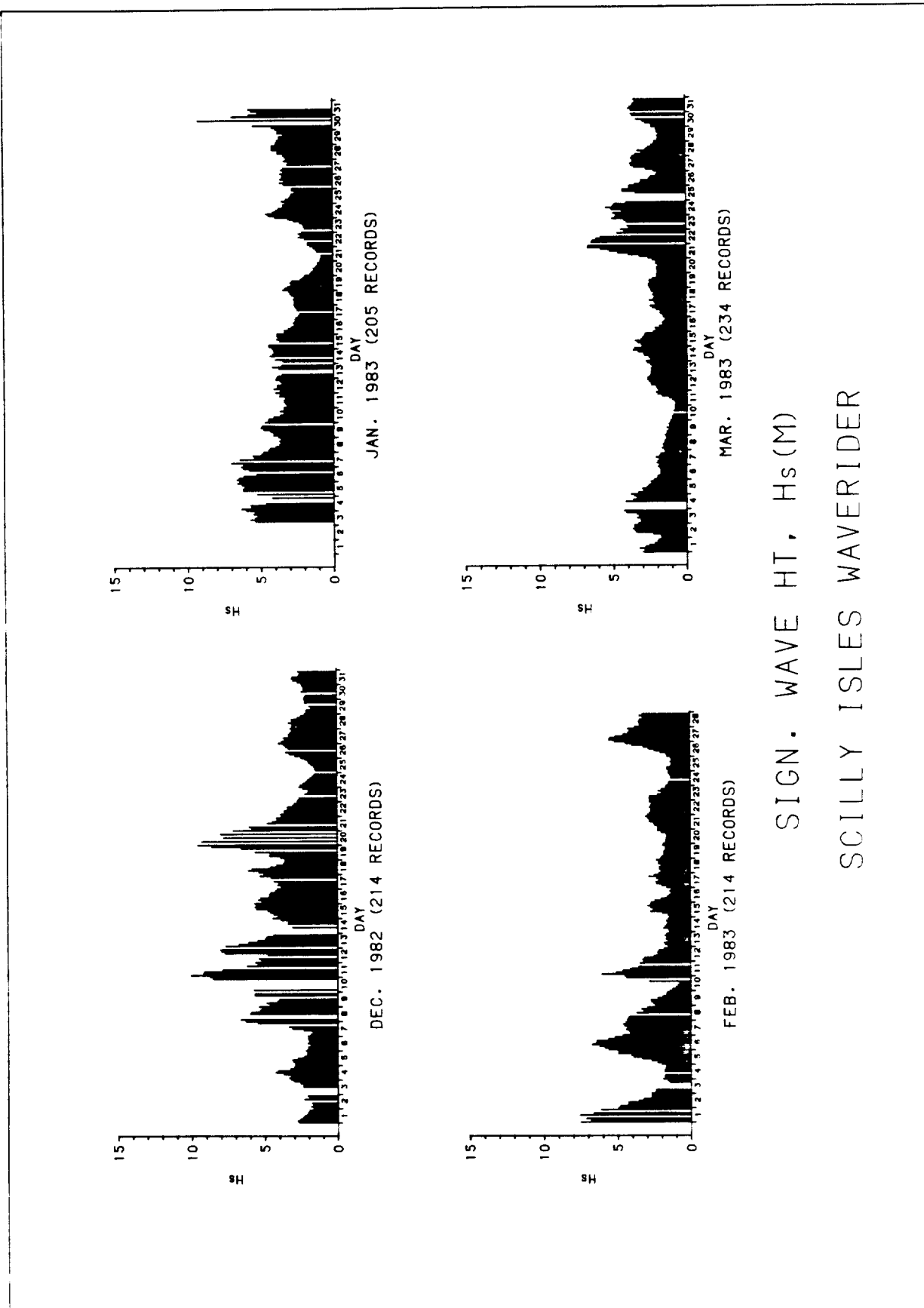
SIGN. WAVE HT. Hs(M)
SCILLY ISLES WAVERIDER

Fig.2(k)



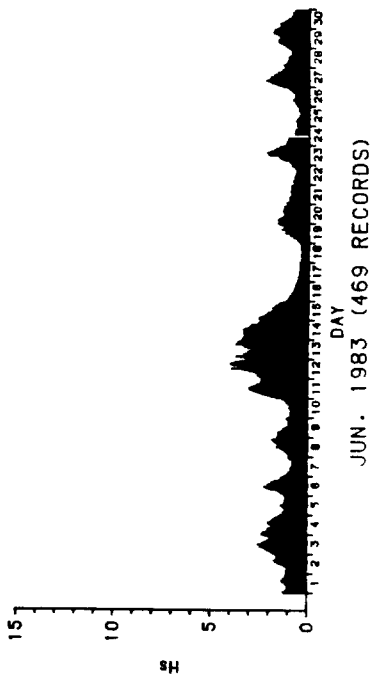
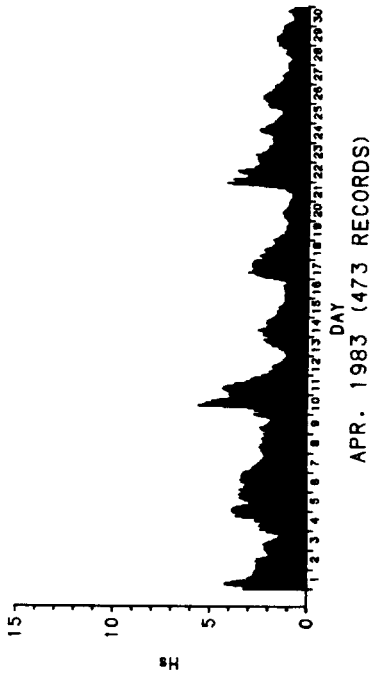
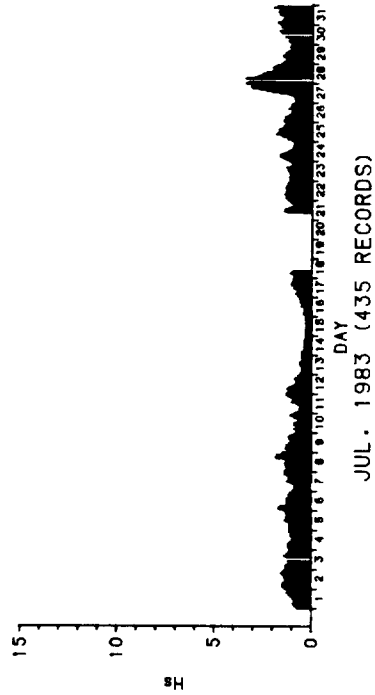
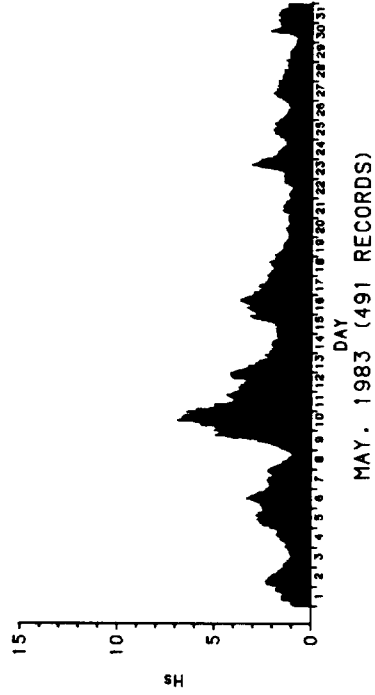
SIGN. WAVE HT, Hs(M)
SCILLY ISLES WAVERIDER

Fig.2(1)



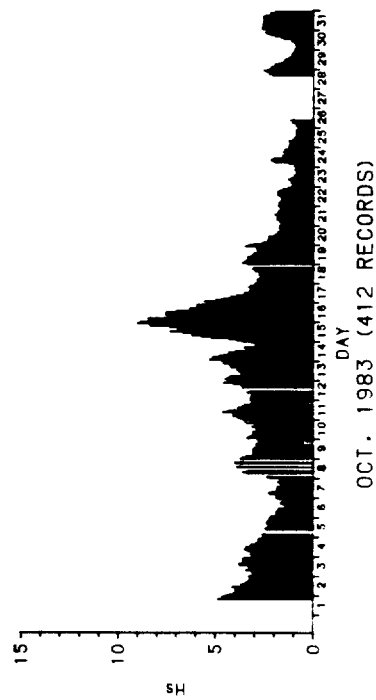
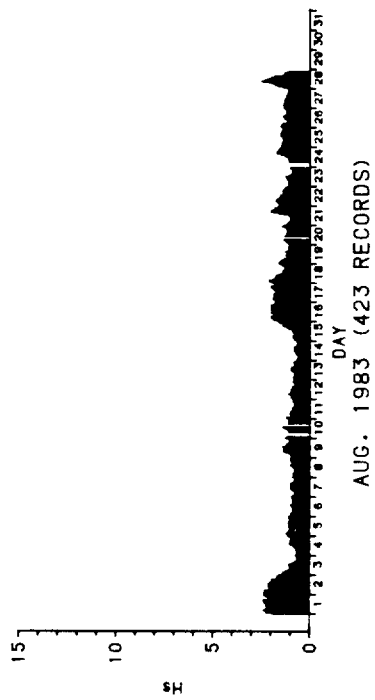
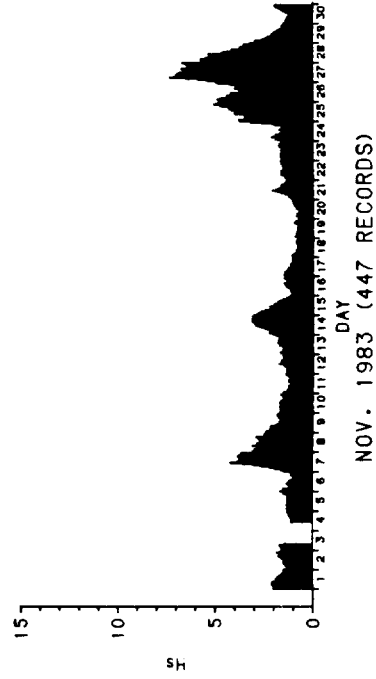
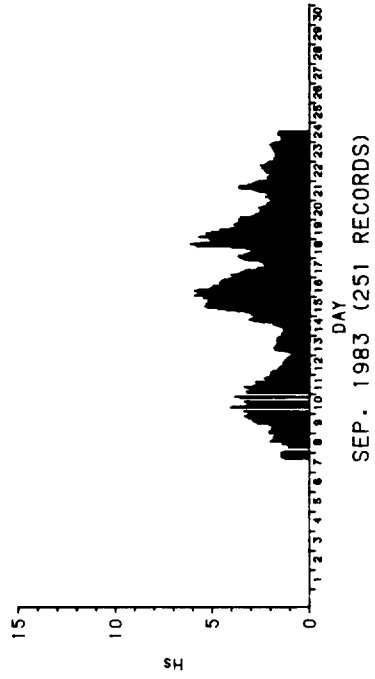
SIGN. WAVE HT, Hs(M)
SCILLY ISLES WAVERIDER

Fig.2(m)



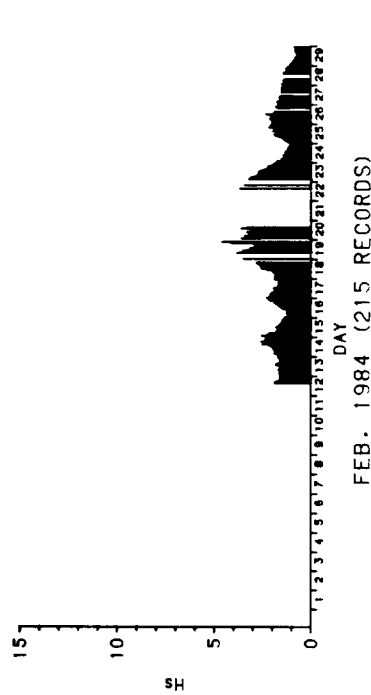
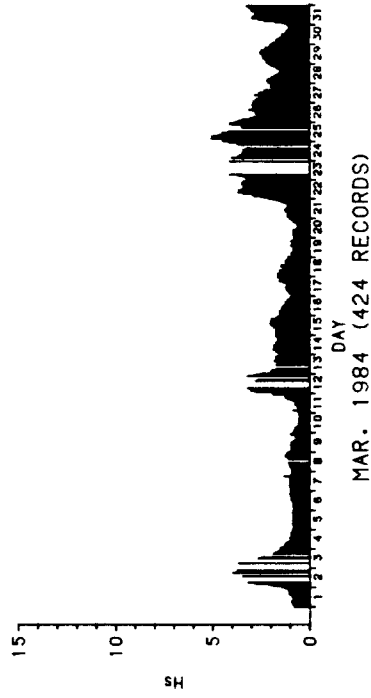
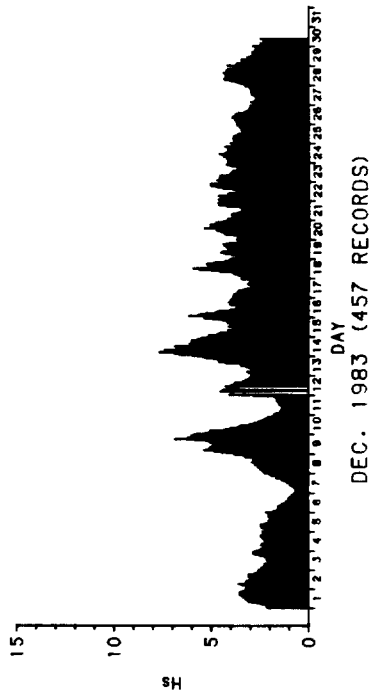
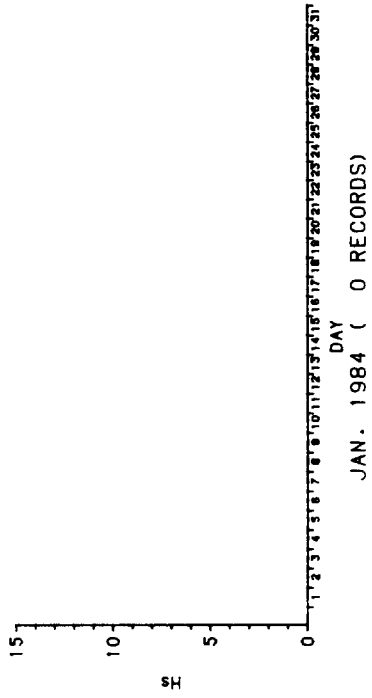
SIGN. WAVE HT, H_s(M)
SCILLY ISLES WAVERIDER

Fig.2(n)



SIGN. WAVE HT, Hs(M)
SCILLY ISLES WAVERIDER

Fig.2(o)



SIGN. WAVE HT, Hs(M)
SCILLY ISLES WAVERIDER

Fig.2(p)

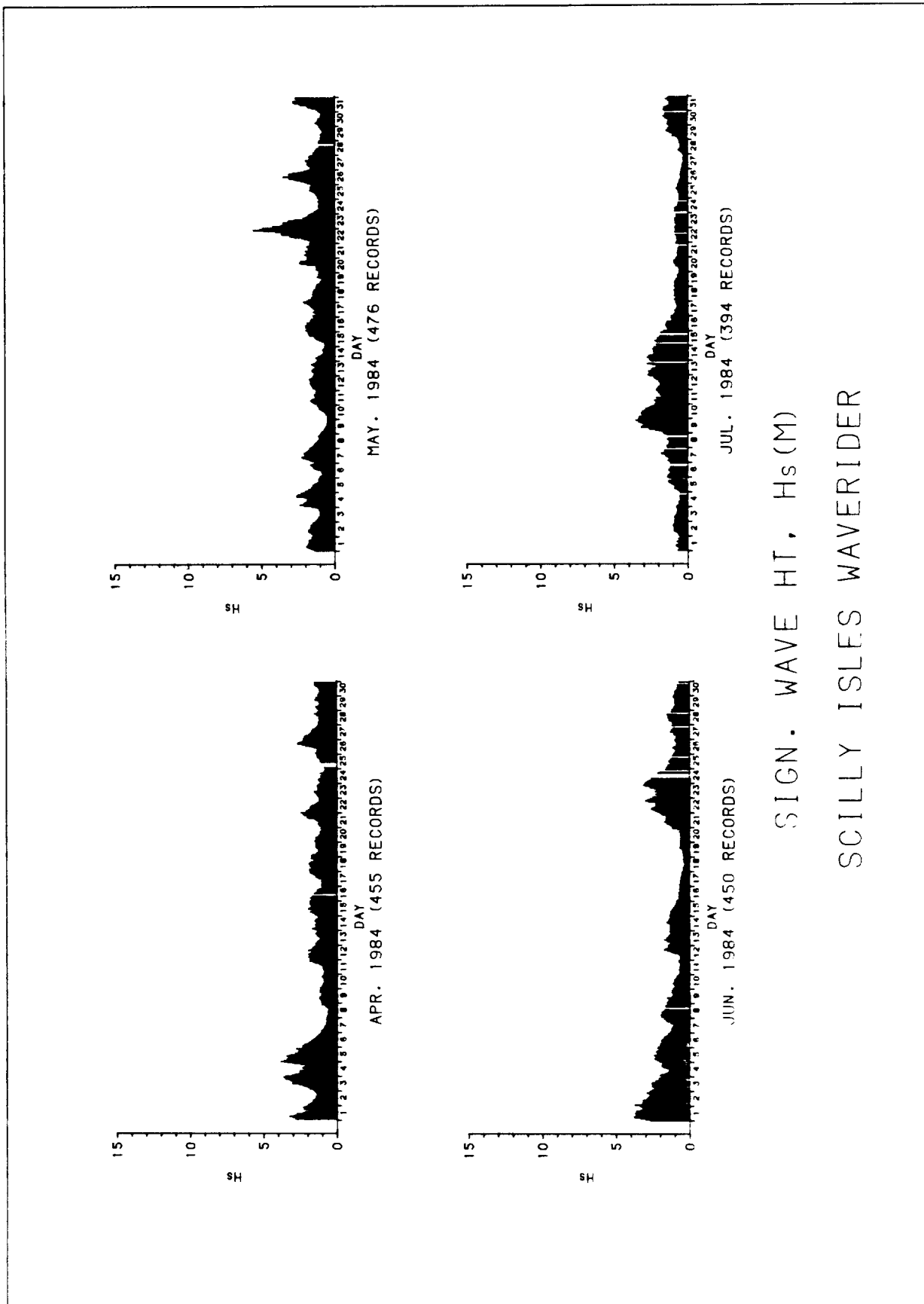


Fig.2(q)

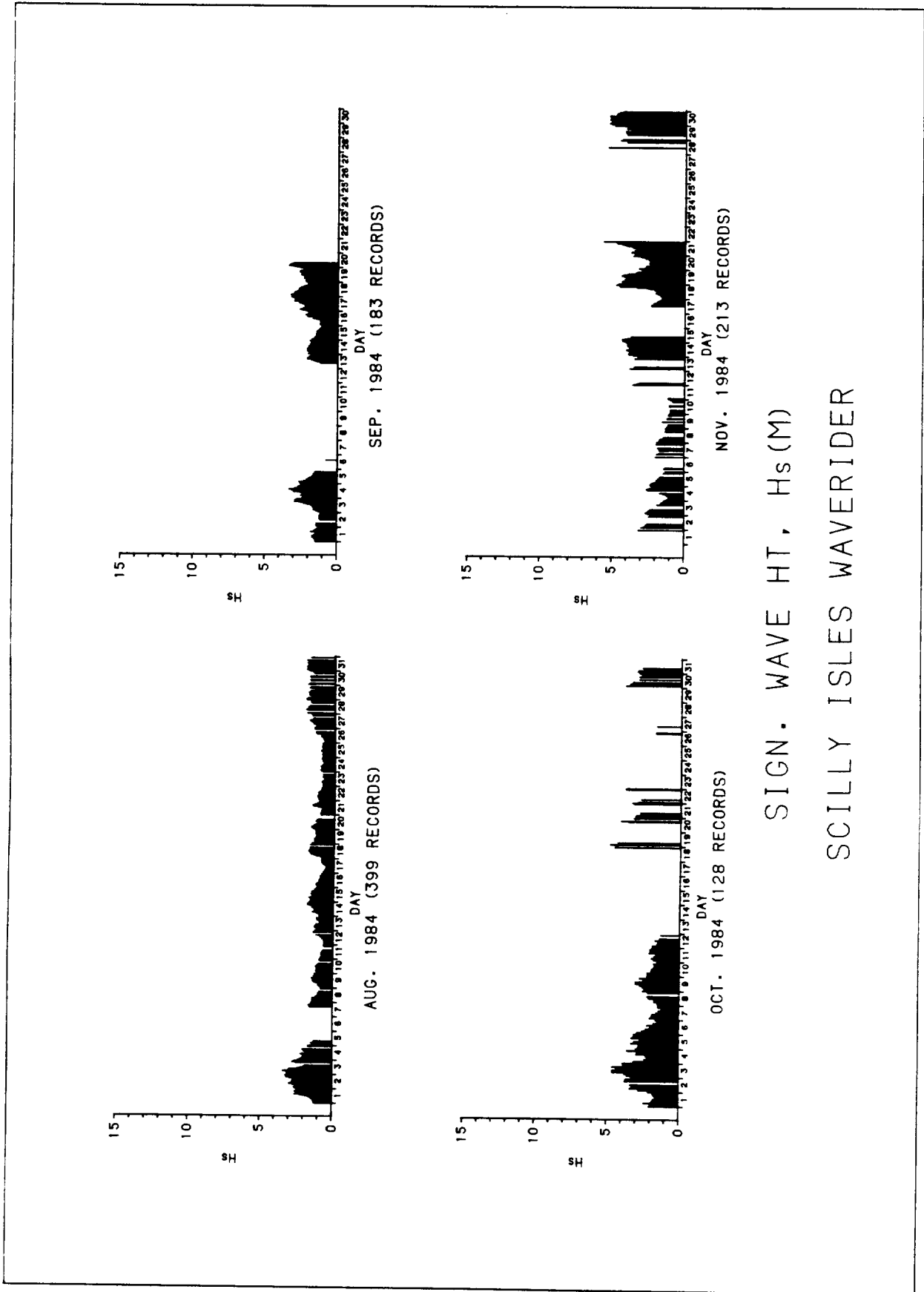
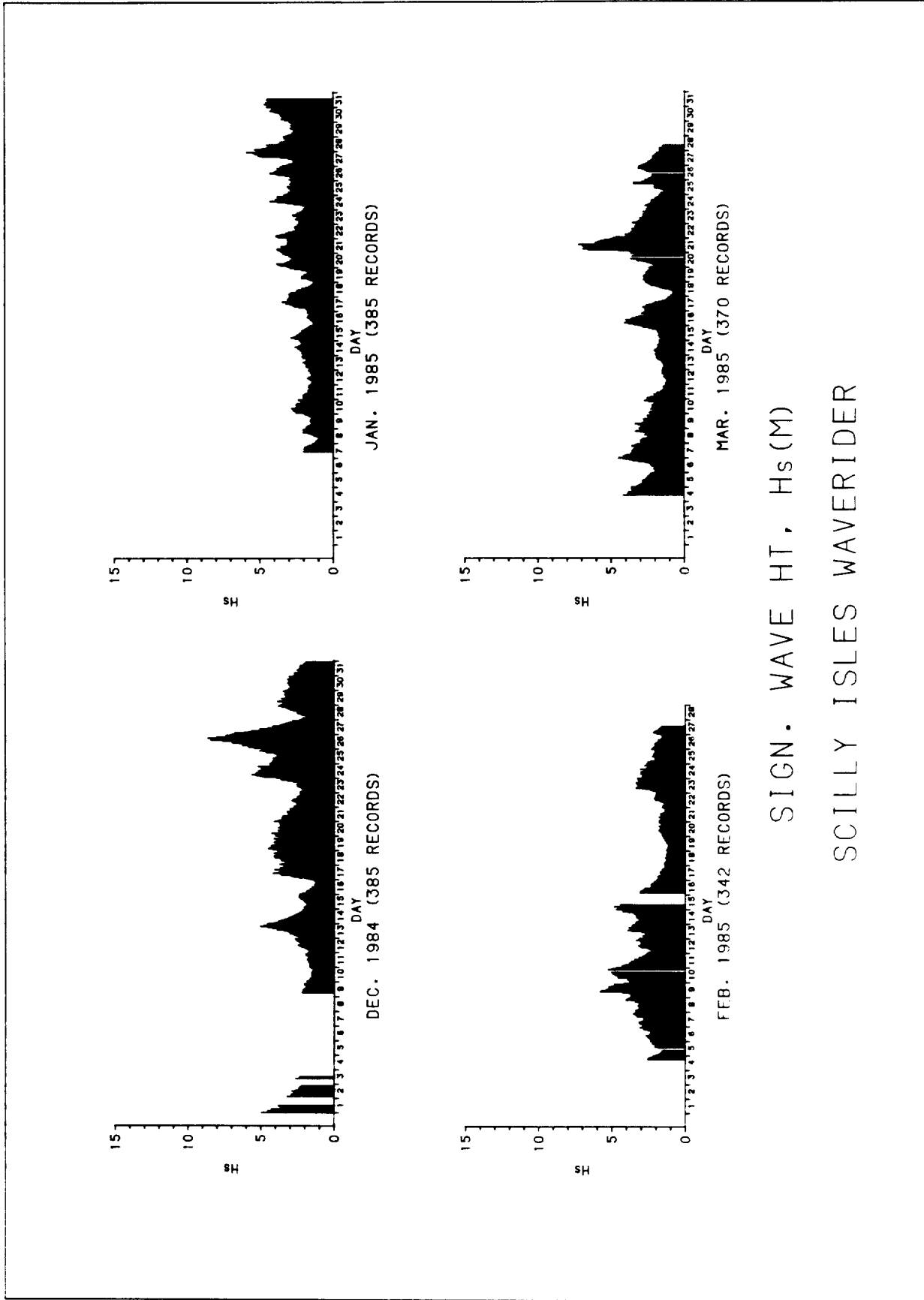
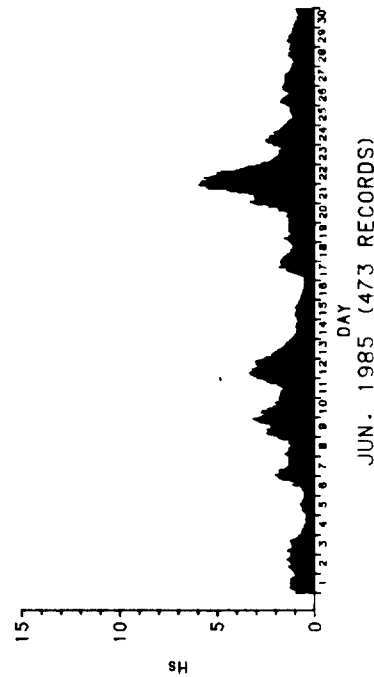
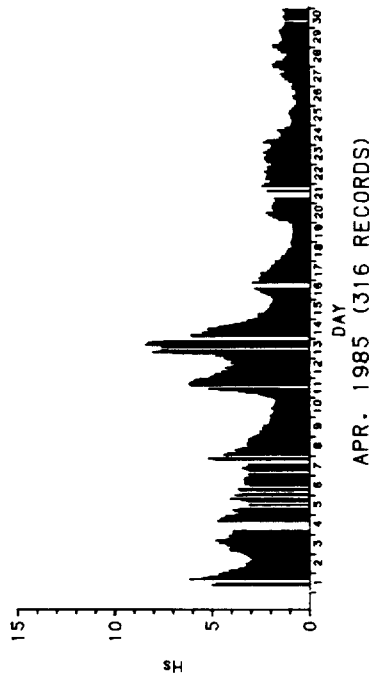
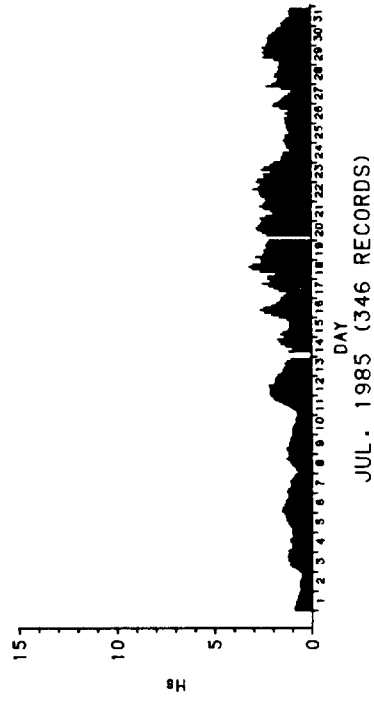
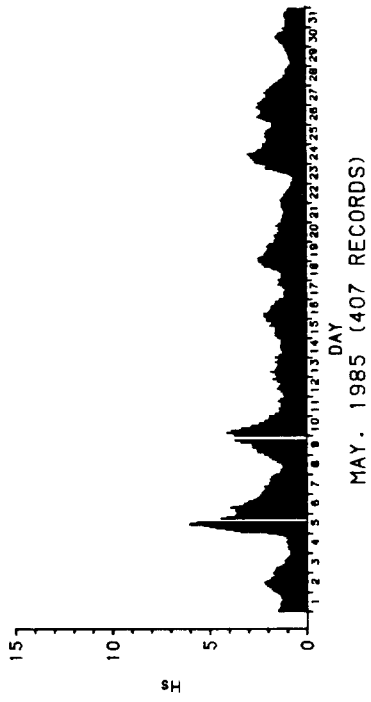


Fig.2(r)





SIGN. WAVE HT, Hs(M)
SCILLY ISLES WAVERIDER

Fig.2(t)

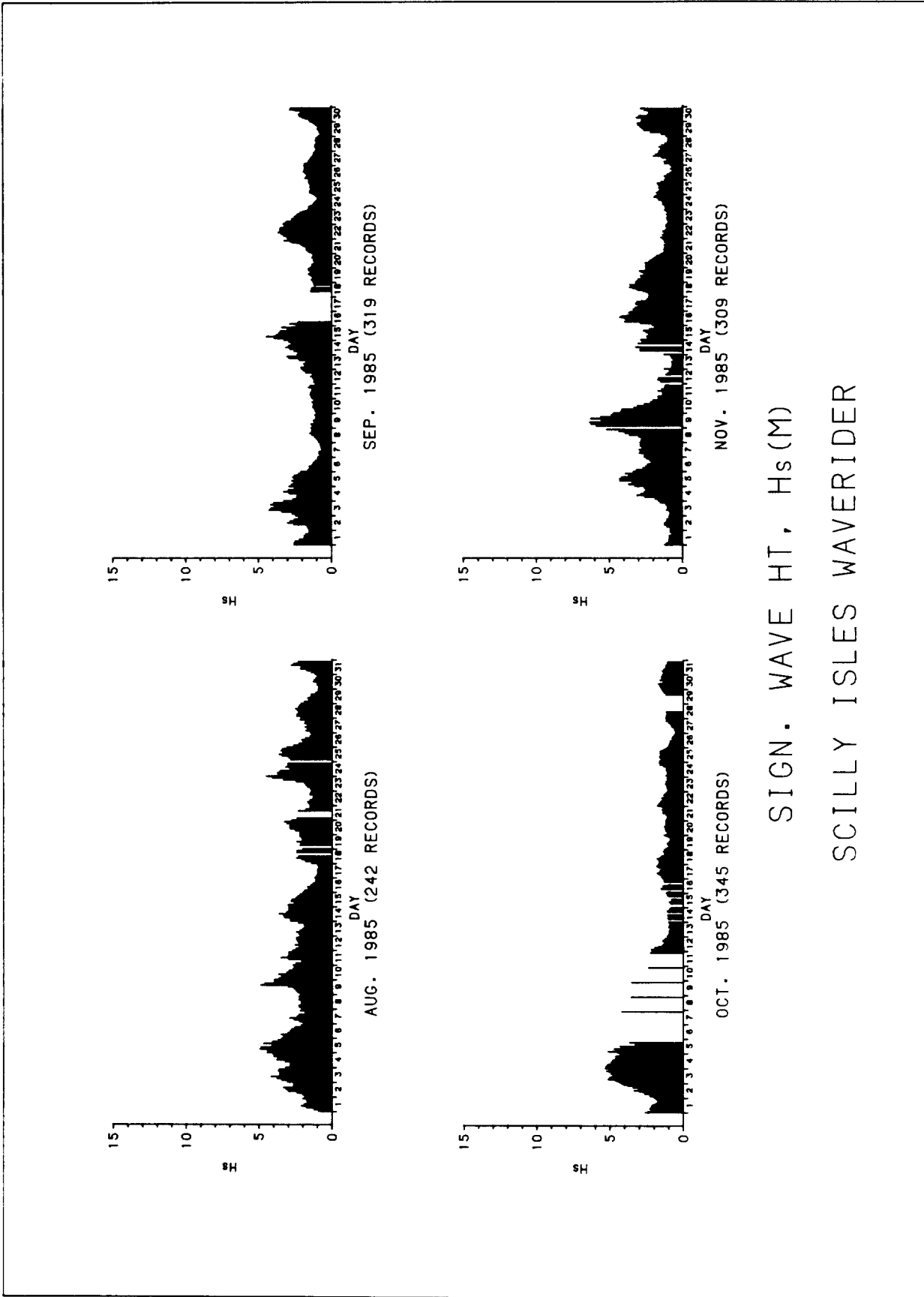
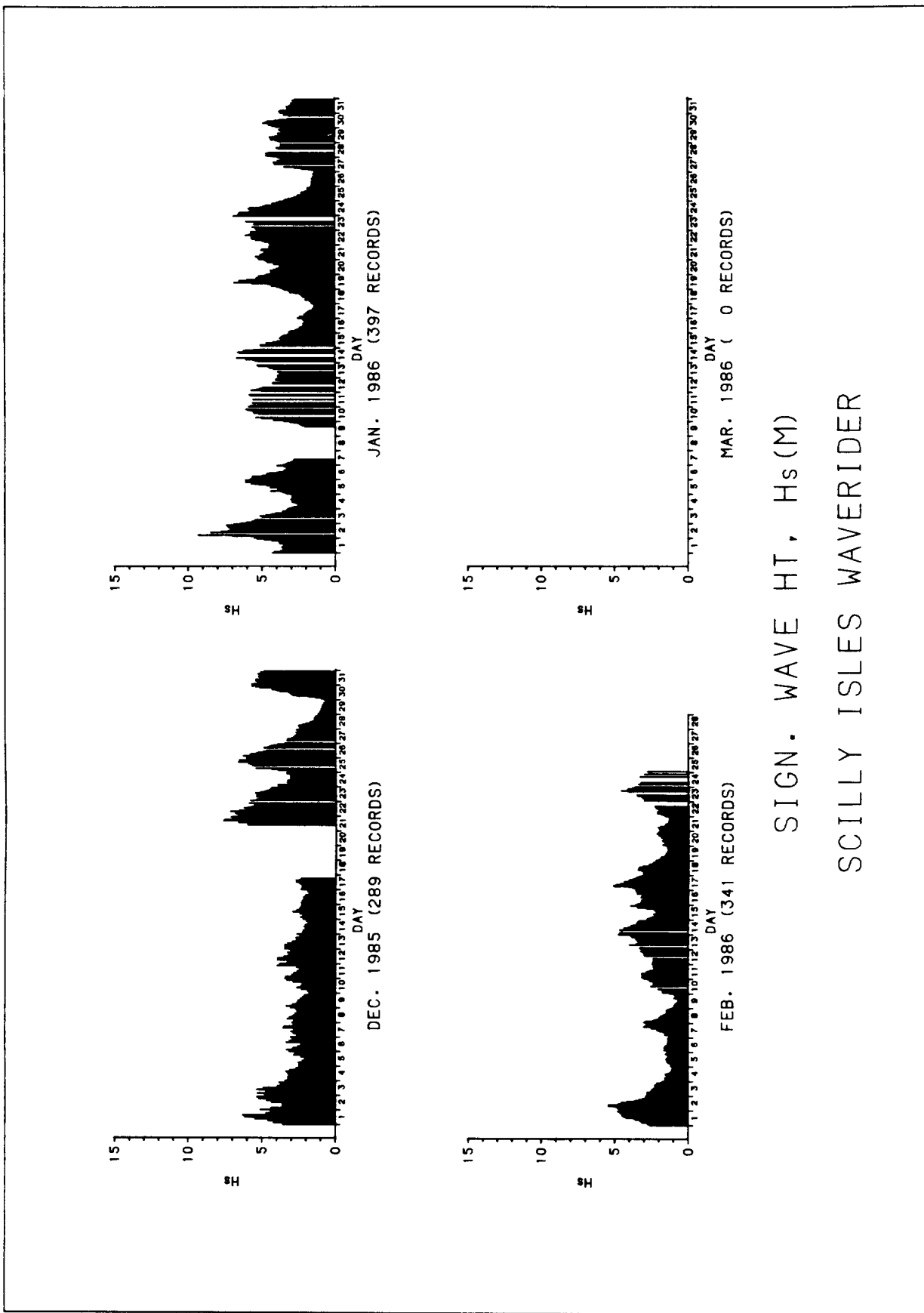
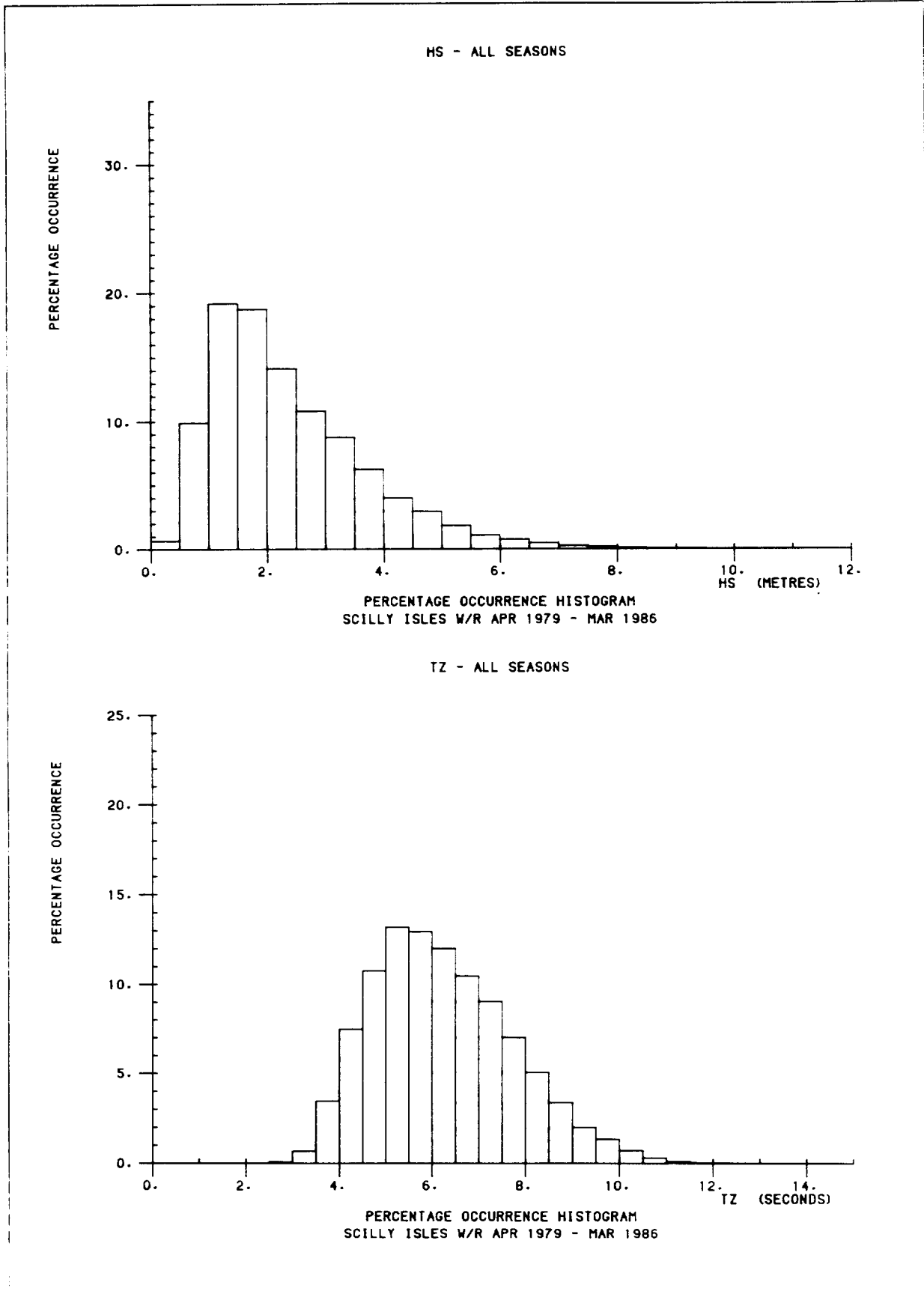
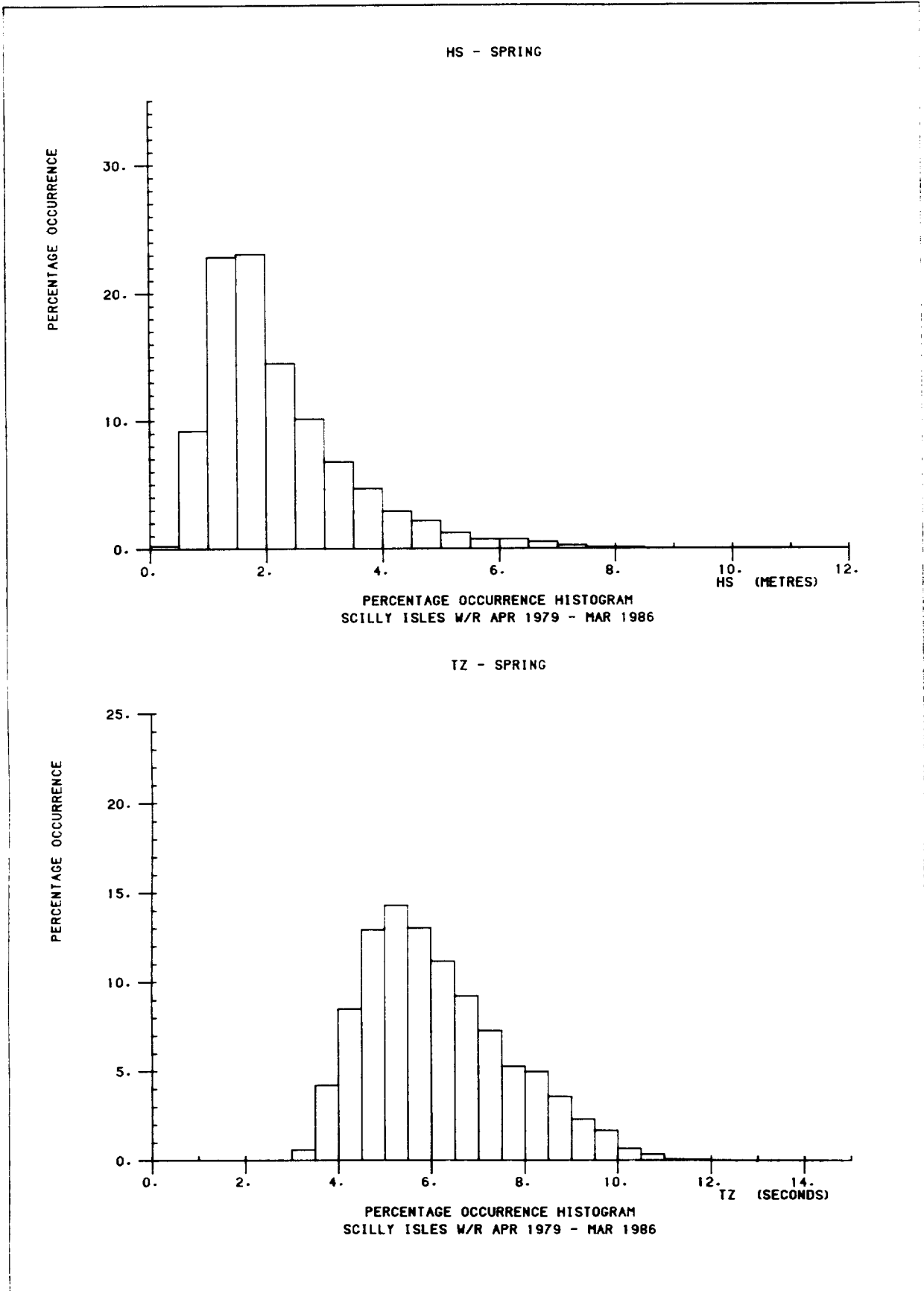


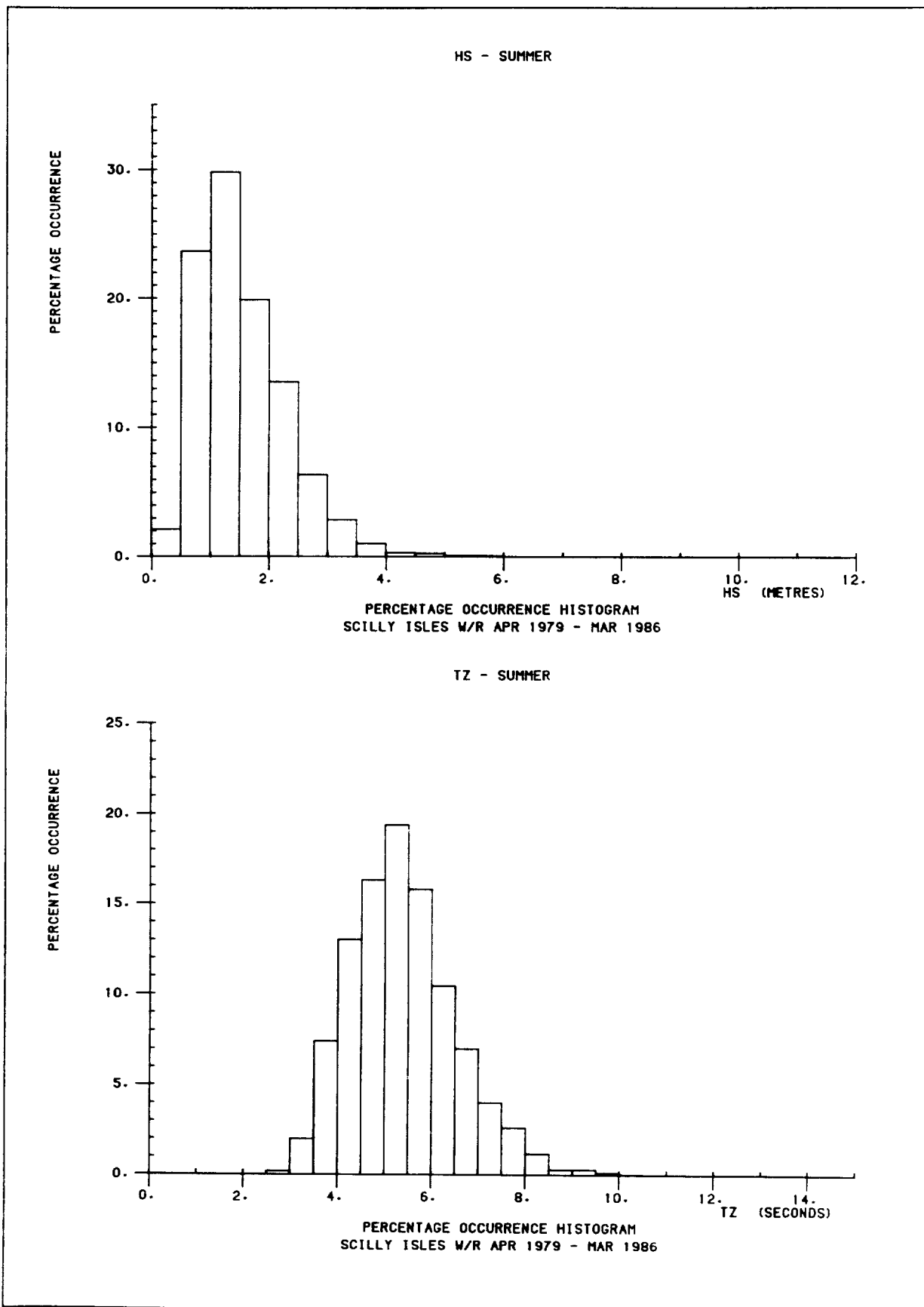
Fig.2(u)



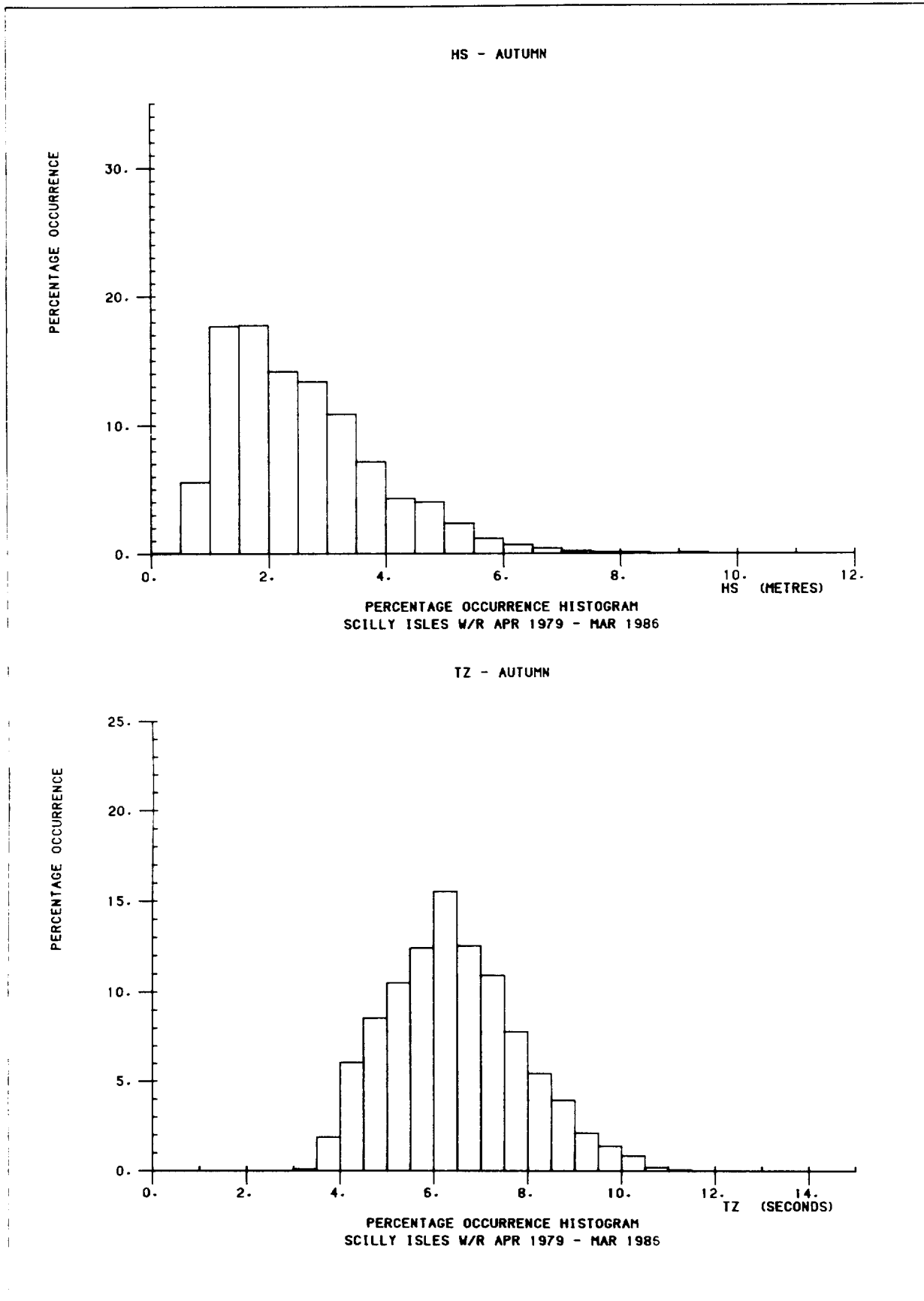
SIGN. WAVE HT, Hs(M)
SCILLY ISLES WAVERIDER







Figs.3(g) & (h)



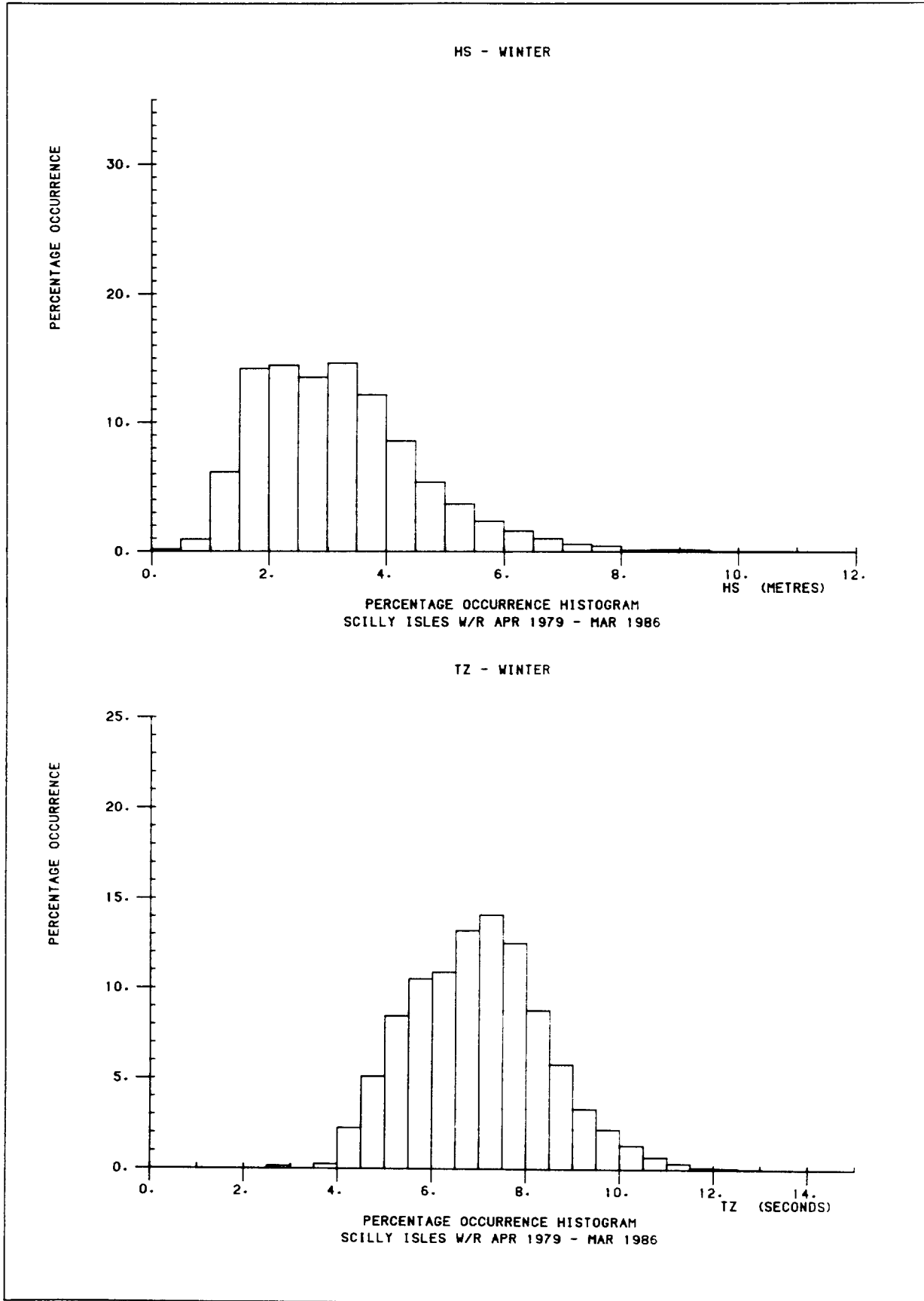
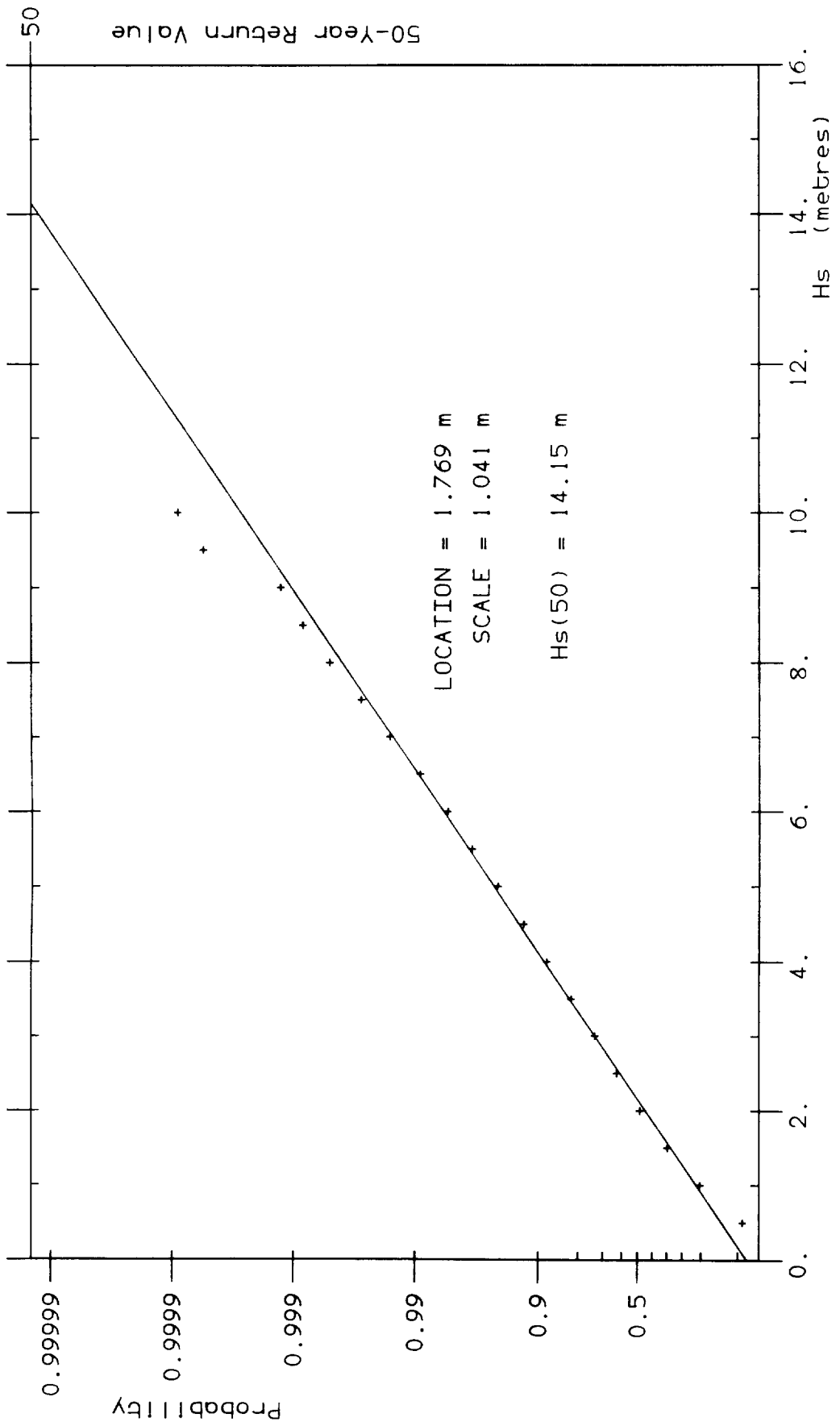
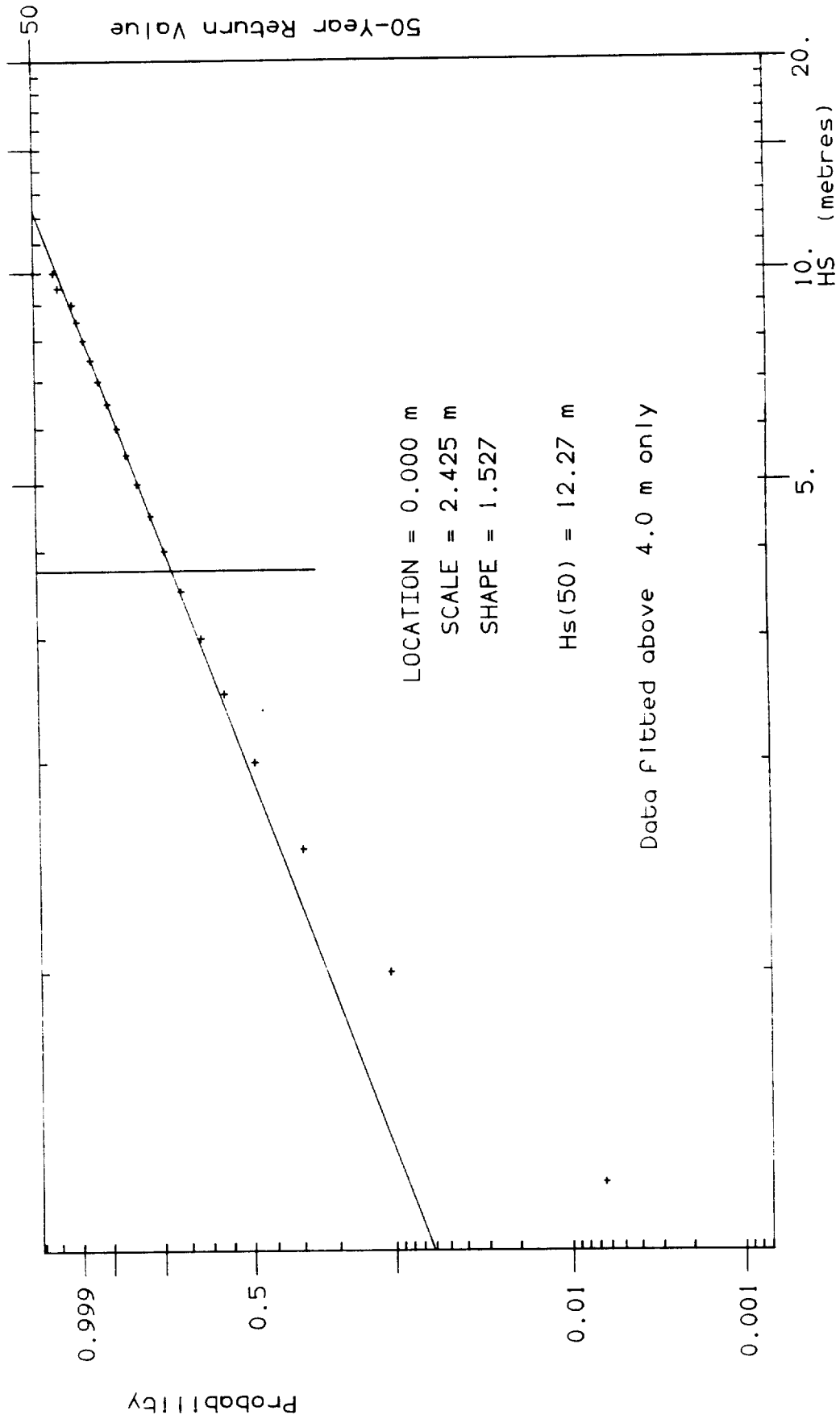


Fig.4



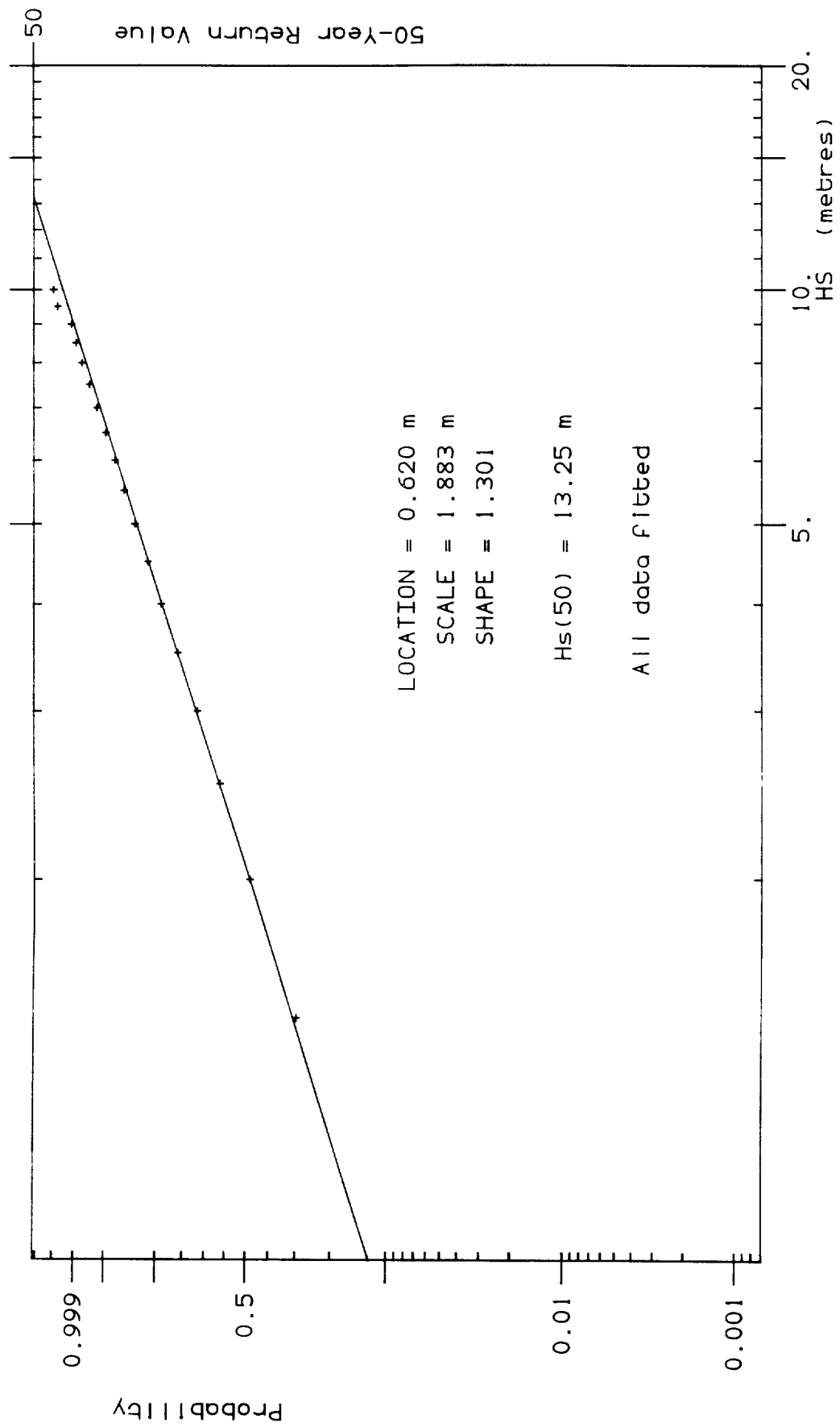
CUMULATIVE HS PROBABILITY on FT-1 SCALE

Fig.5



CUMULATIVE HS PROBABILITY on WEIBULL SCALE

Fig.6



CUMULATIVE HS PROBABILITY ON WEIBULL SCALE

Fig.7

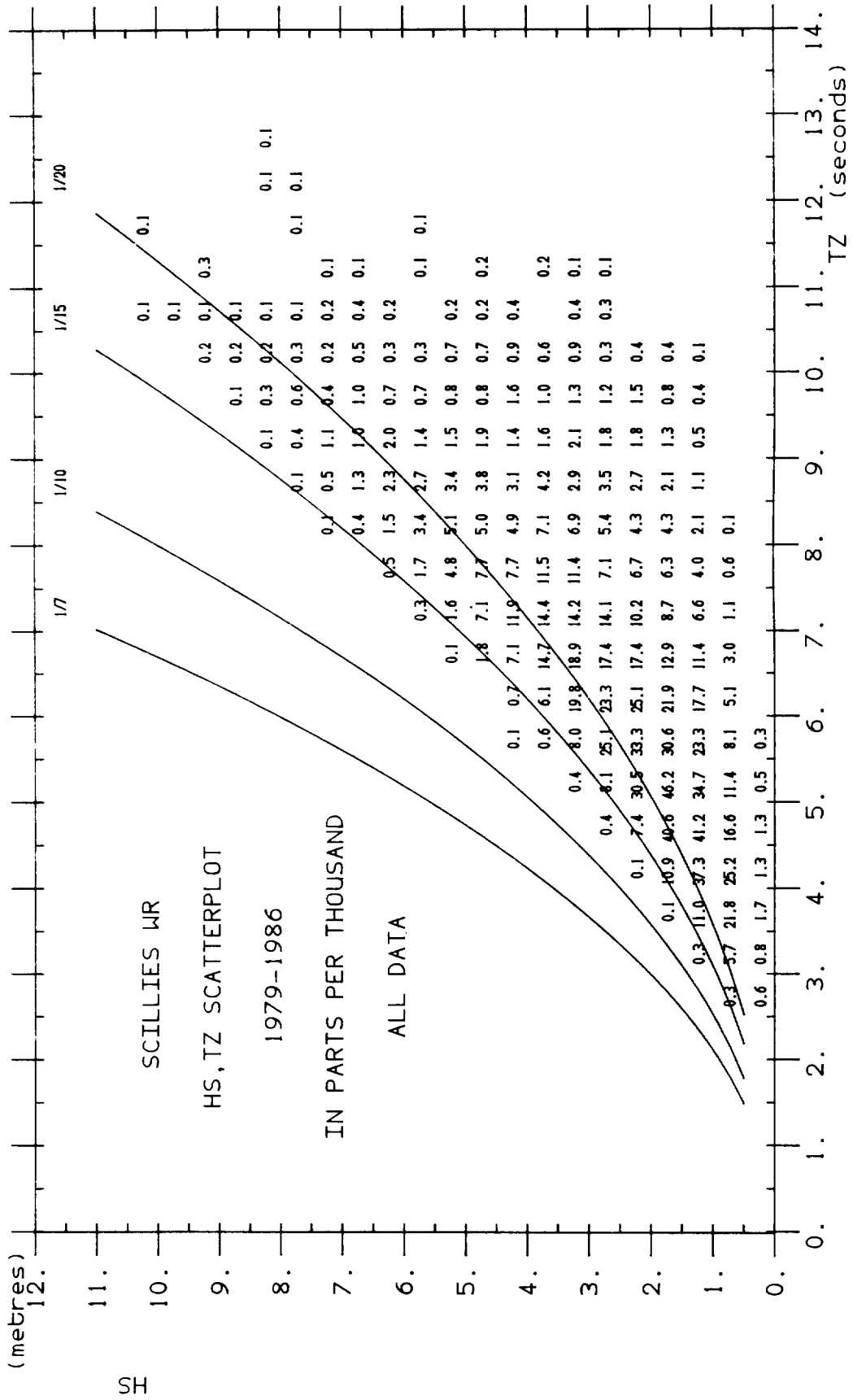


Fig.8

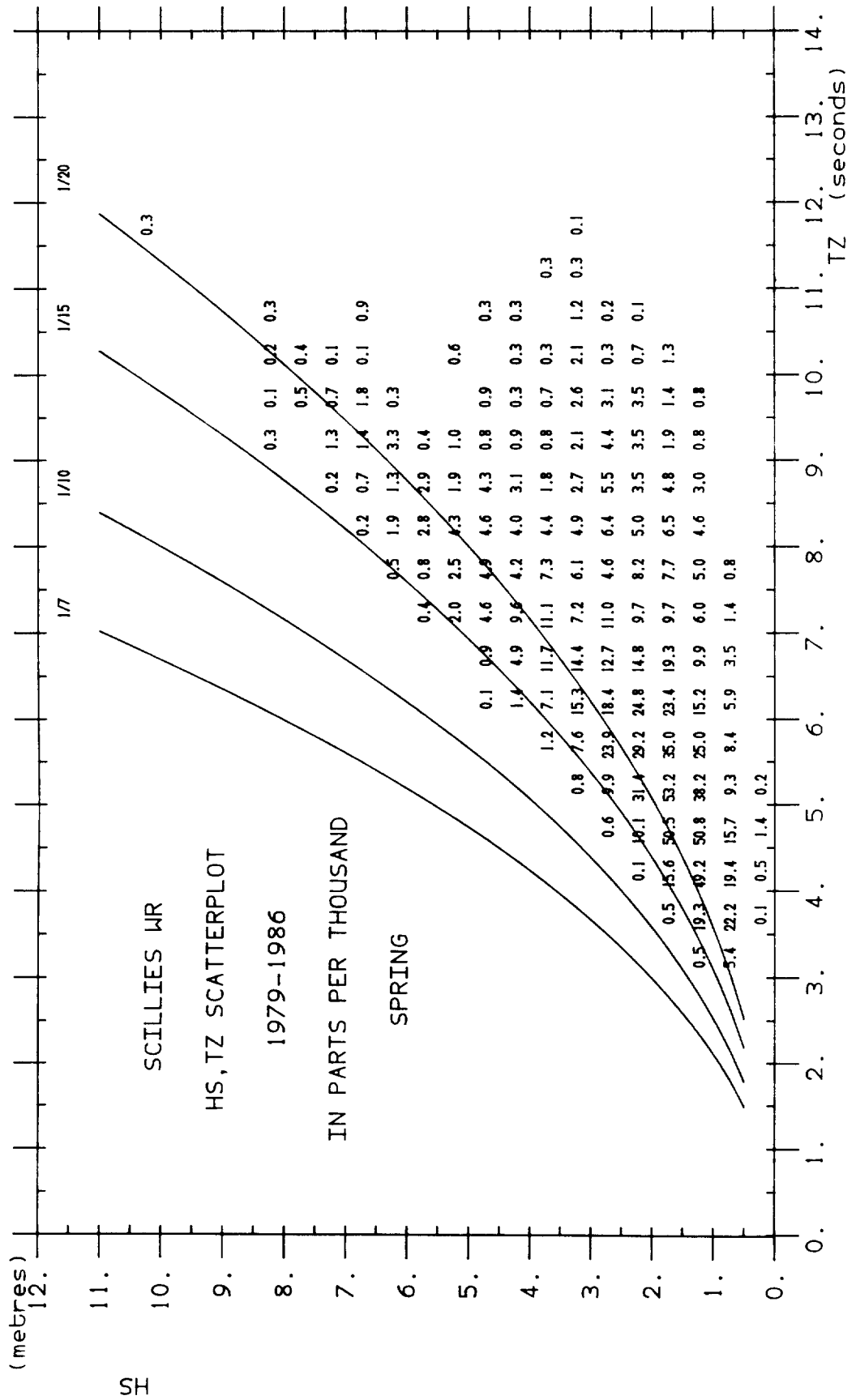


Fig.9

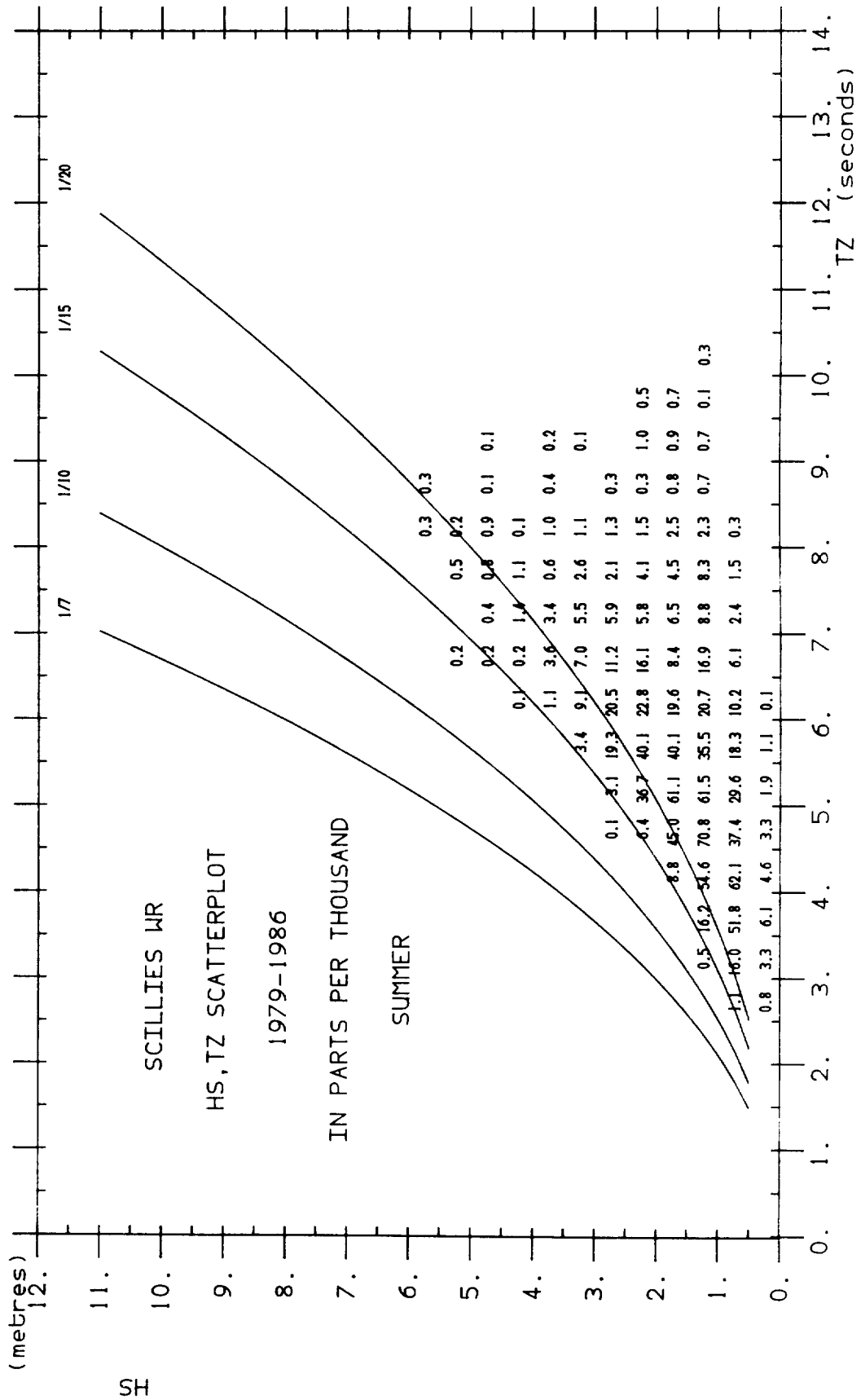


Fig.10

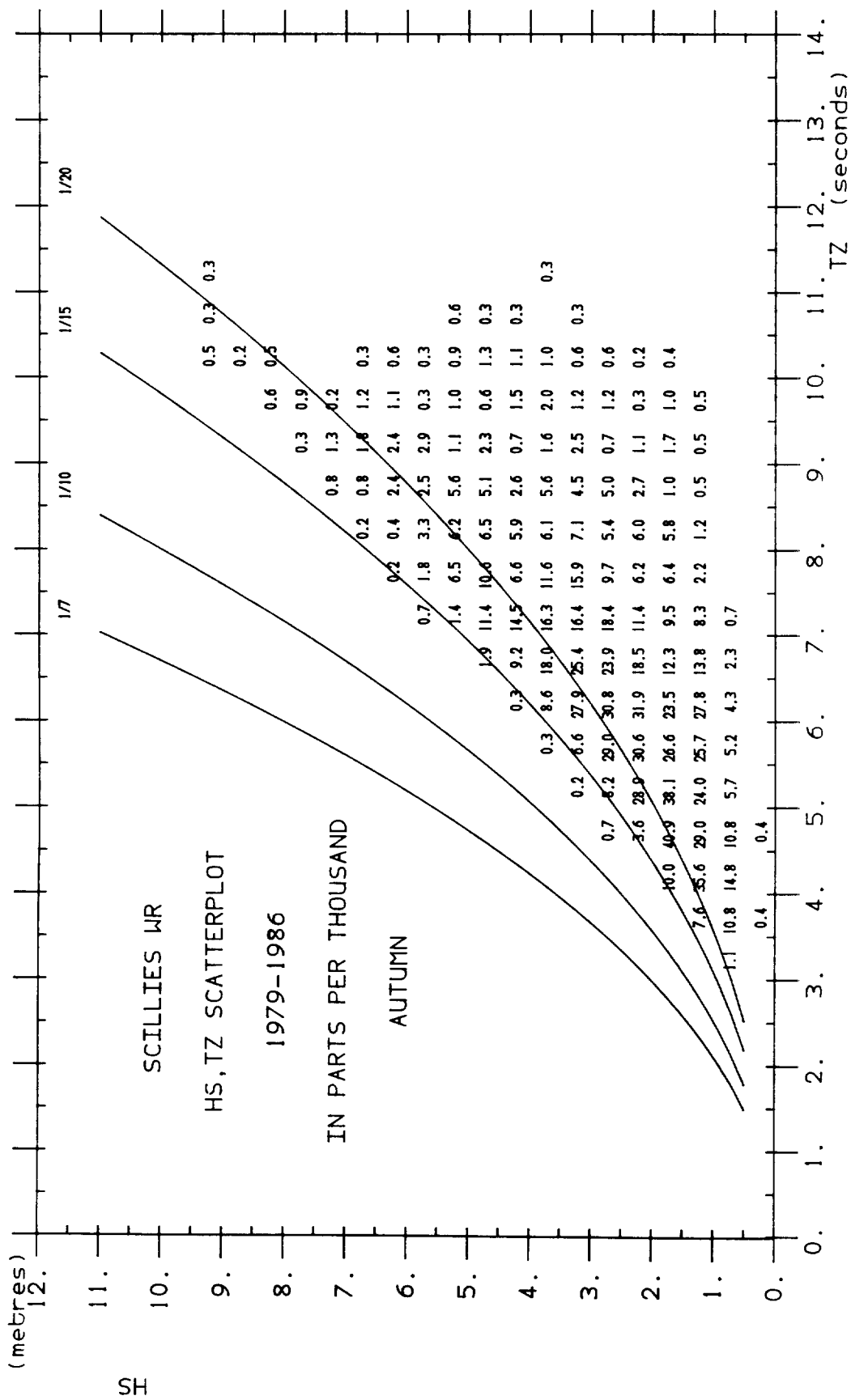
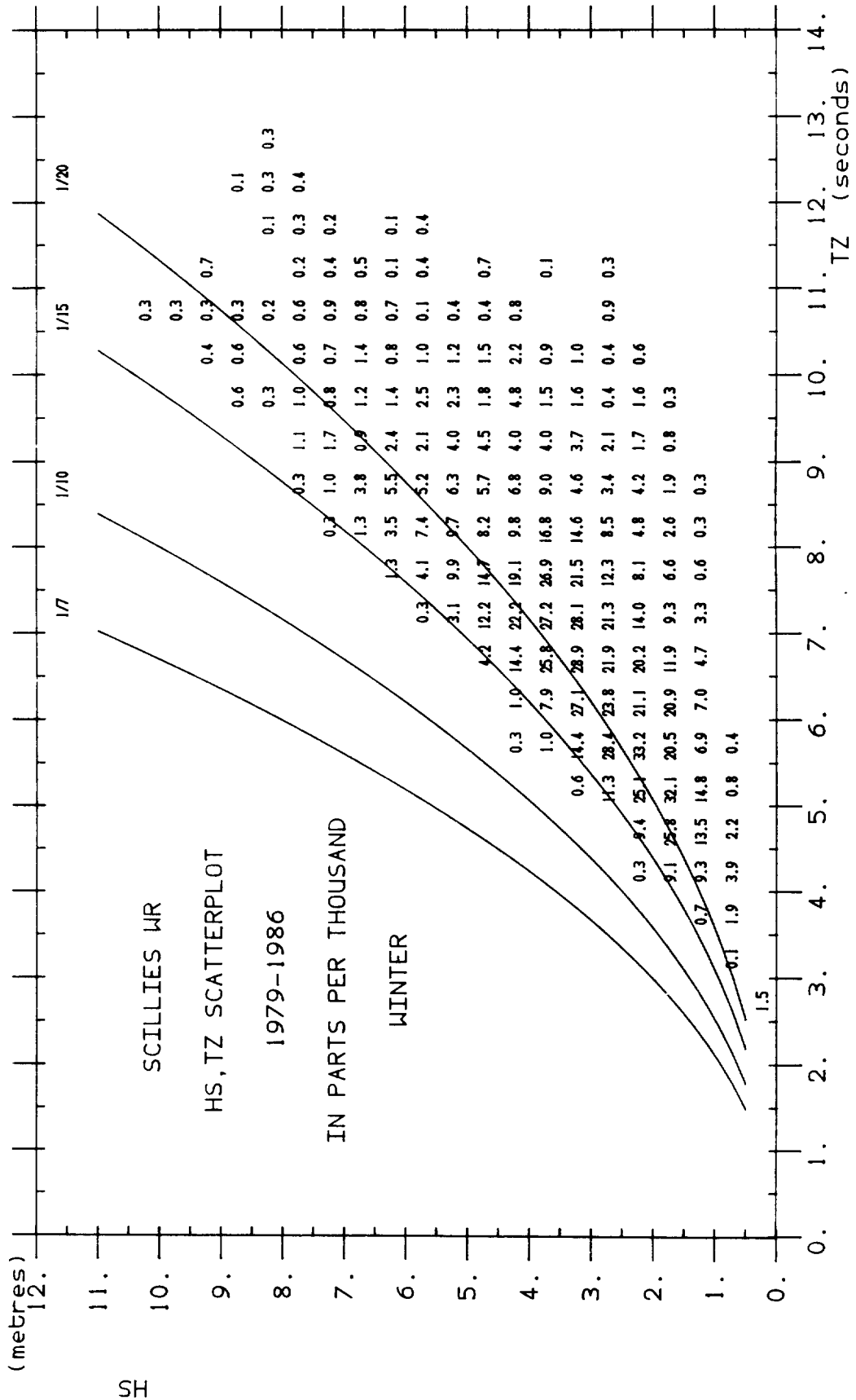
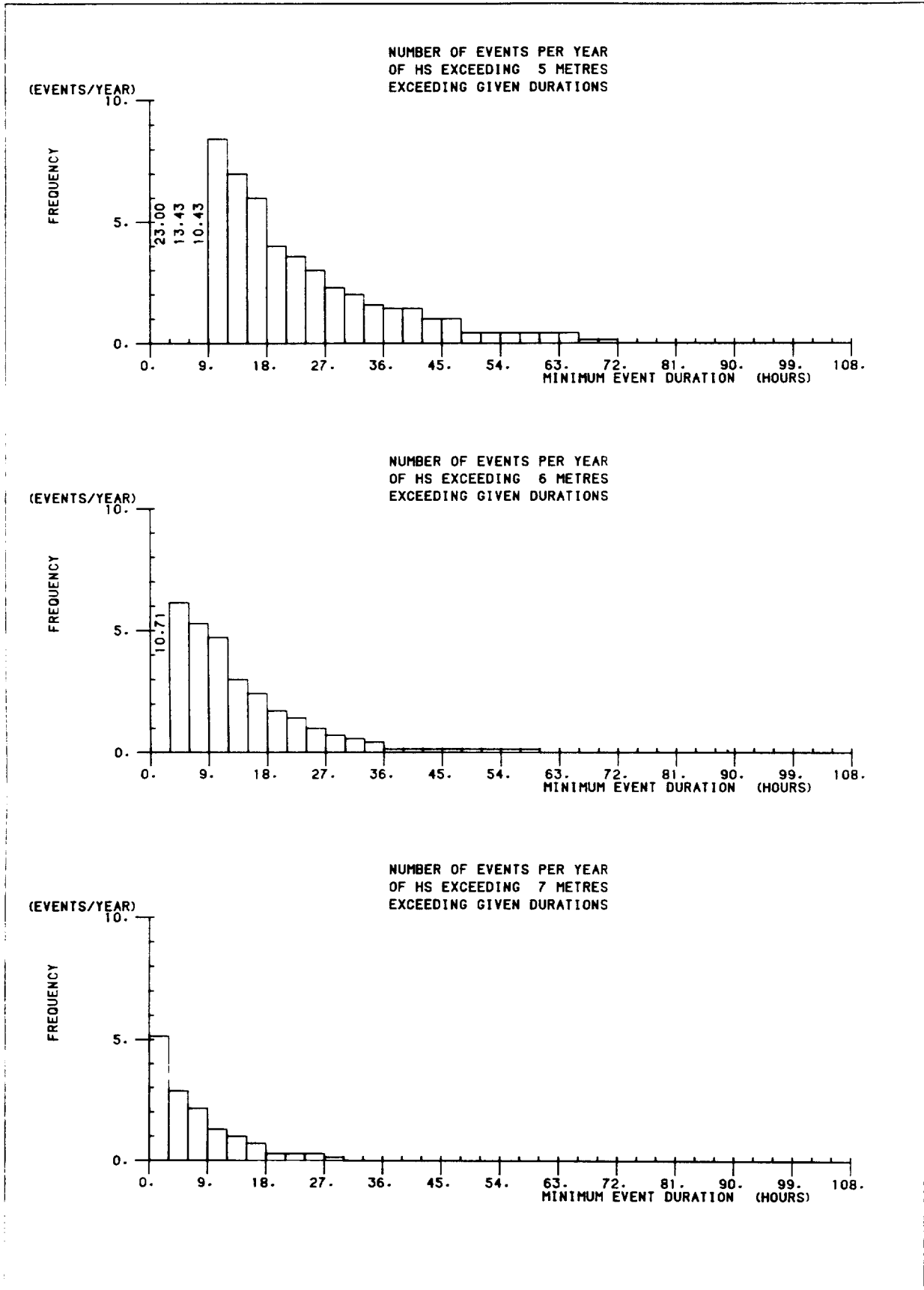


Fig.11



Figs.12(a), (b) & (c)



Figs.12(d), (e) & (f)

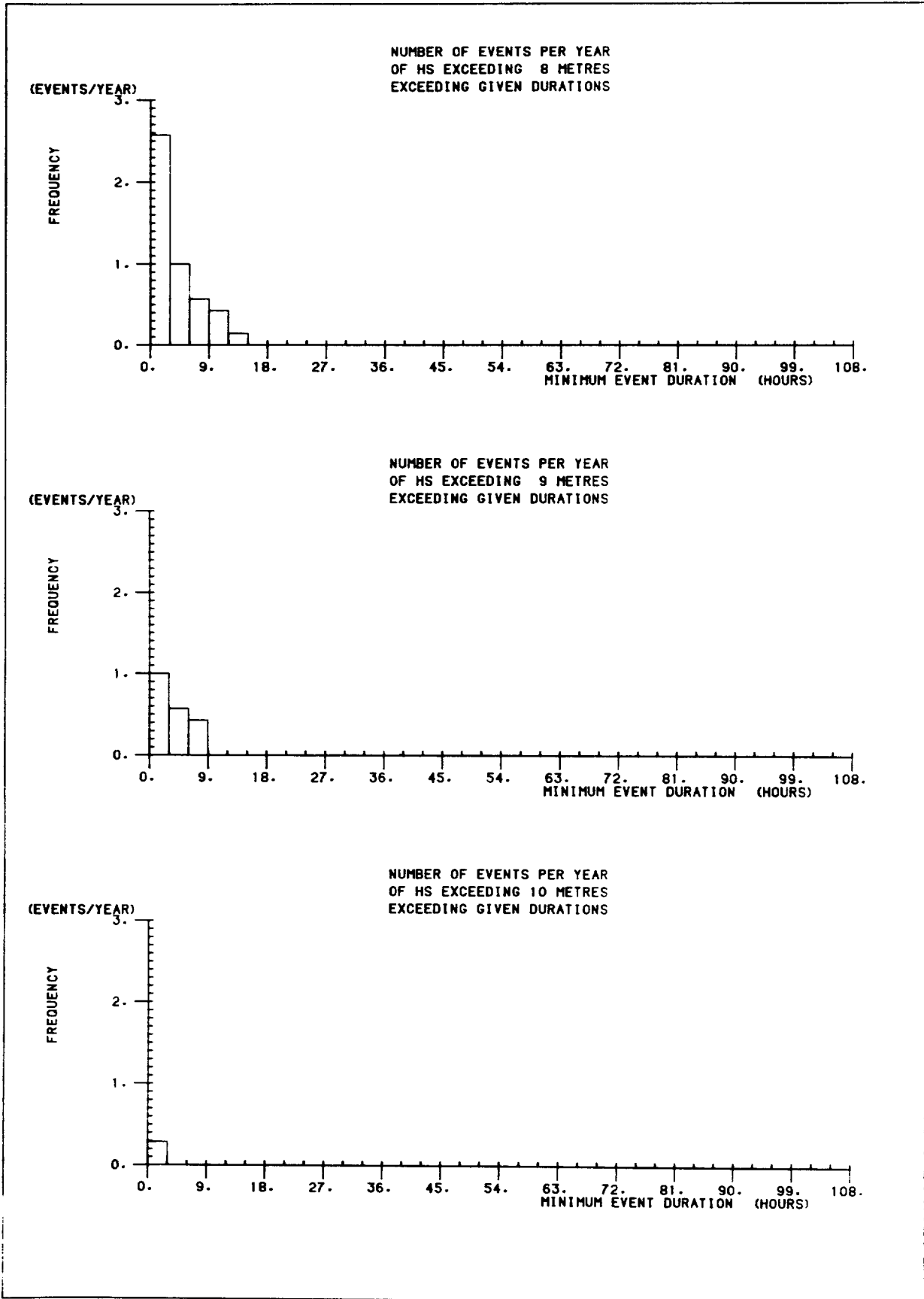


Fig.13

