

I.O.S.

WAVES RECORDED AT ST. GOWAN LIGHT VESSEL

by
B C H FORTNUM

**Data for August 1975 to July 1976
and January 1977 to December 1978
at position 51°31'N, 005°00'W**

Summary Analysis and Interpretation Report

**Report No 125
1981**

**NATURAL ENVIRONMENT
INSTITUTE OF OCEANOGRAPHIC
SCIENCES
RESEARCH COUNCIL**

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in it have been financed by the Departments of Energy and of Industry

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1. INTRODUCTION

1.1 Site description

The site at which the wave measurements were taken is shown on the map in figure 1.1. It is approximately 10 kilometres south of the south-western tip of Wales, at position $51^{\circ}30.5'N$, $004^{\circ}59.8'W$. Although it is moored in water of depth about 49 metres, there are several shoals in its vicinity; in particular a shoal to the north-east has a water depth above it of about 7 metres. The tidal currents in the area are quite strong, the maximum current being over 2 knots (1 metre/sec), with directions of approximately 100° and 280° . Since the vessel is stationed close to several shoals, it is probable that it experiences even stronger currents, which may occasionally cause an increase in the steepnesses of the waves to an unusually high level. This effect would be most pronounced for short, low-period waves.

1.2 Description of measuring and recording systems

The vessel is fitted with a shipborne wave recorder as described in TUCKER (1956), which provides information about the water surface elevation. This information is recorded for a 12-minute period (usually) once every 3 hours by a pen on paper chart rolls. The expression for the correction factor which is applied to the wave height data is given in Appendix IV, together with a table of its values for various values of T_z , using 1.62 metres as the depth of the pressure transducer on the vessel. The correction factor is greater than 2 for T_z of 4.3 seconds and less. Since the expression for the correction factor is rather inaccurate for these low values of T_z , data with T_z values of 4.5 seconds and less are not considered to be reliable. However all the data have been included in the presentations in this report, with a warning shown on those figures which, if used without regard for this limitation on the data, might yield misleading information.

1.3 Details of calibration and maintenance

During the period covered by this report an NIO 4753 (valve) shipborne wave recorder was deployed on the lightvessel.

(i) 5 August 1975 - Instrument installed on LV no. 10 following laboratory calibration in June 1975.

(ii) 19 August 1976 - LV no. 10 docked at Holyhead for re-fit. SBWR removed and calibration checked. Result of check was an overall change in sensitivity of -1.7% since June 1975.

(iii) 20 October 1976 - SBWR re-installed in LV no. 10. Vessel returned to St. Gowan station in December 1976.

(iv) 19 October 1979 - LV no. 10 docked at Swansea for re-fit. SBWR removed and calibration checked. Result of check was an overall change in sensitivity of -0.5% since August 1976.

1.4 Wave data coverage and return (figures 1.4(A) - 1.4(I))

The periods covered by the data are 1 August 1975 to 31 July 1976, and 1 January 1977 to 31 December 1978. For this period 718 of the 8768 possible chart records were either missing or classified as invalid, resulting in a data return of 91.8%; and 1.34% of the valid records were calms (see Appendix II). No attempt has been made to correct any bias which may have resulted from missing/invalid records, because of the uncertain reliability of available techniques. (Simple gap-filling by

linear interpolation, up to a maximum of 7 consecutive records, has been carried out for the purpose of persistence calculations only: see section 3.6.) The approximate times when missing/invalid records occurred may be derived from the plots in figure 1.4 which show Hs as a time series. On these plots each vertical line represents a valid record, and the height of the line is proportional to the value of Hs for that record: therefore these plots also indicate the variation of Hs with time.

2. WIND DATA - COMPARISON WITH THE LONG-TERM AVERAGE

The meteorological station nearest to the wave measurement site is Milford Haven ($51^{\circ}43'N$, $005^{\circ}02'W$) from which wind data have been analysed from the period August 1964 to December 1978. Winds approaching from directions which have very limited fetches associated with them have not been considered, so that only winds in the sector from 200° to 280° have been considered in this report (including a proportion of calms and variables). The data used are three-hourly synoptic wind speeds.

2.1 Monthly variation of wind speeds (figure 2.1)

For each month, the mean of the monthly means of wind speed is plotted. Comparing monthly means for the 'long-term' wind data and for the wind data from the years when waves were recorded, only March appears to show a significant and consistent difference (individual years about 10%-20% higher than the 'long-term' mean) although for two of the years November has a higher wind speed than the 'long-term' mean (by about 23% and 18%).

2.2 Yearly variation of wind speeds (figures 2.2(A) and 2.2(B))

The year-to-year variability of wind conditions is illustrated in these figures. It shows, for each year, the maximum value of wind speed, and also the means of the next N highest wind speeds, where $N = 5, 10, 20, 50, 100$ (thus the highest 186 wind speeds are represented). It can be seen that for 1975/6, 1977 and 1978 the maximum wind speeds are higher than the maximum wind speeds for most of the other years, whilst the mean wind speeds for these same years lie in the middle or even the lower part of the range of mean wind speeds. This suggests that the worst storms during the period of wave recording were amongst the most severe of those for the 14 years considered.

3. WAVE DATA - DESCRIPTION AND DISCUSSION OF THE PRESENTATIONS

Where figures show seasonal data, the seasons are defined as follows:

spring - March, April, May

summer - June, July, August

autumn - September, October, November

winter - December, January, February

The maximum value of Hs in these three years of data is 9.1 metres; the associated value of Tz is 9.8 seconds, and of Hmax(3hr) is 17.1 metres. However, the largest value of Tz recorded is 13.7 seconds, with an associated Hs of 4.2 metres.

3.1 Statistics of variations of wave heights

3.1.1 Monthly variation of Hs (figure 3.1.1)

For each month, the mean of the significant wave height is calculated and plotted separately for each year. The summer months (June, July and August) show little variation in the mean Hs between years. For December, the yearly variation is very large (the 1978 mean is 2 times greater than the 1975/6 mean); and October, also, has a large yearly variation (the 1977 mean is more than twice the 1978 mean). The winter month of January has the smallest variation when expressed as the ratio of range to mean.

3.1.2 Yearly variation of Hs (figure 3.1.2)

The year-to-year variability of wave conditions is illustrated in this figure. It shows, for each year, the maximum value of Hs, and also the means of the next N highest values of Hs, where N = 5, 10, 20, 50, 100 (thus the highest 186 values of Hs are represented). From this figure there appears to be very little yearly variation in the highest values of Hs, although the first of the three years shows slightly lower values of the highest wave conditions than the latter two years.

3.2 Statistics of wave heights

3.2.1 Occurrence of Hs (figures 3.2.1.1-3.2.1.5)

The percentage occurrence of Hs is shown on histograms. The most frequently occurring values of Hs may be seen, from figure 3.2.1.5, to lie between 0.5 and 1.5 metres, accounting for 37% of the total.

3.2.2 Exceedance of Hs and Hmax(3hr) (figures 3.2.2.1-3.2.2.5)

These graphs may be used to estimate the fraction of the time during which Hs was greater than, or less than, a given height. For instance, from figure 3.2.2.4 it may be seen that during winter the significant wave height exceeded 4 metres for approximately 16 per cent of the time.

3.3 Design wave heights

The methods used to calculate the design wave height (the most probable height of the highest wave with a return period of 50 years) are described in Appendix III. The results obtained by the different methods are given below.

3.3.1 Weibull distribution of Hs (figure 3.3.1)

The parameters of the Weibull distribution which most closely fits the data are $A = 0.24$ metre, $B = 1.91$ metres and $C = 1.39$, and this distribution is represented by the straight line in figure 3.3.1. Extrapolation of this line to a return period of 50 years gives a value of Hs of 11.6 metres. The value of Tz associated with this Hs is approximately 10.5 seconds, resulting in a value for the design wave height of 21.7 metres.

3.3.2 Fisher-Tippett I distribution of Hs (figure 3.3.2)

The parameters of the Fisher-Tippett I distribution which most closely fits the data are $a = 1.28$ metre⁻¹ and $b = 3.28$, and this distribution is represented by the straight line in figure 3.3.2. (The parameters have been chosen for a best fit to the top 7 data points.) Extrapolation of this line to a return period of 50 years gives a value of Hs of 11.9 metres. The value of Tz associated with this Hs is approximately 10.5 seconds, resulting in a value for the design wave height of 22.3 metres.

3.3.3 Fisher-Tippett III distribution of Hs (figure 3.3.3)

The parameters of the Fisher-Tippett III distribution which most closely fits the data are $A = 34.0$ metres, $B = 32.6$ metres and $C = 30.9$, and this distribution is represented by the straight line in figure 3.3.3. Extrapolation of this line to a return period of 50 years gives a value of Hs of 11.7 metres. The value of Tz associated with this Hs is approximately 10.5 seconds, resulting in a value for the design wave height of 21.9 metres.

3.3.4 Individual wave model (figure 3.3.4)

The value of steepness used in the wave-by-wave method of determining design wave heights (as described in Appendix III) is 1:18, and the inverse mean period is 0.15 Hz. Using these values and the Fisher-Tippett III parameters given in section 3.3.3, the data which appear in figure 3.3.4 are obtained; by interpolation the design wave height is found to be 23.6 metres. A higher value of design wave height is expected from this method than from the methods described above, for the reasons stated in Appendix III.

3.4 Statistics of wave periods

The percentage occurrence of Tz is shown on a histogram.

3.4.1 Occurrence of Tz (figures 3.4.1.1-3.4.1.5)

The most frequently occurring values of Tz in the data set lie between 6.0 and 7.0 seconds (29% of the total), and all values of Tz lie between 2.5 and 14.0 seconds (figure 3.4.1.5).

3.5 Statistics of wave height and period combined

These figures (sometimes called "scatter" plots) show the numbers of wave records having particular combinations of values of Hs and Tz. The numbers of wave records are presented as parts per thousand (the total number of valid observations being shown on each figure), except for those which would be less than one part per thousand; these are shown instead as single occurrences and are distinguished by being underlined.

3.5.1 Occurrences of Hs and Tz combined (figures 3.5.1.1-3.5.1.5)

On these figures points of equal occurrences are joined by contour lines to give an indication of the bivariate probability distribution of Hs and Tz, and to illustrate the correlation between them. A wave "steepness" (as defined in Appendix III) can be calculated for each (Hs,Tz) pair. A line is drawn on figure 3.5.1.5 showing a "steepness" of 1:12, which is the limiting "steepness" for the main body of the data. (Wave "steepnesses" as shown in this figure are less than the maximum of 1:7 for an individual wave, since Hs and Tz are parameters averaged over a number of waves most of which have steepnesses less than this maximum.)

3.6 Statistics of persistence of wave conditions

These figures show the means and standard deviations of the durations of storms and calms against each threshold value of Hs, and also the percentage of the total duration occupied by each event. Gaps in the data series of 7 or less records are filled (for the purpose of persistence calculations only) by linear interpolation; larger gaps are not filled, effectively reducing the series to a number of smaller sub-series, each with a correspondingly smaller total duration. 'Split' seasons (those in which the months are not consecutive) are not used in the persistence calculations. (For storms, the curves showing percentage of time occupied by the events are, for all practical purposes, the same as those showing percentage exceedance of Hs as described in section 3.2.2.)

3.6.1 Persistence of calms of Hs (figures 3.6.1.1-3.6.1.5)

Information about, for example, calms of Hs less than 0.7 metres at the St. Gowan station during winter can be derived from figure 3.6.1.4. The mean duration of such calms was approximately 15 hours (with a standard deviation of 21 hours); they occupied about 7% of the total duration of 4173 hours, i.e. about 290 hours; and therefore there were 19 or 20 such calm events during this period. Since this represents two seasons, there were on average about 10 such events each winter.

3.6.2 Persistence of storms of Hs (figures 3.6.2.1-3.6.2.5)

Similar information can be derived for storms. For Hs of 3.5 metres during summer, figure 3.6.2.2 shows that the mean duration of such storms was approximately 4 hours (with a standard deviation of 3 hours); the total time occupied was a little over 1% of 4392 hours, i.e. about 50 hours; and therefore the number of such storm events in this period was 12 or 13. Since this represents two seasons, there were on average about 6 such events each summer.

4. ACKNOWLEDGEMENTS

Contributions have been made towards the collection, analysis and presentation of the St. Gowan wave data by several members of the Applied Wave Research Group and of the Instrument Engineering Group, both based at the Taunton laboratory of the Institute of Oceanographic Sciences. Thanks are due to Trinity House for permission to install the shipborne wave recorder in the St. Gowan Light Vessel; and also to the Meteorological Office for supplying the wind data.

5. REFERENCE

TUCKER M J 1956. A shipborne wave recorder. Transactions of the
Institute of Naval Architects 98 , 236-250.

APPENDIX I

Method of system calibration

I.1 Since there are two types of transducer in the shipborne wave recorder system, it is necessary to divide the calibration procedure into two sections. First the accelerometers are removed from the ship mountings and each is inserted into a rig which allows the transducer to be driven through a vertical circle of diameter 1 metre. The transducer is mounted in gimbals and maintains a vertical attitude during rotation. Two rotation rates are applied: 12 and 18 second periods which are derived from a crystal oscillator. The transducer is connected to the electronics unit in the usual way, and the calibration signal is displayed on the chart recorder. However, because a 1 metre 'heave' is small compared with the wave-heights usually experienced at sea, a precision amplifier (contained in the electronics) is switched into the circuit, converting the 1 metre into an apparent 10 metre signal. The output signal can then be read from the chart record and any corrections to instrument sensitivity made.

The pressure units cannot be easily subjected to a dynamic test since this requires the application of a sinusoidally-varying pressure. Therefore for routine re-calibration a static test is applied. Each pressure unit is fixed to the test rig and a series of discrete pressure levels is applied from a reservoir via a regulator valve. Each pressure level is set manually with the valve by reference to a precise pressure transducer contained within the calibrator unit. The output voltage of the transducer is monitored in the SBWR electronics unit and compared to the original laboratory calibration. Any changes in sensitivity are then compensated for by adjustment of the input amplifier gain.

Full re-calibrations are usually only possible when the ship comes into dock for its 3-yearly re-fit.

I.2 Monthly checks

All lightvessel crews are asked to drain water through the valve assemblies to ensure that no blockage prevents the water pressure being transmitted to the pressure sensors, and then to take a test record, on a monthly basis. The test record consists of a short length of pen-trace with all transducers turned off (electrically), followed by a few minutes recording with each transducer on its own. The record thus produced shows two heave records (one from each accelerometer) which should look broadly similar; and also the pressure traces, which may not agree so well, but when compared with other monthly test records should exhibit no systematic error. These tests are not direct checks on calibration accuracy but are often good indicators of a fault condition developing.

APPENDIX II

Definitions of wave parameters and method of analysis of wave chart data

II.1 The technique used to analyse the wave data was that proposed by TUCKER(1961) and DRAPER(1963), and reviewed by TANN(1976). A twelve-minute record is taken every three hours, and from this the following parameters may be derived.

II.1.1 T_z - the mean zero up-crossing period. This is defined as the duration of the record divided by the number of zero up-crossings N_z . (A zero up-crossing is considered to occur when the trace crosses the mean line in an upward direction.)

II.1.2 H_s - the significant wave height. This is defined as 4σ where σ is the standard deviation of the record. (An estimate of σ is obtained from N_z and from the excursions of the two highest maxima, and of the two lowest minima, from the record mean.) For a narrow band random process this parameter approximates closely to the mean height of the highest one-third zero up-cross waves (see LONGUET-HIGGINS(1952)). Comparison between the two definitions is made by GODA(1970,1974). (A zero up-cross wave is defined as the portion of the wave record between two zero up-crossings, and its height is the vertical distance between the highest and lowest points on the wave.)

II.1.3 $H_{max}(3hr)$ - the most probable height of the highest zero up-cross wave in the recording interval of three hours. (This is derived from N_z , and the duration of the recording interval.) The parameter $H_{max}(3hr)$, which is the mode of the distribution, should not be confused with the expected height of the highest wave in three hours, which is the mean of the distribution. The mean of the distribution is typically 3 per cent higher than the mode (see TANN(1976)).

(A 'calm' record is one for which the sum of the height of the highest crest and of the lowest trough is less than 1 foot or 0.3 metre.)

II.2 References

- DRAPER L 1963. Derivation of a 'design wave' from instrumental records of sea waves. Proceedings of the Institution of Civil Engineers 26, 291-304.
- GODA Y 1970. Numerical experiments on wave statistics with spectral simulation. Report of the Port and Harbour Research Institute 9, No 3, 3-57.
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TANN H M 1976. The estimation of wave parameters for the design of offshore structures. Institute of Oceanographic Sciences, Report No 23.

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APPENDIX III

Details of methods used for calculating design wave heights

III.1 By finding the long-term distribution of H_s

III.1.1 H_s is used as a measure of the "sea-state" (i.e. the intensity of wave activity), and it is sampled every 3 hours. It is assumed that a set of H_s data for one year, or an integral number of years, is representative of the wave climate.

For each value of H_s , the probability that this value will not be exceeded is calculated; this probability is then plotted against H_s . The axes are scaled according to a long-term distribution, so that data with a perfect fit would appear as a straight line on the diagram. This procedure is carried out using long-term distributions defined in the following ways

Weibull

$$\text{Prob}(H_s \leq h) = \begin{cases} 1 - \exp\left[-\left(\frac{h-A}{B}\right)^C\right], & \text{for } h > A \\ 0 & , \text{ for } h \leq A \end{cases}$$

where B and C are positive, and A represents a lower bound on h .

Fisher-Tippett I (first asymptote)

$$\text{Prob}(H_s \leq h) = \exp[-\exp(-ah+b)].$$

Fisher-Tippett III (third asymptote)

$$\text{Prob}(H_s \leq h) = \begin{cases} \exp\left[-\left(\frac{A-h}{B}\right)^C\right], & \text{for } h \leq A \\ 1 & , \text{ for } h > A \end{cases}$$

where B and C are positive, and A represents an upper bound on h . (See FISHER AND TIPPETT(1928) and GUMBEL (1958) for the derivations of these distributions.)

For each long-term distribution the best-fit straight line is drawn; this line is then extrapolated to the desired probability (see section III.1.2) and the corresponding value of H_s is read off as the "design sea-state".

III.1.2 To calculate the "sea-state" which will be exceeded only once in N years, a storm duration of D hours needs to be assumed. The probability that a randomly chosen time will be within this storm is then

$$\frac{D}{24 \times 365.25 \times N}$$

IOS uses $D = 3$ hours (this choice is discussed in section III.1.5) which gives

$$\begin{aligned} \text{Probability} &= \frac{3.422 \times 10^{-4}}{N} \\ &= 6.845 \times 10^{-6} \quad \text{for } N = 50 \text{ years.} \end{aligned}$$

III.1.3 The value of T_z for the "design sea-state" is required before the highest wave in the storm can be calculated. This is derived from the bivariate distribution of H_s and T_z (figure 3.5.1.5). A line is drawn across this at the "design sea-state" value of H_s and the most likely value of T_z (the modal value) is then estimated using extrapolations of the probability contours.

III.1.4 The most probable value of the highest zero-up-cross wave in the storm is then derived by assuming that the heights of such waves follow a Rayleigh distribution whose probability density function is

$$\text{prob}(h) = \frac{2h}{(H_{rms})^2} \exp\left[-\left(\frac{h}{H_{rms}}\right)^2\right]$$

where $H_{rms} \approx \frac{H_s}{\sqrt{2}}$.

Exact theory is not available for zero-up-cross wave heights, but this distribution has been found to be an adequate fit to measured data. If there are n waves in the recording interval (3hr), then the probability that the highest wave, H , in three hours is less than h is

$$\text{Prob}(H \leq h) = \left\{ 1 - \exp\left[-\left(\frac{h}{H_{rms}}\right)^2\right] \right\}^n$$

with a corresponding probability density function

$$\frac{2n}{(H_{rms})^2} h \exp\left[-\left(\frac{h}{H_{rms}}\right)^2\right] \left\{ 1 - \exp\left[-\left(\frac{h}{H_{rms}}\right)^2\right] \right\}^{n-1}$$

The most probable value (the mode) of this probability density function is usually used and is given by

$$H_{max}(3hr) = H_{rms} \sqrt{\Psi}$$

where Ψ is a function of T_z which may be found using either figure 7 or equation 6.1-2 in TANN(1976).

III.1.5 In choosing the value of storm duration D , it should be noted that the effect of increasing D is to decrease the value of H_s for a given return period N . However, it also increases the ratio of $H_{max}(3hr)$ to H_s . It is found that in practice these effects roughly cancel and typically the value of $H_{max}(3hr)$ changes by only 3 per cent for a change of D from 3 to 15 hours. The choice of D is therefore not critical.

Many details of the above procedures may be found in TANN(1976).

III.2 By a wave-by-wave method

III.2.1 BATTJES(1970) shows that the probability that a randomly chosen wave will have a height H greater than h is

$$\text{Prob}(H>h) = \frac{\int_0^\infty \int_0^\infty R(h, H_s) T_z^{-1} p(T_z, H_s) dH_s dT_z}{\int_0^\infty \int_0^\infty T_z^{-1} p(T_z, H_s) dH_s dT_z} \dots\dots\dots(1)$$

where $R(h, H_s)$ is the Rayleigh cumulative probability function and $p(T_z, H_s)$ is the joint probability density function of H_s and T_z .

III.2.2 TANN makes the following suggestion in an unpublished manuscript. In order that values of H_s higher than those actually measured may be represented in the calculation of this probability, the values of H_s are assumed to have a long-term cumulative probability function $F(H_s)$, and a probability density function $f(H_s) = F'(H_s)$.

For each value of H_s throughout the long-term distribution, an average value of T_z^{-1} is used (denoted by $\overline{T_z^{-1}(H_s)}$). It is defined as

$$\overline{T_z^{-1}(H_s)} = \int_0^\infty T_z^{-1} \frac{p(T_z, H_s)}{P(H_s)} dT_z$$

where $P(H_s) = f(H_s)$.

Therefore

$$\int_0^\infty T_z^{-1} p(T_z, H_s) dT_z = \overline{T_z^{-1}(H_s)}$$

which, when substituted into equation (1), allows the probability of exceedance to be written

$$\text{Prob}(H>h) = \frac{\int_0^\infty R(h, H_s) \overline{T_z^{-1}(H_s)} f(H_s) dH_s}{\int_0^\infty \overline{T_z^{-1}(H_s)} f(H_s) dH_s}$$

The value of $\overline{T_z^{-1}(H_s)}$ used with each value of H_s is chosen to satisfy the condition of constant wave "steepness", where the relationship between "steepness"(1:s), water depth(d), H_s and T_z is

$$T_z = \sqrt{\frac{2\pi s H_s}{g}} \coth\left(\frac{2\pi d}{s H_s}\right)$$

The value for the steepness used in this report is given in section 3.3.4.

The long-term distribution used in the computation for this report is the Fisher-Tippett III extreme-value distribution, whose probability density function is

$$f(H_s) = \frac{C}{A-H_s} \left(\frac{A-H_s}{B}\right)^C \exp\left[-\left(\frac{A-H_s}{B}\right)^C\right]$$

The constants A,B,C are determined graphically as described in section III.1.1, and their values as used in this report are given in section 3.3.3.

III.2.3 Thus the probability of a wave exceeding each particular wave height may be found, and this probability may be converted into a return period of N years using the formula

$$N = \frac{1}{365.25 \times 24 \times 3600 \times T_{ave}^{-1} \times Prob}$$

where $T_{ave}^{-1} = \frac{1}{\text{average period}}$.

The value of the average wave period is contained in section 3.3.4. Since T_{ave}^{-1} is a non-analytic function of Prob, the simplest way of solving the problem is to calculate Prob for various values of h, calculate N for each of these values of Prob, and then interpolate to find the height h corresponding to the required value of N (in this case 50 years).

Whereas the method described in section III.1 assumes that the highest wave in a 50-year period will come from the most stormy 3-hour period in 50 years, the individual wave method takes into account the probability that storms other than the highest may provide the wave with a 50-year return period. Consequently the height of a 50-year wave as estimated by this method is likely to be greater than that estimated from the method of using a long-term distribution of Hs.

III.3 References

- BATTJES J A 1970. Long-term wave height distribution at seven stations around the British Isles. National Institute of Oceanography, Internal Report No A44.
- FISHER R A AND TIPPETT L H C 1928. Limiting forms of frequency distribution of the largest or smallest member of a sample. Proceedings of the Cambridge Philosophical Society 24, 180-190.
- GUMBEL E J 1958. Statistics of Extremes. New York: Columbia University Press. 371 pp.
- TANN H M 1976. The estimation of wave parameters for the design of offshore structures. Institute of Oceanographic Sciences, Report No 23.

APPENDIX IV

The correction factor applied to wave heights

Two corrections need to be applied to the wave height data:

- (i) to compensate for the frequency response of the electronics; and
- (ii) to compensate for the hydrodynamic attenuation of the pressure fluctuations.

These are combined into a single correction factor C, which is dependent upon Tz, and upon the depth of the pressure transducer below the mean water level. The correction factor is

$$C = 0.83 \left\{ 1 + \frac{1}{77.44\omega^2} \right\}^3 \exp \left[\frac{2.5\omega^2 d}{g} \right]$$

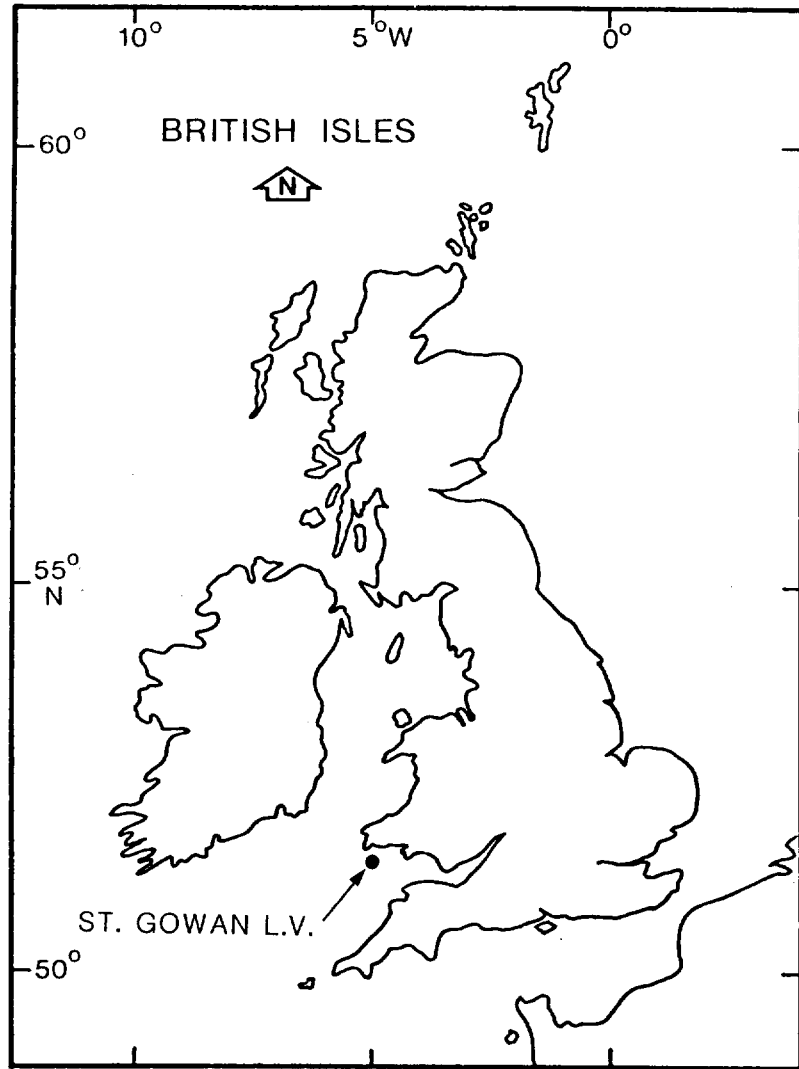
where d is the depth of the pressure transducer,

and $\omega = \frac{2\pi}{Tz}$.

The value of Hs calculated by the method outlined in Appendix II is multiplied by C to obtain the corrected value of Hs.

A table showing the values of C for various values of Tz (and with d = 1.62 metres for the lightvessel on the St. Gowan station) is given below.

Tz(sec)	C
2.5	11.224
3.0	5.076
4.0	2.311
4.5	1.871
5.0	1.610
6.0	1.327
7.0	1.185
8.0	1.104
10.0	1.025
12.0	0.996
15.0	0.974
20.0	1.040



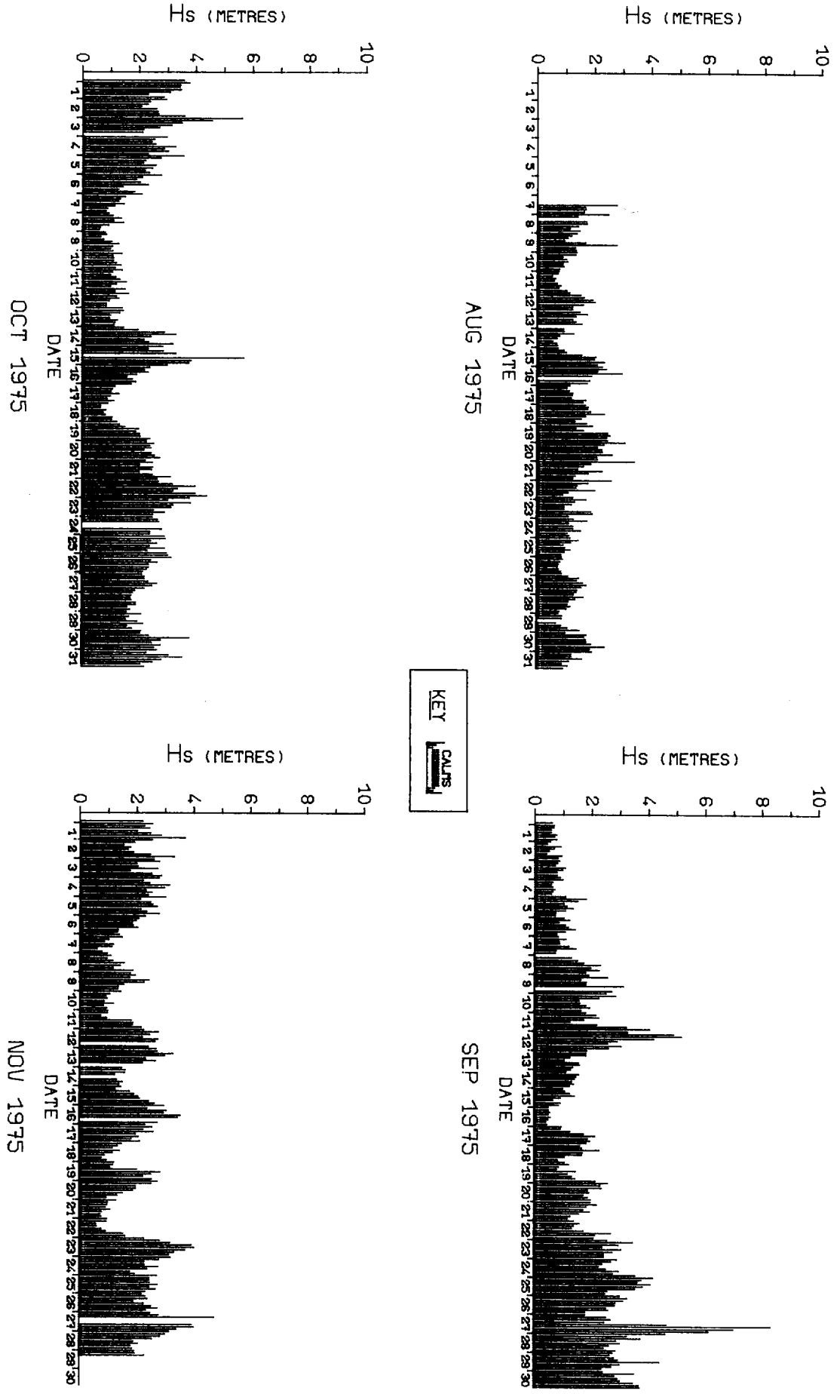
LOCATION MAP OF ST. GOWAN WAVE RECORDER

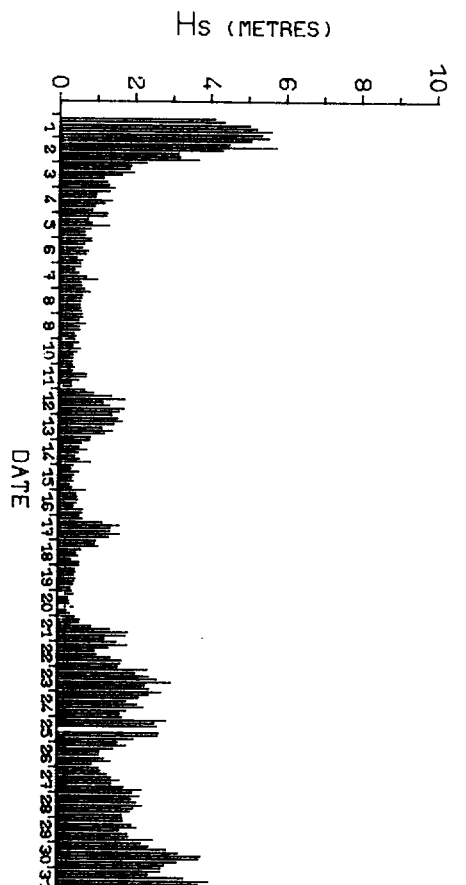
FIG 1.1

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

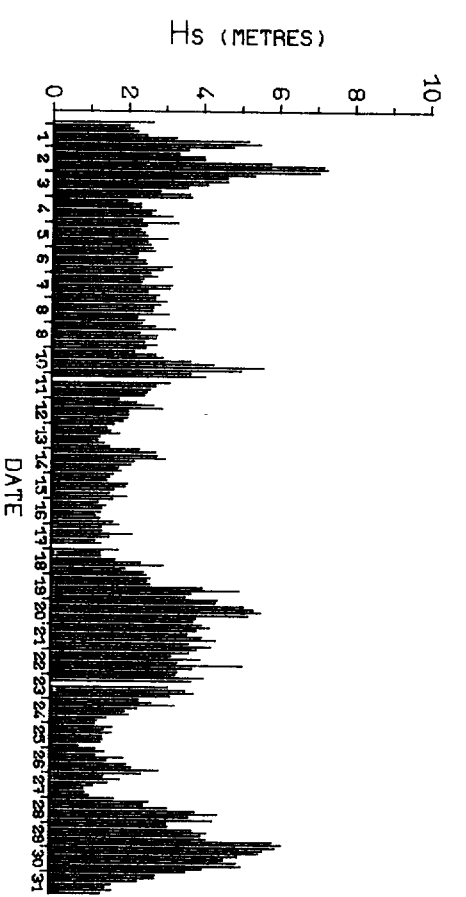
FIG 1.4(A)

TIME SERIES OF HS



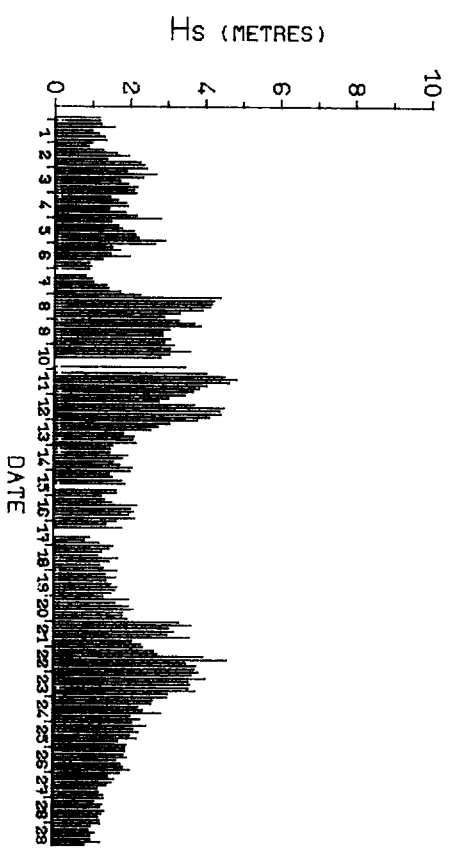


DEC 1975

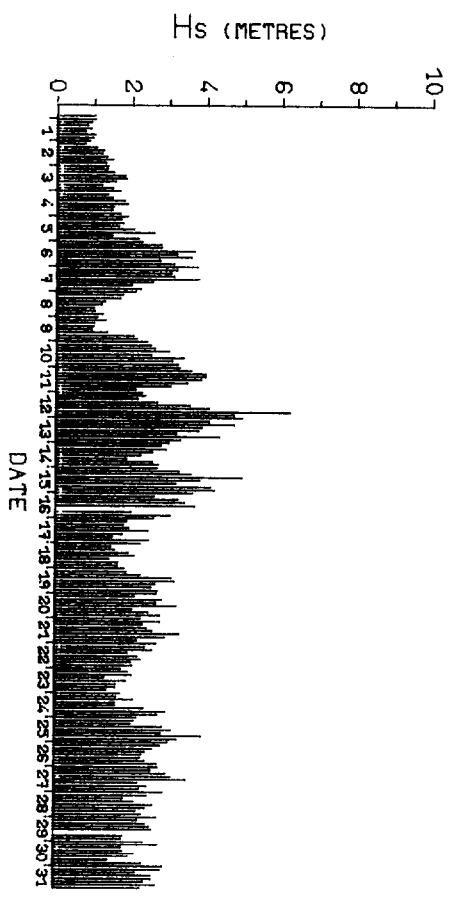


JAN 1976

KEY
CALMS



FEB 1976

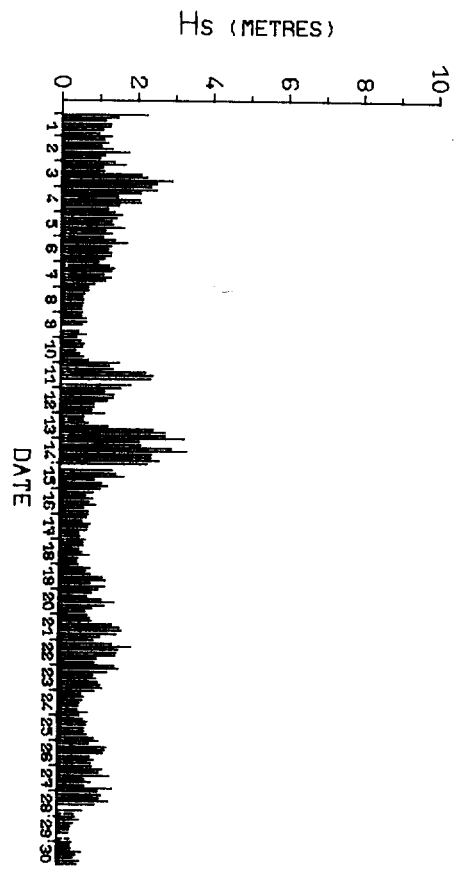


MAR 1976

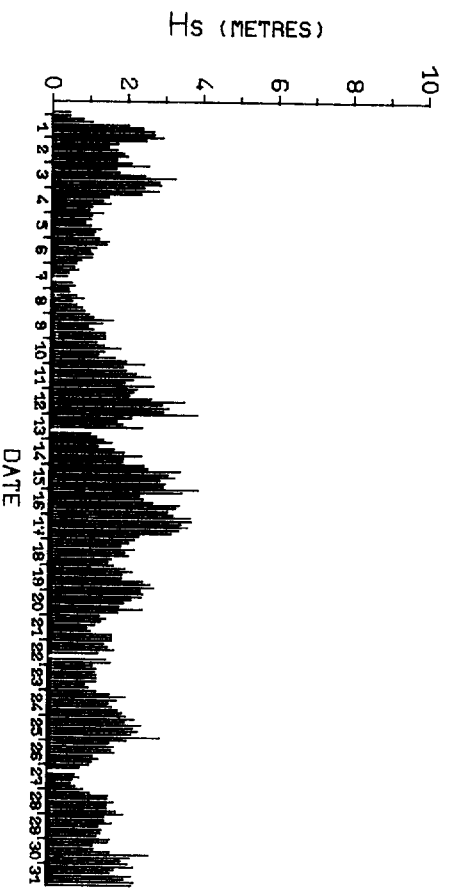
TIME SERIES OF HS

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978


FIG 1.4(B)

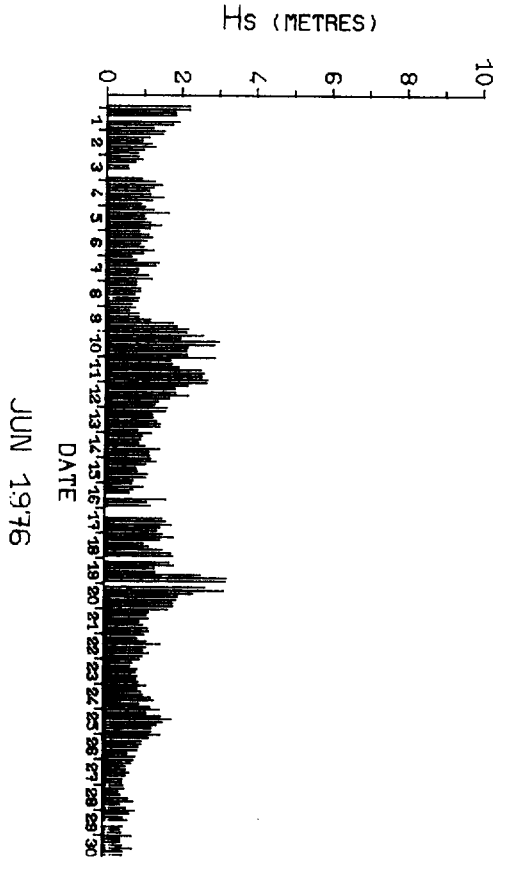


APR 1976

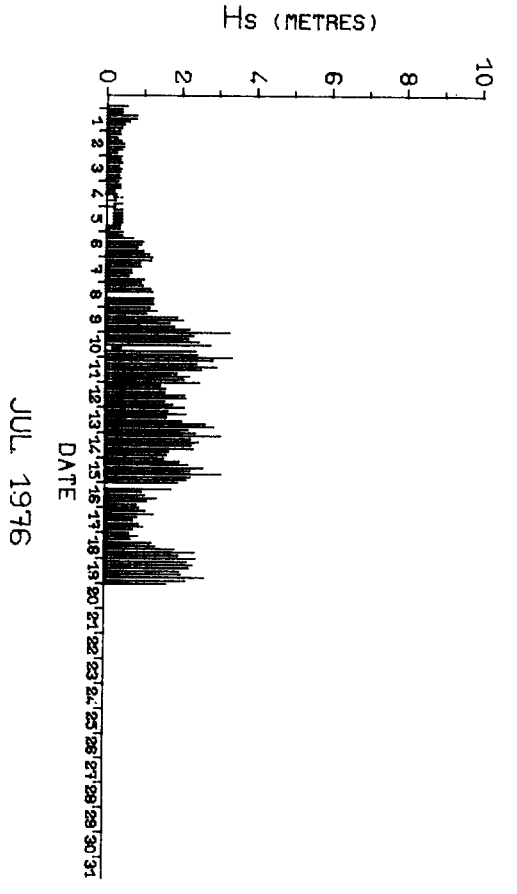


MAY 1976

KEY
 CALMS



JUN 1976



JUL 1976

TIME SERIES OF HS

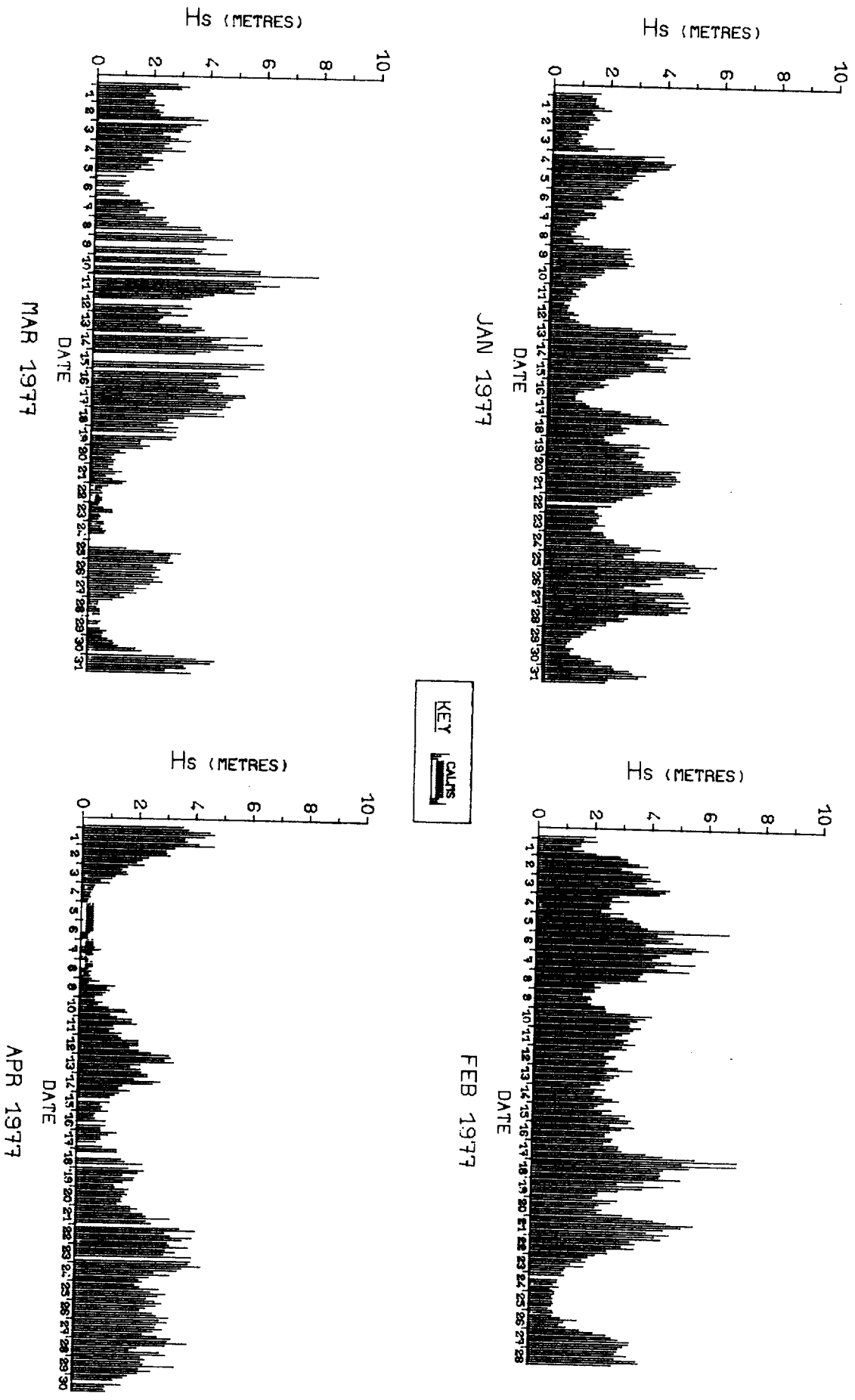
ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

FIG 1.4(C)

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

FIG 1.4(D)

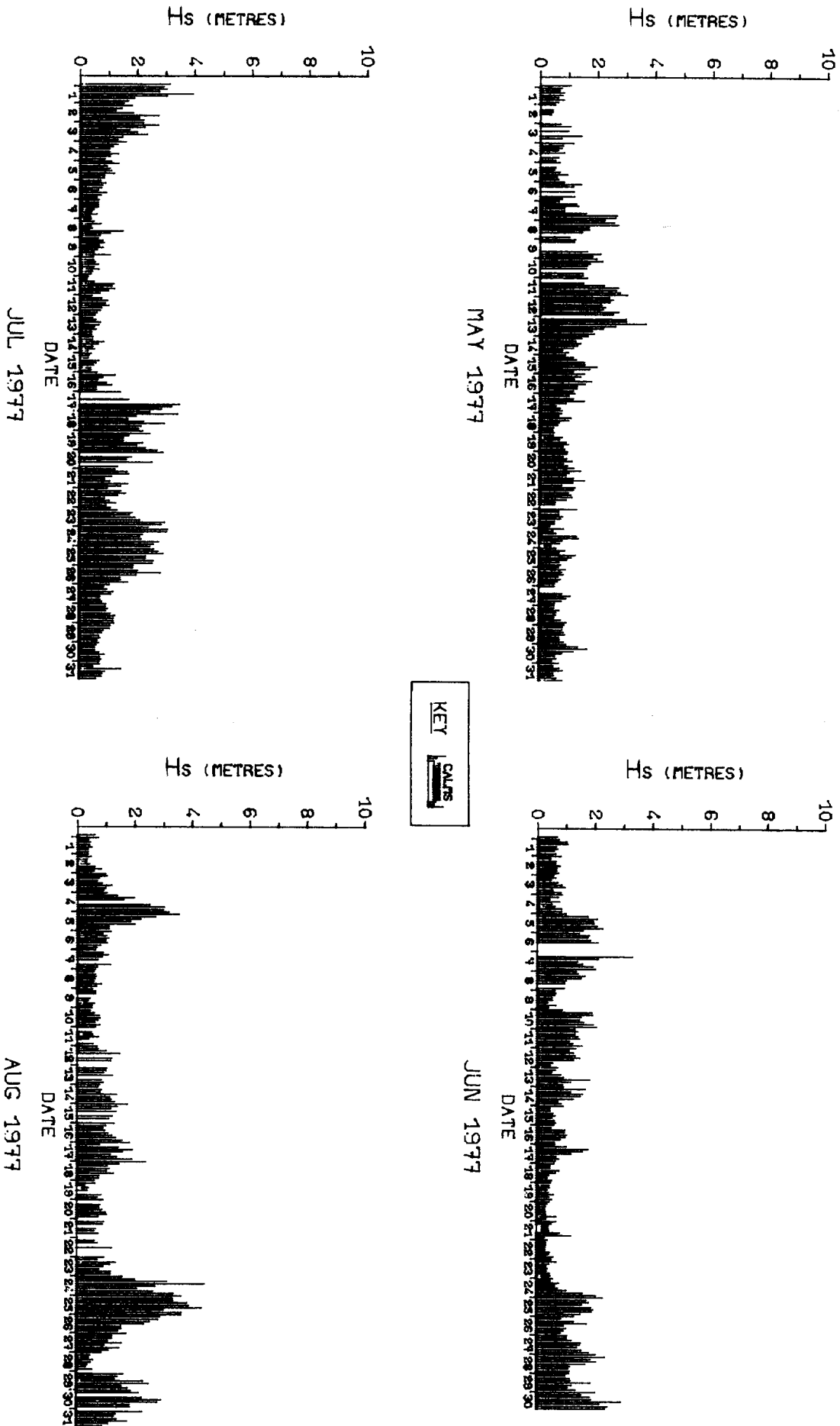
TIME SERIES OF HS



ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

FIG 1.4(E)

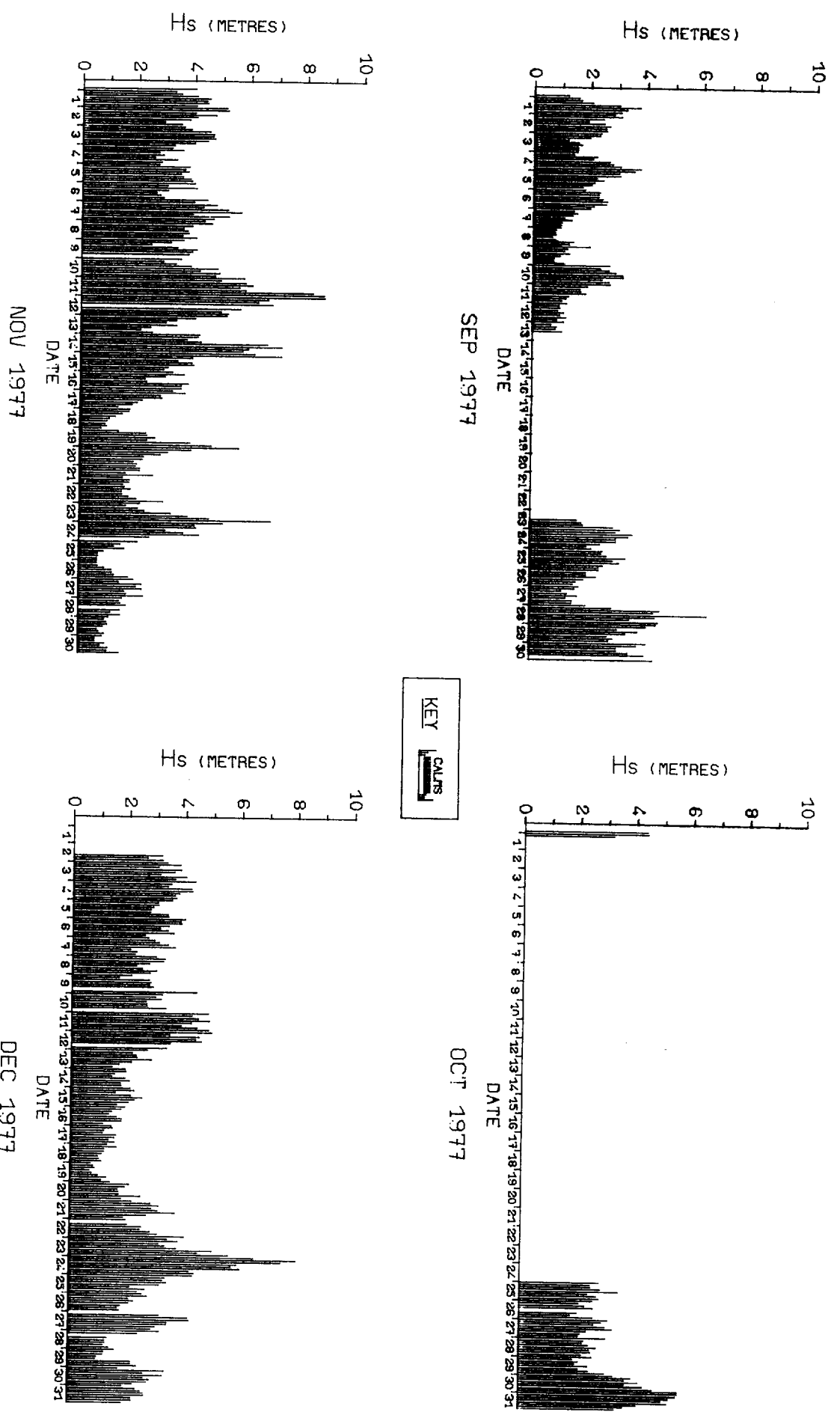
TIME SERIES OF HS

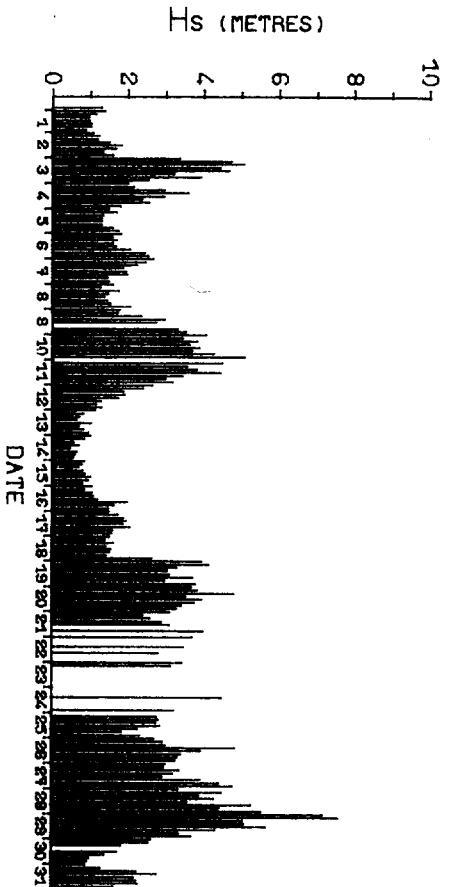


ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

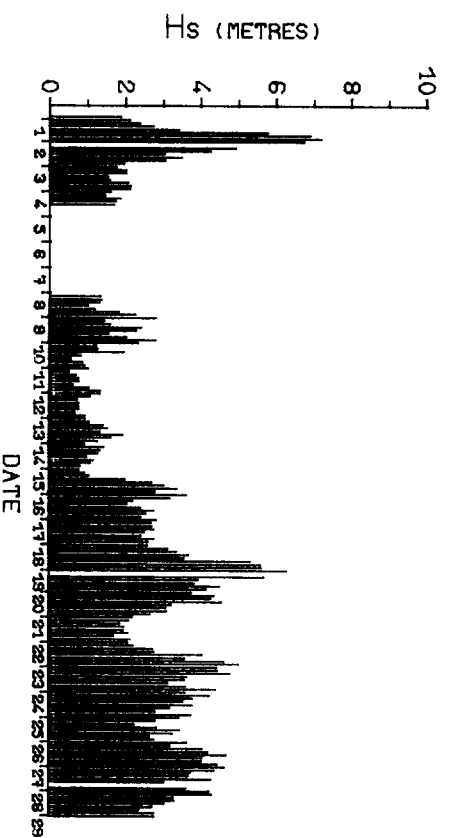
FIG 1.4(F)

TIME SERIES OF HS




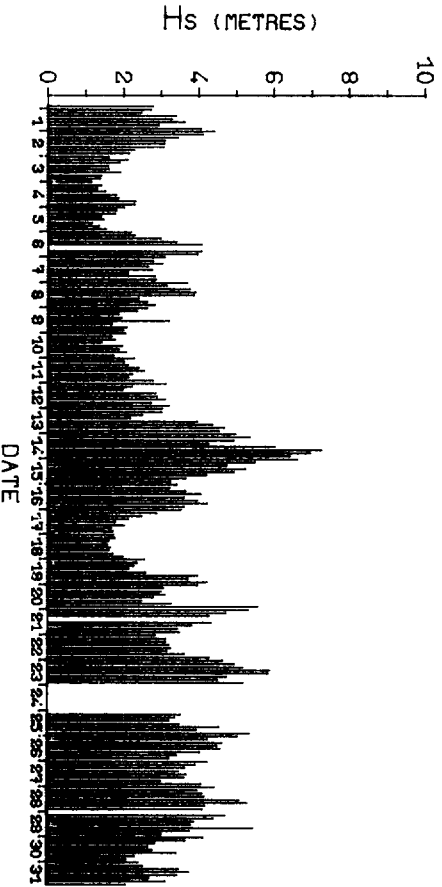


JAN 1978

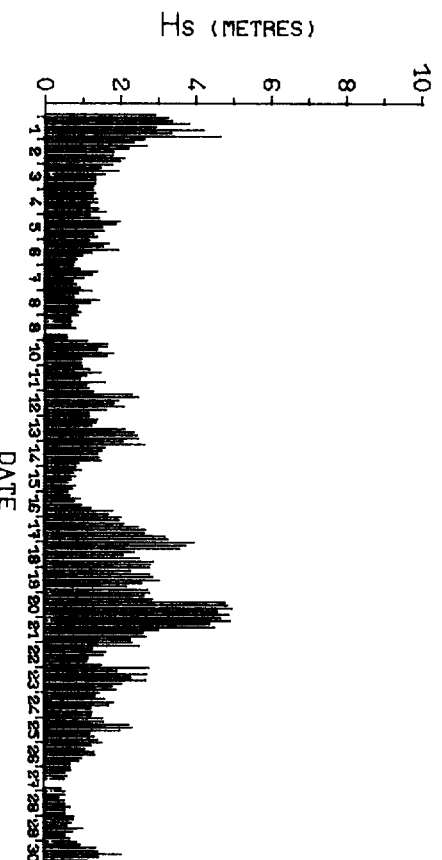


FEB 1978

KEY
 GAPS



MAR 1978

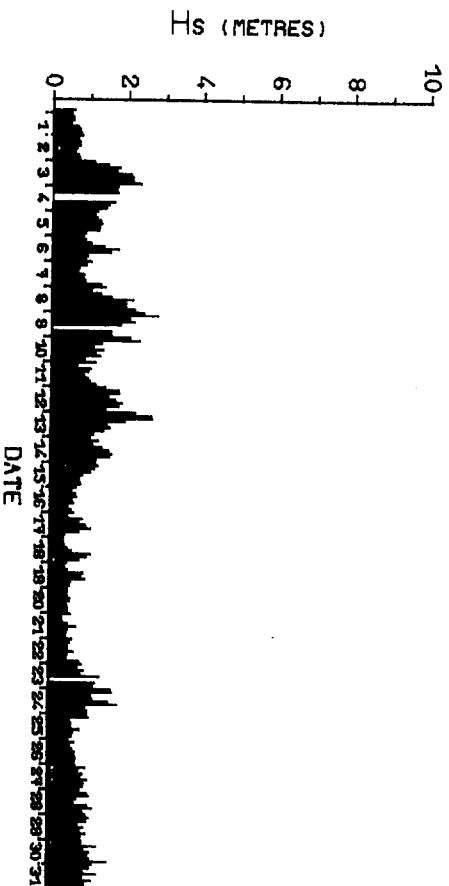


APR 1978

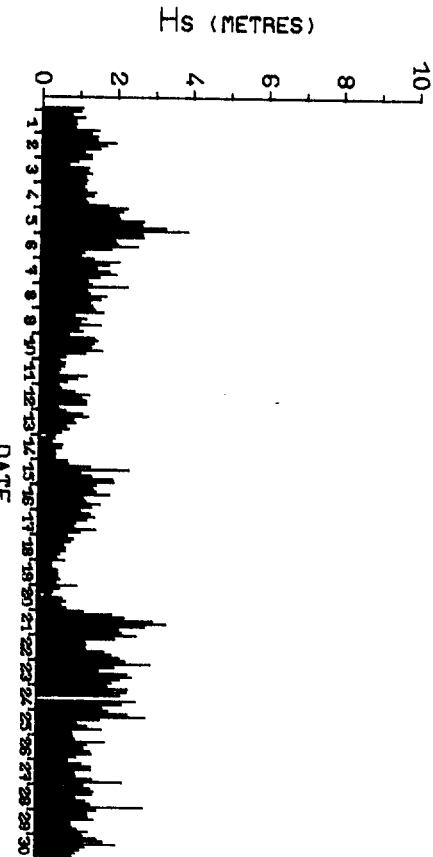
TIME SERIES OF HS

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

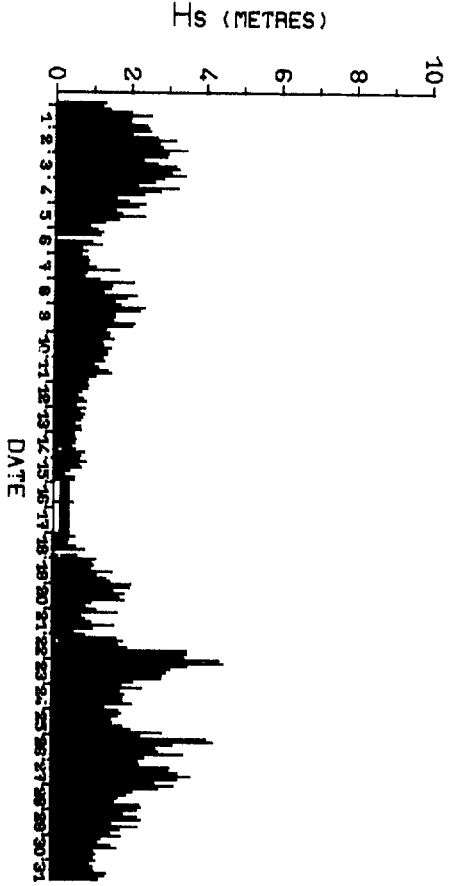
FIG 1.4(G)



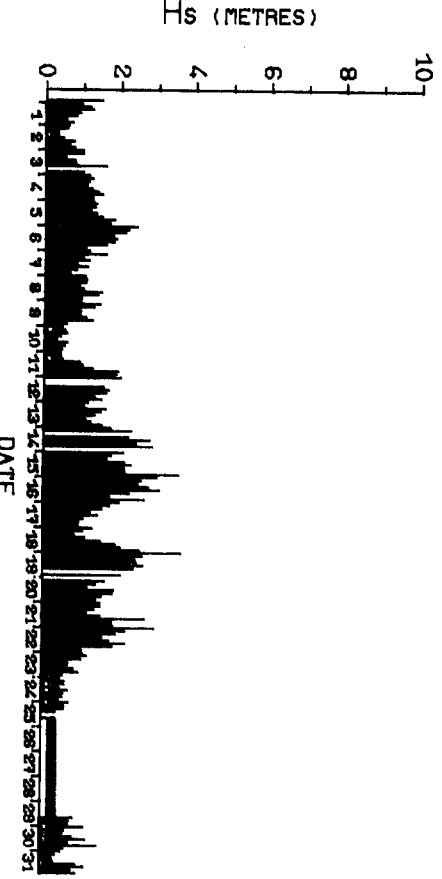
MAY 1978



JUN 1978



JUL 1978

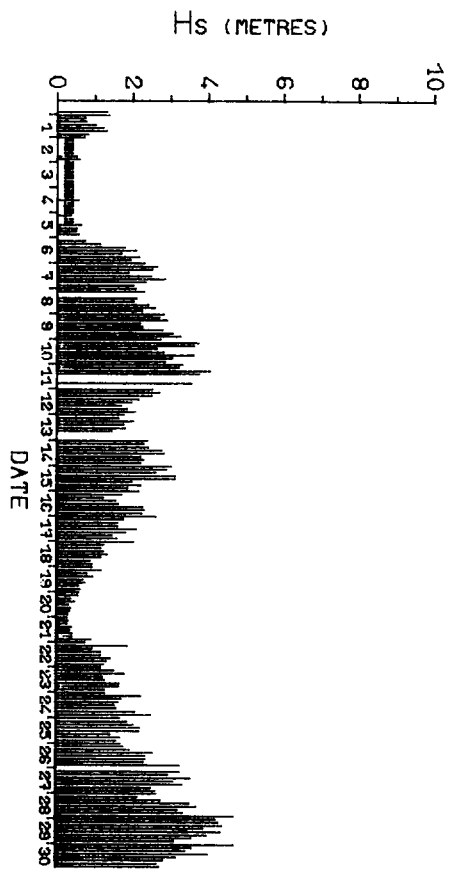


AUG 1978

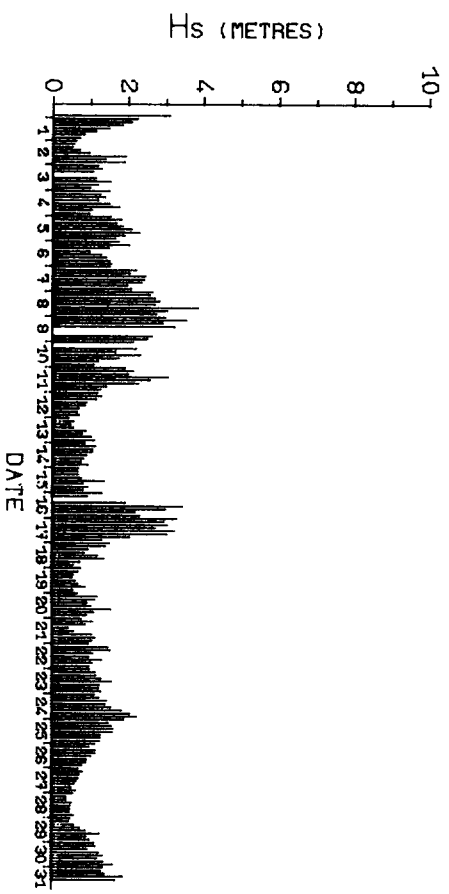
TIME SERIES OF HS

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

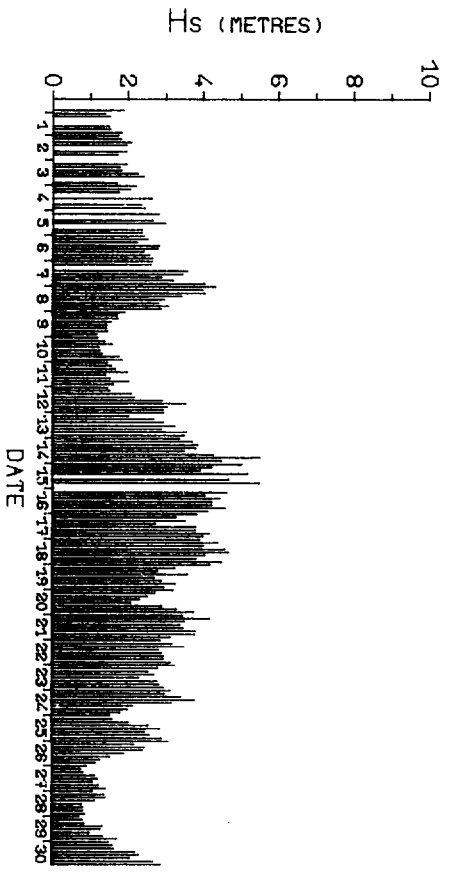
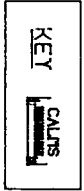
FIG 1.4(H)



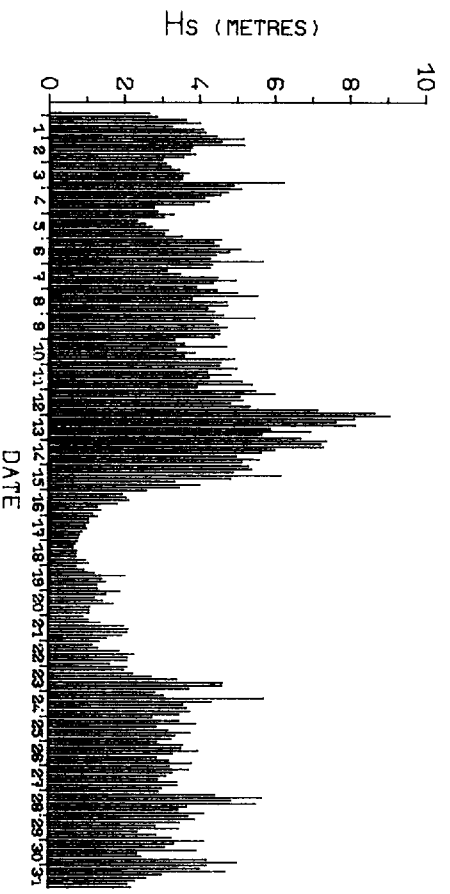
SEP 1978



OCT 1978



NOV 1978



DEC 1978

TIME SERIES OF HS

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

FIG 1.4(1)

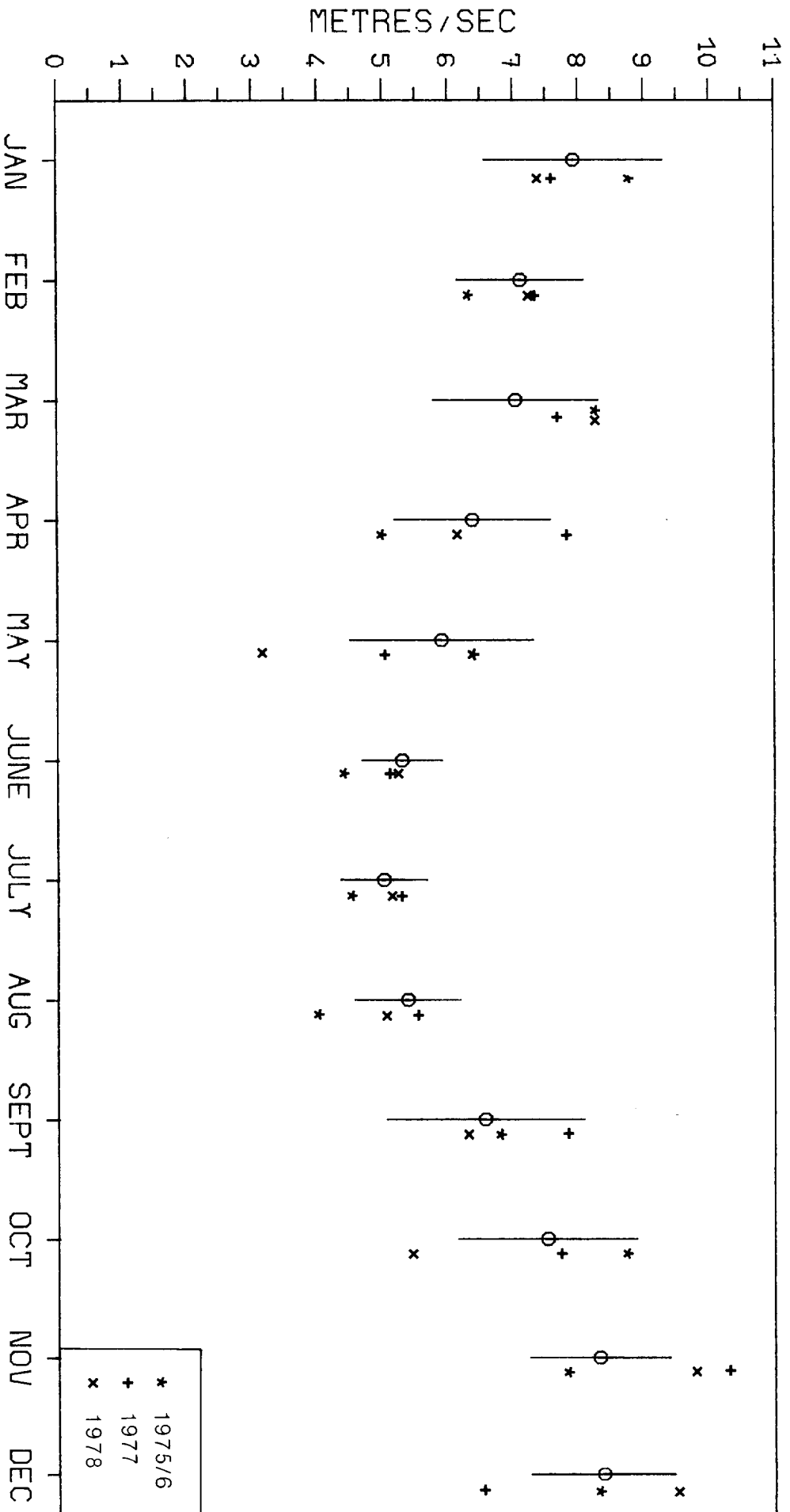
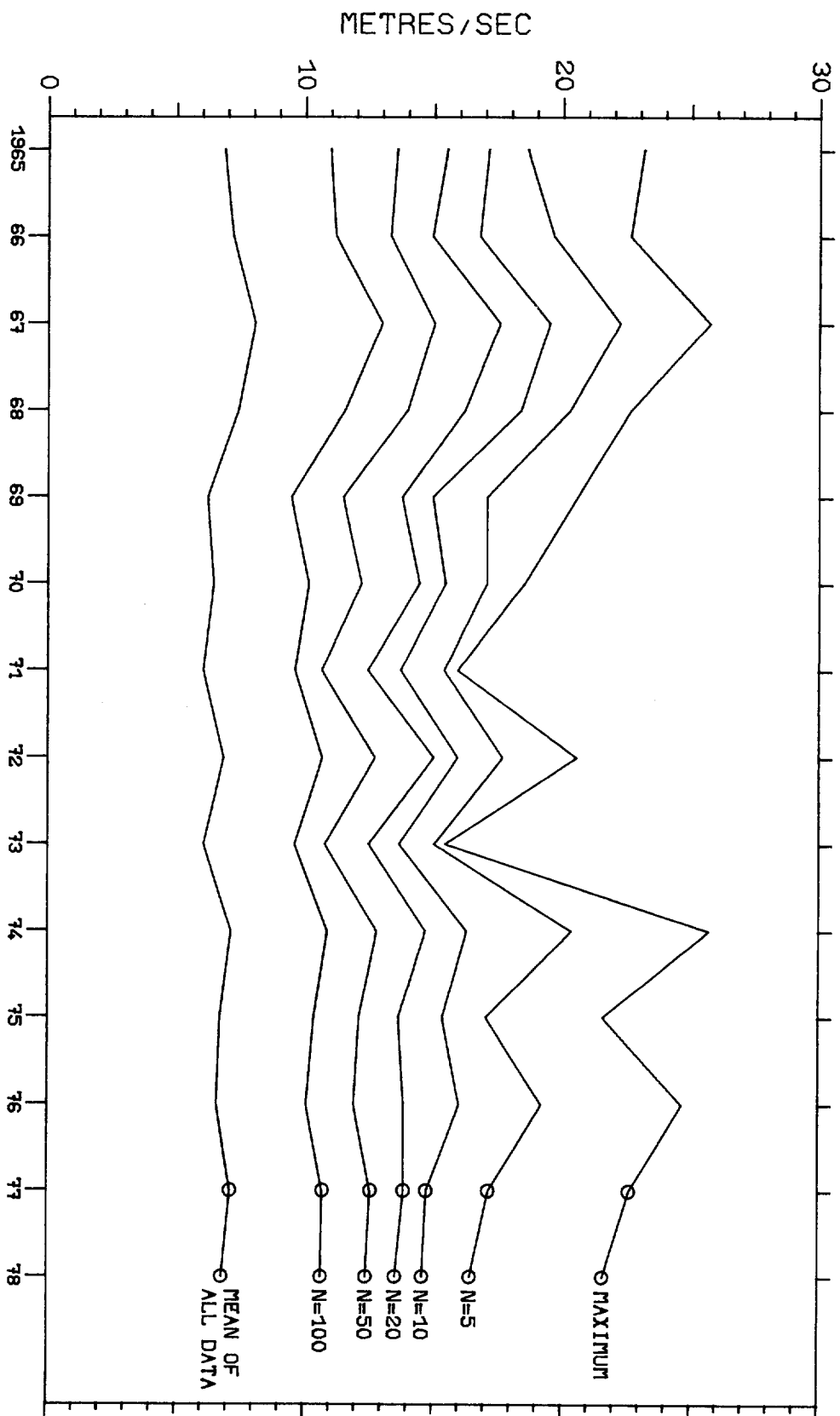


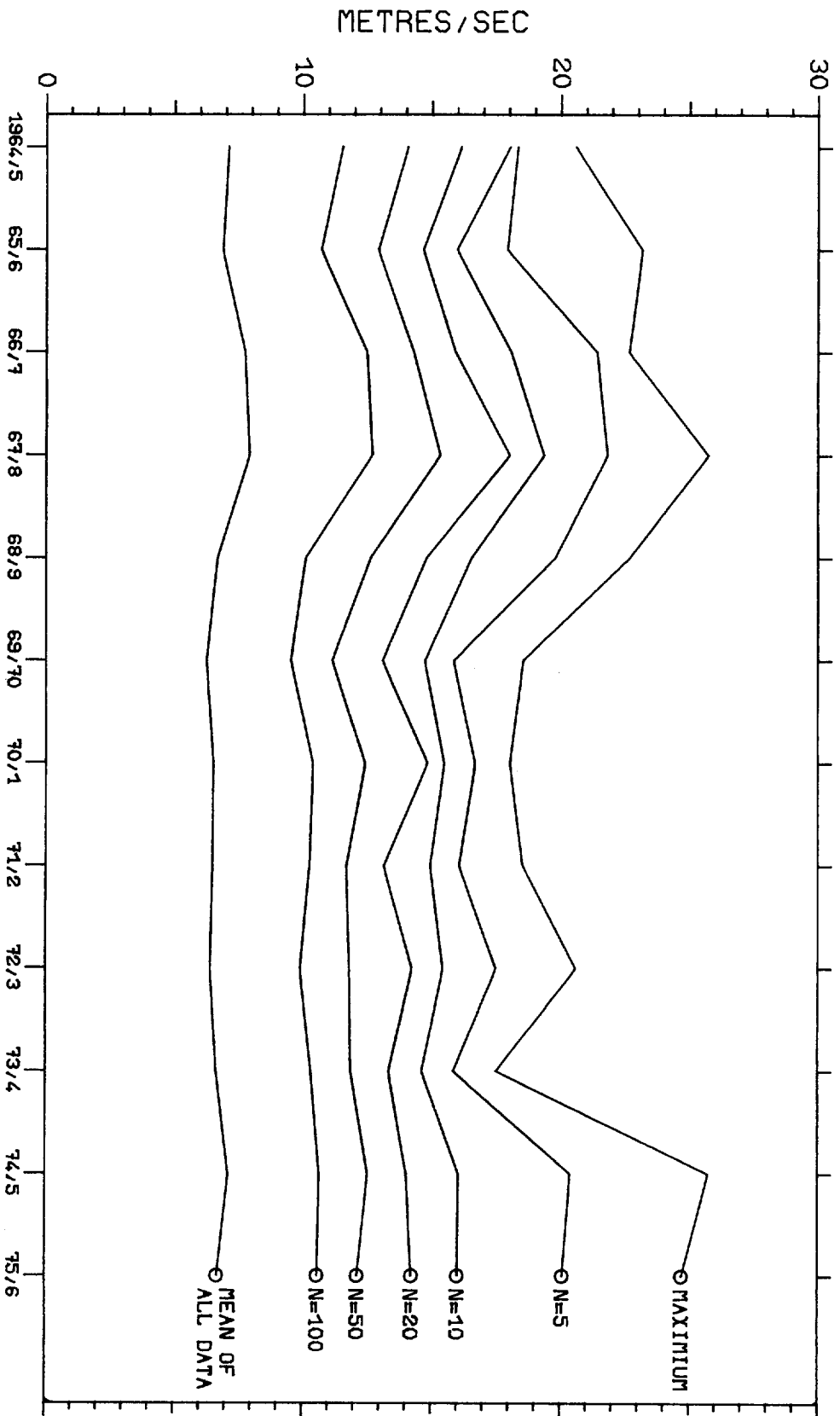
FIG 2.1

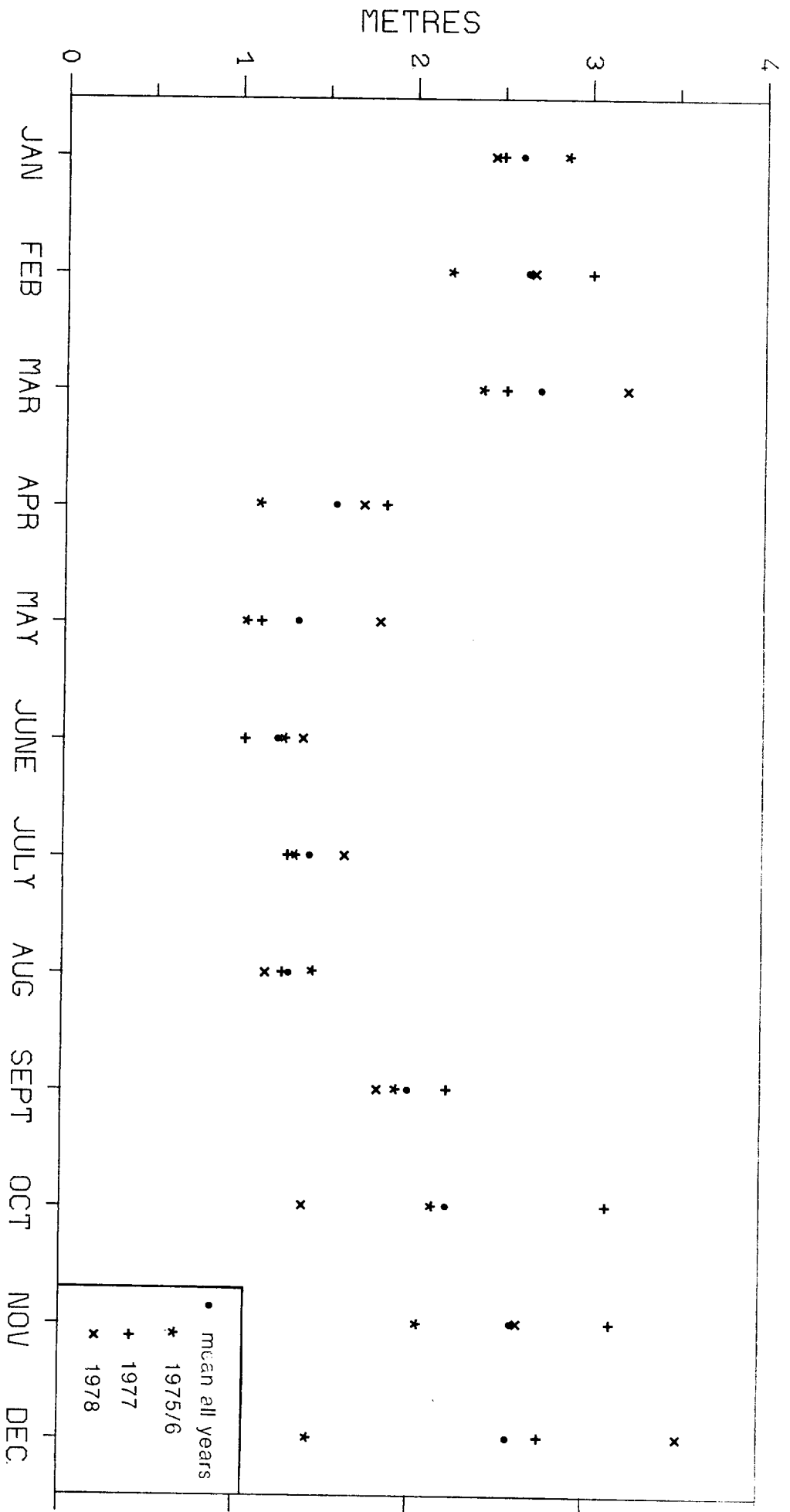


MEAN OF N LARGEST VALUES OF WIND SPEED
MILFORD HAVEN JAN 1965 - DEC 1978

FIG 2.2(A)

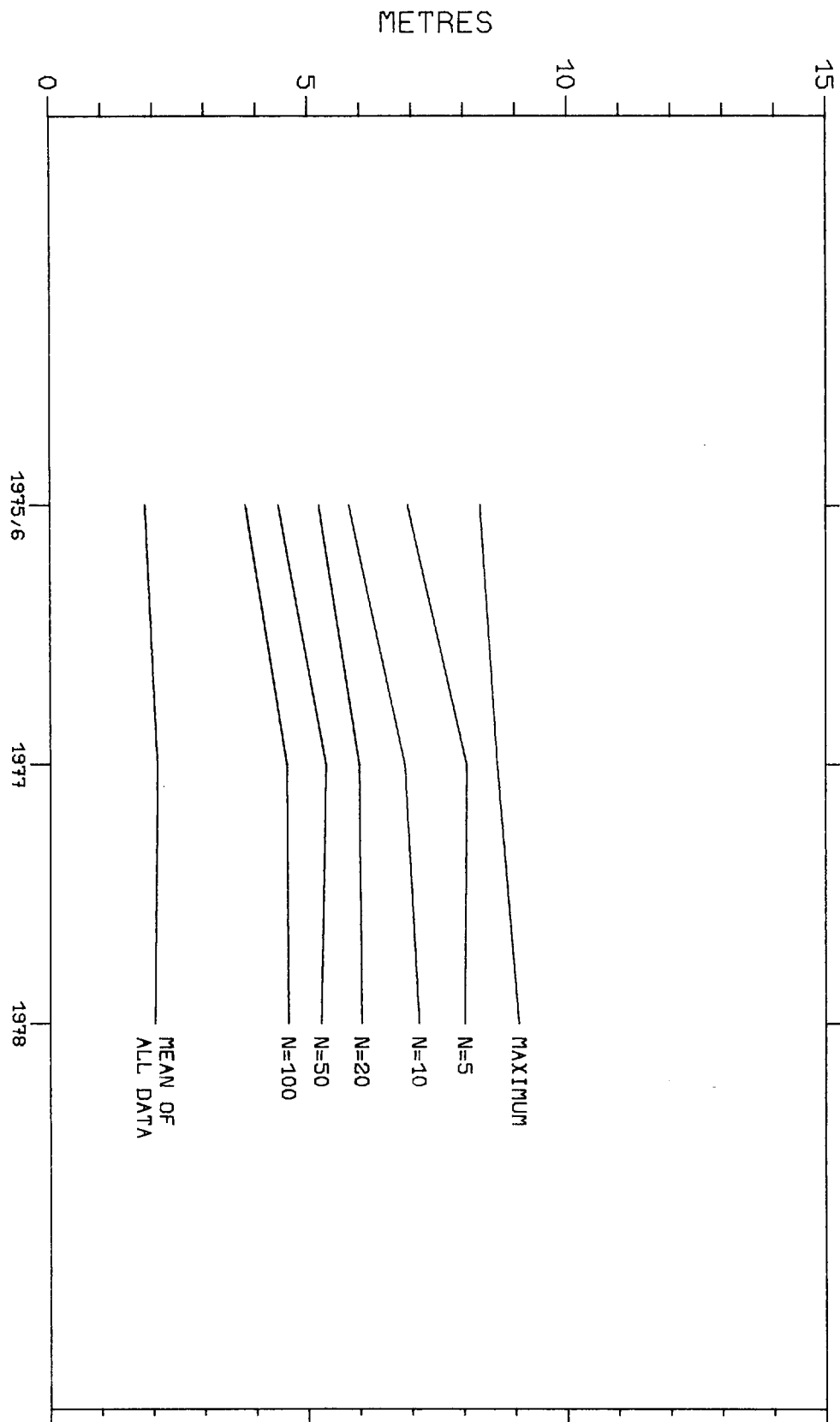
MEAN OF N LARGEST VALUES OF WIND SPEED
MILFORD HAVEN AUG 1964 - JUL 1976





MONTHLY MEAN OF Hs FOR EACH YEAR
 ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

FIG 3.1.1

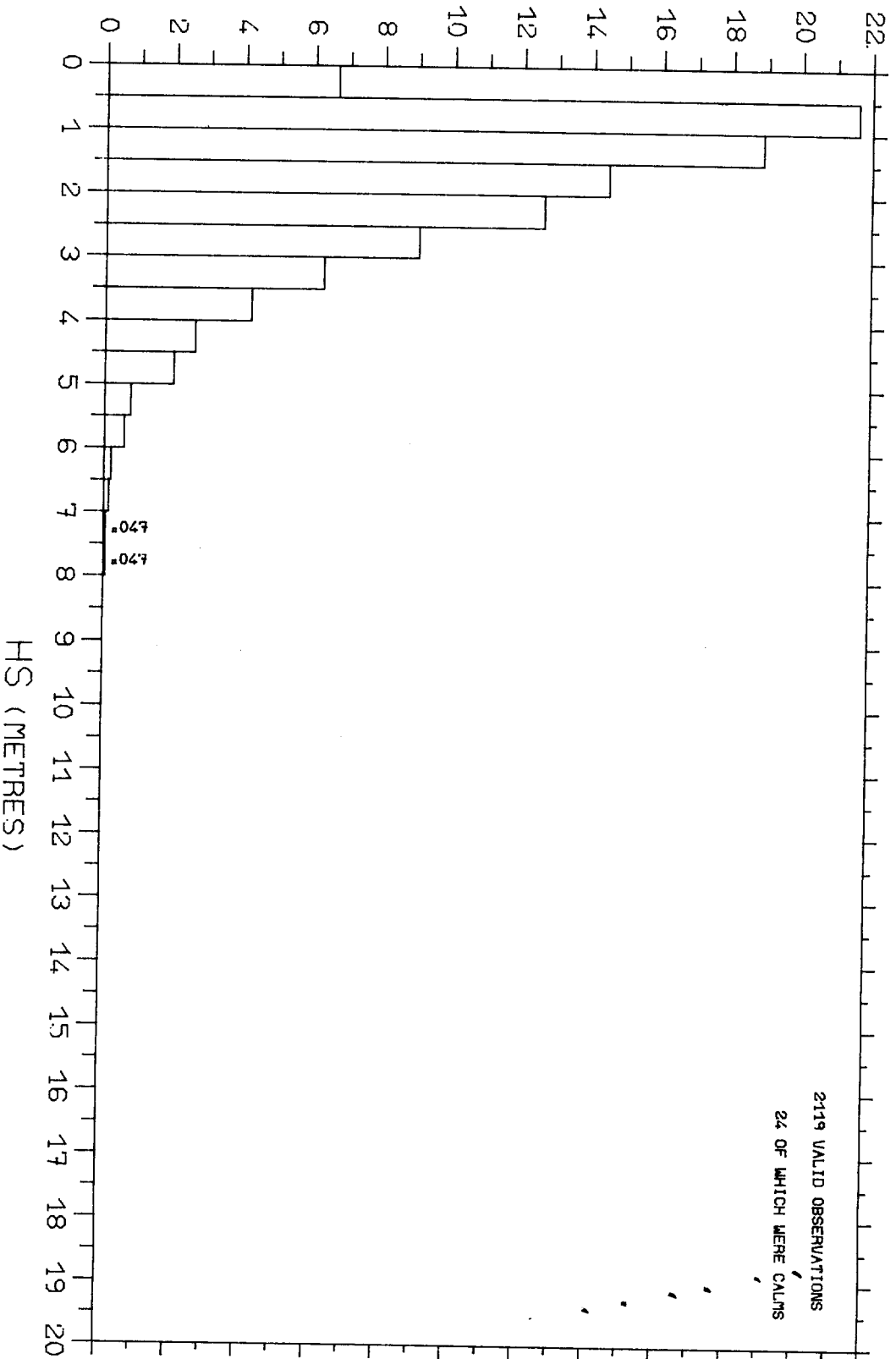


MEAN OF N LARGEST VALUES OF H_s

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

FIG 3.1.2

PERCENTAGE OCCURRENCE

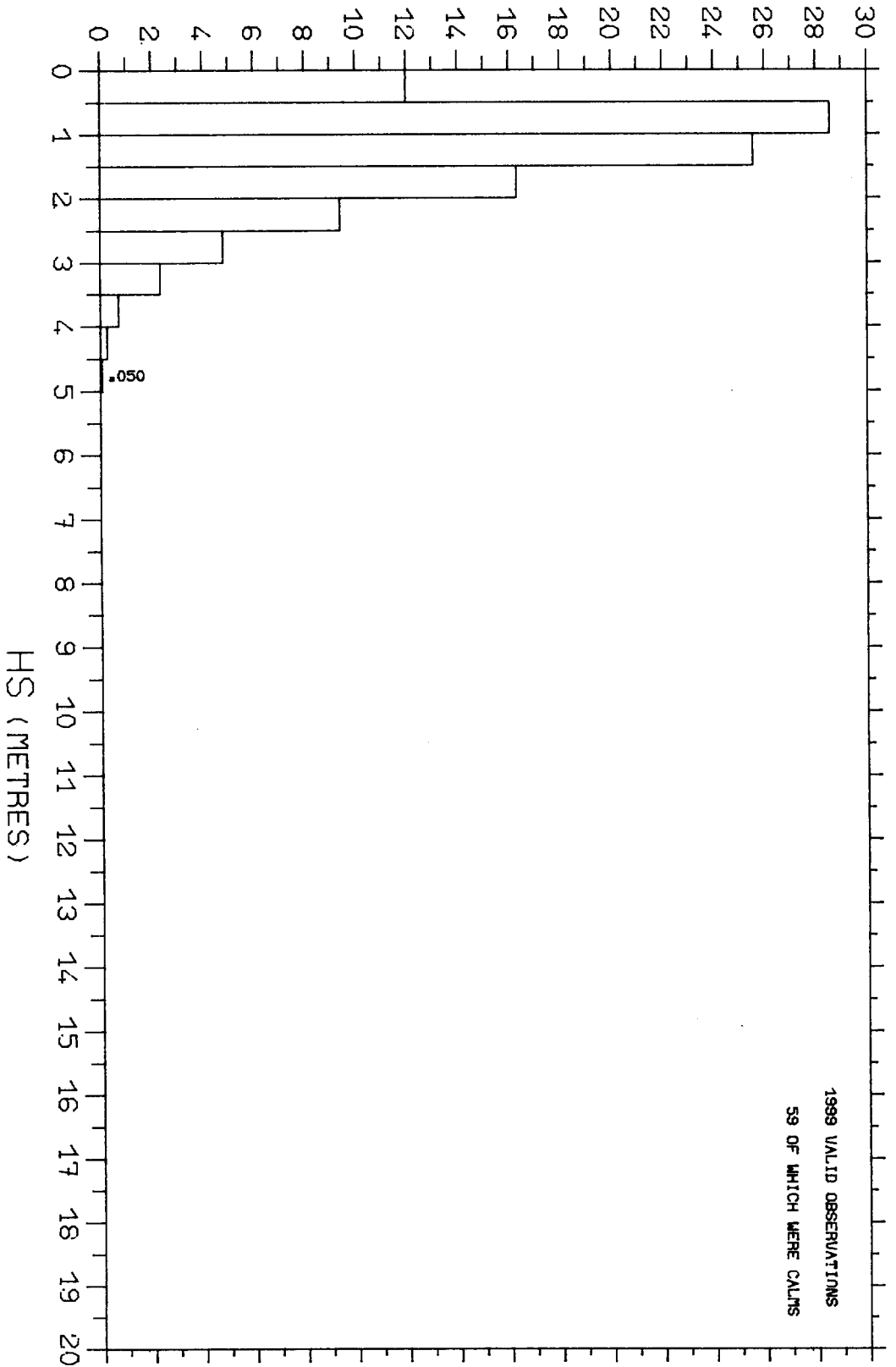


PERCENTAGE OCCURRENCE HISTOGRAM

ST. GOWAN LV 1975/6, 1977, 1978 - SPRINGS

FIG 3.2.1.1

PERCENTAGE OCCURRENCE

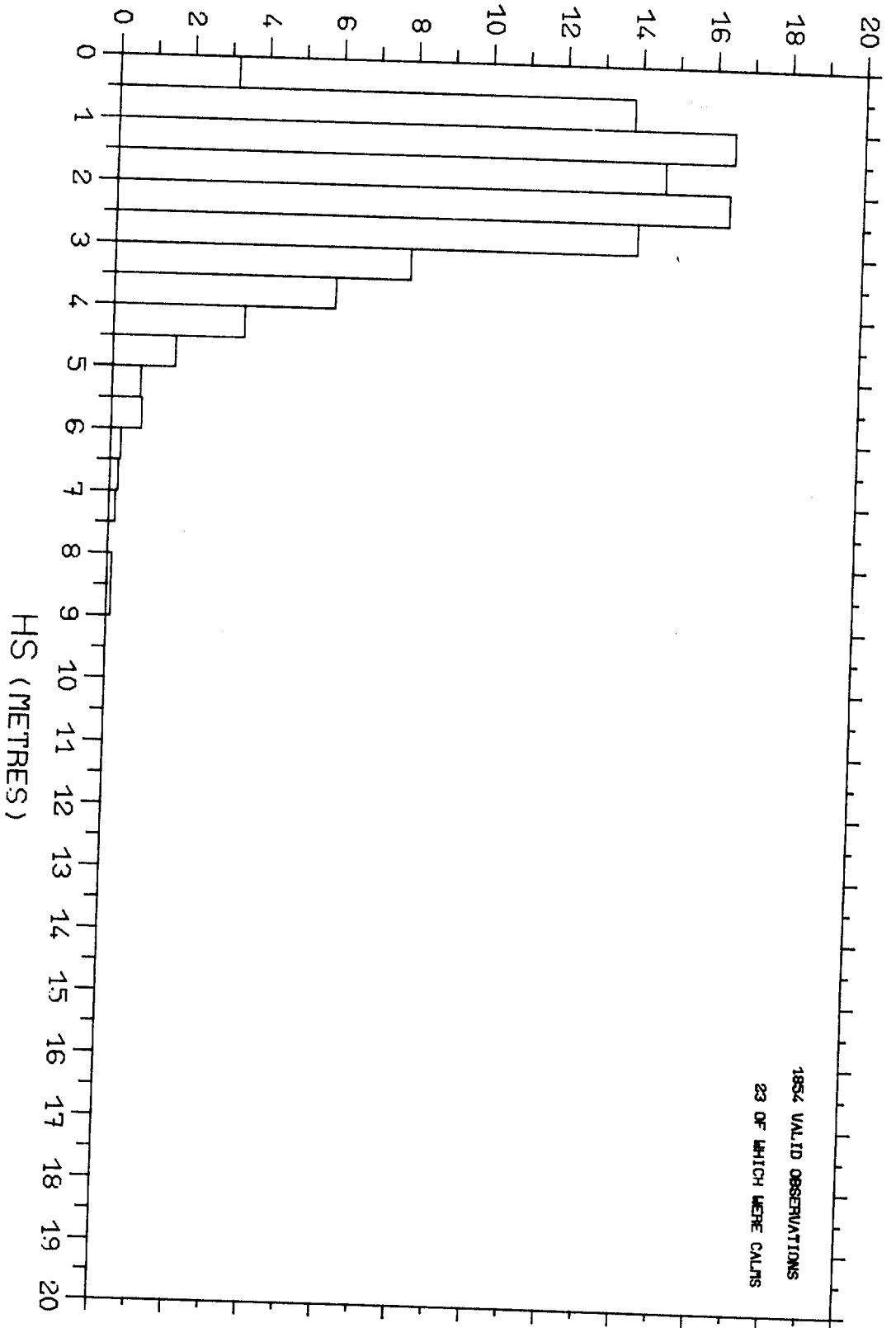


PERCENTAGE OCCURRENCE HISTOGRAM

ST. GOWAN LV 1975/6, 1977, 1978 - SUMMERS

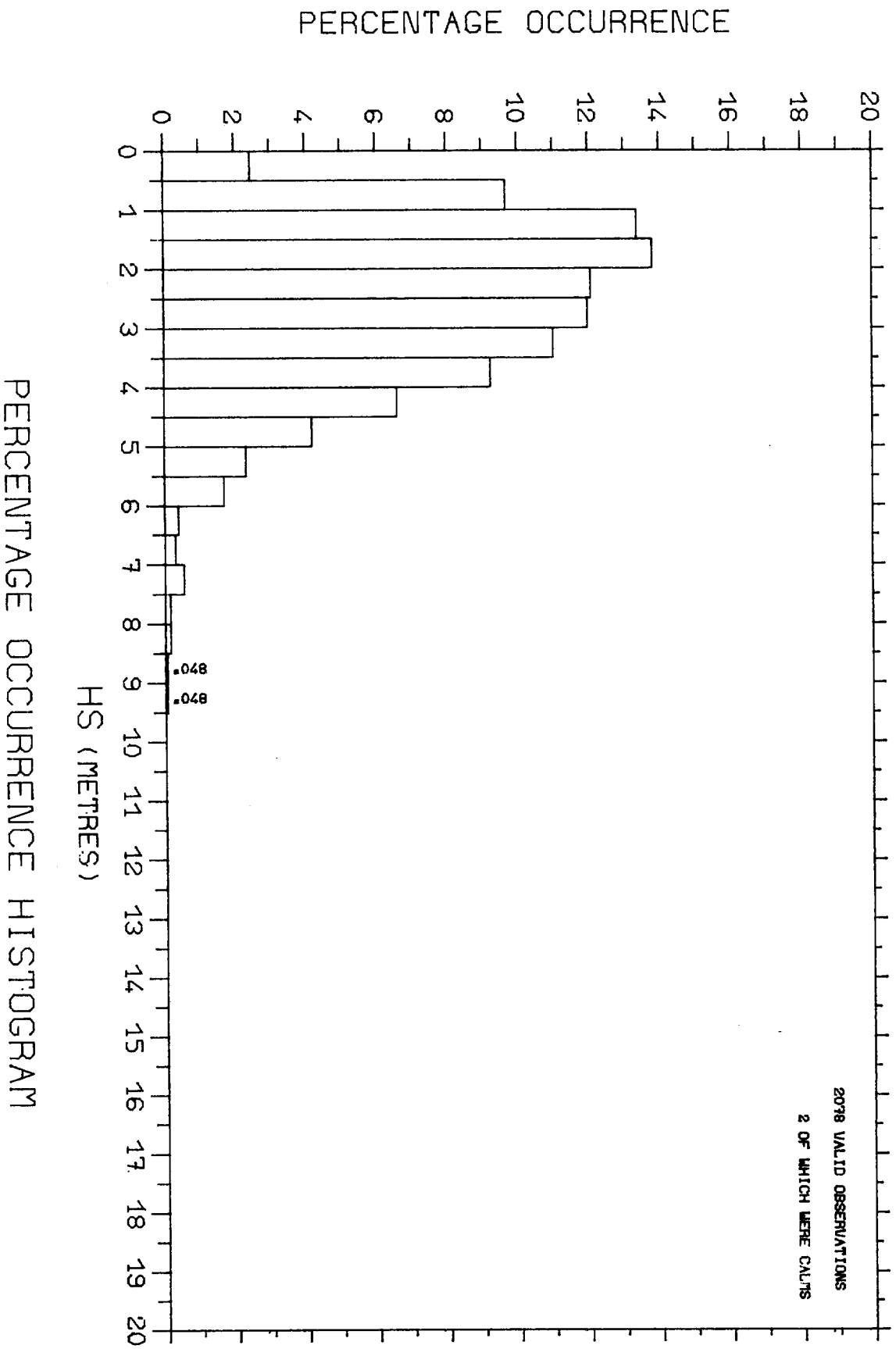
FIG 3.2.1.2

PERCENTAGE OCCURRENCE



PERCENTAGE OCCURRENCE HISTOGRAM

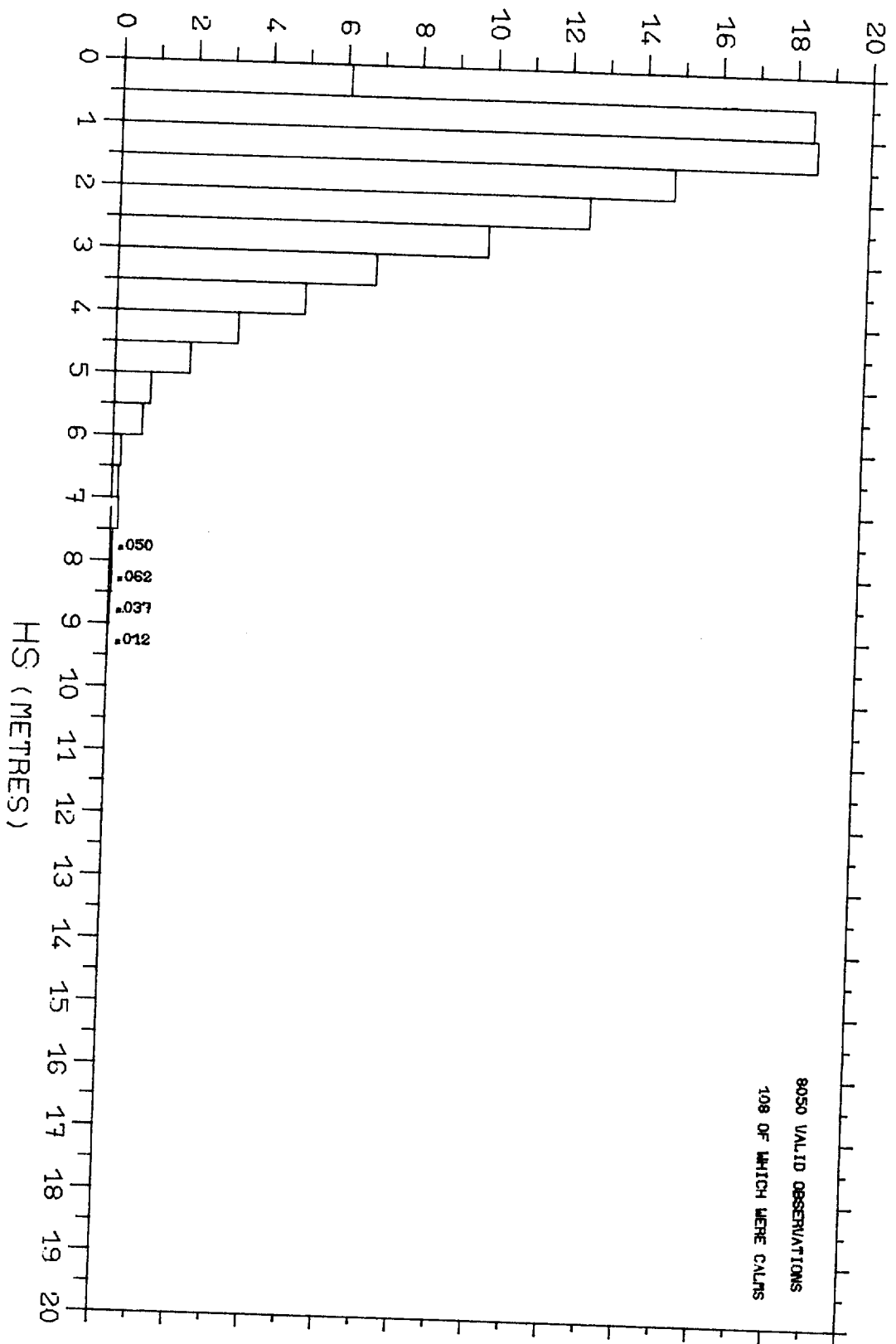
ST. GOWAN LV 1975/6, 1977, 1978 - AUTUMNS



ST. GOWAN LV 1975/6, 1977, 1978 - WINTERS

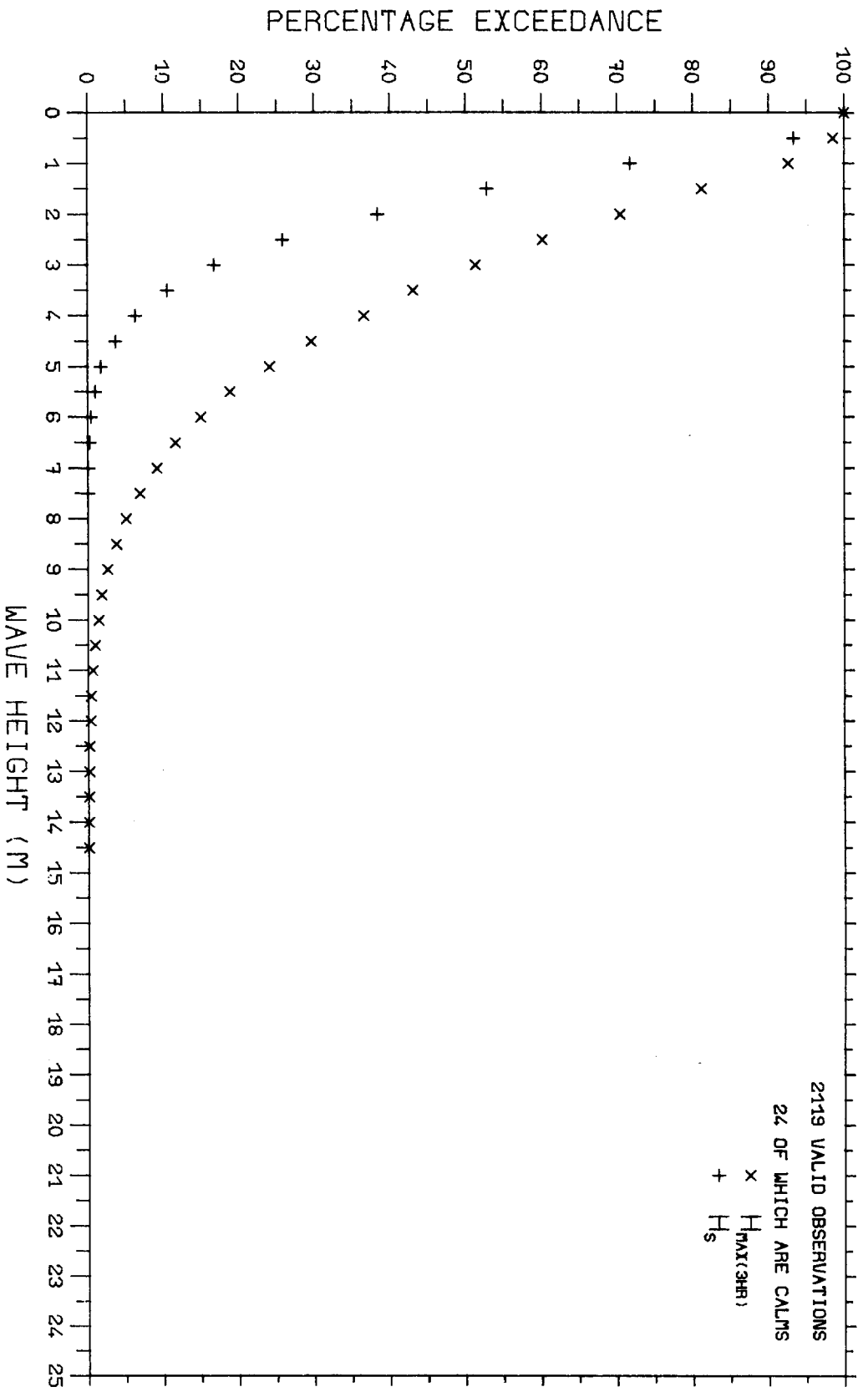
FIG 3.2.1.4

PERCENTAGE OCCURRENCE



PERCENTAGE OCCURRENCE HISTOGRAM

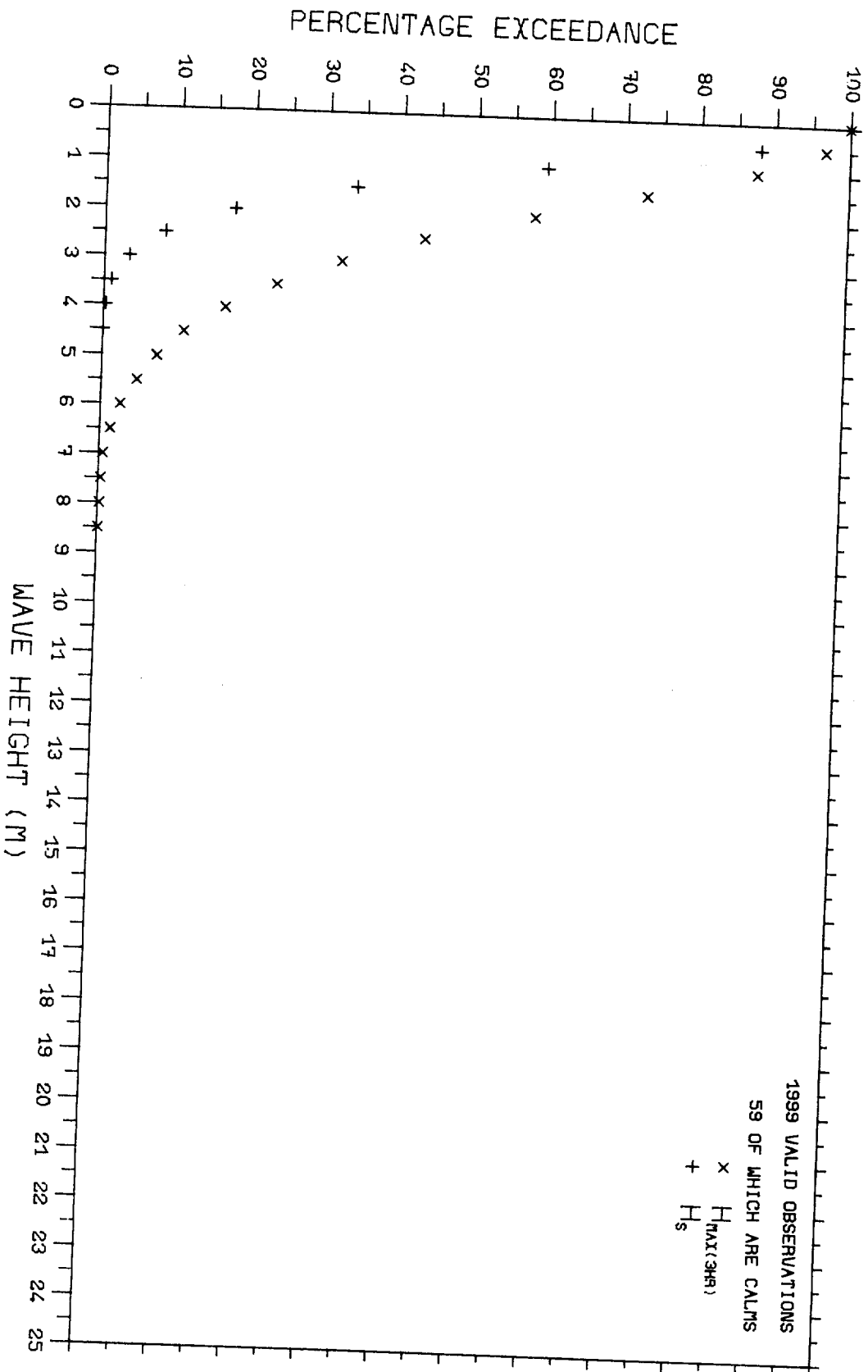
ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978



PERCENTAGE EXCEEDANCE OF H_S AND H_{MAX(3HR)}

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978 - SPRINGS

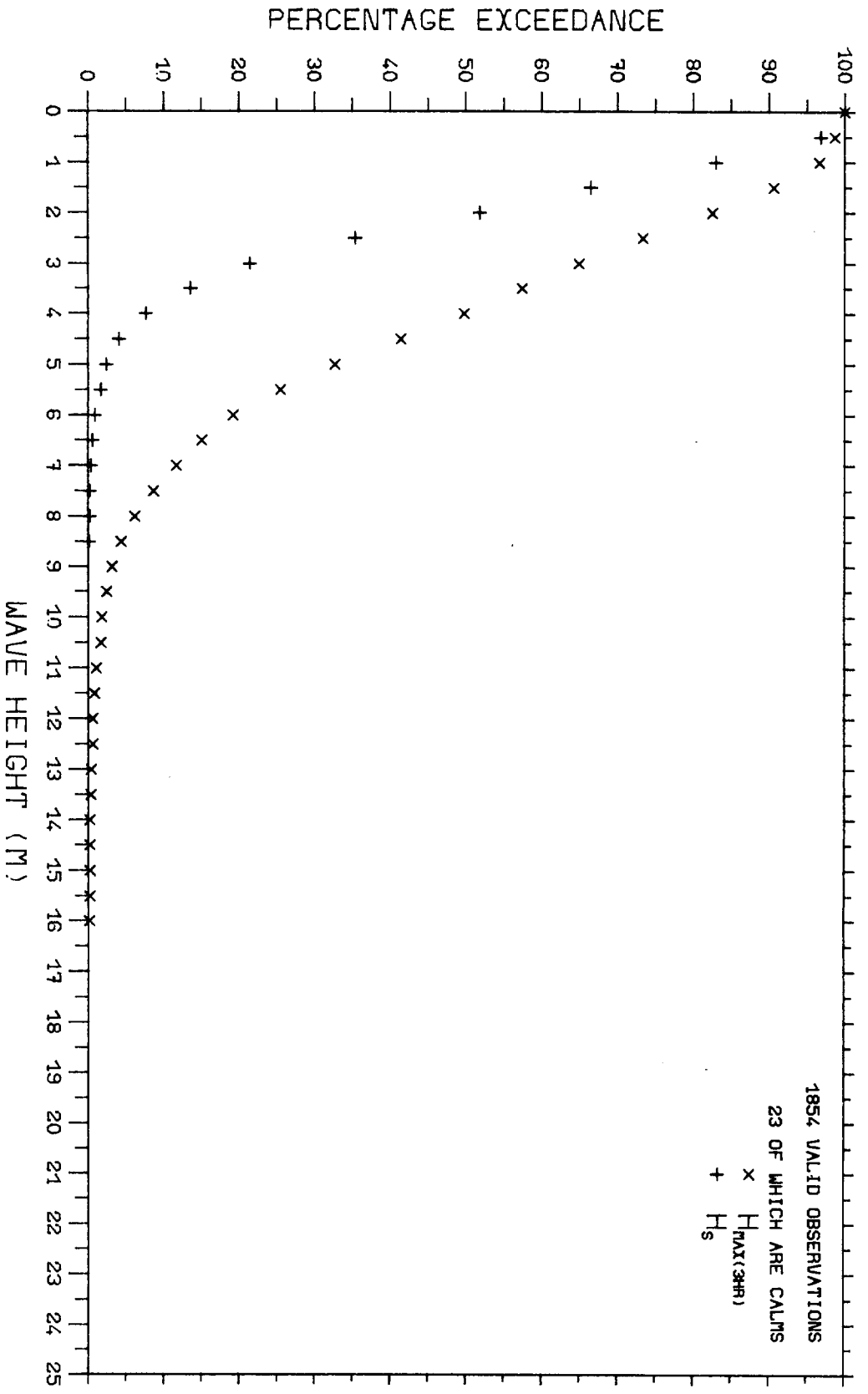
FIG 3.2.2.1



PERCENTAGE EXCEEDANCE OF H_s AND $H_{max(3hr)}$

ST. GOMAN LV AUG 1975 - JUL 1976, 1977, 1978 - SUMMERS

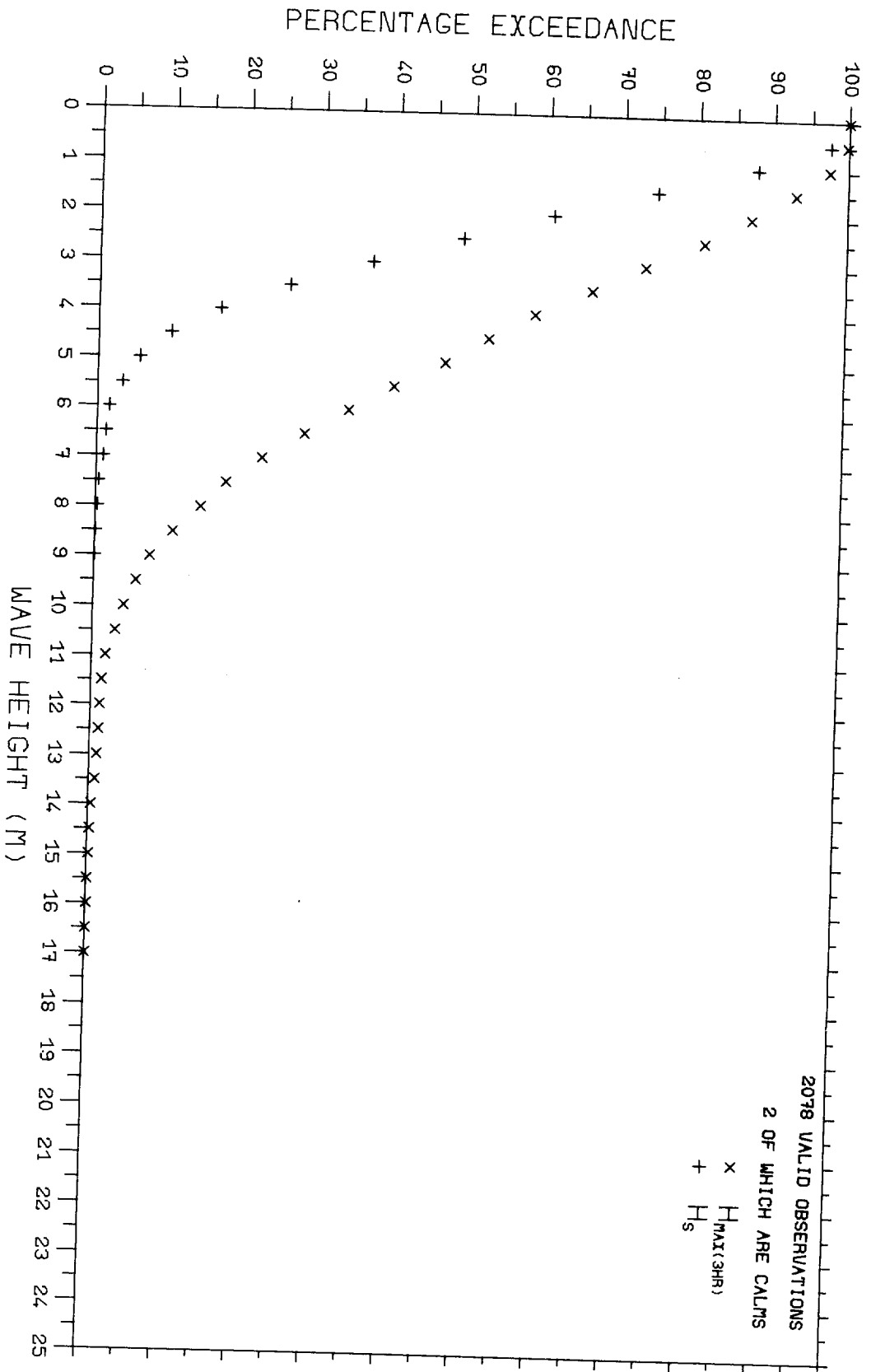
FIG 3.2.2.2



PERCENTAGE EXCEEDANCE OF H_s AND $H_{MAX(3HR)}$

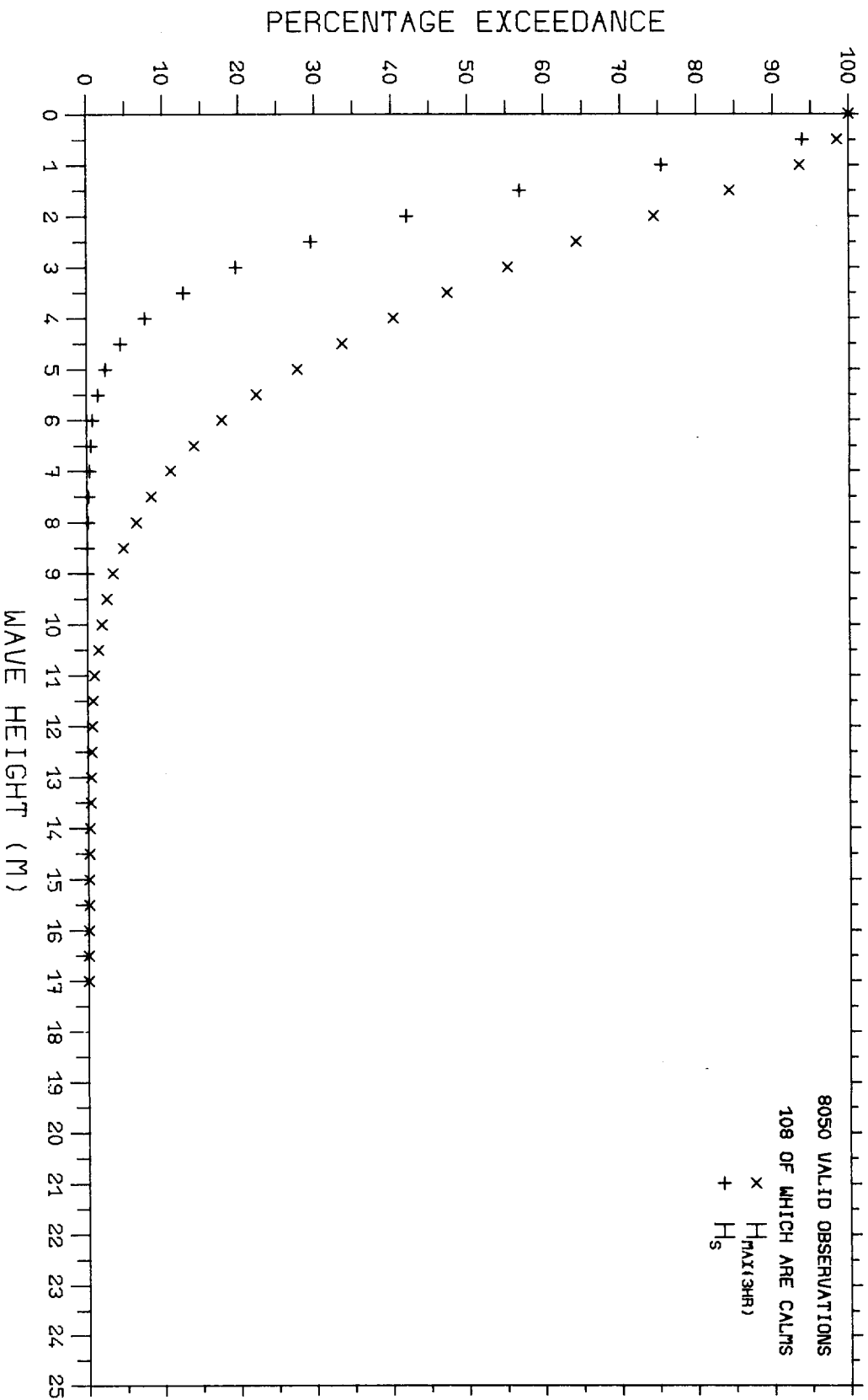
ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978 - AUTUMNS

FIG 3.2.2.3



ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978 - WINTERS

PERCENTAGE EXCEEDANCE OF H_s AND $H_{\text{MAX}(3HR)}$

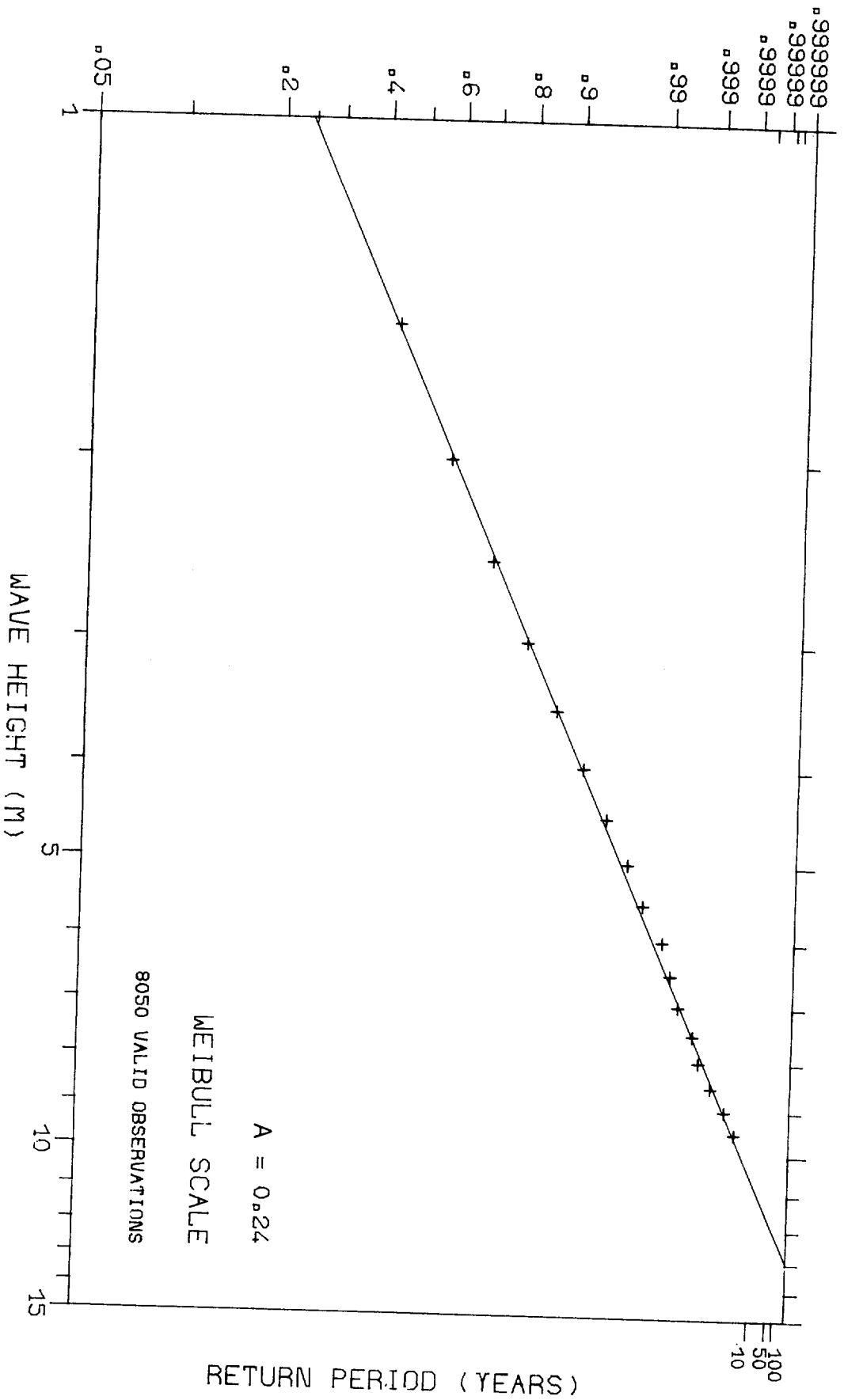


PERCENTAGE EXCEEDANCE OF H_s AND $H_{MAX(3HR)}$

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

FIG 3.2.2.5

PROBABILITY OF NON-EXCEEDANCE



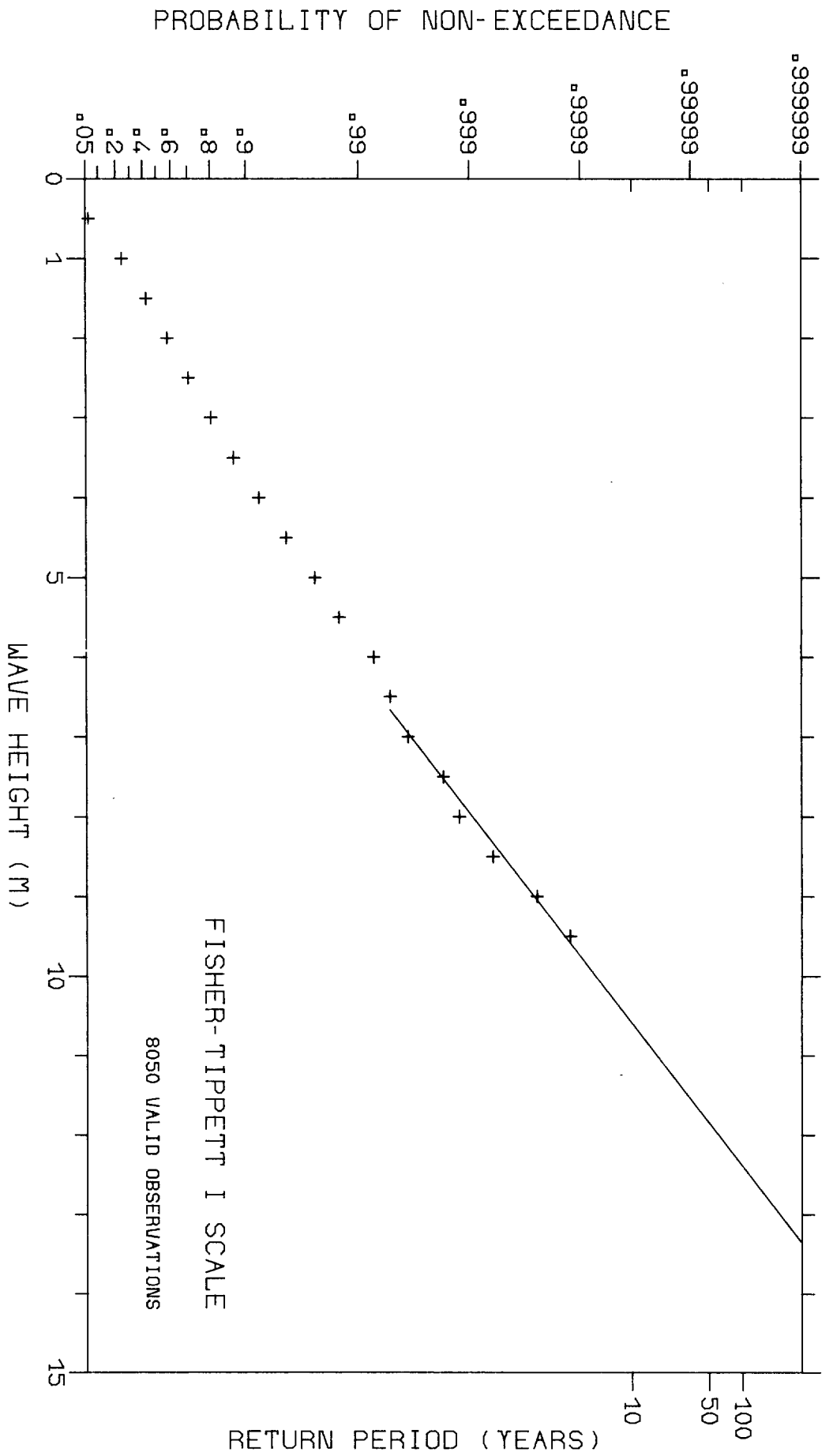
CUMULATIVE DISTRIBUTION OF WAVE HEIGHT, HS

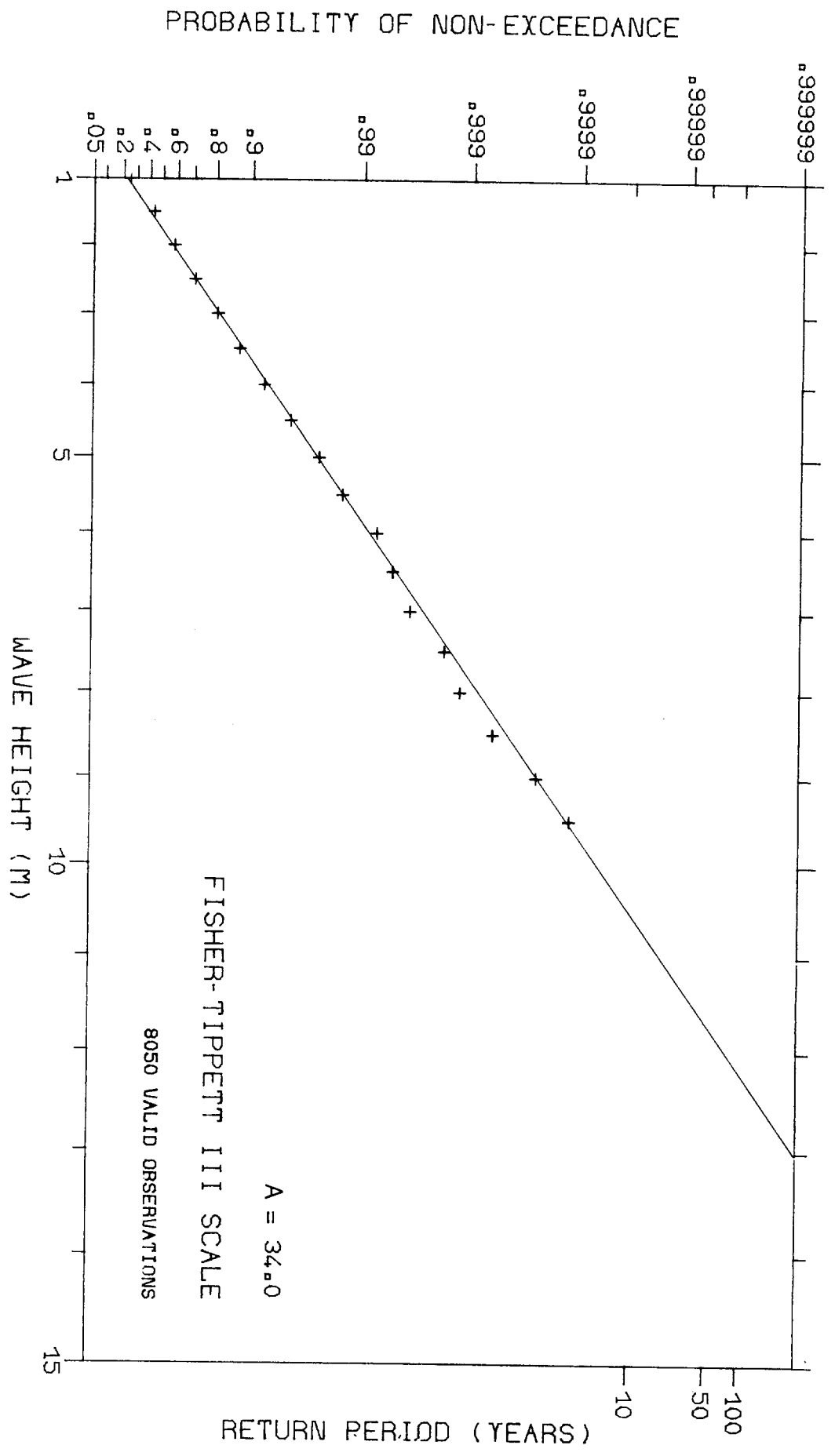
ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

FIG 3.3.1

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

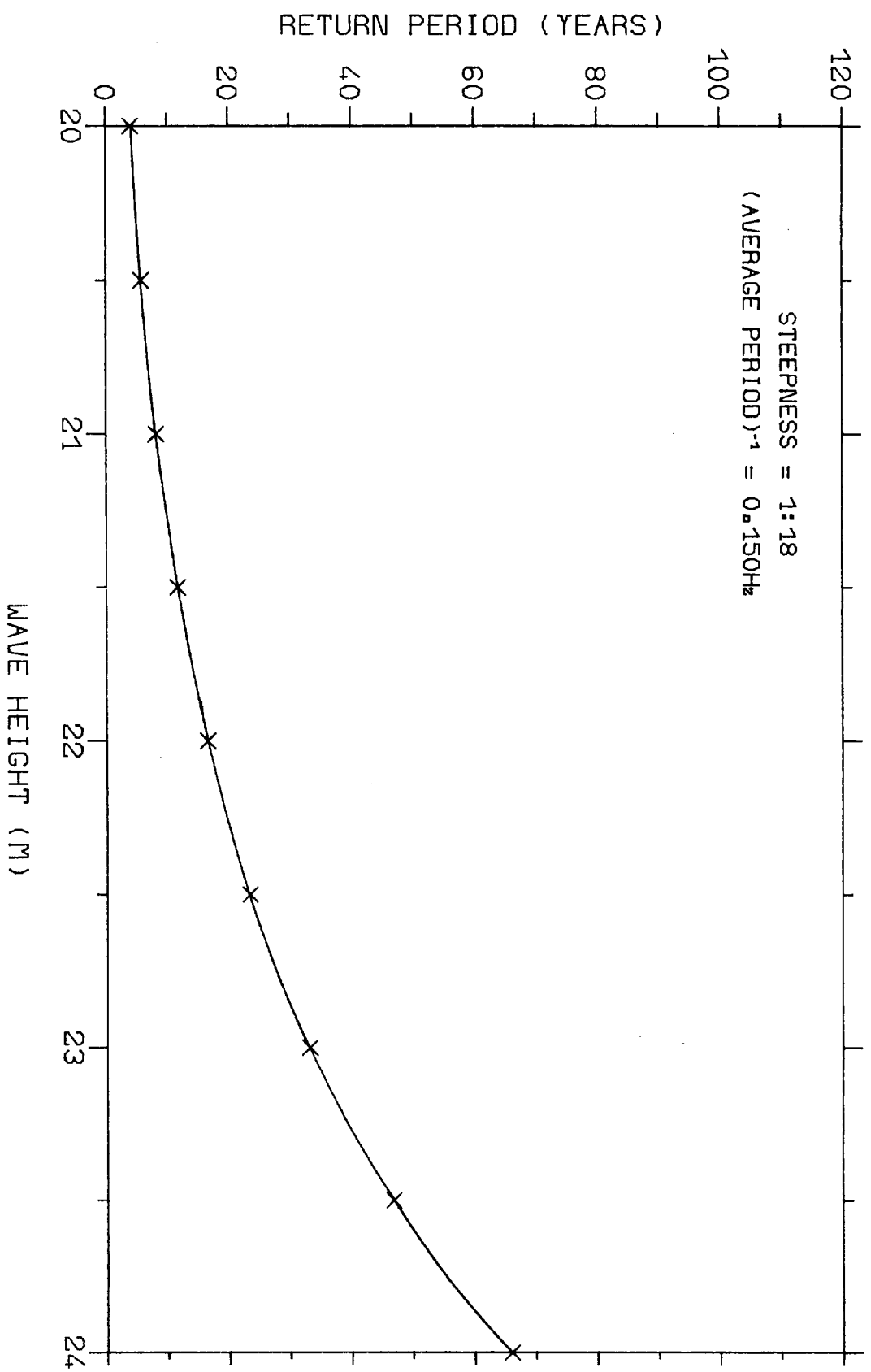
FIG 3.3.2





CUMULATIVE DISTRIBUTION OF WAVE HEIGHT, HS
ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

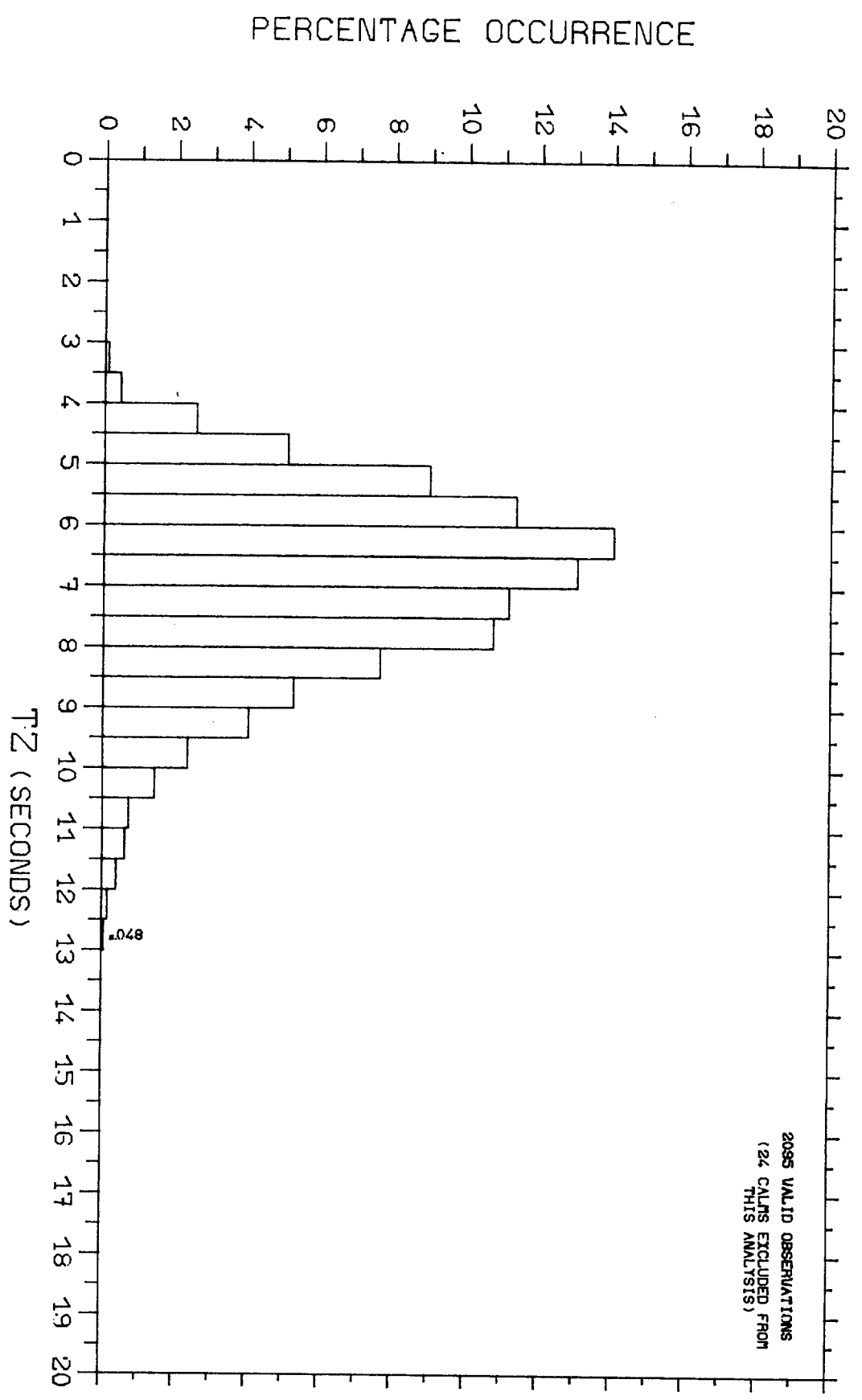
FIG 3.3.3

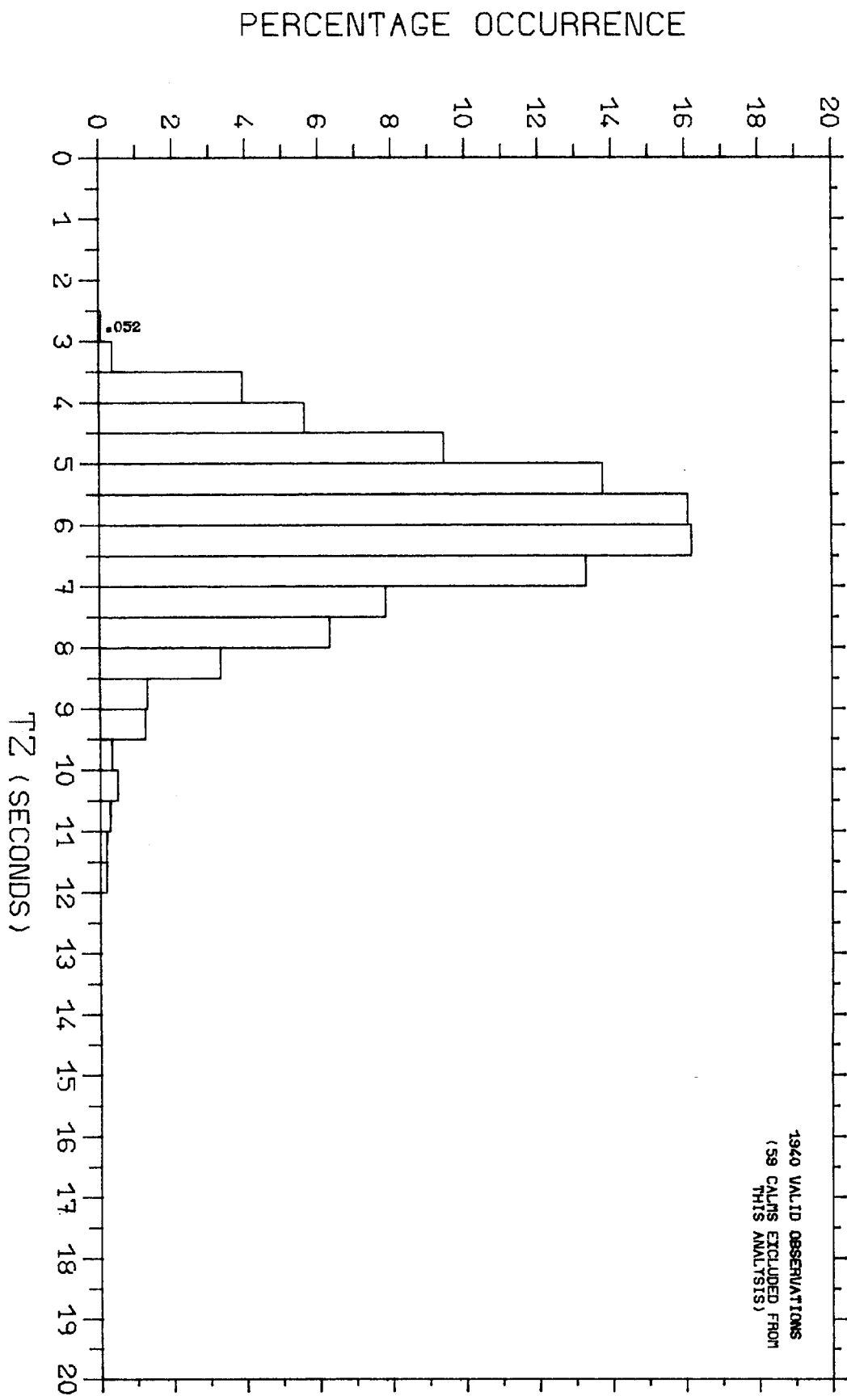


RETURN PERIOD v. WAVE HEIGHT - INDIVIDUAL WAVE MODEL
ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

ST. GOWAN LV 1975/6, 1977, 1978 - SPRINGS

FIG 3.4.1.1

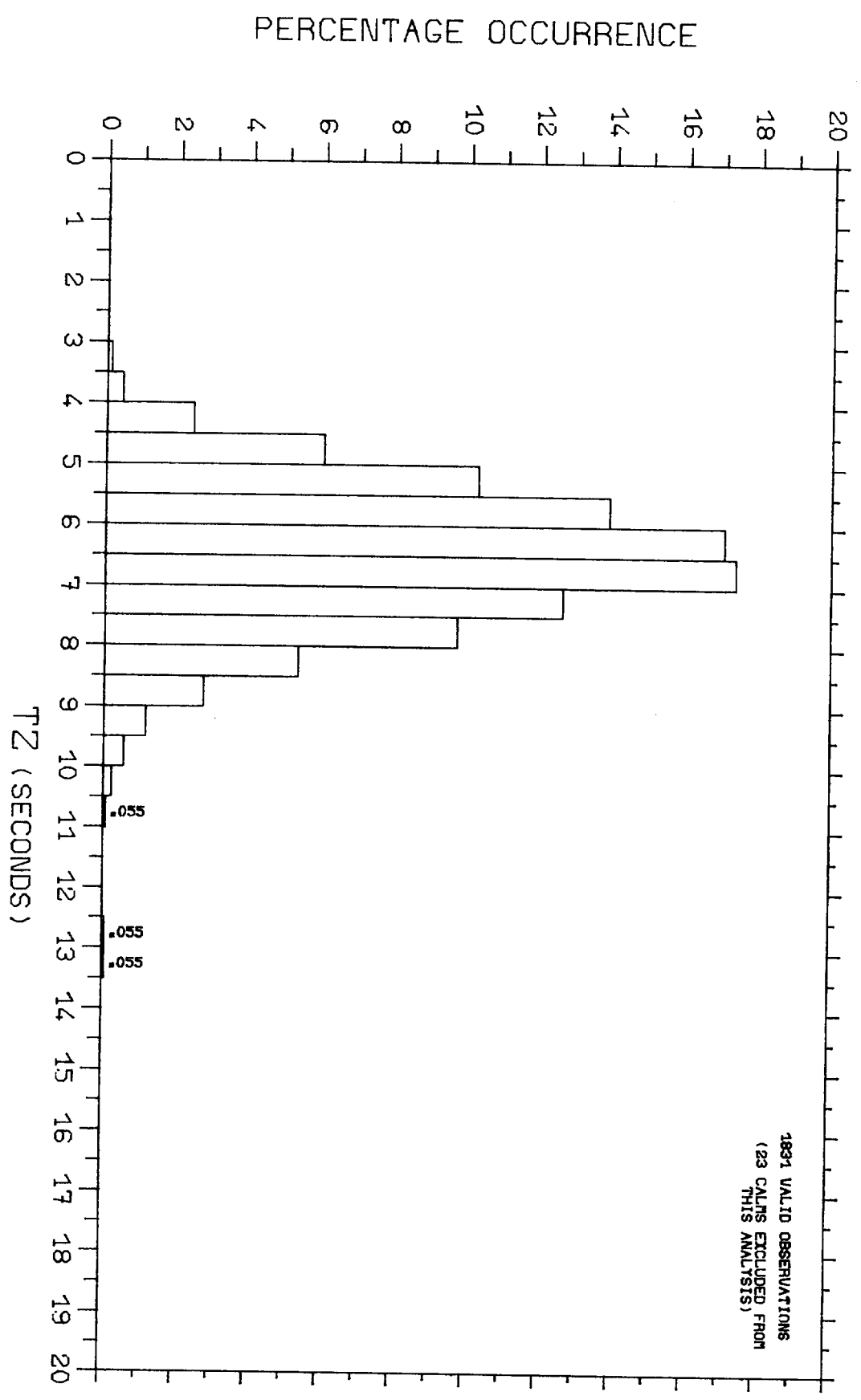




PERCENTAGE OCCURRENCE HISTOGRAM

ST. GOWAN LV 1975/6, 1977, 1978 - SUMMERS

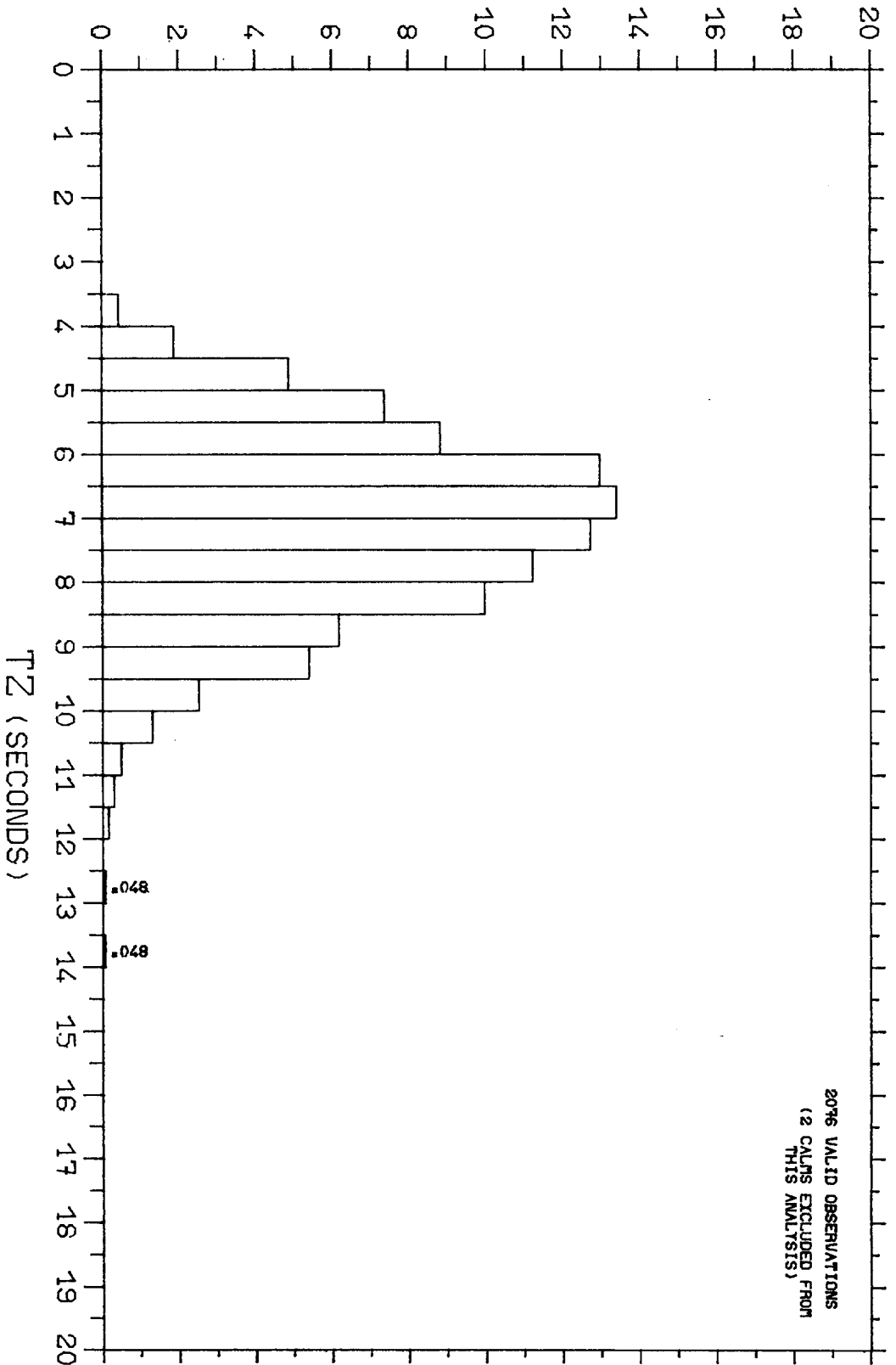
FIG 3.4.1.2



PERCENTAGE OCCURRENCE HISTOGRAM
ST. GOWAN LV 1975/6, 1977, 1978 - AUTUMNS

FIG 3.4.1.3

PERCENTAGE OCCURRENCE

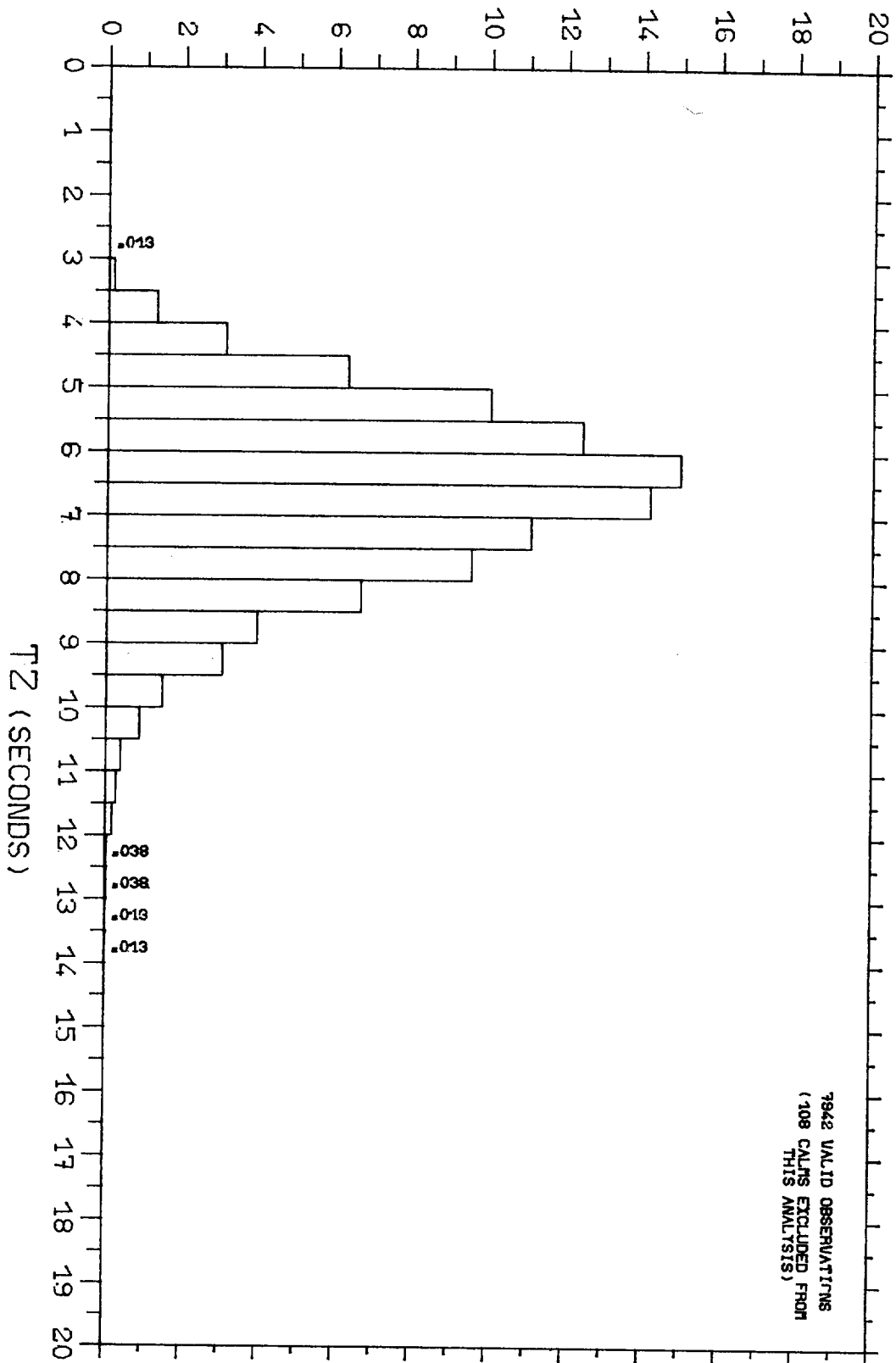


PERCENTAGE OCCURRENCE HISTOGRAM

ST. GOWAN LV 1975/6, 1977, 1978 - WINTERS

FIG 3.4.1.4

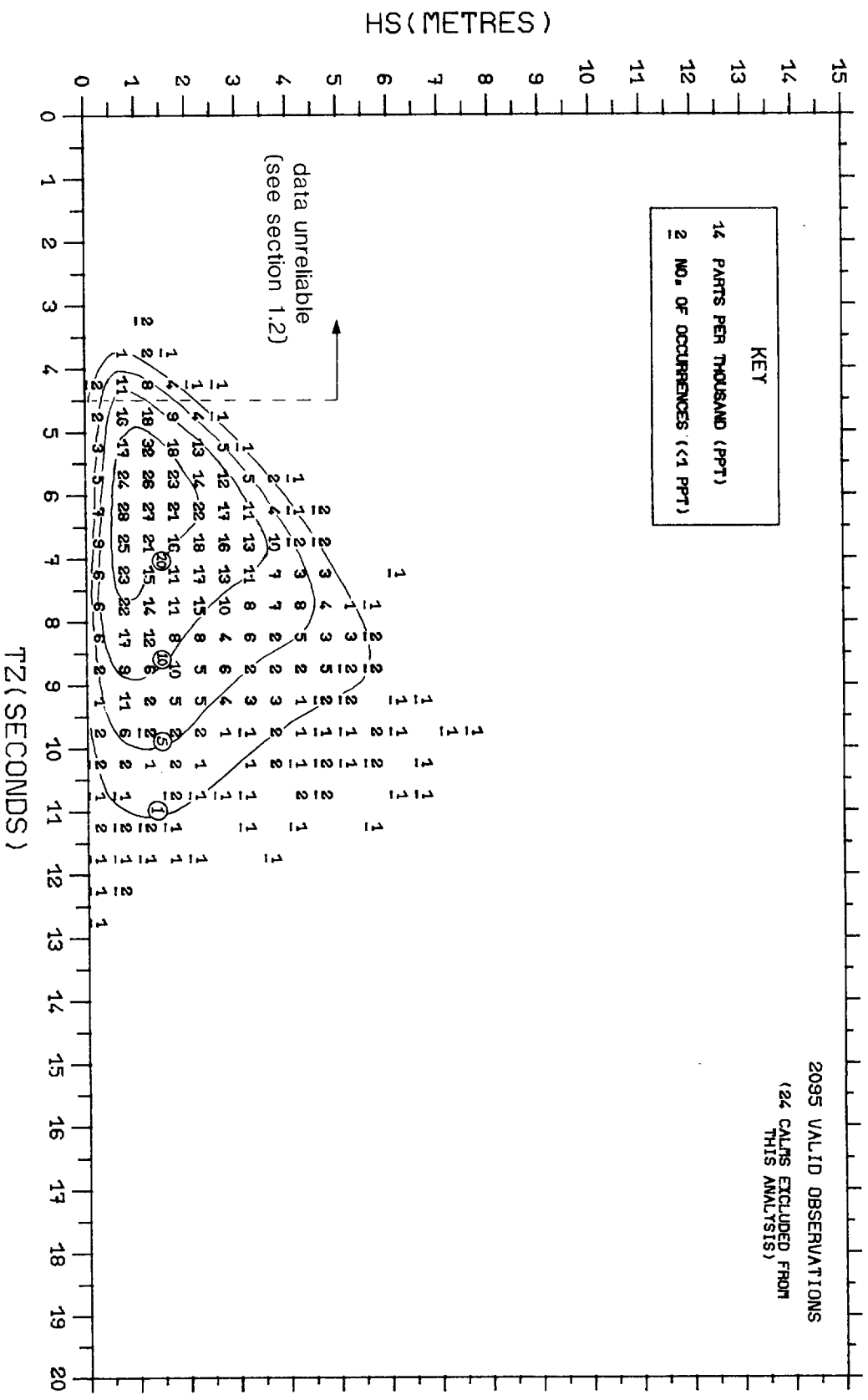
PERCENTAGE OCCURRENCE



PERCENTAGE OCCURRENCE HISTOGRAM

ST. GOWAN LV AUG 1975 - JUL 1976, 1977, 1978

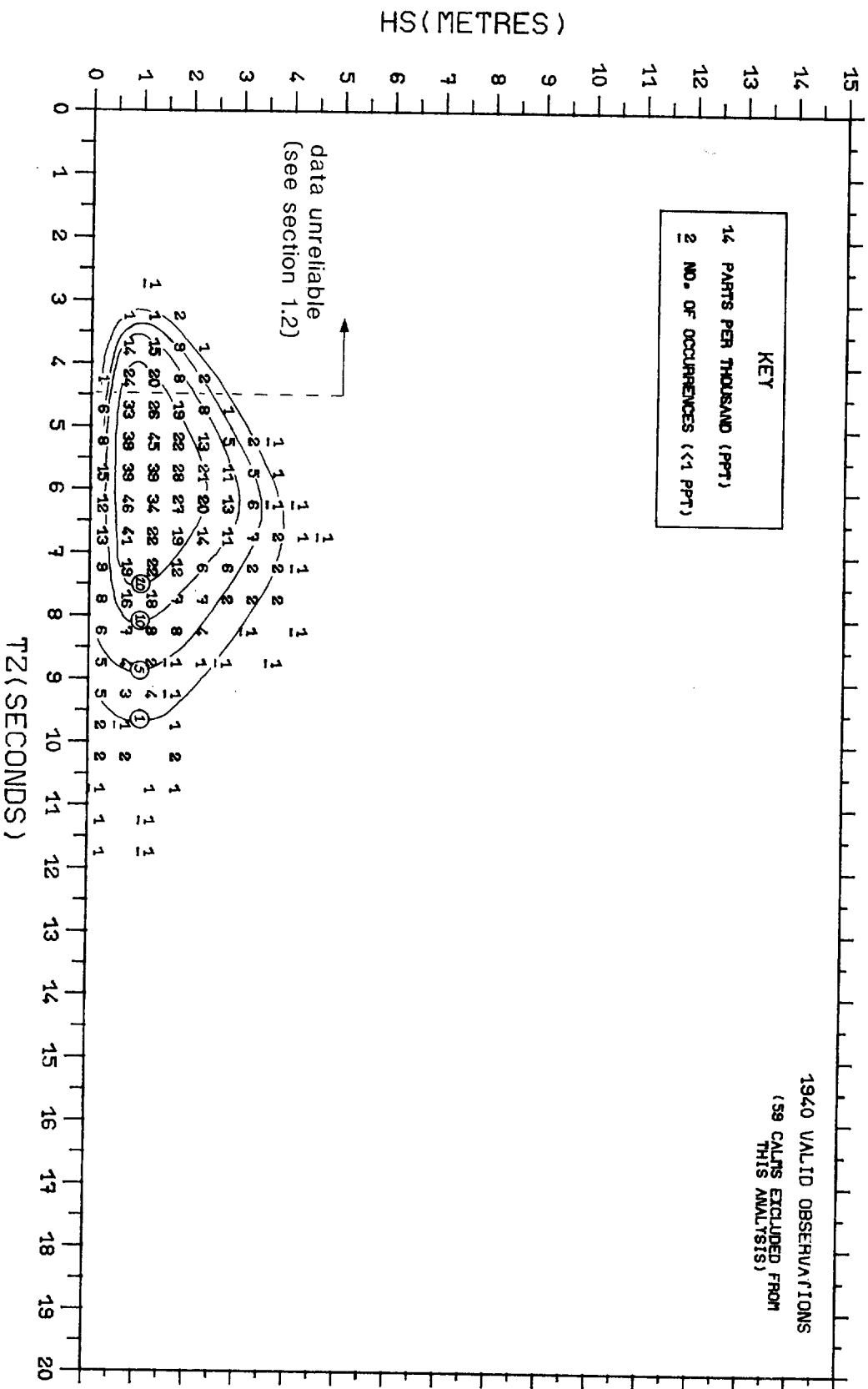
FIG 3.4.1.5



SCATTER PLOT OF HS AND TZ

ST. GOWAN LV 1975/6, 1977, 1978 - SPRINGS

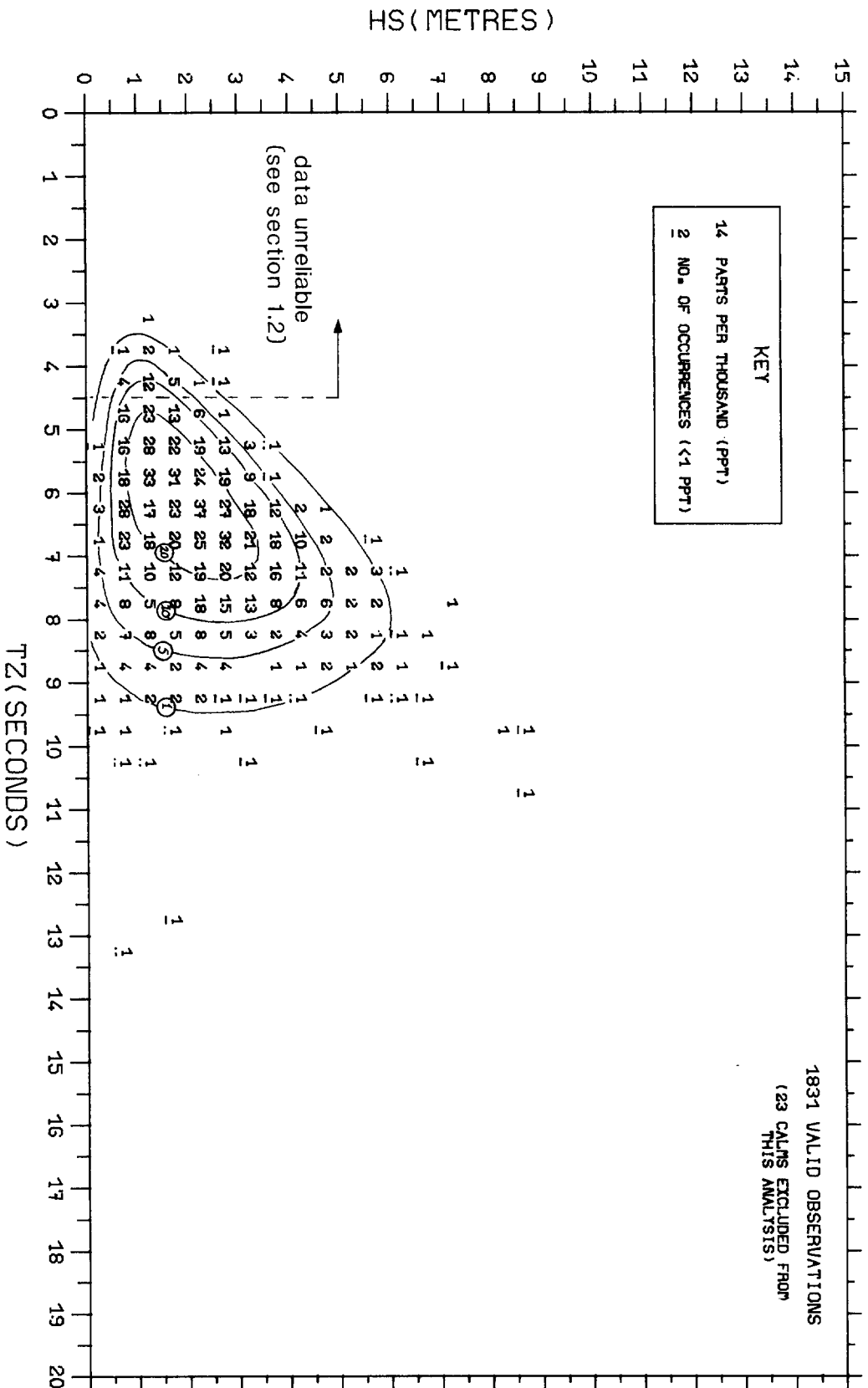
FIG 3.5.1.1



SCATTER PLOT OF HS AND TZ

ST. GOWAN LV 1975/6, 1977, 1978 - SUMMERS

FIG 3.5.1.2

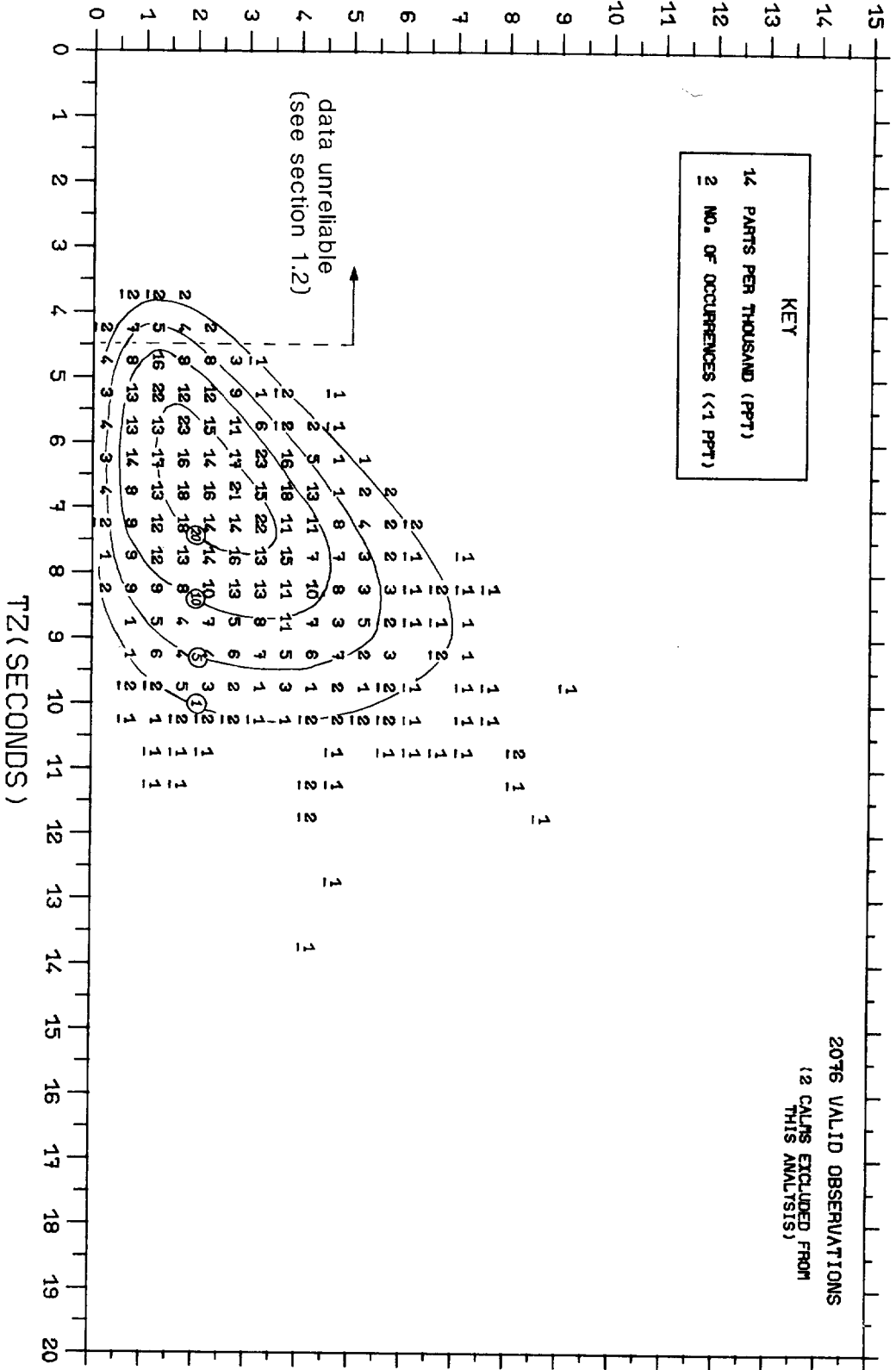


SCATTER PLOT OF HS AND TZ

ST. GOWAN LV 1975/6, 1977, 1978 - AUTUMNS

FIG 3.5.1.3

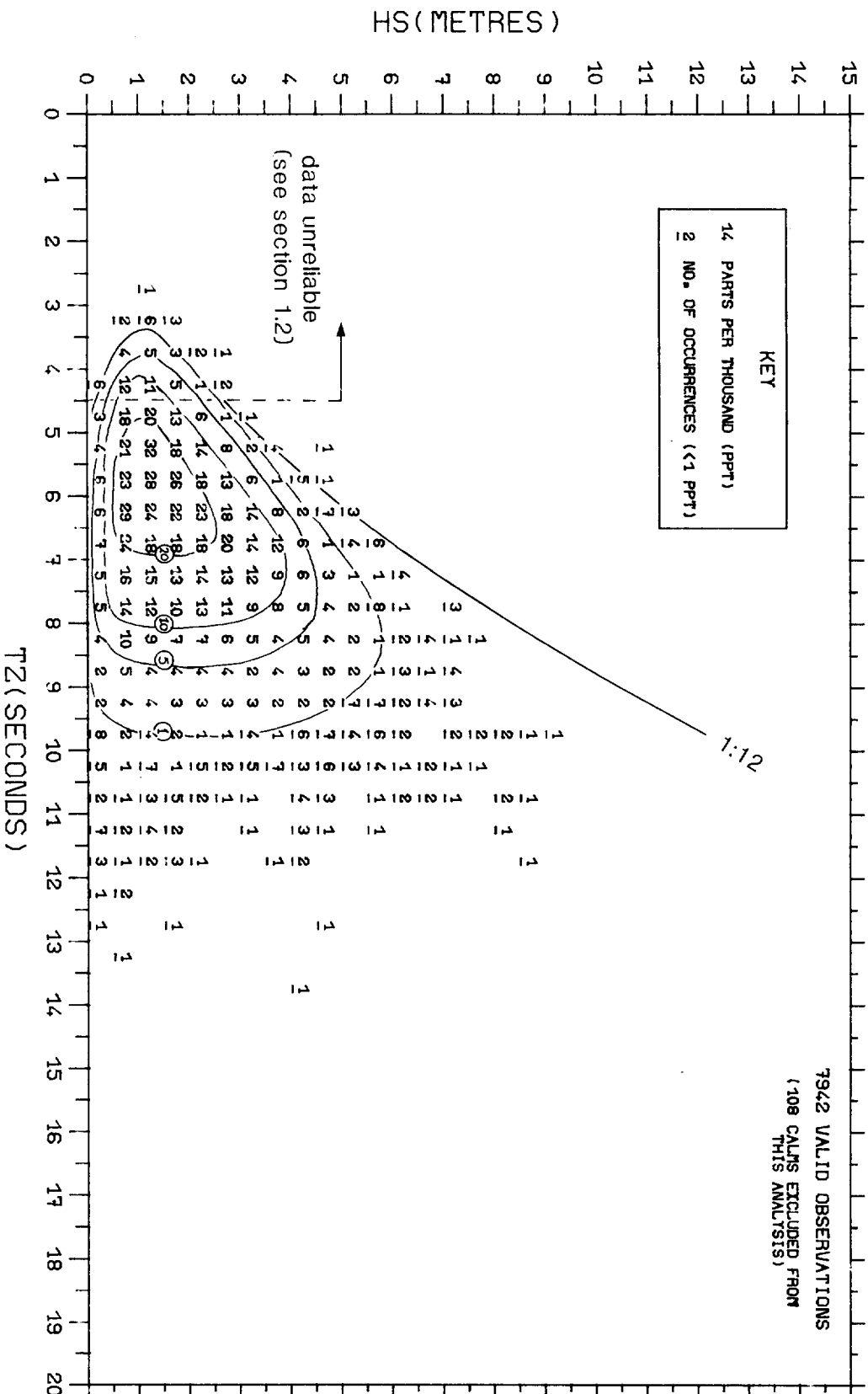
HS (METRES)



SCATTER PLOT OF HS AND TZ

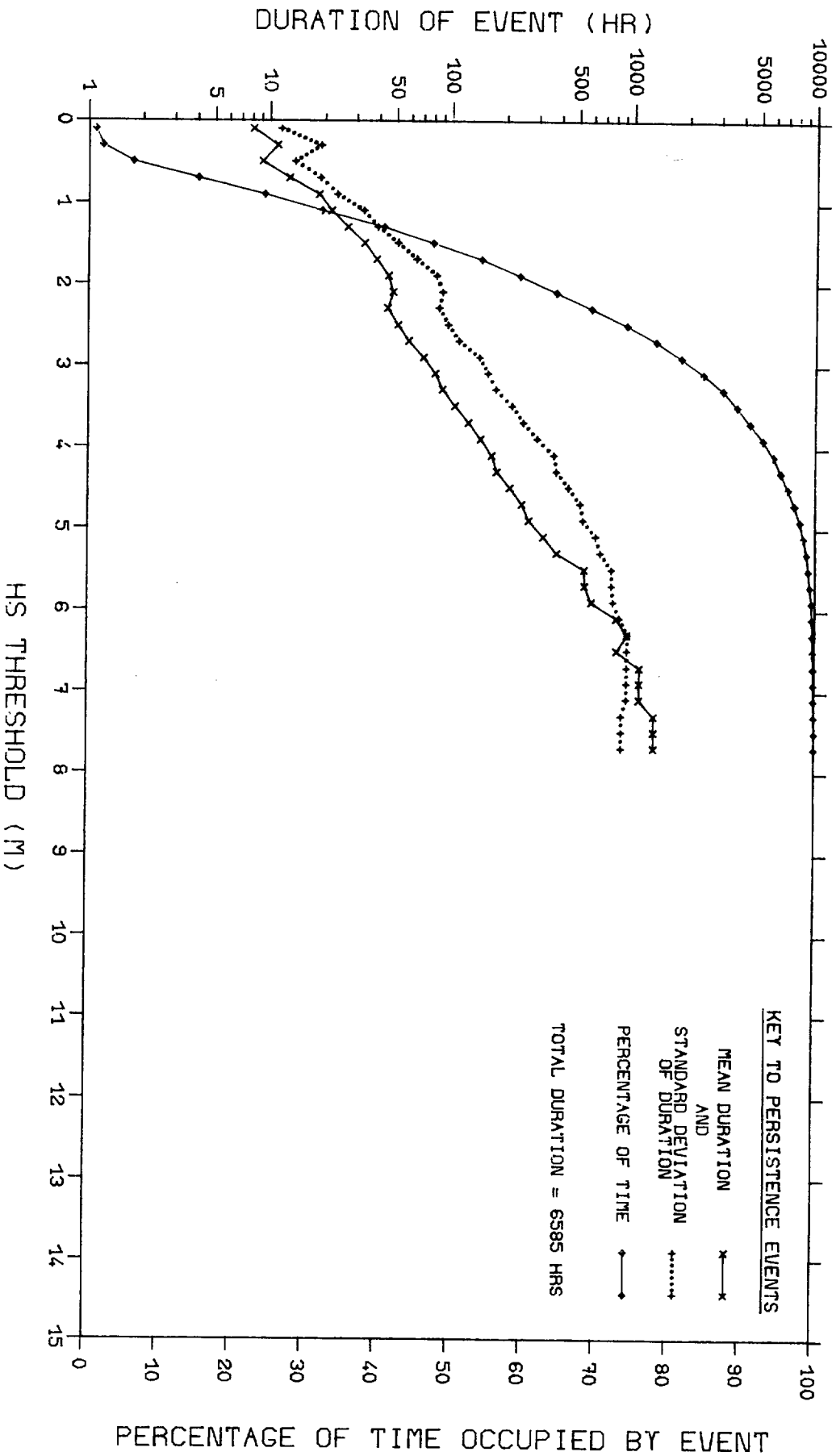
ST. GOWAN LV 1975/6, 1977, 1978 - WINTERS

FIG 3.5.1.4



SCATTER PLOT OF HS AND TZ
ST. GOWAN LV 1975/6, 1977, 1978

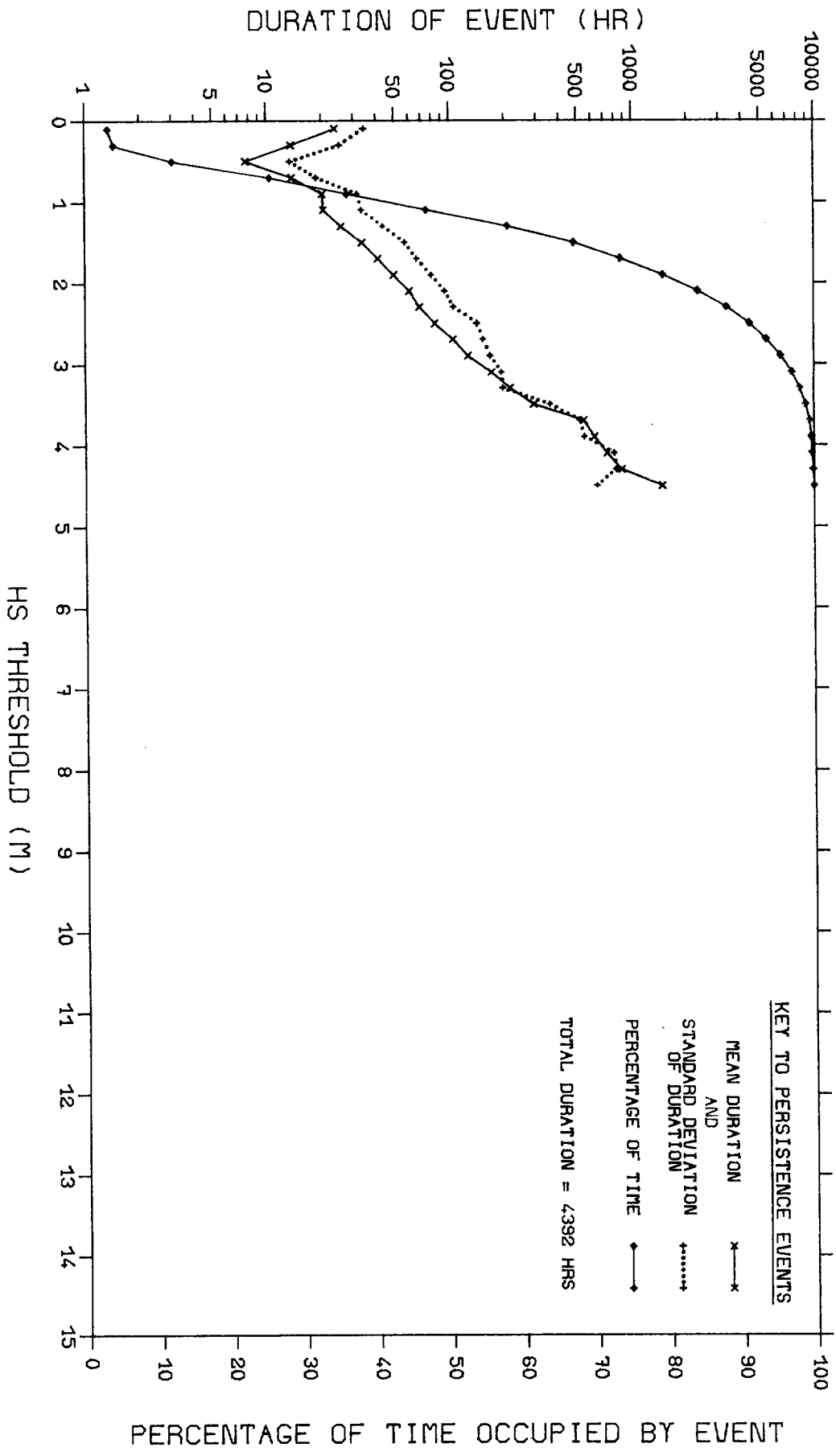
FIG 3.5.1.5



ST. GOMAN LV 1975/6, 1977, 1978 - SPRINGS

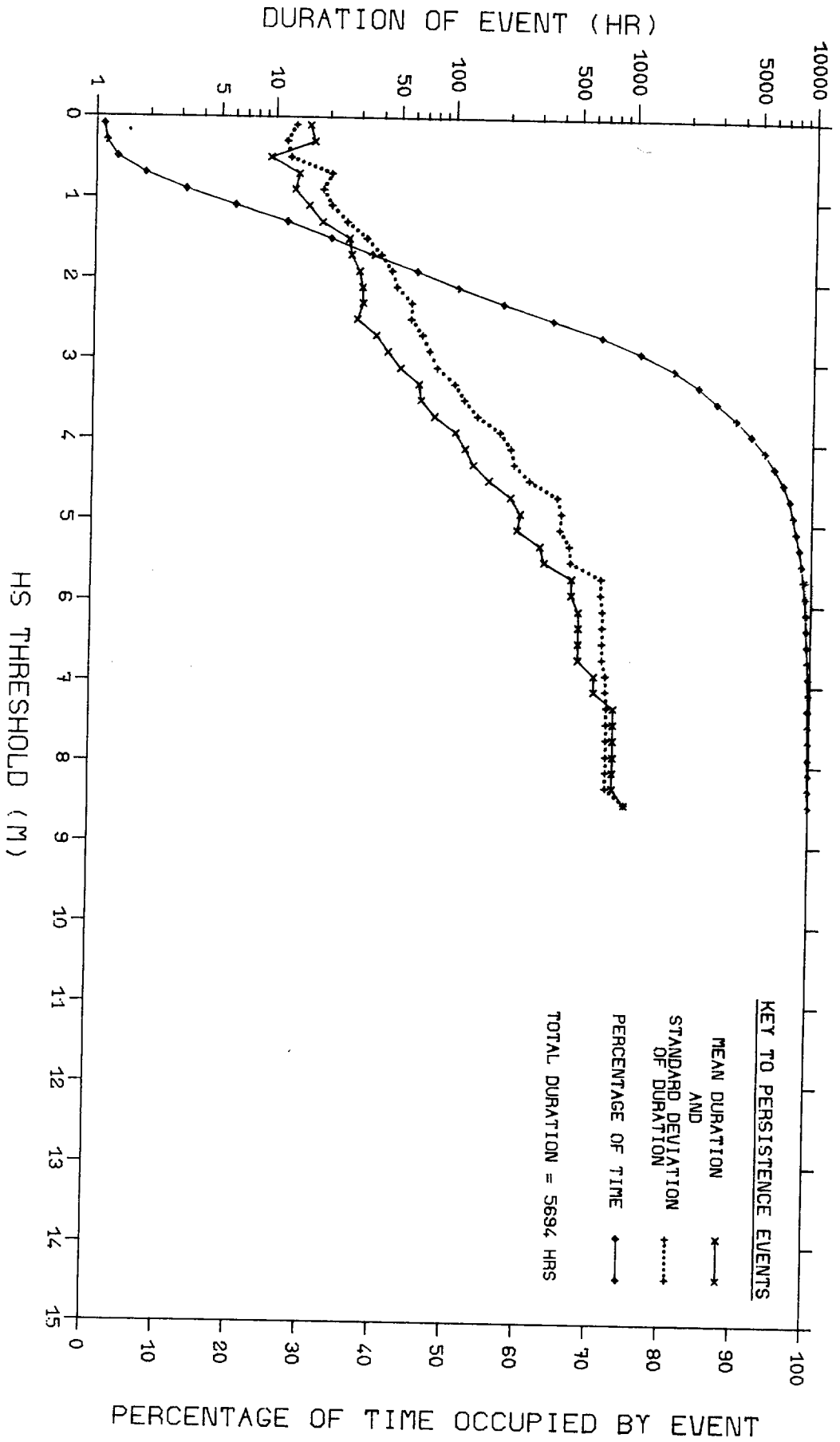
CALMS

FIG 3.6.1.1



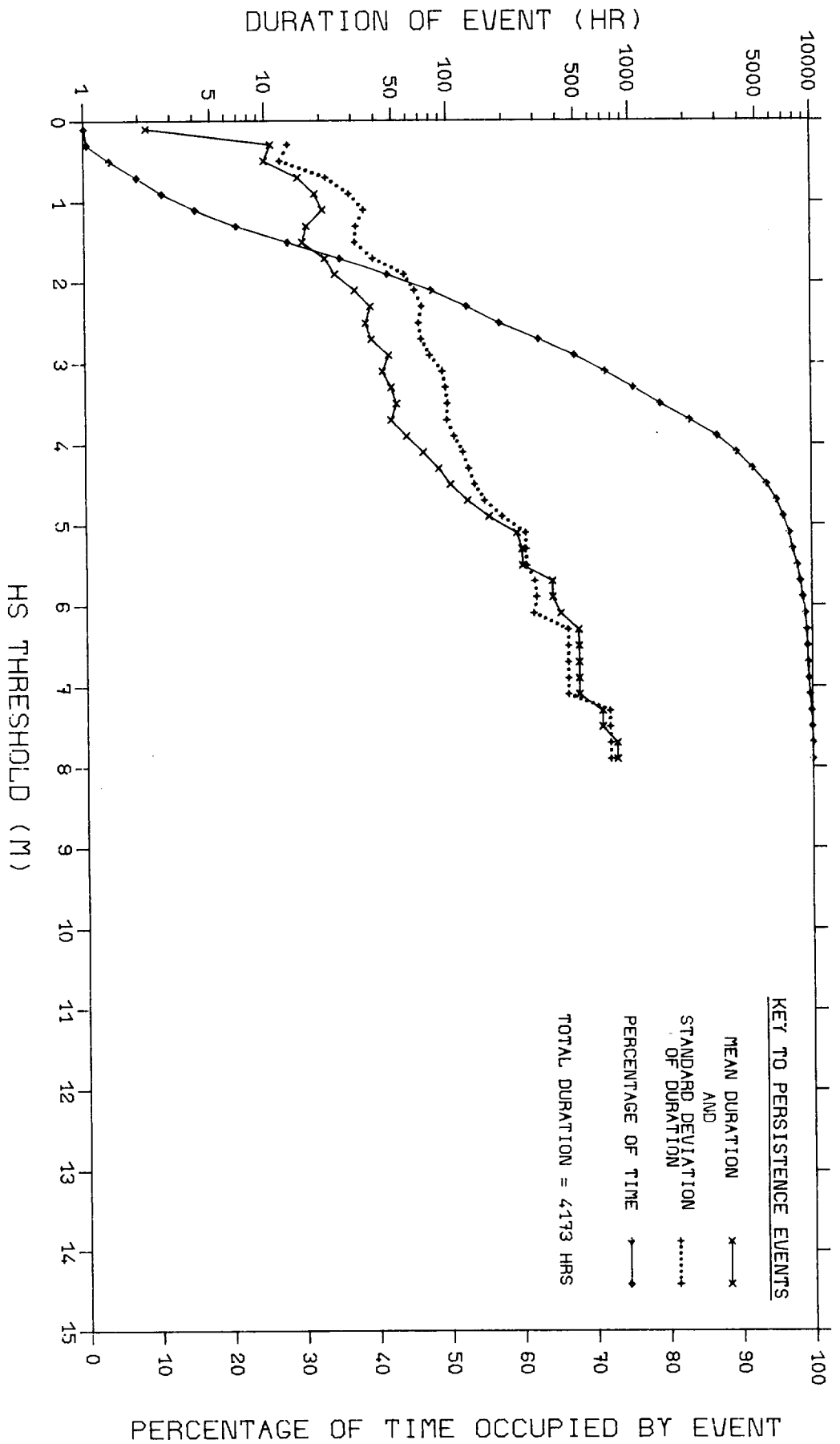
ST. GOWAN LV 1975/6, 1977, 1978 - SUMMERS
CALMS

FIG 3.6.1.2



ST. GOWAN LV 1975/6, 1977, 1978 - AUTUMNS
CALMS

FIG 3.6.1.3



ST. GOWAN LV 1975/6, 1977, 1978 - WINTERS

CALMS

FIG 3.6.1.4

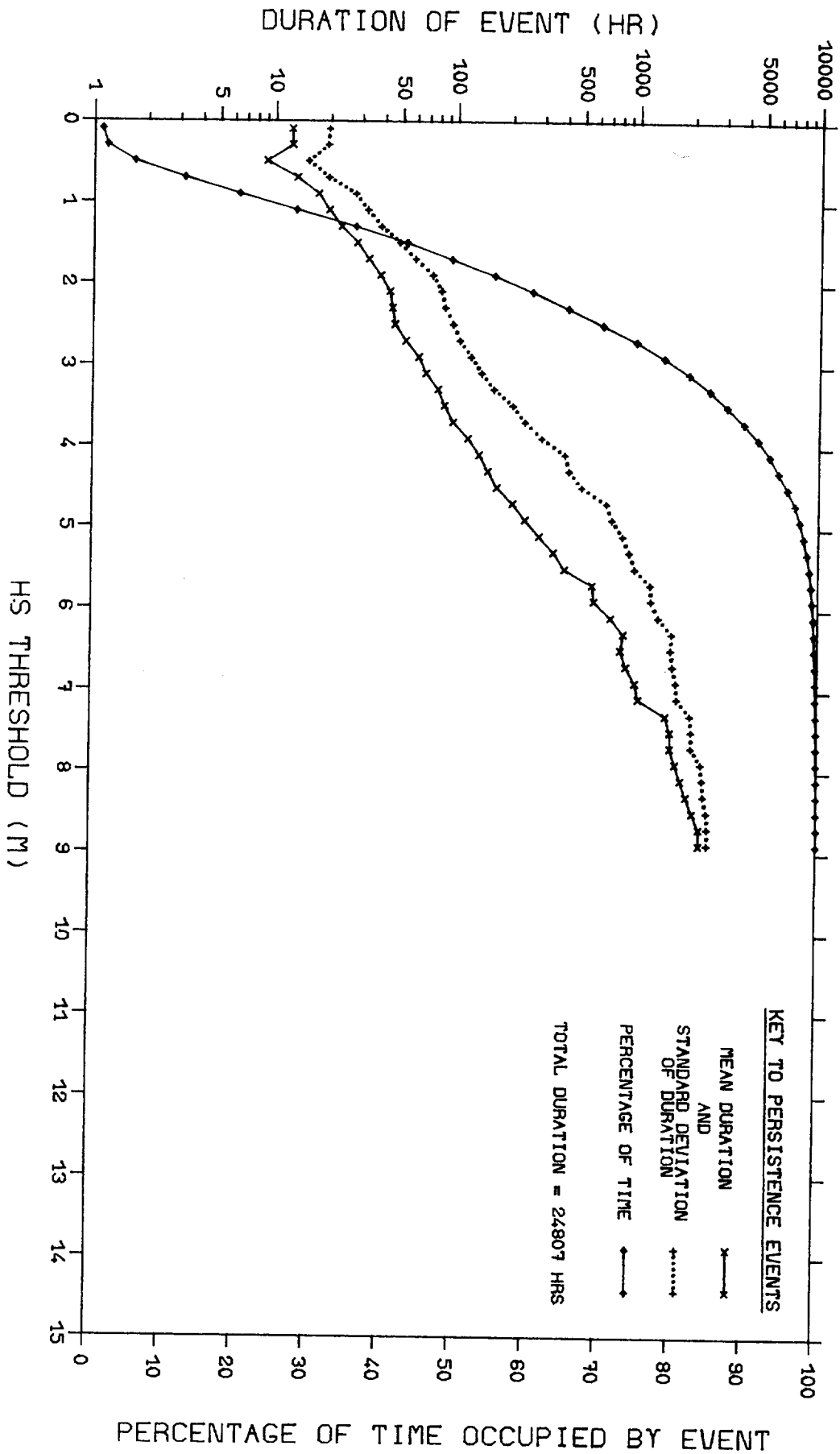
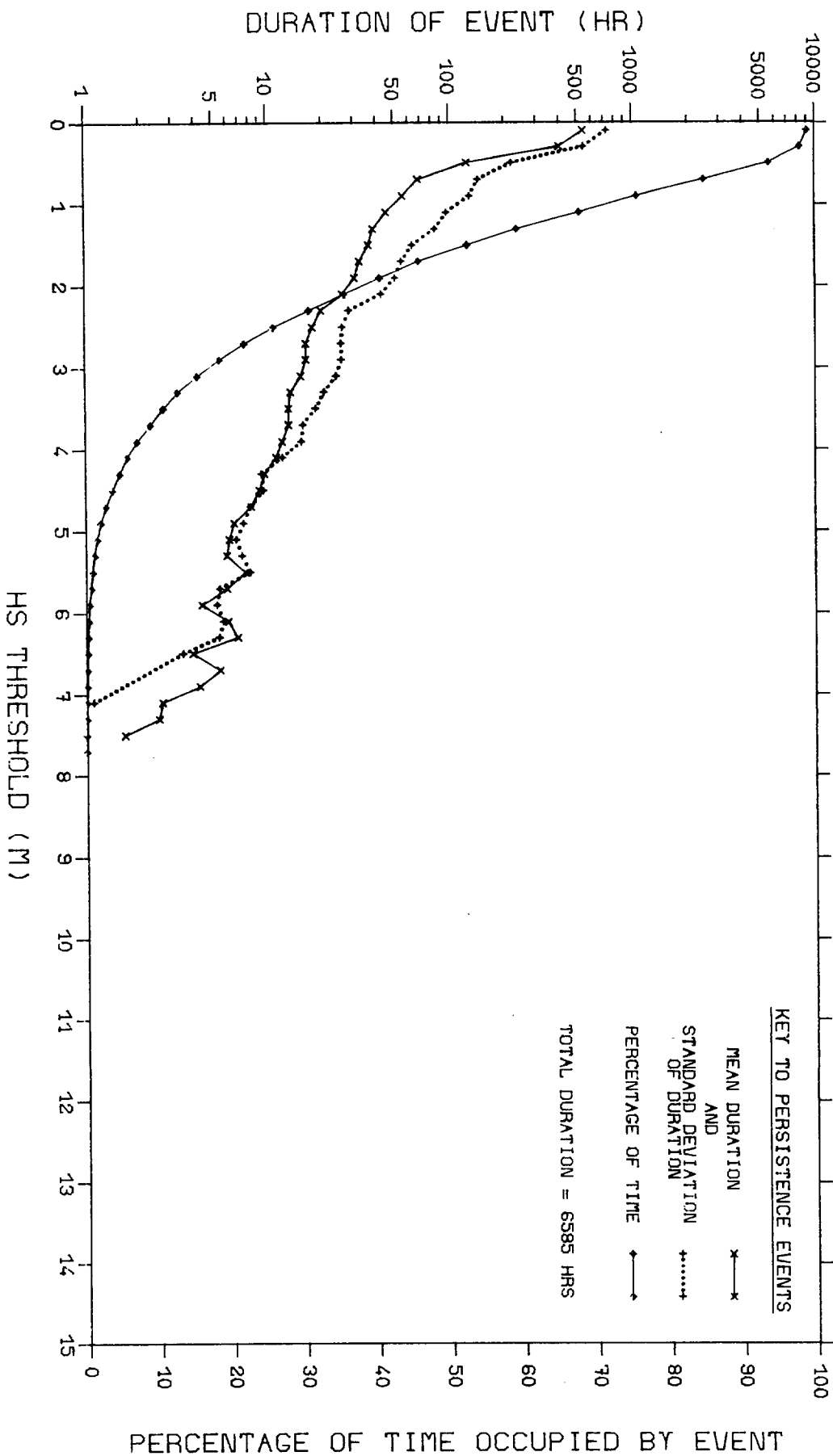
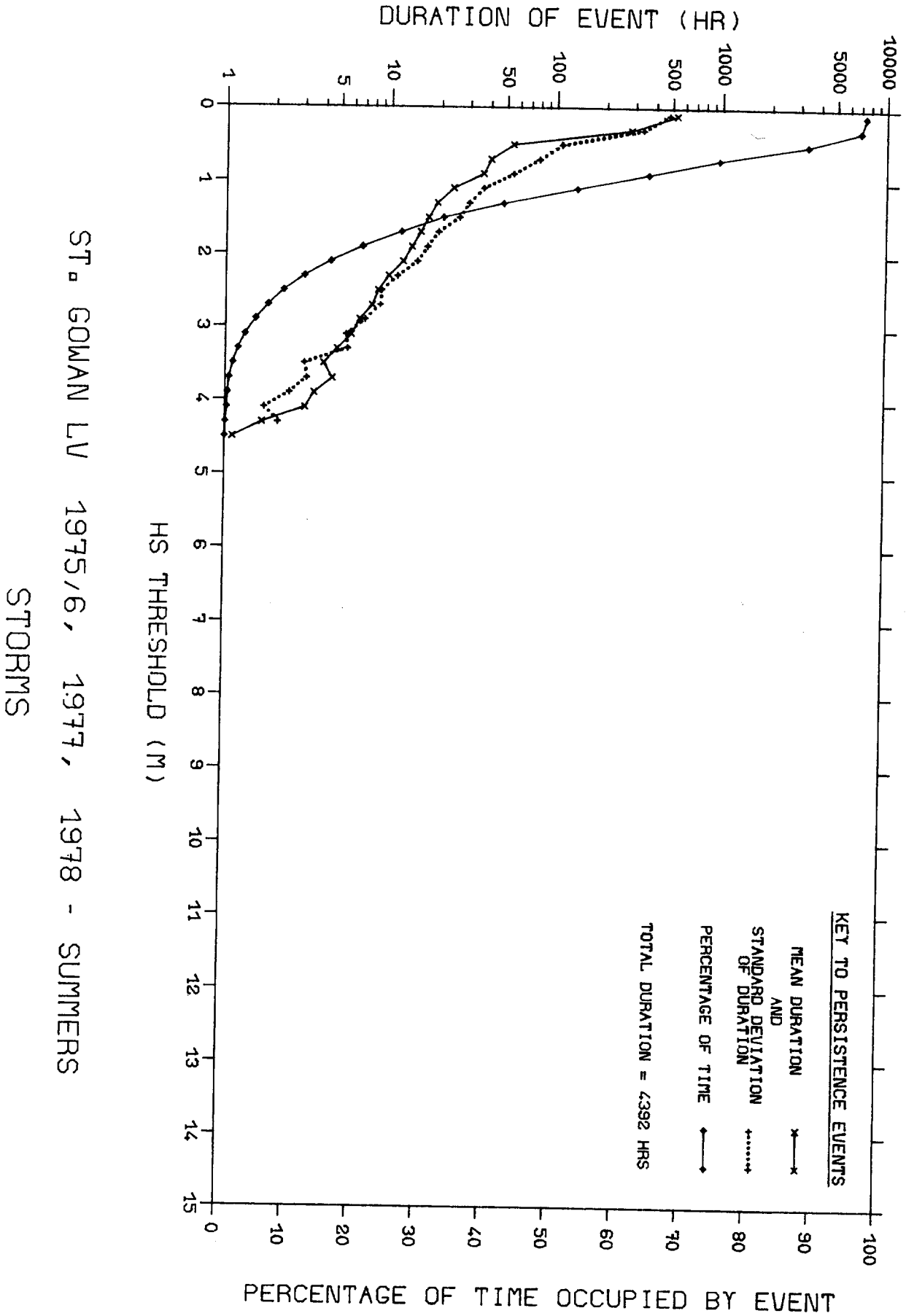


FIG 3.6.1.5



ST. GOWAN LV 1975/6, 1977, 1978 - SPRINGS STORMS

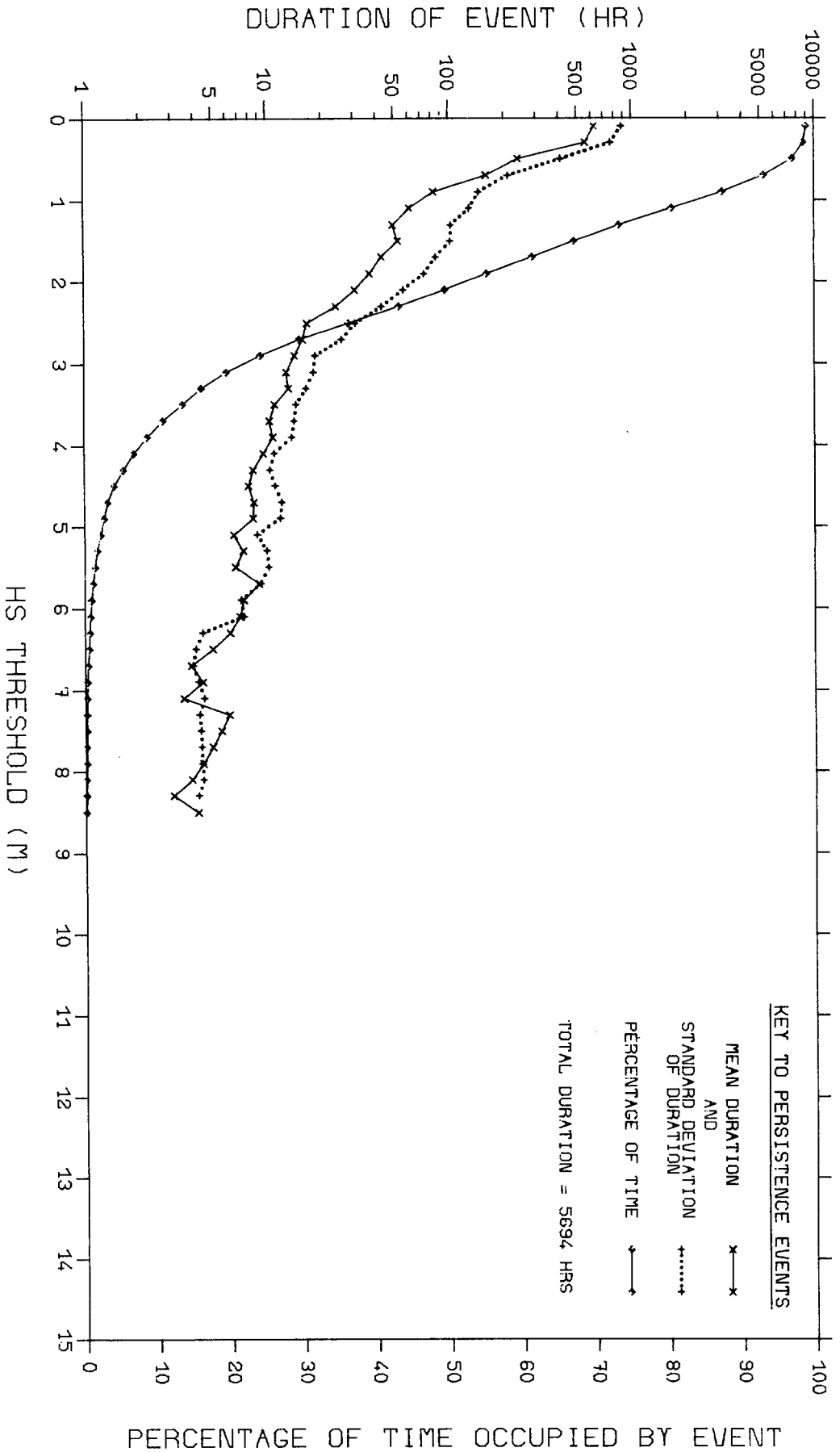
FIG 3.6.2.1



ST. GOWAN LV 1975/6, 1977, 1978 - SUMMERS

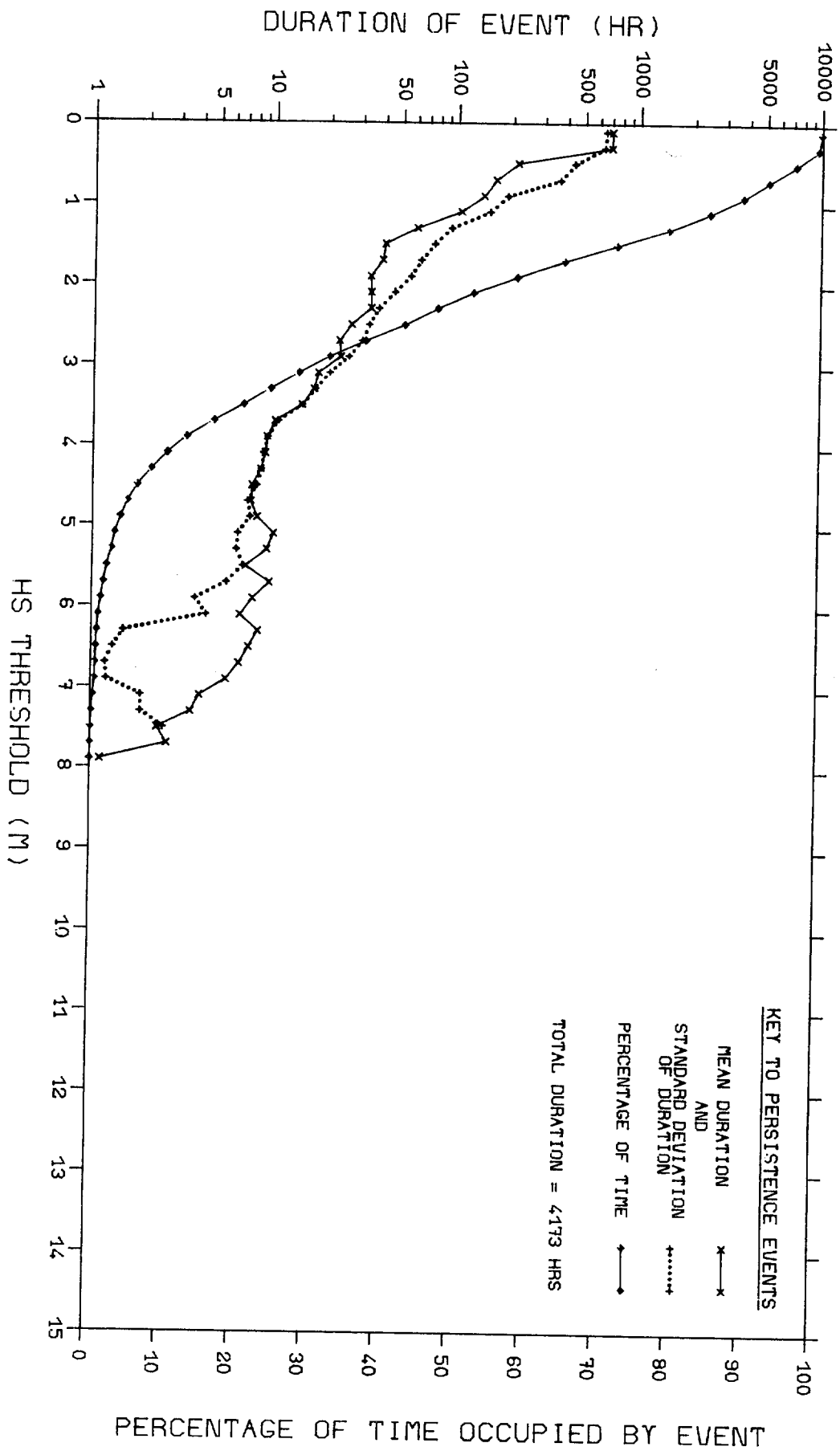
STORMS

FIG 3.6.2.2



ST. GOWAN LV 1975/6, 1977, 1978 - AUTUMNS
STORMS

FIG 3.6.2.3



ST. GOWAN LV 1975/6, 1977, 1978 - WINTERS

STORMS

FIG 3.6.2.4

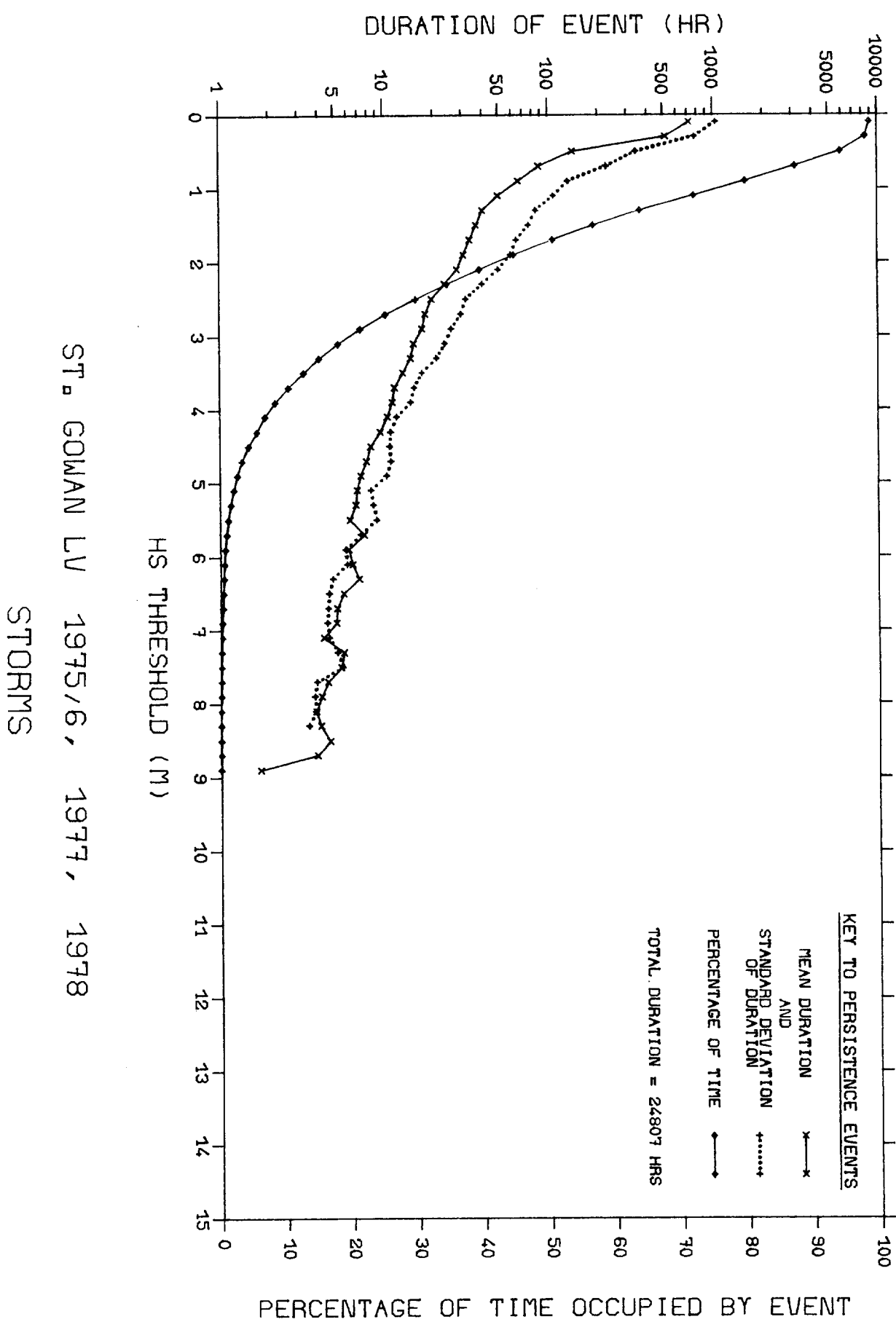


FIG 3.6.2.5