

**I.O.S.**

**WAVES AND CURRENTS  
NEAR THE  
CONTINENTAL SHELF EDGE**

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## SUMMARY

The abrupt depth increase which characterises the edge of many continental shelves determines a reduced horizontal length scale and a localised transition from shelf seas to the deep ocean. Particular forms of motion which may arise from the steep slopes include topographically guided currents along the slope, shelf-break upwelling, topographic Rossby waves and internal lee waves in the tidal current. The ocean/shelf mismatch may lead to a clear separation of water types, substantial reflection (from the shelf-edge neighbourhood) of all oceanic and shelf motions with periods greater than a few hours, and interaction between barotropic and baroclinic motions.

## 1. INTRODUCTION

The region near the continental shelf edge is of particular interest to oceanographers for the occurrence of motions of both oceanic and shelf origin. Moreover, it is often convenient to model the ocean or shelf separately from each other; shelf-edge boundary conditions correctly representing the transmission, trapping or reflection of various motions are then required. In addition, the reduced topographic length scale and steep slopes are associated with waveforms and motions peculiar to the neighbourhood of the shelf edge.

These various processes lead us to consider a region extending oceanwards at least to the continental slope below the main oceanic stratification (be this a seasonal or permanent thermocline or of some other origin). In the (rare) absence of such stratification, we might characterise the shelf edge region topographically as an area of steep sea floor slope (relative to typical slopes on the shelf inshore), ie.

$$h|\nabla h|^{-1} \ll \text{distance to coast}$$

and include a strip of similar width on either side. Such a characterisation is usually appropriate on the coastal side. Baroclinic motions often set an internal Rossby radius of deformation scale

$$R_I = [gh_1h_2(\rho_2 - \rho_1)/(h_1 + h_2)\rho_2 f^2]^{1/2} = O(10 \text{ km})$$

for a two-layer model with upper/lower layer densities and depths  $\rho_1, h_1 / \rho_2, h_2$  respectively;  $g$  is gravitational acceleration and  $f$  the Coriolis parameter  $2(\text{earth's spin}) \sin(\text{latitude})$ . Since  $R_I$  or the topographic scale  $h|\nabla h|^{-1}$  is usually much less than the shelf width, we have a clear intuitive notion of a narrow shelf edge region seaward of the main shelf area. Ultimately, however, it is important not to

impose an arbitrary definition but to let the various physical processes determine their own domains of interest.

Several length scales are indicated by the equations of motion. The momentum and continuity equations for an inviscid non-diffusive stratified sea of depth  $h(\underline{x})$  may be non-dimensionalised using the following scales: horizontal velocity  $U$ , time  $T$ , Coriolis parameter  $f$ , gravity  $g$ , reduced gravity  $g' \equiv g\Delta\rho/\rho$  (where  $\rho$  is a mean density and  $\Delta\rho$  a characteristic density difference), depth  $H$ , characteristic bottom slope  $\alpha$  and the earth's radius  $R$ . The pressure and vertical velocity scales are determined in terms of these by the momentum and continuity equations respectively. Seven independent length scales result; non-dimensional combinations make the choice somewhat arbitrary and the following is merely conventional: particle excursion  $UT$ , depth  $H$ , gravity 'wavelength'  $(gH)^{1/2}T$ , Rossby 'wavelength'  $R_p \equiv R_E/(f_0T)$ , internal Rossby deformation radius  $R_I \equiv (g'H)^{1/2}/f_0$  and the topographic scale  $H/\alpha$ . The length scale  $L$  in any particular context is normally the minimum of the last four, except that  $R_I$  does not influence barotropic motion.

Six non-dimensional ratios result. The Rossby number  $\epsilon \equiv U/f_0L$ , or  $UT/L$  for motions above the inertial frequency, measures the non-linearity of the motion. Typical small oceanic values often indicate the intensity rather than the primary character of the flow. However, the small shelf-edge topographic scale tends to increase  $\epsilon$ , encouraging non-linear effects. Energy may be transferred to different frequencies or to mean currents, and to wave motions from unstable mean currents, although the sea floor slope itself is

generally stabilising. The aspect ratio  $\delta \equiv H/L$  is usually small except for (i) internal motions at frequencies (several cycles per hour) comparable with  $N \equiv (-g/\rho \, d\rho/dz)^{1/2}$ , the Brunt-Väisälä or buoyancy frequency; (ii) high frequency surface waves with periods of less than a minute or so in a depth of 200m (say). However, the latter decay downwards and are less influenced by the sea floor. Equivalently, only surface waves with periods greater than one minute experience amplitude increases exceeding 10% on entering 200m depth from the deep sea, i.e. only small  $\delta$  'long' surface waves are of concern in a study of shelf edge phenomena. Small  $\delta$  is associated with approximately hydrostatic pressure. The internal Froude number  $F \equiv U/(g'H)^{1/2}$  relates the flow speed  $U$  to the long internal gravity wave speed  $= O(g'H)^{1/2}$ , e.g.  $[g h_1 h_2 (\rho_2 - \rho_1) / (h_1 + h_2)]^{1/2}$  in a two-layer model. Sufficiently rapid flows ( $F > 1$ ) are supercritical in the hydraulic sense, e.g.  $U = 0.2$  m/sec if there is a 25m surface layer with a density reduction of  $0.2 \sigma_T$  units, conditions typical of tidal currents and the initial spring stratification on the North-west European shelf (Cartwright 1976; Ellett and Martin 1973). The divergence parameter  $D^2 \equiv L^2 / (T_g^2 H)$  is small when the length scale  $L$  is much less than the gravity wavelength  $(gH)^{1/2} T$ . Surface displacements then play a negligible role in the mass balance, leading to the common approximation of negligible horizontal divergence for motions of sufficiently long period  $2\pi T$ . At the other extreme, ray theory (e.g. Shen, Meyer and Keller, 1968) may be applied to gravity waves when  $H/\alpha \gg (gH)^{1/2} T$ , i.e. ambient conditions change little for the gravity wave in one wavelength. The intermediate case of strong interaction,



$(gH)^{1/2} T \sim H/\alpha \sim 10\text{km}$  (say), occurs for gravity wave periods  $2\pi T$  of several minutes (23.4 min if  $H = 200\text{m}$ ). The motion is then horizontally divergent but the gravity wave form is strongly affected by the sloping bottom. Finally, two ratios may be formed from the internal deformation radius  $R_I$  (typically  $O(10\text{km})$  on the shelf), the topographic scale  $H/\alpha$  (typically  $O(10\text{km})$  near the shelf edge), and the Rossby 'wavelength'  $R_B$ .  $R_B \lesssim 10\text{km}$  for barotropic Rossby waves of period  $\gtrsim 1$  year. However, Rossby waves depend on potential vorticity gradients, to which the depth gradient contributes overwhelmingly near the shelf edge. Hence  $R_B$  is best compared with  $R_I$  away from the shelf edge and large depth gradients. Then baroclinic Rossby waves are only possible at sufficiently low frequencies:  $R_B \lesssim R_I$ . Higher frequency Rossby waves must be barotropic; these longer waves then tend to see the shelf edge region as a scarp, where they are almost totally reflected (section 5 below). The comparability of  $R_I$  and  $H/\alpha$  implies matching of the bottom slope with the characteristic slope of internal waves near the inertial or tidal frequencies. Strong internal tide generation may result (section 9 below), and topographically trapped waves below the inertial frequency may be significantly modified by the stratification.

Diffusion is essential to the description of steady state upwelling (for example). Indeed, spatially varying eddy viscosity is the agent of one shelf-edge upwelling mechanism described at the end of chapter 4. Apart from this and double-diffusive effects, however, diffusion tends to control the final form rather than the initial possibility of the various phenomena. Accordingly, we do not

explicitly treat diffusive length scales, which are usually less than  $L$  and, subject to the above, play a passive role. In particular, the onset rather than the steady state of 'conventional' upwelling is considered. This has the advantages of simplicity and a clearer separation of cause and effect.

Longshelf variations in the shelf topography or the flow may define a separate length scale  $\Lambda$ , generally greater than the cross-shelf scale  $L$  near the shelf edge. Its dynamical significance for the flow depends on the time scale  $T$ ; increased geostrophic guidance at long periods  $T \gg f_0^{-1}$  gives longshelf variations an enhanced role, roughly as  $f_0 T L / \Lambda$ . Near and below the inertial period, significant local effects generally require  $\Lambda / L = O(1)$ . However, shelf variations of length scale  $\Lambda$  strongly affect waves of length  $\Lambda$  or more, no matter how large  $\Lambda / L$  may be.

The two time scales  $T$  and  $f_0^{-1}$  combine as  $T f_0$ , indicating the period of the motion relative to an inertial period. The following sections are roughly in order of decreasing  $T f_0$ , although the separate treatment of individual phenomena takes priority. As hinted above, a lower limit  $2\pi T \sim 1$  minute is set where the shelf - edge topography ceases to have any substantial effect on the motion.

## 2. WATER MASSES

The small depths and consequent reduced thermal capacity of continental shelf seas are liable to increase the temperature effects of summer heating and winter cooling in comparison with the adjacent ocean. (A famous example of the latter is the localisation of Antarctic Bottom Water formation to the Weddell Sea, where winter

cooling and ice formation over the shelf play a key role: Sverdrup, Johnson and Fleming 1946, pg 609-612). The input of fresh river-water at the coast may help to form shelf water distinct from that of the adjacent deep sea. Moreover, stirring from below by the stronger tidal current sharpens the seasonal thermocline in the Celtic Sea (say) relative to that in deeper water.

A well-documented example of the abrupt change of water type (front) which may ensue near the shelf edge is furnished by the Middle Atlantic Bight (figure 1). Wright (1976) used a shelf water criterion  $T < 10^{\circ}\text{C}$  (or  $S < 35\%$  for summer-heated surface water) to find that the shelf water boundary at the bottom remained within 16km ( $\pm 20\text{m}$  in water depth) of the 100m depth contour 84% of the time. The position of the boundary on the sea surface is more variable but may be quite coherent and sharp; Voorhis, Webb and Millard (1976) show a striking satellite photograph from late winter when the shelf water is coldest.

Any fresh water contribution to the shelf water tends to make it less dense than the adjacent ocean, at least in summer (all the year off New England). The boundary between water masses then slopes off-shore from the bottom to the surface, and there is an associated near-surface geostrophic flow, cyclonic about the deep sea. Flagg and Beardsley (1978) find that the geostrophic guidance of the depth contours may render the New England shelf-edge flow effectively stable after dissipation, in contrast with strong instability if it were over the interior shelf. They suggest that this may cause the shelf edge location of the front.

Shelf water formed as a mixture of slope water and continual fresh water input must eventually be released to the deep sea at a rate balancing its production. Wright (1976) suggested that the observed occurrence of detached parcels of shelf water in the slope water could suffice. Voorhis et al. (1976) observed interleaving of the shelf and slope water as the latter lightened by spring warming. Such interleaving enhanced shelf/slope water exchanges by turbulent diffusion sufficiently to account for a substantial part of the shelf water release. Morgan and Bishop (1977) observed a tongue of shelf water extending over the slope in association with a Gulf Stream ring there, and estimated that such events might be frequent enough to release 10% of the shelf water. This estimate was greatly increased by Smith (1978), who shows a striking satellite photograph of a Gulf Stream ring entraining shelf water. The frequency and scale of this process has been further studied by Halliwell and Mooers (1979) using satellite photographs. A mooring at 250m depth on the Scotian shelf edge also suggested a compensating 'return' flow of water and certainly a flux of heat and salt onto the shelf deeper in the water column (Smith, 1978).

By contrast, a summer infra-red satellite photograph of surface temperatures off Washington and British Columbia in the NE Pacific shows no indication of the shelf edge position (see Mysak, 1977). The North-west European shelf edge has no rapid transition from shelf to slope water at any season, and the position of the shelf edge could not usually be estimated from temperature and salinity measurements in the overlying water, e.g. I.C.E.S. (1962), Ellett and Martin (1973).

(However, cooler surface water near the shelf edge has often been observed in satellite photographs (Pingree, 1979): see section 11). During the summer, thermal stratification above the shelf edge depth extends well onto the shelf and shows topographic influence only in a sharpening of the lower side of the thermocline by the stronger tidal currents on the shelf (figure 2). The underlying water over the shelf edge only warms significantly from its late winter temperature when autumnal mixing through the total shelf depth occurs. This suggests that late winter conditions largely control the absence of any shelf edge fronts. Then the temperature and salinity increase smoothly from coastal to deep sea values across the whole width of the shelf. By contrast with the New England context, milder winters, less influence from fresh water (which tends to be geostrophically guided through the eastern North Sea to the Norwegian coast), greater mixing by tidal currents and winter storms, and in some places an extremely irregular shelf edge probably all play a part in avoiding any abrupt changes near the NW European shelf edge. Moreover, in average or cold winters the Celtic Sea shelf water may cool to become as dense as the adjacent slope water, and locally cascade over the shelf edge (Cooper and Vaux, 1949). Such an event further encourages shelf/slope exchange by drawing compensatory slope water over the shelf elsewhere.

As suggested in the New England context, the geostrophic guidance furnished by the shelf edge and adjacent steep slope generally inhibits exchange between the water masses to either side, which thus acquire local characteristics, particularly over the shelf. Such differing water types, and the convergence of depth contours where the

shelf break is sharp, may nevertheless encourage strong geostrophic flows there. Furthermore, the major western boundary currents often follow a scarp for a large part of their well-defined course. Examples are the Florida current/Gulf Stream north to Cape Hatteras, and the Agulhas current off S. Africa, particularly where the continental slope is steep (Pearce, 1977). Where such strong currents lose the shelf-edge guidance, they appear to be major sources of eddies (Wyrski, Magaard and Hager, 1976). However, the Gulf Stream rings associated with shelf-water release in the Middle Atlantic Bight (Morgan and Bishop, 1977; Smith, 1978) themselves remain seaward of the shelf edge, reducing their potential for inducing shelf/slope exchange. While still guided by the shelf edge, a strong current may be unstable, meandering and spinning off small eddies. However, the effects of these are also observed to diminish rapidly inshore of the shelf edge (Pearce, 1977; Lee and Brooks, 1979), perhaps on the internal deformation scale  $R_T$  (see section 5).

### 3. LONG-PERIOD WAVES

The steep slopes near the shelf edge imply a large potential vorticity gradient. A fluid column displaced up or down the slope, and conserving potential vorticity, acquires relative vorticity. This is manifested as displacements by adjacent columns; the result is long-shelf propagation of the disturbance in a cyclonic sense about the deeper water. Such wave motions are known as continental shelf waves (Robinson, 1964) and, over a mid-ocean scarp, as double-Kelvin waves (Rhines, 1967; Longuet-Higgins, 1968). It is known (Huthnance, 1975) that for a straight shelf with any monotonic depth profile there

is an infinite set of wave modes, with 1,2,... offshore modes. Each mode  $n$  has a dispersion relation  $\sigma = \sigma_n(k)$  for the frequency as a function of longshore wavenumber  $k$ , which for small  $k$  gives a constant phase speed  $d\sigma_n/dk$ . The frequencies  $\sigma_n$  decrease with  $n$  from  $\sigma_1 < f$ . In unstratified conditions the (barotropic) current magnitude varies spatially roughly as  $h^{-1/2}$ ; the modal structure is distributed roughly as  $(h^{-1} dh/dx)^{1/2}$  and is therefore concentrated near any well defined shelf edge (figure 3). Shelf waves and double Kelvin waves are thus seen to be manifestations of the same phenomenon, differing only in whether the coast is far enough from the shelf edge for the motion at the latter to be regarded separately (the double Kelvin wave).

All the above qualitative conclusions extend to stratified conditions (Huthnance, 1978a), particularly since the stratification is effectively weak ( $R_T \ll$  shelf width) on most shelves broad enough to have a well-defined edge. A few of the modes may take the form of internal Kelvin waves at the coast if  $R_T > H/\alpha$  there. However, the majority of the modes cluster near the shelf edge; here there is a reduced topographic scale  $H/\alpha$  and quite possibly  $R_T \gtrsim H/\alpha$ , so that the wave modes include an element of gravity wave dynamics (for  $R_T \gg H/\alpha$  they adopt internal Kelvin-like wave forms against the steep slope). For short long-shelf wavelengths, they take the form of bottom-trapped waves (Rhines, 1970) of frequency  $\sigma \approx N\alpha$  and located near the sea-floor maximum of  $N\alpha$  ( $N^2 \equiv -g/\rho \, d\rho/dz$ ). This maximum is usually near the shelf edge. Its representation is a severe test of any model.

Continental shelf waves are most readily generated by wind stress along the shelf (Adams and Buchwald, 1969), and so typically have periods of several days. A simple picture of the way a long shelf wave is built up by local forcing as it progresses has been developed by Gill and Schumann (1974). Waves forced meteorologically have been observed along the coast of Australia (Hamon, 1966), North Carolina (Mysak and Hamon, 1969), Oregon (Cutchin and Smith, 1973) and elsewhere, albeit often only in coastal sea level. The Oregon interpretation is supported by current meter measurements. Wind induced currents of 0(10-20cm/sec) appear to be typical. Shelf waves contribute to K1 diurnal tidal elevations at the edge of the north west Scottish shelf which are twice as great as at the adjacent coast (Cartwright, Huthnance, Spencer and Vassie, 1980).

Bottom trapped waves have probably been observed near site D, where  $N\alpha \approx 1/8$  cycle per day (Thompson and Luyten, 1976). They can also exist above the inertial frequency and, for example, perhaps play a part in the large quarter-diurnal tidal currents noticed by Gordon (1979) off north west Africa. These occurred near the bottom in 400m of water where  $N\alpha$  was near 4 cycles per day, a large value (but not unusual, see section 9) resulting from the conjunction of substantial stratification with the steepest portion of the shelf profile.

Shelf-wave forms also play an important role in low frequency oceanic-scale motions, by which they are forced in order to satisfy in detail the boundary condition of no flow through the oceanic 'sidewall' i.e. the continental slope and shelf (Allen, 1976; Huthnance, 1978b). Smith and Louis (1979) have observed waves in the



Nova Scotia surface front (0(100km) seaward from the shelf break) induced by warm core Gulf Stream eddies further offshore and to the west.

A longshore current, unlike stratification, may affect the qualitative modal distribution (Niiler and Mysak, 1971); it advects the waves and slower modes may even be reversed to propagate anticyclonically about the deep sea. Moreover, its shear modifies the background potential vorticity; if the gradient of the latter is anywhere reversed a new set of modes may exist, also propagating in the reverse sense. Potential vorticity extrema roughly corresponding to shear maxima may also lead to instability of the modes at some longshore wavenumbers. This is the counterpart to the instability of plane shear flow, and the depth gradient has a generally stabilising effect. A mean current which varies in depth and is therefore associated with sloping isopycnals may also be baroclinically unstable. Resulting meanders in the Gulf Stream and Agulhas current have already been mentioned in section 2. Examples from eastern ocean boundaries are fluctuations in the Norwegian Current of period 2-3 days (Mysak and Schott, 1977) and wave forms in sea-surface temperature and salinity off British Columbia, apparently deriving from the California undercurrent (Mysak, 1977).

Current knowledge of the scope for wave-wave interaction and instability of larger amplitude shelf waves has been considered by Mysak (1980). These include the interesting possibility of resonant subharmonic response to a travelling wind field (Barton, 1977).

Mysak (1980) also reviews the effects on shelf waves of topographic irregularities and long-shelf variations. If the latter have a spatial scale  $\Lambda$  longer than the wavelength, the shelf wave evolves as it progresses to adopt the local mode form while maintaining its frequency and the longshelf energy flux. Short-scale irregularities scatter energy into other modes. At the sides of canyons in the shelf edge (for example), upwelling may occur.

Indeed, upwelling (see the following section) is an important phenomenon associated with shelf waves; it may also be propagated along the shelf by them. Other physical roles of topographic Rossby waves derive from their ability to transfer momentum (Garrett, 1979) and heat, salt and nutrients onto the shelf (Smith, 1978).

All these long waves are manifested as currents (predominantly along the shelf at frequencies much below the inertial frequency) and as internal elevations of magnitude  $O[\min(UH/Lf_0, U/N)]$  rather than as surface displacements, which are of magnitude  $O(f_0 UL/g)$ ,  $L$  being the local topographic scale  $H/\alpha$ . For  $L \sim 10\text{km}$  near the shelf edge this is only 1 cm for a 10 cm/sec current, but coastal values may be greater with  $L$  representing the shelf width. The whole topic of shelf waves has recently been reviewed by Mysak (1980).

#### 4. UPWELLING

The 'classical' upwelling scenario is a coast on the east side of an ocean with an equatorward wind stress  $\tau$ . Coastal upwelling is required to supply the offshore Ekman transport (e.g. Defant, 1961). A simple two-layer model with wind stress acting on the whole upper-layer depth (Yoshida, 1955; see Defant) nicely illustrates the

development of this condition, including an equatorward coastal jet in the upper layer. The phenomenon may occur for all coastal orientations provided only that there is a longshore wind-stress component in the above sense; downwelling occurs if this is reversed.

A similar two-layer model (figure 4; in fact considered earlier by Defant (1952) with an oscillatory onshore wind) demonstrates the onset of upwelling over a sea-floor scarp parallel to the coast. If this model shelf is wide ( $\gg R_T$ ) and the ocean depth is large, such shelf-edge upwelling is a fraction  $h_1/[h_2 + (h_2 H)^{1/2}]$  of the coastal rate  $\tau(g\rho\Delta\rho h_1 H/h_2)^{-1/2}$  according to the model. Under favourable conditions the interface may thus rise some metres per day. The upwelling decays to either side of the scarp over the respective internal deformation radii  $R_T$ . Likewise the offshore and longshore currents tend to their uniform-depth values on the same scale. The shelf edge upwelling arises because (i) the total shoreward flux summed over both layers is zero and (ii) relative to the lower layer flow, the upper layer offshore flux is (wind stress)/f far ( $\gg R_T$ ) from boundaries. In the deep lower layer off the shelf, the compensating inshore flow is slow by (i). Hence (ii) implies that the offshore flow in the upper layer is faster off the shelf than on the shelf. The shelf-edge upwelling feeds this upper layer divergence.

Csanady (1973) used a similar model with a sloping shelf to find the response to longshore and onshore winds. Johnson (1980) also considers a time-dependent model to find shelf-break upwelling proportional to (change of bottom slope)/(shelf depth).

Steady state calculations by Lill (1979) confirm the existence of upwelling in a homogeneous sea just offshore from a smooth shelf break. Steady state upwelling in various shelf edge contexts has also been considered theoretically by Tomczak and Käse (1974), Johnson and Killworth (1975) and Hsueh and Ou (1975). Johnson and Manja (1979) considered a level shelf meeting a slope. Over the slope, horizontal diffusion increases the onshore transport in the bottom boundary layer beneath a steady longshore flow. This tends to give bottom boundary layer convergence at the shelf break. However, they found that in the (final) steady state the consequent shelf break upwelling cannot be accommodated by the water above. Instead, there is shear in the longshore current over the shelf break. Wunsch (1970) pointed out that the stable oceanic temperature stratification, combined with a condition of zero heat flux through the sea floor at the continental slope, implies a dip of the isotherms there, and a tendency for upwelling in a thin layer against the slope; however, this may be complicated if the salinity stratification is unstable.

Gill and Clark (1974) emphasised that upwelling may also be caused by propagating long waves (section 3) generated elsewhere on the shelf. Bang and Andrews (1974) observed a strong upwelling - associated current just seaward of the S.W. Africa shelf break, which they believed to be amplified by a wind profile intensifying inshore. Off northwest Africa the coastal upwelling is observed to migrate to the shelf edge in a few days if the favourable winds persist (Barton, Huyer and Smith, 1977).

Some continental shelves, notably the Celtic Sea and off Southern California, are indented by numerous canyons. Shaffer (1976) has seen some evidence of shelf edge canyons guiding upwelling off N.W. Africa. Peffley and O'Brien (1976) have found numerically that enhanced upwelling occurs on the equatorward side of a canyon near the coast. Moreover, Killworth (1978) has found analytically that a propagating internal upward displacement (upwelling) - i.e. an internal Kelvin wave of finite extent - leaves residual steady upwelling on the equatorward side of a canyon. These results almost certainly apply also to canyons at the shelf edge. Gill and Schumann (1979) considered a two-layer model with a strong upper layer current (Froude number  $O(1)$ ) of speed comparable with that of long internal waves, along a shelf and scarp of varying width. They find that upwelling (surfacing of the lower layer in a coastal strip) may occur at a western ocean boundary, downstream of a narrower shelf section in a poleward current. This appears to be manifested by the Agulhas current south of  $32^{\circ}S$  off S. Africa.

Heaps (1980) has suggested another upwelling mechanism appropriate to the Celtic Sea shelf edge in summer, when the thermocline over the shelf is sharper than off the shelf. This 'slippery' interface beneath the heated upper layer over the shelf facilitates circulation within that layer in a vertical offshore plane. Winds driving coastal downwelling then induce upwelling within the upper layer near the shelf edge where the interface ceases to be 'slippery'.

## 5. EDDIES

A large proportion of the oceanic kinetic energy at frequencies much less than the inertial frequency is found to be in fluctuations with periods of months and length scales of order 100km (the MODE group, 1978). Such 'mesoscale eddies' are thought to be governed by Rossby wave dynamics, i.e. they propagate by the same means as shelf waves (section 3), but the background potential vorticity gradient derives from the latitudinal variation of the inertial frequency  $f$ . Their length scale  $R_B$  and period  $2\pi T$  are related by  $R_B T = O(R_E / f_c)$  (section 1); for sufficiently long periods so that  $R_B \lesssim R_I$ , baroclinic as well as barotropic forms are possible. It is clearly desirable to assess the behaviour of such important oceanic motions on encountering the continental shelf.

Some observational evidence of eddies tending to stop short of the shelf edge has already been mentioned in section 3 (Morgan and Bishop, 1977). The many observations discussed by Lai and Richardson (1977) and Richardson, Cheney and Worthington (1978) also seem to show this. Perhaps related is the decrease, inshore from the shelf edge, of the effects of meanders and eddies spun off from the Gulf Stream on the Georgia shelf (Lee and Brooks, 1979) and the Agulhas current off South Africa (Pearce, 1977). On the shelf, further than perhaps  $O(R_I)$  from the shelf break, the currents tend rather to be driven by local winds.

Kroll and Niiler (1976) considered the transmission of barotropic Rossby waves (allowing for an exponentially sloping sea floor) across changes of bottom slope  $\alpha$ ; any significant change leads to

substantial reflection unless two parallel such changes are appropriately spaced. A two-layer model with a straight vertical scarp but otherwise uniform depth indicates almost total reflection in  $R_E \gg R_B > R_I$ , i.e. at frequencies much less than the inertial frequency (Huthnance, 1980). This **upholds** the barotropic calculation (Le Blond and Mysak 1978, page 204), as expected from the resistance to ageostrophic displacements over the step. At still lower frequencies such that  $R_B \lesssim R_I$  (typically below 1 cycle per year), baroclinic Rossby waves are possible. Then scattering into a transmitted barotropic wave and transmitted and reflected baroclinic Rossby waves (if they exist with matching long-scarp wavenumber) may be substantial, particularly if the baroclinic forms do exist (Huthnance, 1980).

Baroclinic Rossby waves at higher frequencies (i.e.  $R_B > R_I$ ) can exist over a sloping sea floor if they progress cyclonically about the deeper water. Transmission over a scarp may be increased thereby. In general, however, it would appear that substantial reflection of oceanic eddies takes place on encountering the continental slope, and that all but the lowest frequencies will be reflected at a sharply defined continental shelf edge. Transmission at the latter requires that the stratification (probably a seasonal thermocline) extends onto the shelf. The selection of low frequencies implies shorter length scales ( $R_I$ ) over the shelf. However, Kroll and Niiler (1976) point out that these low frequency motions will there be subject to relatively rapid dissipation. Hence even with "transmission" the influence of oceanic eddies may penetrate only  $O(R_I)$  onto the shelf

past a sharp shelf edge. This is to be contrasted with the greater penetration onto the shelf achieved by oceanic-scale motions forcing shelf wave forms (section 3).

Reflection or effective absorption of Rossby waves approaching the shelf implies a transfer of their momentum flux, which appears as a forcing function for the 'mean' pressure and flow fields. Garrett (1979) has suggested that topographic Rossby waves at least partially balance the pressure gradient along the East Australian coast, and are effective in transferring this offshore to drive the East Australia current. One may speculate that the paths of SOFAR floats 2 and 14 during MODE (Riser, Freeland and Rossby, 1978) might indicate a similar process; they were caught in vigorous eddying motions until encountering a strong southward current near  $76^{\circ}\text{W}$ ,  $30^{\circ}\text{N}$  over the foot of the continental slope.

To summarise, it seems unlikely that eddy motions will extend further than  $O(R_I)$  onto the shelf. However, they are another potential source of quasi-geostrophic fluctuating and mean currents guided along the shelf edge. Any associated longshelf gradient of mean pressure (surface elevation) may extend across the shelf to influence the on-shelf circulation.

## 6. INERTIAL MOTIONS

At frequencies close to the local inertial frequency  $f$ , the field equation for internal motion in inviscid, stratified fluid reduces to a constraint on the horizontal structure. For a harmonic constituent  $\exp i(ft+ky)$  freely propagating along a uniform shelf of arbitrary depth profile  $h(x)$ , offshore decay as  $\exp(-kx)$  is required. Since  $k >$



0, this implies cyclonic propagation about the deep sea. The interior dynamics do not constrain the vertical structure, which is determined by the (effectively lateral) sea floor boundary condition  $w = -u \, dh/dx$ ; the perturbation pressure satisfies a second order ordinary differential equation in  $z$  (Huthnance, 1978a). Baroclinic motion is possible if  $Ndh/dx > f$  somewhere on the sloping bottom.

We interpret physically the form of solutions (figure 5) near a shelf edge where a gently sloping shelf meets a steeper continental slope. By conservation of mass, the vertical velocity  $w$  is continuous in  $z$ . Hence the sharp shoreward decrease of  $|dh/dx|$  at the top of the slope implies (through the boundary condition) a sharp increase of the on-offshore velocity  $u$ . Continuing shorewards, the shelf's smaller slope is of increasing influence (a greater fraction of the kinetic energy at upper levels lies over the shelf) and the vertical velocity decreases.

In the shelf-slope system's normal modes, for example, the sea-floor vertical velocity component  $w$  tends to be smaller near the shelf-edge than elsewhere on the slope; matching with its shelf values is part of determining the dispersion relation. Nevertheless, near the shelf edge (near the shelf edge depth particularly) we can expect greater vertical and horizontal velocities than typical shelf values at the inertial frequency, and strong shear between depths. Associated internal elevations are  $O(U\alpha/f)$ , and surface displacements are  $O(fUL/g)$ .

In an unbounded fluid with a non-zero vertical coefficient of kinematic viscosity  $\nu$ , changes in the surface wind stress give rise to

inertial currents, which propagate downwards with speed  $(2\nu f)^{1/2}$  and penetration depth  $(2\nu/f)^{1/2}$ . However, lateral boundaries imply in addition a change of surface slope when the wind stress changes. Krauss (1979) demonstrates that this results in a second set of inertial waves, which occur at all depths within a fraction of an inertial period because the pressure gradient beneath the surface slope acts throughout the depth. In the shelf-edge context this suggests that inertial motions which penetrate down to the shelf-edge depth may then propagate rapidly to much greater depths down the slope, albeit with much reduced amplitude owing to energy constraints.

#### 7. KELVIN AND EDGE WAVES

Long (barotropic) surface waves ( $L \gg H$ ) may be trapped against a straight coast-shelf-slope by the earth's rotation (the Kelvin wave) and by refraction (edge waves) owing to the deeper water and greater local wave speed  $(gH)^{1/2}$  offshore. The Kelvin wave propagates cyclonically about the deep sea, has no offshore nodes, and exists at all frequencies  $\sigma$  below and above the inertial frequency  $f$  according to the longshore wavenumber  $k$ :  $\sigma = \sigma_k(k)$  (Huthnance 1975). Trapping occurs because the Coriolis force piles up against the coast the forward-moving water at the wave crest. Edge waves propagate in either sense with a dispersion relation  $\sigma = \sigma_n(k) > |f|$  for the mode with  $|n|$  offshore nodes;  $n = 1, 2, \dots$  for cyclonic and  $n = 0, -1, -2, \dots$  for anticyclonic propagation about the deep sea. All edge waves have a low wavenumber cutoff where they cease to be trapped because energy is lost to the deep sea as Poincaré waves (Huthnance, 1975) (perhaps only slowly; Longuet-Higgins, 1967). Particular

profiles of depth  $h(x)$  for which the Kelvin and edge wave forms have been calculated include a concave exponential  $l - \exp(-ax)$  (Ball, 1967), a uniformly sloping shelf  $dx/l$  of width  $l$  and maximum depth  $d$  meeting the deep sea of uniform depth  $D$  at a scarp (Mysak, 1968) and the step shelf  $h = d (0 < x < l)$ ,  $h = D (x > l)$  (Munk, Snodgrass and Wimbush, 1970).

If the shelf is broad and much shallower than the deep sea, then the Kelvin wave is controlled by the major depth change at the shelf edge, rather than by the coast. Below and not close to the inertial frequency, this implies a peak amplitude near the shelf edge (figure 6a), i.e. a double Kelvin wave (see section 3; one of the shelf wave modes then usually takes the form of a Kelvin wave against the coast and is essentially confined to the shelf).

At the inertial frequency, the Kelvin wave surface elevation for an arbitrary depth profile  $h(x)$  is  $\exp(i\omega t +iky - kx)$  (figure 6a), where the dispersion relation determines the long-shelf wavenumber  $k$ :  $k^2 = f^2/g\bar{h}$  where  $\bar{h} = 2k \int_0^\infty h \exp(-2kx) dx$  (Huthnance, 1975). Evidently the surface elevation decays offshore, but the on-offshore flux  $hu = if \zeta(0) \exp(kx) \int_0^\infty (h/\bar{h} - 1) \exp(-2kx) dx$  increases steadily from zero at the coast to a maximum near the shelf edge (just offshore from  $h = \bar{h}$ ). The longshore current on the shelf decreases offshore from its coastal value, and may reverse in the region of the shelf edge (figure 6b).

Above the inertial frequency, these same Kelvin wave features typically remain on the broad shelf, except that the offshore elevation decay is not precisely exponential. The on-offshore current maximum near the shelf edge is as large as the longshore shelf

currents (figure 6b). Approximate scales for an ocean depth  $H$ , shelf depth  $h$  and breadth  $L$ , and surface elevation  $\zeta$  are: deep-sea (offshore, longshore) current =  $O[\sigma L/H, (g/H)^{1/2}] \zeta$  respectively; shelf currents =  $O(\zeta) \max[\sigma L/h, (g/H)^{1/2}]$  if the shelf is not too broad ( $\sigma^2 L^2 / gh < 1$ ). Eventually at higher frequencies ( $\sigma^2 L^2 / gh \geq 1$ ) the shelf is seen as broad and the Kelvin wave becomes trapped against the coast (figure 6a).

Edge waves generally have their greatest amplitudes near the coast, particularly when the depth decreases steadily shorewards, e.g. the shelf profile  $h(x) = 1 - \exp(-ax)$  considered by Ball (1967). The amplitude equation for the surface elevation (Huthnance, 1975) implies an offshore distribution of modal structure roughly as  $[(\sigma^2 - f^2)/gh + fk(h\sigma)^{-1} dh/dx - k^2]^{1/2}$ , i.e. as  $(\sigma^2/gh - k^2)^{1/2}$  at high frequencies: this is clearly weighted towards the coast. Hence edge wave elevations and currents at the shelf edge are expected to be no larger than over the shelf as a whole. Shelf-edge and coastal values may be comparable (figure 6c) if the depth is nearly constant across most of the shelf width rather than increasing steadily off-shore. However, edge wave forms decay rapidly seaward of the shelf edge, so that a sharp increase in energy levels shorewards across the shelf edge can be supported. Munk, Snodgrass and Gilbert (1964) found that most Southern California shelf energy between tidal frequencies and  $10^{-2}$  Hz was contained in such trapped wave modes, rather than exchanging energy with the deep sea.

All continental shelf waves, Kelvin waves and edge waves for a straight shelf and slope of any given depth profile are easily

calculated numerically by a method due to Longuet-Higgins (Caldwell, Cutchin and Longuet-Higgins, 1972).

## 8. BAROTROPIC TIDES

The tides in the oceans and largest adjacent seas are the most common and energetic manifestation of the Kelvin waves discussed above. Hence the conclusions of section 7 apply. One surprising result is that at latitudes exceeding  $30^{\circ}$  diurnal tides can be greater at the edge of a broad shelf than at the coast (even without the effects of an additional shelf wave mentioned in Section 3). This appears to be so for the Celtic Sea, where the major O1 and K1 constituents at the shelf edge (Cartwright, Zetler and Hamon, 1979) exceed 'coastal' values at Newlyn, St. Mary's and Brest (Admiralty, 1980) although the phases match. The maximum of on-offshore flux  $hu$  at the shelf edge is important for internal tide generation (section 9).

A major problem in the numerical simulation of ocean or shelf-sea tides is the disparity between oceanic and shelf-sea scales. Oceanic models have difficulty resolving finer shelf length scales, and the inclusion of oceanic depths drastically reduces the time step required by the Courant-Friedrichs-Lewy condition  $\Delta x/\Delta t > (gh)^{1/2}$  for stable forward integration of shelf models (e.g. Richtmyer and Morton, 1967). In both cases the shelf edge forms a natural model boundary, and these problems are circumvented. However, a boundary condition is then required. In the (usual) absence of detailed knowledge of elevations  $\zeta$  or normal fluxes  $hu$  at the shelf edge, some specification of a relation between  $\zeta$  and  $hu$  must be made in an attempt to model the effects of the sea beyond.

(a) Oceanic models.

In oceanic models, representation of the shelf through the shelf edge boundary condition has generally been rudimentary or (more often) absent. Proudman (1941) first demonstrated the significance for ocean tides of energy losses onto the shelf. In his model, the normal flux onto the shelf is  $Hu = (gh)^{1/2}\zeta$ , matching a shoreward-progressive wave on the shelf. Here  $H$  and  $h$  are the deep-sea and shelf-edge depths respectively. If a fraction  $R$  of the shoreward-progressing energy is eventually reflected back to the shelf edge, the modified flux  $Hu = (1-R)/(1+R)(gh)^{1/2}\zeta$  represents this reduced dissipation. Accad and Pekeris (1978) extended this relation to include a continental slope between the deep sea and the shelf; then  $R$  is complex.

In terms of complex amplitudes and  $\exp(i\sigma t)$  time dependence, the general linear boundary condition is  $u - u_c = (a+ib)(\zeta - \zeta_c)$  where  $a, b$  are real and  $u_c, \zeta_c$  are specified (zero in the above examples). The coefficient  $a$ , if positive, represents energy loss onto the shelf;  $b$  represents flow onto the shelf to supply a rising water level there. If both are small relative to  $(g/H)^{1/2}$ , their effects are approximately additive in the ocean. The intuitive notion, that somehow it should be possible to parameterise the influence of the relatively narrow shelf on the oceanic motion, is fully expressed in this one-dimensional (long-shelf) boundary condition. However,  $a$  and  $b$  can be specified with certainty only from a comprehensive model covering the shelf and ocean.

In practice, the value of  $a$  for each section of shelf has to be estimated from any knowledge of the net energy influx from the deep

sea there, perhaps using a local model of shelf tides. For example, Flather's (1976) north-west European shelf calculations give estimates for a along the adjacent North Atlantic boundary.

For a Kelvin wave travelling along a straight shelf, profile  $h(x)$ , with uniform deep-sea depth  $H$ , Miles (1972) finds  $b = \sigma\delta/H + O(\epsilon b)$  if  $\delta \equiv \int_0^\infty (1-h/H) dx$ ,  $\epsilon \equiv \delta\sigma/(gH)^{1/2} \ll 1$ . Positive  $b$  reduces the wave speed to  $(gH)^{1/2}(1-\epsilon f/\sigma)$  and the offshore decay length to  $R(1-\epsilon\sigma/f)$  where  $R \equiv (gH)^{1/2}/f$ . For other forms of motion involving Poincaré waves, the continuity and longshore ( $y$ ) momentum equations imply  $u = i\sigma/H [\chi\zeta + g/\sigma^2 \partial/\partial y (\partial\zeta/\partial y \int_0^\infty h dx)] + O(uX/R)$  for the boundary condition at the edge of the deep sea. Here we assume a narrow shelf of width  $X \ll R$ , and a length scale for longshore variations of at least  $R$ .

In calculations 'from first principles', where observed tidal data are not utilised,  $u_0$  and  $\zeta_0$  are zero. However, if the calculation utilises observations (which should improve the simulation), then the particular boundary conditions  $\zeta = \zeta_0$  or (preferably, but not usually available)  $u = u_0$  are appropriate. Flather (1976) describes an iterative scheme for a shelf model to determine the boundary values at grid points between observations, using  $a = (g/h)^{1/2}$ ,  $b=0$  for pure radiation of  $(u, \zeta) - (u_0, \zeta_0)$  to the deep sea. A similar scheme could interpolate between boundary data in an oceanic model, using the appropriate 'oceanic' values of  $a$  and  $b$  discussed above. If the observational data is in fact coastal ( $\zeta_c$ ), allowance should of course be made for translation to the shelf edge. For a narrow shelf of width  $X$ , Miles (1972) finds that for a Kelvin wave

$$\zeta/\zeta_c = \exp[-X(1+\epsilon\sigma/f)/R] \left\{ 1 - (\sigma^2/f^2 - 1)R^{-2} \int_0^\infty [\delta(H/h-1) - h^{-1} \int_0^\infty (H-h) dx'] dx \right\}.$$

For other wave forms a coarser approximation using the offshore momentum equation is  $\zeta_o = \zeta_c [1 - (\sigma^2 - f^2)/g \int_0^x x/h dx] + if\lambda/\sigma \partial\zeta_c/\partial y$ . Clarke and Battisti (1980) have found that the latter formula reasonably models observed relationships between coastal and open-sea tides along several shelves of moderate width and slow longshelf variation.

(b) Shelf models.

These models have a 'natural' boundary in shallow water near the shelf edge, avoiding the large depth changes and reduced time steps imposed by greater ocean depths on forward time-difference schemes. One might hope that the adjacent deep ocean with its greater length scale  $O(R)$  imposes a large longshelf scale on the shelf-edge tide. Unfortunately, a more precise statement of this notion is that the oceanic surface elevation  $\zeta$  satisfies  $H^{-1}\nabla \cdot (H\nabla\zeta) = O(\zeta/R^2)$  (Lamb, 1932, Art. 193, 207) where  $R = (gH)^{1/2}/f$ . Hence variations of  $\zeta$  along the shelf with length scale  $L \ll R$  decay away from the shelf in the deep sea on the same scale  $L$ . There is no direct implication for the shelf-edge structure, as is evident if continental shelf waves or edge waves constitute part of the tide. Generally, however, the discussion below does imply that the major contribution of the ocean to the shelf-edge elevation is of large scale, i.e. 'smooth'. This does not follow for currents.

The possibility of 'rough' shelf edge tides means that smooth interpolation between (usually) sparse shelf edge observations may be inappropriate, perhaps leading to large or irregular currents. Such was the reason for Flather's (1976) iterative radiation condition already mentioned in (a) above.



Garrett and Greenberg (1977) have considered the influence of oceanic impedance on shelf calculations. The elevation  $\zeta_M$  at  $s$  on the shelf edge  $M$  is  $\zeta_M(s) = \zeta_o(s) - \int_M K(s, \sigma) F(\sigma) d\sigma$  where  $F(\sigma)$  is the offshore flux hu at  $\sigma$  on  $M$ ,  $K(x, \sigma)$  is the elevation at  $x$  in the ocean resulting from unit onshore flux at  $\sigma$  on the shelf edge, and  $\zeta_o(s)$  is the oceanic elevation if  $M$  were a coastal wall.  $\zeta_o(s)$ , being 'purely oceanic', is smooth.  $K(s, \sigma)$  is a linear combination of the infinite set of oceanic normal modes, and significant contributions arise from: (i) modes close to resonance at the tidal frequency considered; such modes have larger spatial scales than any likely shelf model boundary  $M$  and hence this contribution to  $\zeta_M$  is also smooth; (ii) the infinite set of higher frequency modes with smaller spatial scales which represent the effects of deep-sea topography near  $M$ . Since  $\zeta_o$  and (i) are smooth, they can be obtained as a smoothed version of the shelf edge observations which are obviously necessary for any calculation of the co-oscillating shelf tide. The rough contribution (ii), being nearly independent of the distant ocean properties, can be evaluated (approximately) for a semi-infinite ocean. Buchwald (1971) has done this analytically for uniform depth, and Garrett and Toulany (1979) numerically including a continental slope. To the same degree of approximation as our separation of rough and whole-ocean contributions, only the near-field form of  $K$  is required. In a numerical scheme  $\zeta_M = \zeta_o - \int KF$  becomes  $\zeta_{Mp} = \zeta_{op} - \sum \frac{K_{pq}}{2} F_q$ , a boundary condition primarily determining  $\zeta_M$  but allowing trade-off between  $\zeta_M$  and the normal flux  $F$ . The matrix  $K$  has large diagonal terms. Iteration is necessary to adjust  $\zeta_o$  for smoothed contributions from

$\int K F$ , resulting in a scheme similar to Flather's (1976) except for a more sophisticated  $\zeta/F$  relation to represent radiation to the deep sea.

## 9. INTERNAL TIDES

This subject has been reviewed by Wunsch (1975) and Schott (1977). It is generally thought that internal tides are generated from the interaction of the barotropic tide with sea-floor topography, particularly over continental slopes (although this is not now thought to be important as a sink of barotropic tidal energy). Any depth decrease obviously obstructs the lower layer more than the upper in a two-layer model, so that it is clear why baroclinic tidal currents occur, particularly since the barotropic onshore flux is likely to be largest near the shelf edge.

Rattray (1960) considered a two-layer model with a step shelf and coastal wall. This was later extended by Weigand, Farmer, Prinsenber and Rattray (1969) to include friction and good comparison with laboratory experiments, and Rattray, Dworski and Kovala (1969) considered continuous (uniform) stratification. In the latter case, the velocity is singular on the two characteristics of slope  $(\sigma^2 - f^2)^{1/2} / (N^2 - \sigma^2)^{1/2}$  radiating from the shelf edge, including their reflections at the sea surface and the sea floor. Seawards from a shallow shelf, internal tides are confined to the neighbourhood of the relatively narrow band between the two characteristics (figure 7). Rattray et.al. (1969) found the coefficient of the first vertical mode ( $\frac{1}{2}$  cycle) to be comparable with the two-layer result. Evidently the banded and singular solution contains higher modes, but these are more

rapidly dissipated away from the shelf break. The singularity on characteristics is probably also an artifice of the purely linear theory.

Later models (Baines, 1973, 1974; Prinsenberg, Wilmot and Rattray, 1974; Prinsenberg and Rattray, 1975; Sandstrom 1976) consider various forms of stratification and topography. The internal tide stream function in the vertical offshore plane is forced by  $\partial/\partial x (FN^2 z dh'/dx)$  (Baines 1973;  $F$  = onshore flux); this is concentrated as  $d^2 h'/dx^2$  near the shelf edge. Hence more generally, internal tides are concentrated near characteristics from the shelf break if these exist, i.e. if the sea floor slope below the shelf break is greater than the  $M_2$  (say) characteristic slope. This is often the case, e.g. off N.W. Africa (Horn and Meincke, 1976) and New England (Wunsch and Hendry, 1972). A continental slope which approaches the characteristic slope on rising to the shelf edge can reinforce the concentration of motion along that characteristic. Theory (Wunsch, 1968) and observations (Wunsch and Hendry, 1972; Horn and Meincke, 1976) also suggest that strong currents may occur near the bottom where this has the characteristic slope or a little greater. (A slope  $\alpha$  allows Rhines' (1970) bottom trapped waves at frequencies up to  $\alpha N$ , where their energy is likely to be concentrated since  $\alpha N$  is a stationary value (maximum) of their frequency as a function of propagation direction).

If the bottom slope on the shelf is less than the characteristic slope ( $R_1 \alpha / H < 1$  approximately), then the shoreward progressing internal tide is not reflected back towards the shelf edge. In practice it

also appears to be rapidly dissipated during its shoreward progress (Petrie, 1975; Horn and Meincke, 1976). Hence models for internal tide generation can reasonably neglect the coast and impose shoreward radiation as a boundary condition for the shelf internal tide. Baines (personal communication) finds an analytic solution to such a model with uniform stratification, a flat shelf and a uniform continental slope  $\alpha$ . This now appears to contain the essence of internal tide generation when  $\alpha$  exceeds the characteristic slope.

A consequence of the localised internal tide offshore from the shelf edge is its notorious variability and incoherence. Its presence at a moored current meter depends on the characteristics from the shelf edge bracketing or at least coming close to the instrument (Regal and Wunsch, 1973). This will certainly depend upon the season (Sandstrom, 1976) and upon major changes such as upwelling (Hayes and Halpern, 1976). In practice, less obvious stratification changes and perhaps advection by currents also reduce the internal tide's predictability; an experiment designed to span the characteristic from the shelf edge for comparison with theory was only partially successful (Barbee, Dworski, Irish, Larsen and Rattray, 1975). Typical current meter time series from on or near the continental slope do not even show a coherent spring-neap cycle (internal tides, being strongest there, are typically comparable with the barotropic currents). Inability to resolve S2 from M2 may be a factor in the anomalously large S2 first internal mode seen by Gould and McKee (1973) near the Celtic Sea slope. Generation near the shelf edge suggests that the internal tide should be most coherent there,

although spatially complex and still dependent upon season and major stratification changes.

#### 10. INTERNAL WAVES

Internal waves are a universal feature of the oceans (e.g. Garrett and Munk, 1975), but their occurrence on continental shelves will depend on the (often seasonal) stratification there, and on any transmission or generation near the shelf edge.

Theoretical and observational evidence regarding transmission appears to be limited. At frequencies substantially greater than tidal, the characteristic slope  $(\sigma^2 - f^2)^{1/2} / (N^2 - \sigma^2)^{1/2}$  of the waves exceeds that of most continental slopes. It then seems likely that relatively free exchange takes place across the shelf edge, transmission being perfect in a wedge (Wunsch, 1969) and when the topographic length scale  $H/\alpha$  greatly exceeds the wavelength  $O(R_p f/\sigma)$  (Keller and Mow, 1969). Gordon (1978) found a general shoreward decline in internal wave energy and (shoreward) energy flux over the upper slope and shelf off N.W. Africa, with no clear shelf-edge effect. However, large amplitude internal solitary waves arriving from the deep sea are liable to break down into several smaller waves owing to the depth reduction (Djordjevic and Redekopp, 1978).

Of course, any transmission at all depends upon sufficient shelf stratification to support the given frequencies. An interesting question is the extent to which oceanic internal waves may excite barotropic 'medium-frequency' waves, e.g. edge waves on the shelf. The Garrett and Munk (1975) internal wave spectrum implies 3-5 cm/sec near-surface currents at frequencies above  $10^{-4}$  Hz in the ocean;

translated onto the shelf these imply surface elevations of 10-16 cms in 100m of water. Since 5-10 cms is a large medium frequency elevation (Munk, 1962), oceanic internal waves are a potentially important source (even with substantial shelf edge losses), and interaction near the shelf edge merits investigation.

Various internal wave observations appear to be associated with tidal flows near the shelf edge. Bell (1975) finds that harmonics are generated by oscillatory flow over an isolated sea-floor bump, and Blackford (1978) describes a mechanism for generating the first harmonic (e.g. M4 from M2) of tidal flow over a sill. Some such non-linear process probably excited the strong M4 oscillations seen by Gordon (1979) near the bottom at 400m depth on the N.W. Africa slope. These currents, orientated more strongly on-offshore than M2, were probably enhanced by the sea-floor slope nearly equalling the M4 characteristic slope. Shepard (1976) has observed currents up to 50 cm/sec close to the bottom of canyons in the continental shelf off Southern California. They typically have tidal frequencies and phase propagation up the canyon, but nearer the head of the canyon higher frequencies are observed. Observations by Wunsch and Hendry (1972) are consistent with the transfer of internal tidal energy into the remainder of the internal wave spectrum by non-linear interaction.

Groups of short period (eg. 10 min) internal waves at certain phases of the (M2) tide have been seen at many locations, usually in temperature time series, eg. in the Strait of Gibraltar following an internal 'front' (Ziegenbein, 1969), inshore from Stellwagen bank in Massachusetts Bay (Halpern, 1971; Haury, Briscow and Orr, 1979), in

the Strait of Georgia, British Columbia (Gargett, 1976) and in a British Columbia fjord with salinity and currents as indicators (Farmer and Smith, 1978). Apel, Byrne, Prioni and Charnell (1975) discuss satellite observations of internal wave trains separated by one or two tidal periods and propagating inshore from the shelf break in the Middle Atlantic Bight and off S.W. Africa. An explanation supported by laboratory experiments has been given by Maxworthy (1979), and may be applied to the ebb tidal flow over a continental shelf edge and subsequent developments. Tidal flow over an obstacle causes a lee wave under stratified conditions (figure 8a). With sufficient stratification, eg.  $N > 2 \times (\text{tidal frequency})$  for flow of deep fluid over an isolated obstacle (Bell, 1975), the flow is quasi-steady and the lee wave may simply decline when the tidal flow slackens. However, for weaker stratification (higher Froude numbers) the wave persists and tends to maintain its velocity relative to the flow, subsequently propagating with the reversed tidal current, i.e. inshore in the shelf edge context (figure 8b). Being of large amplitude, the lee wave tends to break down into a train of smaller waves, particularly on entering shallower water as already noted (Djordjevic and Redekopp, 1978). This wave train is analogous to an undular bore on a river.

If a lee wave also forms in the reversed tidal stream, that in turn is expected to propagate in the opposite sense, i.e. seawards in the shelf edge context. Here, propagation into deeper water suggests that a wave train need not form; the wave(s) should be of larger amplitude and longer than on the shelf. This has yet to be identified in observations.

In flows sufficiently strong for the lee wave to break and mix, Maxworthy (1979) also found that slackening of the stream brought a collapse of the mixed region and hence solitary waves propagating away in both senses (every half cycle). This high Froude number phenomenon requires a combination of large currents and weak stratification, and is therefore less likely than the wave trains described above and more dependent on the season.

## 11. DISCUSSION

The neighbourhood of the shelf edge sees a rapid change from shelf depths of typically 100-200m to oceanic depths of some kilometres. Accordingly, there is a corresponding rapid change of regime. In contrast with oceanic conditions, wind stress and bottom friction on the shelf are dynamically important for the barotropic flow, which is also strongly controlled by the shelf geometry. Fresh water input and tidal mixing are influential in the formation of the water masses, and generally mixing occurs throughout the depth in winter, when heat loss has an exaggerated effect in the shallower water. On the other hand, the ocean, having a permanent thermocline, can sustain eddy and internal wave fields which would be rapidly dissipated on the shelf. Hence the shelf edge region is interesting, firstly, for the juxtaposition of these different regimes.

Secondly, the depth change plays more active roles. Regarded as a mismatch of the depths on either side, it is responsible for the generation of internal tides, shelf edge upwelling, the reflection of oceanic eddies, and the generation of shelf-wave forms by oceanic-scale motions and eddies. As an overwhelming contribution to



the potential vorticity gradient, it enables continental shelf waves and double Kelvin waves to propagate, the latter also seeing the shelf edge as a replacement for the Kelvin wave's coastal wall. Increasing depth  $h$  offshore implies an increasing long surface wave speed  $(gh)^{1/2}$  with associated refraction towards the shelf, where energy may consequently be trapped as edge waves. Quasi-steady flow of the stratified sea over varying depths causes lee waves.

The shortened horizontal length scale implied by the rapid depth change enables non-linear effects such as M4 tide generation and 'tidal stress' contributions to mean currents (Smith, Petrie and Mann, 1978) to be significant, enhances the generation of internal tides and shortens the length scale of all barotropic currents.

The concentration of depth contours tends to guide and strengthen currents along the continental slope. These may influence flows in the region of the shelf edge, particularly when unstable. Meanders and eddies often dominate shelf-edge flows at subinertial frequencies, although apparently decaying inshore on a length scale  $O(R_f)$ .

Canyons are one feature whose effects are perhaps reduced if they indent the shelf edge rather than approach the coast. The latter context is associated with the piling up of water by onshore wind, with set up from groups of large amplitude swell and with edge waves (Inman, Nordstrom and Flick, 1976); especially in combination, all of these encourage strong flows - turbidity currents entraining sediment - down canyons, but they are much less effective at the distant shelf edge. However, we do expect that shelf-edge canyons may control the position of intensified upwelling. The irregularity they impose on the

shelf edge may also encourage mixing of water masses across it. Shelf-edge irregularities may also induce meanders in currents along the slope, such as the Gulf Stream downstream from the 'Charleston bump' (Legeckis, 1979; Chao and Janowitz, 1979).

We have not concentrated on secondary effects such as the mean currents which are likely to arise from the difference in water masses and reflection of eddies. A well-known example is the occurrence of 'giant' waves (Smith, 1976) near the shelf edge off the East coast of S. Africa. These are thought to arise at a caustic associated with the refraction of surface waves by the Agulhas current. Although the current is indeed guided by the shelf edge, the direct effect of the depth contrast on the surface waves is minimal. Another example is the observation of a band of cooler surface water 0(30 km) broad over the Celtic Sea shelf edge (Pingree, 1979; R.R. Dickson, personal communication). This is immediately suggestive of shelf-edge upwelling, but (i) the observed temperatures (on June 27, 1979) are about typical of the location and month ( $15^{\circ}$  C: ICES, 1962) within the cooler band, and  $2^{\circ}$  C higher outside, (ii) these observations and associated infra-red images from satellites were obtained (almost inevitably) during calm sunny weather and (iii) the temperature below the seasonal thermocline here is about  $11^{\circ}$  C (I.D. James, personal communication). Hence if upwelling was responsible, it clearly occurred some days previously and subsequent warming had taken place. The conditions and absolute temperatures rather suggest that a thin warmer surface layer had formed on either side of the shelf edge. Such warm surface layers have been observed to play a role in the

spring development of the Celtic Sea seasonal thermocline (James, 1980). Then it would appear that enhanced vertical mixing near the shelf edge (perhaps associated with more energetic internal motions or longshelf currents, or with circulation within the upper layer, as suggested by Heaps (1980): section 4) prevents a thin warm surface layer from forming there.

Whatever its origin, cooler surface water will be associated with surface geostrophic currents along its boundaries. Pingree (1979) suggests that baroclinic instability of those currents results in the apparent eddying along the boundaries of the cooler water. Baroclinic instability in near-shelf edge currents has also been inferred from fluctuations in the Norwegian current (Mysak and Schott, 1977) and waveforms associated with the California undercurrent off British Columbia (Mysak, 1977).

Many interesting problems arise from the shelf edge context which still await solution or deeper understanding. For example, we have only guessed at which factors distinguish the New England shelf edge with a front from the Celtic Sea without. Only the crudest modelling has been attempted of the reflection or otherwise of oceanic eddies approaching a continental slope. Our discussion of shelf-edge boundary conditions for oceanic tide models was restricted to narrow shelves ( $f_0^2 X^2/gh < 1$ ) with slow long-shelf variation, and the most advanced modelling of shelf tides with a shelf-edge boundary condition utilising the oceanic Green's function is very approximate and cumbersome. A better understanding of internal wave transmission and interaction with barotropic motion across the shelf edge is desirable,

as is a greater appreciation of the effects of substantial long-shelf changes of geometry on almost all shelf-edge phenomena.

Acknowledgement.

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## LEGENDS

1. Sketch of Middle Atlantic Bight shelf-edge conditions in late winter (after Wright, 1976; Voorhis et.al., 1976).  
Salinity in ‰ (---), temperature in °C (—). Surface along-shelf current  $\odot$ .
2. Sketch of the North-west European shelf edge in June (after Ellett and Martin, 1973). Salinity in ‰ (---), temperature in °C (—).
3. First two barotropic shelf waves (1,2): elevation  $\zeta/Z$ , longshore current  $v/U$ .  
(a)  $h = Hx/L$  ( $0 < x < L$ ),  $h = H(x > L)$ ,  $k_1 L = 0.415$ ,  $Z_1/U_1 = 0.232 fL/g$   
 $k_2 L = 1.446$ ,  $Z_2/U_2 = 0.0592 fL/g$   
(b)  $h = 0.2H(1 - \exp(-10-5x/L))$  ( $-2L < x < 0.2L$ ),  $h = Hx/L$   
( $0.2L < x < L$ ),  $h = H$  ( $x > L$ ),  $k_1 L = 0.0609$ ,  $Z_1/U_1 = 1.64 fL/g$ ,  
 $k_2 L = 1.196$ ,  $Z_2/U_2 = 0.197 fL/g$ . In both cases  $f^2 L^2 / gH = 0.0025$ ,  $\sigma/f = 0.1$ .
4. Two-layer upwelling. (a) section, (b) Northern hemisphere upper layer plan.  $U_{\bar{z}} \equiv \tau / \rho f h_1$ .
5. First trapped wave at  $\sigma = f$ . Profiles of onshore ( $u$ ), longshore ( $v$ ) and vertical ( $w$ ) velocity components on the sea floor  $z = -h(x)$  where  
 $h = 0.2 Hx/L$  ( $0 < x < L$ ),  $h = -0.6H + 0.8Hx/L$  ( $L < x < 2L$ ),  $h = H$  ( $x > 2L$ ).  
 $u, v, w \propto \exp(-kx)$  where  $kL = 0.97$ ,  $f^2 L^2 / gH < 1$ ,  $N^2 H^2 / f^2 L^2 = 10$ .
6. (a) Kelvin wave elevation profiles for  $\sigma/f = 0.1, 1, 10$  where  $h = 0.05H (1 - \exp(-10-5x/L))$  ( $-2L < x < 0.05L$ ),  $h = Hx/L$  ( $0.05L < x < L$ ),  $h = H$  ( $x > L$ ).  
 $f^2 L^2 / gH = 0.0156$ ,  $k(gH)^{1/2} / \sigma = 1.294, 1.585, 7.20$  respectively.

(b) Kelvin wave onshore ( $u$ ) and longshore ( $v$ ) velocity profiles when  $\sigma=f$  in (a).

Elevation/velocity scale =  $0.0534 (H/g)^{1/2}$ .

(c) All edge wave elevation profiles for  $\sigma/f = 10$  and the same depth profile.  $k(gH)^{1/2}/\sigma = 1.136, 2.575, 3.753, 4.309$  respectively for modes 4,3,2,1.

7. Sketch of internal tide generation. Characteristics (---), barotropic tide  $\xrightarrow{U}$ , phase lead (+), lag (-). After Prinsenbergh and Rattray (1975).
8. Sketch of (a) shelf edge lee wave and (b) subsequent internal wave train propagating inshore (after Maxworthy, 1979).

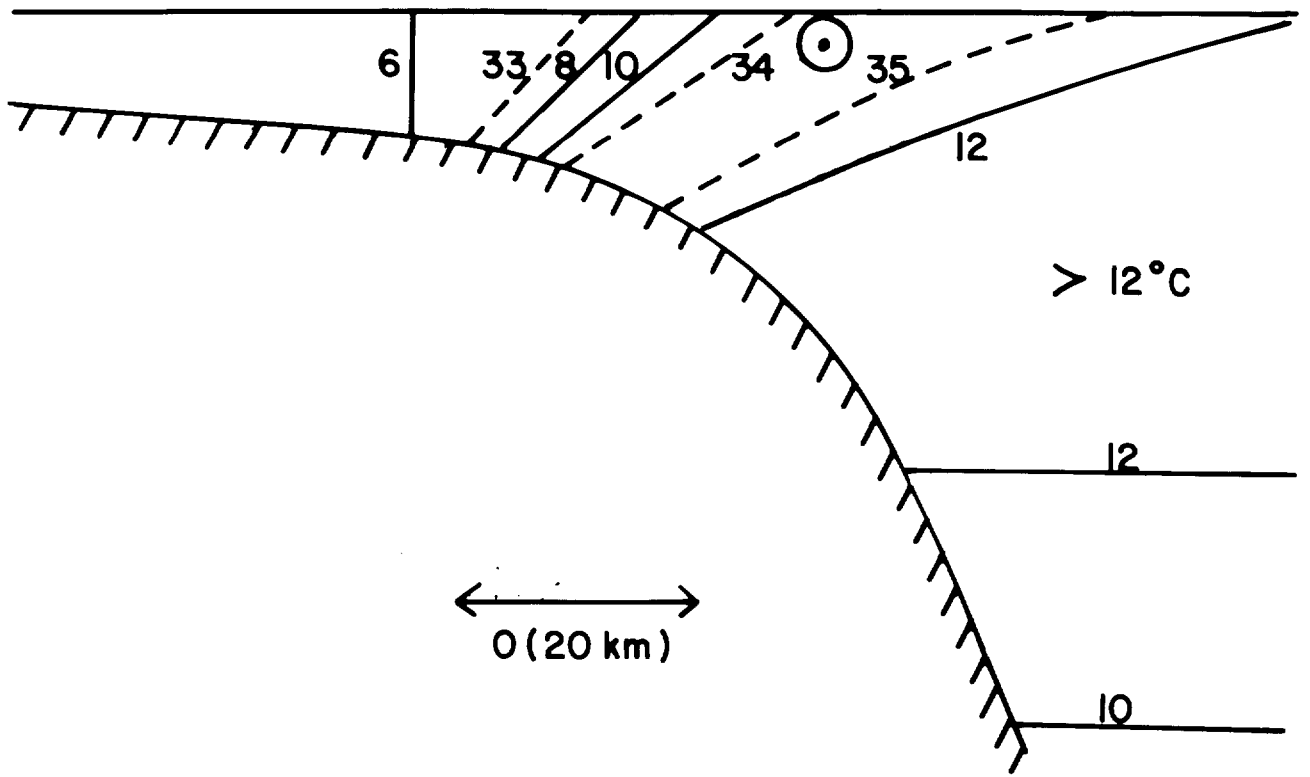


Figure 1

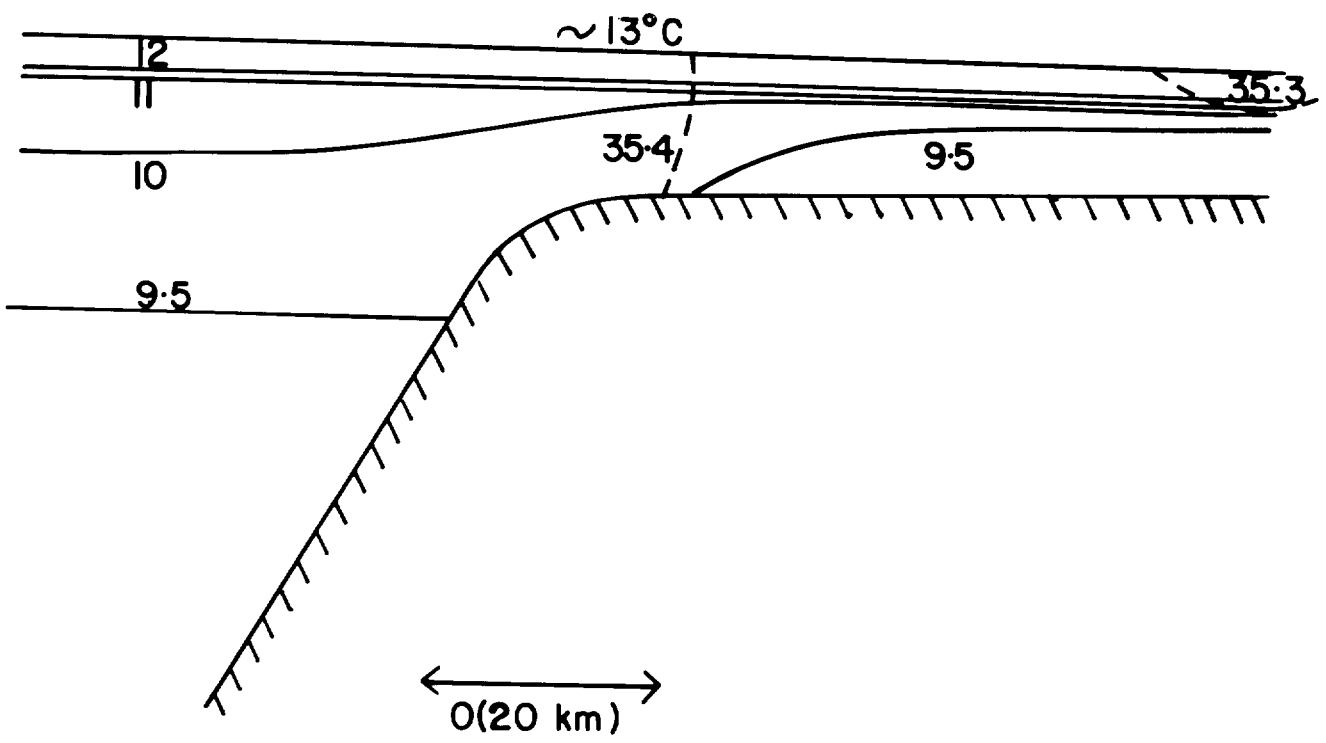


Figure 2

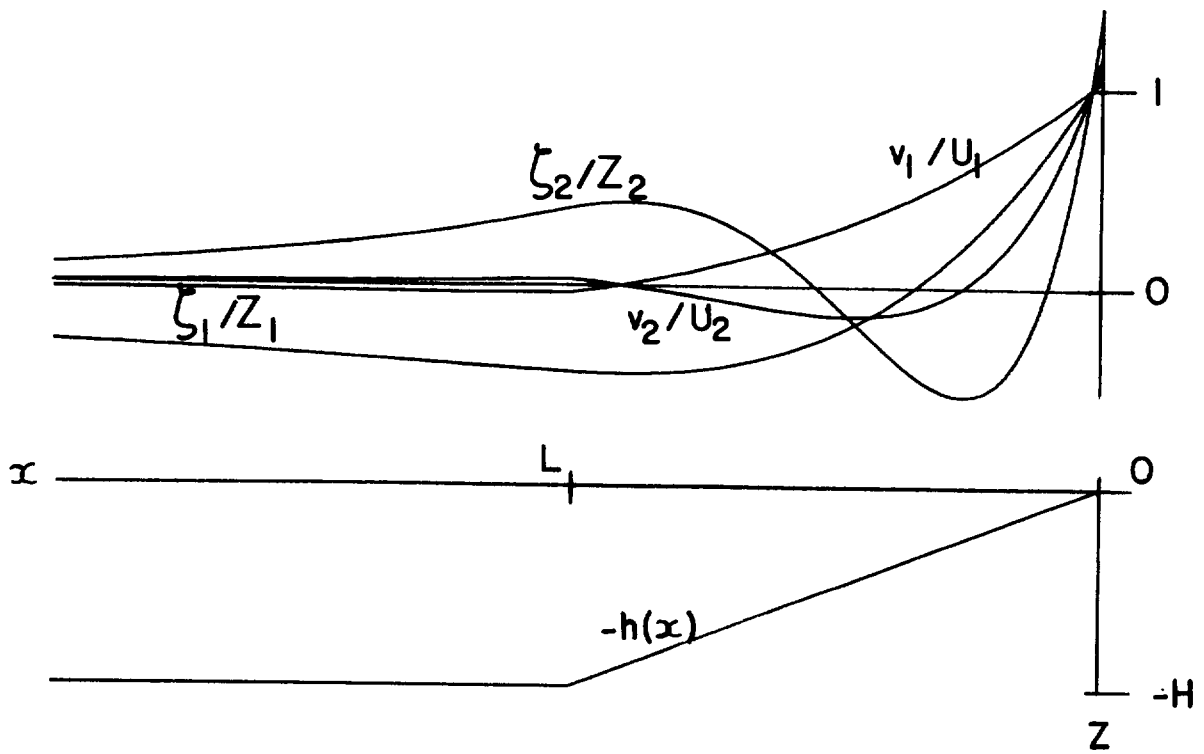


Figure 3(a)

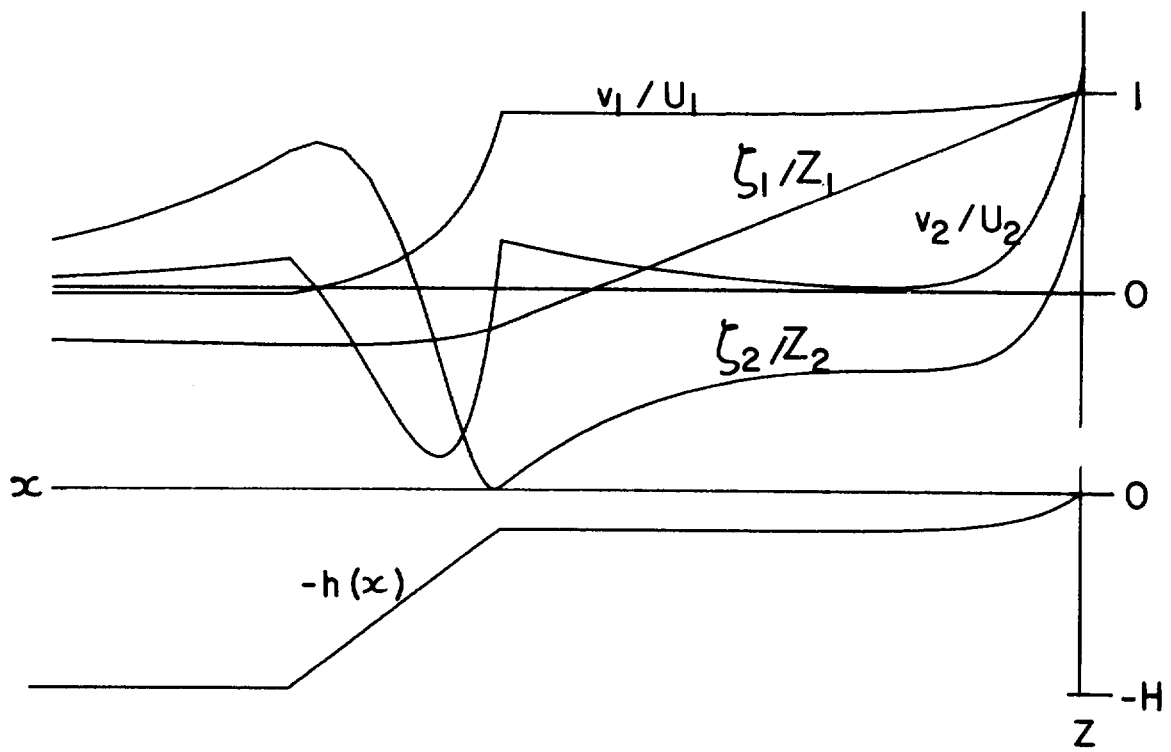


Figure 3(b)

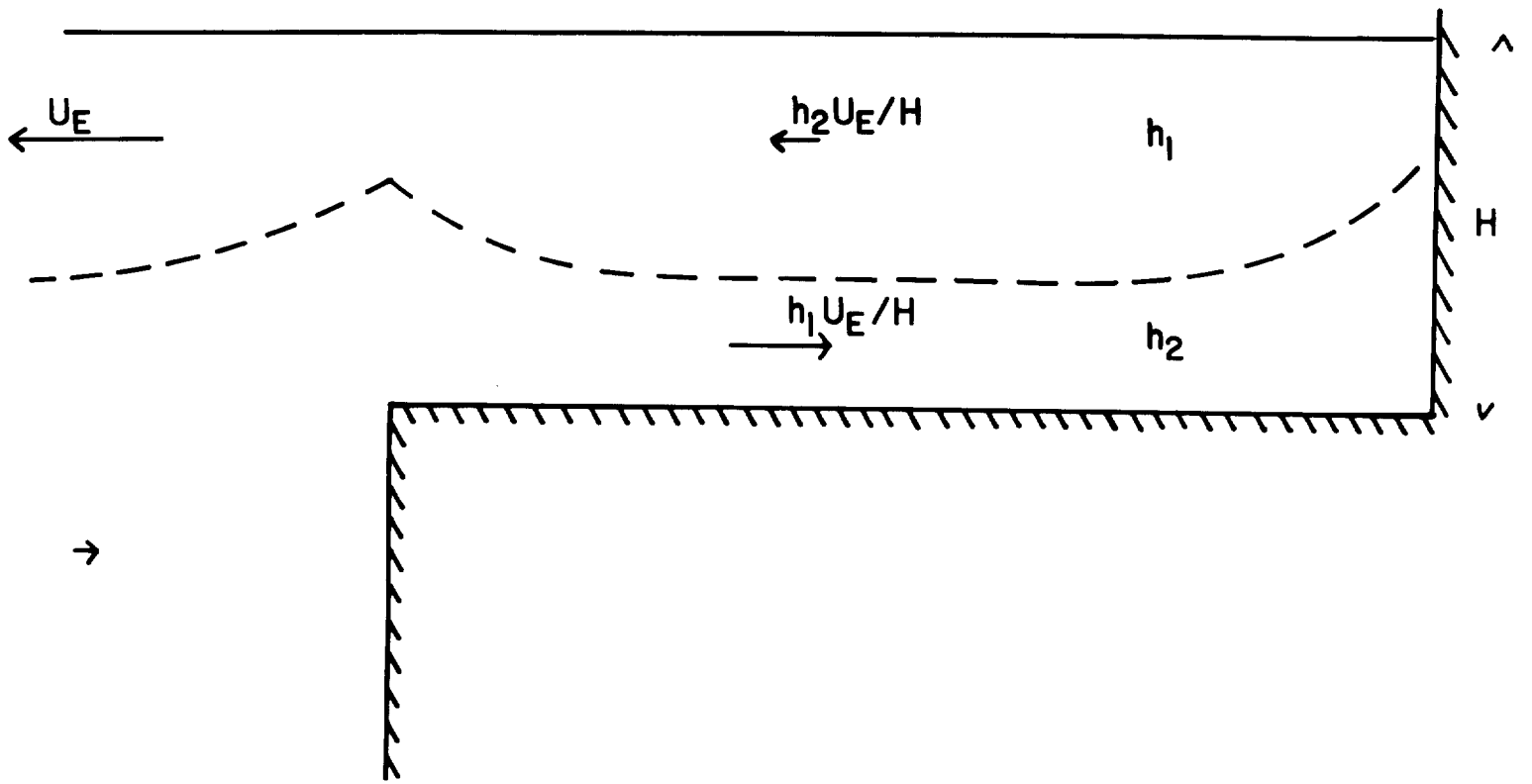


Figure 4(a)

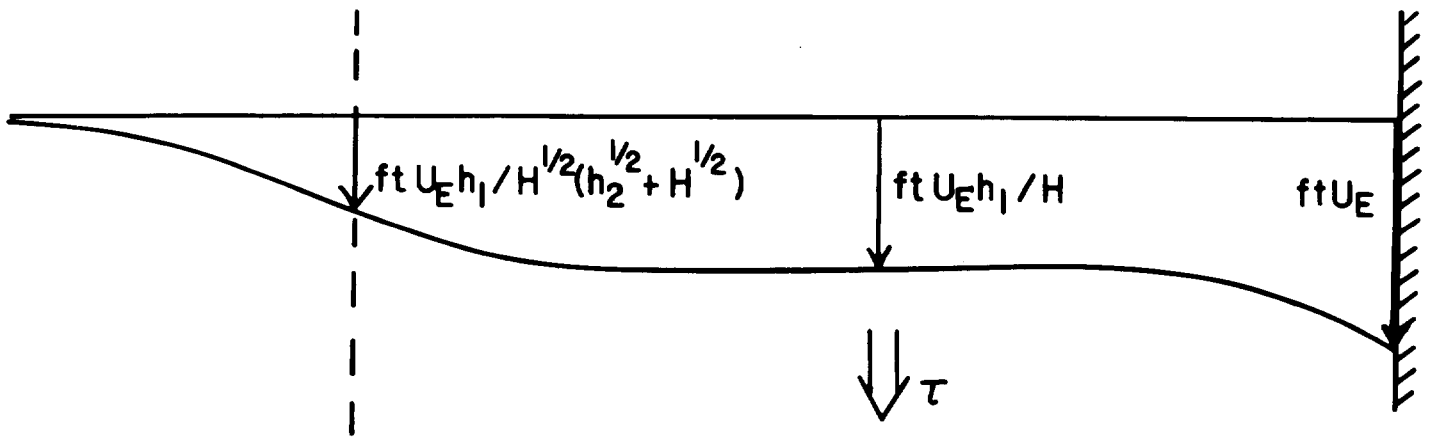


Figure 4(b)

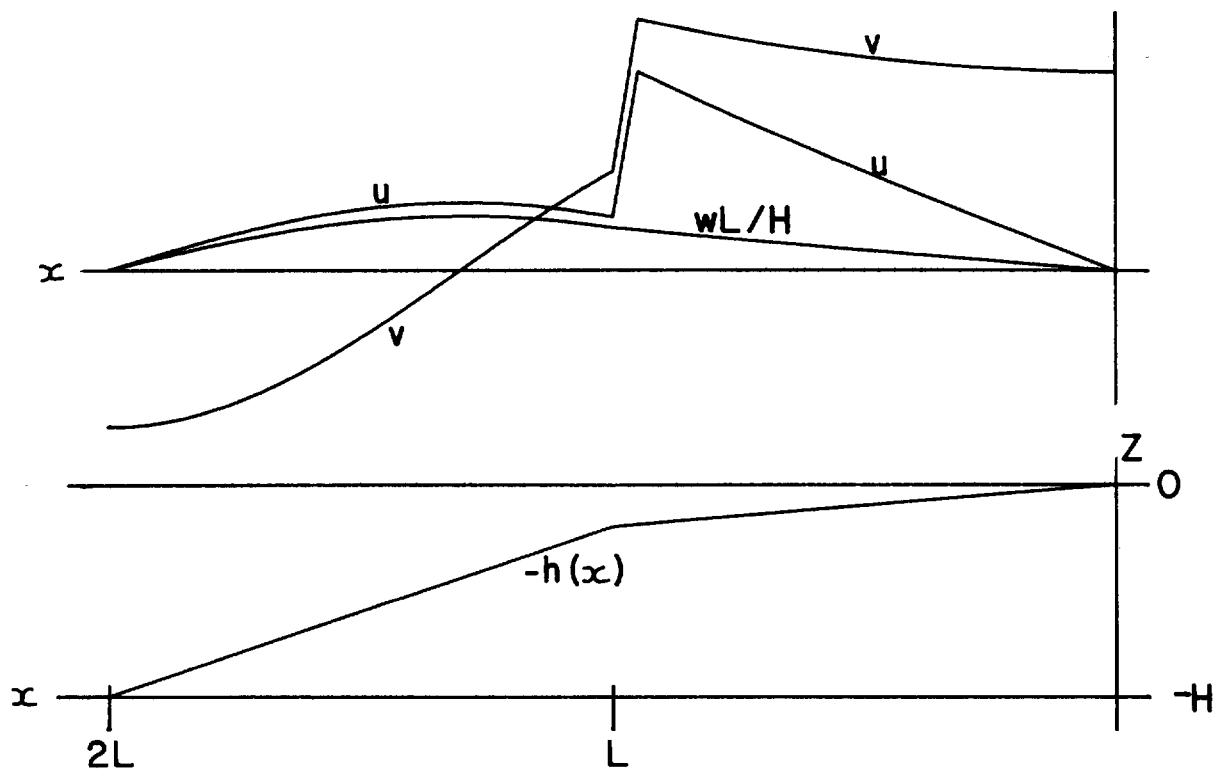


Figure 5

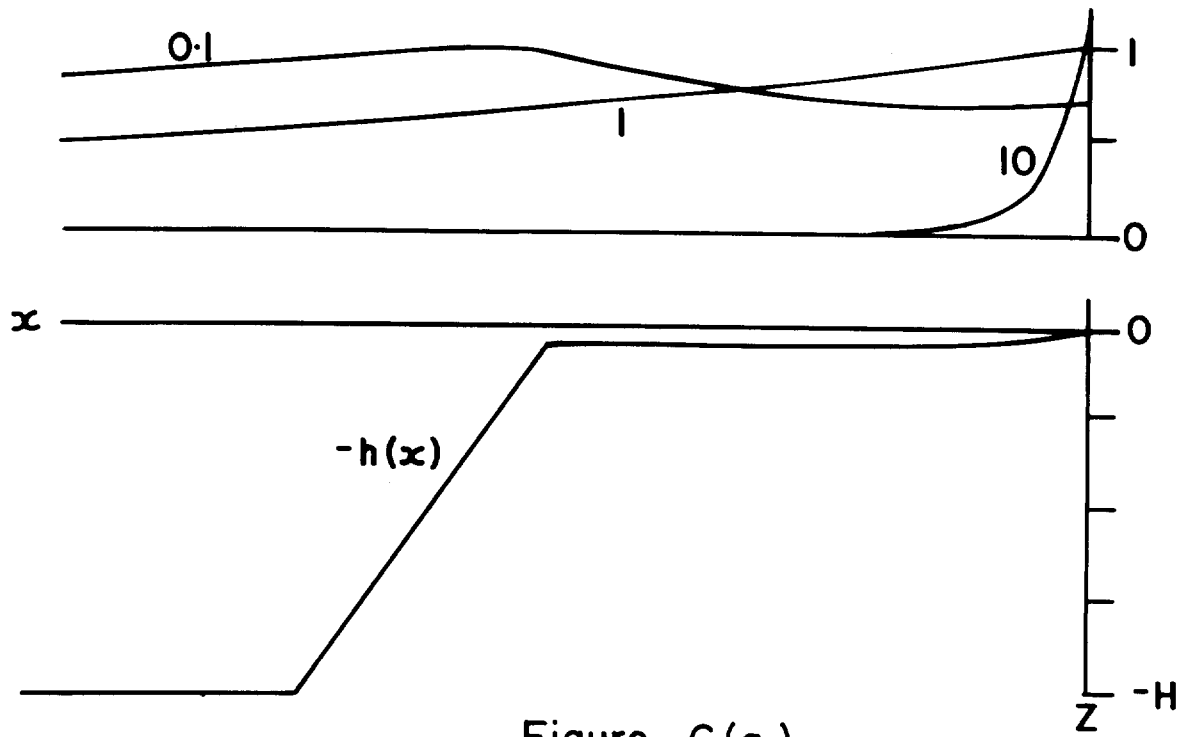


Figure 6(a)

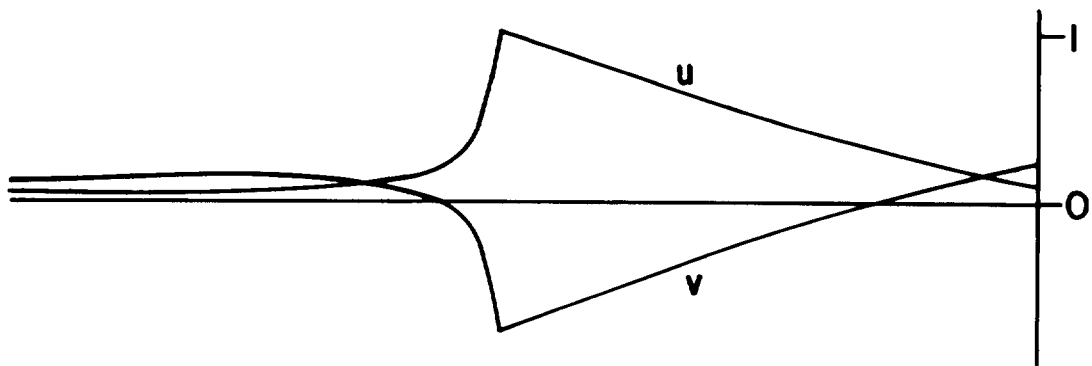


Figure 6(b)

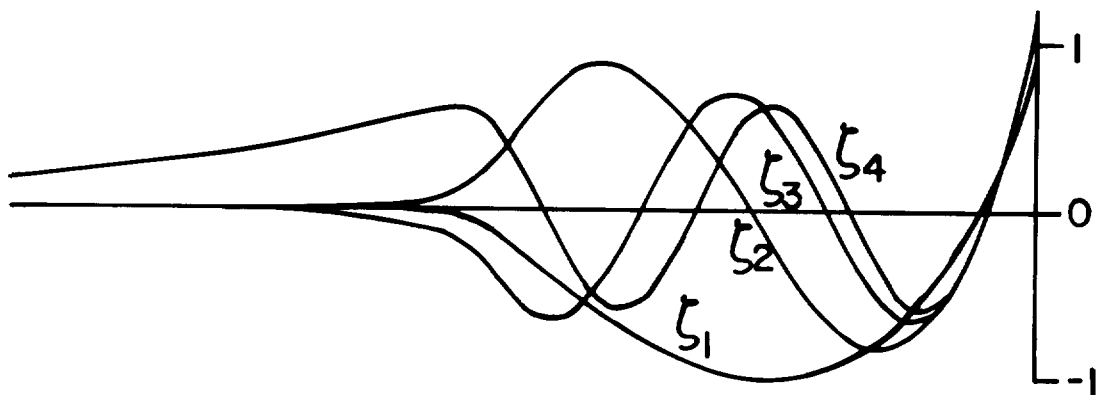


Figure 6(c)

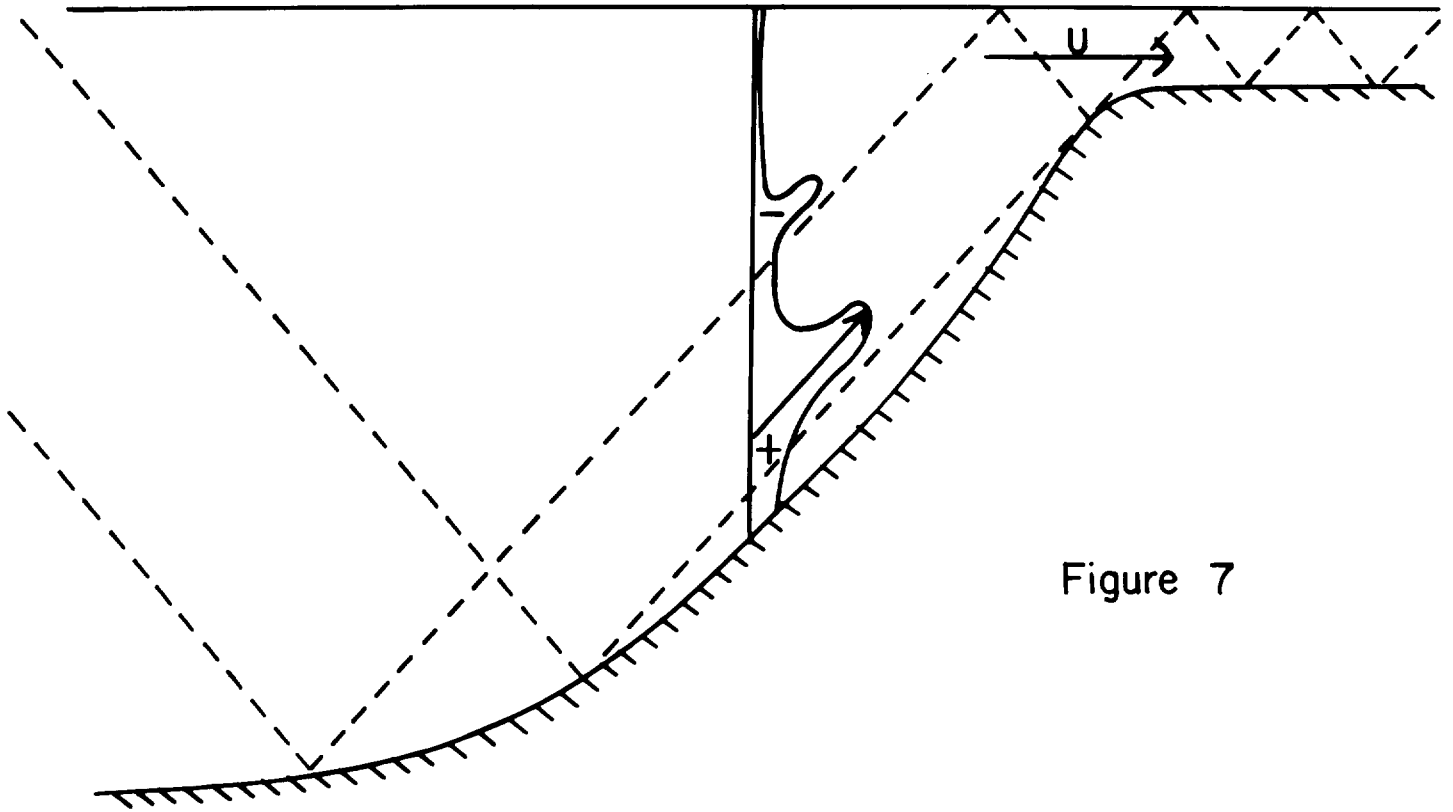


Figure 7



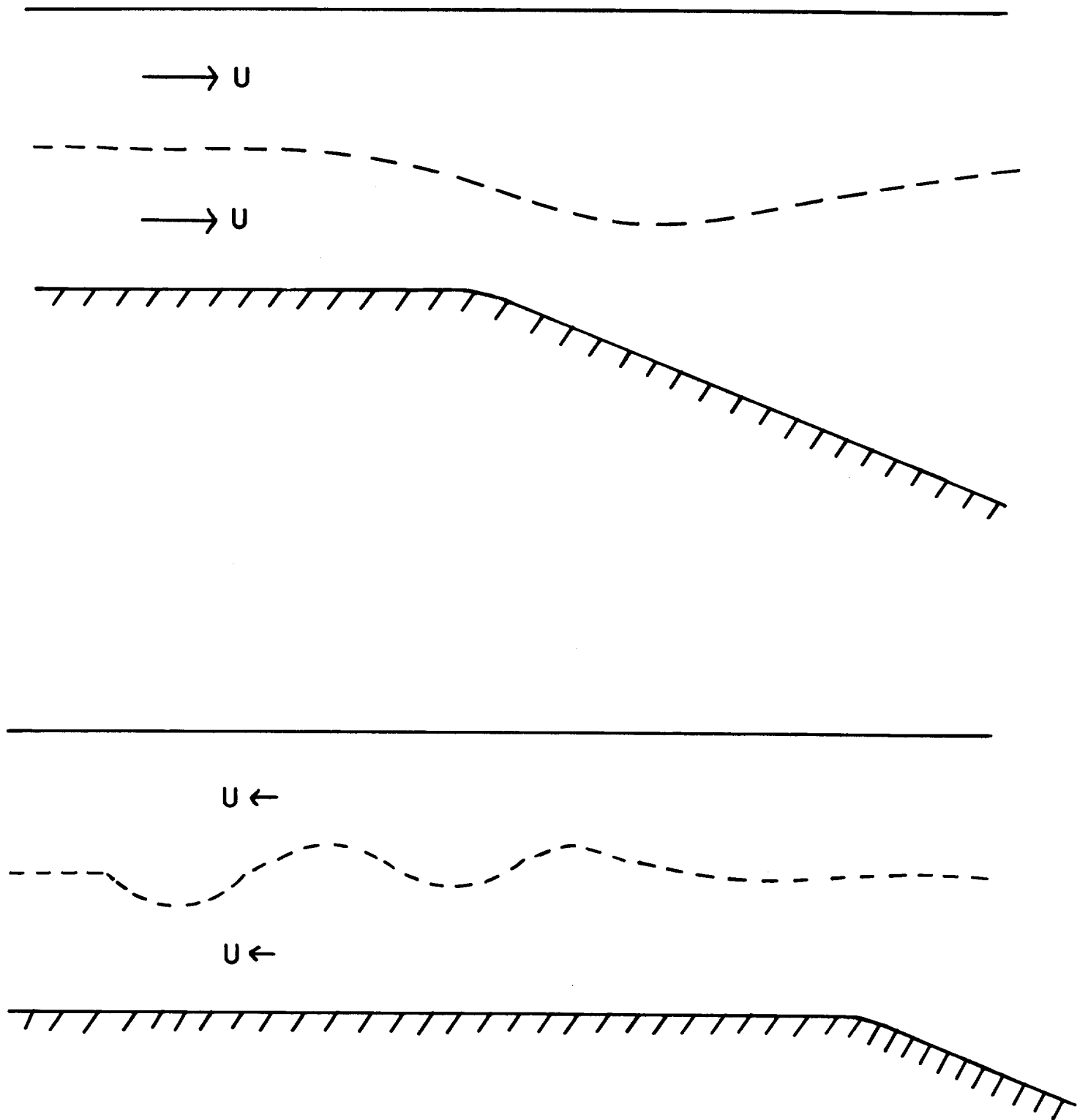


Figure 8