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THE INTERACTION OF SURGE AND TIDE IN THE NORTH SEA AND RIVER THAMES

D. PRANDLE and J. WOLF

REPORT NO. 64

1978

NATURAL ENVIRONMENT RESEARCH COUNCIL

INSTITUTE OF OCEANOGRAPHIC SCIENCES

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ABSTRACT

An examination of surges recorded by any of the tide gauges in the River Thames shows that surge peaks tend to occur 3 to 4 hours before high tide and that large peaks seldom, if ever, occur actually on high tide. These characteristics have been repeatedly confirmed by statistical analyses and are of direct relevance to such problems as flood prediction and the determination of the probability of maximum flood levels. The effects may be attributed to the interaction between tidal propagation and surge propagation in the area, accountable in terms of the presence of non-linear terms in the relevant hydrodynamic equations. The phenomenon of surge-tide interaction in this context has attracted the attention of a number of researchers, but their work has not yielded a coherent and satisfying explanation of the mechanisms involved. The present study seeks to clarify some of the processes at work and includes both a statistical analysis of recorded surge data and an examination of surge-tide propagation with the aid of numerical models.

An analysis is given of surges recorded at various British east-coast ports from Lerwick in the northern North Sea southwards to Tower Pier near the head of the Thames. Results show that surge levels are amplified progressively as the surges propagate southwards and that the magnitude of this amplification is sensibly independent of the height of the initial disturbance. Similarly the level of interaction increases progressively from an almost insignificant level at Wick to a maximum at Tower Pier; an interruption in the progression is, however, seen at Lowestoft and reasons for this are suggested. A study of discrete surge events shows that the generation of a surge peak on the rising tide in the Thames can occur irrespective of the phase relationship between tide and surge in the northern North Sea.

The present work employs a one-dimensional numerical model of the Thames with a seaward boundary running approximately from Walton to Margate. This model was used to determine the response and sensitivity of the river to surges of different types. The results show that the amplification of surge levels along the river is sensibly independent of the surge level at the mouth whereas the amplification is strongly dependent on the shape (i.e. the time-profile) of the surge.

A method of identifying the mechanics of interaction in the Thames has been developed here involving two versions of the numerical model mentioned above: one of tidal propagation and the other of surge propagation. These two models

were operated concurrently with a cross linkage, in the form of perturbation terms, by which the tide model was influenced by the surge and the surge model by the tide. An examination of these perturbation terms shows the interaction to be proportional to a product of surge amplitude and tidal amplitude. The results from the models demonstrate the relative significance of the interaction effect of surge on tide and tide on surge, and enable an assessment to be made of the importance of the principal non-linear terms, namely quadratic friction and (representing shallow-water influence) products of the motion involving surface elevation. It is shown that, in the Thames, quadratic friction plays the dominant role by severely damping surge peaks. Also, it is concluded, both from the model studies and from the statistical analysis of recorded surges, that an important component in the observed interaction in the Thames originates outside the River.

Finally, some deductions are made concerning the necessary meteorological conditions under which a large surge could be generated with a peak at, or very near, high tide in the Thames.

1. INTRODUCTION

A storm surge is defined as a variation in sea level resulting from the action of meteorological forces applied to the sea surface. In the North Sea large storm surges occur with some regularity, these being generated by the combined effects of wind and pressure gradients acting both within the North Sea and over the adjacent oceanic regions. A comprehensive review of the physics of storm surge generation in the North Sea has been presented by Heaps (1967). A typical value for the annual maximum surge at Southend is 1.5m ; the amplitude of the Spring tide there is approximately 2.6m and the Neap tide 1.7m. Since the danger level in relation to coastal flooding is specified at around 3.5m above mean sea level, flooding in the Thames is therefore possible whenever a large surge occurs near to the time of tidal high water on a Spring tide. However, mitigating this condition, surge peaks tend to occur 3 to 4 hours before tidal high water and seldom, if ever, coincide with the peak of high water on a Spring tide. There is little doubt now that this characteristic of the surge-tide time series is a manifestation of surge-tide interaction and may be ascribed to the effect of the non-linear terms in the shallow water wave equations, e.g. see Rossiter (1961), Banks (1974).

In addition to the basic geophysical interest, an understanding of surge propagation in the North Sea is required for two related but separate practical purposes. The first concerns the statistical estimation of the likelihood of a particular extreme water level: information required for Civil and Maritime Engineering applications such as the design of sea defence structures and the depth of dredged channels. In determining the relevant statistics for this problem, a more consistent pattern emerges if the probability distributions associated with tide and surge are computed separately. Information on the properties of surge-tide interaction is then required to facilitate the combination of these two separate distributions into a single corresponding distribution relating to total water level.

The second purpose for which information on surge propagation is required concerns the prediction in real-time of potentially dangerous storm surge levels. At present, operational predictions for east-coast ports are computed using statistical formulae derived from recorded surge data. Use is made of forecast meteorological data for the North Sea area and tide gauge data recorded in real time at a number of locations along the east coast. The effectiveness of the various flood contingency plans depends on a flood prediction being issued several hours before the likely flood peak. However, with tide affecting surge and surge affecting tide, extrapolation into the future by some single addition of projected tide and surge profiles is, to say the least of it, unreliable. Too many dangerously high water levels would be predicted unnecessarily in this way, while water levels dangerously high in actuality might be omitted. Hence, for the economic and proper deployment of flood prevention measures, some account of the characteristics of interaction must be taken into account. This is already done to some degree in the prediction formulae presently in use but further improvements are required.

The present interest in the interaction phenomenon stems from the fact that, despite the importance of the phenomenon, there is no clear physical explanation of the underlying mechanics. In addition, some of the results from previous studies of interaction have been misinterpreted.

The approach in the present study may be divided into three parts. The first is concerned with a statistical analysis of five years of recorded surge data from six stations along the east coast of Britain and three stations located in the River Thames. The second and third parts both involve the use of a numerical model of the River Thames. Thus, in the second part the response characteristics of the river are determined for a number of different surge parameters. In the third part the mechanics of interaction are examined using a numerical modelling technique involving two separate models representing tide and surge propagation respectively. The two models are operated simultaneously and an interactive link between them is achieved by a perturbation technique which introduces into each model concurrent values of elevation and velocity as computed by the other. This method enables the separate elements of interaction to be identified, namely: (a) the modification of tidal propagation by the surge; (b) the modification of surge propagation by the tide; (c) shallow water effects and (d) quadratic friction effects.

2. REVIEW

Following the Thames flood of January 1928, Doodson (1929) investigated the meteorological factors present in the generation of North Sea storm surges. In examining the statistics for storm surges recorded at Southend between 1911 and 1928 he noted that surge maxima occurred most frequently in the interval 3 to 4 hours before high water. He also noted that surge values tended to be larger at low water than at high water.

Corkan (1948a) made a comprehensive study of the relationship between surge levels at Southend and the associated meteorological forcing in the North Sea. As a result, he developed the following empirical formula to predict surge levels at Southend :

$$R_S = R_D + \alpha N|N| + \beta E|E| + \gamma n|n| + \delta e|e| \qquad (1)$$

where R_S is the surge level at Southend at time t hours,

R_D the surge level at Dunbar at time (t - 9) hours,

N and E the north and east pressure gradients at a point in the southern North Sea at time t hours,

n and e the north and east pressure gradients at a point in the central North Sea at time (t - 6) hours,

α, β, γ and δ constants determined by fitting the formula to recorded surge data.

Both R_S and R_D are corrected for the local effects of barometric pressure using a statical law.

Corkan (1948b) found that this formula produced a satisfactory prediction for the main part of almost all large surges recorded at Southend, but was unable to account for an oscillatory component : attributed to surge-tide interaction.

The severe flooding and heavy loss of life around the shores of the southern North Sea caused by the storm surge of January 1953 highlighted the importance of the problem of predicting flood levels. One of the fundamental questions raised in the ensuing enquiry (Institution of Civil Engineers 1954) concerned the lack of understanding of the effects of interaction. Basic questions at issue were (a) how does interaction effect the probability distribution of flood levels and (b) can a large surge peak occur on a high tide?

There followed a series of papers on the subject of surge-tide interaction in the Thames Estuary by Proudman (1955a, 1955b, 1957), Doodson (1956), and Rossiter (1961). In considering these papers it is convenient to introduce the equations of one-dimensional motion governing the propagation of tide and surge. Applied to a narrow channel, free from meteorological forcing, the equations may be written,

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\frac{\partial Z}{\partial x} + \frac{KU|U|}{(D+Z)} = 0 , \qquad (2)$$

$$B\frac{\partial Z}{\partial t} + \frac{\partial}{\partial x}\left\{(D+Z)BU\right\} = 0 , \qquad (3)$$

where x is the horizontal axis measured along the length of the channel;

t time;

B breadth of the channel at the level of the water surface;

D depth of the channel below a horizontal datum at approximately mean water level;

Z elevation of the water surface above the same horizontal datum;

U velocity in the x direction - the mean value over a cross section;

K a friction coefficient.

Neglecting variations in the breadth B with changing elevation Z, the non-linear terms in equations (2) and (3) are $U\frac{\partial U}{\partial x}$, $\frac{KU|U|}{(D+Z)}$ and $\frac{\partial}{\partial x}\{(D+Z)BU\}$. Numerical simulations of tide and surge in the Thames (Prandle 1975), using (2) and (3), have indicated that interaction is largely insensitive to the inclusion or omission of the convective term $U\frac{\partial U}{\partial x}$. Hence interaction can be mainly attributed to either the frictional term $\frac{KU|U|}{(D+Z)}$ or the shallow water term $\frac{\partial}{\partial x}\{(D+Z)BU\}$.

Proudman (1955a, 1955b, 1957) derived analytical solutions of the above equations with $(D+Z)$ in the friction term approximated by D. The practical application of his results to explain interaction effects in the Thames is restricted by the number of constraints which qualify the various solutions obtained. However, he demonstrated that, due to the friction term, a surge with a maximum occurring near to the time of tidal high water will be smaller (for the same set of meteorological forcing conditions) than a surge occurring at tidal low water and that the surge height will decrease as the range of tide increases. Similarly he showed that the effect of friction on a positive surge which extended over the period of tidal high water could be to produce a double peak in the time profile of the surge.

Doodson (1956) developed a numerical solution of equations (2) and (3) for the case of an estuary, of constant cross section, terminating at a tidal barrier. The introduction of a tidal barrier enabled him to treat the propagation of tides in the estuary as a boundary value problem. Tidal constituents approximating M_2, M_4, M_6 and M_8 were specified at the barrier and the response at the mouth of the estuary computed. By successive iterations he calculated the appropriate combination of the amplitude and phase of these tidal constituents such that the resulting tide at the mouth approximated a single M_2 oscillation. A surge of approximately 25 hours duration was then superimposed on the prescribed tidal elevations at the barrier and the resulting surface elevations at the mouth computed. This procedure was repeated four times with the peak of the surge occurring at high tide, low tide, rising tide and falling tide respectively. The results showed that, in order to produce the same value of the surge peak at the barrier, a smaller value of surge at the mouth was necessary on the rising tide and a larger value on the falling tide in comparison with the mean of the values required for all four stages examined. Hence, assuming surges originate in the northern North Sea and that in the course of propagating as far as the Thames they suffer interaction with the tide, in the manner described by this model, then the model results seem to confirm observational data (Doodson 1929) that surges in the Thames are larger on the rising tide. The opposing conclusion of Doodson (1956) that the model results do not agree with observational data may, perhaps, have arisen from the assumption that interaction effects are only significant within the River.

Rossiter (1961) developed a numerical model of the River Thames solving (2) and (3) by the initial-value technique. He used this model to simulate the propagation of a surge, introduced at the mouth in the form of a sine wave of 24 hours duration, in combination with a semi-diurnal tide. The simulation was repeated with the surge peak at the mouth specified at four different tidal stages and for two values of the surge amplitude. The model reproduced the observed increase in surge height on the rising tide and it was suggested that the interaction responsible for this was primarily in the form of a phase change in the combined wave profile of tide and surge, attributable to the cumulative influence through time of both shallow water and frictional terms. In the same work, based on an analysis of recorded surge data, it was found that residuals at Aberdeen, Tynemouth, Immingham and Lowestoft revealed no evidence of interaction to any perceptible degree, thus strongly suggesting that the Southend interaction effect is a relatively local one not associated with forces acting over a wider area.

Rossiter used the results from his simulation of the motion of tide and surge to plot the amplification of maximum water level, Z_x, at points along the length of the estuary against the corresponding maximum level, Z_o, at the mouth. The resulting monotonically decreasing curves of Z_x/Z_o against Z_o demonstrated that there was a strong tendency for the higher, more dangerous levels, to be amplified less than the lower ones - apparently confirming that dangerous surge peaks tend to avoid tidal high water. However, the plot conceals the fact that Z_x increases linearly with Z_o so that such confirmation can scarcely be upheld on the basis of the curves alone.

Cartwright (1968) investigated empirical methods of predicting surge-tide interactions. Using a response function approach he produced a regression formula predicting interaction for both Immingham and Southend. His results confirmed the earlier observations that surges are reduced on high and falling tide but are increased on low and rising tide. Keers (1968), using a similar approach to Cartwright, developed an empirical formula to predict the surge height at Southend at time t as a function of surge heights at Lowestoft at times (t - 3) hours and (t - 6) hours. He produced four prediction formulae for surges occurring at high tide, falling tide, low tide and rising tide respectively; the relative magnitudes of the coefficients in these formulae indicate that surges are larger on the rising tide. He also deduced, from an analysis of recorded surge statistics at Aberdeen, Tyne, Immingham, Lowestoft, Harwich and Southend, that interaction increases linearly with surge height and that the degree of interaction at a particular location is highly correlated with tidal range.

Banks (1974) simulated the interaction of tide and surge in the southern North Sea and River Thames with the aid of a numerical model. Results showed that interaction can produce a faster rise to the surge peak, that the major interaction is between the surge and the M_2 tide giving rise to semi-diurnal and quarter-diurnal components in the observed residuals, and that while interaction extends beyond the Thames estuary and the coastal regions, it increases along the river due to the larger tidal ranges and the truncation of the channel dimensions.

3. ANALYSIS OF RECORDED SURGE DATA

Before attempting to analyse the mechanics involved in surge-tide interaction, as it relates to the Thames Estuary, we first establish the main features of surge propagation in the area by an examination of observed surges. To this end, an analysis was made of recorded surge statistics for the following ports : Lerwick, Wick, Aberdeen, North Shields, Lowestoft, Walton, Southend, Tilbury and Tower Pier (figure 1). Observations at these locations mark out the southwards propagation of a surge starting from Lerwick in the northern North Sea and then progressing down the east coast of Britain from Wick as far south as Walton and thence along the Thames from Southend to Tower Pier. The analysis employed 5 years of hourly recordings at each location for the years 1969 to 1973; these records were, on average, 96% complete. Recordings at Immingham were not considered in the present analysis, since there is evidence of strong localised effects on sea levels at this site which lies in the Humber Estuary.

At each port, hourly residual heights were calculated as follows :

$$R_t = O_t - P_t - M \qquad (4)$$

where R_t is the residual elevation, or surge (at time t);

O_t the recorded elevation;

P_t the predicted astronomical tide;

and M the annual mean of O_t, calculated for each year separately.

(a) General analysis

The surge data at each location were analysed to compute probability distributions expressed in terms of percentage exceedances of a particular surge level. The percentage of surges exceeding a value Z_p was denoted by P_z given by $P_z = n/N \times 100$ where n is the number of surge values exceeding Z_p out of a total number of hourly surge values N (approximately 44,000 for the 5 year period). The analysis was performed for positive and negative surges separately. The results are shown in figure 2 in the form of the values of Z_p corresponding to P_z = 0.25, 1, 5 and 20% respectively for the various locations along the path of propagation of the surge. The horizontal scale in figure 2 represents the distance between the tide gauge locations; for convenience this scale was varied on either side of Lowestoft in order to separate the results for the more closely spaced gauges in the Thames. The lines corresponding to the mean positive and mean negative residuals refer to an arithmetic mean. The relatively constant values of Z_p with differing locations indicated by the P_z = 20% lines may be regarded as a reflection of the stationary noise component arising from such effects as (a) errors due to tide-gauge malfunctions or those introduced in digitizing the tidal records, (b) inaccuracies in the astronomical tidal predictions, (c) temporal variations in barometric pressure, (d) set-up due to wind waves and (e) small scale localised disturbances such as variations in river flow.

The values of Z_p for P_z = 0.25% represent surge levels exceeded, on average, by 22 hourly values per year and hence indicate the approximate magnitudes associated with the most severe storms occurring annually. The results for P_z = 1% may be regarded as being representative of the values observed during the propagation of moderate to large storm surges. The variations in Z_p , for these two values of P_z , show a steady and regular increase in amplitude between Lerwick and Lowestoft and thereafter remain reasonably constant between Lowestoft and Tower Pier.

Figure 2 shows that, in general, the statistics for negative surges are similar to those for positive surges. This is perhaps surprising for the case of larger surges since the generating mechanisms for positive and negative surges, respectively, are different.

The accuracy to which the values of Z_p are determined decreases for the smaller values of P_z since the number of surge values exceeding Z_p decreases. A measure of the accuracy in determining Z_p , for Tower Pier, is shown in table 1 by the deviation of yearly averages of Z_p from their mean, taken over the five years of data.

(b) Analysis for different tidal phases

The analysis described in § 3(a) was repeated but, rather than analyse the complete data set as a whole, the data was first separated into distinct subsets according to the timing of any particular observation relative to tidal high water. Each subset was then analysed separately as before. The results are shown in figures 3 and 4. Figure 3 shows the mean surge level for each location at four tidal phases namely; (a) rising tide, $3\frac{1}{2}$ to $2\frac{1}{2}$ hours before high tide (HT); (b) high tide, HT $-\frac{1}{2}$h to HT $+\frac{1}{2}$h; (c) falling tide, HT $+2\frac{1}{2}$h to HT $+3\frac{1}{2}$h and (d) low tide, HT $-6\frac{1}{2}$h to HT $-5\frac{1}{2}$h. The divergence of the four curves is a measure of the degree of interaction at each location. The larger values indicated by the curves for surges on the rising tide clearly illustrate the increase in interaction as surges propagate southwards. Figure 4 shows the variation in the values of Z_p at each tidal phase for the percentage exceedances P_z = 0.25, 1% and 5%. The amount of variation at different tidal phases shown by the curves in figure 4 indicates the degree of interaction present. Both figures 3 and 4 show that interaction can be detected as far northwards as Wick; it then increases continuously as far as Tower Pier. An exception to this continuous increase is the small decrease between North Shields and Lowestoft. One explanation for this behaviour is that the tidal regimes change significantly between these two points: the tides in the central and southern areas of the North Sea are influenced by separate M_2 amphidromic systems. Hence some discontinuity in the ratio of the phase speed of the surge to that of the tide may occur between North Shields and Lowestoft, a discontinuity which might alter the pattern of interaction developed in the northern region. A second explanation centres on the shape of the coastline. Thus, a part of the surge travelling southwards along the northern section of coast may be dissipated in the Wash while an additional component, moving further offshore (Flather 1971) might be expected to pass through to Lowestoft relatively unaffected. Under these circumstances, it is possible that the surge arriving at Lowestoft could have suffered less interaction than that arriving at North Shields.

Our evidence of interaction along most of the east coast contrasts with the statement of Rossiter (1961) that he could find no evidence of interaction to any perceptible degree there. Keers (1968) included Immingham in his analysis of recorded surges and found the magnitude of the interaction there to be about two thirds of the magnitude of the interaction at Southend.

(c) Statistics of individual surges

Although surge statistics have been described in the preceding two subsections, the question remains as to the applicability of these generalised results to the problem of predicting, or indeed understanding, the propagation of any particular surge. This aspect of the problem was examined by analysing the movement of a number of discrete surge peaks, each selected as part of a particular surge event and identifiable at all locations between Lerwick and Tower Pier.

(i) Identification of surge peaks

The method used for the identification of a surge peak was based on an examination of the time series of observed residuals at Southend. After successfully identifying a surge peak at Southend, the particular surge event was only included for subsequent analysis if corresponding peaks could be traced at all the other tide gauge locations both seawards and up-river of this site.

At Southend, the start of a surge was defined to be at time t_1, when the surge height at that time, R_{t_1}, differed from the preceding mean surge level, $\overline{R_N}$, by more than a specified value S_o, where $\overline{R_N}$ denotes the mean surge level averaged over the N hourly values preceding the time t_1. The end of the surge was then defined to be at time t_2 when the surge height, R_{t_2}, no longer exceeded $\overline{R_N}$. Two additional criteria were satisfied in order for a particular surge to be incorporated in the present analysis: (a) the duration of the surge, $(t_2 - t_1)$, had to exceed a minimum value T and (b) the height of the surge peak, R_p, had to exceed a certain minimum value, $\pm S_p$ for positive and negative surges respectively, where R_p is measured relative to the annual mean water level.

A sensitivity analysis was performed to test the effects on this identification procedure of variations in S_o, N, T and S_p. The parameters T and S_p, describing the duration and minimum value of the surges to be considered, may be specified directly from a subjective choice of the form of surge considered to be of interest and, in this respect, the values selected were T = 3 hours and S_p = 30 cm. With these two parameters specified, the number of surges selected was then dependent on the values of S_o and N. By investigating the effects of varying these latter parameters, the values S_o = 15 cm. and N = 48 were chosen on the basis of maximising both the number of surges selected at Southend and the percentage of these surges that were subsequently identifiable from Lerwick to Tower Pier.

In identifying the corresponding surge peaks at other locations, the value of the parameters as specified above for surges at Southend were modified as follows: (a) the initial increment, S_o, was unchanged at 15 cm. except at the more northerly locations where the values S_o = 6 cm. at Aberdeen and North Shields, and S_o = 3 cm. at Wick and Lerwick were adopted; (b) the minimum duration of the surge was specified as $1/2\ (t_2 - t_1)$, i.e. half the corresponding surge duration at Southend; (c) no minimum value for the surge peak was specified; (d) the value of N was unchanged. An additional requirement was introduced concerning the time, T', between the start of the surge at each of these locations and the start of the surge at Southend: minimum values of T' were specified as follows:- Lerwick 24h; Wick 20h; Aberdeen 16h; North Shields 12h; Lowestoft 8h; Walton 4h; Tilbury and Tower Pier 0h.

4. RESPONSE CHARACTERISTICS OF THE RIVER THAMES

An examination of the sea-level response of the River Thames under varying surge conditions was made with the aid of a one-dimensional numerical model. The model extended from a seaward boundary lying along the line of latitude $1^{o}25'$E as far up the river as Richmond Lock where, for the present modelling purposes, a "no-flow" condition was imposed. The model employed an explicit finite difference scheme to solve the shallow water wave equations (2) and (3). The schematic representation of the river is shown in figure 1; a comprehensive account of the numerical scheme is given by Prandle (1974).

The input to this model comprised the specification of surface elevation at the open-sea boundary. The objective of the approach, described in this section, was to determine the sensitivity of the surface elevations along the river to a range of parameters describing differing surge profiles introduced at the open boundary. These surge profiles were simulated in conjunction with a tidal oscillation where simulations extended over the complete range of phase relationships between surge and tide.

(a) Surge parameters

The surge at the mouth of the model, $S(t)$, was assumed to be of the form

$$S(t) = H + j\frac{A}{2}\left(1 - \cos\left\{\frac{2\pi}{P}(t-\theta)\right\}\right) \qquad (5)$$

where H is a change in mean sea level;

A the height of the surge;

P the duration of the surge;

θ the phase of the surge relative to the phase of the tide;

j = 1 for $\theta < t < \theta + P$ and j = 0 at all other times.

The assumption of a sinusoidal time-profile for the prescribed surge seems reasonable taking account of observations of actual surge profiles recorded at the mouth of the Thames. The tidal conditions at the mouth of the model were taken as a semi-diurnal M_2 oscillation (speed denoted by ω_{M_2}) of 180 cm. amplitude. Figure 7 shows a schematic representation of both the surge and tide elevation input. In the following account of the simulation of tide plus surge, the model was run for several cycles with tide alone as input before the inclusion of the surge as given by (5); hence at the start of the surge the elevations and velocities along the river had converged to a cyclic tidal oscillation.

The model was run with the elevation at the mouth $Z_M(t)$, in centimetres, defined by

(1) $Z_M(t) = 180 \sin \omega_{M_2} t$, tide alone;

(2) $Z_M(t) = S(t)$, surge alone;

(3) $Z_M(t) = 180 \sin \omega_{M_2} t + S(t)$, tide plus surge.

The model for tide alone was operated only once while the models of surge alone and tide plus surge were each operated for the following range of parameters :

(i) phase of the surge: $\theta = \theta', 2\theta', 3\theta' \ldots 30\theta'$; where $\theta' = 2\pi/30\,\omega_{M_2}$,

(ii) surge height: A = 60, 120, 180, -60, -120, -180 cm;

(iii) surge duration: P = 3.125, 6.25, 12.5, 25 hours;

(iv) mean water level: H = 0, 60, 120, -120 cm.

Computed residuals at any location along the river were obtained by subtracting the tide, as computed from the model of tide alone, from the results for tide plus surge.

It is implicitly assumed in the present simulation that there is no interaction between the tide and surge at the mouth of the model. While the analysis of recorded data in § 3(b) and § 3(c) showed that some interaction is present at the mouth, the basis of the present approach in examining interaction effects within the Thames remains valid.

(b) Model results

The following examination of the model results is limited to values at Tower Pier; these values may be taken as representative of conditions elsewhere along the river. In the discussion which follows, tidal phases refer to the tidal regime at Tower Pier.

Figures 8, 9 and 10 are contour diagrams constructed to show computed elevations at Tower Pier as a function of time (measured vertically) for varying θ (measured horizontally). Results in figure 8 refer to residuals obtained with

$$P = 25\text{h (diurnal surge)}, \quad A = 180 \text{ cm}, \quad H = 0 ,$$

while those in figure 9 again refer to residuals but with

P = 6.25h (quarter-diurnal surge), A = 180 cm. H = 0.

Figure 10, on the other hand, shows total water level, considering the case of a diurnal surge, i.e.

$$P = 25h, \ A = 180 \text{ cm}, \ H = 0.$$

The object of these diagrams is to demonstrate the effect on surge and total water levels at Tower Pier, of the phase of the input surge, θ. Thus, along any vertical, contour values give levels through time for a particular θ. Horizontal lines mark the times of high and low tide. A straight double line shows the time of the surge peak, for each particular θ, for the case of the surge propagating alone, i.e. the surge without interaction with the tide.

An examination of figure 8 shows that the maximum surge peak occurs on the rising tide. This maximum has a value of 245 cm. which is 50 cm. or 25% larger than the surge peak of 195 cm. obtained for the surge propagation alone. Thus, interaction here is responsible for a significant increase in surge level. However, the figure shows that when the surge peak occurs on the falling tide interaction tends to reduce the value of the surge peak and may produce a double peak.

The results shown in figure 9 correspond to those in figure 8, however the variation in the computed residuals with differing θ for the quarter-diurnal surge is clearly much less pronounced than the variation for the diurnal surge and the maximum surge peak now coincides with low tide. The value of this peak is 410 cm. compared with a surge peak of 366 cm. obtained for the surge propagating alone. Hence, although interaction increases the maximum surge by 44 cm, the effect of interaction for this shorter period surge is less significant than the amplification along the River which occurs with the surge propagating alone - involving a surge peak of 366 cm. at Tower Pier for a surge height of 180 cm. at the mouth. By comparison, for the diurnal surge, interaction increases the surge peak by 50 cm. while the amplification with the surge propagating alone produces a surge peak of 195 cm. at Tower Pier for a surge height of 180 cm. at the mouth.

Returning to the case of the diurnal surge, figure 10 shows that the maximum water level occurs when the surge peak without tide coincides very nearly with high tide. This same result was found for other simulations of surges of varying amplitudes and durations, so that an important conclusion is that although maximum surge peaks tend to occur at certain tidal phases, maximum water levels are realised when the surge peak (without tide) occurs near to tidal high water.

A complete set of results in the form of figures 8 and 10 was obtained for the range of surge parameters previously described; the results of major interest are summarised in figures 11 and 12. Figure 11 shows the maximum surge height at Tower Pier, over all values of θ, plotted against surge duration P for surges of height, A = 60, 120, 180, -60, -120 and -180 cm. where both A and P refer to values at the mouth. The plotted points correspond to surge durations of 25, 12.5, 6.25 and 3.125 hours. The values shown for zero frequency represent the maximum surge values, at Tower Pier, obtained solely from a change in mean sea level H equal to the appropriate value of A; a discussion of the dynamic effects of changing H follows later in this section. The dashed line shown in figure 11, and also in figure 12, indicates the corresponding maximum surge heights for the same range of values of A and surge duration P in the case of surge propagating alone. Figure 11 indicates that the amplification of maximum surge level from the mouth to Tower Pier is, to a first approximation, independent of the magnitude of the surge height at the mouth but is highly dependent on the duration or shape of the time-profile of the surge. Comparison with the equivalent results for surge alone shows that interaction can increase surge heights significantly; this is particularly true for negative surges and surges of short duration.

Figure 12 shows a similar plot to that of figure 11 but with the continuous line now representing the surge level at either maximum or minimum water level over all values of θ. The figure illustrates that, at the time of these extreme levels, interaction tends to reduce surge levels. This effect is more pronounced for surges of shorter duration. Figures 11 and 12 show that surges of longer duration propagate with little amplification of either the maximum surge values or the extreme water levels. An important feature shown in figure 12 is that negative surges are severely damped at the time of minimum water level. This implies an asymmetry between the probability distributions for maximum and minimum water levels.

The effect of variations in mean sea level were investigated by setting A = 0 and H = 60, 120 and -120 cm. respectively in equation 5. Operating both the model for tide alone and the model of tide plus surge, the computed residuals shown in figure 13 were obtained. These residuals thus represent the modification to the tidal propagation due to a sudden change in mean sea level.

Simulations with the above values of H and the whole range of surge parameters examined showed that the major effect of a change in mean sea level was the modification of the tidal propagation as demonstrated here. Figure 13 indicates that the height of the surge peak, measured relative to the change in mean sea level, is approximately linearly proportional to the magnitude of the change in sea level. In addition, these residuals have extreme values on the rising tide, positive for an increase in mean sea level and negative for a decrease.

(c) Comparison of model and recorded results

Figure 14 shows, for Tower Pier, the computed surge levels at various tidal phases obtained by averaging the residuals shown in figures 8 and 9 over the range of values of θ from 0 to 2π. Figure 14 also includes, for comparison, the corresponding distribution of recorded surges as a function of tidal phase as obtained in § 3(b) and shown in figure 4. Whereas the tidal phase distribution for the surge of 25 hours duration is in reasonable agreement with the distributions obtained from observations, the phase distribution for the surge of 6.25 hours duration does not similarly agree. For the other surge periods examined it was found that the results for the surge of 12.5 hours duration were in reasonable agreement with the observed distributions while the results for the surge of 3.125 hours duration were not.

In an attempt to resolve this apparent disparity between model and observed results, a spectral analysis of observed residuals was carried out in order to determine the order of the durations associated with observed surges. A Fast Fourier Transform technique was used to analyse 342 days of recorded hourly residuals for Margate and Tower Pier, where Margate may be considered as representative of conditions at the mouth of the model. The spectra (figure 15) show that most of the surge energy lies in the frequency band corresponding to surge durations of semi-diurnal period or longer, hence the agreement indicated in figure 14 between observed results and results for the computed surge of 25 hours duration.

The spectra also show that the response of the river, defined as the ratio of the surge height at Tower Pier to that at Margate, is close to unity for the surges of diurnal duration or longer but is significantly greater at the frequencies of M_2, M_4, M_6 etc. It may be deduced from the results shown later in §5 that the enhancement of the residual spectra at Tower Pier at these tidal frequencies may be attributed to the surge component generated in the Thames by the modification of the tidal phase due to interaction with the surge. The nature of this additional surge component explains why surge heights in the Thames vary in such a regular fashion with tidal phase and how, at certain tidal phases, interaction increases surge heights.

5. MECHANICS OF INTERACTION IN THE THAMES

Prandle (1975) showed that the numerical model of the Thames employed in §4 can simulate, with reasonable accuracy, the propagation of tide and surge in the river. The present objective is to gain an understanding of the properties of interaction by simulating tide and surge separately while introducing interaction between the two phenomena in the form of perturbation terms. The approach involves the use of two numerical models, operated simultaneously, referred to subsequently as "parallel models". The first model simulates tidal propagation and the second surge propagation. Each model corresponds to the description given in §4(a), with elevation at the mouth specified in terms of tide (for the tide model) and surge (for the surge model).

In this section, as in §4, the model cannot account for interaction which occurs seaward of the mouth.

(a) Parallel models

It is convenient for the purposes of explanation to consider the following simplified equations for flow in the river expressed in terms of cross sectional transport Q (Dronkers 1969) :

$$\frac{1}{B}\frac{\partial Q}{\partial t} + g(D+Z)\frac{\partial Z}{\partial x} + KU|U| = 0 \qquad (6)$$

$$B\frac{\partial Z}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad (7)$$

where the assumption is made that the breadth B does not vary with Z . Expanding equation (6) by writing $Q = BU(D+Z)$ and comparing the result with the more commonly accepted expression of equation (2), it can be seen that the validity of equation (6) depends on the assumptions that the convective term $U\,\partial U/\partial x$ is negligible and that the term $U\,\partial Z/\partial t$ is also negligible in relation to $(D+Z)\partial U/\partial t$. It may be shown that both of these assumptions are valid for small values of Z/D .

It is required, for all locations and at all times, that the elevation and transport in the tidal model, Z_T and Q_T respectively, together with the corresponding values Z_s and Q_s in the surge model should satisfy the relationships :

$$Z_c = Z_T + Z_s \tag{8}$$

and

$$Q_c = Q_T + Q_s \tag{9}$$

where Z_c and Q_c denote elevation and transport, respectively, computed in a combined simulation of tide and surge. Inserting (8) and (9) into equations (6) and (7) produces the following equations for the combined propagation of tide and surge :

$$\frac{1}{B}\frac{\partial}{\partial t}(Q_T+Q_s) + g(D+Z_T+Z_s)\frac{\partial}{\partial x}(Z_T+Z_s) + K(U_T+U_s)|U_T+U_s| = 0 \quad , \tag{10}$$

$$B\frac{\partial}{\partial t}(Z_T+Z_s) + \frac{\partial}{\partial x}(Q_T+Q_s) = 0 \; . \tag{11}$$

Boundary conditions to be satisfied in the solution of these equations are :

$$Z_M = Z_T^{(0)} + Z_s^{(0)} \quad , \tag{12}$$

where Z_M denotes elevation at the open sea boundary, and

$$Q_H = Q_T^{(1)} + Q_s^{(1)} \quad , \tag{13}$$

where Q_H denotes transport at the head of the river. Here $Z_T^{(0)}$ and $Z_s^{(0)}$ are prescribed elevations at the mouth due to tide and surge respectively, while $Q_T^{(1)}$ and $Q_s^{(1)}$ are transports at the head due to tide and surge.

The assumption made in the use of parallel models is that equations (10), (11) and associated boundary conditions may be separated into two parts as follows : for tide,

$$\frac{1}{B}\frac{\partial Q_T}{\partial t} + g(D+Z_T+\underset{\sim}{Z_s})\frac{\partial Z_t}{\partial x} + K\,U_T|U_T+\underset{\sim}{U_s}| = 0 \quad , \tag{14}$$

$$B\frac{\partial Z_T}{\partial t} + \frac{\partial Q_T}{\partial x} = 0 \quad , \tag{15}$$

with boundary conditions $Z_M = Z_T^{(0)}$ and $Q_H = Q_T^{(1)} = 0$, and for surge,

$$\frac{1}{B}\frac{\partial Q_s}{\partial t} + g\left(D + \underline{Z_T} + Z_s\right)\frac{\partial Z_s}{\partial x} + K\, U_s\left|\underline{U_T} + U_s\right| = 0 \quad , \tag{16}$$

$$B\frac{\partial}{\partial t}Z_s + \frac{\partial}{\partial x}Q_s = 0 \quad , \tag{17}$$

with boundary conditions $Z_M = Z_s^{(0)}$ and $Q_H = Q_s^{(1)} = 0.$

In operating the tidal model with (14) and (15), the surge parameters Z_s and U_s which appear in equation (14) are evaluated from the simultaneous operation of the surge model, while in operating the surge model with (16) and (17) the tidal parameters Z_T and U_T which appear in (16) are obtained from the concurrently-running tidal model. Using this parallel model technique to simulate a number of recorded surge events in the Thames, it has been shown that the results from the separate simulations of tide and surge can be combined to give values in close agreement with results obtained from the simulation of tide and surge combined - so that the approach satisfies the conditions (8) and (9). It seems reasonable therefore to regard the additional terms underlined in equations (16) and (18) as representing the interaction between tide and surge. The magnitudes of the four interaction terms are all proportional to a product of tidal amplitude and surge amplitude. Hence for a tide of approximately constant amplitude, the interaction at any particular phase of the tide is linearly proportional to the surge amplitude. Conversely, for a surge of a given magnitude, at any particular location, the degree of interaction is directly proportional to the tidal range at that location; this latter result supports similar deductions made by Proudman (1955a), Keers (1968) and Banks (1974).

(b) Simulation of the surge of 18-21 October 1970

In applying the parallel model technique in the simulation of actual surge events, the equation of motion (2) (with the convective term omitted) was used rather than equation (6). In using equations (2) and (3) to simulate tidal motion, shallow water interaction due to the surge was introduced by modifying D to become $D_T = D + Z_s$ and correspondingly in the model of surge propagation D was modified to become $D_s = D + Z_T$. The interaction due to quadratic friction was introduced into equation (2) by modifying $U_T|U_T|$, as used for the propagation of tide alone, into the form $U_T|U_T + U_s|$ to account for interaction from the surge and, likewise, modifying $U_s|U_s|$, as used for the propagation of surge alone, into $U_S|U_T + U_S|$ to account for interaction from the tide.

A study was made of the storm surge of 18-21 October 1970; this formed one of a number of large surges simulated using the present approach. While the results shown are limited to a 15 hour period at the start of this surge case, these results may be considered as representative of surge-tide interaction over a wide range of conditions.

The models incorporated the predicted tide and the recorded surge at Margate (figure 16) as boundary conditions at the mouth. Computer runs were carried out for

(1) model of tide alone (T);

(2) model of surge alone (S);

(3) model of tide with interaction from model (4) due to shallow water and quadratic friction (T');

(4) model of surge with interaction from model (3) due to shallow water and quadratic friction (S').

The elevations at Tower Pier computed from these four models are shown in figure 17. The difference between the curves for T and T' in figure 17 represents the modification of tidal propagation due to interaction with the surge and similarly the difference between the curves for S and S' represents the modification of the surge due to the tide. The figure shows that the amplitude of the surge peaks are appreciably reduced by interaction and that the phases of both tide and surge are modified by up to about 20 minutes. Interestingly, the figure shows that at the time of high tide, at 15.30h on 19 October, 1970, interaction increases both the tide and surge levels. Calculating the time profile of the total residual $S+I$, where the total interaction $I=T'-T+S'-S$ one finds a peak value on the rising tide. However, this phasing of the peak is more a result of the time profile for S than that for I, thus the surge peak on the rising tide results largely from the time profile for surge heights at the mouth of the model. While figure 17 demonstrates that interaction produces significant modification of phase and amplitude for both the tide and surge, it remains to determine the relative contributions of the major interaction terms, namely shallow water and quadratic friction.

Figure 18 shows the same set of results for models (1), (2), (3) and (4) but with the interactions introduced into models (3) and (4) limited to the terms involving quadratic friction. The results for T' and S' shown in figures 17 and 18 are nearly identical, so that it may be deduced that the major source of interaction in the Thames is due to the quadratic friction term, the most pronounced manifestation of this interaction being the damping of the surge peaks.

Figure 19 shows a similar set of results with interactions in models (3) and (4) limited to the shallow water terms. The results show that the phase of the surge is advanced when the tide rises above mean water level and retarded for low tidal levels. A similar advance in the tidal phase can be seen for positive surge levels. However the amplitude of both tide and surge remains almost unchanged. The magnitude of these phase changes amounts to as much as 20 minutes for both tide and surge. For a tidal amplitude of 300 cm. this phase advance produces an increase in surge height of 50 cm. on the rising tide, which underlines the importance of this mechanism as discussed in §4(c) and as noted by Rossiter (1961).

Results obtained by fictitiously delaying the phase of the surge by three hours at the mouth of the model are shown in figure 20. With both shallow water and frictional interaction taken into account, this figure is to be compared with figure 17 describing the original conditions. Although interaction now decreases both the surge and tidal levels at high water, the maximum total residual, $S' + T' - T$, does coincide with high tide. The maximum water level for the fictitious surge case is 4.95m and is coincident with high tide while the maximum water level for the actual surge, which also occurs at high tide, is 4.27m. The difference between these two values of maximum water level indicates the danger of under-predicting maximum flood levels when a surge peak occurs on high tide since the relevant empirical formulae tend to be based on the more usual occurrence of surge peaks on the rising tide.

7. CONCLUSIONS

1. It has been established, both from statistical analyses of recorded surges and by model studies, that surge peaks tend to occur on the rising tide in the Thames. The statistical analyses have shown that this generation of surge peaks on the rising tide can occur irrespective of the phase relationship between the tide and the surge in the northern North Sea. However, model studies of the effect of phase variation between tide and surge in the Thames have shown that maximum water levels occur when the peak of the surge (without tide) coincides with high tide (figure 10).

2. Surge levels are, in general, amplified progressively as surges propagate southwards; the amplification in any particular case is largely independent of the magnitude of the disturbance but sensitive to its time-profile.

Similarly the degree of interaction between surge and tide increases progressively between Wick and Tower Pier, with one exception in the region of Lowestoft.

An examination of the form of the non-linear terms responsible for interaction has shown that the magnitude of these terms is proportional to a product of surge amplitude and tidal amplitude. This result supports similar deductions made by Proudman (1955a), Keers (1968) and Banks (1974).

3. An examination has been made of the sensitivity of the response of sea level in the Thames to surges defined by a range of parameters with the aid of a numerical model. It has been shown that surges with a duration of a day or more propagate with little amplification of their maximum level while surges of shorter duration may be amplified significantly. A corresponding measure of the response of the river was obtained from observational data by comparing the spectra of recorded residuals at Tower Pier and Margate. The spectra showed similar results, namely that for surges of longer duration the response, defined as the ratio of the surge height at Tower Pier to that at Margate, is close to unity, whereas a marked amplification occurs at the frequencies of M_2, M_4, M_6 etc. This increase in the surge energy at tidal frequencies may be attributed to the residuals arising from the modification of the tidal phase in the presence of a surge. Since this latter effect constitutes a significant component of observed surge levels in the Thames, it follows that surge levels there are importantly dependent on tidal phase and that this interaction component may lead to an increase in surge levels as well as a decrease.

The model study involving a range of surge parameters has demonstrated an important difference between the response for positive and negative surges. Water levels along the river were computed for a range of positive surges together with an equivalent range of negative surges at the mouth of the model combined with a tide of fixed amplitude and with the model operated over a range of phase lags between tide and surge. The results showed that maximum water levels were significantly greater than the corresponding minimum water levels, where these extreme levels were both measured relative to mean sea level (figure 12).

4. A method of resolving the processes involved in interaction has been developed involving two linked numerical models, one of tidal propagation and one of surge propagation. In this way it was shown that, for the Thames, the principal interaction mechanism is the damping of surge peaks through the quadratic friction term. It was also shown that the interactions of surge on

tide and tide on surge are of the same order of magnitude; the contributions from the two major non-linear terms, namely quadratic friction and shallow water involved in these interactions are also, themselves, of the same order of magnitude: this equivalence between orders of magnitude accounts for much of the difficulty in understanding the phenomenon of interaction.

5. It has been shown that an important component in the observed interaction in the Thames originates in the sea areas outside of the river. Thus, at Walton, frequently a significant part of the phase adjustment between tide and surge has already taken place subsequently to be emphasised within the river itself. However, the interaction processes are greatest in the river, with both the friction and shallow water effects becoming progressively more important with decreasing depths along the river. In this manner, the interaction effects tend to become first order and assume a dominant role in the total motion of tide and surge.

6. Two central questions dependent on an understanding of interaction are :

(a) how does interaction affect probability distributions for extreme levels,

and (b) can a large surge peak occur on high tide.

The crux of the first question is that while it may be advantageous to consider the probability distributions for tide and surge separately, a problem arises as to how the separate distributions may be combined to take account of interaction. A possible approach is to divide the tide and surge into subsets according to tidal phase in the manner described in § 3(b); significant variations in surge statistics for different tidal phases have been demonstrated in figure 3 and also by Keers (1968). In each subset, the separate distributions of probability computed respectively for extreme tidal levels and extreme surge levels would be calculated, and the joint distribution of probability for extreme total water levels of that subset deduced. Combination of the results for each subset, would then yield the overall distributions of probability covering all tidal phases. It has been shown (§ 5(a)) that interaction is a function of tidal amplitude and hence, a possible shortcoming in this method of combining surge and tide statistics is that it takes no account of the neap to spring variation in tidal amplitudes. However, since the spectra for recorded residuals (figure 15) shows no clear evidence of a fortnightly constituent, it seems reasonable to neglect the effect of this tidal variation.

The answer to the second question as to whether a large surge peak can occur on high tide in the Thames depends on the definition of a large surge since theory suggests (figures 8, 9 and 20) that surge peaks may coincide with high tide in the river. The generation of a large surge almost invariably entails the action of wind and pressure over the broad expanses of the central and northern North Sea. It has been demonstrated (§ 3(c)) that, when such surges propagate southwards along the east coast of Britain, a peak is generated on the rising tide in the Thames. The most likely mechanism for shifting the surge peak towards high tide is for a locally generated surge component to be superimposed on the surge propagating from the north. However, since it has been shown (figures 3 and 4) that appreciable interaction takes place between Lowestoft and the Thames then, in order for the locally generated component to avoid peaking on the rising tide and continue rising towards high tide, the meteorological forcing must be increasingly effective between rising tide and high tide. An examination of recorded surges at Southend shows two distinct cases when these conditions probably obtained and in which large surge peaks occurred close to tidal high water, namely the surge of January 5-8, 1928 (Corkan 1948a, Doodson 1929) and the surge of December 9-10, 1965 (Synnott 1966). The meteorological conditions in both cases could be categorised as type 1 according to the classification by Doodson (1929), with depressions travelling from Scotland across the North Sea intersecting the coastline in the region of Denmark, with winds over the North Sea, in consequence, veering from south-west to north-west or north. The special characteristic of these two storms, which may well be of significance in relation to Thames sea levels, is that they both travelled rapidly across the North Sea during approximately the time between the preceding tidal high water and the high water on which the surge peak occurred in the Thames.

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Table 1. Means and standard deviations in annual values of percentage exceedances Pz.

	Positive surges				Negative surges			
Pz percent exceedances	20	5	1	0.25	20	5	1	0.25
mean value (cm)	33	59	97	133	-35	-60	-90	-116
standard deviation (cm)	3.2	1.8	9.8	12.3	3.5	4.5	9.6	14.5

Table 1. Means and standard deviation in annual values of percentage exceedances Pz.

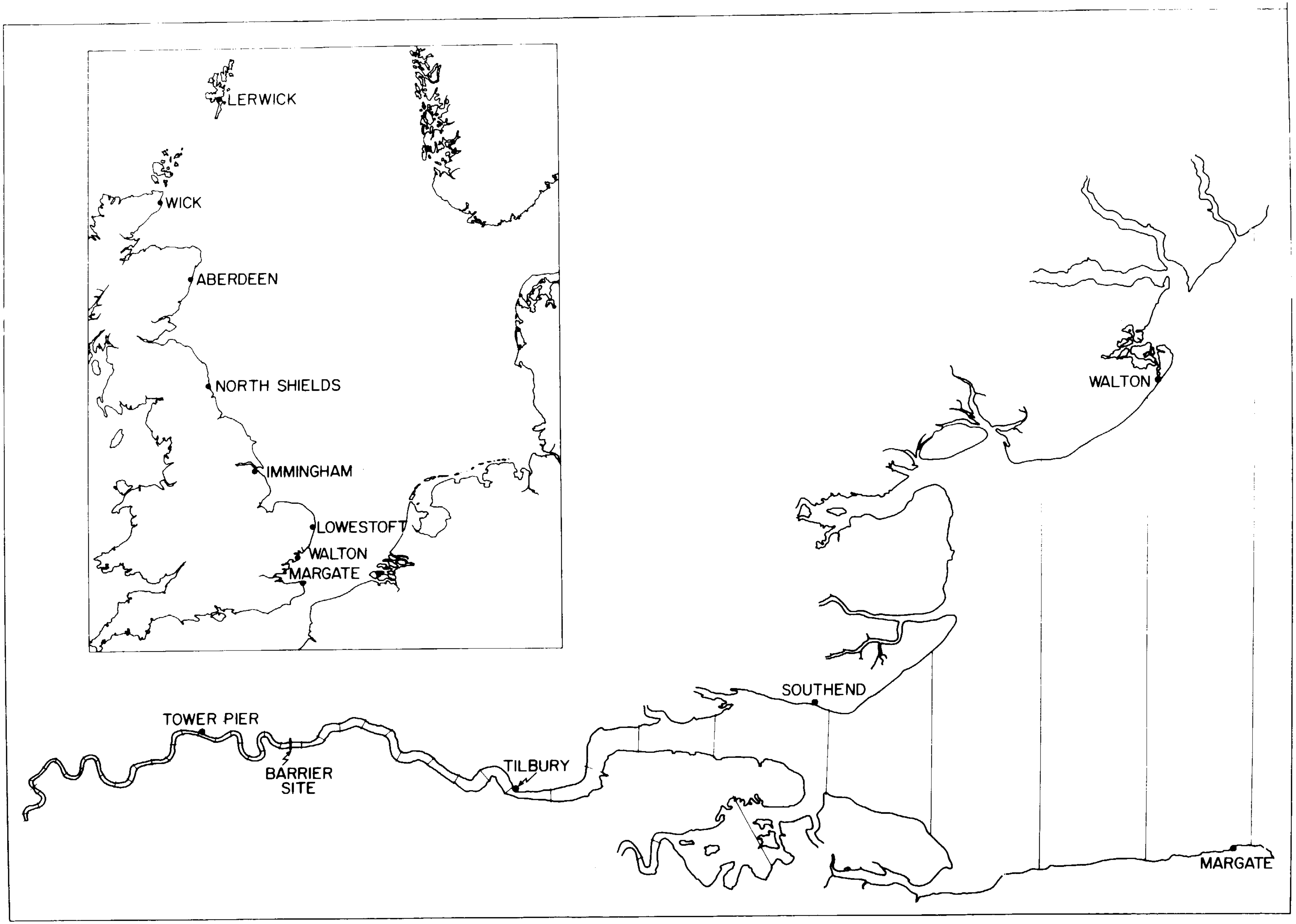

LERWICK
WICK
ABERDEEN
NORTH SHIELDS
IMMINGHAM
LOWESTOFT
WALTON
MARGATE
WALTON
SOUTHEND
TOWER PIER
BARRIER SITE
TILBURY
MARGATE

FIGURE I

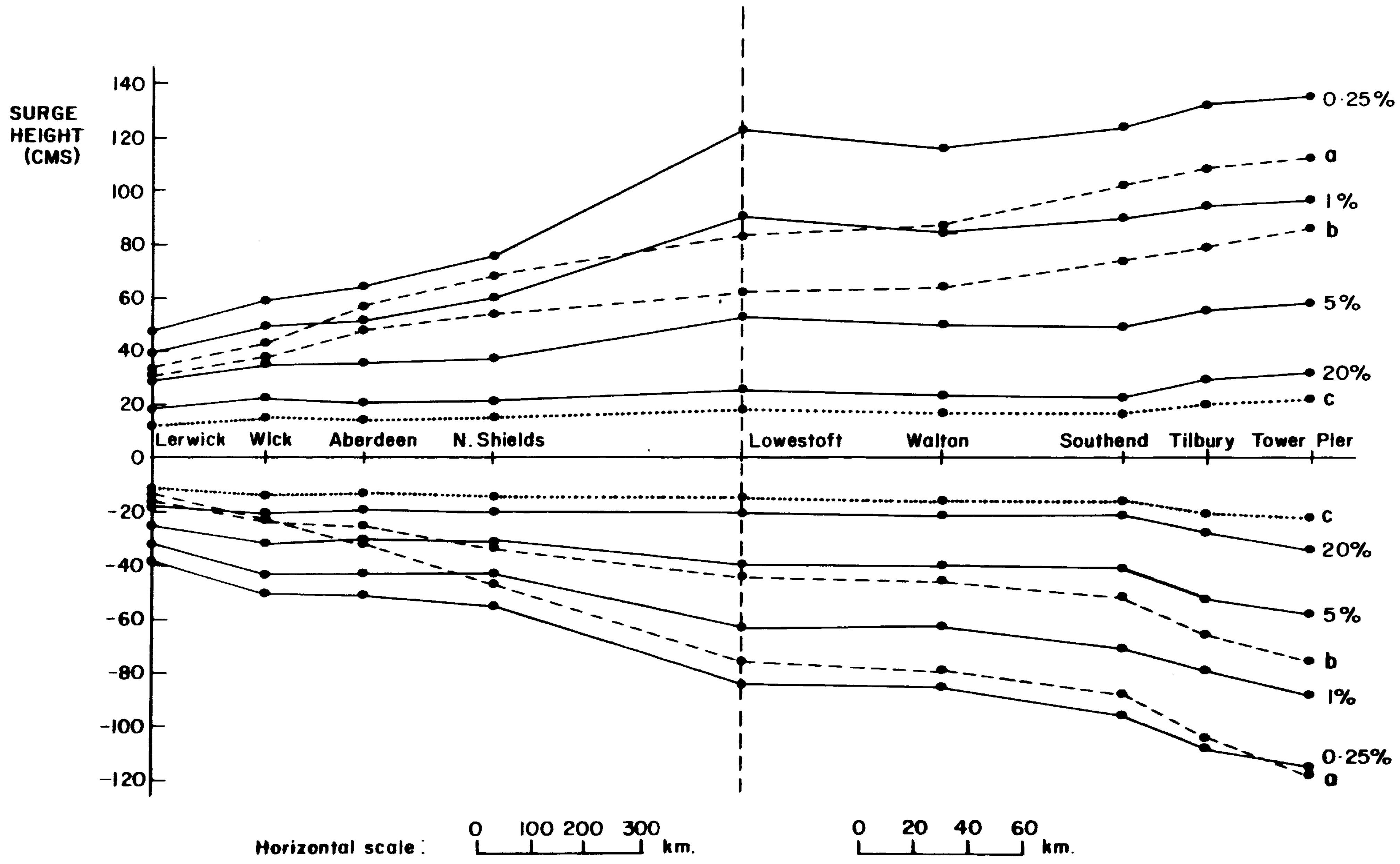

SURGE
HEIGHT
(CMS)
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
Lerwick
Wick
Aberdeen
N. Shields
Lowestoft
Walton
Southend
Tilbury
Tower Pier
0·25%
a
1%
b
5%
20%
c
20%
5%
b
1%
0·25%
a
Horizontal scale:
0
100
200
300
km.
0
20
40
60
km.

FIGURE 2

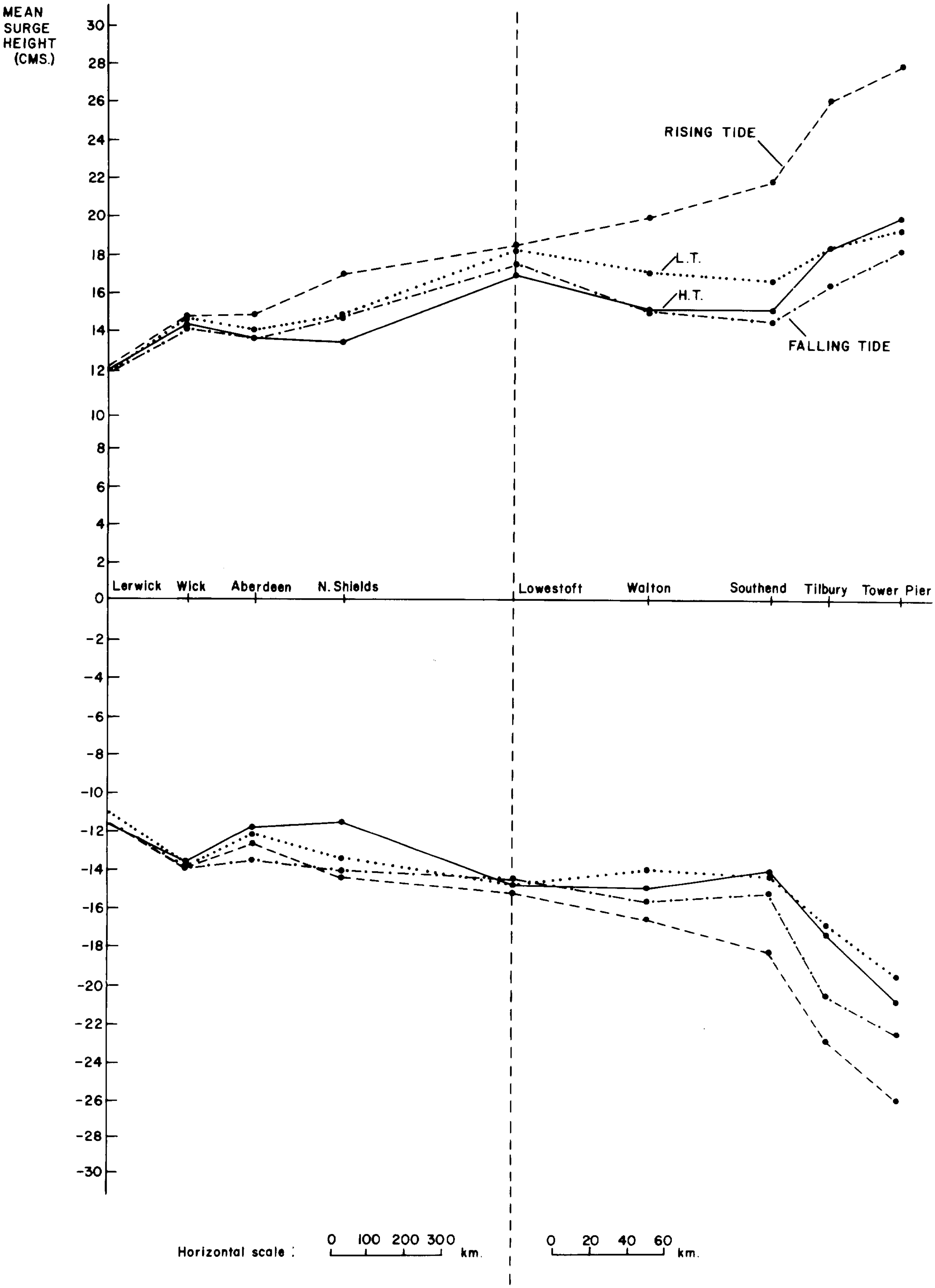

MEAN SURGE HEIGHT (CMS.)
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
-26
-28
-30
RISING TIDE
L.T.
H.T.
FALLING TIDE
Lerwick
Wick
Aberdeen
N. Shields
Lowestoft
Walton
Southend
Tilbury
Tower Pier
Horizontal scale :
0 100 200 300 km.
0 20 40 60 km.

FIGURE 3

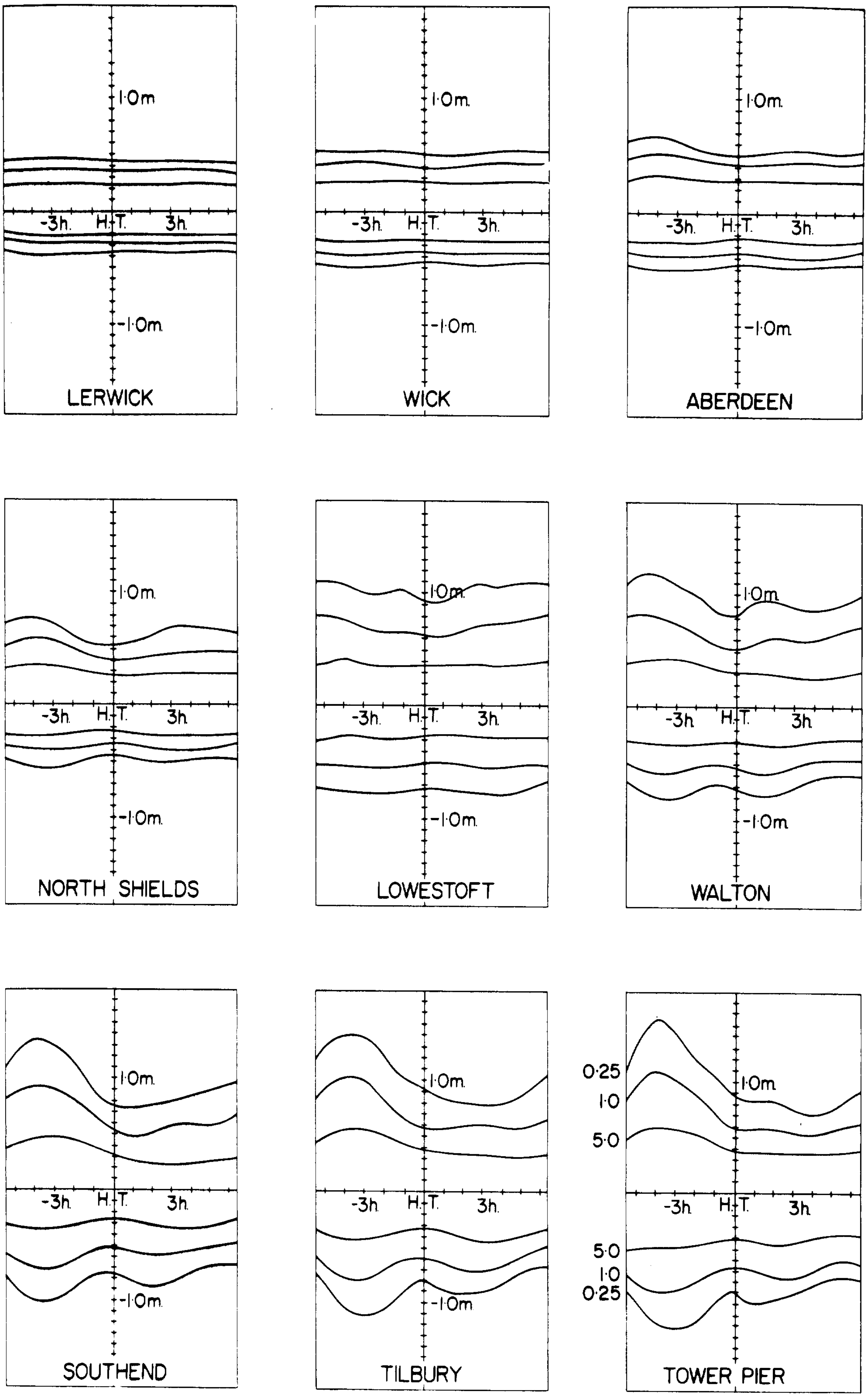

1·0m
-3h.
H.T.
3h.
-1·0m.
LERWICK
1·0m.
-3h.
H.T.
3h.
-1·0m.
WICK
1·0m.
-3h.
H.T.
3h.
-1·0m.
ABERDEEN
1·0m.
-3h.
H.T.
3h.
-1·0m.
NORTH SHIELDS
1·0m.
-3h.
H.T.
3h.
-1·0m.
LOWESTOFT
1·0m.
-3h.
H.T.
3h.
-1·0m.
WALTON
1·0m.
-3h.
H.T.
3h.
-1·0m.
SOUTHEND
1·0m.
-3h.
H.T.
3h.
-1·0m.
TILBURY
0·25
1·0
5·0
1·0m.
-3h.
H.T.
3h.
5·0
1·0
0·25
TOWER PIER

FIGURE 4

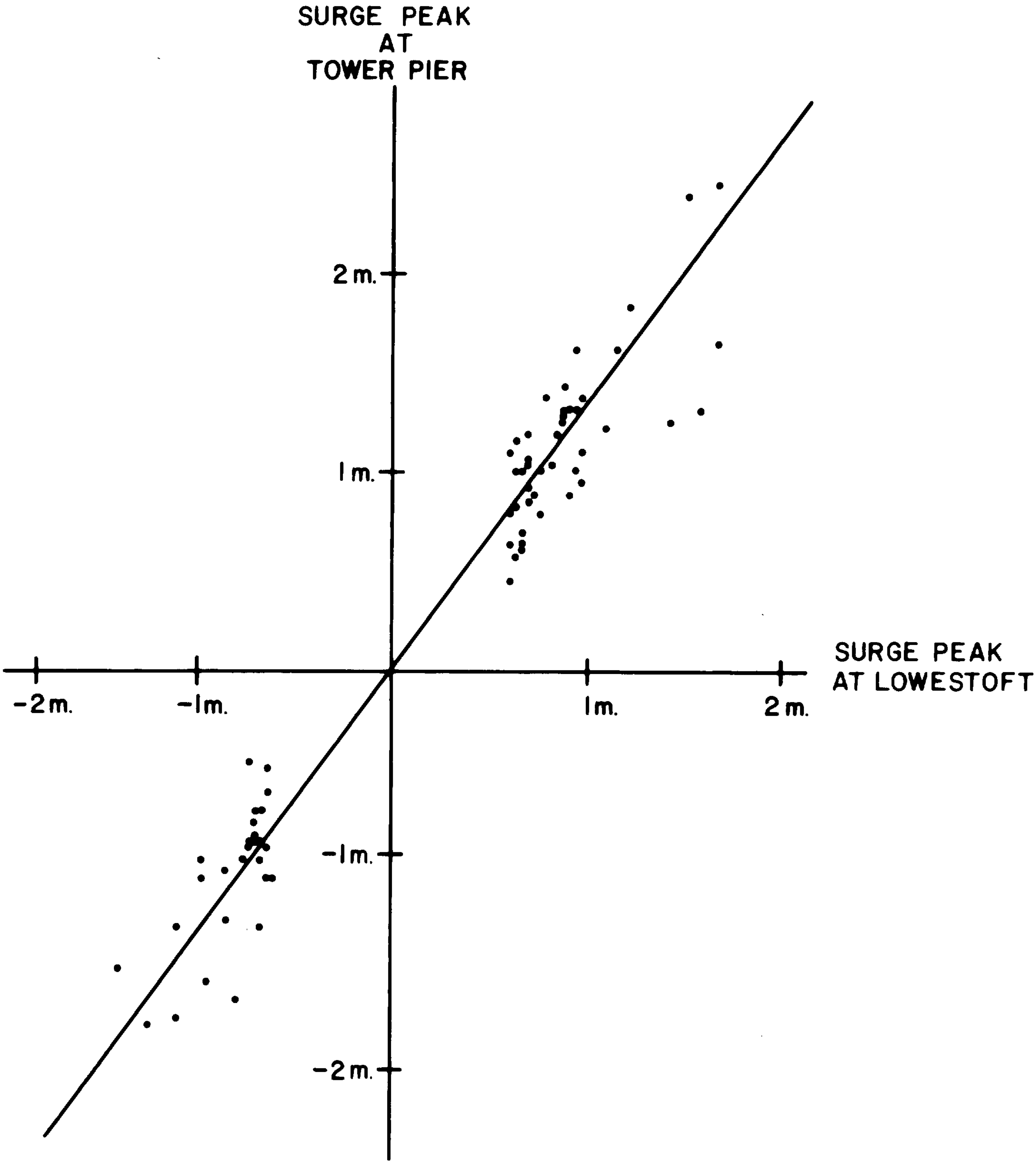
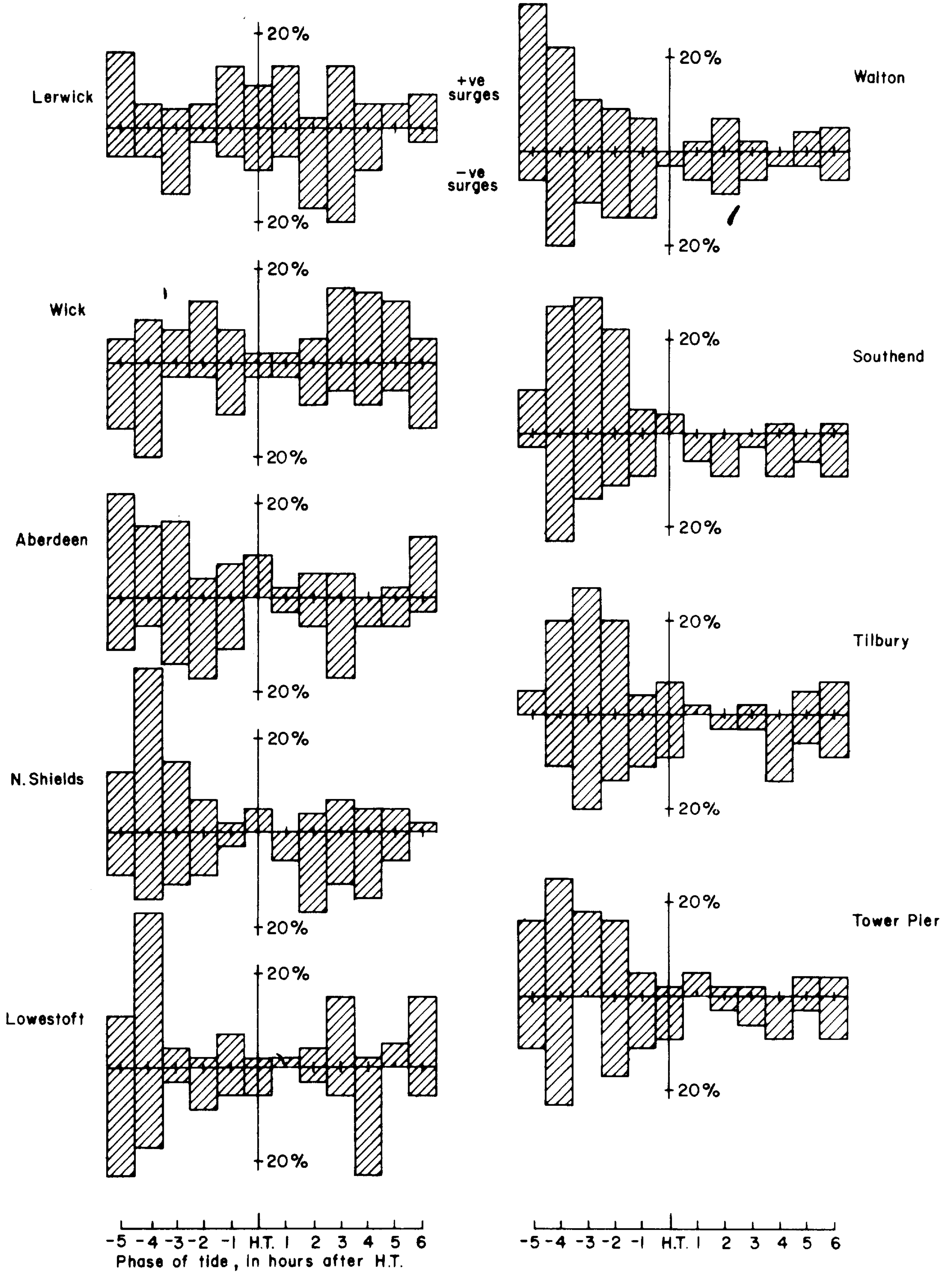

SURGE PEAK
AT
TOWER PIER
2 m.
1 m.
-1 m.
-2 m.
SURGE PEAK
AT LOWESTOFT
-2 m.
-1 m.
1 m.
2 m.

FIGURE 5



Lerwick
Wick
Aberdeen
N. Shields
Lowestoft
Walton
Southend
Tilbury
Tower Pier
+ve surges
−ve surges
20%
-5 -4 -3 -2 -1 H.T. 1 2 3 4 5 6
Phase of tide, in hours after H.T.

FIGURE 6

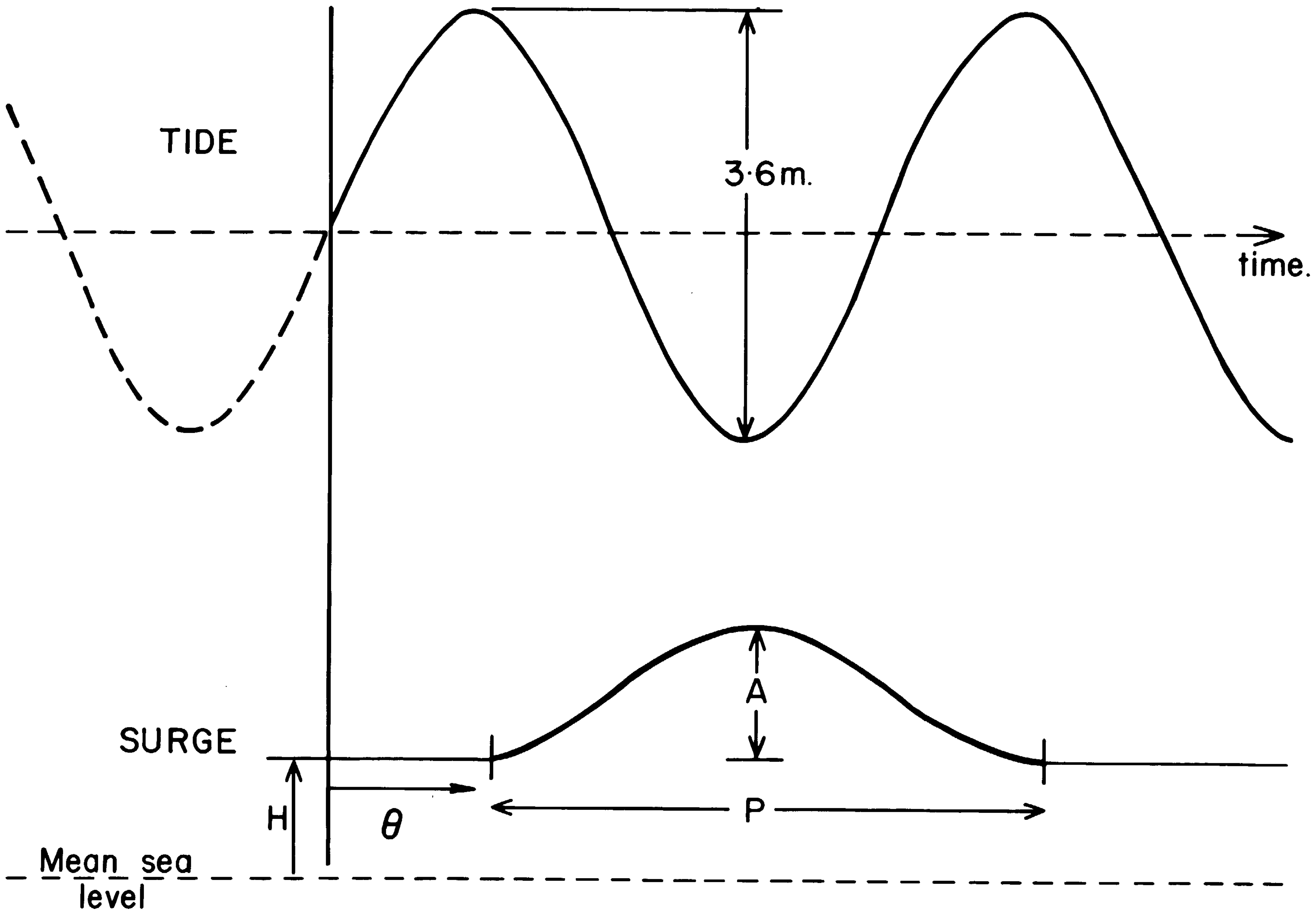


FIGURE 7

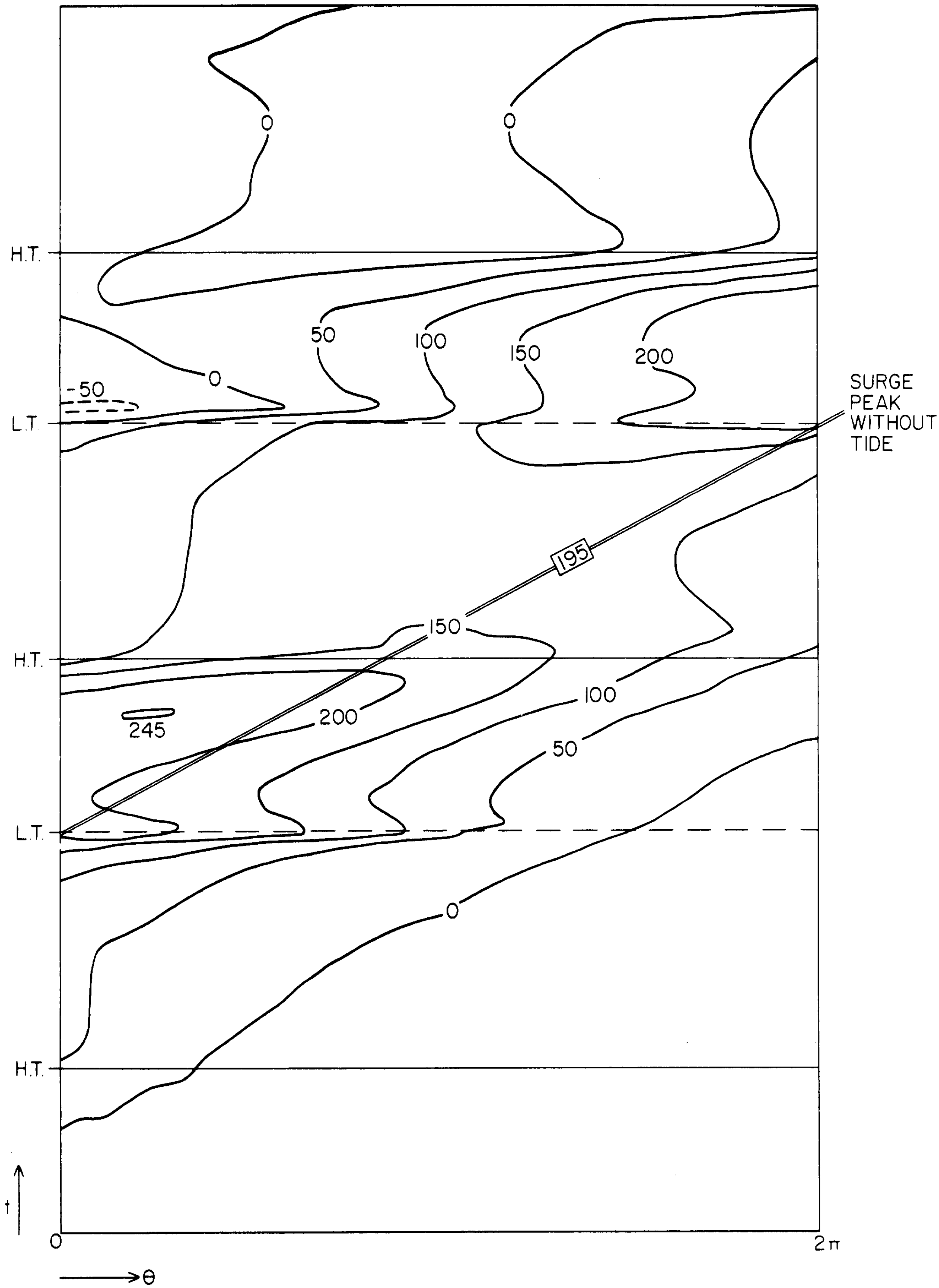

0
0
50
100
150
200
0
-50
SURGE
PEAK
WITHOUT
TIDE
195
150
100
200
245
50
0
H.T.
L.T.
H.T.
L.T.
H.T.
t
0
2π
θ

FIGURE 8

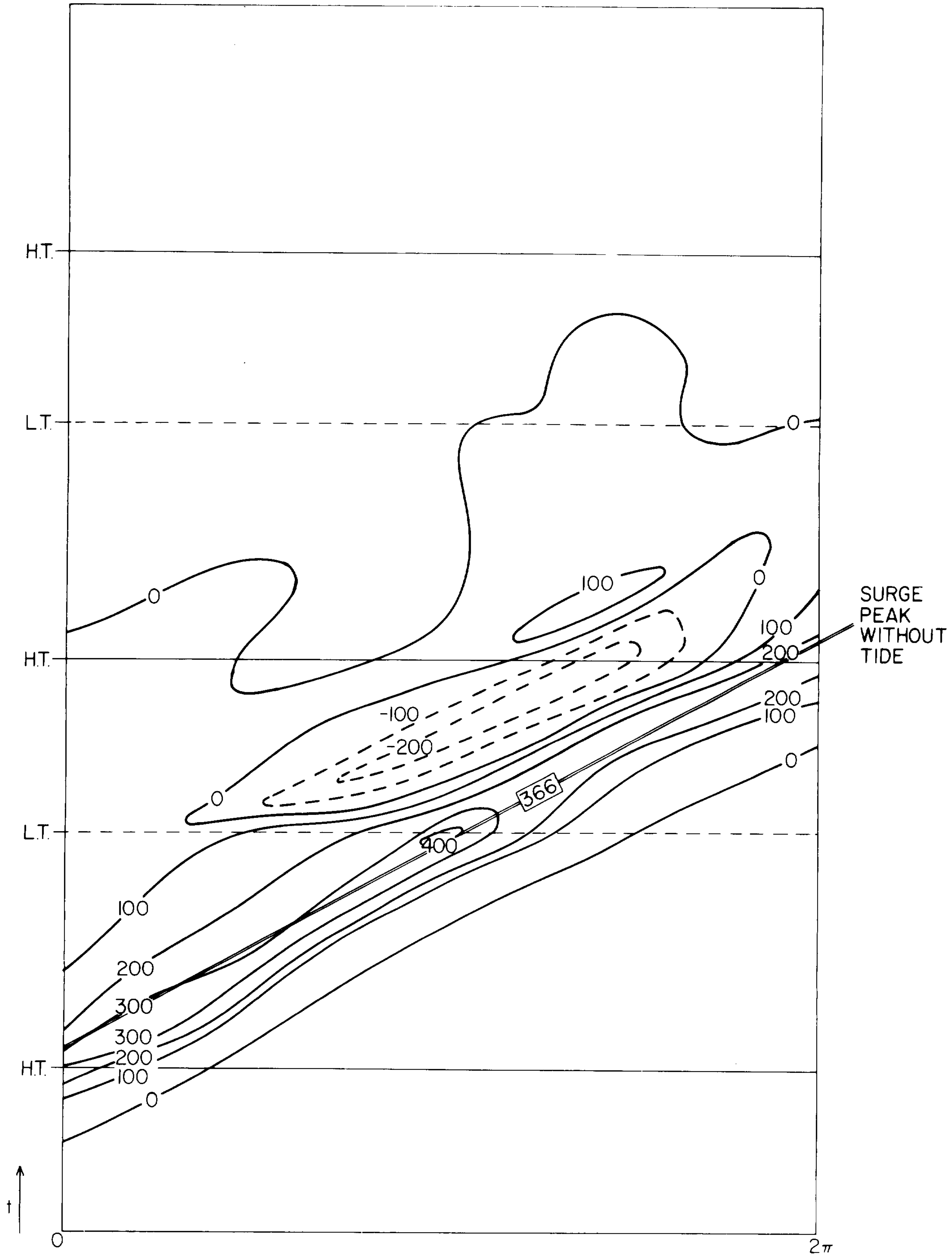
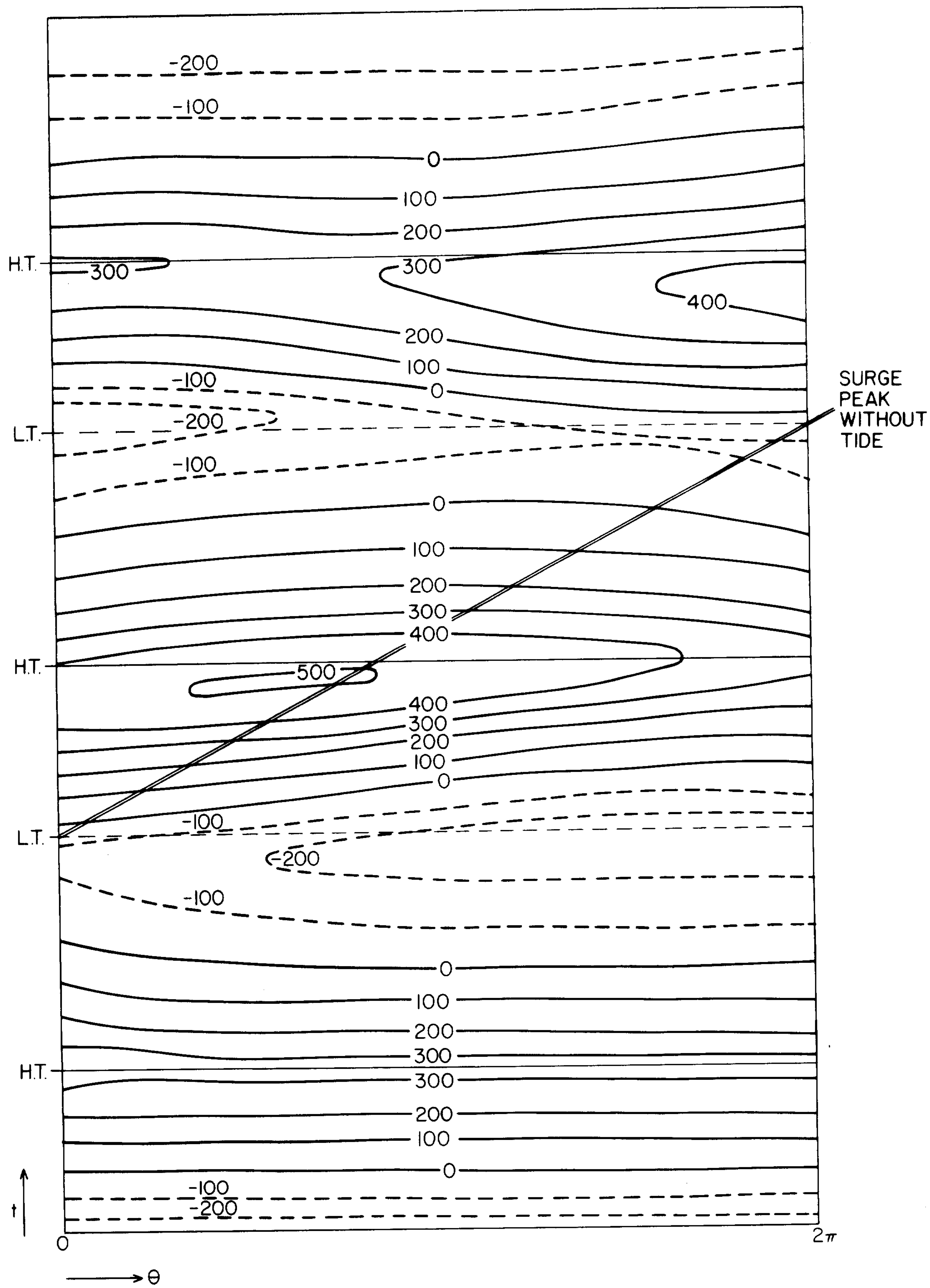

H.T.
L.T.
H.T.
L.T.
H.T.
0
100
-100
-200
0
100
200
200
100
0
366
400
100
200
300
300
200
100
0
SURGE PEAK WITHOUT TIDE
t
0
2π
θ

FIGURE 9



-200
-100
0
100
200
H.T.
300
300
400
200
100
0
-100
SURGE
PEAK
WITHOUT
TIDE
L.T.
-200
-100
0
100
200
300
400
H.T.
500
400
300
200
100
0
-100
L.T.
-200
-100
0
100
200
300
H.T.
300
200
100
0
-100
-200
t
0
2π
θ

FIGURE 10

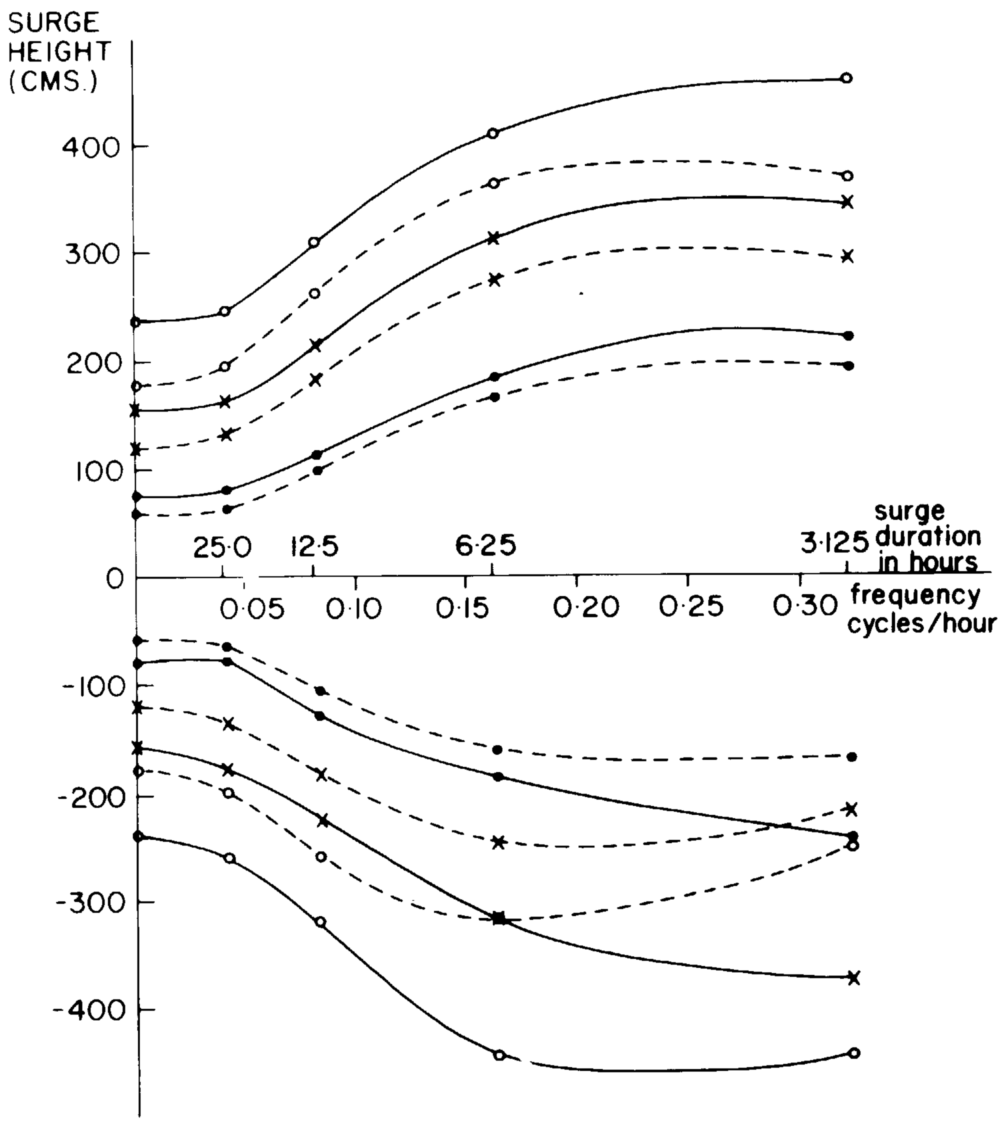


FIGURE 11

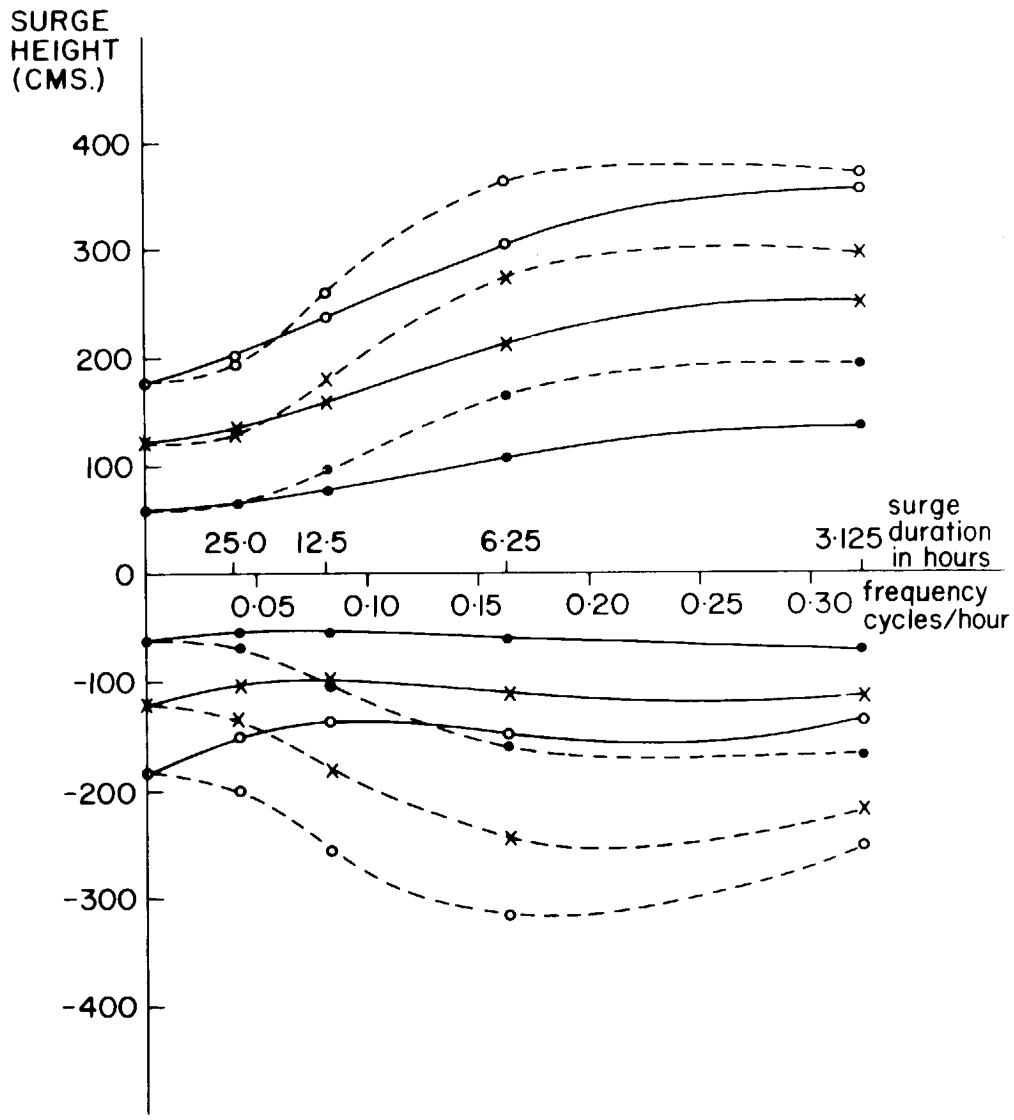


FIGURE 12

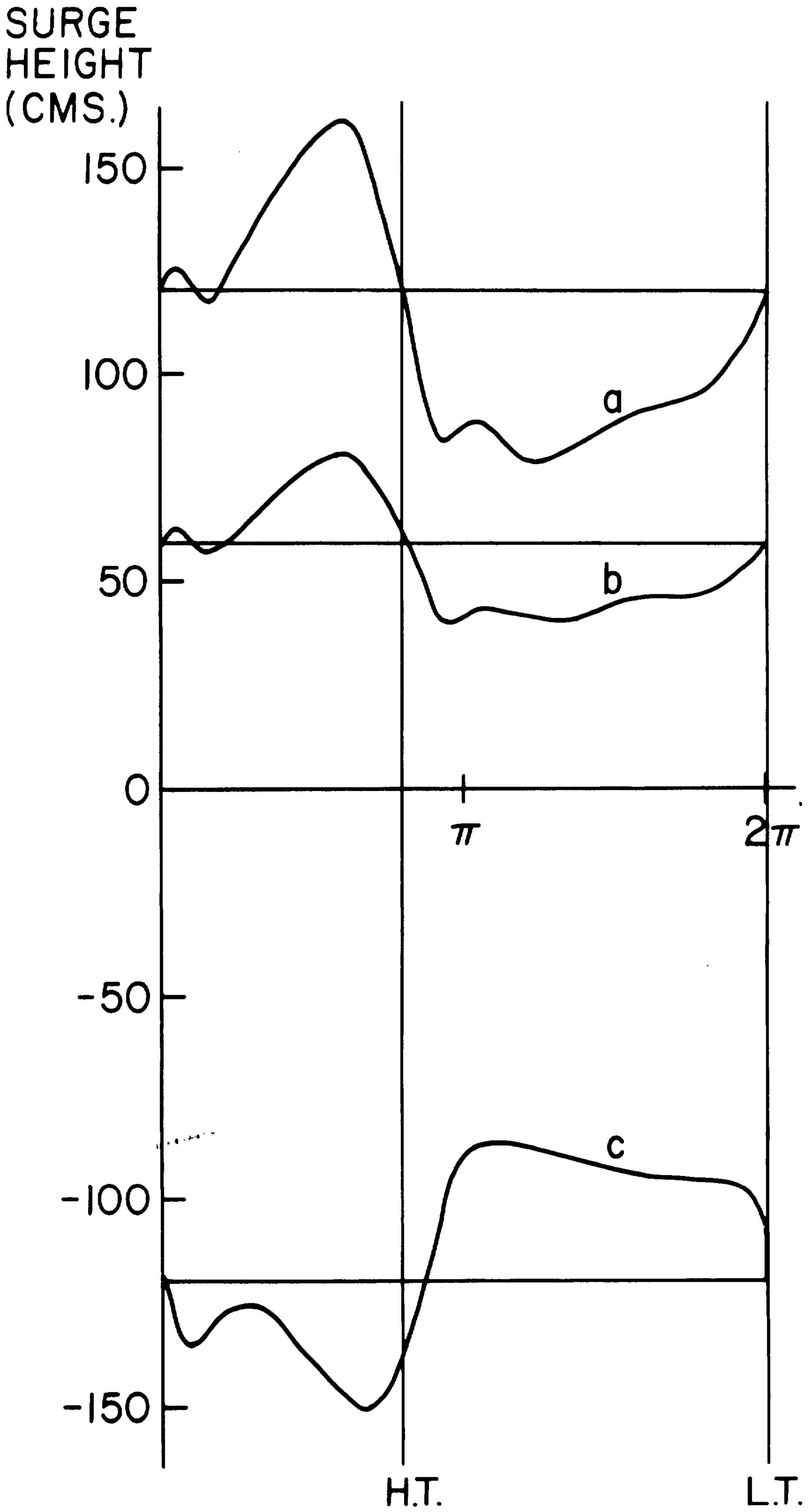


FIGURE 13

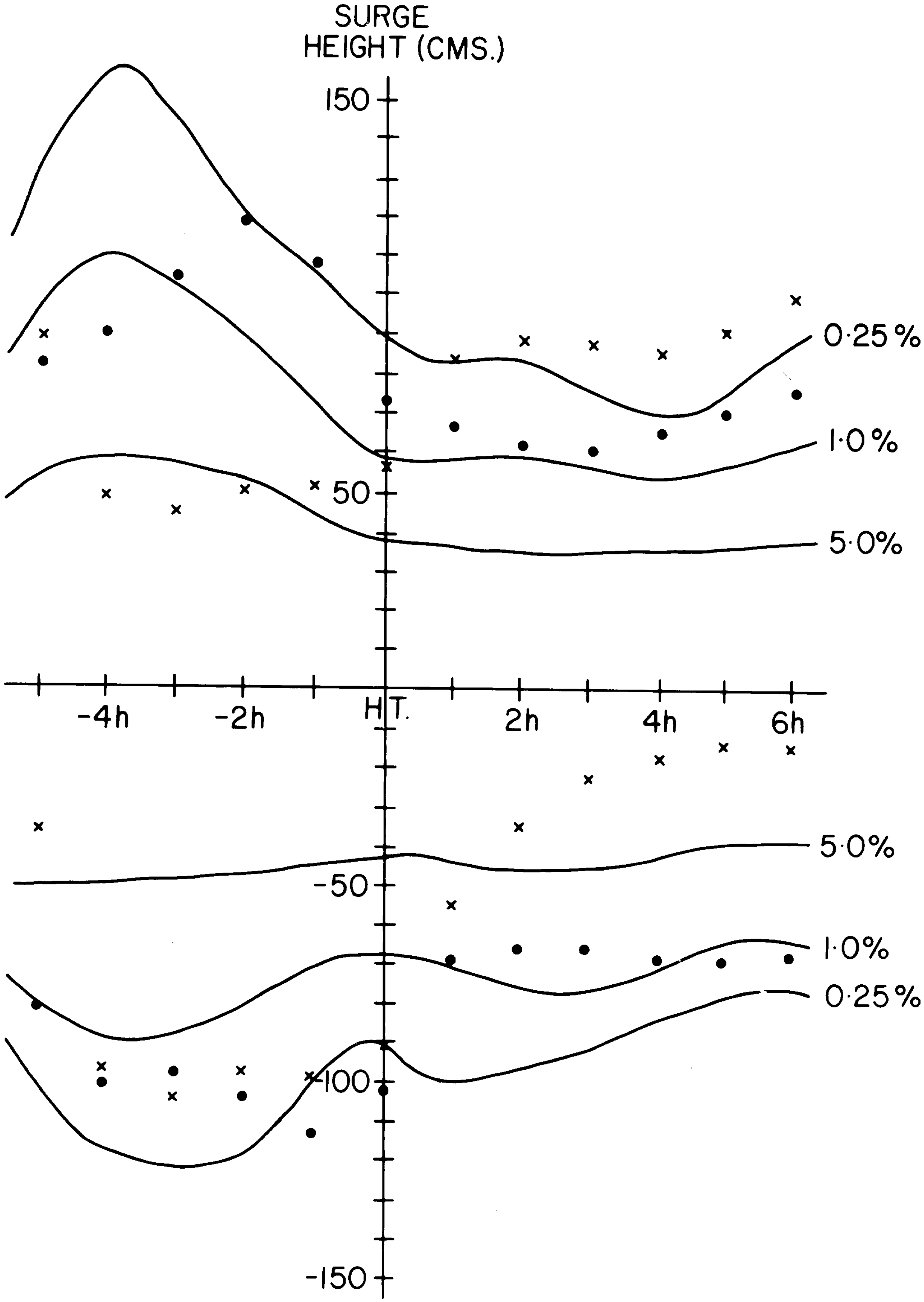

SURGE
HEIGHT (CMS.)
150
50
-50
-100
-150
-4h
-2h
H.T.
2h
4h
6h
0·25%
1·0%
5·0%
5·0%
1·0%
0·25%

FIGURE 14

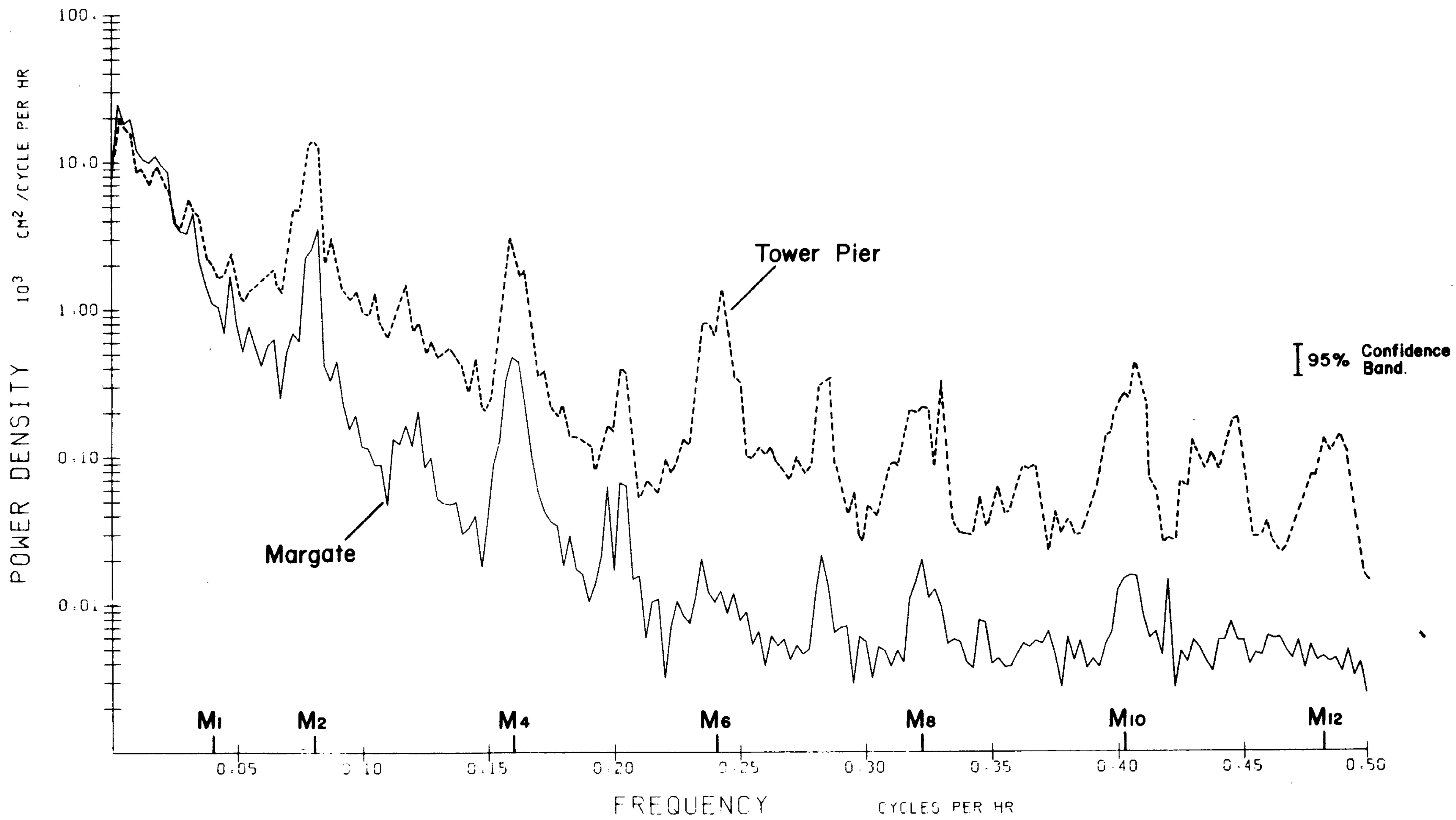

POWER DENSITY 10^3 CM^2 /CYCLE PER HR
100.
10.0
1.00
0.10
0.01
Tower Pier
Margate
95% Confidence Band.
M1
M2
M4
M6
M8
M10
M12
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
FREQUENCY CYCLES PER HR

FIGURE 15

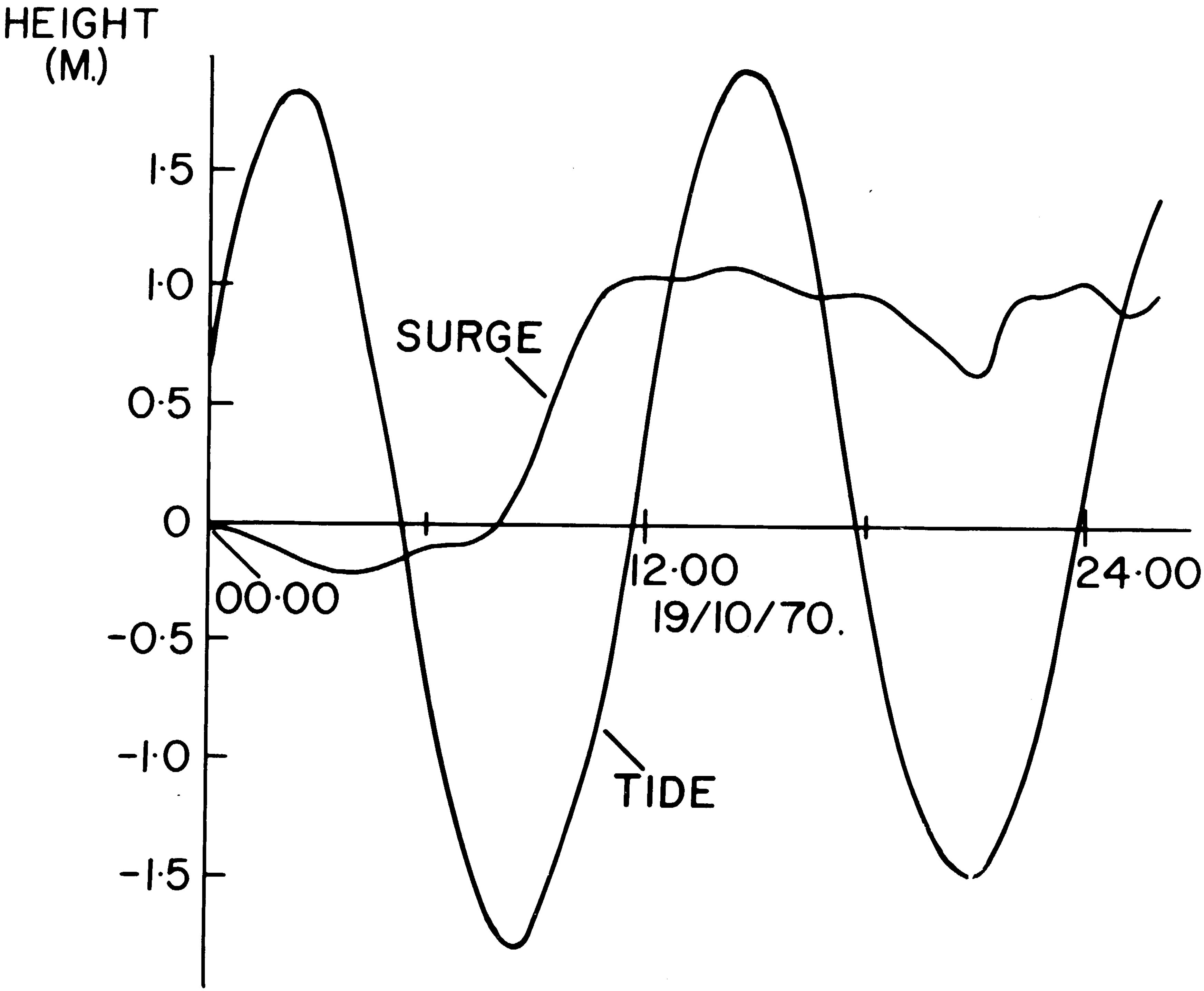


FIGURE 16

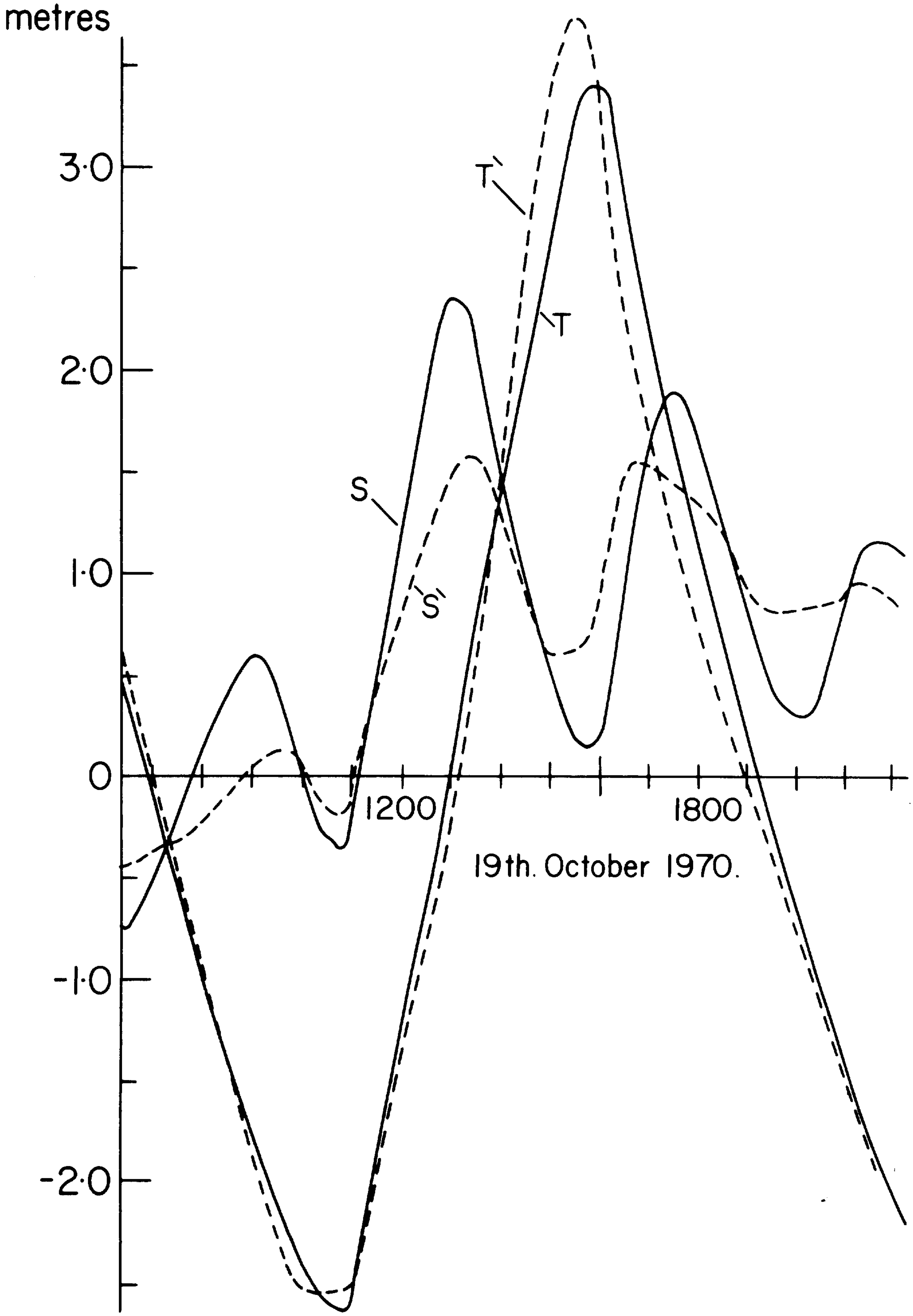


FIGURE 17

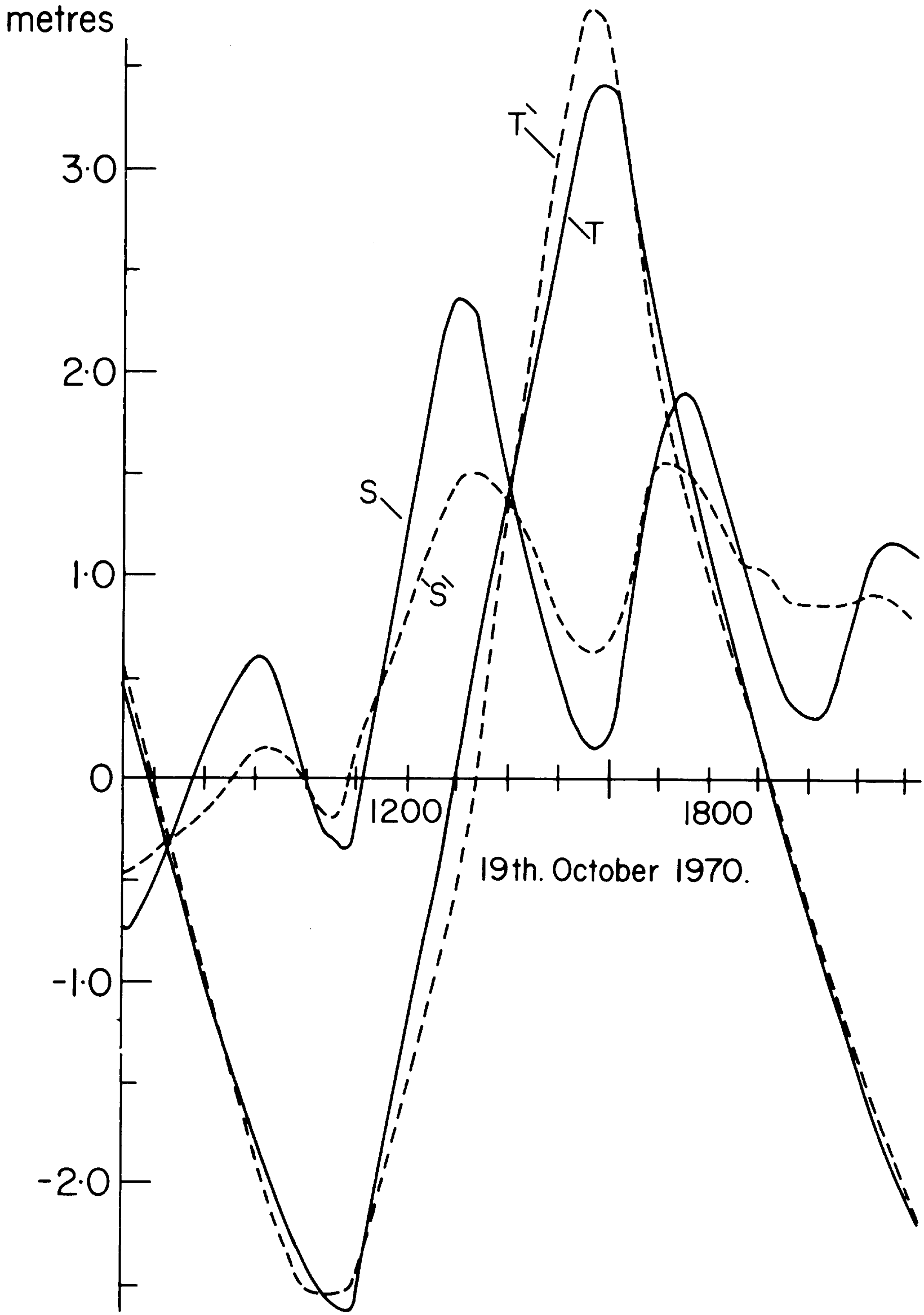


FIGURE 18

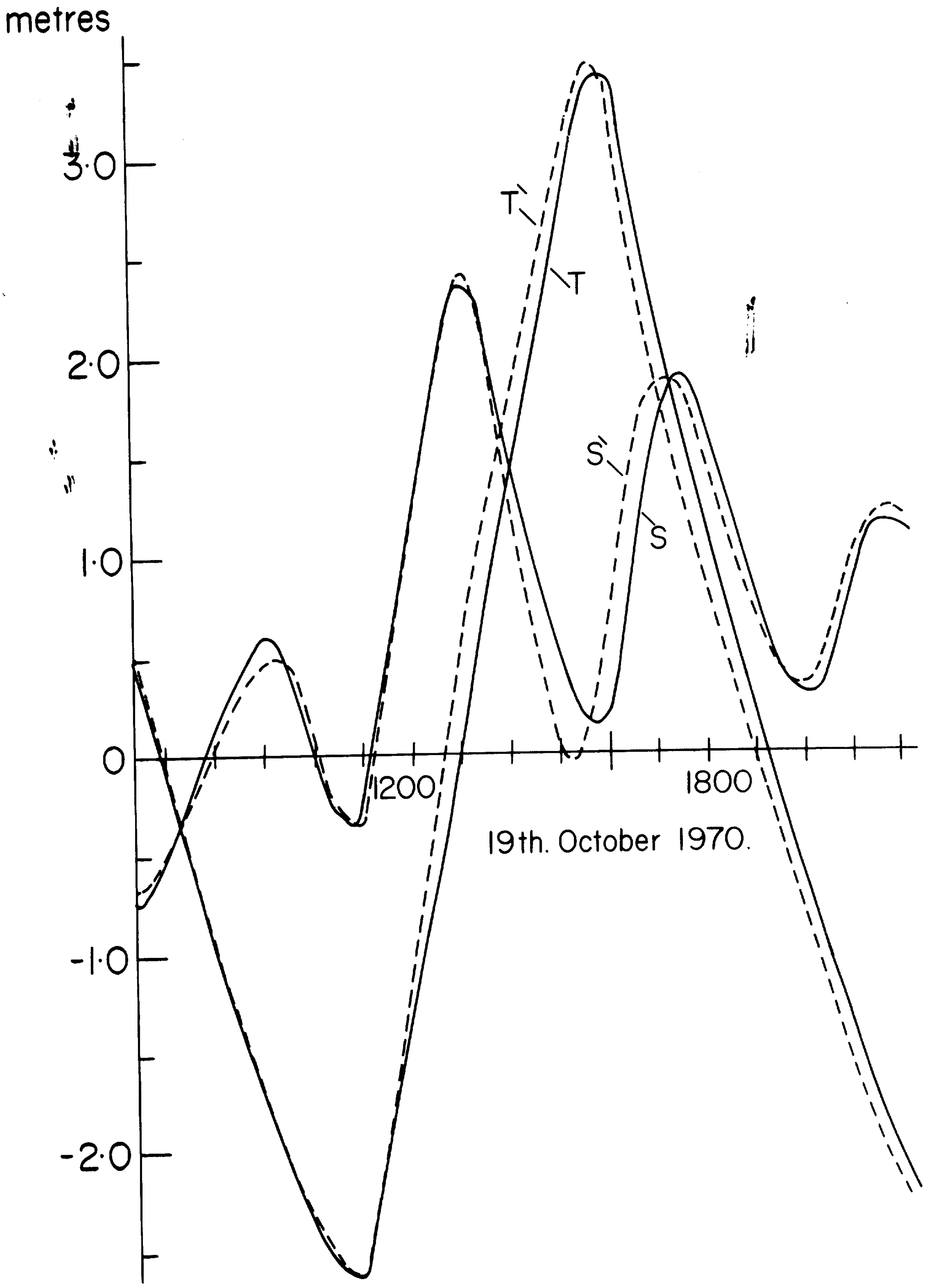


FIGURE 19

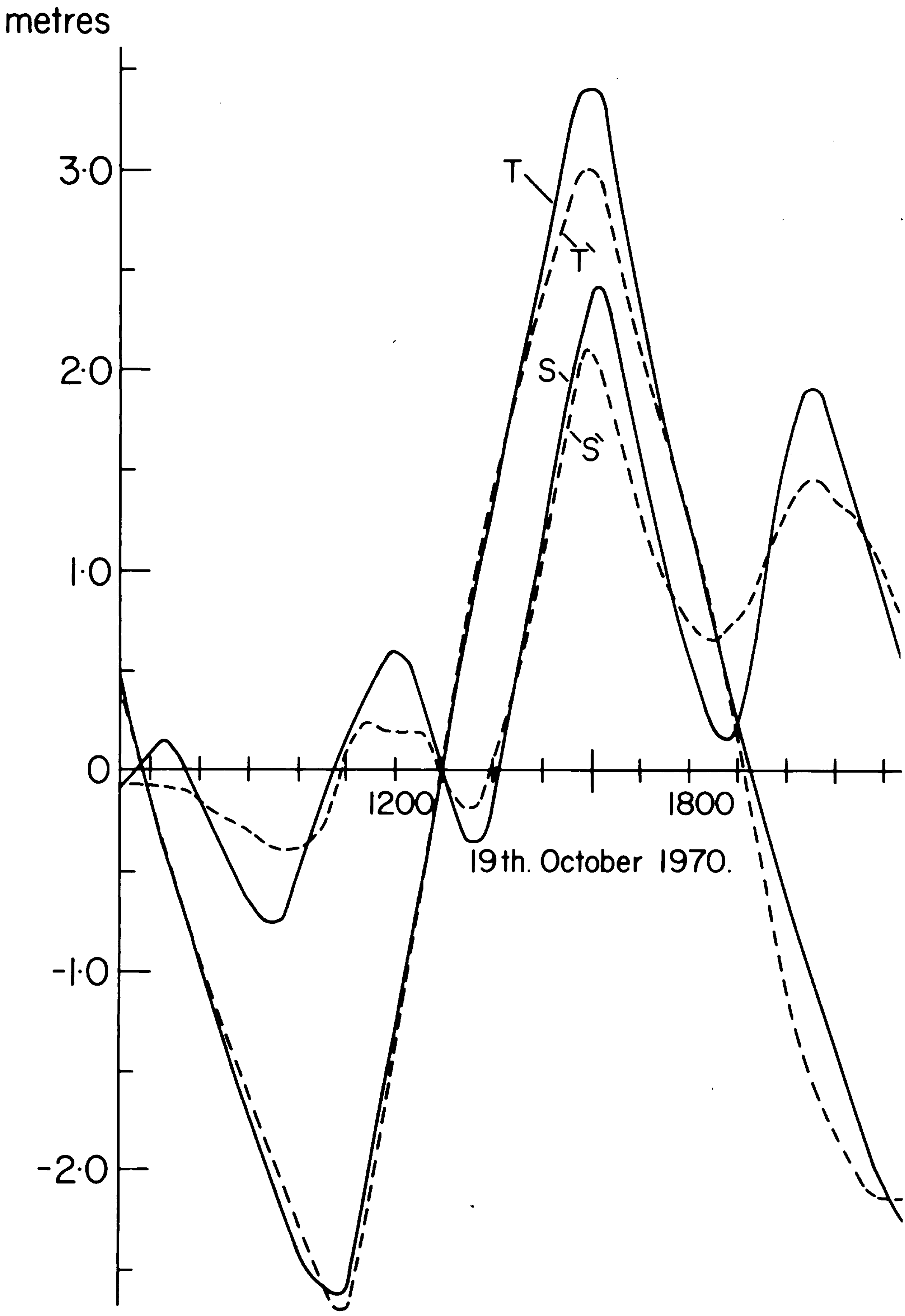


FIGURE 20