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SOME INTERCOMPARISONS BETWEEN WAVE RECORDERS

by

E G Pitt, J S Driver and J A Ewing

REPORT No. 43

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The work described in this report was supported financially by the Departments of Industry and Energy

NATURAL ENVIRONMENT RESEARCH COUNCIL

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CONTENTS

1. ABSTRACT

The report describes two series of comparisons of wave recorders. Part I discusses an experiment which was mounted on a platform in the southern North Sea, to compare simultaneous wave measurements from a Datawell Waverider, Baylor wave gauge and Spectra-Physics Geodolite laser altimeter.

The experimental and logistic aspects of the study are described briefly while retaining what we consider to be the essential information. Descriptions of the principles of operation of the three instruments are given, although in the case of the Baylor wavestaff and the laser altimeter certain important information could not be obtained.

The absolute scaling of the instruments was determined by test calibrations and these revealed some rather large errors.

The data comparisons were made for the most part using spectral analysis techniques, and so the description of the analysis and computational procedures, while by no means complete, is detailed enough to allow confident interpretation of the quantities which have been calculated and compared.

There are approximately 50 simultaneous sample measurements from the three instruments and the information derived from these is presented in a series of graphs. The comparisons specifically between instruments are concerned with the overall relative scaling as indicated by the standard deviation of the wave profile (Fig 5.2.1), and also with the relative output as a function of frequency (Fig 5.4.1). The practical effect of differences in frequency response are illustrated by comparisons of several period and bandwidth parameters (Fig 5.2.2 to 5.2.8), and comparisons of several simply-derived (Tucker-Draper) parameters are presented in Figs 5.2.9 and 5.2.12.

In addition to these comparisons, the relationships between several different parameters as measured by the same instrument were studied. This work included a comparison of the simply-derived statistics with those derived from the spectra (Figs 5.3.1 to 5.3.3) as well as comparisons between other essentially spectrum-based parameters (Figs 5.3.4 to 5.3.7).

As well as the calibration discrepancies already referred to, residual differences in output between the waverider and the other instruments were revealed for which no obvious explanation could be found. In addition, the laser system appeared to show a marked decrease of output with increasing frequency. A number of more subtle effects were revealed which were perhaps related to non-linearities associated with the Baylor wave staff installation.

A number of theoretical relationships between several spectra-based parameters

were well supported by the data.

Part II discusses a limited study of wave measurements taken with a laser altimeter mounted in a helicopter in conjunction with wave spectra measured with a waverider buoy.

2. INTRODUCTION

The Advisory Group on Environmental Data for Offshore Structures in Part II of its submission to the Ship and Marine Technology Requirements Board proposed a project to test and intercompare commonly used wave recorders. The project proposal was designated AGEDOS 02 and entitled 'The calibration and comparison of wave measuring instruments'. It proposed a series of comparisons specifically between the Baylor wave gauge, the Datawell Waverider and the IOS Shipborne Wave Recorder as well as suggesting that less orthodox techniques should be included for evaluation purposes. As well as these 'offshore tests', AGEDOS 02 also proposed a series of 'onshore calibrations' which included the provision of static and dynamic calibration facilities for the Baylor wave gauge.

The reason for this proposal was that there have been very few convincing tests of commonly-used wave meters in conditions of actual use. Those that have been done commonly show discrepancies of 10% or more, which are unacceptable in present day circumstances. It is a sobering thought that the present tests also show such discrepancies.

The present project was undertaken in response to AGEDOS 02 although it falls short of the requirements of that proposal in a number of respects, the main one being that in the event it proved impossible to bring the Shipborne Wave Recorder into the comparisons. Also, the work is directed mainly towards the 'offshore tests' aspects of the proposal so that the construction of onshore test facilities for the Baylor was not undertaken. Because of these circumstances and for other reasons, we consider that this project can only be regarded as the start of a continuing series of tests and evaluations of wave measuring instruments and techniques.

The main impetus for the project was the opportunity to collaborate with the Radio and Navigation Section of the Aircraft and Armaments Experimental Establishment at Boscombe Down in Wiltshire in the hire and use of a laser altimeter. In order to make the optimum use of this very expensive piece of equipment a tightly co-ordinated collaborative programme was arranged between IOS and A & AEE. This programme specified that IOS should have exclusive use of the laser for two weeks during October of 1974, and during this period we arranged for it to be deployed on a gas production platform in the Southern North Sea. In November 1974, IOS took part in a series of trials with A & AEE in which the laser was mounted in a helicopter and flown over the sea near the IOS Waverider installation at the Eddystone.

The programme to be described in this report thus consists of two parts, as shown in Table 2.1.

Of these, Part I yielded the great majority of the usable data, and thus occupies most of the report.

Each part is subdivided into sections dealing respectively with instrument deployment, calibration and performance, data preparation and processing, and presentation and interpretation of results.

TABLE 2.1

Part	In collaboration with	Site	Period	Instruments Involved
I	Amoco	Indefatigable Field, Platform 49/18A. Position 53°22'N 2°34'E	October 1974	Baylor Wave gauge, Datawell W/R Geodolite laser
II	A & AEE	Eddystone light Position 50°11'N 4°16'W	December 1974 - January 1975	Datawell W/R Geodolite laser Mounted in helicopter

3. PART I - MEASUREMENTS AT AMOCO GAS PRODUCTION PLATFORM 49/18A, ON THE INDEFATIGABLE GAS FIELD IN THE SOUTHERN NORTH SEA.

3.1 Description of wave measuring instruments

3.1.1 Baylor wave staff

This instrument is manufactured by the Baylor Company of Texas, USA, and consists of two parallel stainless steel wire ropes each 12.7mm in diameter and separated by 228.6mm. In use the staff is suspended under tension so that it passes through the sea surface, the steel ropes being parallel, co-planar and vertical.

The staff is energised using an AC signal of about 0.65 MHz and the exposed (in-air) part of the staff behaves as a flat twin transmission line terminated at its lower end by the sea, which acts as a low resistance connection. The skin-depth of sea-water to radiation at this frequency is of the order of 0.3m, but since this has the effect of increasing the apparent length of the exposed part of the staff by this (constant) amount it is of no importance from the point of view of measuring waves.

Under these conditions the impedance of the staff as measured at its upper end is given by:

$$Z = j Z_o \tan\left(2\pi L/\lambda\right) \qquad 3.1.1$$

where Z_o is the characteristic impedance of the staff,

L is the exposed length of the staff,

λ is the wavelength of the energising radiation.

Since losses in the line are small, Z_o can be taken as wholly real, and thus Z is essentially inductive. With the dimensions given above Z_o is approximately 430 ohms.

Approximate values of impedance calculated from equation 3.1.1 are:

for L = 5m, Z = 0.51 ohms

L = 22m, Z = 2.04 ohms

These changes of impedance are converted and presented as a voltage which is proportional to the instantaneous level of the water up the staff.

3.1.2 Geodolite 3A laser altimeter

This instrument is manufactured by Spectra-Physics of California, USA, specifically for airborne application.

The instrument uses a continuous wave helium-neon laser to transmit a narrow beam of highly coherent light which is amplitude-modulated. The beam is reflected from the 'target', the sea surface in this case, and focussed by a reflector telescope onto a photomultiplier where it is converted into an electrical signal. The change in phase of the amplitude modulation between the outgoing and received signals is measured and taken as an indication of the distance of the target using the relation

$$D = \frac{C \Delta \phi}{4 \pi f}$$

where D = the distance,

C = speed of light,

f = frequency of amplitude modulation,

$\Delta\phi$ = phase change.

In use the received signal strength is subject to large fluctuations, and in order to deal with these the signal is regenerated using a phase-lock loop. The loop time constant could be selected using a switch; low pass filtering presumably originating from this circuit is evident on the output from the instrument, but we have not been able to obtain enough information from Spectra-Physics to make a correction.

3.1.3 Datawell Waverider

This instrument is manufactured by Datawell at Haarlem in the Netherlands. It consists of a spherical steel float 0.7m in diameter which measures its own motion with an internal passively stabilised accelerometer. The buoy is usually attached to the seabed with a compliant mooring, so that its motion follows the waves more or less accurately. The output of the accelerometer is twice integrated, and this heave signal is used to frequency-modulate a low frequency sub-carrier, which in turn is made to amplitude-modulate a 27 MHz crystal-controlled radio carrier. The buoy carries its own battery power supply and transmits continuously.

The signal from the buoy is received by a simple crystal-controlled receiver, and the sub-carrier is demodulated using a phase-lock loop. The wave profile is recorded on pen and paper chart and the sampling programme is controlled by a mechanical time switch.

3.2 Selection of location

The programme of measurements to be described took place at production platform 49/18A on the Indefatigable gas field, over the period 16 to 24 October 1974. The platform is owned and operated by the Amoco (UK) Exploration Company, and is located approximately 62 miles north east of Great Yarmouth at 53°21'50" North and 2°34'7" East. This position is indicated on figure 3.2.1.

It had been hoped to carry out the measurements from a platform in the northern North Sea, since the chance of experiencing high waves would be greater there than further south. However, since the northern platforms were still in the pre-production phase at that time, it was not possible to mount the project from one of them.

Two companies kindly offered us facilities in the gas field area of the southern North Sea and of these we chose Amoco's 49/18A as being further offshore, clearer of offshore banks and thus better exposed. In the event, we experienced a good variety of conditions with significant wave heights of up to 4 metres. The platform was equipped with a suite of environmental sensors, details of which will be given in 3.3; here it is sufficient to comment that this suite included a Baylor wave staff. To this were added for the purposes of this project a Spectra-Physics Geodolite 3A laser altimeter, which was installed on the platform, and a Datawell waverider buoy which was moored approximately 400m off.

3.3 Siting of instruments on the platform

Figure 3.3.2 shows the general arrangement of the platform which is seen to comprise two structures linked by a bridge. Due to the control room being on the production platform it was considered desirable to site the temporary instrument cabin in the same area. This meant that all recording systems would be in close proximity to each other.

The Baylor wave staff was suspended from the connecting bridge (see figure 3.3.2) and since it was an existing facility it was clearly desirable to site the Geodolite system as close as possible. In the event 22.5m was the horizontal separation between the two points at which waves were being measured. The choice of instrument cabin position was dictated by two main considerations: (1) to minimise the screening effect of the platform over the points of the compass where most wave activity was expected to come from, and (2) to find a suitable area to place the hut bearing in mind accessibility, power supplies and proximity to the Baylor recording system. Given complete freedom of choice from a scientific point of view the cabin would have been placed in the centre of the connecting bridge but for practical reasons this was impossible.

3.3.1 Waverider buoy mooring

The mean depth of water at the platform was close to 21m. The Waverider buoy was deployed by RRS Discovery at a position approximately 400m NW of the platform. The buoy mooring consisted of a 15m rubber cord, 60m of 12mm polypropylene braided rope and approximately 20m of chain with a 20kg Meon anchor. (A review of IOS Waverider mooring techniques is given by HUMPHERY (1975)). The position of the buoy on the sea surface could depart by as much as 75m from a point vertically above the mooring anchor depending on the prevailing wind and current.

3.3.2 Meteorological instrument suite

The meteorological instrument suite installed on the platform consisted of a Munro cup anemometer and wind vane type IM146 and a Kelvin-Hughes marine barograph. The anemometer was situated at the top of the microwave tower at a height of 87m above sea level and the wind direction and speed information was recorded on a Munro paper chart recorder in the control room. The barograph was positioned in the mess-room on the wellhead platform.

A Plessey self-recording current meter was suspended at a depth of 40ft from the SW end of the helideck, which was situated on the wellhead platform.

3.4 Effect of structure on instruments

It is probable that the waves measured by the instruments were affected by the presence of the structure; the difficulty is to quantify this effect.

The laser beam intercepted the sea surface at a position about 3m to the west of the westerly-most corner leg of the production platform (see Fig 3.3.2). It was thus approximately 3 diameters away from this 1m cylindrical member on what was predominantly the 'weather' side.

The situation with regard to the Baylor staff was different - over an arc to the South-west and another between North and East the instrument was shadowed by the structure. The structure consists of a lattice of 1m diameter cylinders at approximately 10m centres.

An attempt was made to detect the effect of the structure in the results by correlating the difference between the laser and Baylor measurements with wind direction, but the results were too scattered for any conclusions to be drawn.

3.5 Installation of equipment on the platform, power supply and switching arrangements

All wave recording equipment was transported to 18A by means of the AMOCO/BRISTOW daily helicopter service. Prior to this, arrangements had been made with a Yarmouth firm to hire the steel instrument cabin and have it fixed in position on the platform. The Waverider buoy was deployed by RRS Discovery two days before the arrival of the remainder of the equipment on the platform.

The Geodolite was mounted vertically over a 12ins square hole cut both through the floor of the cabin and the platform deck grill. The height of the Geodolite front lens above the mean sea level was calculated to be 75ft. Since the system provided an output of 0-10 volts for the range 0-100ft a 5 volt backing off voltage was derived from a stabilised power supply and thus it was possible to set the pen recorder to its 0-5 volt range which corresponded to a height of the laser above the water surface of from 50 to 100 ft. An auxiliary lens was fitted to the beam unit which, by diverging the beam, produced a spot of approximately 8ins diameter at a distance of 70ft.

The Waverider receiver converted the incoming frequency modulated 27.030 MHz carrier into an analogue signal and displayed it on a chart recorder with a full scale deflection of 5 - 0 - 5 metres.

The Baylor system also had a chart recorder whose full scale deflection was equivalent to a water depth of +70ft to +170ft; this system, as with the Geodolite, displayed mean water level as well as wave height whilst the Waverider, being an accelèrometer system, did not.

Figure 3.5.1 is a schematic of the power supply and switching arrangements within the cabin.

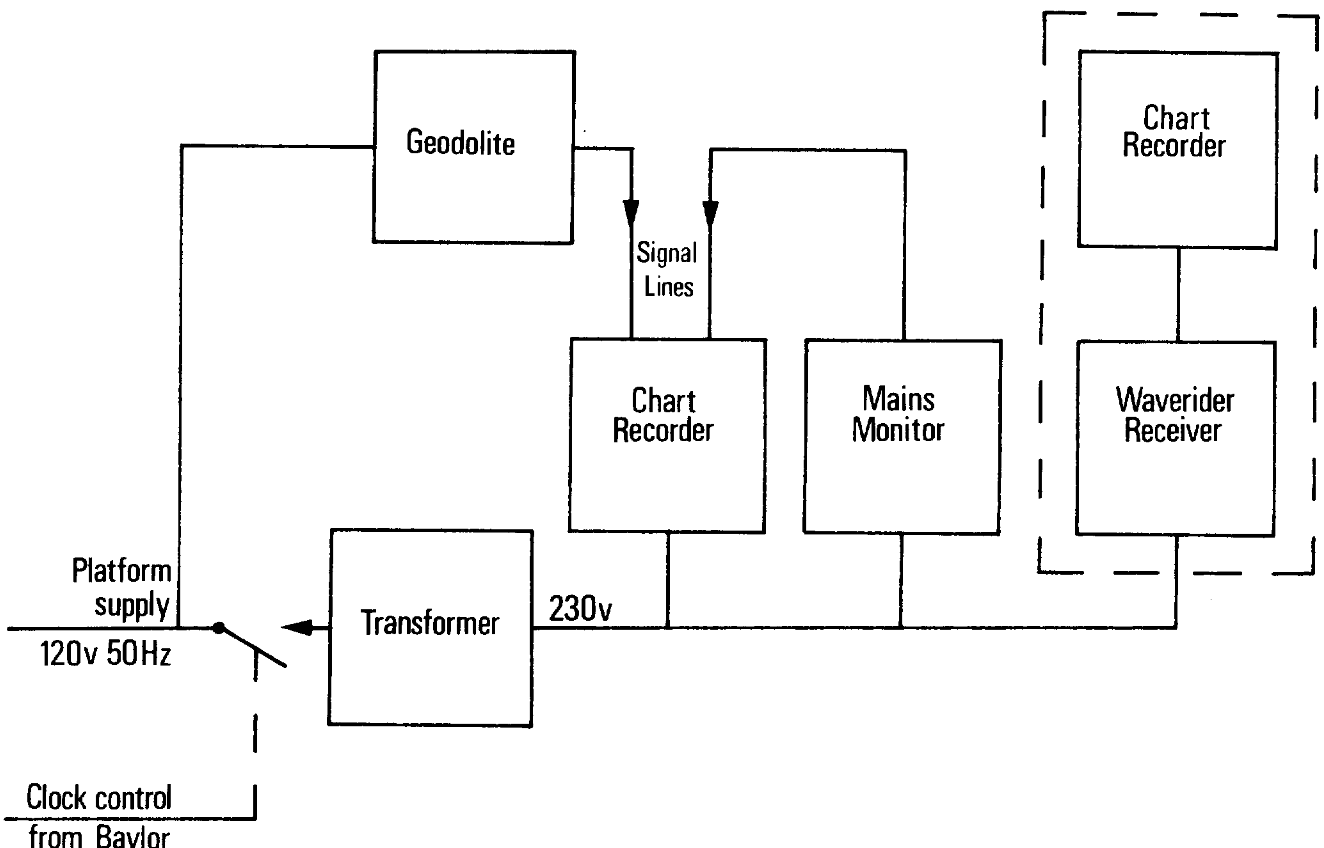

Figure 3.5.1

In order to maintain synchronism in sampling from the three instruments the existing Baylor programme clock was used as the master control. The sampling arrangement of 20 min once every three hours was considered suitable for the experiment. To obtain 'long' records (eg one hour) a manual bypass was operated.

The three chart speeds involved: Baylor 2ins/min; Geodolite 60mm/min; and Waverider 60mm/min were checked at regular intervals by making time marks with the aid of a stopwatch.

The mains test set shown in Fig 3.5.1 served the purpose of monitoring mains voltage and frequency; either parameter could be recorded on the chart recorder.

3.6 Calibration

The Waverider system was calibrated on the 3m test rig of the National Physical Laboratory (now the National Maritime Institute) at Hythe immediately after recovery and was found to have an error of between -8 and -10% over the range of wave periods 3 to 18 seconds. That is, the instrument was indicating displacements which were between 8% and 10% lower than those to which it was subjected. This is an unusually large discrepancy.

The Geodolite system was checked both before and after the experiment simply by setting a target at various fixed distances over the range 0-100ft and noting the corresponding output voltages. No significant calibration error could be detected.

Routine maintenance of the Baylor system on the platform was contracted to Marine Exploration Ltd who carried out a calibration on the platform on 20 February 1975, 4 months after the termination of the experiment. The calibration involved recovering the staff and moving a shorting bar along the parallel wires and noting the chart recorder deflection for each position. The system was found to be reading high by 17%. The calibration information for all three instruments has been incorporated into the subsequent analysis.

3.7 Performance of the instruments

The Baylor staff had been installed at an angle from the vertical of some 25^{o} and although the effect of this on wave height was compensated for there would be some distortion of the wave profile that could not be corrected. With waves of more than about 3ft present water could be seen trailing down the wire as the wave height decreased. The length of this 'trail' could be as much as 1 foot but the effect on the accuracy of the instrument would best be determined by laboratory (wave tank) tests. The Baylor recorder suffered alternately ink starvation and excessive ink supply resulting in occasional loss of record or a rather thick trace. The recorder chart speed was nominally 2 inches per minute but was timed to be 1.7. The programme clock within the Baylor gained about 7 minutes per day. Checks on the power supply showed that a high alternator frequency was responsible for this.

The Geodolite system was subject to much unlocking during the first few days' operation when little wind was present, although some swell could be detected. It was clear that when the water surface was smooth, specular reflection occurred causing very large dynamic variations of reflected laser energy which the receiver input and phase-lock circuitry were unable to cope with. Use of the diverging lens had little effect. When the wind speed exceeded 4 knots this problem largely disappeared. It was noted that the stability of the beam source and associated circuitry improved greatly after 24 hours' operation, and so it was decided to power the system continuously for the two week experiment. The separate potentiometer chart recorder worked well throughout.

Transmissions from the Waverider buoy were received with no interference or interruption throughout the experiment. As with the Baylor, problems were experienced with the chart recorder pen and this resulted in some loss of data. The buoy motion was observed regularly using binoculars and it appeared to be following the waves accurately.

4. DATA ANALYSIS

4.1.1 Manual analysis method

The pen chart records for the three instruments were analysed according to the method proposed by TUCKER (1963) and DRAPER (1967).

The method consists of reading from the pen chart record four height parameters as follows:

A. The highest crest

B. The second highest crest

C. The lowest trough

D. The second lowest trough

All these are measured with respect to the mean of the wave record.

In addition, the number of times the wave trace crosses the mean line is noted, as are the number of crests and the duration of the record.

From these, estimates are made of the following parameters:

m_o – the mean square surface elevation

T_z – the mean zero - crossing period

T_c – the mean crest period

ϵ – the spectral width parameter which is defined by the relation

$$\epsilon^2 = 1 - (T_c / T_z)^2 \qquad (4.1.1)$$

The height parameter quoted in the results is

H_s – the significant wave height, which was evaluated using the relation

$$H_s = 4\sqrt{m_o} \qquad (4.1.2)$$

A discussion of the theoretical basis of these calculations is contained in TANN (1976).

The nominal record length was 20 minutes for all 3 instruments. However, this was reduced to 19.5 minutes in the case of the Baylor by the presence of an automatic start indication sequence on each record. In addition, some records from each instrument were lost or shortened due to pen failures and other instrumental difficulties which have been referred to in section 3.3.

4.1.2 Computer processing of data from manual analysis

The results of the manual analysis - A, B, C, D, the number of zero-crossings, the number of crests and the duration were tabulated. These tabulations were then punched onto paper tape, transferred to magnetic tape, checked and edited.

This data then formed the input to a standard program which performed the calculations required to evaluate H_s, T_z and ϵ . Various other parameters were estimated, in particular the most likely value of the highest wave which would occur during the 3-hour period containing the record - Hmax (3 hr), but these were not used in the comparisons. All the results were output to the lineprinter and magnetic tape.

4.2 Computer processing of data in digital form

4.2.1 Spectrum analysis of wave data. Definition of statistics

In order to give a full account of the relative performances of the instruments it was necessary to compute variance spectra from the records. From these the relative output as a function of frequency could be seen and in addition, various statistical properties of the records could be determined with a lower statistical variability than that obtained when the same parameters are estimated from a simple manual analysis.

A full description of the statistical and analytical basis of spectral methods may be found in BENDAT and PIERSOL (1971); the account given here is limited to that necessary to clarify the definitions of the parameters quoted in the comparisons.

The wave system which is sampled by the three instruments is assumed to be a linear, random process which is time stationary over each sample period. With the above assumptions, and for measurements at a point, the height of the sea surface as a function of time, h(t), can be formulated as follows (see CARTWRIGHT and LONGUET-HIGGINS, 1956):

$$h(t) = \sum_{n=1}^{\infty} c_n \cos(2\pi f_n t + \phi_n) \tag{4.2.1}$$

where the frequencies f_n are densely distributed in the interval $(0, \infty)$ and the phase angles, ϕ_n, are random and uniformly distributed in the range $(0, 2\pi)$. The amplitudes c_n are such that in any small interval of frequency df

$$\sum_{f_n=f}^{f+df} \tfrac{1}{2} c_n^2 = S(f)\, df \tag{4.2.2}$$

S(f) is the (variance) spectrum of h(t).

From the spectrum and its moments various integrated parameters can be obtained. We define the n^{th} moment of the spectrum as

$$m_n = \int_0^{\infty} S(f)\, f^n\, df \tag{4.2.3}$$

in particular

$$m_0 = \int_0^{\infty} S(f)\, df \,, \tag{4.2.4}$$

the total energy in the wave system.

As before we define H_s by

$$H_s = 4\sqrt{m_0} \tag{4.2.5}$$

In addition:

$$T_z = \sqrt{(m_0/m_2)} \tag{4.2.6}$$

$$T_c = \sqrt{(m_2/m_4)} \tag{4.2.7}$$

Other relations are:

$$\overline{T}\ (\text{or TB}) = m_0/m_1 \tag{4.2.8}$$

and

$$\epsilon \text{ (or EPS)} = \sqrt{\left(1 - \frac{m_2^2}{m_0 m_4}\right)} \qquad (4.2.9)$$

Following GODA (1970), a spectral peakedness parameter Qp can be defined thus:

$$Q_p = \frac{2\int_0^\infty f\, S^2(f)\, df}{\left[\int_0^\infty S(f)\, df\right]^2} \qquad (4.2.10)$$

E_o - is the maximum value of the spectrum

F_o - is the frequency at which the spectral peak (E_o) occurs.

4.2.2 Digitisation of the records

Since the primary data recording medium was pen and paper chart, a substantial digitising task had to be undertaken before any detailed computer processing could be done.

All of the usable data from the three instruments was digitised using a D-MAC table interfaced via CAMAC to a PDP11-20 computer. A computer programme called TRACE was written to control the digitising process. This program uses CAMAC handling subroutines also written at Taunton.

The operation of the digitising system was as follows: The operator entered a parameter XINC into the computer which specified the distance along the X or time axis of the wave record between successive digitised points. The operator then fixed the wave record to the table, making sure that the record was indeed parallel with the X-axis of the table, and the origin of coordinates was set on the start line of the record. She then followed the wave trace with the cursor, and as she did so, the Y-value corresponding to each successive integral multiple of XINC was stored in the computer's memory and subsequently written to magnetic disc. The resolution of the system in the X and Y directions was 0.025mm, which on the Waverider chart for example corresponded to 0.025 seconds and 3.75mm respectively. The resolution of the system was thus more than adequate.

4.2.3 Digitising rate

The digitising rate was set at 1 per second, as it was considered that this was adequately fast to prevent aliasing problems in the subsequent spectral analysis, while keeping data storage and handling operations simple.

However, for the statistical analysis of wave heights, a faster sampling rate is preferable - 2 per second at least. This is because digitised points will not in general coincide with the crests or troughs of waves, so that the magnitudes of waves will be systematically underestimated.

Figure 4.2.3 shows the approximate average error due to this effect as a function of the ratio sampling interval : wave period. The error was calculated on the assumption that the crests are parts of a sine wave of period T.

4.2.4 Computational procedure

In section 4.2 a brief account of the considerations underlying spectral methods was given, followed by some definitions of integrated spectral parameters. In this section details of the computations will be given, and these will include a discussion of sampling errors and the associated theory.

The wave records are sampled at intervals of ΔT and are of finite length T, so that $T = N\Delta T$, where N is the number of data points in the record. The amplitude spectrum, and thus the variance spectrum of the process can be obtained from a sample time history by Fourier analysis, and a Fast Fourier Transform (FFT) method was used to accomplish this.

The result of this analysis consists of values of the in-phase and quadrature amplitudes (a_i and b_i respectively) of the Fourier series representation of the sample record. These are available at the fundamental frequency

$$f_0 = \frac{1}{T} = \frac{1}{N\Delta T} \qquad (4.2.4)$$

and integral multiples thereof up to the Nyquist frequency, $1/(2\Delta T)$.

The variance associated with the component whose frequency is if_0, called the i[th] harmonic, is given by

$$\frac{1}{2}\left(a_i^2 + b_i^2\right) = \hat{S}(if_0)\,\Delta f \qquad (4.2.5)$$

This equation is the definition of $\hat{S}$, which is the sample estimate of the spectral density at the frequency if_0.

Δf is the finite element of frequency, is a constant for any given computation and is equal to f_0.

If many independent estimates of $\hat{S}$ are made they will be found to have a very large random fluctuation because they are derived by sampling a random process. If the k th sample estimate is $\hat{S}_k(f)$, then conceptually, S, the spectrum is obtained by averaging over a large number of realisations of the process, ie.

$$S(f) = \left\langle \hat{S}_k(f) \right\rangle_k \qquad (4.2.6)$$

In practical situations one estimates S(f) from just one realisation, and the question then arises as to how well this estimate can be expected to approximate the spectrum defined in Eq. 4.2.6. Since the phases ϕ_n (Eq. 4.2.1) are random, a_i and b_i are uncorrelated, and if further, a_i and b_i are supposed to follow Gaussian distributions, the spectral estimates which are sums of squares of these quantities can be considered as Chi-squared distributed random variables.

With these assumptions it can be shown that the individual estimate considered above (Eq. 4.2.5) has a normalised standard error of 100%. In order to reduce this to acceptable proportions, it is usual to perform some kind of smoothing operation either in the frequency or time domains.

In frequency domain averaging, the record is analysed, and the average of ℓ adjacent spectral estimates is taken as the final estimate, and ascribed to the mid-frequency of the estimates, thus

$$\hat{S}(f_i) = \frac{T}{2\ell} \sum_{j=n}^{n+\ell-1} (a_i^2 + b_i^2) \qquad (4.2.7)$$

where

$$n = (i-1)\ell + 1$$

and

$$f_i = \frac{1}{T}\left(n + (\ell-1)/2\right)$$

In time domain averaging the wave record is divided into q sections of equal length. Each section is analysed, and corresponding estimates from each of the q subsections are averaged to give the final estimate.

Thus

$$\hat{S}(f_i) = \frac{1}{q} \sum_{k=1}^{q} \hat{S}_k(f_i) \qquad (4.2.8)$$

where $\hat{S}_k$ is the estimate calculated from the k^{th} subsection.

The following parameters of these calculations are approximately correct (see BENDAT and PIERSOL, 1971). For frequency domain averaging, the effective bandwidth is given by

$$B_e = \frac{\ell}{T} \qquad (4.2.9)$$

The number of degrees of freedom of each estimate is given by

$$n = 2\ell \qquad (4.2.10)$$

The normalized standard error is given by

$$e_r = \sqrt{(1/\ell)} \tag{4.2.11}$$

The corresponding parameters for time domain averaging are

$$\left.\begin{aligned} B'_e &= q/T \\ n' &= 2q \\ e'_r &= \sqrt{(1/q)} \end{aligned}\right\} \tag{4.2.12}$$

and for combined frequency and time domain averaging

$$\left.\begin{aligned} B''_e &= \ell q/T \\ n'' &= 2\ell q \\ e''_r &= \sqrt{(1/\ell q)} \end{aligned}\right\} \tag{4.2.13}$$

Since the original pen charts were digitised manually, the lengths of the digital records vary a little. In addition, there are several long records. It should be borne in mind in what follows, that the particular FFT program used did not limit the series length to powers of 2.

The routine records are usually about 1186 seconds long, and these were transformed without time sectioning but with frequency domain averaging. As far as possible the resolution was set at 0.01 Hz.

Thus from 4.2.9, 4.2.10 and 4.2.11.

$$\ell \simeq 1186 \times 0{\cdot}01 \simeq 12$$

$$n \simeq 24$$

and

$$e_r \simeq \sqrt{(1/11)} = 29\%$$

The longer records were both time and frequency averaged: for example record number 33, Laser, was 4236 seconds long. It was not possible to transform it 'in one go', and so it was divided into 4 subsections of 1058 seconds.

Then from 4.2.13

$$B_e \simeq 0{\cdot}01 = 4\ell/4232$$

$$\text{ie } \ell \simeq 42{\cdot}34/4 \simeq 10$$

$$n \simeq 80$$

$$e_r = \sqrt{(1/40)} = 16\%$$

The moments of the spectra were calculated from the final smoothed estimates using the formula

$$m_n = \frac{\ell}{T} \sum_i f_i^{\,n}\, \hat{S}(f_i) \tag{4.2.14}$$

The sum is formed for values of the frequency from .05 Hz to 0.5 Hz where the lower limit is necessary to exclude estimates which may be contaminated by low frequency noise.

Q_p was evaluated by

$$Q_p = \frac{2T}{\ell} \sum_i f_i \hat{S}^2(f_i) / \left[\sum_i \hat{S}(f_i)\right]^2 \quad (4.2.15)$$

where the sums are formed over the same range of f_i quoted above.

5. PRESENTATION OF RESULTS

5.1 Basic statistics

Table 5.1.1 shows the basic results of the project. The wind was measured at a height of 87m and recorded on paper chart as part of the existing instrumentation of the platform. The anemograph records were analysed by Marex Ltd and the figures give hourly mean wind speeds in knots, corrected to the standard anemometer height (10m), the directions being in degrees measured from true North.

The values of H_s and T_z shown in Table 5.1.1 are those derived from the spectra.

5.2 Correlations between instruments

Since there are in general three simultaneous measurements available to define each data point, a good way to compare the instruments might be to plot a graph of H_s, for example, on 3-dimensional paper and fit a curve to the data points either by eye or using some numerical technique. In the absence of 3-dimensional paper, and indeed for most practical purposes,one would be interested in the projection of such a curve onto the co-ordinate planes. At least two of the three possible projections would be needed.

In fact no numerical procedure was available for constructing such a curve nor could one be devised. However, if one assumes that the three data sets are linearly correlated then a numerical best fitting procedure is available which gives consistent results.

The three possible graphs are plotted and a best straight line is fitted to each. The procedure used is to minimise the sums of the squares of the perpendicular distances from the data points to the straight line. This technique has been described by YORK (1966), and results in equations (1) of his paper. It is comparatively easy to derive an expression for the slope of the line which in addition is constrained to pass through the origin, and this has the same form as

TABLE 5.1.1

Principal Results from AGP 49/18A: Comparisons of statistics derived from wave spectra

No	October 1974		WIND		Hs(m)			Tz(sec)			Comments
	Day	Time	DIR deg	SP knots	BAY	LAS	WR	BAY	LAS	WR	
5	18	0549	205	21	0.97	0.93	1.00	3.63	3.70	3.43	
6	18	0848	195	23	1.35	1.34	1.49	4.37	4.68	4.36	
7	18	1147	195	27	1.58	1.55	2.08	4.85	4.97	4.77	
8	18	1446	195	33	2.07	2.07	2.39	4.76	5.19	4.67	
9	18	1610	190	34	2.38	2.57	2.95	4.94	5.36	5.16	Long record 4000 secs
10	18	1745	205	32	2.72	2.70	3.07	5.05	5.50	5.54	
11	18	2045	205	27	2.40	2.41	2.76	5.61	6.11	6.07	
12	18	2344	255	21	1.97	1.92	2.41	5.55	6.03	5.61	
13	19	0243	300	29	1.80	1.95	2.39	4.90	5.33	5.05	
14	19	0542	300	27	2.47	2.58	-	5.46	5.57	-	Chart recorder failure
15	19	0841	295	27	2.46	2.65	2.89	5.11	5.72	5.41	
16	19	0916	295	27	2.52	2.51	2.85	5.20	5.61	5.21	
17	19	1142	290	21	2.28	2.00	-	5.54	5.76	-	Chart recorder failure
18	19	1440	275	17	1.92	1.85	-	5.68	5.66	-	Chart recorder failure
19	19	1740	260	18	1.80	1.70	1.88	5.11	5.29	5.01	
20	19	2036	265	15	1.59	1.66	1.84	4.70	4.97	4.84	
21	19	2335	310	15	1.40	1.40	1.62	4.86	5.00	4.89	
22	20	0235	335	14	1.26	1.18	1.27	4.89	5.10	4.90	
23	20	0534	330	08	1.02	1.00	1.11	4.85	4.89	4.71	
24	20	0833	310	12	-	1.05	1.29	-	4.67	4.57	Chart recorder failure
25	20	0900	310	12	-	-	-	-	-	-	Records too short to analyse
26	20	1132	300	18	1.24	1.19	1.41	4.32	4.72	4.36	
27	20	1431	295	20	1.46	1.25	1.55	4.63	4.97	4.75	
28	20	1731	330	15	1.35	-	1.45	4.81	-	4.95	Laser record too short to analyse
29	20	2030	320	17	1.42	1.39	1.82	4.74	4.92	4.72	
30	20	2329	325	18	1.18	1.27	1.53	4.61	4.69	4.56	
31	21	0228	325	16	1.51	1.47	1.56	4.95	5.23	4.75	
32	21	0527	310	22	2.07	1.90	2.17	5.34	5.59	5.39	
33	21	0824	305	29	2.46	2.65	3.05	5.43	5.81	5.55	Long record 4000 secs
34	21	1124	310	20	2.11	2.29	2.45	5.49	5.79	5.41	
35	21	1422	315	13	-	1.93	2.15	-	5.69	5.36	Long record 4000 secs Baylor record faulty

Table 5.1.1 - page 2

No	October 1974		WIND		Hs(m)			Tz(sec)			Comments
	Day	Time	DIR deg	SP knots	BAY	LAS	WR	BAY	LAS	WR	
36	21	1721	285	13	-	1.93	2.17	-	6.21	5.81	Baylor record faulty
37	21	2020	120	03	1.75	1.94	1.93	5.82	5.46	5.81	
38	21	2319	085	18	-	1.69	1.92	-	4.84	5.06	Baylor record faulty
39	22	0218	065	18	-	1.60	1.54	-	5.14	4.74	Baylor record faulty
40	22	0516	035	17	-	1.66	2.04	-	5.19	4.90	Baylor record faulty
41	22	815	020	24	2.51	2.48	2.96	5.78	5.99	5.63	Long record 3700 secs
42	22	1115	010	27	2.98	3.06	3.73	5.81	6.44	6.14	
43	22	1414	355	28	3.13	2.83	3.32	5.55	6.01	5.62	
44	22	1713	355	27	3.11	-	3.50	5.81	-	6.03	Chard recorder failure
45	22	1805	345	29	-	-	-	-	-	-	Chart speed check, not analysed
46	22	2012	340	30	3.33	3.38	3.62	6.22	6.60	6.17	
47	22	2311	355	31	3.80	3.78	4.05	6.08	6.49	6.16	
48	23	0210	360	30	3.58	3.37	4.24	6.01	6.37	6.34	
49	23	0509	360	30	3.93	3.58	-	6.35	6.82	-	Chart recorder failure
50	23	0808	360	31	3.96	3.75	4.22	6.56	7.05	6.78	Long record 4000 secs
51	23	1107	005	28	3.73	3.73	4.41	6.61	6.86	6.84	
52	23	1405	005	26	3.17	3.14	3.44	5.83	6.56	6.04	
53	23	1705	005	26	3.16	3.00	2.94	5.93	5.91	-	
54	23	2004	005	21	2.73	2.46	2.59	5.81	6.28	6.02	
55	23	2303	360	18	2.42	2.27	2.44	5.50	6.06	5.65	
56	24	0202	350	18	2.05	1.85	2.22	5.35	5.62	5.29	
57	24	0501	330	15	1.82	1.63	1.61	5.17	5.66	5.18	

York's equation (1) but with the U's and V's - which are the data values minus their individual means - replaced by the data values themselves.

5.2.1 Summary of comparisons between instruments

Figures 5.2.1 to 5.2.8 each show a set of 3 graphs as described above which present the results derived from the spectra, and Table 5.2.1 summarises these results.

For a straight line drawn in three dimensional space, X, Y, Z, we have the relation $\frac{\Delta X}{\Delta Y} \cdot \frac{\Delta Y}{\Delta Z} \cdot \frac{\Delta Z}{\Delta X} \equiv 1$. Thus, if the three slopes derived as described above are self-consistent, their product should be close to unity. The values in the column labelled 'product of slopes' shows that this is indeed the case.

The 'standard error' quoted is simply the square root of the mean square perpendicular distance of the data points from the fitted line, and it therefore has the dimensions of the data variable.

Figures 5.2.9 to 5.2.12 show similar graphs for the manually derived data and Table 5.2.2 summarises these results.

5.3 Comparisons between various parameters as measured by the same instrument

Table 5.3.1 summarizes the results of comparing quantities derived from spectral methods with those from the manual analysis. As before the best fit line which has been constrained to pass through the origin is shown.

In addition two other correlations were tried. These are: (a) TB against Tz (both from the spectrum), for all three instruments, and (b) TB against $^1/F_o$ for all three instruments. For both these comparisons the best fit line (not constrained to pass through the origin) is shown. The graph of TB against $^1/F_o$ is rather difficult to interpret due to the large amount of scatter. If the best line through the origin is fitted, then the results shown as the penultimate entry in Table 5.3.1 are obtained.

5.4 The spectra

5.4.1 Relative responses of instruments

Fig 5.4.1 shows three comparative graphs on which are plotted the means and standard deviations of the square roots of the ratios of the spectral densities for each pair of instruments, for each of 46 frequencies. Since the spectral estimates were not in general evaluated for identical frequencies, the measured values have been linearly interpolated to a standard set of frequencies.

Thus the i^{th} plotted point has been formed as follows: (see Page 24)

TABLE 5.2.1

Summary of spectrum-based comparisons

Variable	Instruments	Slope	Standard Error (see text)	Product of slopes	Figure No.
Hs	WR : BAY LAS : WR BAY : LAS	1.123 0.878 1.020	0.146 0.109 0.098	1.006	5.2.1
Tz	WR : BAY LAS : WR BAY : LAS	1.028 1.047 0.932	0.112 0.131 0.132	1.003	5.2.2
Tc	WR : BAY LAS : WR BAY : LAS	1.051 1.058 0.901	0.097 0.123 0.149	1.002	5.2.3
TB	WR : BAY LAS : WR BAY : LAS	1.018 1.042 0.946	0.114 0.126 0.113	1.003	5.2.4
QP	WR : BAY LAS : WR BAY : LAS	1.117 1.018 0.884	0.197 0.199 0.234	1.005	5.2.5
EPS	WR : BAY LAS : WR BAY : LAS	0.976 0.993 1.032	0.011 0.014 0.014	1.000	5.2.6
EØ	WR : BAY LAS : WR BAY : LAS	1.560 0.773 0.838	1.452 1.042 0.894	1.011	5.2.7
FØ	WR : BAY LAS : WR BAY : LAS	1.010 0.990 1.004	0.010 0.009 0.008	1.004	5.2.8

TABLE 5.2.2

Summary of comparisons using manually analysed results

Variable	Instruments	Slope	Standard Error (see text)	Product of slopes	Figure No
Hs	WR : BAY LAS : WR BAY : LAS	1.032 0.861 1.114	0.230 0.201 0.259	0.990	5.2.9
Tz	WR : BAY LAS : WR BAY : LAS	1.002 1.015 0.979	0.223 0.323 0.349	0.996	5.2.10
Tc	WR : BAY LAS : WR BAY : LAS	0.926 1.015 1.061	0.219 0.467 0.466	0.997	5.2.11
EPS	WR : BAY LAS : WR BAY : LAS	1.126 1.014 0.875	0.053 0.060 0.067	0.999	5.2.12

TABLE 5.3.1

Summary of comparisons of various parameters measured by the same instrument

Variables	Instrument	Slope	Intercept	Standard Error (See text)	Figure No
Hs(M : S)	BAY	1.073	0	0.120	
	LAS	0.981	0	0.170	5.3.1
	WRB	0.988	0	0.101	
Tz(M : S)	BAY	1.048	0	0.149	
	LAS	0.998	0	0.241	5.3.2
	WRB	1.030	0	0.116	
Tc(M : S)	BAY	1.183	0	0.190	
	LAS	1.001	0	0.367	5.3.3
	WRB	1.044	0	0.243	
EPS(M : S)	BAY	0.861	0	0.038	
	LAS	1.018	0	0.048	5.3.4
	WRB	0.996	0	0.040	
Tz : TB	BAY	0.857	0.356	0.035	
	LAS	0.928	0.020	0.049	5.3.5
	WRB	0.890	0.214	0.033	
1/FØ : TB	BAY	2.324	-5.638	C.345	
	LAS	1.919	-3.961	C.325	5.3.6
	WRB	1.898	-3.458		
1/FØ : TB	BAY	1.357	0	0.499	
	LAS	1.217	0	0.422	5.3.7
	WRB	1.306	0	0.493	
1/EPS : QP	BAY	0.632	0	-	
	LAS	0.602	0	-	5.3.8
	WRB	0.607	0	-	

NOTE: Hs(M : S) means a comparison between Hs derived from the spectra and Hs derived from manual (simple) analyses.

$$R_i = \frac{1}{N}\sum_{i=1}^{N} \left\{ S_i(B)/S_i(L) \right\}^{\frac{1}{2}}$$

where S_i is the i^{th} interpolated spectral estimate (corresponding to frequency (0.01 x i) Hz); B, L refer to Baylor or Laser; and N is the number of pairs of spectra available. The standard deviation of R_i is taken over N in the usual way.

The most striking feature of these graphs is the smaller amount of energy at high frequencies shown by the laser compared with the other two instruments.

It should be noted that the main contribution to the total energy occurs in the frequency range 0.1 Hz to 0.2 Hz, and the values outside this range are both less important and less reliable. Even so, the relative response of the Baylor compared with the laser changes by about 40% over the whole range, which is very substantial.

5.4.2 Quality of the spectra

Figure 5.4.2 shows two spectra plotted as log E against f. Series 12 was of the usual length (1186 sec) while series 9 was a long record (4000 sec). Several aspects are worth noting:

(1) The high frequency tail of the spectrum of series 9 is smoother than that of series 12. This is due to the improved descriptions of the spectrum obtained when the number of degrees of freedom of each estimate is increased.

(2) The remarkably close agreement between the Baylor and the Laser in the neighbourhood of the spectral peak in the spectrum of series 12. This was noticeable on many of the spectra.

(3) The somewhat higher noise levels at high and low frequencies in the spectrum of series 9 than are evident in series 12.

(4) The low frequency noise levels on the Baylor in series 9 - these are rather higher than was usual. Figure 5.4.3 shows that a portion of the spectrum of series 9 which is to the right of the spectral peak, and figure 5.4.4 shows the spectrum of series 12. The figures are plotted as log E against log f, and the straight line has a slope of -5. Again, the spectra of the longer record are smoother than those of series 12. There is evidence of some contamination by high frequency noise in these spectra, although the effect is not worse than is normal in this type of computation.

5.5 Discussion of results

5.5.1 Comparison of quantities derived from the spectra, Table 5.2.1 and associated figures

Hs - Figure 5.2.1

The laser and Baylor agree very well. The Waverider scaling appears to exceed that of the other two by about 12%. This difference is difficult to account for. The possibility of systematic differences in the wave activity due to the separation of 400m perhaps caused by wave focussing was looked into. However, the scale of bathymetric features in the area is very large compared with the separation of the instruments, and this idea can probably be discounted.

Tz - Figure 5.2.2

The correlation between instruments is good with the laser reading highest, the Waverider next and the Baylor giving the shortest periods.

Tc - Figure 5.2.3

There is a fair degree of correlation between the instruments, although the range of periods measured was rather small. Here again the laser gives the longest periods followed by the Waverider with the Baylor giving the shortest.

Since this period is a function of the fourth spectral moment, m_4, the frequency response of the instrument at higher frequencies and the presence of noise at high frequencies have more effect than is the case with Tz. This probably accounts for the larger amount of scatter and the greater differences between instruments than is shown with Tz.

TB - Figure 5.2.4

The graphs show good correlation between instruments. The relationship laser greater than Waverider which is greater than Baylor is maintained, although in this case the differences are marginally smaller than with Tz, and at worst are only about 5%. The highest moment in the definition of TB is m_1, so that it is less sensitive to the high frequency performance of the instrument than is the case with Tz or Tc.

Qp - Figure 5.2.5

There is tolerably good correlation between instruments, and the laser and Waverider show good agreement. The Baylor values are about 12% lower than the other two instruments. This is an interesting result (see discussion of Eo).

ϵ - Figure 5.2.6

The range of values measured is rather small, but even so the correlation between instruments is good.

ϵ is a function of m_4 which for spectra with f^{-5} dependence does not converge. It is thus much affected by the high frequency performance of the measurement system and the (high frequency) truncation frequency used in the calculation of the moments (see, for example, RYE (1977)). COUNT and ROBINSON (1976) demonstrate the effect of instrumental response for a simplified measurement system.

Eo - Figure 5.2.7

The values plotted in these graphs reveal considerable differences between instruments. In order to investigate these we should attempt to eliminate effects due to scaling errors in the instruments. If we assume that the departure from unity of the slopes of the lines in Figure 5.2.1 (H_s) are due entirely to systematic scaling differences, then the relative output scaling of the three instruments can be expressed in terms of the ratios

Waverider	:	Baylor = 1.123
Laser	:	Waverider = 0.878
Baylor	:	Laser = 1.020

if we now divide the slopes shown in figure 5.2.7 by the squares of these numbers, we will have corrected for systematic scaling differences. The ratios for E_o then become

Waverider	:	Baylor = 1.237
Laser	:	Waverider = 1.003
Baylor	:	Laser = 0.805

The laser and Waverider thus agree very well, while the Baylor measured peak variance values which are about 20% lower.

It should also be noted that the scatter about the fitted lines is in all cases within the expected statistical variability of the estimates. These results taken in conjunction with those for Q_p suggest that the Baylor instrument was introducing some distortion into the measurements. The fact that it was installed at 25^o to the vertical would certainly introduce nonlinearities, in that the measurement position would fluctuate by an amount proportional to the wave height.

Fo - Figure 5.2.8

There is very good agreement between the instruments. The scatter reflects the usual statistical variability as well as that the records from the three instruments were not of exactly the same length.

5.5.3 Comparison of quantities derived from the manual analysis

Hs - Figure 5.2.9

The results are much more scattered than those derived from the spectra (Figure 5.2.1). The results for the Waverider and the laser support those of the spectrum-based comparisons while those which include the Baylor show some differences (see Section 5.5.4 ahead).

Tz - Figure 5.2.10

There is more scatter than is the case with the results derived from the spectra; however the fitted lines show very good agreement between instruments.

Tc - Figure 5.2.11

These graphs show a great deal of scatter, and moreover the range of measured values is greater than that of the spectrum-based values. The results perhaps indicate the difficulty of accurately identifying turning points on chart records.

EPS - Figure 5.2.12

The range of values measured was rather restricted, and the correlation between instruments poor. The results for the laser and Waverider agree, while the Baylor shows some discrepancies.

5.5.4 Comparisons between various wave parameters measured by the same instrument. Table 5.3.1 and associated figures

These comparisons are not directly between the three instruments, but study of them gives some insight into their relative performances.

5.5.5 Comparisons between results of manual and spectral analyses

Hs - Figure 5.3.1

There is good agreement between the results of the manual analysis and the spectral analysis in the case of the laser and Waverider. For the Baylor, however, the manual results are 7% higher than the spectral. The results for the Waverider and laser confirm conclusions by previous workers that the RICE (1944, 1945) and CARTWRIGHT and LONGUET-HIGGINS (1956) statistics allow one to estimate m_o accurately using the manual technique. The results for the Baylor suggest some distortion of the wave profile, although the discrepancy is not great.

Tz - Figure 5.3.2

The graphs show good agreement between manual and spectral results. These results disagree with those of GODA (1974) who suggests that Rice's theory requires a correction of +20% (ie, the mean zero crossing period measured from wave records is greater than that estimated from the spectrum).

Tc - Figure 5.3.3

These results show a good deal of scatter as might be expected from what has already been said in Sections 5.5.2 and 5.5.3. The fitted lines have in effect been constrained to pass through the origin and the centroid of the data, and have slopes not far removed from unity. It is clear from these that with shorter period waves the estimates from the manual analysis are less than those from the spectrum, while the reverse is true of the longer period waves.

EPS - Figure 5.3.4

The results for the laser and the Waverider show fair agreement, although the correlation is not good, the results for the Baylor show substantial differences.

Tz : TB - Figure 5.3.5

The graphs show excellent correlation between these quantities for all three instruments. For a Pierson-Moskowitz spectrum $T_z/T_B = \Gamma(\frac{3}{4})/\pi^{\frac{1}{4}} = 0.920$. This is close to the average value of the results of the three instruments of 0.892.

1/FO : TB - Figures 5.3.6 and 5.3.7

These graphs show rather a lot of scatter. However if the results for the line through the origin (Fig 5.3.5) are taken, the mean of the slopes is 1.311. This is close to the value for a Pierson-Moskowitz spectrum, which is given by

$$\frac{1}{f_o T_B} = \frac{\Gamma(\frac{3}{4})}{(\frac{4}{5})^{\frac{1}{4}}} = 1.296$$

1/EPS : Qp - Figure 5.3.8

For a narrow band spectrum the spectral width parameter and the spectral peakedness parameter have the approximately inverse relation

$$\frac{1}{\epsilon} \simeq \sqrt{\frac{\pi}{6}}\, Q_p = 0.72\, Q_p$$

(see EWING (1973)). The main feature of Figure 5.3.7 is the very small variation of $^1/\epsilon$ with Qp. Qp appears to be a more reliable parameter than ϵ for describing the width of spectra as also noted by RYE (1977).

5.6 Conclusions

The three instruments, while producing broadly comparable results, showed some important differences in several aspects of their performance. These may be categorized under the headings:

Scaling
Frequency response
Linearity

5.6.1 Scaling

The calibrations for both the Waverider and the Baylor wave staff were carried out after the project, and in the case of the Baylor wave staff, after an interval of some months.

The departures from the nominal sensitivities found in both these instruments were rather large: approximately -9% in the case of the Waverider, and approximately +17% in the case of the Baylor. The laser system did not show any significant departure from the nominal calibration when tested statically.

In view of these results our first conclusion must be to emphasise the need for careful and regular calibration of wave recording instruments.

The calibrations were used in the subsequent calculations, and the comparisons then showed good agreement between the Baylor and the laser, but an apparent 12% error in the Waverider. We cannot explain this result except in terms of systematic differences in the wavefields at the platform and the Waverider site, but as has been discussed previously such a difference seems unlikely.

5.6.2 Frequency response

The most striking aspect of the relative frequency response was the marked fall-off in energy measured by the laser with increasing frequency. In view of this the scaling of the instrument must be specified at a particular frequency. No discrepancy between the nominal and actual scaling was revealed by static tests, but the (unspecified) falling frequency response meant that the laser could not be adopted as a standard with which the other instruments were compared. By contrast the relative response of the Baylor and Waverider appears to vary little with frequency.

5.6.3 Linearity

The calibration of the Baylor and the laser revealed no appreciable departure from a linear response. The Waverider was calibrated at one amplitude only, over a range of frequencies and this gave a check on the linearity of some parts of the instrument.

Suspending the staff at an angle to the vertical introduced non-linearities as has been discussed. It is recognised of course that practical considerations may make it difficult to deploy the staff both vertically and in a well-exposed position.

5.6.4 Relationships between parameters

For many practical purposes the height parameters of a wave field may be inferred from just an accurate estimate of the standard deviation of the wave profile, and this in turn requires only that the instrument scaling is accurately known near to the peak frequency of the wave spectrum and that the amplitude response is reasonably linear. However, the accurate determination of period and bandwidth parameters depends on the overall frequency response and linearity of the measurement and recording system. In view of this it is perhaps worth stressing that the wave information used for both the manual and spectral analyses were derived from the same paper charts, so that in the manual:spectral comparisons the effectsof recording system responses (including that of the pen recorder) have been eliminated.

6. PART II - MEASUREMENTS OF WAVES FROM A HELICOPTER USING A LASER ALTIMETER

6.1 Introduction

Remote measurements of waves using radar and laser systems in aircraft have usually been difficult to interpret because of the unknown directional characteristics of the waves which are required in the accurate transformation of spectra measured by airborne systems to a fixed frame of reference. These difficulties can be overcome if the measurements are made from a helicopter which is able to maintain a stationary position in the hover mode.

It was decided to test these ideas in collaboration with A & AEE by mounting a laser system in a Sea King helicopter and by making comparisons with Waverider measurements taken near Eddystone light tower.

6.2 Instrumentation

The Geodolite laser altimeter and Datawell Waverider buoy have been described in Part I of this report. IOS provided a system to measure the motions of the helicopter using the transducers available in a pitch-roll buoy. The motions measurements were therefore of pitch, roll and vertical acceleration using a gyro mounted system.

The helicopter motions were recorded on magnetic tape. The laser wave measurements were recorded on an analogue recorder supplied by A & AEE together with event markers to provide synchronisation of the two sets of measurements. The Waverider signals were transmitted and recorded at Fort Bovisand, Plymouth, on a chart recorder.

6.3 Analysis

The laser traces were digitized at 1 sec intervals. Measurements of the motions of the helicopter were sampled at 0.1 sec and subsequently used at 0.5 sec intervals in the spectral calculations. The spectra were computed by taking the Fast Fourier Transform of the total record of 700 sec duration and forming spectral estimates at a frequency resolution of 0.01 Hz from the average over 7 adjacent harmonics. This process yields spectral estimates with 14 degrees of freedom.

The Waverider traces were digitized at 1 sec intervals over the 30 min recording period and the spectrum estimated using the ensemble averaging method described in Part I to give values at 0.01 Hz interval with 36 degrees of freedom.

Let H denote the vertical displacement of the helicopter from its mean position, ℓ the distance of the sea surface as measured by the laser and h the wave elevation then, at any time t

$$h(t) = \ell(t) - H(t). \tag{6.3.1}$$

It can be shown that the spectra of h, ℓ and H are related by

$$S_h(f) = S_\ell(f) + S_H(f) - 2\,Co_{\ell H}(f) \tag{6.3.2}$$

where $Co_{\ell H}$ is the co-spectrum of ℓ and H and f is the wave frequency. Equation (6.3.2) was used to estimate the wave spectrum. (The influence of the time difference between laser and motions recording systems was taken into account when using equation (6.3.2)). The spectrum of the vertical motion of the helicopter was obtained from the spectrum of vertical acceleration using

$$S_h(f) = \frac{S_a(f)}{(2\pi f)^4}\ . \tag{6.3.3}$$

6.4 Discussion of results

Two sorties were made to Eddystone tower on 4 December 1974 and 22 January 1975. Table 6.4 gives details of the measurements and results for the analysis of the significant values of the waves and heave motion obtained from the spectra.

Table 6.4

Record number	Date and time of measurements	Wind speed and direction	Significant values (m)			
			Wave-rider	laser (uncorrected)	laser (corrected)	heave
1	4 Dec 1974;1230 h.	20-25kts;290°	1.4	1.7	1.4	0.7
2	22 Jan 1975;1600 h.	25-35kts;245°	3.3	4.2	-	1.6

The spectral measurements made during Record 1 are shown in Fig 6.4.1. There is satisfactory agreement between the Waverider spectra and laser measurements, (after allowing for helicopter motions) for frequencies greater than 0.1 Hz. The significant wave heights from the two systems are in close agreement for frequencies greater than 0.1 Hz.

It was not possible to carry out the full correction procedure of equation (6.3.2) for Record 2 since the event marks for synchronising the two systems did not register on the laser trace. However the heaving motion was small compared to the laser signal over most of the important frequency range so that a comparison can be made between the laser and Waverider spectra as shown in Fig 6.4.2. As in the case of Record 1 there is reasonable agreement at frequencies above about 0.1 Hz but below this frequency the laser measurements show a sharp peak in energy which is not in either the Waverider or the motion spectra. The maximum pitch or roll measured during Record 2 was about $\pm 3.5^{\circ}$ which is not large enough to cause differences of the magnitude observed in the laser measurements. Other laser records taken with the helicopter moving at 60 knots were badly contaminated by noise. D B Ross (private communication) believes a longer time constant of about 20 milliseconds should have been used during the experiment to reduce the noise level and that the helicopter should have flown at 100 ft or higher to better suit the focal length of the laser optical system. However it is difficult to see how contamination by noise can produce the sharp peak observed in the laser spectrum for Record 2.

6.5 Conclusions

It has not been possible to obtain satisfactory agreement between laser measurements from a helicopter and Waverider measurements except for a limited high frequency region of the wave spectrum. The reasons for the differences at low frequencies are not known but they could have been caused by the use of too short a time constant in the laser system resulting in excessive noise or by flying the helicopter at a level below the minimum focal length of the optical system.

The use of a helicopter as a platform for making remote wave measurements at a fixed point is encouraging since it was possible to maintain station during strong winds with significant vertical motions of less than 2m.

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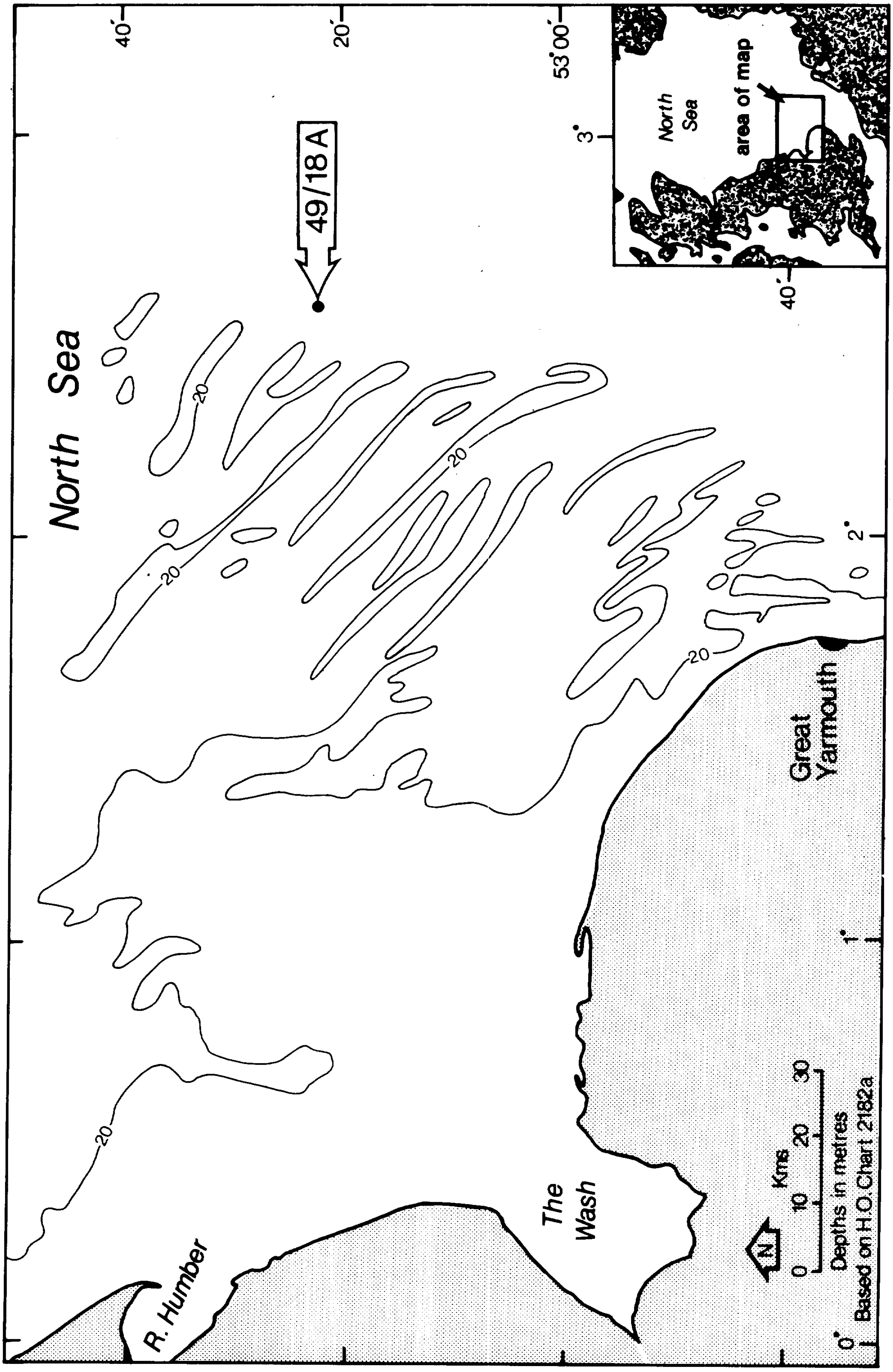
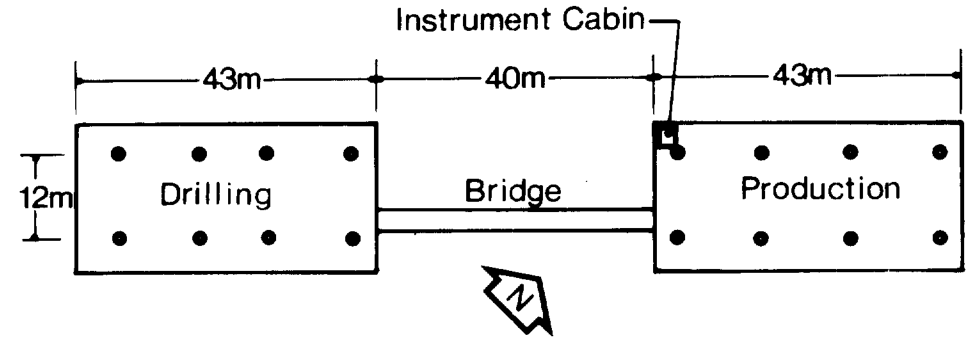
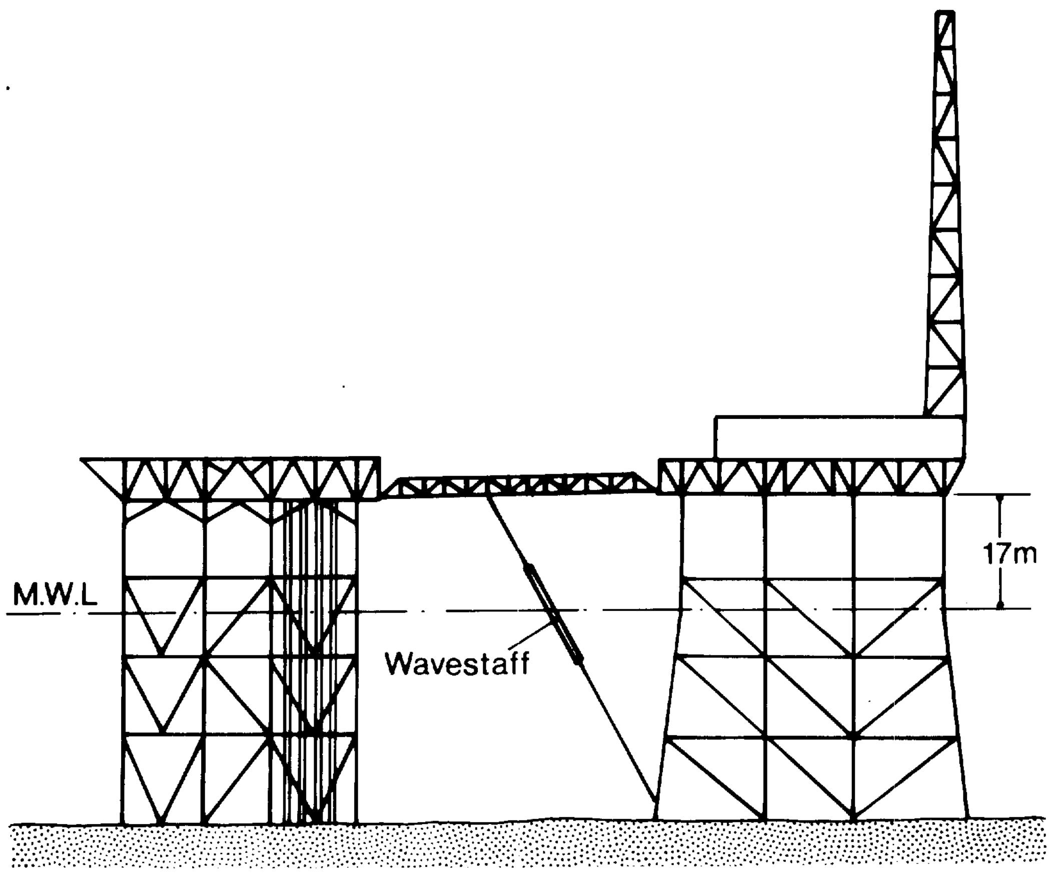
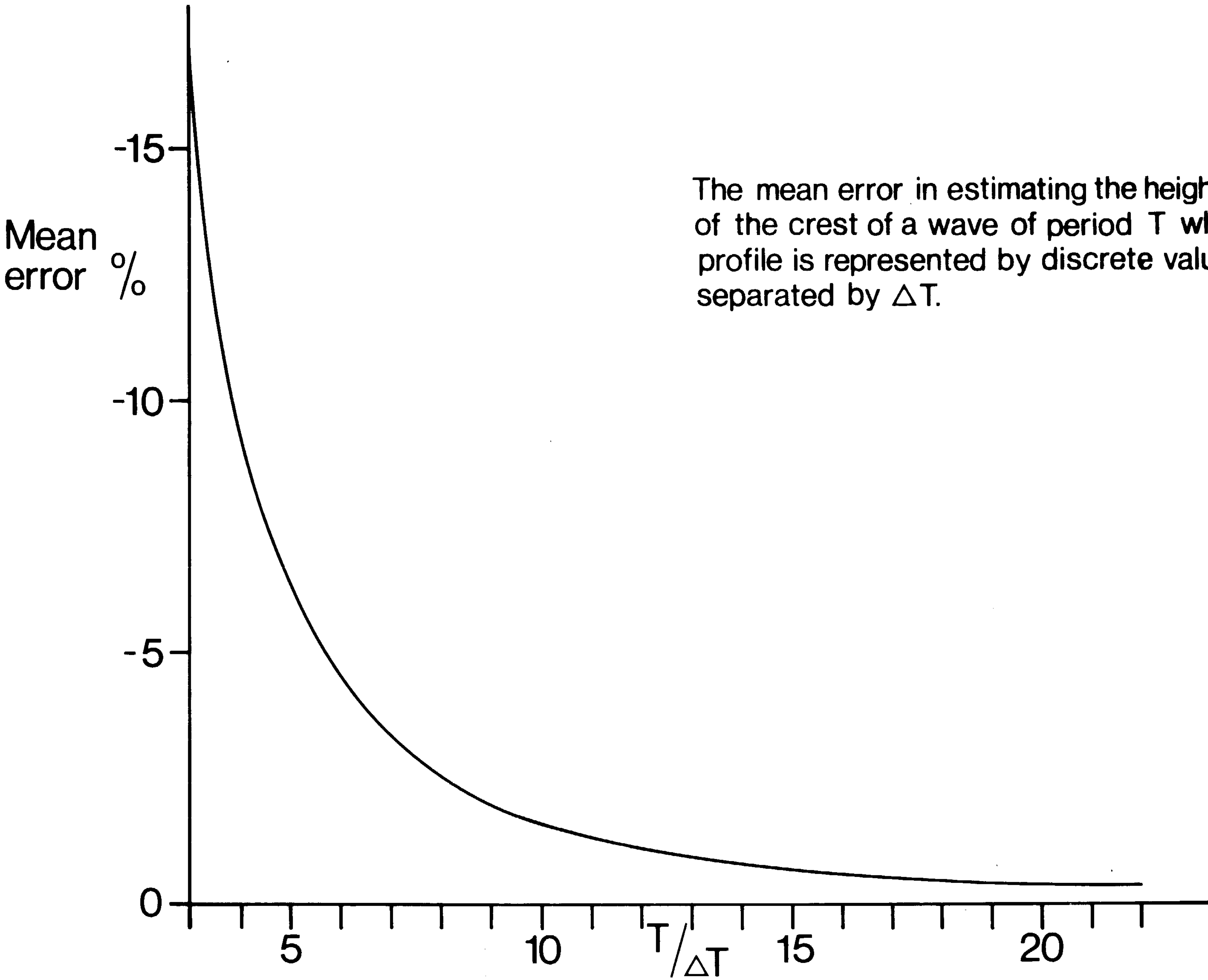

North Sea
49/18 A
20
Great Yarmouth
The Wash
R. Humber
North Sea
area of map
Kms
0 10 20 30
Depths in metres
Based on H.O.Chart 2182a
40'
20'
53°00'
3°
2°
1°
0°
40'

3-2-1

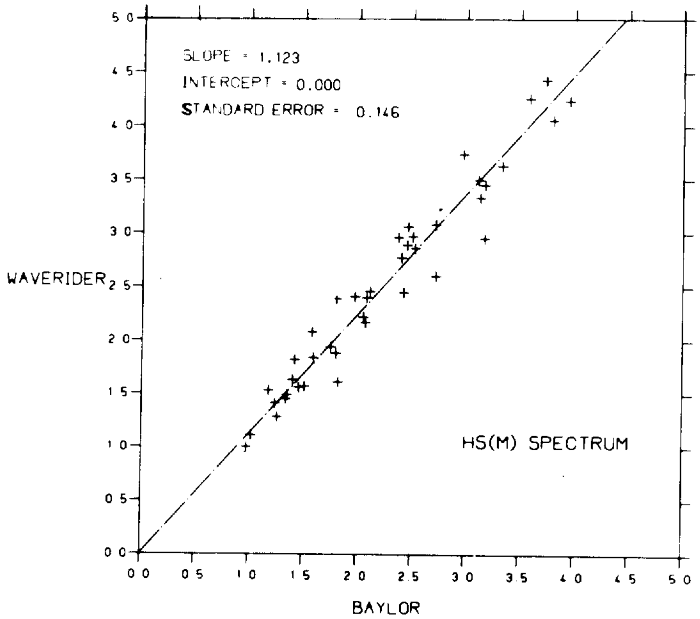




AMOCO Platform 49/18 A



The mean error in estimating the height of the crest of a wave of period T whose profile is represented by discrete values separated by ΔT.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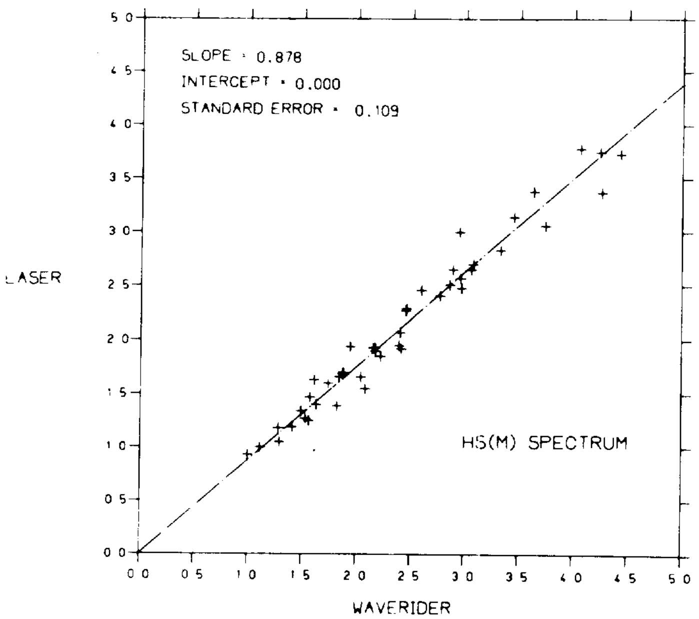
BAYLOR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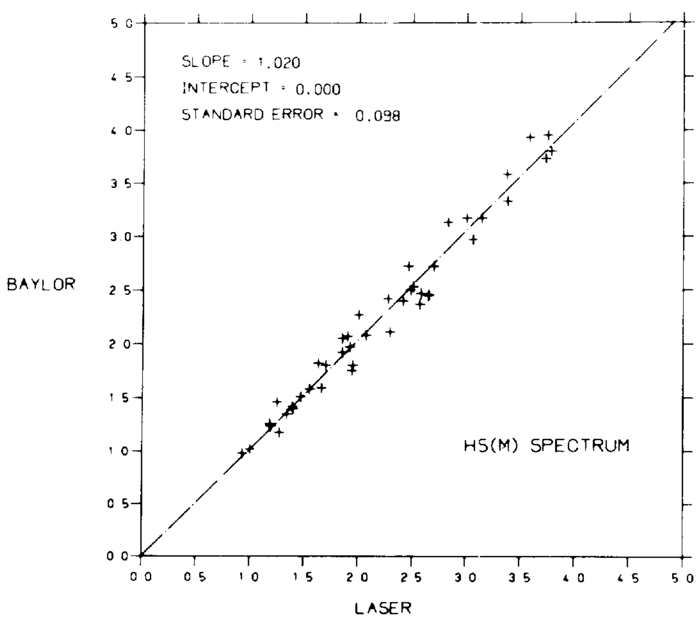
WAVERIDER

5-2-1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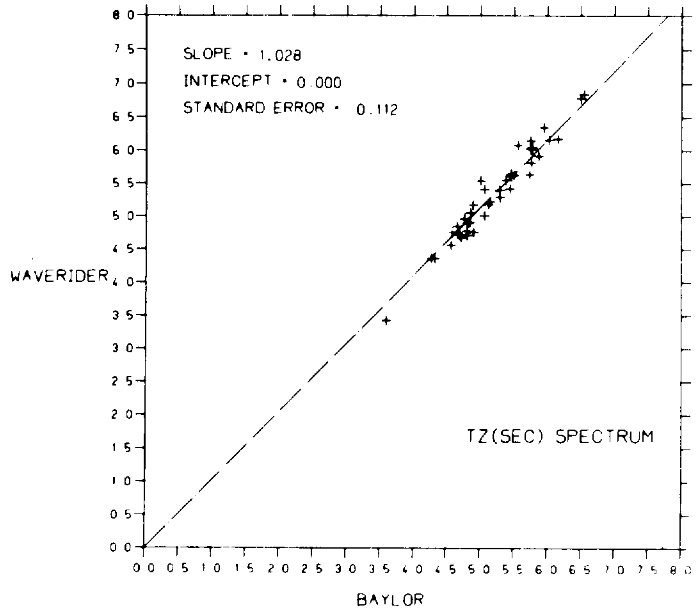
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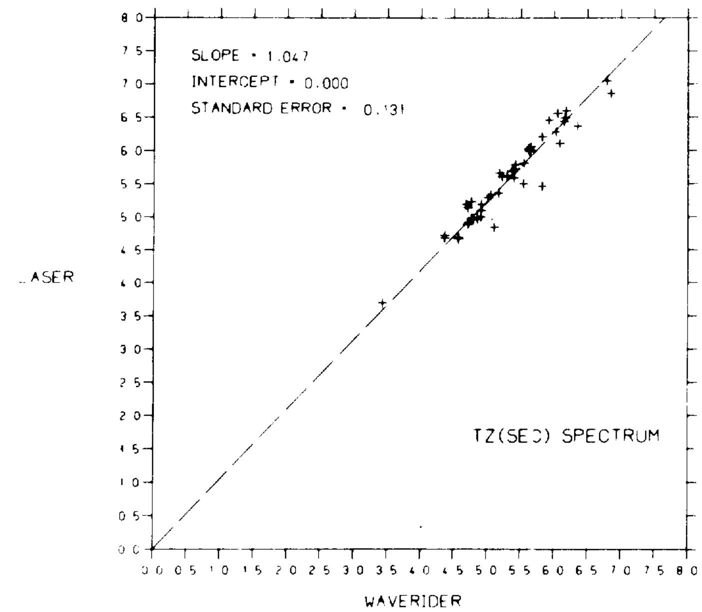
WAVERIDER
BAYLOR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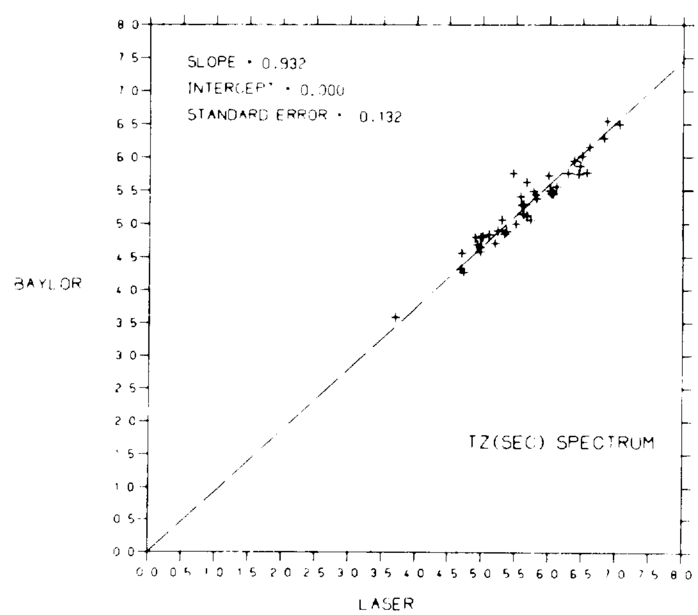
WAVERIDER

5-2-2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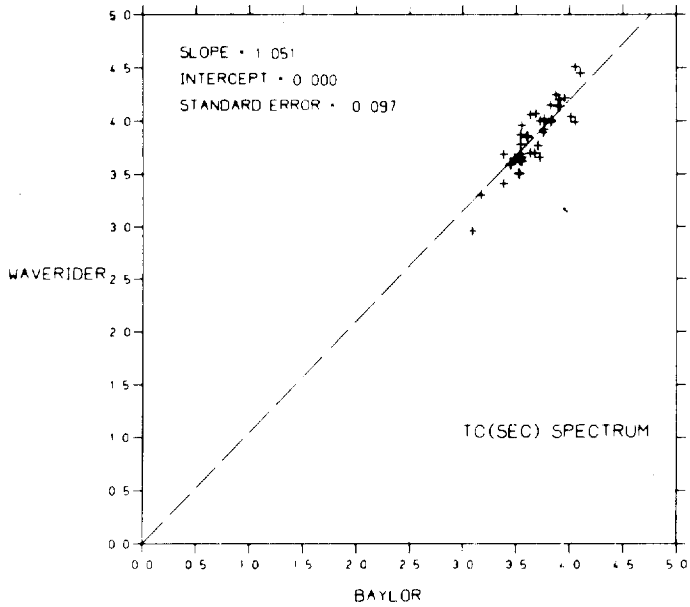
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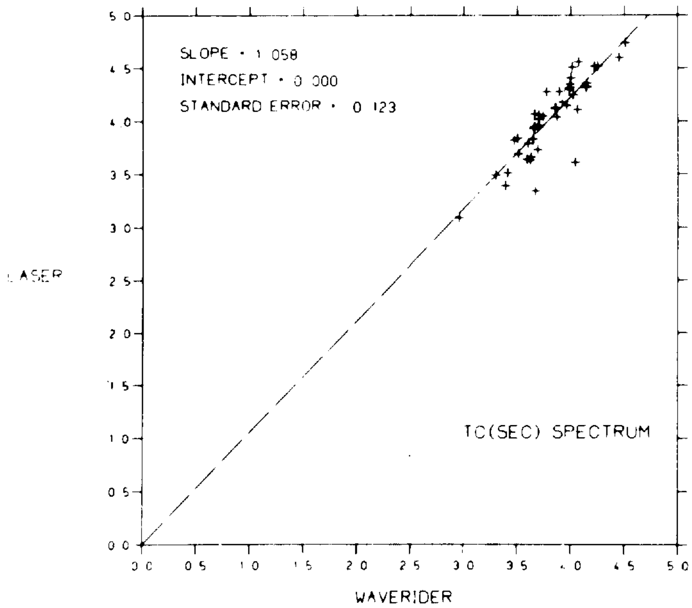
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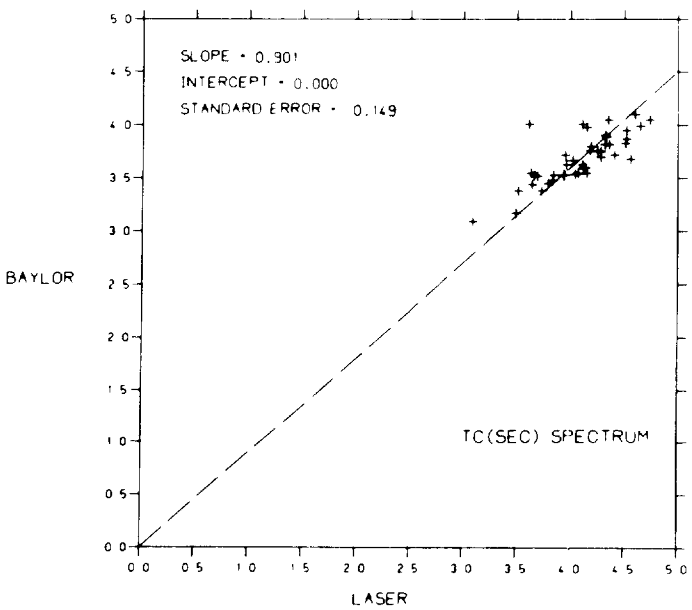
5-2-3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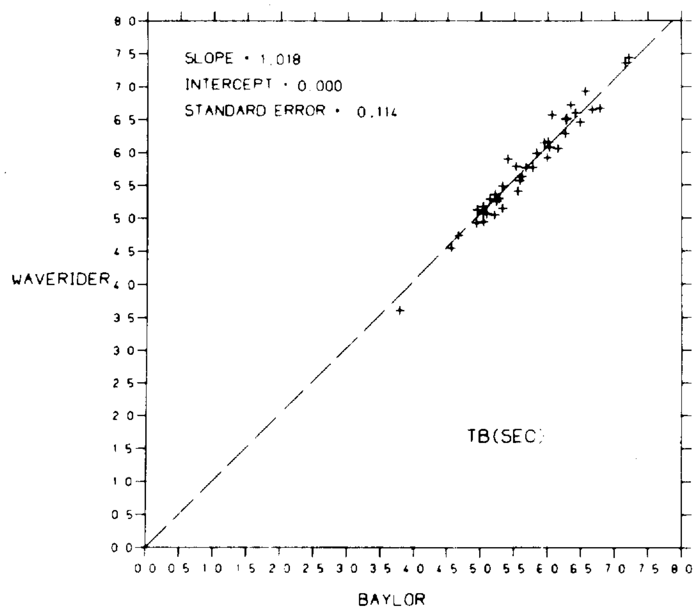
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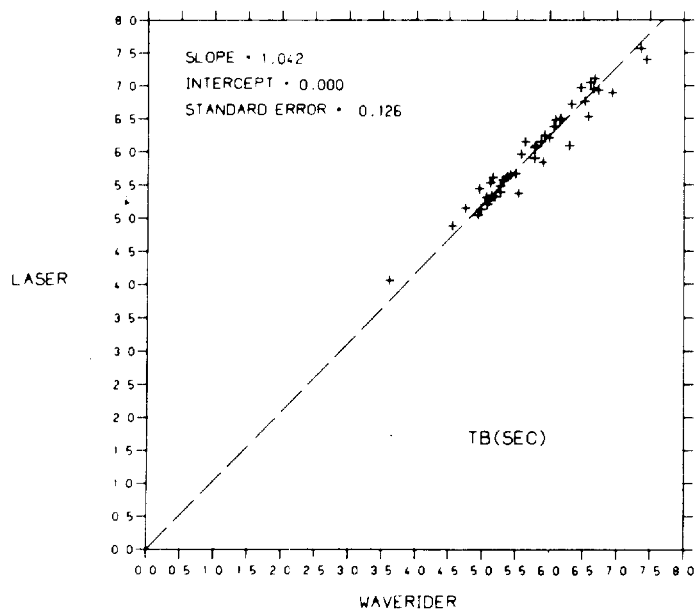
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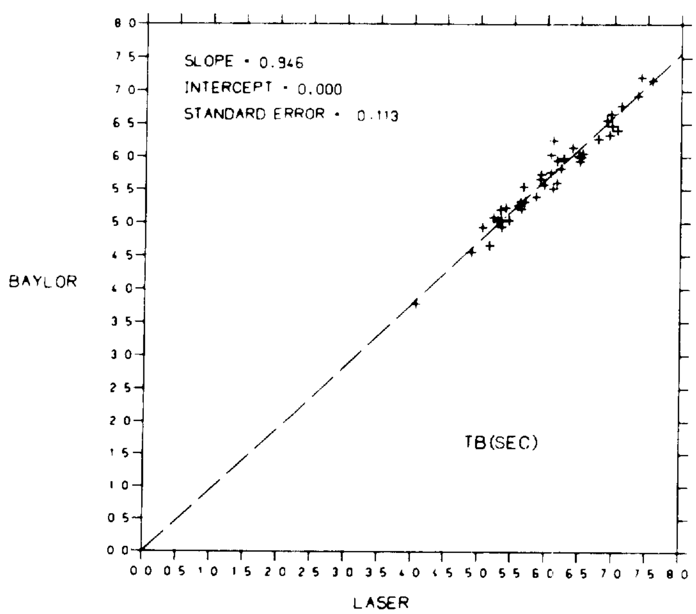
5-2-4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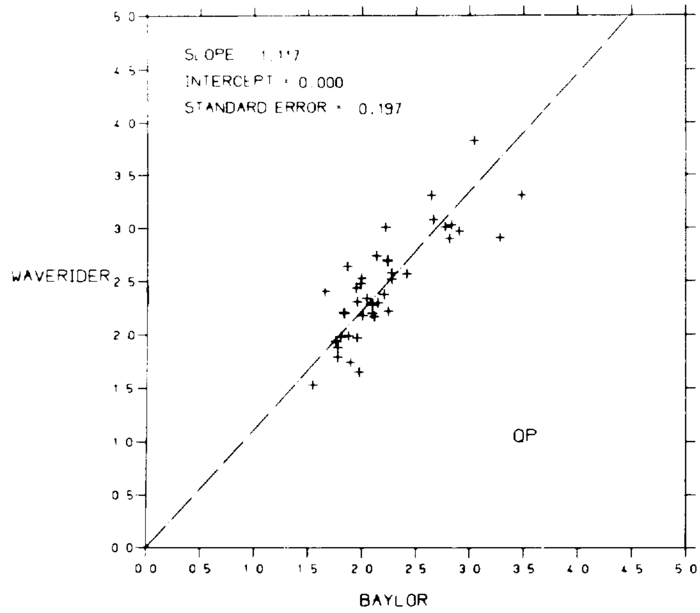

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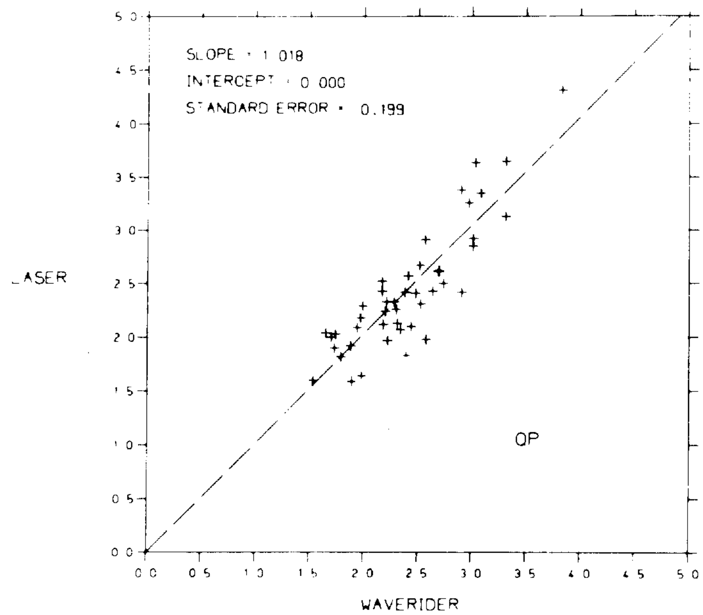

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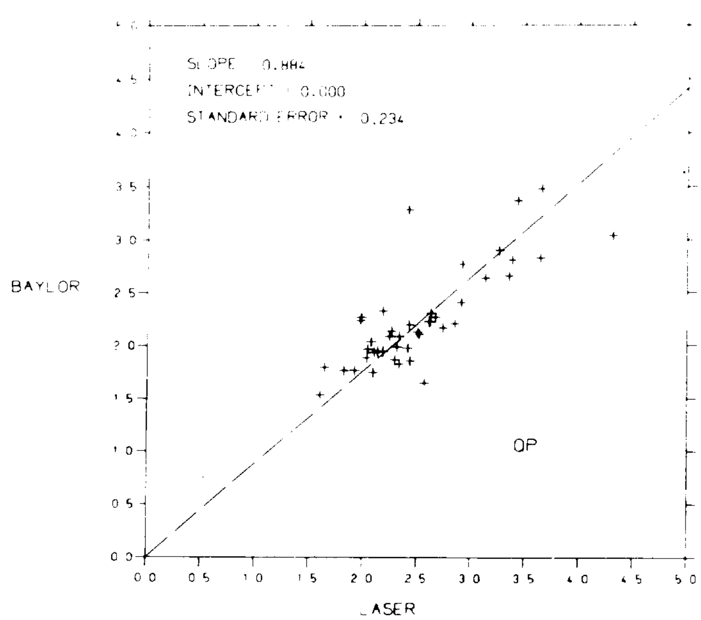
5-2-5



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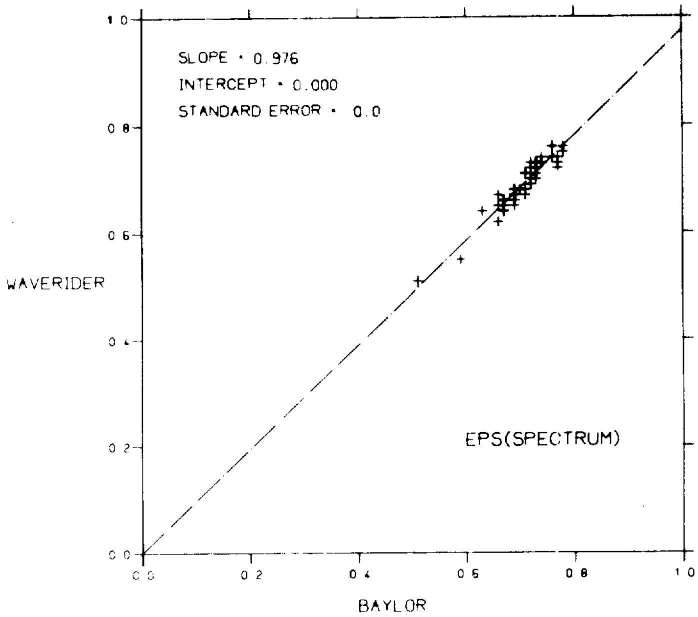
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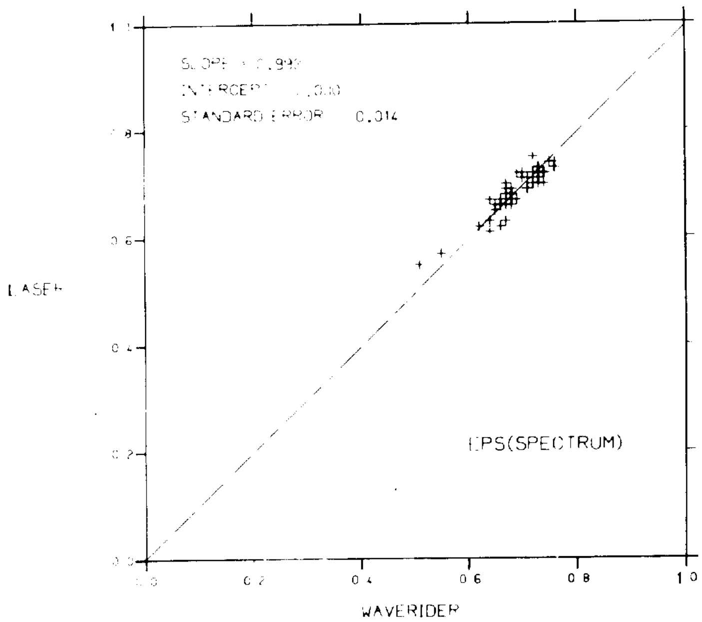
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INTERCEPT
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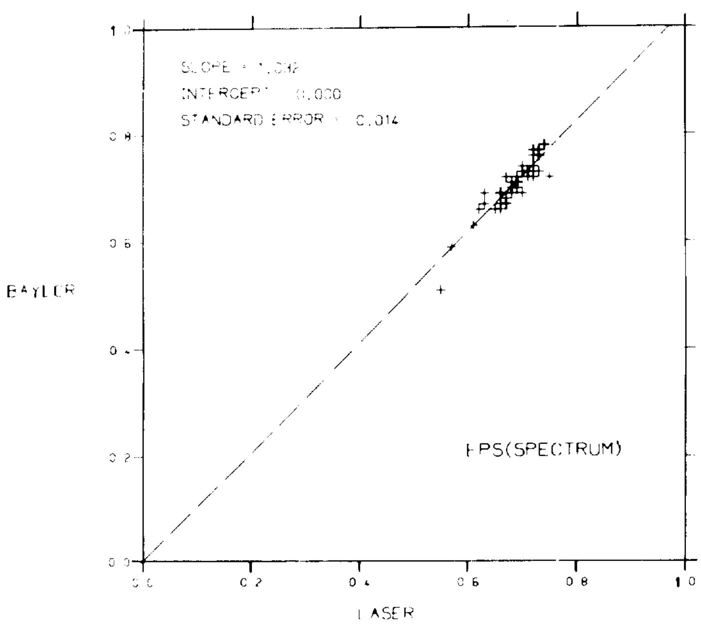
5-2-6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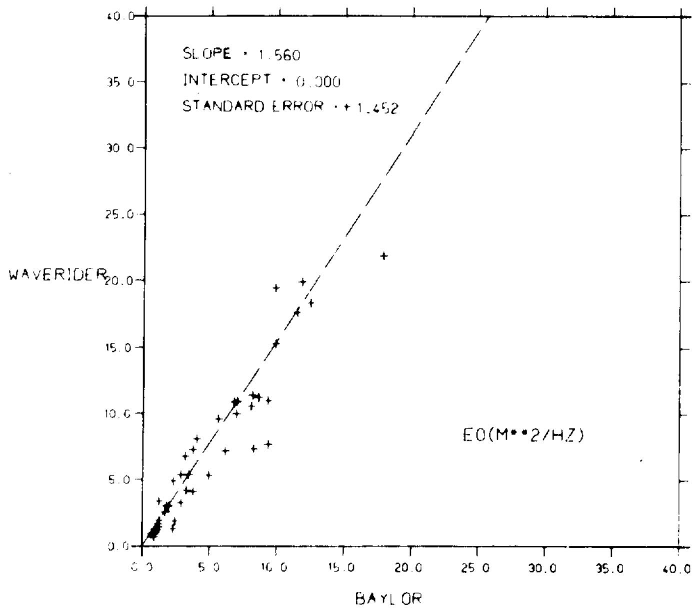
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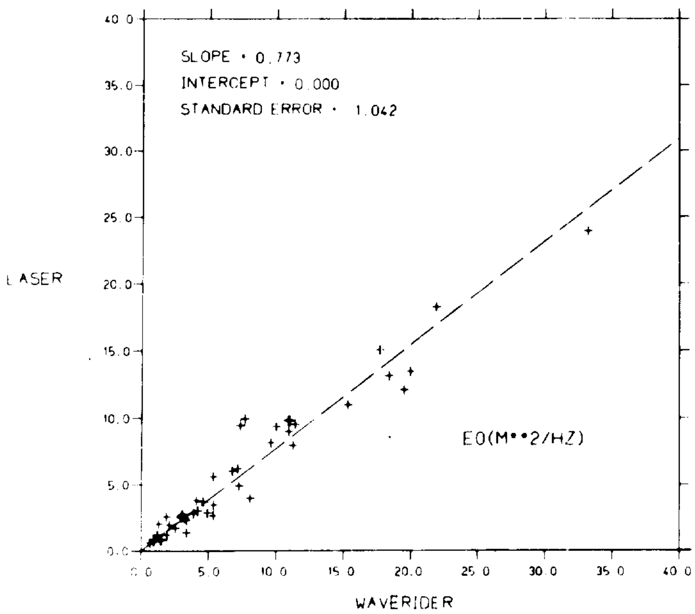
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WAVERIDER
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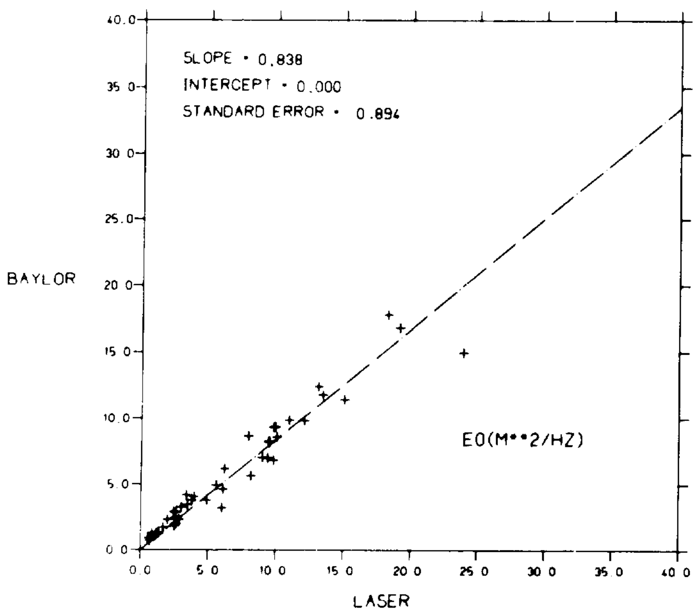
5-2-7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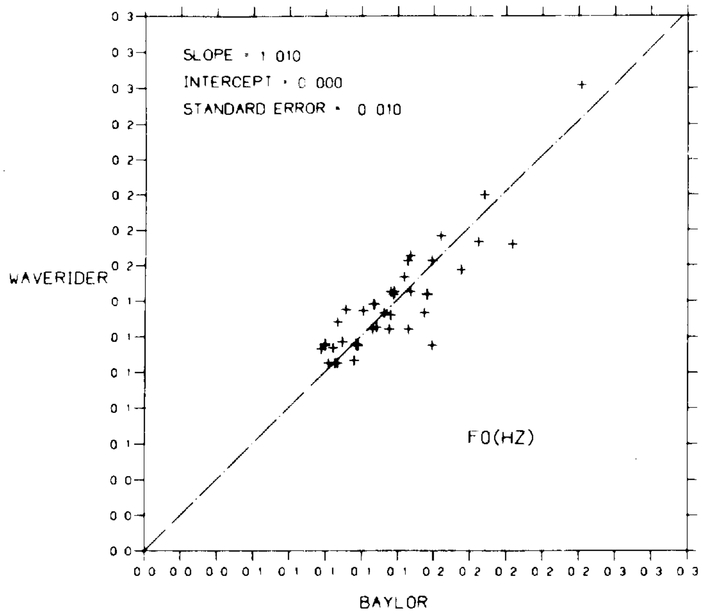
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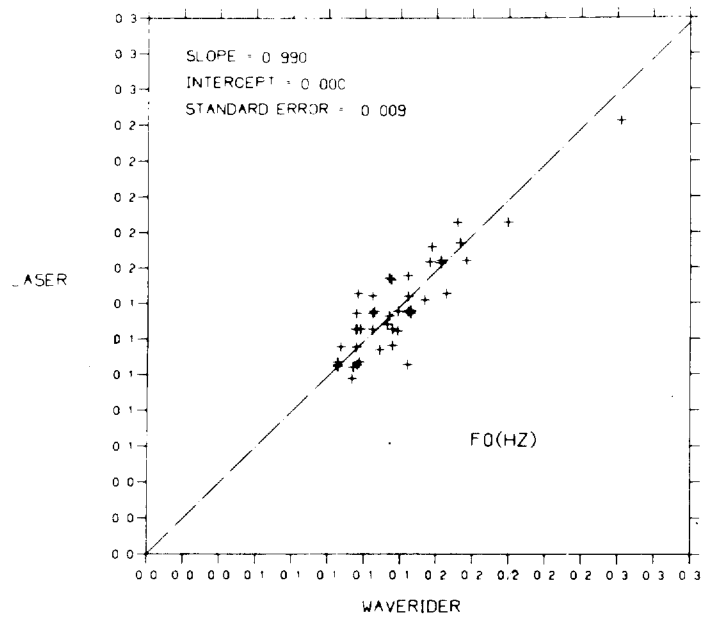
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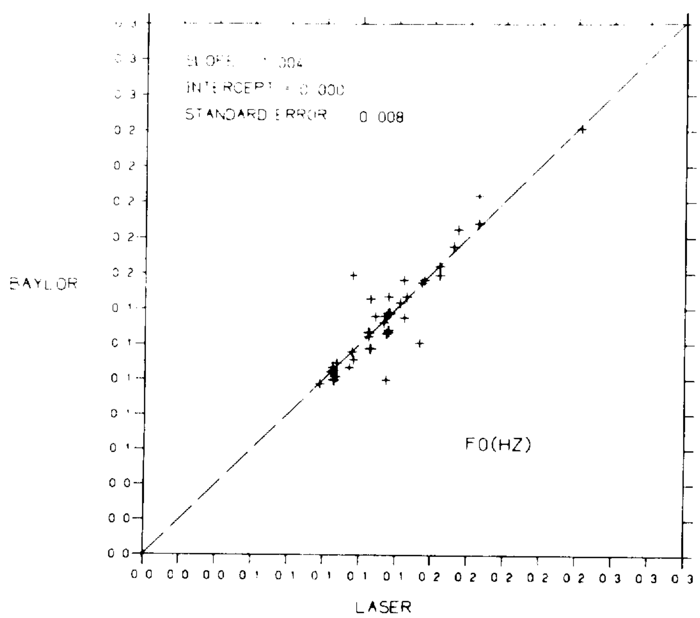
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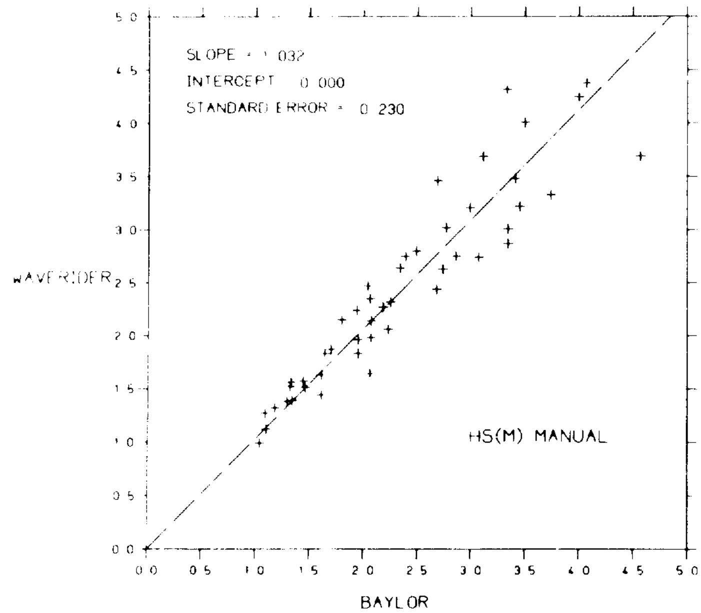
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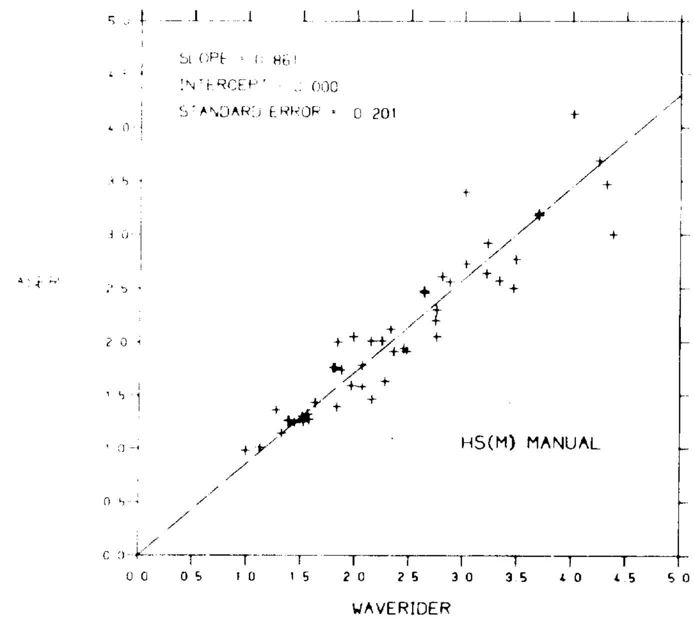
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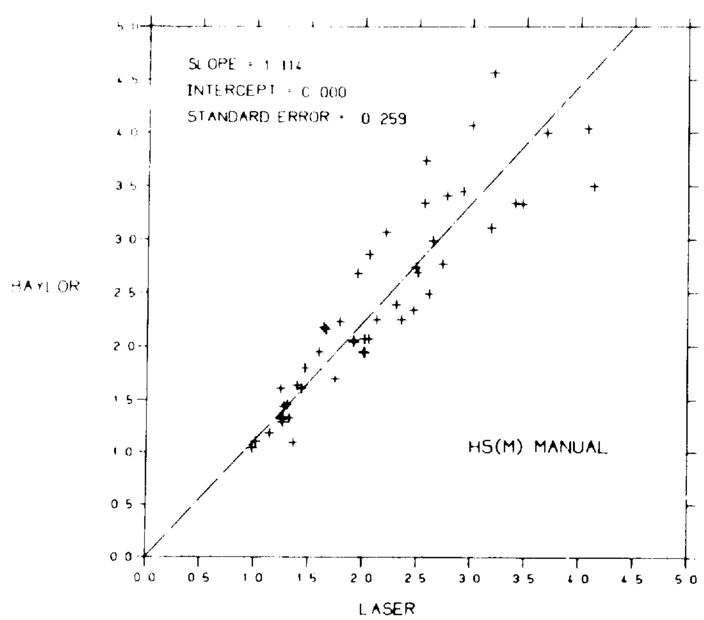
5-2-9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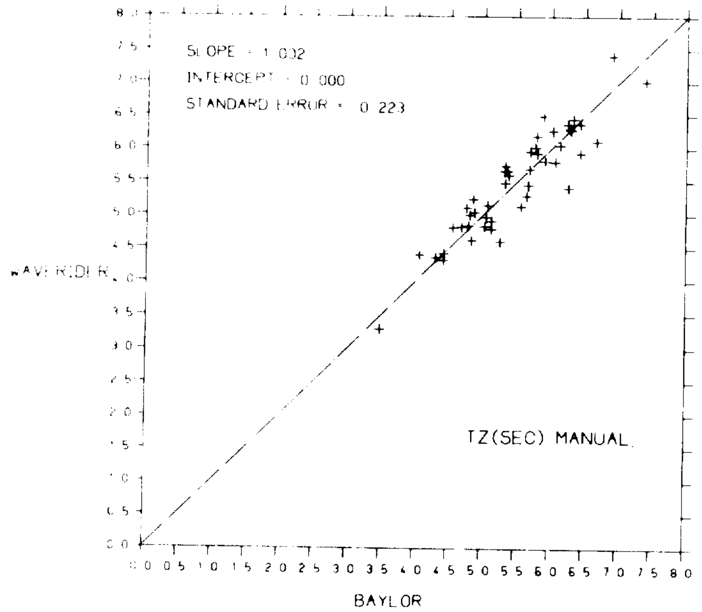
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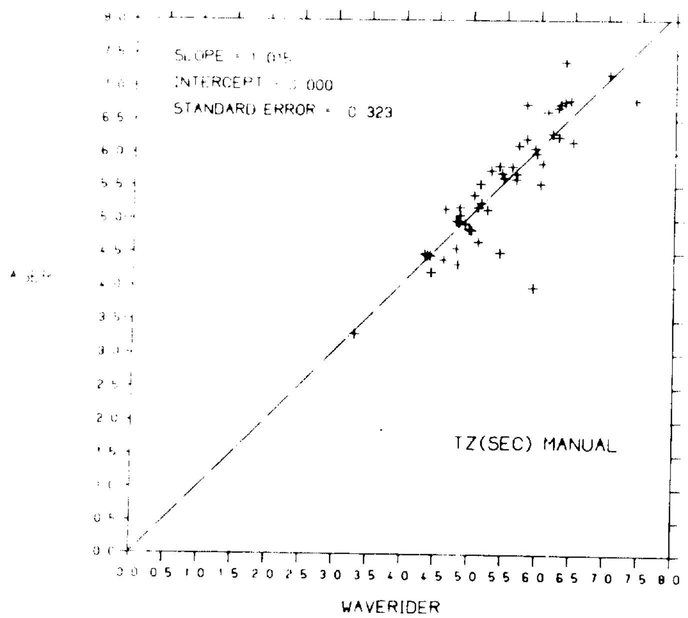
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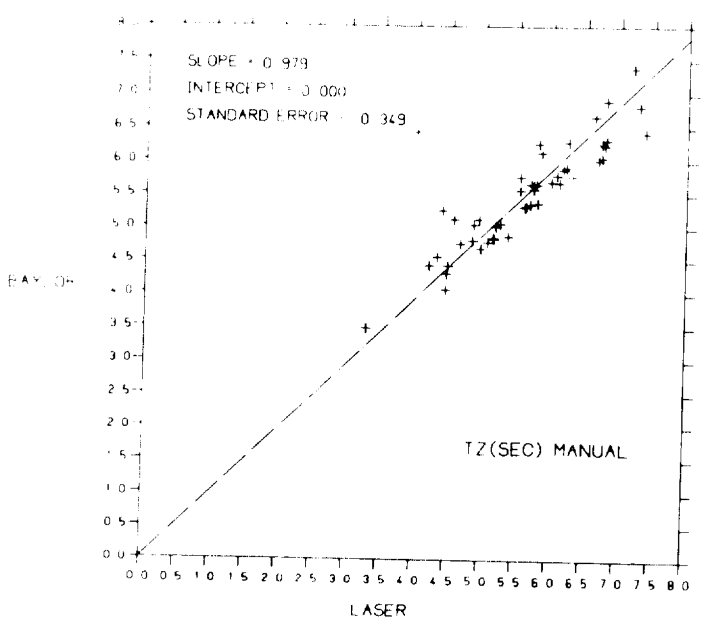
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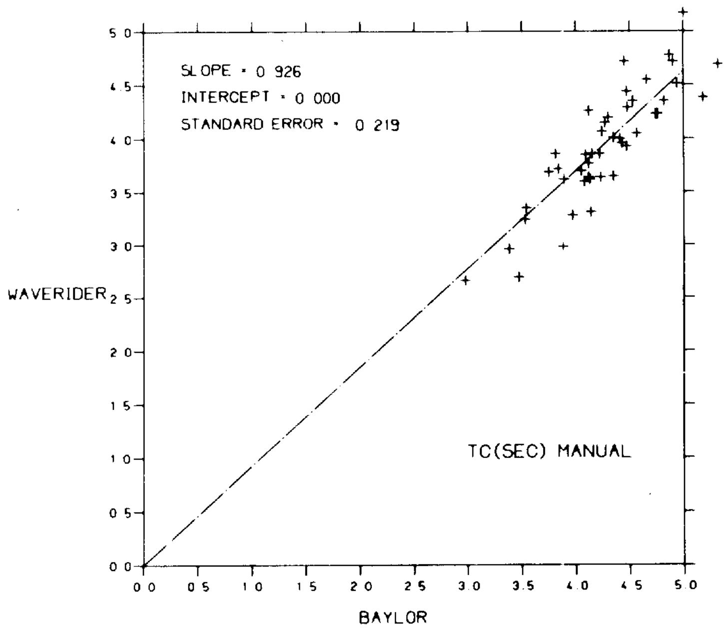
5-2-10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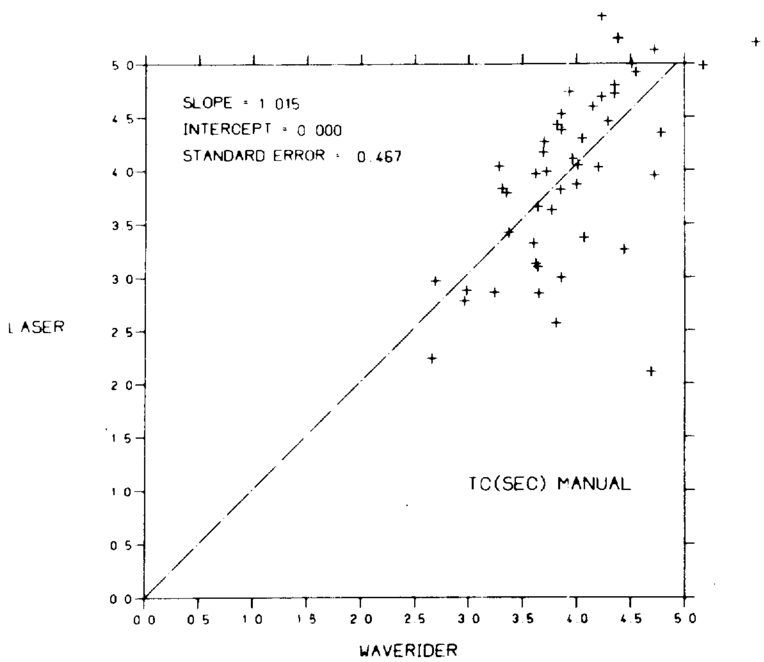
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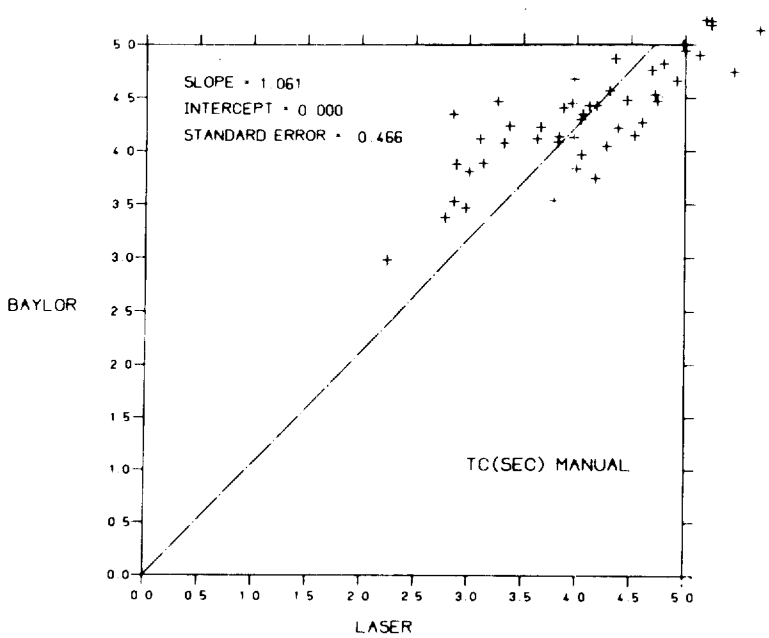
5-2-11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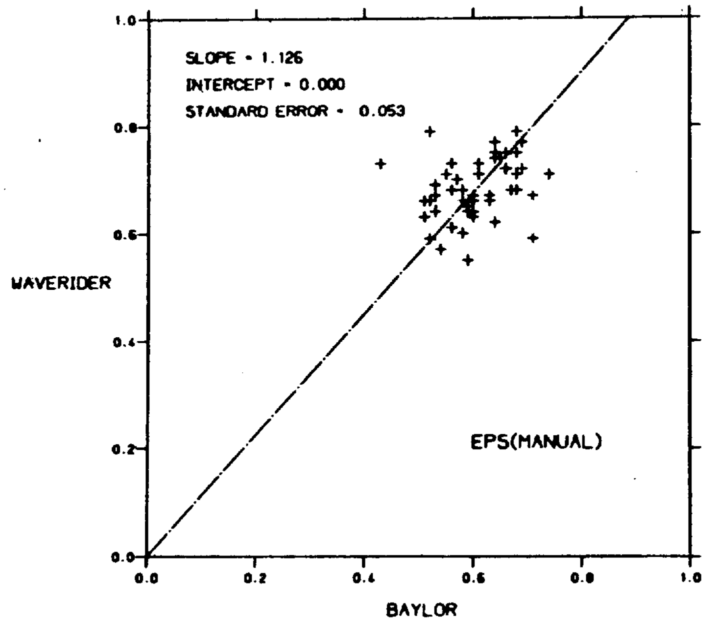
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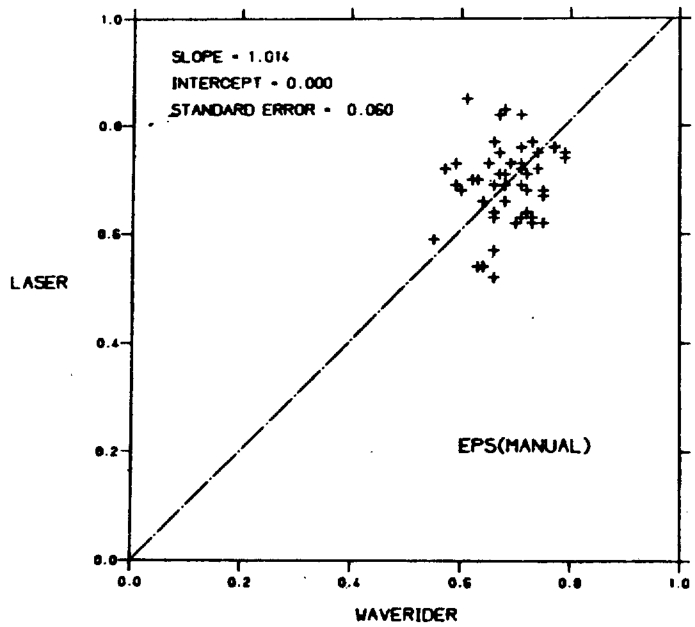
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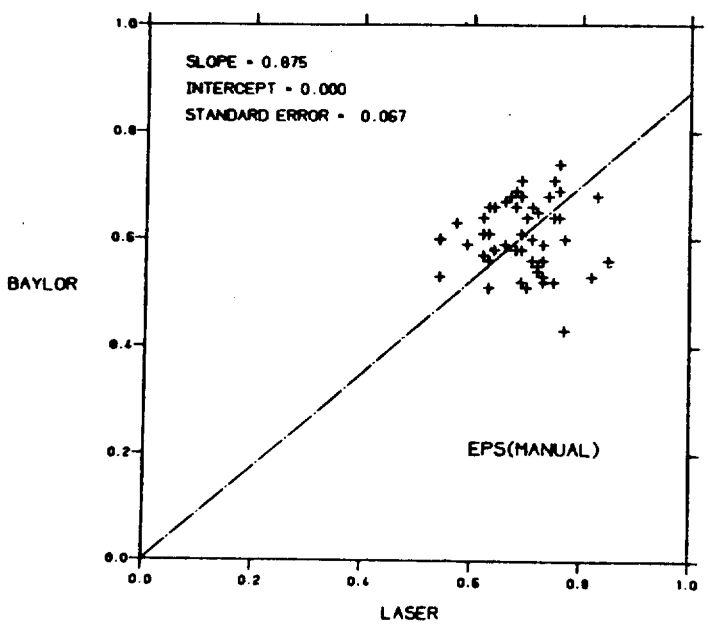
5-2-12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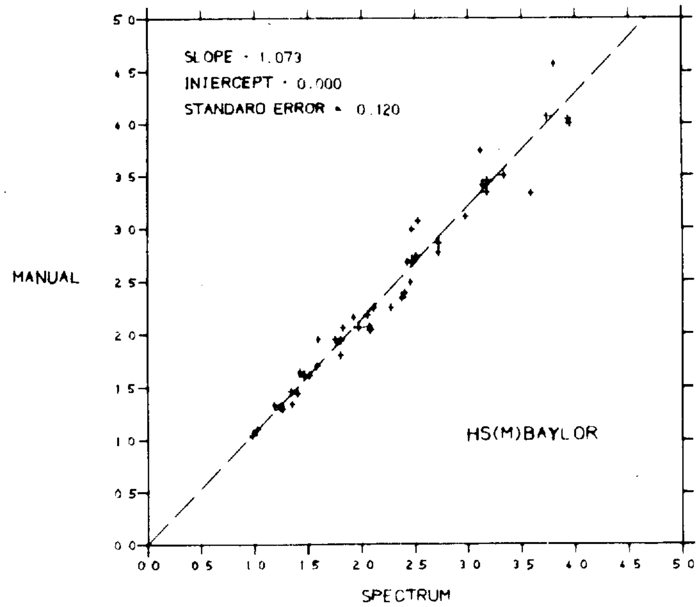
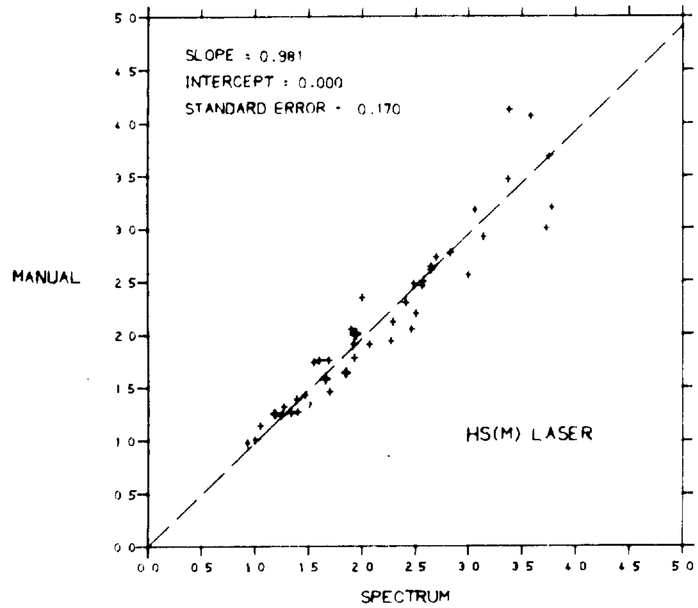
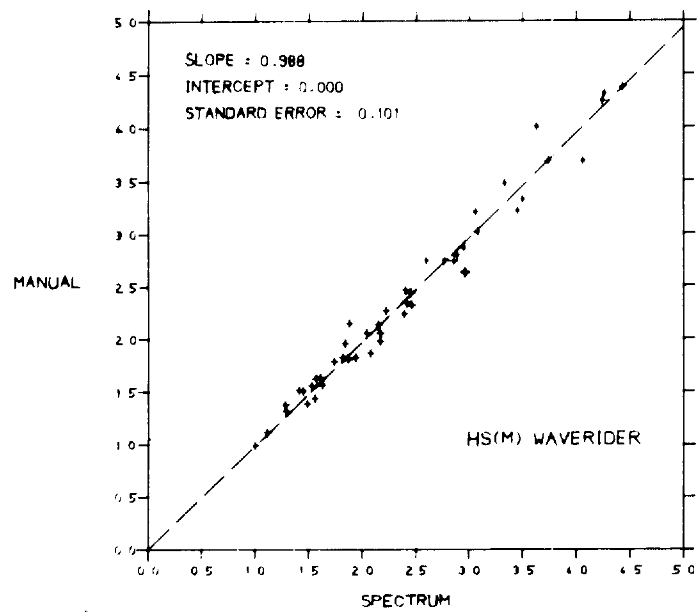
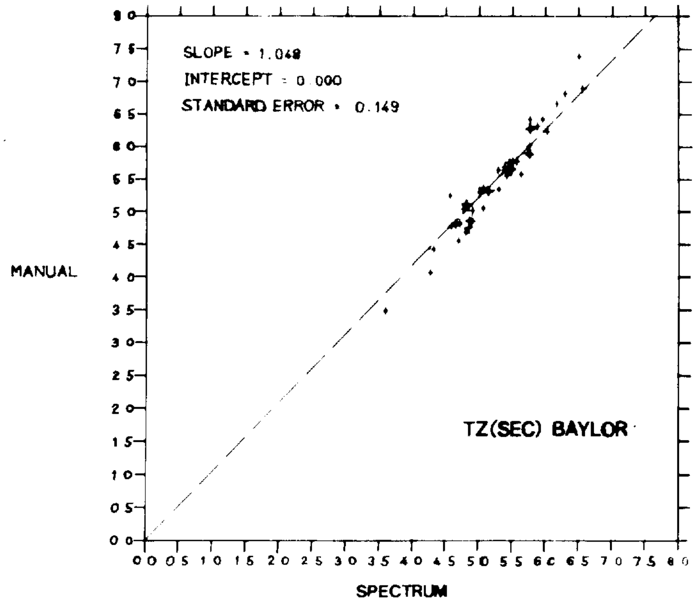
0.8
1.0


SLOPE · 1.073
INTERCEPT · 0.000
STANDARD ERROR · 0.120
HS(M)BAYLOR
MANUAL
SPECTRUM

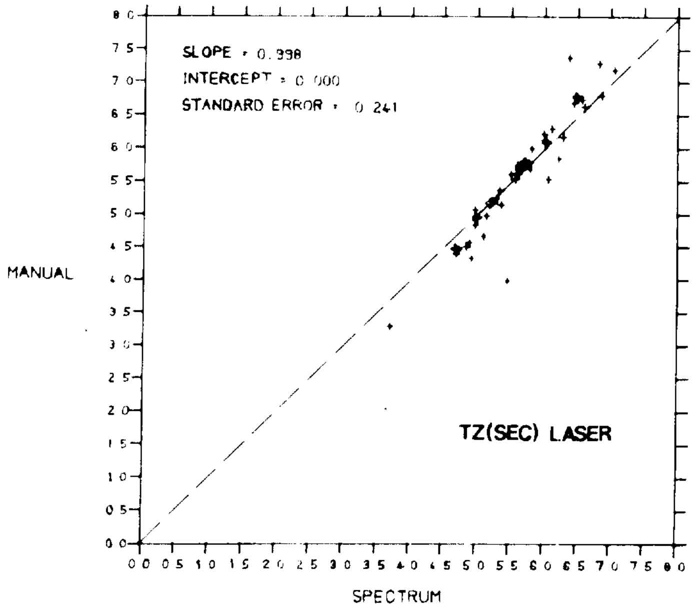

SLOPE : 0.981
INTERCEPT : 0.000
STANDARD ERROR · 0.170
HS(M) LASER
MANUAL
SPECTRUM

5-3-1


SLOPE : 0.988
INTERCEPT : 0.000
STANDARD ERROR : 0.101
HS(M) WAVERIDER
MANUAL
SPECTRUM

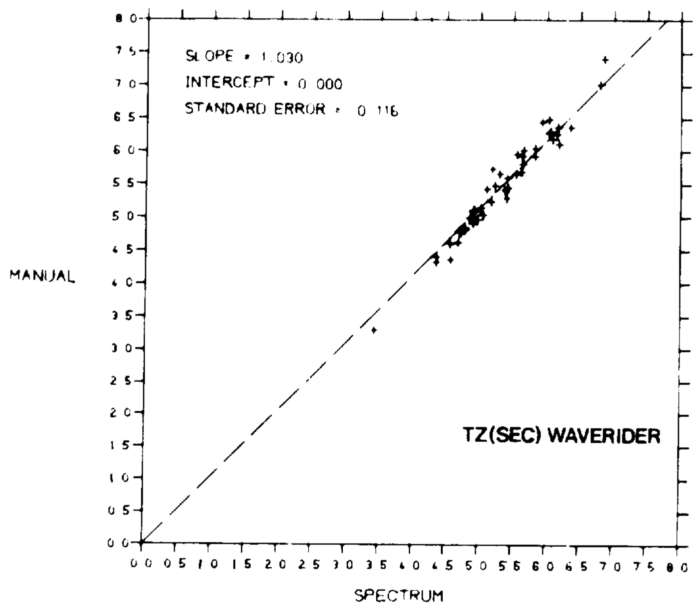


SLOPE = 1.048
INTERCEPT = 0.000
STANDARD ERROR = 0.149
TZ(SEC) BAYLOR
MANUAL
SPECTRUM

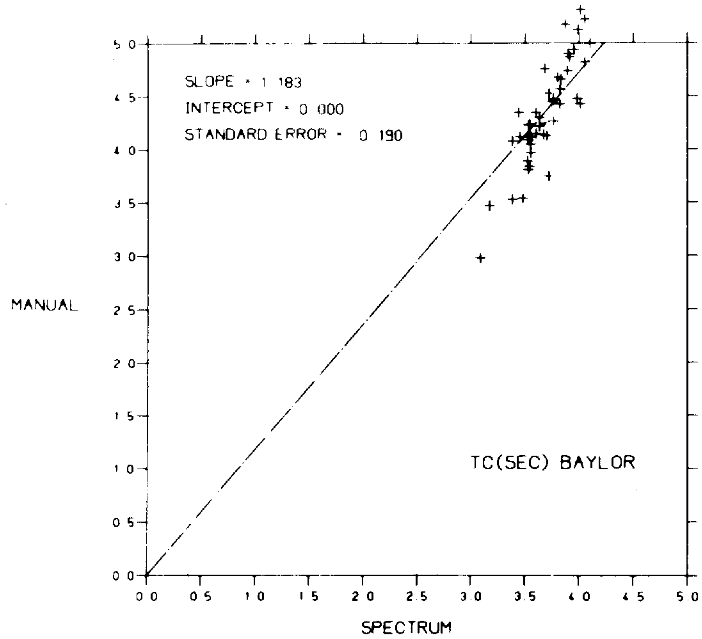


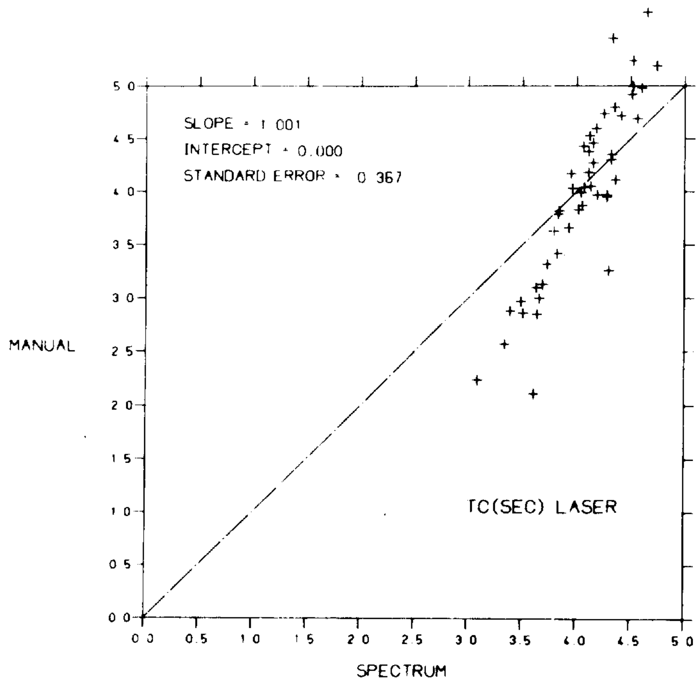
SLOPE = 0.998
INTERCEPT = 0.000
STANDARD ERROR = 0.241
TZ(SEC) LASER
MANUAL
SPECTRUM

5-3-2

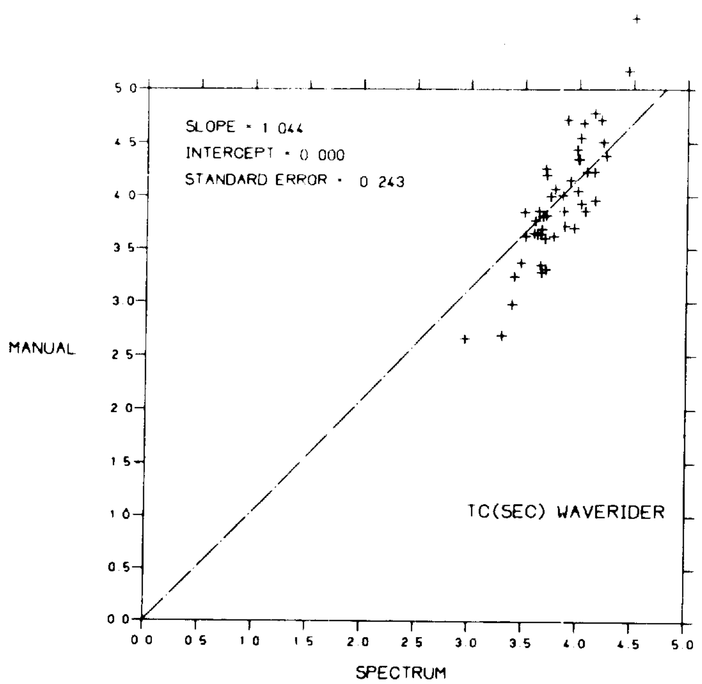


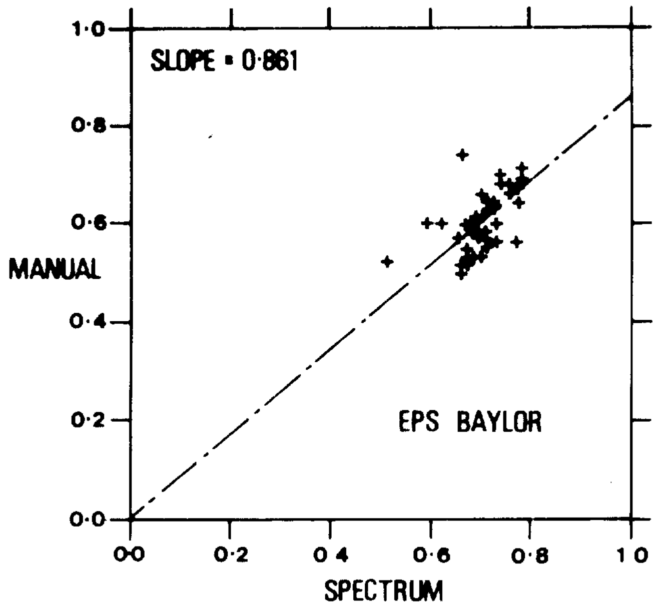
SLOPE = 1.030
INTERCEPT = 0.000
STANDARD ERROR = 0.116
TZ(SEC) WAVERIDER
MANUAL
SPECTRUM

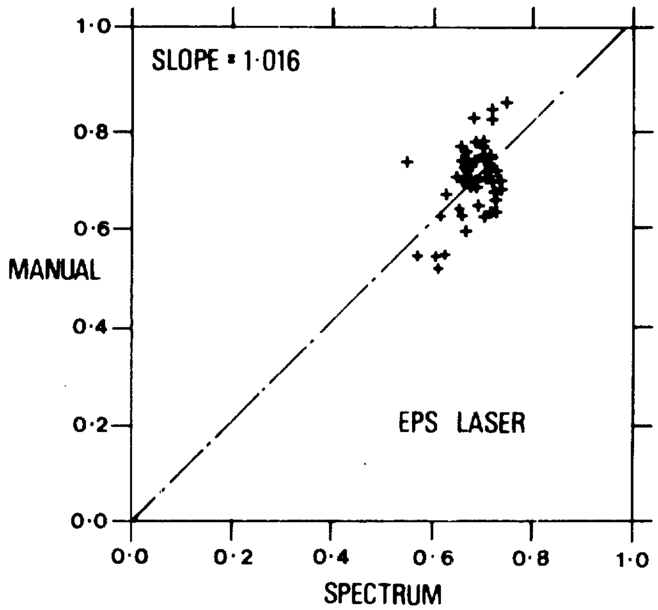

SLOPE = 1 183
INTERCEPT = 0 000
STANDARD ERROR = 0 190
TC(SEC) BAYLOR
MANUAL
SPECTRUM


SLOPE = 1 001
INTERCEPT = 0.000
STANDARD ERROR = 0 367
TC(SEC) LASER
MANUAL
SPECTRUM

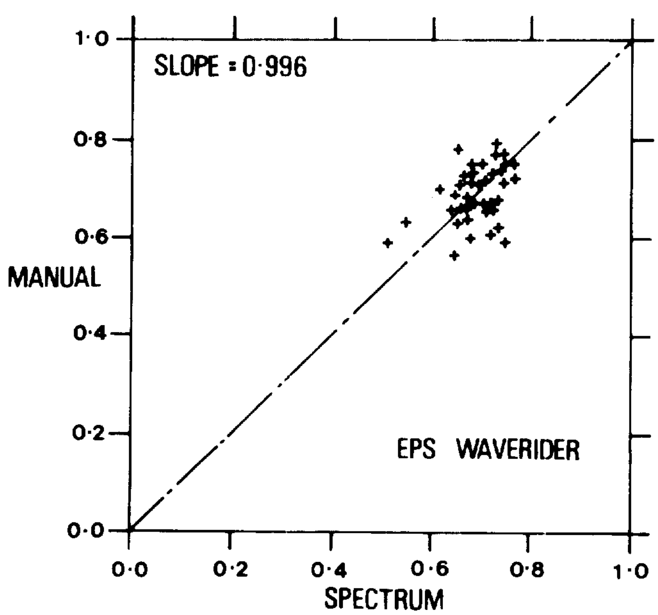
5-3-3

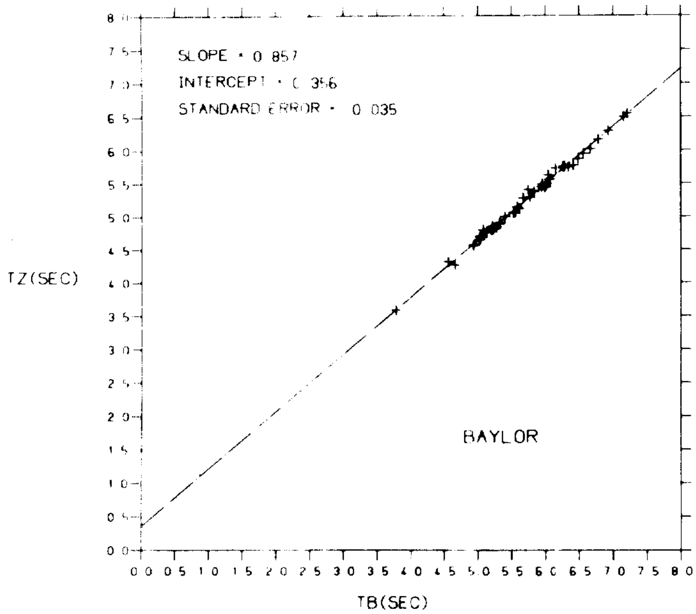

SLOPE = 1 044
INTERCEPT = 0 000
STANDARD ERROR = 0 243
TC(SEC) WAVERIDER
MANUAL
SPECTRUM

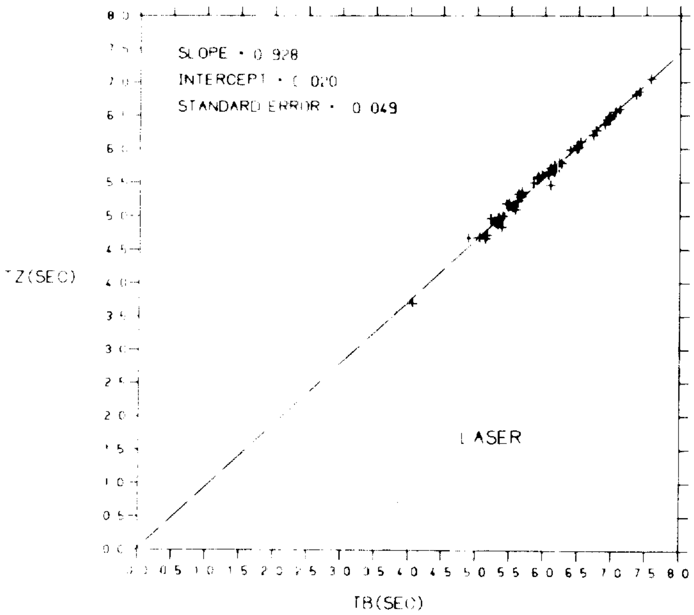

SLOPE = 0·861
MANUAL
SPECTRUM
EPS BAYLOR
1·0
0·8
0·6
0·4
0·2
0·0

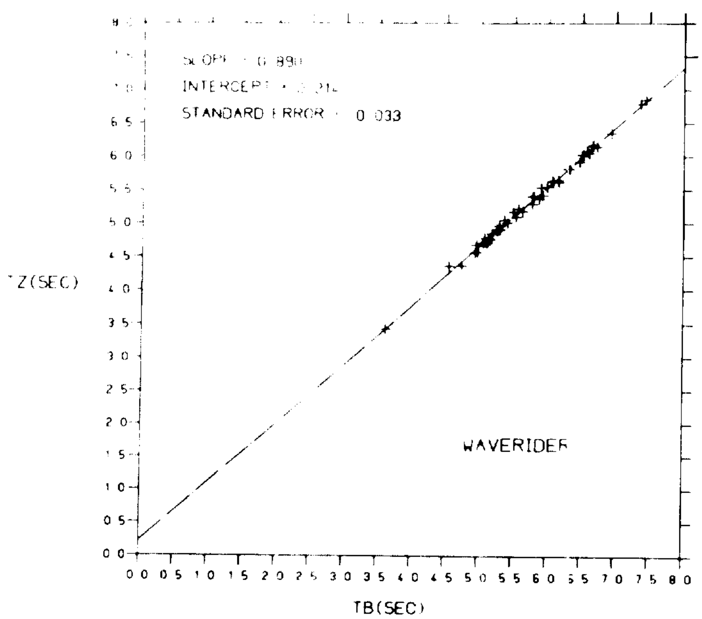

SLOPE = 1·016
MANUAL
SPECTRUM
EPS LASER
1·0
0·8
0·6
0·4
0·2
0·0

5-3-4

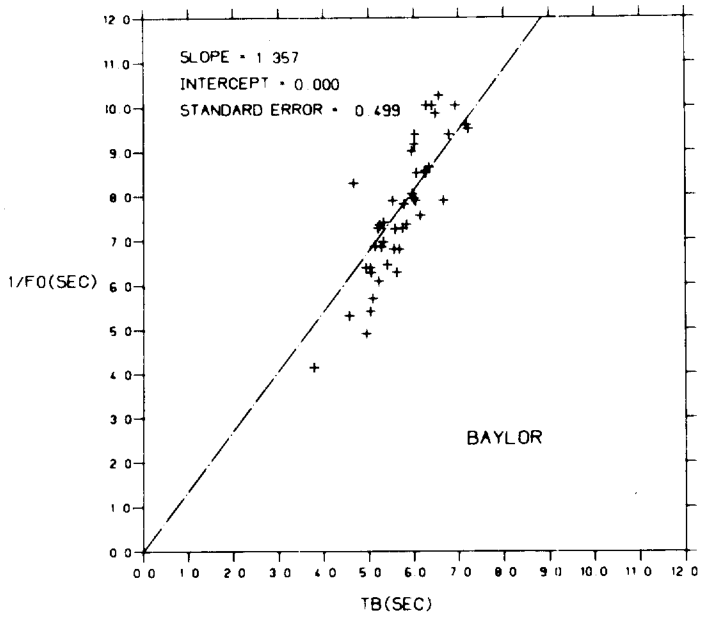

SLOPE = 0·996
MANUAL
SPECTRUM
EPS WAVERIDER
1·0
0·8
0·6
0·4
0·2
0·0

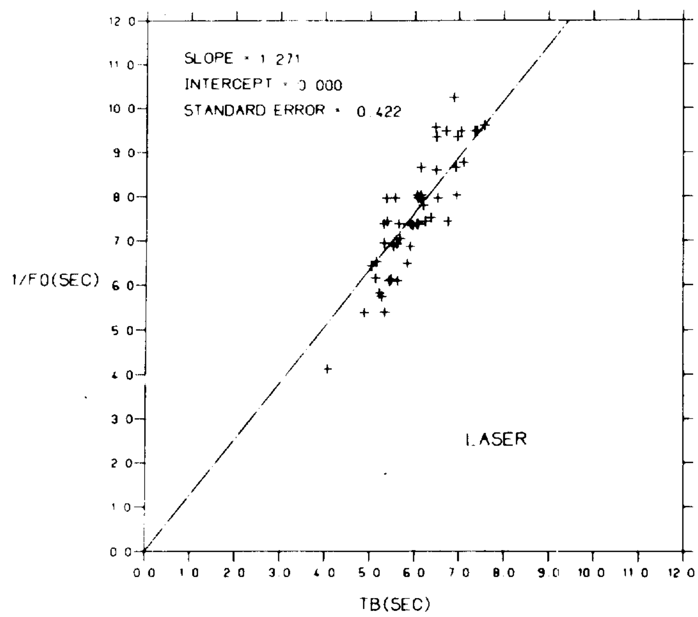

SLOPE • 0 857
INTERCEPT • C 356
STANDARD ERROR • 0 035
TZ(SEC)
TB(SEC)
BAYLOR

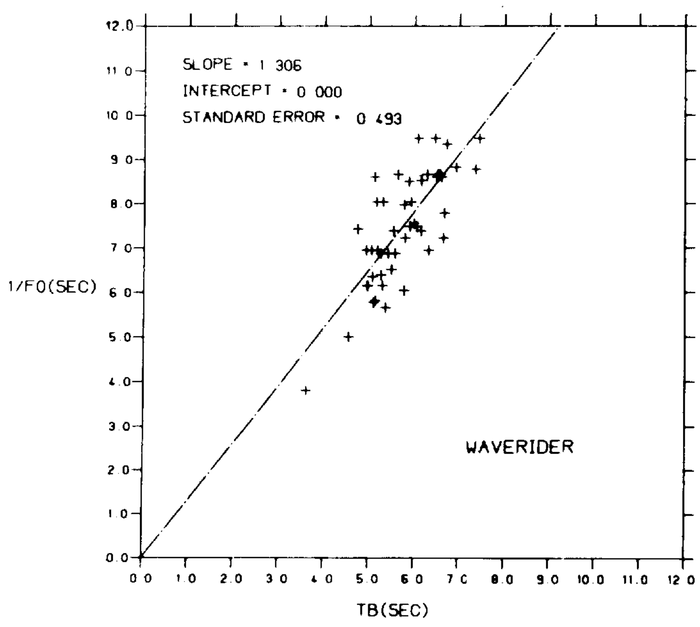

SLOPE • 0 928
INTERCEPT
STANDARD ERROR • 0 049
TZ(SEC)
TB(SEC)
LASER


INTERCEPT
STANDARD ERROR 0 033
TZ(SEC)
TB(SEC)
WAVERIDER

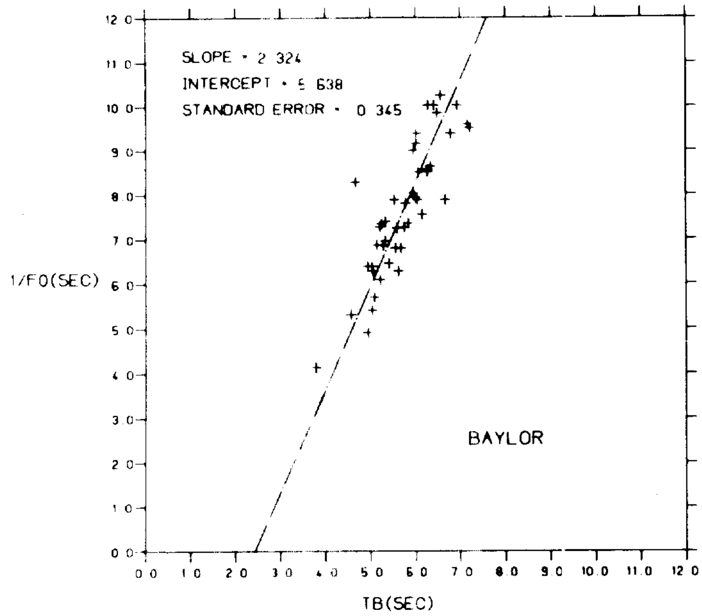
5-3-5

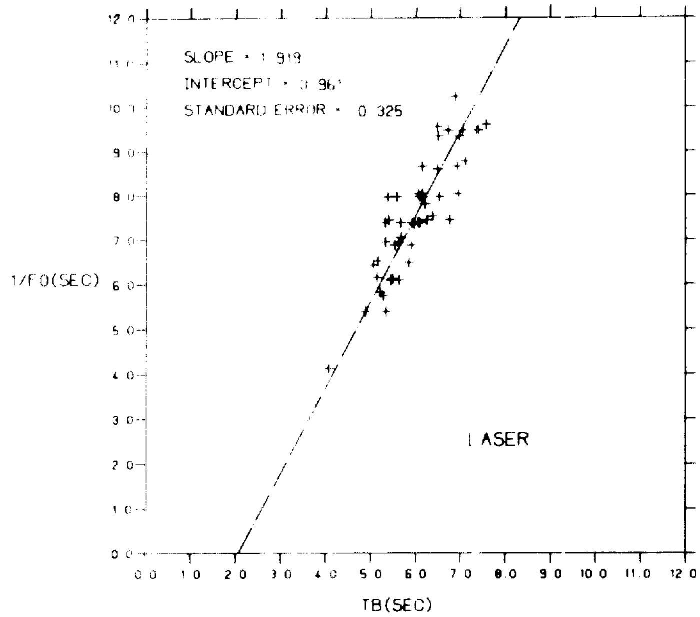

SLOPE = 1.357
INTERCEPT = 0.000
STANDARD ERROR = 0.499
BAYLOR
1/FO(SEC)
TB(SEC)
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0

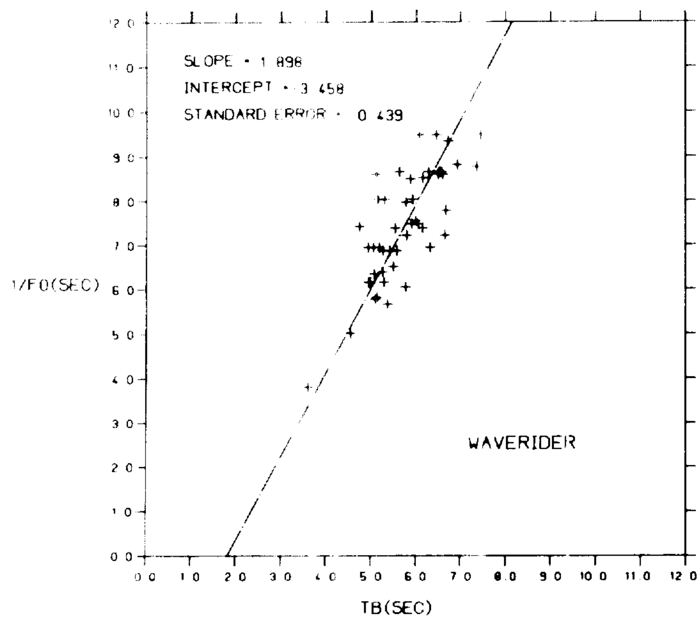

SLOPE = 1.271
INTERCEPT = 0.000
STANDARD ERROR = 0.422
LASER
1/FO(SEC)
TB(SEC)
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0


SLOPE = 1.306
INTERCEPT = 0.000
STANDARD ERROR = 0.493
WAVERIDER
1/FO(SEC)
TB(SEC)
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0 11.0 12.0

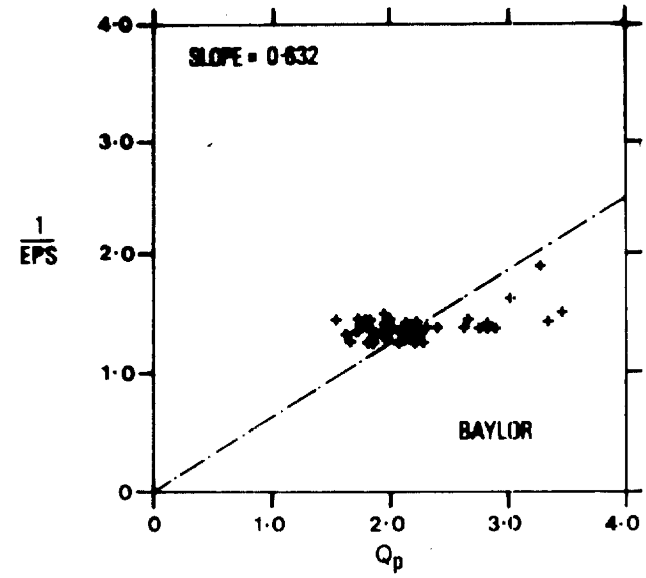
5-3-6

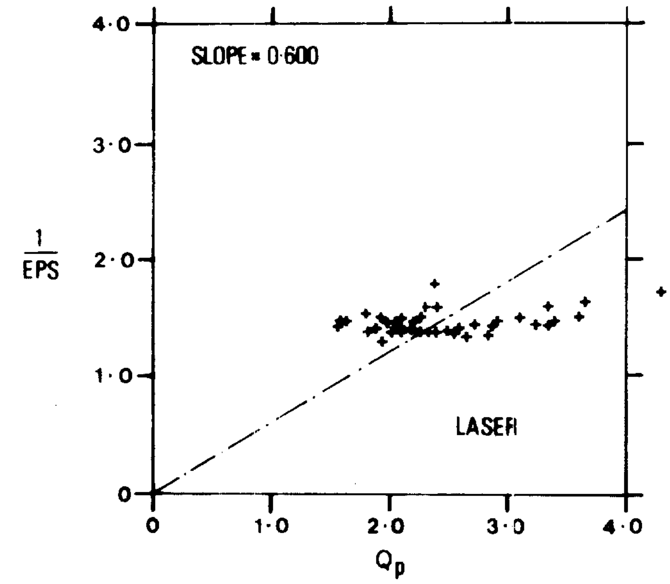

SLOPE = 2.324
INTERCEPT = 5.638
STANDARD ERROR = 0.345
BAYLOR
1/F0(SEC)
TB(SEC)

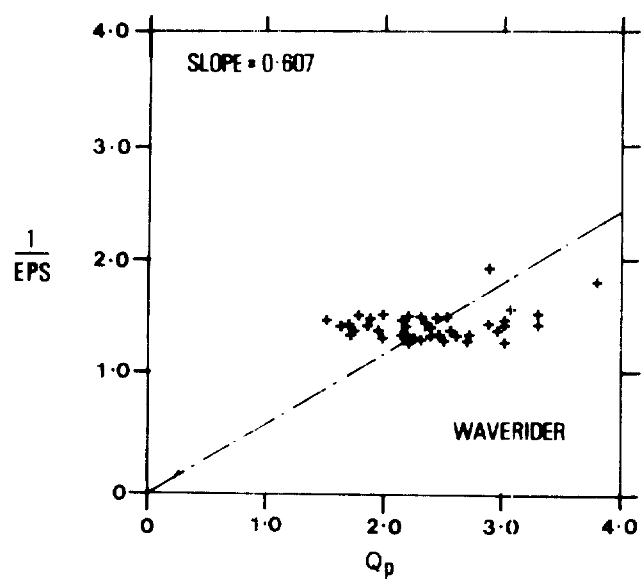

SLOPE = 1.919
STANDARD ERROR = 0.325
LASER
1/F0(SEC)
TB(SEC)

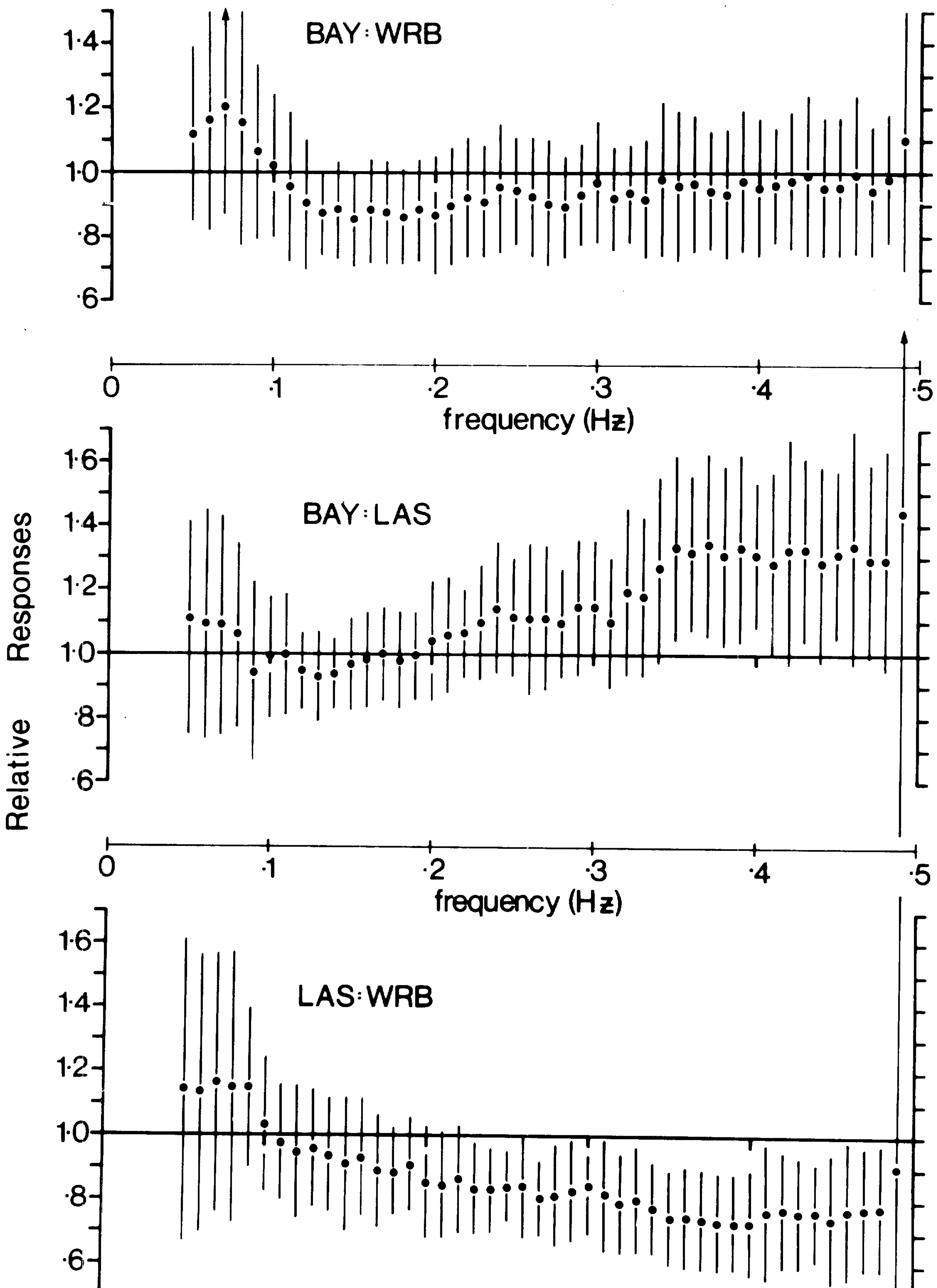

SLOPE = 1.898
INTERCEPT = 3.458
STANDARD ERROR = 0.439
WAVERIDER
1/F0(SEC)
TB(SEC)

5-3-7

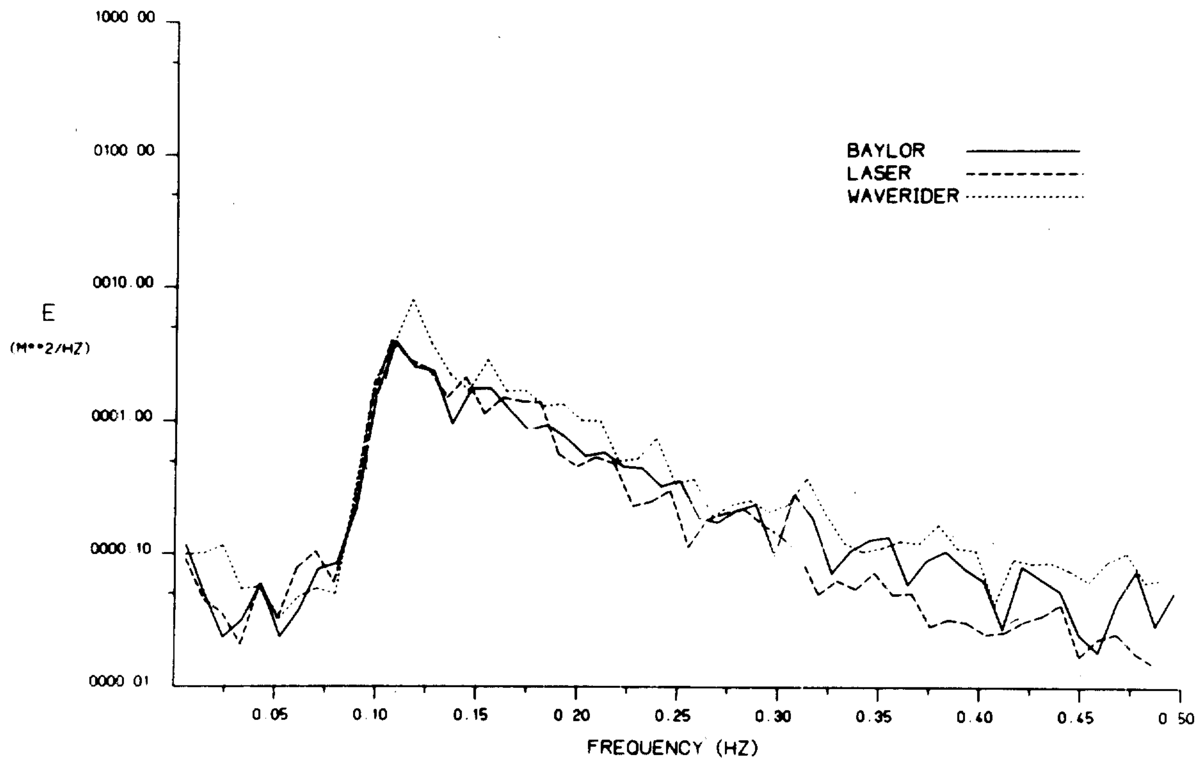

SLOPE = 0·632
BAYLOR
1/EPS
Qp


SLOPE = 0·600
LASER
1/EPS
Qp


SLOPE = 0·607
WAVERIDER
1/EPS
Qp

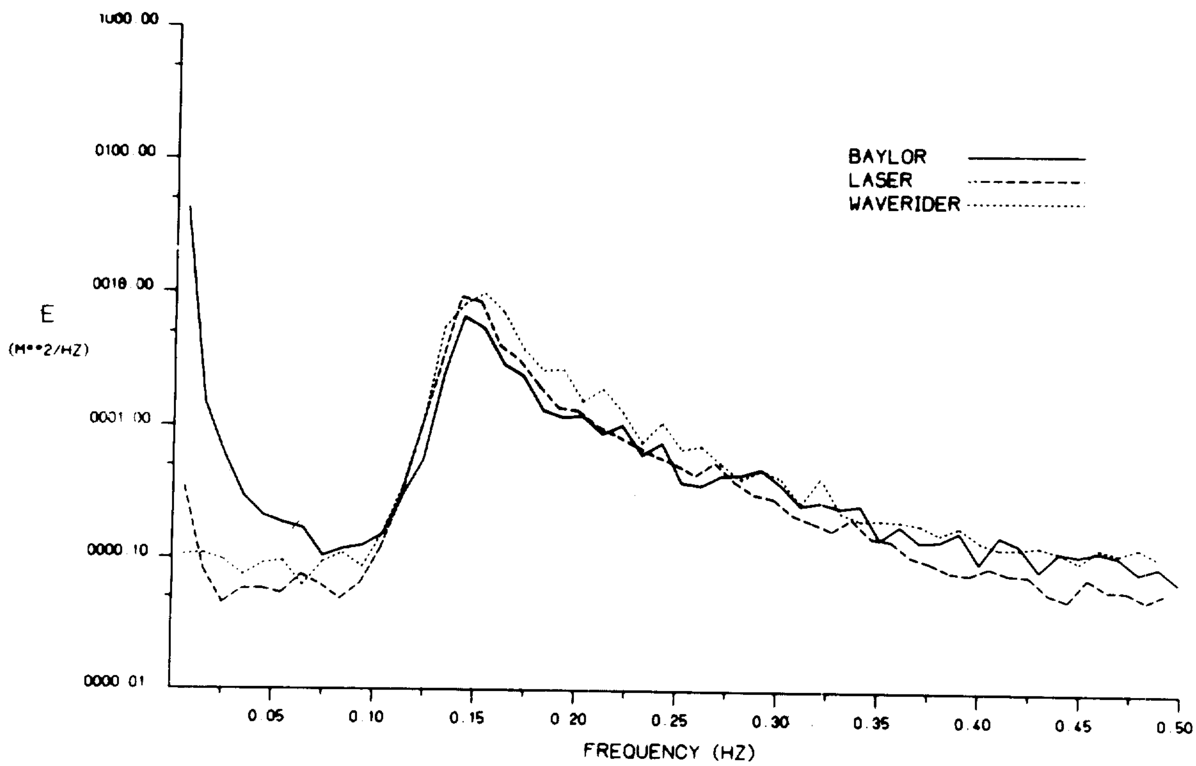

BAY : WRB
1·4
1·2
1·0
·8
·6
0
·1
·2
·3
·4
·5
frequency (Hz)
BAY : LAS
1·6
1·4
1·2
1·0
·8
·6
Relative Responses
0
·1
·2
·3
·4
·5
frequency (Hz)
LAS : WRB
1·6
1·4
1·2
1·0
·8
·6

5-4-1

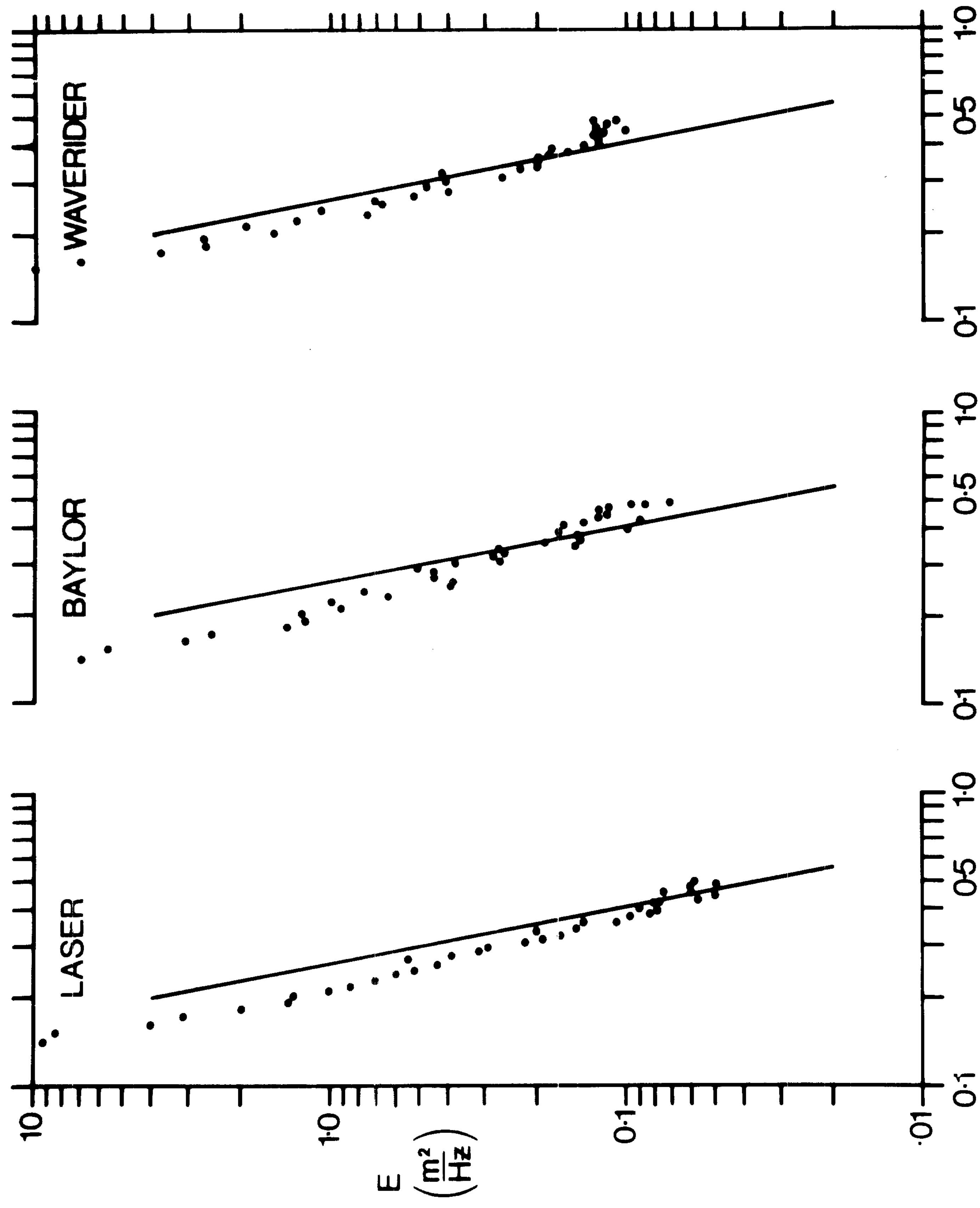


AMOCO PLATFORM 49-18A OCTOBER 1974 SERIES NO. 12

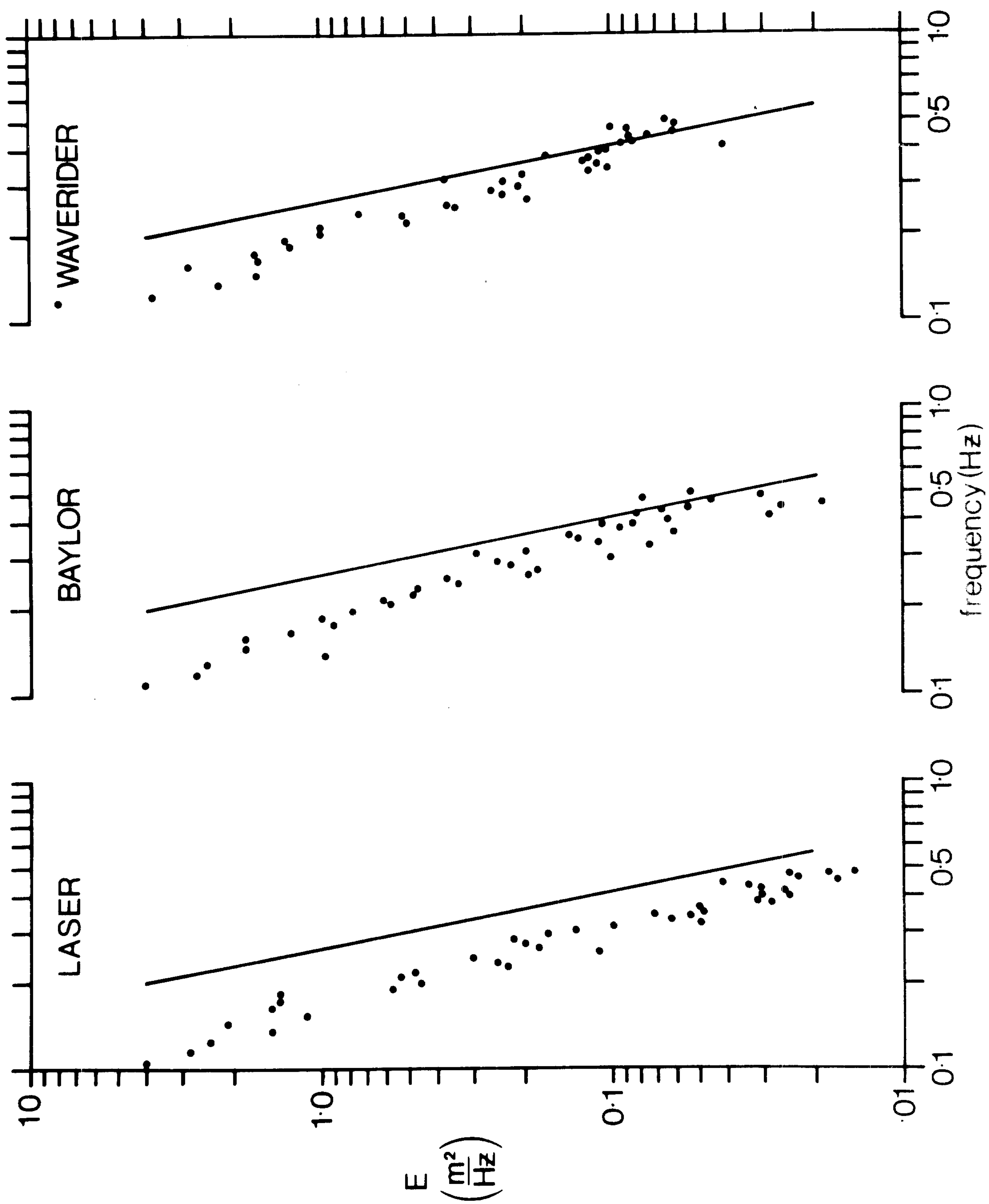
5-4-2



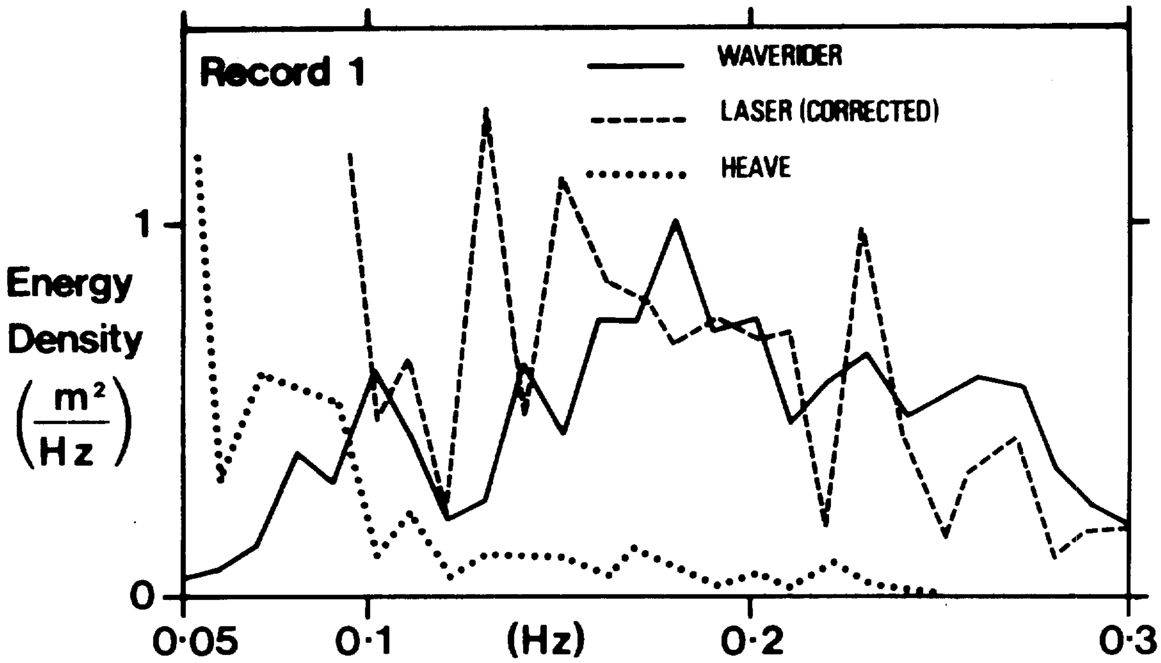
AMOCO PLATFORM 49-18A OCTOBER 1974 SERIES NO. 9


LASER
BAYLOR
WAVERIDER
E $\left(\frac{m^2}{Hz}\right)$
10
1·0
0·1
·01
0·1
0·5
1·0
frequency (Hz)

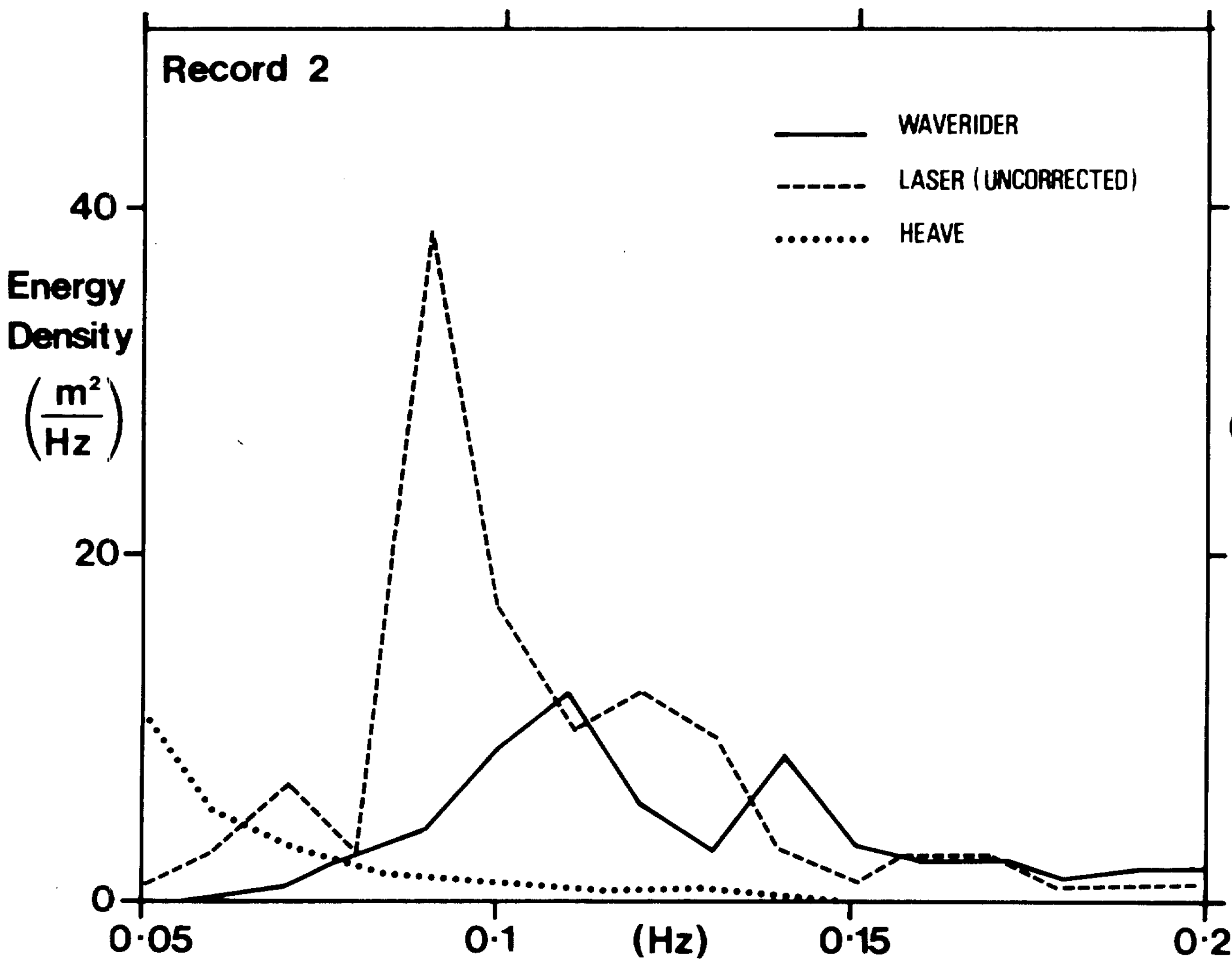
5-4-3


LASER
BAYLOR
WAVERIDER
E $\left(\frac{m^2}{Hz}\right)$
10
1·0
0·1
·01
0·1
0·5
1·0
frequency (Hz)

5-4-4


Record 1
WAVERIDER
LASER (CORRECTED)
HEAVE
1
0
Energy Density (m²/Hz)
0·05
0·1
(Hz)
0·2
0·3

6-4-1


Record 2
WAVERIDER
LASER (UNCORRECTED)
HEAVE
40
20
0
Energy Density (m²/Hz)
0·05
0·1
(Hz)
0·15
0·2

6-4-2