INSTITUTE OF HYDROLOGY

MOVEMENT OF HEAT IN SOILS

by

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ABSTRACT

This report is a review of the large and varied literature on soil temperature. Its aim is to place the key literature in context and into a consistent structure.

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1. INTRODUCTION

The thermal behaviour of soils has consequences in many fields of environmental research and technology. As the surface which intercepts much of the incoming solar radiation, the soil is an important source of heat for the lower atmosphere, while its porous structure provides micro-environments which are insulated from the extremes of heat and cold at the surface, and yet freely ventilated and provided with water and nutrients.

In biology and ecology the soil temperature is an important controlling factor, having obvious relevance to seedling emergence and the activity of soil organisms (which from the agricultural point of view will include pests and diseases). Keen and Russell (1921) likened conditions in an English soil in summer to those in a $20^{\circ}C$ laboratory incubator. In winter conditions are less favourable, but even then the soil provides a certain amount of thermal insulation for dormant organisms.

The thermal properties of soil are sometimes of importance in engineering applications; for instance Murray and Whalley (1954) measured thermal properties with the aim of predicting temperatures of oil flowing through a buried pipeline, while Fluker (1958) was interested in the soil as a source of heat for heat pump systems. Research into thermal migration of moisture is important for frost heave prediction.

Much of the work relating to soil heat, and particularly to the vertical flux of heat through the soil, has been concerned with the energy balance of the land surface and with the size of the soil heat flux term in evaporation equations. The combination method of evaporation prediction depends strongly on the surface heat budget, and in some circumstances heat storage in the soil can make a significant contribution to this term in the equation.

It is not surprising, in view of the breadth of the field of application and the number of disciplines involved, that soil temperature has a large and varied literature, with no consistent notation. This report is an attempt to place some of that literature (but by no means all of it) in context and to force upon it a first approximation to a consistent structure.

2. STUDIES OF SOIL TEMPERATURE

The variation of soil temperature with season and time of day has long been of interest, and many series of measurements were obtained in the last century. Baver (1956) and Geiger (1965) give a number of

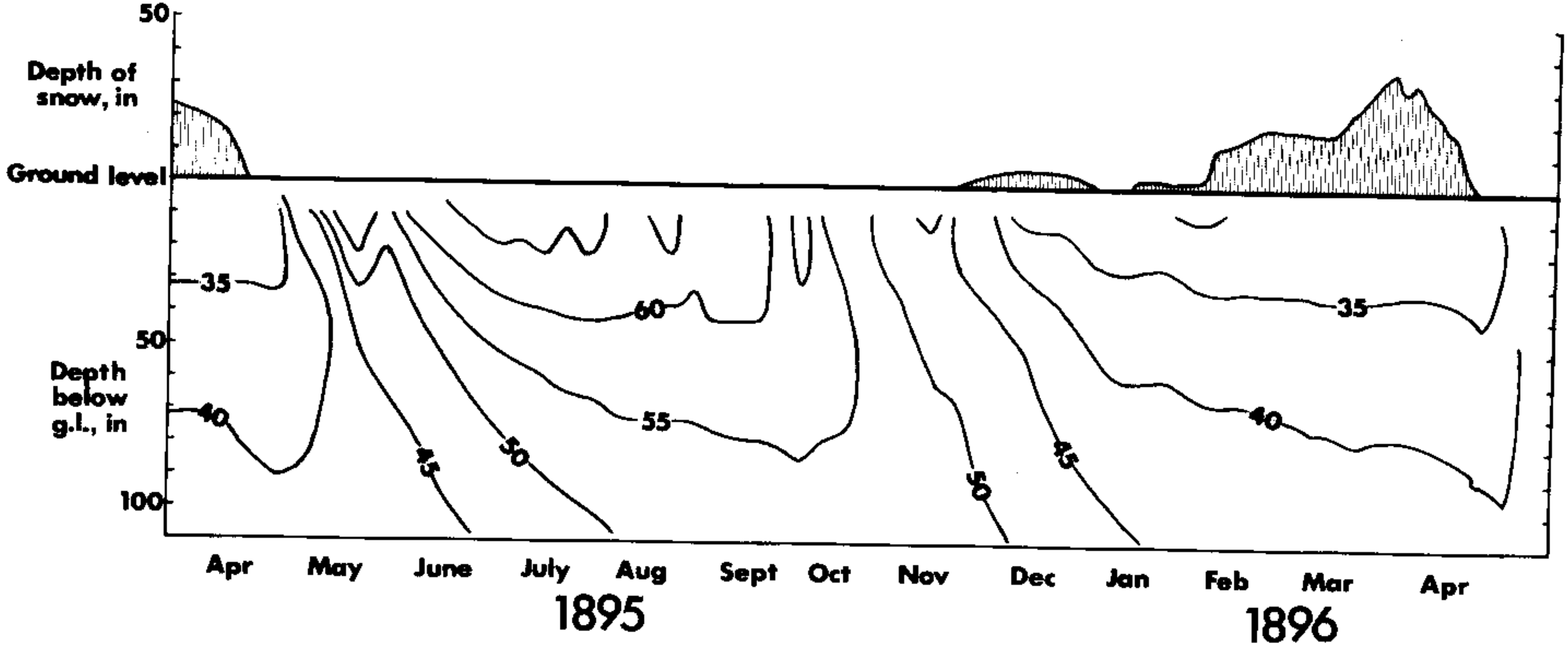

FIGURE 1 Isothermal diagram for a one-year period, showing propagation of the annual wave of soil temperature, and the insulating effect of a snow cover (Callendar & McLeod 1896). Temperatures are in ^{o}F.

references to European work, while Fitton and Brooks (1931, cited by Richards *et alia* 1952) collated data from many sources in the U.S.A.

A particularly detailed study was performed at McGill University, Montreal, by Callendar and McLeod (1896), using platinum resistance thermometers buried at seven depths in the soil for a period of 18 months. They recorded daily temperatures at 1230h at depths between 25 mm and 2.74 m, noting in their discussion of the results that this temperature represented a daily minimum at 0.25 m, a maximum at 0.5 m and just above the mean at 0.1 m. Fluctuations of period of the order of ten days, caused by weather conditions, were smoothed out at 0.5 m. It was possible to observe the effect of snow lying on the ground over the winter, insulating the ground from heat losses (Figure 1).

Measurements at Rothamsted, Hertfordshire, at the single depth of 0.15 m, were described by Keen and Russell (1921). They discussed the relationship between soil and air temperature for the diurnal cycle, and presented diagrams showing the type of variation observed over the seasons. In winter, for instance, fluctuations in air temperature were extremely irregular, while soil temperatures varied smoothly. In summer, clear days were characterised by strong periodic variations which were also observed in the soil (Figure 2). The data were subjected

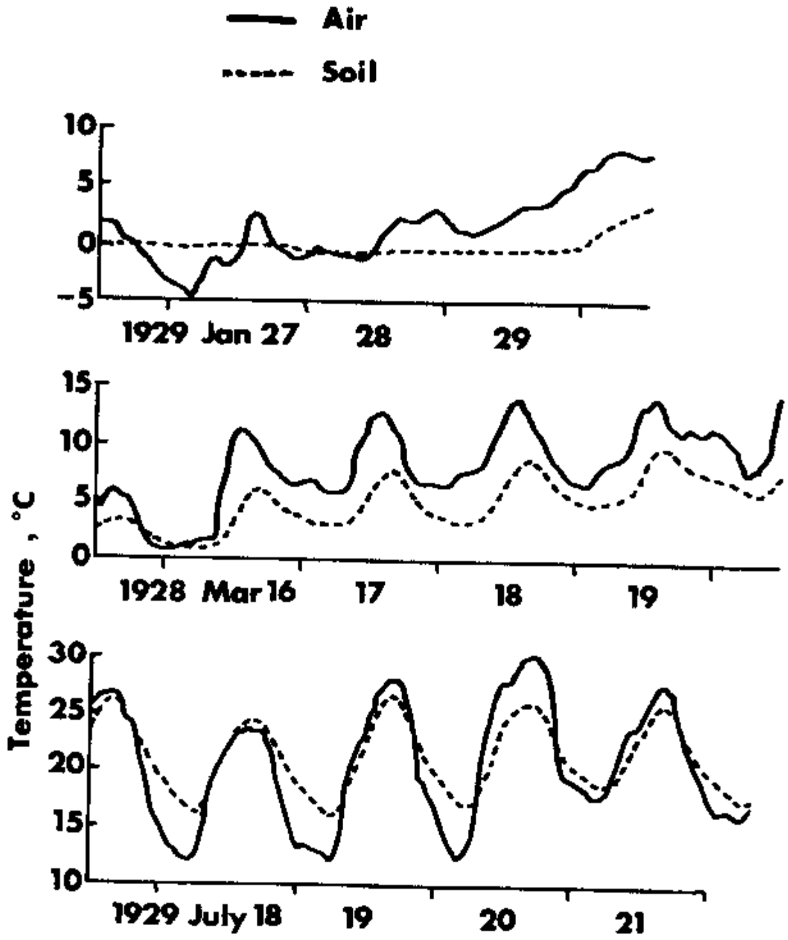

FIGURE 2 Relation between bare soil temperature at 0.1 m depth and temperature in screen at 1.22 m for winter, spring and summer (Keen 1931).

to a detailed analysis, but the lack of thermometers at other depths, non-destructive techniques for soil moisture measurement or instantaneous radiation measurements meant that the analysis of the effects of sunshine, rain and soil moisture was somewhat inconclusive. Valuable results which did emerge were the comparisons between the 0.15 m thermograph and simpler measurements: for example the 0.3 m soil thermometer in its steel tube gives a reading at 0900 h representative of the minimum temperature at 0.15 m, while the maximum air temperature measured in a screen at 1.5 m above ground level gave an estimate of the maximum soil temperature at 0.15 m. Soil temperatures at this depth varied in such a way that the daily mean could be estimated as the average of the daily maximum and minimum.

The effect of the changing seasons on the diurnal fluctuations of soil temperature was further discussed by Keen (1931) with reference to the Rothamsted results. In winter the smooth variation in soil temperature is due to the high specific heat of the soil (the effect of moisture) and a low level of solar radiation. In spring the increase in radiation brings about a steady increase in the daily mean soil temperature, while the reduction in specific heat as the soil dries out causes an increase in the magnitude of diurnal fluctuations. In summer the temperature range is limited by a decrease in the thermal conductivity of the soil with the decrease in moisture content.

Cultivation, by creating an open-textured layer at the soil surface, changes the thermal properties of the soil profile and hence the soil temperature. West (1933) investigated the effect of cultivation on two adjacent plots at Griffith, New South Wales. The mulch layer, 0.13 m deep, caused a damping of the diurnal temperature variation and a reduction in the seasonal maximum by about 3 °C.

Data acquisition systems and computers have made possible the collection of much greater quantities of data, and more sophisticated analysis methods (see, for example, the studies conducted by Fluker 1958, Pearce and Gold 1959). Carson (1963) used soil temperature data from Argonne, Illinois, which had been obtained from resistance thermometers at depths of up to 8.84 m below ground level. Air temperature was recorded as a ten minute average once an hour. Fourier analysis was then used to investigate the attenuation of annual and diurnal components with depth. A considerable amount of data reduction was necessary to extract the relevant information from the mass of data collected.

Another study using modern electronic equipment was that of Wierenga *et alia* (1970), who measured the effects on soil temperature of irrigation with warm and cold water. The principal effects observed were a reduction in the mean temperature due to evaporative cooling, and reduction in the range of fluctuations as a result of the increase in heat capacity of the moist soil. In this experiment the computer was used to simulate soil temperature variation under certain simplifying assumptions.

The relationship between soil and air temperatures was studied by Kalma (1971), with special reference to conditions in the upper 20 mm, where seed germination takes place. Over a 14 year period, daily maximum and minimum air temperatures were recorded, and soil temperatures were measured by a mercury-in-steel thermograph with the centre of its 18 mm diameter bulb at 13 mm below a bare soil surface. The experimental site was at Katherine Research Station in the Northern Territory of Australia, a tropical savannah area. Minimum air and soil temperatures taken over pentads, five-day intervals, were found to be closely correlated and approximately equal, but a graph of air maximum against soil maximum showed a natural division of the year into five climatic periods which could be explained on the basis of seasonal climatic changes.

Significant seasonal effects were also noted by Krishnan and Kushwaha (1972), working in the arid zone of India at Jodhpur. The effect of the southwest Monsoon was apparent, causing a depression of the annual maximum in soil temperatures.

In Arctic regions the soil temperature assumes an obvious importance, and the thermal behaviour of the soil can promote or inhibit the formation of permafrost conditions. Luthin and Guymon (1974) related soil temperatures to the vegetative cover, and found that vegetation and permafrost interacted in a complex way. For instance, peat-forming vegetation could insulate the soil from summer solar radiation, while the surface peat, frozen during the winter, had a much higher winter thermal conductivity and allowed the escape of heat. Thus the soil temperature was lowered and permafrost would develop, impeding drainage and encouraging the spread of the mosses which formed peat.

Richards *et alia* (1952) reviewed the instruments which have been used in studies of soil temperature.

(i) Mercury-in-glass thermometers are useful for single daily readings, or for maximum and minimum measurements, and for calibration standards. Two patterns are widely used: the long-stem thermometer whose bulb is permanently in position at the given depth, and the lagged thermometer, which is suspended in a steel tube which serves to integrate the soil temperature over a significant depth.

(ii) Mercury-in-steel thermographs have the advantage of providing a continuous record without the need for electric power or electronics. A thermograph using differential expansion of metals was used by Keen and Russell (1921).

(iii) Thermocouples have been a very popular choice, because of their advantages of small size, cheapness and stability. The cold junction, often a problem with thermocouples, is usually positioned at the base of the profile, where temperatures can be expected to be constant.

(iv) Resistance thermometers were used in an early study by Callendar and McLeod (1896). Their advantage over thermocouples is accuracy of measurement, and in certain applications their large spatial extent combined with low heat capacity. Suomi (1957) built 'temperature integrators', large nickel coils which could give a spatial mean of temperatures in the upper soil.

(v) After initial problems with drift, thermistors are coming into use as a cheap, accurate temperature sensor with a much higher thermal coefficient of resistance than the metals used for resistance thermometers.

3. THEORETICAL ANALYSIS OF SOIL TEMPERATURE FLUCTUATIONS

The temperature of the soil is determined by the flow of heat upwards from the interior of the earth and by conditions at the ground surface. The flow of geothermal heat is so small (about 0.04 Wm^{-2}) that for most purposes it may be neglected (Diment and Robertson 1963). The geothermal temperature gradient associated with this flow is about 0.03^{o}C per metre (Keen 1931), and gradients of annual mean temperature of about this order should be expected in the soil.

There being no significant sources of heat within the soil, the soil surface is the most important source of variation in soil temperature. Solar radiation reaching the soil surface is reflected in part, the remainder being used to heat the soil and the air and to evaporate water (Penman 1948). Where vegetation is present, the plant surfaces intercept most of the incoming radiation, and the soil receives much of its heat from re-radiation and by conduction from the air. Some of the heat is re-radiated at long wavelengths, according to Stefan's Law, the soil surface behaving as a black body.

When the soil surface is heated, energy is transported vertically by conduction through the soil by the downward thermal gradient. At night the thermal gradient near the surface reverses, and heat is returned to the surface to be radiated, lost by conduction to the air or used for evaporation from the soil surface. Deeper in the soil the temperature distribution takes the form of a wave which propagates downwards, being attenuated rapidly. It is a feature of the equation of heat conduction that this wave involves no net transport of energy (Figure 3).

In addition to the strong diurnal wave, which is most prominent on clear summer days, there is an annual wave which penetrates to a much greater depth. Keen (1931) gave a detailed account of the propagation of diurnal and annual waves, and other accounts may be found in more recent general works by Geiger (1965), Rose (1966) and Monteith (1973).

The homogeneous conductor model

The observed variations in soil temperature at a site bear a strong resemblance to temperature variations inside a semi-infinite homogeneous conductor subjected to a periodic surface temperature, and this model, attributed to Fourier, has been used extensively to describe soil phenomena, with varying degrees of success (Thomson 1861, Callendar and McLeod 1896, McCulloch 1959, Carson 1963, Krishnan and Kushwaha 1972).

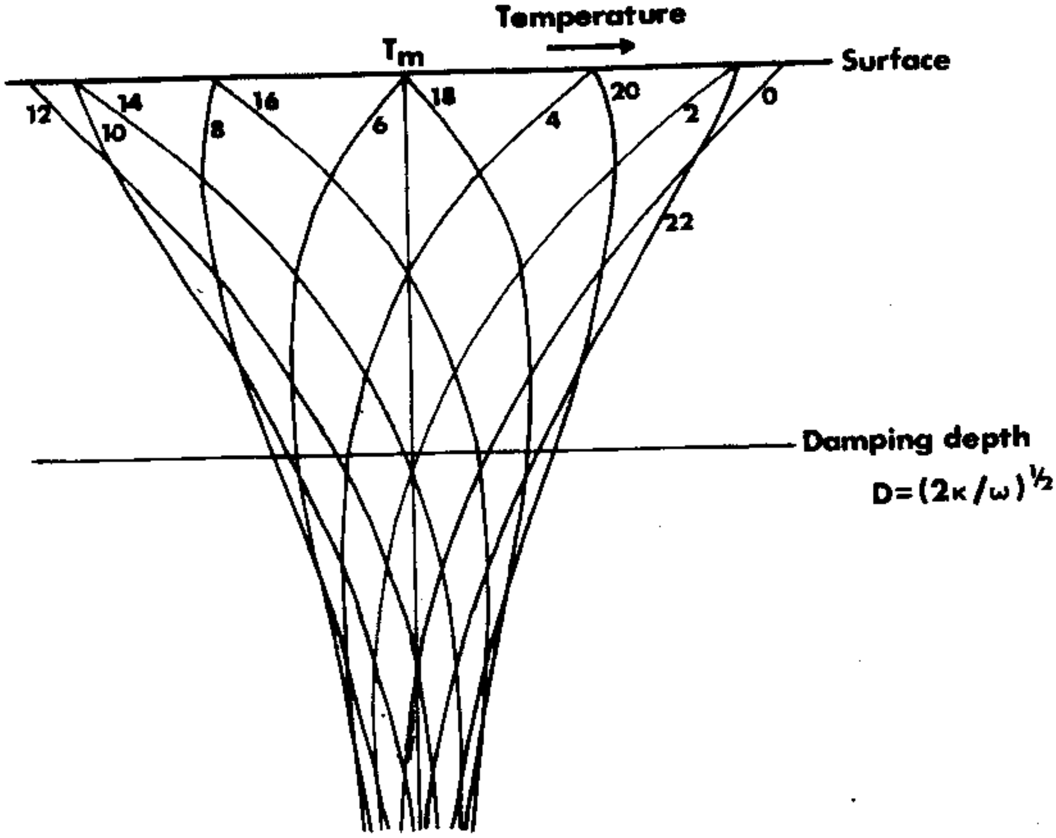

FIGURE 3 Downward propagation of diurnal temperature wave in a homogeneous soil. The figures beside each curve are in hours after the moment of maximum surface temperature.

The equation for the one-dimensional conduction of heat in a solid is

$$\frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z}\right) = \rho c \frac{\partial T}{\partial t} \qquad (1)$$

where T is temperature, λ the thermal conductivity of the medium, ρ its density and c its specific heat. The assumption of homogeneity results in the simplified equation

$$\frac{\lambda}{\rho c} \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \qquad (2)$$

The combination $\lambda/\rho c$ is referred to as the thermal diffusivity of the material and is usually denoted by κ. Applying the boundary condition at $z = 0$,

$$T(0, t) = T_o \cos \omega t + T_m \quad (3)$$

and seeking solutions with periodic form and angular frequency ω, results in a solution

$$T(z, t) = T_o \exp(-z/D) \cos(\omega t - z/D) + T_m \quad (4)$$

which has the required form of an attenuating wave propagating downwards (i.e. in the direction of z increasing). The distance D, sometimes referred to as the 'damping depth' (van Wijk & de Vries 1963), is given by

$$D = (\frac{2\kappa}{\omega})^{\frac{1}{2}} \quad (5)$$

and is the depth in which the wave amplitude is reduced by a factor of e. It is obvious from (5) that low frequency oscillations, such as the annual variation of surface temperature, will penetrate to greater depth than high frequency oscillations such as the diurnal wave. This expression also provides an explanation of the observation that minor irregularities in the diurnal wave are rapidly smoothed out, so that the diurnal variation in soil temperature becomes more closely sinusoidal at greater depths.

The equation of heat conduction (2) is linear, so that solutions of the form (4) may be superposed to give the solution for any general periodic boundary condition, expressed as a Fourier series (van Wijk and de Vries 1963).

For any given Fourier component with angular frequency ω, the amplitude of the soil temperature variation

$$A(\omega, z) = A_o(\omega) \exp(-z/D) \quad (6)$$

while the phase

$$\Phi(\omega, z) = z/D \quad (7)$$

Where the homogeneous conductor model is valid, plotting the natural logarithm of the amplitude against depth will yield a straight line whose gradient is -1/D, while a plot of the phase angle in radians will yield a straight line with gradient 1/D. Using equation (5) it is possible to derive two estimates of κ, which should be equal. Deviations from the homogeneous conductor model are immediately evident, either as a failure to plot on a straight line, or as a discrepancy between the amplitude and phase estimates of κ. The method may be used when temperatures at only two depths are available (McCulloch 1959), but is most accurate when used with data from several depths (West 1952). (Figure 4). Serious discrepancies between the amplitude and phase estimates imply that the homogeneous conductor model is inappropriate (Lettau 1954, McCulloch and Penman 1956).

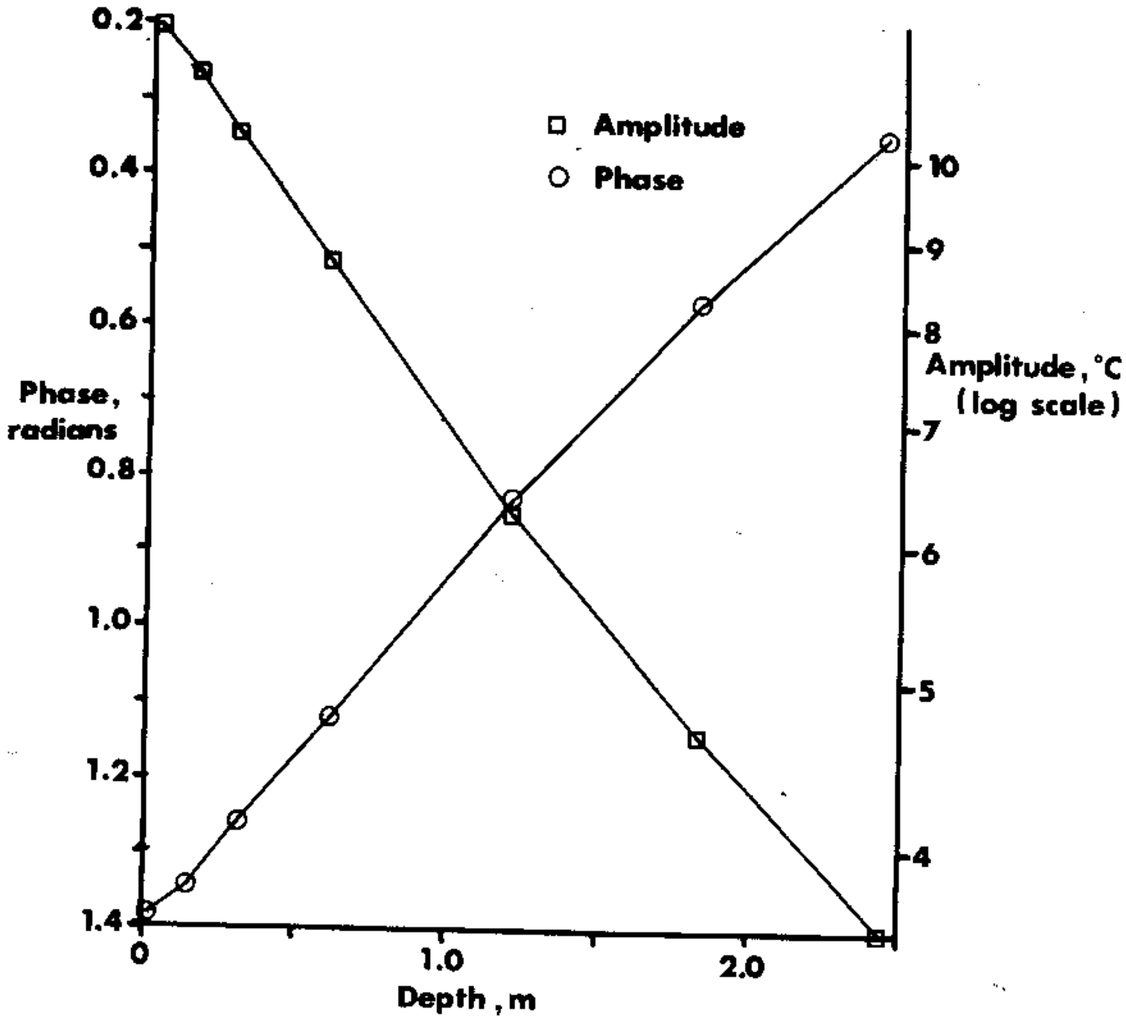

FIGURE 4 The phase shift and the natural logarithm of the amplitude of a diurnal temperature wave plotted against the depth. A set of results obtained for a homogeneous soil by West (1952).

If the flux of heat into the surface, rather than the temperature, is used as a forcing function for the differential equation (2), the result is similar (van Wijk and de Vries 1963). The flow of heat is given by

$$H = -\lambda \frac{\partial T}{\partial z} \tag{8}$$

and for a temperature field described by (4) the heat flux is

$$H(z,\ t) = \frac{\lambda\sqrt{2}}{D}\ T_o \exp(-z/D) \cos(\omega t - \frac{z}{D} + \frac{\pi}{4}) \qquad (9)$$

The heat flux wave has a phase lead of $\pi/4$ on the temperature wave, and the amplitudes are in the ratio

$$\frac{\lambda\sqrt{2}}{D} = \omega^{\frac{1}{2}}(\lambda C)^{\frac{1}{2}} \qquad (10)$$

where $C = \rho c$ is known as the heat capacity (or volumetric heat capacity) and the product λC is known as the contact coefficient. $(\lambda C)^{\frac{1}{2}}$ is sometimes referred to as the thermal admittance of the soil. The periodic solution outlined above applies generally to annual fluctuations, which have a very strong first harmonic component, and to diurnal fluctuations recorded on a sequence of clear days, when the variation of soil temperature is accurately described by the first two Fourier components (McCulloch 1959). Deviations from periodicity caused by cloudiness, present either as irregularities on a single daily record, or as variations from day to day, invalidate the periodic model, and a general time-varying model must be adopted. This solution is also applied in certain cases to the determination of soil properties, particularly where rapid temperature changes are necessary to minimise the effects of vapour transfer.

The full solution of (2) with general boundary and initial conditions consists of two terms, the first depending on the initial condition, of temperature *versus* depth, in the soil. In the analysis of temperature fluctuations in the soil, the initial condition rapidly becomes ineffective, and the forcing condition at the boundary is much more important, while in artificial conditions created in the laboratory the initial condition is invariably one of constant temperature.

The relationship between the temperature in the soil and at the surface for a general boundary condition may be expressed in two ways

(i) as a *convolution integral* solution

$$T(z,t) = \int_0^t T(0,t-\tau)\ \frac{z}{2(\pi\kappa\tau^3)^{\frac{1}{2}}}\ \exp\ \frac{-z^2}{4\kappa\tau}\ d\tau \qquad (11)$$

where τ is an integration variable.

This is a linear superposition of solutions, of the type commonly encountered in systems theory (eg the unit hydrograph) and was used by Langbein (1949) to predict soil temperatures from near-surface temperatures. Langbein's model consisted of the summation of the terms of the form

$$\Delta T(z,t) \;=\; T(0,t-\tau)\,\frac{2}{\sqrt{\pi}} \int_0^{z/2\sqrt{\kappa\tau}} \exp(-\beta^2)\;d\beta \qquad (12)$$

which is a discretised approximation to the integrand in equation (11). The values of the probability integral were used in tabulated form.

Hasfurther and Burman (1974) used a similar convolution model to predict soil temperature at 25 mm depth from air temperature (as ten day averages). They assumed that air and soil temperatures were related linearly in a general way, and found the terms of the summation by Fourier transformation.

(ii) in *Laplace transforms*. Van Wijk (1963) noted that the Laplace transform could be used to determine the diffusivity K from non-periodic soil temperature data. The Laplace transform L(f) of a function f(t) is given by

$$L(f) \;=\; F(p) \;=\; \int_0^{\infty} e^{-pt}\, f(t)\; dt \qquad (13)$$

Detailed discussion of the Laplace transform can be found in many mathematical texts, but the treatment of the diffusion equation (2) requires only the condition of linearity ($L(mf + ng) = mL(f) + n\,L(g)$ where m and n are independent of t) and the differential relations

$$L(\frac{\partial f}{\partial t}) \;=\; p\,L(f) - f(z,0) \qquad (14)$$

and

$$L(\frac{\partial^2 f}{\partial z^2}) \;=\; \frac{\partial^2}{\partial z^2}\,L(f) \qquad (15)$$

where the transformation is in the t-variable.

Equation (2) then becomes

$$\kappa\,\frac{\partial^2}{\partial z^2}\;L(T) \;=\; pL(T) - T(z,0) \qquad (16)$$

If the zero of the time scale is taken at a point where T(z,t) is uniform, the term − T(z,0) can be eliminated by subtracting T(z,0) from the temperature data. The solution of (16) is then

$$L(T) \;=\; K\,\exp(-z\,\sqrt{p/\kappa}) \qquad (17)$$

where K is a constant.

To find κ, the natural logarithm of $L(T)$ for a given value of p is plotted against z, (van Wijk 1963). The value of p is in principle arbitrary, but the values chosen by van Wijk (1963), van Wijk and Goedkoop (1963) and van Wijk and Derksen (1966) are between four and ten times the reciprocal of the period of record. A detailed discussion of the role of the variable p was given by van Wijk and Bruijn (1964a).

Layered conductor model

The homogeneous conductor model has the appeal of simplicity, but it often fails to fit observed data or gives inconsistent estimates of diffusivity. For instance, several workers have found different values of κ from considerations of the amplitude and phase of diurnal fluctuations in the soil (Lettau 1954, McCulloch and Penman 1956, Rider 1957). Failure is frequently attributed to the existence of a layer near the surface, usually caused by cultivation or evaporative drying, with different thermal properties from the deeper soil horizons.

Van Wijk and Derksen (1963) considered a soil consisting of two layers, the lower layer extending to infinite depth, as an approximation to a soil in which the upper few centimetres were wetter or drier than the deeper soil, where a tilth was maintained by cultivation or where a mulch of a different material was added to the surface. A notable example was the addition of a sand layer to a peat soil to inhibit ground frost. Laplace transform theory was applied to this problem by van Wijk and Bruijn (1964b).

The algebraic manipulations leading to the solution are somewhat cumbersome, resulting from the requirement of the solution in the upper layer to satisfy two boundary conditions rather than one. Although the qualitative predictions of the model are in agreement with the observed behaviour of soils under cultivation, a quantitative comparison would be difficult, as many different quantities, such as evaporation and albedo, are changed by cultivation and mulching.

Inhomogeneous conductor models

Thomson (1861), in analysing a long series of data from stations around Edinburgh, noticed that the agreement between amplitude and phase estimates of diffusivity, and between estimates of diffusivity for different depth ranges, was not good, and attributed these discrepancies to inhomogeneity in thermal properties. He suggested that a method of including inhomogeneity in the model was to force solutions of the form

$$T(z,t) = A \exp(-Pz) \cos(\omega t - Qz) + T_m \qquad (18)$$

where P and Q were functions of z which could be found by substitution into the differential equation (1), and values of λ and C as functions of depth could be derived. However, the values he found by this method were physically unrealistic, and further use of this model had to wait for more accurate and extensive data.

Heat flux	1 Wm^{-2}	≡	0.0239 mcal $cm^{-2}s^{-1}$
		≡	2.06 langleys day^{-1}
	1 mcal $cm^{-2}s^{-1}$	≡	41.8 Wm^{-2}
	1 langley day^{-1}	≡	0.484 Wm^{-2}

Two combinations of the fundamental properties are also of interest: the diffusivity $\kappa = \lambda/C$, which governs the movement of temperature waves in the soil, and the contact coefficient λC, which determines the heat flux for a given periodic temperature field.

Heat capacity C

The heat capacity of a soil depends only on the materials present and on their relative proportions, and not on the arrangement of the soil particles and the water- and air-filled voids. De Vries (1963) gave a formula based on the quantities of mineral soil, organic matter and water present in unit volume

$$C = 0.46\,x_m + 0.60\,x_o + \theta \qquad \text{cal cm}^{-3}(^{\circ}\text{C})^{-1} \qquad (27)$$

which in S.I. units is

$$C = 1.92\,x_m + 2.51\,x_o + 4.18\,\theta \quad \text{MJ m}^{-3}(^{\circ}\text{C})^{-1} \qquad (28)$$

where x_m, x_o and θ are the proportions (by volume) of mineral soil, organic matter and water respectively.

Direct measurement of heat capacity is possible by calorimetry. Two methods are available: the method of mixtures, in which a quantity of material of known specific heat and temperature is mixed with the sample, and the electrical heating method, in which a known amount of heat is developed electrically, and the temperature of the sample measured. Murray and Whalley (1954) used the method of mixtures, with aluminium pellets as the reference standard. Heat transfer between standard and sample was slow, and some extrapolation was necessary to allow for the effects of cooling during mixing. Wierenga *et alia* (1969) used water as a standard, suspending their oven-dried soil in about five times its mass of water. The calorimeter was then heated electrically, increasing its temperature by about 0.4°C in two minutes. The de Vries formula (27) was found to underestimate the measured heat capacity by about 13% for this soil (Yolo silt loam). Högström (1974) suspended a core sample above an electric lamp in a Dewar flask, and measured the rate of increase in temperature of the soil over a period of two hours.

When the specific heat of dry soils is to be determined, account must be taken of heat of wetting. Patten (1909) performed separate experiments in which oven-dried soils were added to water at the same temperature, and found significant amounts of heat developed in this

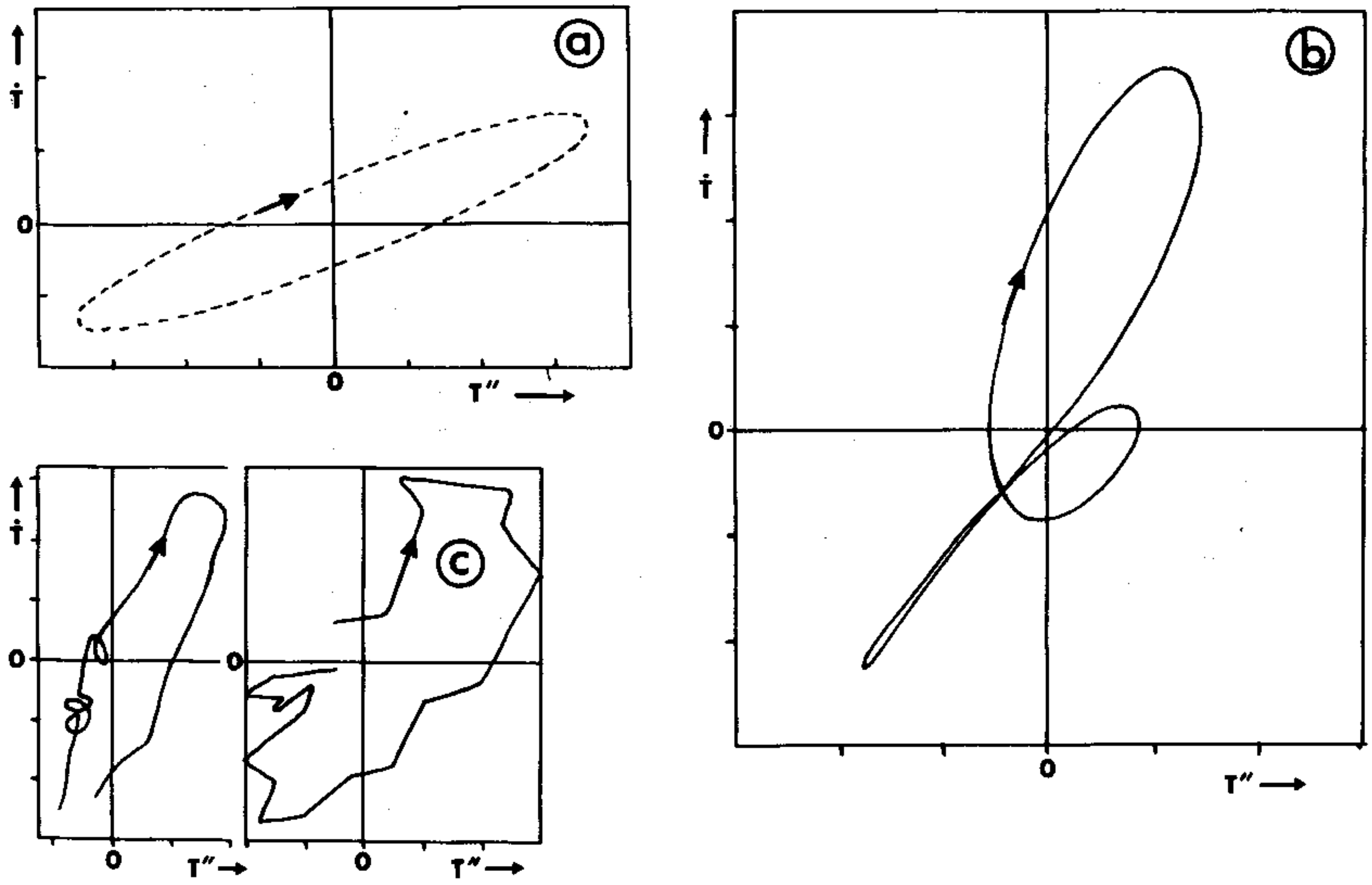

FIGURE 5 (a) For an inhomogeneous soil, plotting $\partial T/\partial t$ ($\dot{T}$) against $\partial^2 T/\partial z^2$ (T") should yield an ellipse.
(b) Lettau (1954) concluded from this graph that the model did not account for the observed facts.
(c) Fuchs & Hadas (1972) obtained a better agreement.

moisture transfer.

Experiments to determine the thermal conductivity of moist soils incur problems with the thermal transfer of water and the consequent change in thermal properties with time. Standard laboratory methods for measuring the conductivity of poor conductors must be modified to take account of this, and a number of approaches have been devised.

Patten (1909) used a transient method, in which one end of a soil column was heated, and rates of change of temperature with space and time were measured at some distance from the source of heat. Direct substitution of $\partial^2 T/\partial x^2$ and $\partial T/\partial t$ in the diffusion equation gave a value for the diffusivity κ. The heat capacity was measured by calorimetry, and the thermal conductivity λ computed from $\lambda = \kappa C$. The effect of thermal transfer of moisture was assumed to be initially localised to the heated end of the column, so that measurements at 50 mm from this face were unaffected.

W.O. Smith and Byers (1938) measured the conductivity of dry soils, using a modification of the standard laboratory method. The oven-dried samples, packed to a known bulk density, were contained in shallow wooden frames with plastic film end covers. An electric heater mounted between two identical samples supplied heat at a known rate, and the temperature drop across the samples was measured by thermocouples.

An attempt by W.O. Smith (1940) to measure conductivities of moist soils by the same method served to demonstrate the effects of moisture transfer. At intermediate moisture contents (around 10% moisture volume fraction) the thermal gradients took longer to stabilise, and samples were found to have developed large moisture gradients, the warm sides being drier.

The use of a cylindrical probe to measure thermal conductivity *in situ* has a long history: Al Nakshabandi and Kohnke (1965) state that it was first suggested by Schleiermacher in 1888. A form of probe which has been used widely is that due to de Vries (1952c). A long electrically heated wire was used as a line source of heat in the soil, and the temperature very near to the wire was measured by a thermocouple. The solution of the diffusion equation for a line source 'switched on' at time zero is

$$T(r,t) = \frac{q}{4\pi\lambda} \left(-\mathrm{Ei}(-r^2/4\kappa t)\right) \qquad (32)$$

where q is the rate of heat production, r is the distance of the measuring point from the source, κ is the diffusivity and t is time. Ei(-x) is a tabulated integral which for small values of x may be approximated by $-0.5772 - \ln x$. Thus if $r^2/4\kappa t$ is small

$$T(r,t) = \frac{q}{4\pi\lambda} \left(c(r) + \ln t\right) \qquad (33)$$

where c(r) is independent of time. Plotting T against ln t gives a

4. DETERMINATION OF THE THERMAL PROPERTIES OF SOIL

Soil is a composite medium consisting of a matrix of mineral and organic particles, with interstices containing water or air. The continuum physics of soils and other composites is based on the premise that the particles and the pores are small when compared with some typical length scale of the medium (for instance the depth of a soil horizon) and distributed in a random fashion. When these conditions are satisfied (and it is easy in the case of soils to imagine systems which do not satisfy these criteria, e.g. blocky and prismatic soil structures) bulk properties of the medium may be defined.

The two fundamental thermal properties are specific heat and thermal conductivity. In soils, the specific heat is used in the form of a heat capacity C =ec, the product of the density and specific heat of the complete soil system in bulk, i.e. solid particles, water and air. Water in particular has a high specific heat, and so is an important contributor to the heat capacity.

The heat capacity in S.I. units is the heat required (in Megajoules) to raise the temperature of one cubic metre of the soil by one Celsius degree (without change of moisture content)

The thermal conductivity depends on the arrangement of the soil particles as well as their thermal conductivities and exhibits a variation with moisture content. Patten (1909) found that the thermal conductivity of a medium sand rose rapidly with moisture content when the sand was in a relatively dry state, then more slowly up to its greatest value at saturation. For loamy and organic soils the rise was more gradual. Al Nakshabandi and Kohnke (1965) found a similar behaviour but there were strong similarities between sand, silt loam and clay soils when the conductivity was considered as a function of soil moisture tension.

The thermal conductivity in S.I. units is the heat flow (in watts) through an area of one square metre of soil subjected to a temperature gradient of one Celsius degree per metre

Conversions from S.I. units to the more familiar c.g.s. units are as follows:

Heat capacity	1 MJ $m^{-3}(^{o}C)^{-1}$	≡	0.239 cal $cm^{-3}(^{o}C)^{-1}$
	1 cal $cm^{-3}(^{o}C)^{-1}$	≡	4.18 MJ $m^{-3}(^{o}C)^{-1}$
Thermal conductivity	1 W $m^{-1}(^{o}C)^{-1}$	≡	2.39 mcal $cm^{-1}s^{-1}(^{o}C)^{-1}$
	1 mcal $cm^{-1}s^{-1}(^{o}C)^{-1}$	≡	0.418 $Wm^{-1}(^{o}C)^{-1}$

sandy soil. Agreement with the result obtained by direct substitution was good. West (1952) calculated two estimates of diffusivity from thermographs and mercury-in-glass thermometers up to 2.44 m deep in a soil which varied from sandy loam on the surface to clay at depth. A final value, incorporating both phase and amplitude, was obtained by least squares fitting to equation (4). Murray and Whalley (1954) also used both the phase and amplitude of the annual wave, using lagged mercury-in-glass thermometers at depths of 0.6 m, 1.37 m and 2 m in soils varying from clay and sand to massive limestone. Pearce and Gold (1959) obtained essentially the same values of κ from the amplitude and phase of an annual wave, using thermocouples at ten depths between 0.05 m and 0.9 m in a clay soil.

The agreement between phase and amplitude estimates of diffusivity is not always good, however, particularly in the upper horizons of the soil, where thermal properties can be expected to vary over a wide range. McCulloch and Penman (1956) reported a variation in diffusivity and conductivity of an order of magnitude in the top 0.1 metre. Carson (1963) stated that, in general, those investigators who used the annual temperature record reported realistic and consistent results, whereas those using the daily cycle did not. He ascribed this to non-diffusive heat transfer processes in the topsoil, but it could equally be due to inhomogeneity (Lettau 1954, McCulloch and Penman 1956). It is significant that the study of Wierenga *et alia* (1969), which used the diurnal wave with some success, was conducted on a deep silt loam with a remarkable homogeneity of composition and bulk density.

Vertical inhomogeneity was incorporated into the diffusion equation with periodic boundary conditions by Lettau (1954) and McCulloch and Penman (1956). Forcing the solution to be periodic yielded expressions for the diffusivity at a given depth, in terms of values and derivatives of the phase and amplitude of the temperature wave. Lettau (1954) went on to ascribe deviations from his model to time-dependent properties, or to non-diffusive processes in the upper layers of the soil.

The analysis of Lettau (1954) and McCulloch and Penman (1956) requires accurate temperature measurements at a number of depths. Where these data are not available, a simple method of combining phase and amplitude estimates of diffusivity is useful. McCulloch (1959) used a simple formula

$$\kappa = \frac{\omega a}{b(a^2 + b^2)} \qquad (35)$$

where the temperature at depth z is given by

$$T(z,t) = T_m + A \exp(-az) \cos(\omega t - bz) \qquad (36)$$

The formula (35) was obtained by McCulloch and Penman (1956) by use of the assumption that a and b (constants in the homogeneous conductor model) varied only weakly with depth.

way. These preliminary experiments allowed corrections to be made for heat of wetting.

Thermal conductivity λ

Theoretical computation of thermal conductivity is a much more difficult problem than the estimation of heat capacity, as the arrangement, size and shape of soil particles and voids is obviously important. De Vries (1952a, 1952b) compared models of the analogous electrical properties of granular media due to Maxwell and others, and concluded that the theory of Maxwell, Burger and Eucken was most applicable to soils over a wide range of moisture contents.

The soil is considered as a continuous medium in which particles of solid material are dispersed in a random fashion. If the particles are considered as randomly oriented ellipsoids, then the thermal conductivity

$$\lambda = \frac{x_o \lambda_o + \Sigma\, k_i x_i \lambda_i}{x_o + \Sigma\, k_i x_i} \tag{29}$$

where x_i is the volume fraction of the i'th constituent, λ_i is its thermal conductivity and k_i is a shape factor. k_i is defined as follows

$$k_i = \frac{1}{3}\, \Sigma_j \left(1 + \left(\frac{\lambda_i}{\lambda_o} - 1\right) g_j^i\right)^{-1} \tag{30}$$

where the summation is over the three principal axes of the ellipsoids, λ_o is the thermal conductivity of the continuous constituent, and g_j^i is a quantity known as the depolarising factor of the ellipsoid in the direction of the j'th axis. The definition of the g_j^i is simplified by the relationship

$$g_1^i + g_2^i + g_3^i = 1 \tag{31}$$

and by the choice of two of the g_j^i as equal (i.e. the ellipsoids are prolate or oblate spheriods, spindle-shaped or disc-shaped respectively). De Vries (1963) showed that for dry soils, taking the continuous constituent as air, formula (29) could be used with a correction factor of 1.25 and $g_1 = g_2 = 0.125$ (consequently $g_3 = 0.75$) for the solid particles to give an estimate of the thermal conductivity. Skaggs and E M Smith (1968) conducted tests on Maury silt loam, and found that the conductivity of the dry soil was approximated by equation (29) with a correction factor of 1.65.

Extension to moist soils demands a variation in g_j^i with water content, as the shape of the air space changes from connected sinuous channels to discontinuous spherical bubbles. For moist soils water is considered as the continuous phase, with solid and air 'particles' interspersed in it. De Vries assumed that g_1 for air would vary from 0.035 for dry soils to 0.333 for wet soils (representing spherical particles). The thermal conductivity of the air-filled pores is enhanced by thermal material with known thermal properties, at a temperature T_1 initially constant, is brought into contact with the soil, which is at temperature T_2. A thermocouple at the contact surface records the temperature $T_o(t)$. If there is no initial thermal gradient in the soil, the temperature at the contact face

$$T_o(t) = \frac{T_1\sqrt{\lambda_1 C_1} + T_2\sqrt{\lambda_2 C_2}}{\sqrt{\lambda_1 C_1} + \sqrt{\lambda_2 C_2}} \tag{37}$$

where subscript 1 refers to the block, 2 to the soil. In practice defective thermal contact causes the constant value given by (37) to be reached slowly (after about two minutes), while a temperature gradient in the soil introduces a term in $\sqrt{t}$, discussed by van Wijk and Belghith (1967). From known values of T_1, T_2, λ_1 and C_1 and the measured (limiting) value of $T_o(t)$ the contact coefficient of the upper soil layer may be calculated. Stigter (1969 and 1970) set out the limitations of the method for the measurement of heat flux (by the use of the term in $\sqrt{t}$) and soil moisture (by calibration against gravimetrically determined samples). Schneider (1969) gave examples of the results obtained with the block method on dry sandy soils.

5. MIGRATION OF SOIL MOISTURE UNDER THE INFLUENCE OF THERMAL GRADIENTS

A temperature gradient in a moist porous medium causes movements of moisture from the warm zone to the cold. The thermal gradient produces gradients of water vapour pressure and capillary potential, which tend to move water in both the liquid phase and the vapour phase. Although the phenomenon is well understood in this qualitative fashion, the relative orders of magnitude of liquid and vapour transfer, and the significance of thermal migration in the field, have long been debated.

Patten (1909) recognised that the imposition of a thermal gradient in a moist soil could induce the movement of soil water. For this reason, he adopted a transient method for the determination of thermal properties, and worked outside the region of steep thermal gradients.

Boujoucos (1915) set up strong thermal gradients across closed soil samples, and measured the flux of moisture by determining the change from an initially uniform moisture content. He attempted to allow only vapour transfer by inserting a 6 mm air gap, and finding no transfer at all, concluded that movement was in the liquid phase, induced by changes in surface tension.

In his attempt to measure the thermal conductivity of moist soils, W.O. Smith (1940) ran head-on into the problem of moisture transfer.

straight line with gradient $q/4\pi\lambda$.

Hadas (1974) investigated the effect of poor thermal contact on the performance of a cylindrical probe, and concluded that *in situ* probes could be used only in soils which were not well aggregated. Applying low thermal outputs for short durations would minimise errors due to thermal transfer of moisture.

Although natural fluctuations of soil temperature are often used to evaluate the thermal diffusivity of the soil, the conductivity cannot normally be found directly by this method. However, McCulloch and Penman (1956) derived an equation which can be used to define the variation of conductivity with depth, and its absolute value given one value at a known depth. Using the notation of equation (19), letting $p = -a_n'/a_n$ and $q = \alpha_n'$

$$\frac{d}{dz}\ln\lambda = \frac{1}{\lambda}\frac{d\lambda}{dz} = \frac{p^2 - q^2 - \partial p/\partial z}{p} \quad (34)$$

Given one value $\lambda(z_o)$, the value of λ at any depth z can be found by numerical integration.

Thermal conductivity and diffusivity may be found by subjecting the soil surface to a short burst of radiation from an electric lamp. Van Wijk (1963) described a device incorporating a heat flux plate and several soil thermocouples, which was placed in the upper layer of soil and irradiated. Laplace transforms of temperatures measured at the upper and lower surfaces of the heat flux plate and at several depths in the soil were used to determine the diffusivity and the conductivity. The laboratory calibration of the heat flux plate, obtained under steady conditions, was not applied, but instead the Laplace transform method was used to determine the instantaneous flux of heat through the plate.

Thermal diffusivity κ

The thermal diffusivity of a medium may be determined from measurements of temperature alone. The simplest technique, in principle, is that employed by Callendar and McLeod (1896) and Patten (1909), who measured the spatial and temporal derivatives $\partial^2T/\partial z^2$ and $\partial T/\partial t$ and substituted these values into the diffusion equation. This apparent simplicity hides the very real experimental difficulties in such a determination: the evaluation of $\partial T/\partial z^2$ requires very accurate absolute measurements of temperature at known locations.

By far the most popular method for diffusivity measurement is that based on Fourier's homogeneous conductor with a periodic boundary condition Variations of the method encountered in the literature include the use of annual, diurnal and semi-diurnal waves, and estimation of κ from phase, amplitude or both. Thomson (1861) used the phase and amplitude of the annual wave, measured at depths between 0.91 m and 7.32 m in sandstone. Callendar and McLeod (1896) made use of the amplitude of the annual wave, measured at four depths from 0.5 m to 2.74 m in a by air gaps and wire mesh screens. A wire screen technique had been used by Bouyoucos (1915) but his experiment came in for criticism from W.O. Smith (1943) and Taylor and Cavazza, who found vapour flux, transmitted through the air gap while liquid flow was prevented, to be very significant.

Hadley and Eisenstadt (1955), working with radioactive tracers in a medium composed of spherical glass beads, confirmed the existence of a liquid-vapour cycle in a closed system when the moisture was above a critical value, which they identified with the moisture content at which connected capillary threads existed.

A more complete theory of concurrent heat and moisture movement was developed by Philip and de Vries (1957), who set out to account for the observed large value of vapour transfer and its dependence on moisture content. An extension of the simple theory of gas diffusion in porous media was proposed, in which enhancement was effected by transfer through 'liquid islands': short stretches of capillary water with condensation and evaporation at opposite faces. This model accounted for the observed liquid-vapour cycle in the closed system where moisture exceeded that critical value at which the liquid conductivity falls to very low levels. The effect of the model on the diffusion equation was to add two terms, the first an enhancement of the thermal conductivity to a value λ^* which included a distillation effect, and the second a convective term depending on the moisture gradient:

$$C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda^* \frac{\partial T}{\partial x}\right) - L\rho_o \frac{\partial}{\partial x} \left(D_{\theta vap} \frac{\partial \theta}{\partial x}\right) \qquad (39)$$

where θ is moisture volume fraction, ρ_o is the density of water and L is the latent heat of vaporisation at temperature T. $D_{\theta vap}$ is a diffusivity for vapour transport by moisture gradients.

Philip and de Vries' explanation of vapour flux enhancement received some confirmation from Woodside and Kuzmak (1958), who conducted model tests to determine the magnitudes of vapour pressure gradients within pores, and found these to be several times larger than those used in the simple diffusion model.

The formulation of Philip and de Vries had not taken account of heat of wetting, or of the transfer of sensible heat by liquid and vapour phases. This was rectified with some loss of simplicity by de Vries (1958a). De Vries argued that, in cases where moisture changes were caused by applied temperature gradients, changes in liquid and vapour content could be of the same order, and it was necessary to couple the equations governing heat and moisture transport.

Laboratory verification of transfer of moisture by temperature gradients is mostly confined to closed systems, like the sealed samples used by W.O. Smith (1940). However, Kuzmak and Sereda (1957a and 1957b), by experiments with porous plates in close proximity and with salt tracers, were able to show that the movement of water by thermal gradients was in the vapour (or vapour with liquid islands) phase. They were

An interesting variation on this theme was used by Jackson and Kirkham (1958). Moisture moving under the influence of thermal gradients introduces non-linear behaviour into the equations. The smaller the temperature gradients, the less significant will be the migration of moisture. Jackson and Kirkham's approach was to subject a soil sample in the laboratory to artificially produced periodic boundary conditions. Temperatures were measured inside the soil sample and diffusivity estimates made for each of a number of frequencies of the forcing condition. As the frequency increased, the attenuation of the wave became more effective and smaller temperature gradients were observed within the soil. Extrapolation to infinite frequency gave a value of the thermal diffusivity with no moisture migration.

The Laplace transform method has been used by van Wijk and his associates, at Wageningen and Tunis. Applications have included determination of the diffusivity of

(i) sandy clay soil on a cloudy day with short periods of bright sunshine (van Wijk 1963)

(ii) sand and peat subjected to pulses of light from an electric lamp (van Wijk 1963, van Wijk and Derksen 1966)

(iii) sandy soil on a day with definite deviations from a sinusoidal variation of temperature (van Wijk and Derksen 1966)

(iv) air in a vegetative canopy during the day (van Wijk and Goedkoop 1963)

The obvious advantage of the Laplace transform method is its ability to deal with non-periodic variations in temperature. However the method requires an initial condition which is somewhat restrictive. In (i), (ii) and (iii) the soil temperature was initially uniform, while in (iv) unstable atmospheric conditions ensured thorough mixing of the air at the beginning and end of each day. Feddes (1971) found that the Laplace transform method was very sensitive to deviations from this initial condition, with departures of $1^{o}C$ having a large influence on the results.

Hadas (1968a) used two methods for determination of the diffusivity in the laboratory. A difference was observed between estimates of κ from a sinusoidal boundary condition and a square wave, analysed by Laplace transforms. However, the initial condition for the Laplace transform method was not mentioned.

Contact coefficient λC

The product λC is known as the contact coefficient (van Wijk 1963, van Wijk and Derksen 1963) and is important in problems where heat flux is considered. The square root of λC is sometimes known as the thermal admittance (Stearns 1969). Van Wijk (1964) proposed a method for the measurement of λC in the field. A 0.1 m cube of plastic the vapour phase), and β^* is equal to $-\, d\theta/d(\ln \Theta)$ in the final steady state.

Cary (1965) set up a column experiment in which moisture tension gradients and thermal gradients could be controlled, and measured flows of heat and water across the sample. Separate determinations were made of vapour flow and liquid flow due to thermal and moisture gradients. Enhancement of vapour transfer was found to be very effective: movement of vapour through the soil was greater than through still air. A model of liquid movement by thermal gradients was proposed. Water molecules exist in free and bonded states: the lower the temperature the more molecules are bonded. At any given cross-section, more molecules are free to make the transition from warm to cold than are free to move in the reverse direction. The result is a net transfer of water molecules down the temperature gradient. As thermal movement has been observed in saturated and unsaturated media, Cary considered that this mechanism was more important than variation in surface tension, which would affect only unsaturated media. Comparing water flows induced by thermal and moisture gradients, Cary found that for his loam sample a temperature gradient of 50° C/m at a soil moisture tension of 50 mm Hg would move as much water through the soil as a tension gradient of 147 mm Hg per metre. At a suction of 340 mm Hg this thermal gradient was equivalent to a tension gradient of 18400 mm Hg per metre. In most cases the transfer of water in the vapour phase was less than the thermally-induced transfer in the liquid phase.

The equation proposed by Cary for the flow of heat in an unsaturated soil was

$$J_q = -\left(\rho_o QK + \frac{\beta D h e_s g}{R^2 \Theta^2}\right)\frac{\partial \phi}{\partial x} - \left(\lambda + \frac{\beta D h e_s L^2}{R^2\ \Theta^3}\right)\frac{\partial T}{\partial x} \qquad (44)$$

where ρ_o is the density of water

Q is the heat of transport (of the order of 100 J/kg)

K is the unsaturated hydraulic conductivity of the soil

β is an enhancement factor

D is the diffusivity of water vapour in air

h is the relative humidity of the pores

e_s is the saturation vapour pressure

g is the acceleration due to gravity

R is the gas constant

Θ is the temperature in $^{\circ}$K

ϕ is the moisture potential expressed in height of a water column

λ is the thermal conductivity of the soil (in the absence of vapour transfer)

L is the latent heat of vaporisation of water

In closed soil samples there was substantial transfer of water at intermediate moisture contents, so that a large moisture gradient was set up to oppose the temperature gradient in the eventual steady state. In a later paper Smith (1943) set out to explain the phenomenon, which in his experiments had been much more significant than Bouyoucos had suggested. Smith's theory involved the movement of capillary globules, triggered by evaporation and condensation on opposite faces. This explained the dependence of moisture movement on water content, and also the observed changes caused by fragmentation of samples.

De Vries (1950) calculated the heat transfer associated with the evaporation, diffusion and condensation of water vapour in moist soils. It was found that the effect of vapour transport was an increase in the thermal conductivity of the soil of a few per cent at ordinary temperatures. Terms which were non-linear in the temperature gradient were found to be insignificant in magnitude. In de Vries' notation the diffusion equation becomes

$$C \frac{\partial T}{\partial t} = (\lambda + \alpha\phi f_1(t)) \frac{\partial^2 T}{\partial x^2} + \alpha\phi f_1(T) f_2(T) \left(\frac{\partial T}{\partial x}\right)^2 \qquad (38)$$

where $f_1(T)$ and $f_2(T)$ are functions of temperature, latent heat of vaporisation and saturated vapour pressure, and $\alpha\phi$ is a function of the soil structure and water content. $f_1(T)$ was strongly dependent on temperature, and the additional conductivity could vary from 0.1% to 45%, its value at 50°C being five times that at 20°C. Vapour transfer would therefore be far more important in tropical than in temperate regions. Hursh and Pereira (1953), in a survey of the soil hydrology of the Shimba Hills, Kenya, discussed the possibility of upward movement of vapour by large nocturnal temperature gradients near the soil surface. This process would expose a much greater proportion of soil water to evaporation, and would be a significant factor in the comparison of the effectiveness of grassland and forest for water conservation.

Gurr *et alia* (1952) assessed the contribution of liquid and vapour flux to the temperature effect by measuring the concentration of a soluble salt in the soil water. The salt should be convected by liquid flow, but left behind on evaporation, so that in the absence of osmotic effects the salt would act as an inert tracer of liquid flow. It was found, in fact, that movement of water by thermal gradients could take place against osmotic gradients. Liquid flow occurred in the direction of cold to hot, while vapour moved in the opposite direction. However, this was in a closed system, where an initial movement of water in both liquid and vapour phases set up a moisture gradient which then induced a reverse flow of liquid, resulting in a dynamic equilibrium with vapour flow exactly opposing liquid flow. The quantity of vapour flow was found to be several times the value predicted by Penman (1940) from an analogy with gas diffusion through porous media.

This enhancement of vapour flux was also encountered by Taylor and Cavazza (1954) in an experiment with a soil column divided into slices for vapour flux enhancement by some mechanism.

A more direct measurement of soil water flux was performed by Jackson *et alia* (1973), who measured moisture content and evaporative flux at half hour intervals in bare soil lysimeters. They found a rapidly fluctuating plane of zero moisture flux, with rapid drying of the soil surface in the morning caused by upward flux above 10 mm depth and downward flux below that depth. Changes in direction of the moisture flux at a given depth could take place up to four times a day.

Cassel *et alia* (1969) set out to test the Philip and de Vries (1957) and Taylor and Cary (1964) theories against laboratory evidence obtained from horizontal soil columns under imposed temperature gradients. It was found that the Philip and de Vries equation fitted observed moisture fluxes best, with Penman's (1940) diffusion model (without enhancement) giving an estimate about five times too small, and Taylor and Cary's prediction being about 10-40 times too small.

Fritton *et alia* (1970) used Philip and de Vries' approach to compute the upward flux of water in a soil column subjected to radiation at its upper surface. A similar column where wind was used as an evaporating agent was found to follow the simple law of isothermal diffusion.

Laboratory column experiments are generally performed with imposed constant thermal gradients. Westcot and Wierenga (1974) inserted soil columns into a field plot, and observed the response of the columns to natural daily temperature fluctuations. A comparison was made of observed temperature variations with predictions made by a CSMP model (Wierenga and de Wit 1970) with and without a vapour flux component. The vapour flux was computed using Philip and de Vries (1957) theory with an enhancement factor to allow for microscopic temperature gradients across air-filled pores, and the 'liquid island' effect. The comparison indicated that the vapour flux contribution to soil heat flux should be included in estimates made for dry soils, and that heat flux plates in particular were unlikely to record the true flux of heat in the surface layers, as they impeded vapour transfer.

Jury and Miller (1974), in an experiment designed to measure all the transport coefficients L_{ik} in equation (42) for the coupled flow of heat and moisture, obtained values for thermal conductivity within $\pm$ 7% of de Vries (1963) estimates. The movement of heat by isothermal moisture transport was found to be negligible in this experiment. The other cross-coupling coefficient, governing the movement of liquid moisture by thermal gradients, was several times larger than the value predicted from surface tension considerations (compare with Cary 1965).

Jackson *et alia* (1975) conducted experiments on the lysimeter site described in an earlier paper (Jackson *et alia* 1973). Calculated water fluxes and heat fluxes were compared with measured values, and it was found that consideration of the vapour fluxes made no significant improvement to the agreement obtained from isothermal moisture flux and pure conduction. Kimball *et alia* (1976a), analysing data from the same site, attempted to modify the de Vries (1963) formulation, particularly the variation of the air shape factor g_j with unable to detect vapour movement induced by moisture gradients. Müller-Stoll and Lerch (1963) conducted experiments on soil columns, in which moisture transported by thermal gradients was removed from the system by transpiring plants. They concluded that in some climatic regions, notably in dry lands, but also where soils were frozen over winter, upward vapour transfer at night and in winter may be important in maintaining higher moisture levels in the upper soil than would otherwise exist.

Using a theory advanced by Krischer and Rohnalter in 1940 de Vries (1963) proposed a simple method for estimating the thermal conductivity of a soil taking into account the phenomenon of vapour transfer. Using the ellipsoid model described above, an estimate was required of the conductivity of the vapour- and air-filled pores. This conductivity was considered as the sum of two components, one due to normal heat conduction, the other λ_v due to vapour transport. Krischer and Rohnalter's formula for λ_v^s where air in the pores is saturated with water vapour, was

$$\lambda_v^s = \frac{LDp}{M_W R \Theta (p-e_s)} \frac{de_s}{d\Theta} \tag{40}$$

where L is the latent heat of vaporisation, D the diffusion coefficient of water vapour in air, p the atmospheric pressure, e_s the saturation vapour pressure, M_W the molecular weight of water, R the universal gas constant and Θ the temperature in degrees absolute (^oK).

For soils where the air is not saturated with water vapour (at very low moisture contents) the value of λ_v^s was modified:

$$\lambda_v = h\lambda_v^s \tag{41}$$

where h is relative humidity expressed as a fraction.

Taylor and Cary (1964) applied the techniques of irreversible thermodynamics to the problem. From a very general development of linear equations of the form

$$J_i = \sum_{k=1}^{N} L_{ik} X_k \quad (i = 1,2....N) \tag{42}$$

where J_i is the flux of the i'th component, L_{ik} the phenomenological coefficient due to the k'th driving force affecting the i'th flux, and X_k the driving force for the k'th component, Taylor and Cary derived an equation for the movement of moisture in a sealed horizontal column

$$J_w = - D_\theta \left(\frac{\partial\theta}{\partial x} + \beta^* \frac{\partial}{\partial x} \ln \Theta \right) \tag{43}$$

where D_θ is a diffusion coefficient equal to the sum of Philip and de Vries' coefficients $D_{\theta liq}$ (for the liquid phase) and $D_{\theta vap}$ (for

1. The evaporation rate from a water table is either the evaporation rate from saturated soil subjected to the same conditions or a function only of the water table depth (and the soil characteristics), whichever evaporation rate is the lesser.

2. The heat flux has only a small effect on evaporation in fairly moist soils (water table within 1 m of the surface) - ie. an isothermal moisture flow model works - but exerts a growing (and ultimately dominant) influence as the soil becomes drier.

3. For water tables more than 2 m from the surface, a downward heat flux inhibits evaporation more than an upward one increases it.

Conclusion (3) indicates that evaporation from the soil may be least when meteorological conditions are apparently most favourable for evaporation ie seasonally in early summer, or diurnally in the late morning, and that it would be futile to attempt to relate evaporation from dry soils to evaporation from wet soils or open water under the same climatic conditions.

Neglect of the soil heat flux term in the surface heat budget leads to a seasonal imbalance in the assessment of evapotranspiration by an energy balance or combination method. Where daily evapotranspiration is computed, only the daily net loss or gain of heat is significant, and this shows a seasonal variation. Edwards and Rodda (1970) attempted to explain a marked seasonal imbalance observed in the Ray catchment by calculating a heat flux term. Although the phase of the imbalance was matched, the heat flux component did not account for its amplitude. Thom and Oliver (1977) were able to show that a combination of a revised aerodynamic term in the Penman evapotranspiration equation and a heat flux term could explain the imbalance both in phase and in amplitude.

As the net radiation is the source of the energy transmitted as conducted heat through the soil, several authors have taken the natural step of correlating soil heat flux with net radiation. Fuchs and Hadas (1972) found that, in a deep sandy loam at Gilat, Israel, approximately 30% of the net radiation falling on bare soil was conducted into the soil. This figure was the same for wet and dry soils, indicating that the larger thermal admittance ($\sqrt{\lambda C}$) of the wet soil was balanced by an increased evaporation term, wind and air temperature conditions being similar. However, they stressed the importance of direct measurement of soil heat flux in heat balance studies.

The approach used by Idso *et alia* (1975) was to derive a relationship between the available energy flux, R_N-G, as a fraction of net solar radiation, and soil moisture in the upper 20 mm of soil for a smooth bare loam soil at Phoenix, Arizona. The relationship was found to hold true for different seasons. Available energy was found to vary greatly with water content, the soil heat flux being most important for dry soils, where the limited availability of moisture controlled the evaporation rate.

The additional term in the thermal conductivity is similar in form to that put forward by de Vries (1963). The gradient of the saturation vapour pressure curve

$$\frac{de_s}{d\Theta} = \frac{L M_w e_s}{R\Theta^2} \qquad (45)$$

from the Clausius-Clapeyron equation (see for example Shuttleworth 1975), and the Cary expression is almost identical with de Vries' expression with the exception of the β coefficient, which arises from the structure of the porous medium, and from 'liquid islands' as in the Philip and de Vries model.

The next logical stage in the analysis of thermal moisture flux was the application to field conditions, where temperatures vary widely over the day, and the movement of moisture is of great importance. Cary (1966) used the above theory to predict vertical movements of water, in the liquid and vapour phases, induced by a diurnal temperature variation. The conclusion was that although thermal moisture flux is not an important component in bulk soil hydrology, locally the effects could be very important, for instance in re-establishing capillary moisture contact in the critical regions around plant roots. The vapour flux would also be important in the transport of salt in the soil, as the return phase of a 'salt pump' operating in the upper horizon of the soil. Where a frozen zone exists in the soil, thermal flux tends to add water and contribute towards 'frost heaving' of the soil.

Hadas (1968b) subjected soil samples to a periodic heating (from 4 minutes to 32 minutes period) and found that moisture transfer was in excess of that predicted by Cary (1966). He concluded that the theory was as yet incomplete, and that the factor β, in particular, demanded detailed examination.

In a later paper, Hadas (1969) attributed the increase in moisture flux above that predicted by Philip and de Vries and Cary to mass flow. Pressure variations in the soil air could cause convection of vapour by air movements, a process which would be significant for short-period fluctuations.

Vapour movement in the field was measured by Rose (1968a and 1968b) at Alice Springs, using gravimetric water content determination and thermistors for temperature measurement in the upper 130 mm of soil. Over a period of several days, the movement of moisture as isothermal liquid and vapour flux (in response to tension gradients), liquid flux arising from temperature gradients and gravitational liquid flux were calculated, the only unknown being vapour flux induced by temperature gradients. The temperature-induced and gravitational liquid fluxes proved to be very small by comparison with isothermal flux and temperature-induced vapour flux. An involved error analysis produced the, albeit tentative, conclusion that there was evidence method was employed by Monteith (1958) who immersed the plate in soil from his experimental site. Later investigators (Fuchs and Tanner 1968, Mogensen 1970, Biscoe *et alia* 1977) also used methods where the flux plate was embedded in a granular medium, usually sand. Fuchs and Tanner used a symmetrical arrangement where the heat generated by an electric heater, placed centrally, passed through one heat flux plate on each side, the symmetry ensuring that half of the heat developed passed through each plate. Biscoe *et alia* used a secondary heater, whose output was controlled to ensure zero heat flux through the back of the primary heater. Mogensen set up a temperature gradient along a column of material of known conductivity, the temperature difference being maintained by two circulating water baths. A similar apparatus was used by Brach and Mack (1969) who sandwiched a heat flux plate between two water baths, and used the known dimensions and thermal conductivity of the plate to compute the flow of heat for a given temperature difference. Idso (1972) used a technique originally devised for the calibration of long-wave radiation meters. The heat flux plate is located near to a heated or cooled blackened metallic plate in a controlled environment. The radiation received by the heat flux plate is re-radiated from both surfaces at very nearly equal rates, so that the flow of heat through the plate must be equal to half the net radiation received by the plate. Good agreement was found between the radiative method of calibration and a conductive method.

Once heat flux plates are installed in the soil profile, usually by horizontal insertion from a pit which is backfilled and left to equilibrate over a period, it is very difficult and time-consuming to check on their attitude, thermal contact with the soil, depth and calibration. Roach (1955) unearthed a heat flux plate after one year of operation at 0.15 m depth and found that it had sunk 20 mm and tilted by 15°. Anchoring the plate securely will ensure a correct attitude, but thermal contact will be lost if the surrounding soil subsides. Stearns (1969) used Lettau's (1954) model of heat diffusion in inhomogeneous media to check on the calibration and depth of heat flux plates *in situ*. Högström (1974) computed the change in heat content of the soil below the heat flux plate over a period of one hour, and equated this quantity to the heat flux through the plate.

A more fundamental problem of the heat flux plate was mentioned briefly by Monteith (1958). If the thermal conductivity of the plate is different from that of the soil, the temperature field in the soil will be disturbed by the presence of the plate, and more or less heat will flow through the plate than would have flowed through the same area of soil in the absence of the plate. Portman (1958) proposed an exponential form for this heat flow ratio:

$$f = \frac{q_t}{q_s} = \exp\left(a \frac{L}{W} (1 - \varepsilon^{-1})\right) \qquad (47)$$

moisture content, but concluded that a consistent analysis including vapour transfer was not possible.

Evidence supporting Philip and de Vries' model was provided by Hadas (1977) who compared predicted and observed values of the thermal conductivity of moist soil, and found that the de Vries model predicted transfer of heat by vapour quite accurately in steady conditions, but underestimated it under nonsteady conditions. Predicted values were close to those observed in moist soil (moisture volume fraction 15%) but not for dry soil (moisture volume fraction 5%).

6. SOIL HEAT FLUX

In many contexts it is not the temperature of the soil that is of importance, but the flow of heat induced by temperature gradients. The soil heat flux assumes this importance in meteorology and hydrology as a component of the energy budget at the ground surface.

Soil heat and the energy budget

Neglecting the effects of vegetative canopies, the energy budget of the ground surface may be stated as:

$$R_N = LE + H + G \qquad (46)$$

where R_N is net radiation, L is the latent heat of vaporisation, E the actual evapotranspiration rate, H the flux of sensible heat to the atmosphere and G the soil heat flux. The partition of the net radiation input into latent heat flux, sensible heat flux and soil heat flux has often been determined in the field, but a satisfactory model for theoretical prediction of the relative magnitude of the terms does not exist.

Penman (1948 and 1963) in setting up a heat budget for the ground surface, assumed that storage of heat in the soil would be negligible over a period of several days, and reduced the heat budget to net radiation, evaporation and transfer of sensible heat by the atmosphere, although he admitted (Penman 1957) that heat transfer to the soil may be important in some times, places and conditions.

Soil moisture was known to affect evaporation and transpiration in a somewhat uncertain way (Penman 1957), and Philip (1957) put forward a theory to quantify the control exercised by the soil on the rate of evaporation from a bare soil surface. This theory necessitated a partition of incident energy at the soil surface, and an explanation of soil moisture conditions based on the ideas of Philip and de Vries (1957). Heat flux was an important factor in the discussion, leading to the following conclusions:

Calorimetric and combination methods for the determination of soil heat flux

There are two ways of using measured soil temperatures and thermal properties to compute the soil heat flux. These are the temperature gradient method and the temperature integral method. The first method uses a known thermal conductivity and a measured soil temperature gradient at a given depth to give the flux of soil heat at that depth. The second method consists of an integration of the heat content of the soil below the given depth, and differentiation with respect to time to give the heat flux. The difficulty of integrating the heat content for the deeper levels has given rise to the 'combination' methods, which use an estimated heat flux at a lower boundary.

The temperature gradient method

The heat flux induced by a thermal gradient is

$$G(z) = -\lambda\frac{\partial T}{\partial z} \tag{50}$$

Staley and Gerhardt (1957) used a needle probe to determine the thermal conductivity λ, while Kimball *et alia* (1976 b) computed λ by de Vries' (1963) estimation method. Although the temperature gradient method is sufficiently accurate to estimate the heat flow at depth, for instance as a lower boundary condition for the calorimetric method (Kimball and Jackson 1975, Kimball *et alia* 1976b), the measurement of λ for the upper layers of soil is not easy, and the thermal conductivity may show pronounced variations in the horizontal direction (Buettner 1957, Scharringa 1976). Thus for surface heat flux, which is the quantity of most interest to the hydrologist and meteorologist, the temperature gradient method would be impractical.

The temperature integral method

The temperature integral method is based on the relation

$$G(z) = \frac{\partial}{\partial t}\int_z^{\infty} C(\xi, t)T(\xi, t)d\xi \tag{51}$$

which expresses the heat balance for the column of soil below the reference depth z. The integral is usually evaluated as the sum of the heat contents of a number of layers. The obvious problem of (51) is the extension to infinity of a necessarily finite sequence of layers. This is overcome in the truncated form of (51)

$$G(z) = \frac{\partial}{\partial t}\int_z^{z_o} C(\xi, t)T(\xi, t)d\xi + G(z_o) \tag{52}$$

In the presence of vegetation, the transfer of heat to the soil is modified by the effects of the vegetation canopy, which intercepts most of the incoming radiation, and the layer of relatively still air within the vegetation. Monteith (1958) performed experiments with heat flux plates beneath three crops, potatoes, wheat and grass, and found that while the flux into the soil during the day was about 20% of net incoming radiation, the flux out of the soil at night was of the same order as the net outgoing radiation, and often almost equal to it. Daytime heat flux showed a decrease with the development of the crop (due to the interception of incoming radiation by the canopy), while nocturnal fluxes were strongly dependent on windspeed, demonstrating the effect of a downward sensible heat flux. Monteith went on to estimate possible errors in evaporation figures consequent on neglect of the soil heat flux term, and concluded that during prolonged periods of fine weather potential evaporation may be overestimated by 7%.

Equation (46) describes vertical transfer of energy, such as would be expected on a uniform plane land surface. In the real world horizontal transport of energy may be important. De Vries (1958b) calculated the transfer of heat from a mass of warm air moving laterally from an area of different thermal conditions, and concluded that advective energy could be very important, particularly for irrigated plots in arid zones.

Measurement of soil heat flux - the heat flux plate

An obvious need in the study of the surface energy budget is a simple method for the measurement of the vertical flux of heat through the soil. Hydrologists and meteorologists, encouraged by the relative unimportance of the net daily heat flux, have favoured the use of a single sensor, which can be linked easily to a data acquisition system.

The heat flux plate measures the flow of heat by recording the temperature difference across a thin plate of material whose thermal conductivity is similar to or greater than that of the soil. Deacon (1950) described the construction of a heat flux plate consisting of a thin disc of bismuth cast between two copper discs. The bismuth-copper sandwich behaved as a thermocouple, producing an e.m.f. of about 86 μV for a heat flux of 1 KW m^{-2}.

This early form of heat flux meter was superseded by plates made of an insulating material, e.g. Paxolin or glass, with multiple thermocouples on the faces. Deacon (1950) constructed a glass heat flux plate with an e.m.f. of about 1700 μV for a heat flux of 1 KW m^{-2}, while Monteith (1958) used a Paxolin plate with a calibration of 17 mV for 1 KW m^{-2} flux.

Calibration of the plates is usually performed in the laboratory, in a medium with a similar thermal conductivity to that of soil. Some workers have chosen to omit this medium: for instance Deacon (1950) placed the disc in contact with a copper block with heaters arranged to give a known vertical flux through the plate. A variation of this determination of $G(z_o)$ in the temperature-integral method.

7. SUMMARY

The scientist makes progress in his field by putting forward hypotheses and testing them against observed reality. The simplest hypotheses fit some of the facts, perhaps in a qualitative way, but at every stage there are unaccountable discrepancies requiring the next stage of sophistication in the working hypothesis.

Fourier's model of conduction of heat in the soil gives a qualitative explanation of the facts, but spatial inhomogeneity and non-conductive processes lead to deviations from the simple model. Inhomogeneity may be included in the model in a vertical sense, but a deterministic description of a three-dimensionally inhomogeneous system would be difficult to construct and of doubtful value. Models of non-conductive processes such as moisture migration must be generally applicable to be useful, and current understanding of the topic falls far short of a model valid for all soils in all climates.

It is important to maintain a clear perspective in this field as in any field of research. A model which fails at an extreme may be valid for many purposes - for instance a good estimate of the net daily heat flux may be obtained by ignoring heat storage in the upper soil layers, and computing flow at depth. No improvement in the estimate would result from a detailed understanding of processes in the top few centimetres. The various theories of thermal moisture migration may be most useful at the present for determining whether or not thermal transfer may be neglected.

There is a growing movement in the hydrological literature to incorporate probabilistic concepts into deterministic models, and to question the confidence attached to the values of parameters in environmental systems. The manifest inhomogeneity of soils could well be described in a statistical way, with the result that predictions would carry uncertainties, and modifications of the model would be made on the basis of sensitivity analysis. The extension of current models to cover horizontal variation of boundary conditions and physical properties will certainly require this type of approach, as will the application of profile data to the regional scale.

where q_t is the heat flow through the plate (transducer)

q_s is the heat flow through undisturbed soil

L/W is the ratio of thickness to diameter of the plate

$\varepsilon = \lambda_t/\lambda_s$ where λ_t and λ_s are the conductivities of the plate material and soil respectively

and a is a positive constant. Philip (1961) developed an exact theory for a plate in the form of an oblate spheroid, and proceeded to the limiting form of a thin disc, deriving the formula

$$f = \frac{1}{1 - 1.92\,\frac{L}{W}\,(1 - \varepsilon^{-1})} \qquad (48)$$

Philip set out a number of conditions to be satisfied by heat flux plates to minimise the variation of the flux ratio f.

(1) The meter should be made as thin as possible and placed with its minimum dimension in the direction of the heat flow in the medium

(2) The meter should be calibrated for the range of soil thermal conductivities to be encountered in its use

(3) The thermal conductivity of the meter material should be made as great as possible.

Philip considered poor thermal contact to be a serious source of possible errors, particularly as the presence of gaps between the meter and the soil would be unknown to the observer. Fuchs and Hadas (1973) assessed a heat flux plate designed by Tanner, and concluded that the aluminium plates with which it was faced were extremely valuable in decreasing its contact resistance, presumably by lateral transfer of heat.

The proper use of Philip's or Portman's equations, or an empirical calibration of the plate for differing soil thermal conductivities, demands a knowledge of the thermal conductivity of the soil at the time of measurement. Cary (1971) devised a method, using two heat flux plates, which would eliminate this requirement. If the meters are identical in all but their conductivities λ_1 and λ_2, then the heat flux through the soil

$$q_s = A\, q_1 \left(\frac{n-1}{n-m}\right) \qquad (49)$$

where q_1 and q_2 are the heat fluxes through the two meters

$$m = q_1/q_2$$
$$n = \lambda_1/\lambda_2$$

and A is a constant to be determined by calibration.

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The heat flux at the depth z_o may be evaluated by any of a number of methods. Swinbank (1948) used equation (52), with $G(z_o)$ assumed zero, in an attempt to estimate the thermal conductivity of the soil, by combining the temperature gradient and temperature integral estimates of $G(z)$. However the results obtained were very scattered.

Carson and Moses (1963) used the temperature integral method to estimate the total heat stored during annual and diurnal cycles. The heat capacity at various depths in the soil profile was measured by calorimetry, and temperature data were obtained using the network of resistance thermometers described by Carson (1963). Heat flux over the day was found to follow the net radiation quite closely.

When heat flux plates are inserted in the soil profile, estimates of the surface heat flux must be corrected for the heat storage occurring in the soil above the plates. Fuchs and Tanner (1968) used the temperature-integral method to correct for the flux divergence in this layer, and found that the phase shift between surface heat flux and the flux at a depth of 75 mm was about 90 minutes, while the amplitudes were in the ratio of approximately 1:0.7. Where daily heat flux measurements were required the heat storage above the heat flux plate would be insignificant.

The heat flux at the reference depth z_o may be measured by a heat flux plate. This is described as a 'combination method', and Hanks and Jacobs (1971) compared the combination method with a 'calorimetric method', which involved the assumption that the heat flux below some reference depth was zero. Their conclusion was that the calorimetric method was best, but Hanks and Tanner (1972), after a more detailed analysis of the results, stated that an accurate calorimetric determination required measurements down to at least four metres, or an estimate $G(z_o)$, and that either method could be used with care.

Kimball and Jackson (1975) proposed a 'null-alignment' method, in which a plane of zero heat flux was used. An estimate of the thermal conductivity λ^* was first made for the depth z_o (at say 0.2 m). The heat flux at depth z was then estimated by

$$G^*(z) = \frac{\partial}{\partial t} \int_z^{z_o} C(\xi)T(\xi)\,d\xi - \lambda^* \left.\frac{\partial T}{\partial z}\right|_{z_o} \tag{53}$$

Temperature profile data were then used to define a depth z_n at which the temperature gradient, and hence $G(z)$ was zero. A better estimate of the heat flux is then obtained by

$$G(z) = G^*(z) - G^*(z_n) \tag{54}$$

In computing the surface heat flux, Kimball *et alia* (1976b), used an estimated λ at a depth of 0.2 m, and concluded that the sophistication of the null-alignment method was not required for this case, but estimates of λ by de Vries' method were sufficiently accurate for the

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Lettau (1954) considered the amplitude a_n and phase α_n of a Fourier component

$$T_n(z,t) = a_n \cos(n\omega t - \alpha_n) \tag{19}$$

to vary with z, as a consequence of the variation of diffusivity with depth. Substituting the expression (19) into the diffusion equation (1) gives the two simultaneous equations

$$\lambda' a_n' + \lambda(a_n'' - a_n(\alpha_n')^2) = 0 \tag{20}$$

$$\lambda' a_n \alpha_n' + \lambda(2a_n'\alpha_n' + a_n\alpha_n'') = -\rho c n\omega a_n$$

in which the prime signifies differentiation with respect to z. λ may be eliminated from the equations to give

$$\kappa = \frac{\lambda}{\rho c} = \frac{n\omega a_n a_n'}{a_n a_n''\alpha_n' - a_n^2 (\alpha_n')^3 - 2(a_n')^2\alpha_n' - a_n a_n'\alpha_n''} \tag{21}$$

Lettau's equation involved phase variables from the wave of heat flux corresponding to (19), but these were derived by a single differentiation of (19), so that (21) represents a more direct development. Equation (21) is, as Lettau stated, an exact equation yielding values of the diffusivity at any depth, but it must be noted that a large amount of experimental data is required to evaluate the second derivatives to a sufficient degree of accuracy.

A consequence of the spatial variation of λ is that $\partial^2T/\partial z^2$ is no longer in phase with $\partial T/\partial t$. A plot of one derivative against the other will yield an ellipse rather than a straight line. Lettau obtained a more complicated variation, demonstrating that other processes were at work. Fuchs and Hadas (1972) obtained curves which were closer approximations to ellipses (Figure 5). If independent measurements of heat flux are available, for instance from heat flux plates, Lettau's theory may be used to calculate values of $\lambda(z)$ and $C(z)$. If the heat flux is supposed to consist of Fourier components

$$G_n(z,t) = b_n \cos(n\omega t - \beta_n) \tag{22}$$

then the relations for $\lambda(z)$ and $C(z)$ are

$$\lambda(z) = \frac{b_n \sin(\alpha_n - \beta_n)}{a_n \alpha_n'} \tag{23}$$

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and

$$C(z) = \frac{b_n \beta'_n}{n\omega\, a_n \cos(\alpha_n - \beta_n)} \qquad (24)$$

Stearns (1969) used expressions (23) and (24), but found that deviations from $\alpha_n - \beta_n = \pi/4$ were very small. Fuchs and Hadas (1972) found fair agreement between values of $\lambda(z)$ and $C(z)$ computed from equations (23) and (24) and those obtained by heat flux plates and estimates based on the soil constituents.

McCulloch and Penman (1956) developed a formula from (20), using as variables $p = -a'_1/a_1$ and $q = \alpha'_1$. With this change of variable, (21) becomes

$$\kappa = \frac{\omega p}{q(p^2 + q^2) - (pq' - p'q)} \qquad (25)$$

A second model by Lettau (1962) was an attempt to find an analytical solution to the diffusion equation with spatially varying thermal properties, such as may be found in the upper part of the soil. The model was applied to the moon's crust, where it was thought that a zone of low thermal conductivity was due to the impact of micrometeorites. Although in principle an analytical solution for smoothly varying thermal properties is a better approximation to reality than is the layered system described above, in practice difficulty in integrating the equations led to a 'special case' solution where the heat capacity was supposed uniform with depth. The more general case must be solved numerically. Hadas and Fuchs (1973) used a curve-fitting procedure, and obtained agreement with measured values of the soil heat flux, although the prediction of temperature was less accurate.

Convection of heat by percolating water

An interesting extension of the homogeneous conductor model is the use of diurnal waves to measure vertical percolation of water. Suzuki (1960) and Stallman (1964) devised a method for estimating percolation velocities using the periodic solution of the convection diffusion equation:

$$\lambda \frac{\partial^2 T}{\partial z^2} - v c_o \rho_o \frac{\partial T}{\partial z} = c\rho \frac{\partial T}{\partial t} \qquad (26)$$

where v is the bulk velocity of fluid movement (percolation rate), c_o and ρ_o are the specific heat and density of water, and c and ρ are the specific heat and density of the soil including the contained water. The term in v causes a discrepancy between the amplitude and phase estimates of κ, and from this difference the percolation rate may be calculated. However, as Lettau (1954) demonstrated, there are other possible causes of such inconsistencies.