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**ATTENUATION OF PEAK GROUND
ACCELERATIONS FOR RECENT UK
EARTHQUAKES OF MAGNITUDE ≤ 4.6 ML.**

J.D.J Bott,

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British Geological Survey
Murchison House
West Mains Road
Edinburgh, EH9 3LA
Scotland, UK

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SUMMARY

I have measured empirical horizontal and vertical peak ground accelerations for earthquakes from between 1992 and 2001 with magnitudes between local magnitude (ML) -0.7 and 4.6, and for distances of 1 to 867 km. The majority of the ground accelerations were measured from short-period velocity records by removing the instrument response and differentiating the time series. All seismograms were first filtered between 1 and 30 Hz to reduce instabilities in the processing due to amplification of low and high frequency noise. Some higher frequency accelerations ($f > 30$ Hz) may have been filtered out from the records but only result in a small reduction in the peak acceleration values. Data recorded in the Scottish Borders was investigated preferentially due to the higher dynamic range of the 16-bit system installed there. The resulting empirical data are quite scattered for a given event, resulting from the varying effects of source radiation pattern, path and site effects. However, ground acceleration generally decays exponentially with distance. Peak ground accelerations are about an order of magnitude lower than predicted from published empirical attenuation relations currently used in the UK hazard studies, for distances greater than 100 km. The rate of amplitude decay at these distances is also much higher than predicted.

In this study various other aspects of the decay of peak ground accelerations are investigated, such as the effect of source depth (some coalfield events were included in the analyses since these generate more surface waves than earthquakes from greater depths), site effects, path effects and underlying geology. A special study was made of unsaturated near-field peak ground accelerations, to describe this portion of the attenuation curve. The full attenuation curves for distances both less than and greater than 100 km were only achieved for small magnitude events (magnitude between 2 and 3 ML) due to the limited amplitude range available for the current instrumentation in the UK. These attenuation curves show a similar pattern of a slower rate of decay at distances less than 100 km, and a more rapid decay at distances greater than 100 km. This is similar to what has been observed for Canada (Mereu and Atkinson, 1992) for a range of magnitudes (mb 3.5 to 5.0). Ratios between horizontal and vertical peak ground accelerations were also calculated where both readings were available. The average ratio found to be about 1.6 and no strong dependence on distance or magnitude was observed, despite the larger scatter in the near field, where the measured accelerations are most affected by the earthquake radiation pattern.

The decay of the PGA with distance for larger events appears to be more rapid than for the much smaller earthquakes (magnitudes ≤ 2.5 ML), however this might be related to the observable portion of the attenuation curve available from on-scale UK measurements. It is hoped that by observing the decay of accelerations for small events at distances less than 100 km, the rate of decay for larger events at these distances might be estimated, for which few unsaturated measurements are available in the UK. However, one has to be careful in making such extrapolations, because ground motions from small earthquakes tend to attenuate much faster than those from larger earthquakes. This is because small earthquakes produce a larger proportion of high frequency ground

motion, and high frequencies attenuate much faster than low frequencies. Also the duration of the highest ground accelerations is much less for smaller events.

BACKGROUND

The attenuation of peak ground accelerations for UK earthquakes is not known with great certainty, there being only a few on-scale measurements of large events during the instrumental period. In early seismic hazard studies for the nuclear industry (PML, 1984, 1988) attenuation relations were developed for the UK, however data used to construct these relationships were for events of surface-wave magnitude (M_S) ≥ 4.0 recorded in Europe and the western United States. During the past 30 years, the UK seismographic network has slowly developed, during which time only 2 earthquakes of magnitude over 5 ML have occurred. Unsaturated records from these events are only available for stations at distances of about 100 km and over (Marrow, 1992). This is due to the limited dynamic range available for the seismographic instrumentation installed in the UK at the time. The limited dynamic range, and the desire to detect the more frequent small earthquakes as well as the larger infrequent ones, has prevented the collection of on-scale records of larger events at close distances. This type of data would be necessary to determine UK specific attenuation relations.

The largest earthquake to occur in or near to the UK was the magnitude 6.1 ML Dogger Bank event that occurred some ___ km offshore of ___ in 1931. Similar-sized or larger events are possible in the UK in the future. For seismic hazard studies, it is important to be able to estimate the levels of ground acceleration to expect for this size of event in order to design sensitive facilities to withstand its potential ground shaking. To address the problem of having so few on-scale measurements for larger events, the BGS have installed ___ strong-motion instruments across the UK in the last 8 (?) years, in the hope of capturing on-scale ground accelerations of future moderate or large earthquakes. So far, only 22 three-component strong motion records have been written from these instruments, with the maximum acceleration being 62 mm/sec² for a magnitude 3.8 ML earthquake in Cornwall in 1996.

In the meantime, therefore, any available information on the attenuation of ground acceleration with distance for the UK is helpful in estimating the ground shaking of future large events in the UK, until a reasonable empirical data set has been collected. It has been noted before (REFS ___) and reiterated in this study, that the ground accelerations measured in the UK, are much lower than those predicted using the attenuation relations currently used in seismic hazard studies (PML, 1988; Dahle, 1991; Ambraseys and Bommer, 1991). However, measurements have generally only been measured in the far field, at distances generally greater than 100 km. It has also been noted (REFS ___) that published empirical ground motion relationships generally emphasize ground motions values for distances less than 100 km and so the portion of the attenuation curve at larger distances is not well-understood and the attenuation could be underestimated (Musson and Winter, 1996). At this stage, therefore, one cannot extrapolate to the near field and assume that the near-field accelerations will also be smaller. Therefore we cannot recommend reducing design ground motions based on the

few distant observations, as many factors could affect the level of shaking in the near field.

In this study I have collated unsaturated ground acceleration data for many events occurring between 1992 and 2001 over a wide magnitude range (-0.7 to 4.6 ML), to try to determine the form of the attenuation curve with distance for smaller events in the UK. I then use a simple RVT stochastic model to determine synthetic attenuation curves for several orders of earthquake magnitude and try to match these with the empirical data. The stochastic modeling requires input values for various attenuation parameters such as Q_0 , κ_0 and κ_1 as well as the frequency dependence of attenuation. Choice of stress drop also appears to be an important factor for the attenuation of ground acceleration particularly in the near-field, with low stress drop events producing lower ground accelerations in the near field compared to high stress drop events. Source depth has a large effect on attenuation with distance, with deeper events having with lower accelerations in the near field but acceleration decaying more slowly with distance compared to shallow events. Shallow events usually have high accelerations close to the source and a more rapid decay of acceleration with distance. This is a well-known observation from macroseismic surveys and has been used to estimate the depth of events in the UK from the areas encompassed by the various isoseismal lines (Musson, 1996).

The PGA measurements from across the UK are examined to see if there is any difference in attenuation with underlying geology. Site effects are investigated to see if any particular seismograph stations consistently produce high or low ground accelerations for a range of earthquake sizes and distances. Variation due to source radiation pattern could not be investigated due to too few events in the UK having reliable focal mechanisms. Radiation pattern effects will account for some of the large scatter observed in the data, but appear to strongly affect the near field compared to the far field. All unsaturated ground accelerations for distances less than 20 km were measured for the time period 1992-2001, to investigate the shape of the attenuation curve at these distances mostly for earthquakes of magnitude less than about 2.5 ML. Above this magnitude very little data is available due to instrument saturation and only data from low gain or strong motion instruments is available. The only exception to this are measurements of earthquakes from the Scottish Borders where the instrumentation has a higher dynamic range (16-bit) compared to the rest of the UK network (12-bit). Unfortunately, not too many large earthquakes occur in the Borders, but this network does provide on-scale measurements of earthquakes of magnitudes up to 3.0 ML at distances less than 100km that have occurred in this region since 1995. This size of earthquake would saturate other SP instruments in the UK at these distances.

The ratio between peak horizontal and vertical ground accelerations is also investigated for UK earthquakes and its dependence on epicentral distance and magnitude. Ground motion levels perceptible to humans are estimated based on the expected felt area limits for various magnitudes for the UK.

METHODOLOGY

Earthquakes from as many geographic locations as possible were selected and for all available magnitudes. The data set is therefore somewhat limited by the sizes of events, and their locations and stations that recorded them. Data was taken preferentially for earthquakes since 1995, with some events from between 1992 and 1995. Some earthquakes prior to 1995 were recorded on Geostore equipment (?) for several networks and these recordings have a more limited frequency range (1-16 Hz) compared to measurements post-1995 (1-30 Hz), after upgrading the recording equipment. More measurements from smaller magnitude events were taken due to the larger number of events of this size recorded during the chosen time period. Data from the region around the Borders network also features heavily due it having a higher dynamic range than the other UK networks where larger on-scale accelerations could be recorded.

All seismograms for each event were first visually filtered using the Seisan Program Mulplt, to remove all events that were saturated, did not have a clear signal, were noisy or contained spikes. The seismograms were bandpass filtered between 1 and 30 Hz, using a ____ filter, at the same time as the ground motion is calculated to reduce the amplification of low and high frequency noise. The Geostore records from prior to 1995 were bandpass filtered between 1 and 16 Hz, due to the limitations in the recording equipment. However, typical recordings of small earthquakes in the UK, tend to have peak values around 10-15 Hz, suggesting that filtering between 1 and 30 Hz is perfectly satisfactory for the measurement of PGA's. Some additional analyses were performed on recordings of several events to investigate the effect of varying the frequencies of the bandpass filter used. It was found that varying the bandpass filter from 1-30Hz to 1-25, 1-20 Hz and 1-15 Hz does not significantly reduce the peak ground acceleration estimate for these distances.

Peak ground accelerations were calculated for the whole seismogram. This was achieved by interactively selecting a window around the earthquake time series, starting just prior to the P-wave onset and ending when the signal was of the same order as the background noise. The program Seisan then estimates the ground accelerations of the time-series bounded by the selected window, by first removing the instrument response, resulting in a displacement seismogram, and then differentiating the time series twice to produce an acceleration seismogram. The peak accelerations in nm/sec^2 can be read from the plot. These values were recorded in a database in mm/sec^2 , along with information about the earthquake and recording station, and its epicentral distance in kilometres. Minimum detectable ground accelerations on the SP instruments are about 1 to 10 mm/sec^2 , about a factor of 100 less than those detectable on the strong motion instruments. All accelerations measured on the strong motion instruments were also included in the database.

PEAK GROUND ACCELERATIONS FOR THE UK

Figure __ illustrates the attenuation of PGA with distance for a range of earthquake magnitudes for the UK. The earthquakes have been divided into 0.5 magnitude unit bins

and distinguished on colour. Vertical accelerations are depicted as open diamonds and horizontal ones as open circles. Attenuation of separated horizontal and vertical ground accelerations are shown in Figures _ & ___.

Looking at some of the empirical data in more detail, it is noted that the two magnitude 2.7 ML events (Bargoed and Jura) have similar ground accelerations with distance, though the Bargoed event has more near-field data. This near-field data suggests a flattening of the slope closer in to the source than one would predict from observing the Jura data alone. However, a comparison of the Arran (magnitude 4.0 ML) and the Sennybridge events (magnitude 3.6 ML) we can see that these two produce very similar ground accelerations at distances greater than 100km. Their depths are 19 km and 14 km respectively. For the Hereford and Boston events (both 2.8 ML) the ground accelerations are significantly different with the Boston event having higher ground accelerations for most distances greater than 100 km. This could be due to varying attenuation across the two regions (Wales versus eastern England) or may be a result of a poor estimation of one of the earthquake magnitudes and/or depths.

CF events have much more rapid attenuation than deeper events, due to their shallow sources. Attenuation of GM's from CF events and equivalent tectonic events of similar size is compared.

Ground accelerations at certain stations as a function of distance and magnitude might indicate possible site effects. We compare the ground accelerations at closely spaced stations for the same event over a wide distribution of magnitudes and source to site distances.

Ratios between peak horizontal and vertical ground accelerations were calculated for all event-station pairs having measurements from both of the horizontal components as well as the vertical component. The distribution of 3-component instruments and all earthquakes for which ratios are available are shown in Figure ___.

STOCHASTIC MODELLING

The Program Criatt, available with the Seisan software (REF) can be used to calculate stochastic RVT ground motions given the correct regional attenuation parameters for a simple point Brune source model. This is an important methodology which might be useful here in the UK, which like the EUS, where it has been applied with some success, has few recorded ground motions from large magnitude events. Stochastic ground motion modelling has been used successfully in the EUS. The resulting relations compared favorably with actual recorded data. The program CRIATT was written by Mario Ordaz and is based on the model of Boore et al. (1983; 1989) which can be used to estimate ground motion parameters as a function of M_w (or M_s) and hypocentral distance, R . The ground motions are assumed to be band limited, stationary and of finite duration. The resulting tables of ground motions as a function of magnitude and distance are output in gals (cm/sec^2).

The RVT stochastic model assumes a Brune w^{-2} source model. The ground motions depend on a frequency independent term ($4\pi R_{vo} F V / 4\pi p B^2$), a geometrical spreading term (a function of source-to-site distance), the source function (which is frequency dependent) and a diminution function (also frequency dependent). The constant related to radiation pattern, R_{vo} , is taken as an average value of 0.55, the free surface correction $F = 2.0$, the vector of two horizontal components V is 0.707, the density is p , and shear wave velocity is B . The diminution factor, $D(f)$, accounts for loss of energy due to friction and scattering, i.e. attenuation. At distances less than a critical distance, R_c , the records are dominated by S-waves ($1/R$) but at $R > R_c$ then the records decay at $1/(R R_c)^{1/2}$. $D(f)$ requires $Q(f)$ and $P(f)$ where $Q(f) = Q_0 f^e$ (regional attenuation where f = frequency and e , the frequency exponent of attenuation is ≤ 1.0). $P(f) = e^{-\pi k f}$ and is an additional attenuation term related to near surface losses of energy where k , or κ is high frequency decay factor (see Singh et al., 1982). This is also described in more detail in the Seisan manual (Havskov, 1999?). The program has several output options, for pga, for periods of 2.5 secs (0.4 Hz) or for PGV.

MODELLING RESULTS AND COMPARISON WITH UK DATA

In this study, only the peak ground accelerations were measured and calculated using CRIATT. I first tried to match the RVT model with some of the data collected with the recommended numbers provided in the example in Seisan (stress drop = 100 bars, $\kappa_0 = 0.02$, $Q = 500$, $\kappa_1 = 0.0$, $\epsilon = 1.0$). However, the results didn't match decay of measured peak ground acceleration for the UK for the small magnitude events at distances greater than 100 km ($M_L < 4.0$). The rate of decay of the data is much higher than that for the computed curves. I then computed multiple models to test which parameters effect the resulting curve the most, keeping all other parameters the same. It appears that ϵ and Q_0 affect the curve the most in terms of the level of acceleration and the slope at these distances (Figures __ and __). Factors such as κ_0 and κ_1 had little effect and stress drop had a small effect on the ground acceleration level at these distances but not on the gradient of the curve. Stress drop appears to affect the near field accelerations more. I was able to match the empirical data I collected for some 1999 events at distances greater than 100 km for small magnitude events using the following parameters: stress drop of 100 bars, Q_0 of 500, $\epsilon = 0.3$, $\kappa_0 = 0.02$, $\kappa_1 = 0.04$. The reasoning for choosing these values was purely subjective as to what fit the data at this stage.

Comparing the stochastic attenuation relation for various magnitudes with the UK data, we can match the two at distances greater than 100 km using the following parameters to describe attenuation. However, for distances less than 100 km the ground motions do not decay quite so rapidly as the stochastic relations predict. The rate of decay is more similar to that indicated from other empirical attenuation relations. Something about attenuation relations just concentrating on distances less than 100km so don't see the rapid attenuation with distance that we commonly observe in the UK for our unsaturated data for larger events. The attenuation is still however, much lower than predicted from these relations at distances less than 100 km by a factor of __.

The empirical attenuation for small events in the UK, follows a similar pattern to that observed in ENA (Canada). Mereu and Atkinson (1992) found that amplitude data for a range of magnitudes, when it was normalized by magnitude, gave a tri-linear curve, with a flat region between about 70 and 120 km, which corresponds to the Moho bounce region. The UK data appears to mimic this same form – at least for small magnitude events. Whether the larger UK earthquakes have a similar attenuation curve is still debatable, the fact that in Canada the form of the attenuation curve was actually magnitude independent indicates that maybe it does.

Can we fit the curve for distances below 100 km with a different set of attenuation parameters, or is the model just not sophisticated enough. Another thing observed in Canada was that the Brune source model for a simple circular source did not match the data and that a rectangular source, resulting in two significant corner frequencies also had a strong affect on the attenuation with distance. Possibly one influences the near-field attenuation and the other influences the far-field attenuation curve. Not sure – totally speculative right now. Need to do some reading.

COMPARISON WITH OTHER ATTENUATION RELATIONS

In order to justify the values used I compared these values with those used by others using the stochastic RVT methodology. Atkinson and Boore (1997) computed stochastic ground motions for crustal earthquakes in the Cascadia region and used a stress drop of 50 bars, Q of $380f^{0.39}$ (epsilon of 0.39) and κ of 0.011 based on data collected from the region (Atkinson, 1995; 1996). Also Toro et al. (1997) used stochastic modelling techniques to estimate ground motions for the CENA using the following parameters: stress drop of 120 bars, κ_0 of 0.003, 0.006 and 0.012 (3 values) appropriate for hard rock sites (for soil sites it recommends using an amplification factor). The actual Q values were not mentioned but they reference a couple of papers (EPRI, 1993; Abrahamson et al., 1995). Epsilon was not discussed in their paper. Another factor that needs to be specified is T_d which is a distant dependent term for dispersion and the default in the Seisan example was 0.05 secs ($T = T_0 + T_d$, where $T_0 = 1/f_0$ for source duration). For the Atkinson and Boore (1997) study for the Cascadia region, they used $T_d = 0.0$ for $R < 50$ km and $T_d = 0.07$ for distances greater than 50 km.

I am able to fit the Arran data (ML 4.0 estimated M_S or M_W 3.6) using the above parameters of stress drop 100 bars, $Q = 500$, $\kappa_0 = 0.02$, $\kappa_1 = 0.04$, and epsilon = 0.3 for between MW 3.5 and 4.0 (See figure __). The curves model the decay of peak ground acceleration with distance better than those of other published attenuation relations used in the UK (PML, Ambraseys and Bommer, and Dahle). At near distances the curves appear to converge with the published attenuation relations. More data is needed for the near field for larger events to understand how ground accelerations attenuate, but using stochastic modelling techniques may be the best way to go for now for the UK, since we need to move forward with the attenuation relations.

Atkinson and Boore (1995) also computed a stochastic ground motion model for the ENA for $M \geq 4.0$ and for distances 10-500 km. They found that their empirical data fit the curves for M 4-5 events. They discuss the divergence from a bi-linear curve of acceleration decay with distance. Their model was based on work of Atkinson and Mereu (1992). They also discuss the difference in the source model from the Brune model that they use and this is based on Atkinson (1993). They found that they needed to have two corner frequencies to describe the larger events based on empirical data (see discussion below). They use $Q = 680f^{0.36}$ (thus $\epsilon = 0.36$ and $Q_0 = 680$). They also describe the dispersion with distance (T_d). Instead of using kappa they use a f_{\max} model (Hanks, 198_) to describe near site attenuation.

Atkinson and Mereu (1992) describe the decay of spectral amplitude (acceleration) with distance for earthquakes M 3.0- 6.5 and for distances out to 1700 km for SE Canada and focal depths 5-30 km. They find that the fall off can be modelled with a hinged trilinear regression. They use spectral amplitudes of windowed S arrivals (with noise removed for distant events) for a range of frequencies (1-10 Hz) and normalize them for earthquake size to get the general shape of the amplitude decay. They normalised for site effects as well as source effects and found no frequency dependence, the same shape of decay for amplitudes for all frequencies between 1 and 10 Hz being observed. The decay followed the relation $1/R^{1.1}$ for R between 0 and 70 km, then amplitudes were constant between an R of 70 and 130 km, then for $R > 130$ km the decay followed the relation $1/R^{0.5}$. They explain the lack of decay from 70 to 130 km to coincide with the “Moho bounce” region and the switch over of dominance from direct waves to Lg waves. This Moho bounce was also noted for the Loma Prieta earthquake and the reason for high ground motions in the Bay Area, at such a large distance from the event. They also investigate the effect of depth on decay of ground motion and found that for deeper events (not much data for shallow ones) that the effects of lower attenuation can be seen for distances less than 40 km. They saw no difference between data collected from different seismotectonic provinces in their data set. As more data is available for the UK, these types of analysis would be possible to determine if our measured ground accelerations show the same sort of distance decay and to see if it is frequency independent.

Atkinson (1993) also investigated source spectra for the same dataset of 100 earthquakes. They compared the data to the Brune source model but found they had to modify their source model to include two corner frequencies. This shows that the sources are not simple circular Brune sources but more complex rectangular shaped faults. They found that they couldn't measure Brune stress drop for the small events ($M < 4$) because all corner frequencies were around 10 Hz despite the size, thus implying a decrease in stress drop with earthquake size. However, their data was band limited to frequencies between 1 and 10 Hz. For the larger events they did not have enough band width to measure the low frequency spectral levels and so had to rely on other data to determine the moments and stress drops for their data set. Two of the larger events in the region, Saguenay and Mont Laurier both had high calculated stress drops of around 500 bars. They also regressed felt area against high frequency spectral amplitude and found that there was a clear relation between the two.

MOMENT MAGNITUDES AND STRESS DROPS FOR 1999 EARTHQUAKES

Looking at spectra for the Arran earthquake and some other earthquakes from 1999, I noticed some trends. First of all, the Arran earthquake appears to have two corner frequencies on many stations (one around 2-3 Hz and one around 10 Hz. This would imply that we have a rectangular fault rather than a typical circular Brune source model. The Brune stress drop varies with corner frequency chosen from very high (greater than 500 bars) to very small (less than 50 bars). Again the measurements of moment magnitude are generally very similar and lie between M_W 3.0 and 3.5. There is one estimate that is about 3.8 MW at station BBH (150 km away). Though I am accounting for attenuation using $Q = 500$, $\kappa = 0.01$ and $\alpha = 0.4$ (frequency dependence of Q). Stress drop varies significantly with location chosen for the corner frequency, of course since it is a function of one over the source radius cubed and the source radius is directly linked to the corner frequency chosen. For the Arran aftershock I got very similar size to M_L 1.6 value (around M_W 1.5-1.8). When I compared my picks with those of Maureen using SPS she got about the same. However, her magnitudes for Arran were significantly higher than the ones I calculated by about 0.3-0.4 magnitude units. There is a difference in the way Q is accounted for too, and in Seisan there is a term to describe the decay of Lg waves at distances greater than 100 km being different than the direct waves. Maureen used an equation $Q = 90 + 110f^{1.2}$ whereas I used $Q = 500f^{0.4}$, figures that appear reasonable compared to those used in Norway (?) and those used in the Eastern North America.

Looking at two events in the Borders region I measure an average M_W of 1.9-2.0 for both M_L 1.9 and M_L 2.1 events at distances less than about 60 km. Again for the Caernarvon earthquake (M_L 3.1) I get a M_W of between 2.6-2.8 for stations at distances greater than 100 km. There could be a distance or size dependence on these calculations.

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