Svensson, C.; **Jones, D.A.**. 2010 Review of methods for deriving areal reduction factors. *Journal of Flood Risk Management*, 3. 232-245.
10.1111/j.1753-318X.2010.01075.

Review of methods for deriving areal reduction factors

Cecilia Svensson and David A. Jones

Centre for Ecology and Hydrology, Wallingford, OX10 8BB, UK

16 July 2010

Abstract

The design of hydraulic structures requires knowledge of how much rain is likely to fall within a certain amount of time, and over a specific area. Point rainfalls are only representative for a very limited area, and for larger areas the areal average rainfall depth is likely to be much smaller than at the point of maximum observed depth. The estimation of areal reduction factors (ARFs) is concerned with the relationship between the point and areal rainfalls. This relationship has been found to vary with, for example, predominant weather type, season and return period. Traditionally, ARF estimates are based on empirical methods though, more recently, a range of analytical methods have been applied. The review has found that no method is unambiguously correct. However, the traditional data-intensive, empirical, fixed-area methods still have advantages, including probabilistically correct ARF estimates and applicability over a comprehensive range of spatial and temporal scales. Although the analytical techniques try to put ARF estimation on a sounder scientific basis, they tend to rely on simplified assumptions and/or are only applicable within limited scales. The use of radar is problematic because of inhomogeneities and short data records, as well as possible biases in the ARF estimates.

Keywords: Areal rainfall, areal reduction factor, design rainfall, methodology, rainfall

Corresponding author: Cecilia Svensson, csve@ceh.ac.uk

Accepted for publication in Journal of Flood Risk Management (Chartered Institution of Water and Environmental Management, Blackwell Publishing)

The definitive version is available at www.blackwell-synergy.com

http://onlinelibrary.wiley.com/doi/10.1111/j.1753-318X.2010.01075.x/abstract

Introduction

Areal reduction factors are used in the construction of design rainfall events which are needed for the design of hydrological structures. A design rainfall event consists of a specification of a set of rainfall depths varying in space as well as time. There are two philosophically different ways to approach this specification. Firstly, observed events can be transposed or entire rainfall fields can be generated by a computer model, as a stochastic series of space- and time-rainfalls that reproduces the observed behaviour (e.g. Seed et al. 1999). Secondly, a simplified representation can be used. For example, an observed temporal rainfall profile can be combined with a uniform spatial rainfall distribution to obtain a design rainfall event. This review paper is concerned with the derivation of the magnitude of this spatial rainfall from point rainfalls, via the concept of an areal reduction factor (ARF). The ARF denotes the ratio between the areal average rainfall and a point rainfall. There are several different ways to define this ratio, which can lead to ARFs with different properties.

The use of ARFs is convenient because networks of raingauges with long series, which are needed for accurate rainfall frequency estimation, are generally sparse, and do not allow for an appropriate characterisation of the associated spatial rainfall patterns. Denser networks may be available for more recent decades, or for special study areas, and these datasets can be used to study spatial rainfall variability. The spatial information can subsequently be combined with a point rainfall frequency estimate from the long-term dataset, to obtain an areal rainfall frequency estimate. Methods for estimating point rainfall magnitudes are reviewed in Svensson and Jones (submitted).

An outline of how various factors influence the ARF is given below. A description then follows of methods used to estimate ARFs, and suitable methods for re-examining ARFs in a modern context are suggested.

Factors influencing the areal reduction factor

Several different issues affect the ratio between the spatial average rainfall over an area, and a point rainfall in that area. These issues include factors relating to the characteristics of the rainfall itself, but also to the physical characteristics of the catchment, and to the data and methods used to derive the ARF.

Factors relating to rainfall characteristics

Different synoptic weather types produce different spatial rainfall patterns (e.g. Huff and Shipp 1969; Skaugen 1997; Einfalt et al. 1998). Skaugen (1997) classified daily rainfall events in southeast Norway into convective showers and frontal rainfall, and concluded that the spatial averages for large-scale frontal events do not reduce much in magnitude with increasing area, whereas for small-scale convective events they do. Using a more detailed classification, Huff and Shipp (1969) found that the decay in spatial correlation is smaller in storms occurring in low pressure centres than at the fronts associated with midlatitude cyclones, and that it is greatest in air mass storms.

Skaugen (1997) found that the difference in ARF curves (ARF plotted against area) between convective and frontal events in Norway becomes more pronounced for higher return periods (i.e. for more extreme rainfall events). ARFs for both convective and frontal events decrease with increasing return period, but the rate of decrease for convective events is considerably greater than for frontal events. Using data from North Carolina and New Jersey in the United States, Allen and DeGaetano (2005a) also found that the areal rainfall is smaller compared with the corresponding point rainfall (i.e. the ARF decreases) at higher return periods. Similar results were obtained for Texas by Asquith and Famiglietti (2000). In contrast, when analysing areal rainfall in Switzerland, Grebner and Roesch (1997) found that ARFs were independent of return period, at least for areas greater than 500 km^2. For smaller areas there was some variation between the ARF curves for different return periods, but the authors thought this may be caused by the limited ability of the network to detect centres of convection (about 1 gauge per 100 km^2) and the shortness of the reference period (13 years).

Allen and DeGaetano (2005a) found that the ARFs are smaller in the warm season than in the cold, presumably in response to increased convection in summer. Huff and Shipp (1969) found a similar seasonal difference, in that the decay with distance of spatial correlation patterns of precipitation was greater in May-September than in the cold season. The decrease in ARF with increasing return period may also reflect the importance of convection in producing very heavy point rainfalls.

Skaugen (1997) noted that the point rainfall extremes associated with the convective type tends to occur inland, whereas the maxima of the large-scale events usually occurred nearer the coast. Other investigators also note that ARFs vary with geographical location and climate, presumably because of a difference in the predominant rainfall generating mechanisms. For example, Omolayo (1993) suggests that 1-day ARFs are generally higher in the United States than in Australia, and Zehr and Myers (1984) suggest that ARFs decline more rapidly in the semi-arid southwestern United States than in the rest of the country. The latter finding is supported by Asquith and Famiglietti (2000) who found that ARFs are higher in eastern United States than in Texas.

ARFs reported for the United Kingdom (UK) in the Flood Studies Report (NERC 1975) show more sharply decreasing ARFs with increasing areas for shorter durations than for longer ones (Figure 1). This feature was also noted by Ramos et al. (2005) when investigating rainfalls of 6 to 90 minutes duration in Marseille, France. Sivapalan and Blöschl (1998) explain that the rationale for this is that short-duration events (i.e. convective) are small in areal extent. Other authors do not find much variation in ARF with the duration of rainfall, probably because they are not studying as wide a range of different durations as presented in the Flood Studies Report (which lists durations from 1 minute to 25 days), and/or are considering comparatively long durations. Clark and Rakhecha (2002) investigated heavy rainfalls of one to three days' duration in India for areas up to 20,000km^2, and did not find any real difference in ARFs between these durations. Huff (1995) studied shorter durations, between 3 and 24 hours, in the midwest of the United States, and came to the same conclusion. He attributed the similar behaviour to the large dependence between heavy rainfall events of different durations. In his study, most of the 24-hour storms were also found in the shorter-duration samples.

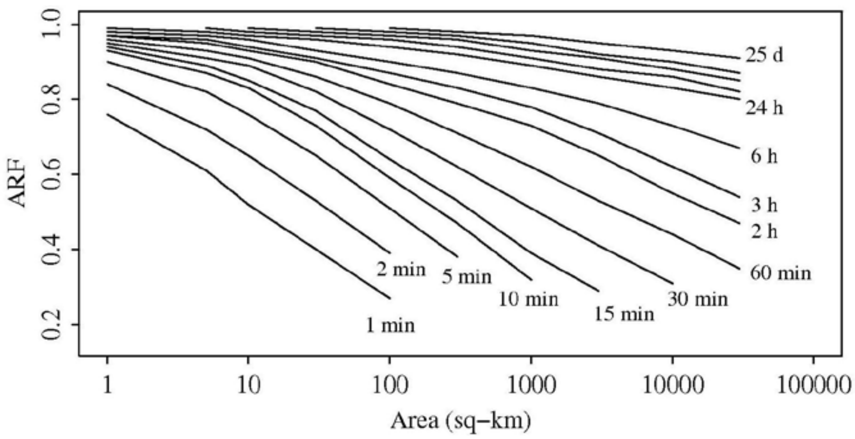

Figure 1. Areal reduction factors for precipitation in the United Kingdom presented in the Flood Studies Report (diagram derived from tabulated values in NERC (1975)).

Factors relating to catchment characteristics

So far, research suggests that the effect of catchment characteristics, such as catchment shape, topography and urbanisation, on ARFs is small.

An elongated catchment shape would result in different ARFs depending on whether the typical rainfall isohyets (resulting from the shape of the typical rainfall system and/or its direction of movement) were aligned along the catchment or perpendicular to it. However, when investigating rainfall fields and ARFs from a theoretical multifractal perspective, Veneziano and Langousis (2005) conclude that the effect of catchment shape is generally small, and also notes that very highly elongated catchments are rare.

Leeward and windward effects of hills and mountains on rainfall may affect the ARFs. Thiessen polygon and inverse distance weighting methods used to compute areal precipitation do not directly account for topography. Because raingauge networks tend to be sparser at higher elevations (e.g. Prudhomme and Reed 1999) they may not adequately represent areal precipitation at high altitudes. Allen and DeGaetano (2005a) suggested adjusting for this when spatially interpolating rainfall amounts. However, they found that topographical rainfall biases appear to be insignificant for the estimation of ARFs.

Huff (1995) noted that there may be a difference in the reduction factor between urban areas and the surrounding rural areas. Eight storms in Chicago were found to have a slower rate of decrease in the reduction factor within 500 km^2 of the urban storm centre compared with 67 rural storms. For larger areas, the rate of decrease for urban storms exceeded that for the rural storms. However, this sample of storms is rather small, and natural variability in spatial rainfall characteristics is large, so Huff concluded that this anomaly could also be due to natural variation rather than an urban rainfall effect.

Factors relating to data and methodology

Because of the temporal variability in rainfall, the periods of data collection may influence the ARF estimates (e.g. Asquith and Famiglietti 2000). Asquith and Famiglietti (2000) also noted that three overlapping raingauge networks around Houston, Texas, did not give the same ARFs, and concluded that differing precipitation-monitoring networks cannot be indiscriminately combined. However, as far as only station density is concerned, Allen and DeGaetano (2005a) conclude that for North Carolina and New Jersey the influence of differences in station density and interpolation method appear to be insignificant.

The use of different methodologies to estimate ARFs is likely to result in different ARF estimates. The next section comprises a review of ARF estimation methods, including the use of radar data.

Methods for ARF estimation

Methods for estimation of areal reduction factors include empirical and analytical methods. In many countries the current design guidelines are based on empirical methods, including in the UK where they were issued in 1975 (NERC 1975). However, since then several new analytical methods have been proposed. These include methods based on correlation analysis, crossing properties, scaling relationships and storm movement. Radar rainfall data have also become available in many parts of the world, and at improving spatial and temporal resolutions. Empirical and analytical methods for ARF estimation, as well as the potential for use of radar rainfall data, are discussed in this section. Notations have sometimes been changed compared with the original source documents in order to keep them reasonably consistent within this review.

Empirical methods

Empirical methods are generally data intensive and computationally laborious, but they largely don't rely heavily on distributional or other assumptions about the rainfall process.

Geographically fixed versus storm-centred approaches

Empirical ARF estimation can be divided into two categories, "geographically fixed" or "fixed-area" approaches on the one hand, and "storm-centred" approaches on the other hand (e.g. Omolayo, 1993).

In the storm centred approach, the region over which the areal rainfall is estimated is not fixed, but changes for each storm. The centre point for the approach is the point observing the maximum rainfall, which also changes for each storm. The areal reduction factor is given by

$$ARF = P_{area} / P_{point}, \qquad \text{(Eq. 1)}$$

where P_{area} is the areal storm rainfall enclosed by a selected isohyet (the rainfall in the enclosure is everywhere at least as large as the value of the isohyet), and P_{point} is the maximum point rainfall at the storm centre.

Asquith and Famiglietti (2000) note that storm-centred approaches have not seen widespread application, partly because of difficult implementation on multi-centred storms. In contrast, the fixed-area approach takes an extreme value of the areal average rainfall over a geographically fixed area (such as a catchment) and divides it by a corresponding value of the point rainfall that is typical for the area.

Omolayo (1993) also points out that storm-centred approaches are not correct for estimating areal rainfall of a particular frequency from point rainfalls. This is because extreme point rainfalls and extreme areal rainfalls are unlikely to be produced by the same storm, or storm type. For example, localised convective events may produce very heavy point rainfalls, but may not result in a large areal rainfall. Omolayo (1993) suggests that to obtain the probabilistically correct ARF for a duration D, the T-year areal rainfall over a region, P_{area}, of size A should be divided by the (w_i-weighted) average T-year point rainfall, P_i, of all the gauges i in the same region:

$$ARF(A,D,T) = \frac{P_{area}(A,D,T)}{\frac{1}{\sum_i w_i} \sum_i \left[w_i \; P_i(D,T) \right]}. \qquad \text{(Eq. 2)}$$

Thus, this necessitates a fixed-area approach. Note that the ARF defined in this way (Eq. 2) is radically different from that for the storm-centred approach (Eq. 1).

A variant of the fixed-area approach is described by Yoo et al. (2007), who apply a mixed distribution based on the concept of rainfall intermittency (wet and dry periods, with a continuous Gamma distribution fitted to the wet periods) for estimating rainfall return periods, T. This method uses all the daily data available, rather than the traditional way of fitting an extreme value distribution to the annual maxima series. For ARF purposes, only the 1-day duration is studied, and subsequently a function is fitted to the empirically estimated ARFs for various areas A, as

$$ARF(A,T) = 1.0 - M e^{-(a A^b)^{-1}}.$$

The parameters M, a and b are estimated for each return period considered.

Storm-centred approaches may be used for probable maximum precipitation (PMP) purposes, because PMP does not have an associated frequency estimate. However, the storm-centred approach does generally not result in a conservative ARF estimate. Sivapalan and Blöschl (1998) note that storm-centred ARFs are usually somewhat smaller than geographically fixed ARFs. There are probably at least two reasons for this. If the ARFs are derived from storms with a heavy point rainfall, then these storms may be dominated by convective events with a limited areal extent. Also, the heaviest point rainfall of a storm may often be located outside the boundaries used for a fixed-area approach.

The US Weather Bureau method

Many "traditional" empirical methods disregard any effect of return period on the ARF, although such effects are now acknowledged. The method developed by the US Weather Bureau in Technichal Paper No. 29 (U. S. Weather Bureau 1957-1958) remains the most commonly used method in the US (Allen and DeGaetano 2005a) although alternative methods have been proposed in the intervening years (see below). The US Weather Bureau method has the advantage of being intuitive to apply, although it is somewhat laborious. It does not take return period into account, as only short rainfall series from dense networks were available at the time the method was developed. The gauges in the study were nearly uniformly spaced, but weighted rainfall estimates could be recommended for uneven networks. The method relates the mean of the annual maximum areal rainfall series to the mean of the annual maximum point measurements at all stations, i, and in all years, j. With the annual maximum point rainfalls denoted P_{ij}, and the point measurements making up the annual maximum areal event denoted P'_{ij} , the ARF is calculated as:

$$ARF_{TP29} = \frac{\sum_j \sum_i w_i P'_{ij}}{\sum_j \sum_i P_{ij}} . \qquad \text{(Eq. 3)}$$

Small-scale study

A three-year experimental monitoring program in France focused on ARF estimation using a dense network of 9 gauges covering up to 4 km^2 for durations from 5 minutes to 4 hours (Desbordes et al. 1984). The 58 largest events, as measured by resulting discharge at the urban catchment outlet, were studied using two methodologies. These methods resemble, but do not quite conform to, the above concepts of storm-centred and fixed-area methods. For the "storm-centred" approach the largest point rainfall recorded in the study area for each event and duration was noted, and the concurrent areal rainfall (calculated using Thiessen polygons) was divided by it. The authors note that the actual centre of the storm would probably in most cases be located outside of the study area, and the areal reduction factors would thus most likely recede more quickly than those estimated from the available data. The ARFs for each duration were averaged separately before being analysed.

For the “fixed-area” approach the largest point rainfall of a given duration at each gauge for each event was selected, and the concurrent areal rainfall of the same duration was divided by it (the “storm-centred” ARFs are a subset of the fixed-area set). Some of these ARFs are therefore greater than 1. The ARFs for the individual events and gauges were subsequently averaged for each duration. These averaged ARF estimates are greater than the “storm-centred” ones, but it is not clear how they would compare with those of more standard methodologies like, say, the US Weather Bureau method. One may speculate that they would be larger because all the point rainfalls have been used, and because of the frequent occurrence of the selected events (on average 19 per year). The large number of selected events makes good use of the data collected during the short experiment, but it is at the expense of events not being representative of extreme rainfalls. It seems unlikely that the ARFs would be strictly probabilistically correct (i.e. relating an areal rainfall of a certain return period to a point rainfall of the same return period), but the magnitude of the difference is unknown.

The initial selection of the 58 events based on discharge most likely captures the largest areal rainfalls, and because of the small size and high gauge density of the study area probably also the largest point rainfalls. However, in a larger area it would be safer to select rainfall events based on the rainfall rather than the discharge, as isolated heavy rainfalls may not have caused extreme runoff.

Methods used in the United Kingdom

The fixed-area method currently used in the UK, presented in the Flood Studies Report (NERC 1975), does not take return period into account, as the effect was assumed to be small. In this method, the annual maximum areal rainfall over a particular region is found, and the point measurements, P'_{ij}, at station i and year j of these areal events are noted. Independently, the annual maximum point rainfalls, P_{ij}, at each station i for year j, in the region are noted. For each region of area A and for each duration D, the ARF is calculated as

$$ARF_{FSR} = \frac{1}{IJ}\sum_{j}\sum_{i}\frac{P'_{ij}}{P_{ij}}$$

where I is the total number of stations in the region, and J is the record length (years). This is a simplification of the US Weather Bureau method described above, and was adopted for computational convenience. It makes the assumption that “an average of ratios” can safely approximate “a ratio of averages” (NERC 1977), which may be considered as somewhat unorthodox.

Bell (1976) re-examined the ARFs in the Flood Studies Report with regard to, among other things, the assumption that ARFs are independent of return period. He did so by fitting frequency distributions to the areal and point annual maximum rainfall series, and then calculating the ARF as the ratio between the areal and point rainfall estimates of the same return period. The method involves ranking the annual maximum (Thiessen-weighted) areal rainfall series, and then also ranking the annual maximum point rainfall series at each individual station in the area. To obtain a single point rainfall frequency curve that is representative for the area, the Thiessen-weighted mean of annual maximum point rainfalls of the same rank were computed. Frequency distributions were then fitted to the areal and point rainfall series and the ARFs calculated for different return periods. Bell (1976) found evidence for more rapidly decreasing ARFs with increasing return period, but concluded that using the ARFs calculated according to the Flood Studies Report (NERC 1975) resulted in conservative ARF estimates.

Essentially following Bell's (1976) method, Stewart (1989) re-evaluated the ARFs for an upland area in northwest England. Stewart introduced a standardisation of the rainfall through division by the mean annual maximum rainfall, so that the ARF's were derived using rainfall growth curves rather than actual rainfall frequency curves. This has the effect of allowing locational variations to be represented by differences in the mean annual maximum values, while return period effects are represented through the growth factors. For this particular region, the ARFs for daily durations, derived using raingauge data, are lower than those presented in NERC (1975). Similarly to Bell's study, ARFs were found to decrease with increasing return period, and this rate of decrease increases with area. ARFs were shown to be weakly correlated with latitude and showed a stronger relationship with Standard Average Annual Rainfall (SAAR).

Variations of Bell's method have also been applied to rainfall in Australia by Siriwardena and Weinmann (1996) and Porter and Ladson (1993). However, the current Australian design guidelines (ARR 2001), suggests using ARFs derived by the United States Weather Service for Chicago (Myers and Zehr 1980) or Arizona (Zehr and Myers 1984), depending on which Australian climatic zone the ARFs will be applied to. This method is discussed below.

National Weather Service method

The current national United States approach to ARF estimation is outlined in NOAA Technical Report NWS 24 (Myers and Zehr 1980). It is based on frequency analysis of annual maximum rainfall at pairs of stations, and the distance between them. The definition of the ARF, taking into account the return period of the rainfall, is a restatement of Eq. (2) as

$$ARF_{NWS}(f, \Delta t, A) = \frac{X_A(f, \Delta t, A)}{X_A(f, \Delta t, 0)},$$

where $X_A(f, \Delta t, A)$ is the areal average rainfall of frequency f and duration Δt over area A, and $X_A(f, \Delta t, 0)$ represents point rainfall of frequency f and duration Δt. The precipitation magnitudes X at frequency f are estimated using Chow's (1951, 1964) general equation for frequency analysis,

$$X(f) = \mu + K(f)\sigma$$

,

where μ is the mean and σ is the standard deviation of the population of X's, and K(f) is the frequency factor for the Gumbel fitting of a Fisher-Tippet type I distribution.

Rather than estimating μ and σ from samples of point and areal rainfalls directly, the method was developed to take advantage of series of annual maximum two-station average rainfall, and from certain other statistics derived for station-pairs. The μ and σ are estimated for station-pairs at various distances and for various durations, and a smoothing surface is then fitted to each parameter in the distance-duration-space. From these statistics, upper and lower bounds of the moments μ and σ of the "true" areal average rainfall are estimated by (a rather involved) theory developed for the purpose. The positions of the areal moments between these bounds are then set by a calibration with the moments of a limited number of annual maximum multi-station averages.

This method explicitly takes into account the variation of ARF with return period. It uses statistics of station-pairs and small five-station networks, which reduces the need for large, dense networks with concurrent data observations. Because the station-pairs and five-station networks are located at random locations within an area, there is an assumption of isotropy in the spatial rainfall field. Hence, the case of elongated catchments with rainfalls typically aligned in one direction or other is not considered (a feature not unique to this method). However, it is questionable whether the complicated methodology used is justified as precipitation observations become more plentiful with time.

Annual-maxima centred method

Asquith and Famiglietti (2000) present an annual-maxima centred approach to ARF estimation and apply it to study areas in Texas, United States. It uses the concurrent rainfall measurements surrounding a point annual maximum, and does a pair-wise calculation of the ratio of rainfall between each surrounding gauge and the target (annual maximum) gauge. The ratios are then plotted against the distance between the gauge-pairs, and a function is fitted. This is done separately for each return period, i.e. those ratios surrounding, say, five-year or greater annual point maxima are plotted together. ARFs can be estimated from the function by spatially integrating the ratios for a user-specified area. It is labour-saving as it does not require prior spatial averaging of precipitation, explicit determination of spatial correlation coefficients or explicit definition of a representative area of a particular storm in the analysis. However, its application requires a dense network of gauges in the study area. The method results in more rapidly decreasing ARFs than the US method presented in Technical Paper No. 29 (U. S. Weather Bureau 1957-1958). This seems reasonable for two reasons. Firstly, other authors have found faster declining ARFs in the drier southwestern United States than in the eastern areas for which the Weather Bureau based their estimates (e.g. Zehr and Myers 1984). Second, the storms are selected based on heavy point rainfalls, which means that the largest areal rainfall events may not be represented at all. The population of events selected will not be as focussed on heavy point rainfalls as a storm-centred approach, but is still likely to result in more rapidly decreasing ARFs than fixed-area estimates, as discussed previously. Thus it seems questionable that the method will result in a "probabilistically correct" ARF, as this ARF multiplied by the point rainfall of a particular return period may not result in an areal rainfall of the same return period, but more likely in a less extreme one. However, the size of the discrepancy is unknown, and, for any method, a small difference may be acceptable when considered in conjunction with a method's other advantages.

Spatial correlation structure

This collection of methods that are based on estimates of the spatial correlation of the rainfall field are perhaps more elegantly formulated than the traditional empirical methods. However, they rely on assumptions such as isotropy and particular statistical distributions of the rainfall process. They still require a reasonable amount of data to estimate the model parameters, and hence the ARF.

Rodriguez-Iturbe and Mejía method

Rodriguez-Iturbe and Mejía (1974) introduced a method of estimating ARFs using the correlation, ρ_d, between two gauges separated by a "characteristic correlation distance" in the study area. The characteristic correlation distance, d, measures the mean separation between two points randomly chosen in the area, and thus depends on the size and shape of the area. The ARF depends only on the correlation, and is calculated as

$$ARF_{RIM} = \sqrt{\rho_d} \; .$$

The method assumes a particular spatial correlation structure, either an exponentially decaying function or a Bessel-type correlation structure. Other assumptions include the point precipitation being both isotropic and Gaussian with a zero mean. This distribution is not a typical characteristic of extreme, shorter-duration precipitation. When the precipitation is non-Gaussian, there will not be an exact correspondence between the frequency factors of the point and areal precipitation extremes. That is, the method will have a problem associated with it that is similar to that of storm-centred approaches. However, whereas typical storm-centred approaches generally result in more quickly receding ARFs, this may not be the case for the Rodriguez-Iturbe and Mejía method. Because this method uses all precipitation data, rather than just extreme events, it is not certain that the likely result will be an un-conservative ARF estimate, as less extreme events may be spatially more evenly distributed than heavier events.

Sivapalan and Blöschl method

Sivapalan and Blöschl (1998) point out that in the Rodriguez-Iturbe and Mejía method the mean of the areally averaged rainfall does not change with the averaging area. Sivapalan and Blöschl (1998) consider this feature to be more appropriate for the parent rainfall process rather than to the extreme value process, and therefore propose an extension to the Rodriguez-Iturbe and Mejía method. The new method makes use of the spatial correlation structure while looking at the extreme value distributions rather than the parent distributions only. The authors consider it appropriate for rainfall systems which are large relative to catchment area, because it cannot handle finite storm extent or partial coverage of the catchment.

The parent distribution of rainfall intensity at a point is assumed to be exponential. The spatial correlation, ρ_p, of point rainfall intensity is assumed to be isotropic and of the exponential type:

$$\rho_p(r) = \exp\left(-\frac{r}{\lambda}\right),$$

where r is the distance between two points and λ is the spatial correlation length (interpreted as a measure of the spatial extent of the rainfall field). When the point rainfall process is exponentially distributed, the areally averaged rainfall process is approximately gamma distributed. To the upper tail of the cumulative gamma distribution of areally averaged rainfall intensity, the authors then fit an exponential function. Through the Gumbel (1958) theory of extremes, the parameters of this exponential function are also the parameters of the Gumbel distribution of areally averaged extreme rainfall intensity. The parameters are also functions of the scaled catchment area, A/λ^2, and this dependence is used to show how catchment intensity-duration-frequency (IDF) curves change with catchment area. The parameters of the catchment IDF curves for the particular case of zero catchment area (A = 0), are matched with the parameters of observed point IDF curves, resulting in absolute values (rather than functions) of the parameters for the catchment IDF curves. For this particular case, this is only possible if the observed point rainfall extremes also follow a Gumbel distribution. Hence, this particular formulation of the method does not cater for the many cases where extreme point rainfall is better fitted by other distributions. Similar to the Rodriguez-Iturbe and Mejía method, there is also an assumption that the correlation structure of the extreme rainfall is the same as for the parent ("average") rainfall process, which seems unlikely.

Even after some simplification the final expression of ARF is complex, but shows dependence on catchment area, spatial correlation length, duration and return period. There is only a weak dependence on duration, whereas in empirical methods this is generally a major control on the ARF. Instead, it is the spatial correlation length, λ, that is critical in the proposed method, and the authors note that λ and duration are often closely related to each other and to storm type. For very large return periods, the ARF is a function of catchment area and correlation structure only. The authors argue that the correlation length is a more direct and pertinent measure of storm type and governing precipitation processes than duration is, and that correlation length is therefore more relevant to the estimation of ARFs. However, for hydrological applications the concentration (response) time of the catchment, and hence the rainfall duration, as well as its spatial extent, is the design criterion. The concentration time does not only vary with catchment area, but also with other catchment characteristics, such as geology, and density and configuration of the drainage network.

Although the authors refer to the method as "fixed area" rather than "storm-centred", they use two individual storm events to verify their model, rather than using long and spatially dense data sets which would be required for an empirical estimate of fixed area ARFs, particularly if variation with return period is to be shown. The spatial correlation length, λ, is estimated visually from isohyetal patterns of the two storms, one small-scale convective event (λ = 1-2 km, duration ~ 4 hours) and one large-scale frontal event (λ = 60-120 km, duration ~ 96 hours). The suggested ranges of correlation lengths result in ARF curves that partly envelope the empirically derived storm-centred ARFs.

Omolayo's method

Omolayo (1989, referenced in Srikanthan 1995) assumes that rainfall depths are log-normally distributed in space, and uses the average spatial correlation coefficient, ρ, to estimate the ARF as:

$$ARF_{Omolayo} = \exp\left\{K_T \sigma \left(\sqrt{\frac{1+(n-1)\rho}{n}} - 1\right)\right\},$$

where K_T is a frequency factor corresponding to return period T, σ is the standard deviation of rainfall depth in the log domain, and n is the number of rainfall stations. This ARF varies directly with spatial correlation coefficient and inversely with return period, standard deviation and the number of raingauges. The ARF depends on the number of stations used, and on the area because ρ will depend on the area. The ARF has a lower bound which is reached for ρ = 0. Tabulated values of these lower bounds given in Srikanthan (1995) for typical values of T, n and σ seem high.

For an assumption of normal distribution, the expression of the ARF is

$$ARF_{Omolayo,2} = \sqrt{\frac{1+(n-1)\rho}{n}} \quad ,$$

which for large n reduces to

$$ARF_{Omolayo,3} = \sqrt{\rho} \quad .$$

This is similar to the expression derived by Rodriguez-Iturbe and Mejía, except that in this case the correlation coefficient is averaged over the raingauges rather than representing the value at a particular separation distance.

Crossing properties

The method described in this section takes a completely different approach to the estimation of ARFs compared with the empirical and correlation methods. An elaborate statistical framework for describing the spatio-temporal behaviour of rainfall is used, which involves several idealised assumptions. However, the calibration of the model requires less data than is needed for ARF estimation using the previously described methods.

The method of Bacchi and Ranzi (1996) is based on an underlying probabilistic model involving the crossing properties of extreme rainfall and on spatial and temporal integrals of this field. "Crossing properties" here refers to the local behaviour, in terms of spatial and temporal derivatives, of the fields at points where a given threshold is crossed. The rainfall field is considered in space (two dimensions) and time, and a type of peaks-over-threshold approach is applied in these three dimensions. As the threshold, b, increases, the average number, μ_b, of local rainfall maxima above the threshold per unit "volume" in the space-time domain is proportional to the rate of crossings of the threshold. The authors assume that the process of crossings converges to a Poisson process: however, while they do concede that corrections would be needed if clustering of local maxima occurs, they do not include any in their analysis. The ARF is calculated as

$$\mathrm{ARF}_{B\&R}\left(A, D, T\right) = \frac{P(A, D, T)}{P(A_1, D, T)} = \frac{b(A, D, T)}{b(A_1, D, T)}$$

where P is the rainfall of duration D over the area A or A_1 which has return period T. Here A_1 denotes the area of a "point" approximated by a 1 km square radar pixel. The notation using b in the second form corresponds to using b for the threshold in the underlying theory.

Let $X_{A,D} = X_{A,D}(x, y, t)$ denote the rainfall field accumulated locally over a duration D and over an area A. If the Poisson counting of exceedances holds, the probability that the maxima do not exceed the threshold b within an overall time-period D_0 and within an overall spatial domain A_0 is given by the following exponential relationship:

$$F_{A,D}(b) = \Pr(\text{no } X_{A,D} \text{ exceeds } b) = \exp\left(- \mu_b A_0 D_0\right).$$

Here the dependence of μ_b on the area A and duration D of the accumulated rainfall field is suppressed from the notation. The function $b(A, D, T)$ is found by solving

$$F_{A,D}(b) = (KT)^{-1} \qquad \text{or} \qquad \mu_b A_0 D_0 = \log(KT),$$

for b, where K is the constant to convert the time-units to years. After some further development of the theory, and some data analysis, a formula of the following type is derived:

$$b(A, D, T) = \left\{ \frac{dA^{\gamma} D^{\delta} A_0 D_0}{\log(KT)} \right\}^{1/(3+\beta_{A,D})},$$

where d, γ, δ and $\beta_{A,D}$ are data-fitted constants obtained using observed values of $X_{A,D}$ from a given data set, and where $\beta_{A,D}$ varies with A and D.

In order to arrive at their expression for the ARF, Bacchi and Ranzi (1996) needed to make a number of assumptions (apparently unjustified in strict terms although the assumptions hold for Gaussian fields). These relate to crossing-properties taking each of the x-, y- and t-directions separately and include assumptions about statistical independence between statistics for these directions. The assumptions used are in addition to the assumption inherent in the use of a Poisson model for the occurrence of large rainfalls.

The data analysis is limited to a single storm event in a period covering 4 days, which seems to be a very small amount of data from which to derive estimates of extreme rainfall frequencies. Nevertheless, the functional form of the relationship of $ARF_{B\&R}(A, D, T)$ is of some interest. In particular the results suggest that ARF is proportional to a power (close to zero) of $\log(KT)$, which would imply a fairly slowly-changing function: the parameter values found by Bacchi and Ranzi (1996) imply a slow decay (i.e. a small negative power).

The model is calibrated using radar images of an occluded front passing over the eastern Po Valley, Italy. That is, the model is calibrated using images from a single, large-scale storm. The authors consider the method to be applicable for small urban catchments no greater than a few square kilometres and with durations up to a few hours. These limited scales are presumably necessary for the maxima to be assumed independent in time and space, i.e. for the Poisson assumption. The authors note that the local maxima in the storm used for calibration showed clustering features. They conclude that a more extensive analysis of observed rainfall fields is needed to investigate the dependence of the reduction factor on different meteorological regimes and to improve the statistical soundness of the relationships presented. The use of radar data is discussed in a separate section below.

Scaling relationships

It has been noted that empirical ARF curves often display scale-invariant (scaling) behaviour in space and time within particular limits, the scaling regime, which indicates that a multifractal analysis may be successful (e.g. Veneziano and Langousis 2005). The attractive feature of a multifractal approach is that statistical properties of complex geophysical data can be characterised over a wide range of scales in terms of a few parameters (e.g. Davis et al. 1994). Scaling relationships therefore seem to hold some promise for the development of a theoretical framework for ARF estimation.

In a review paper, Veneziano et al. (2006) discuss different definitions of multifractality and outline the theory for deriving ARFs based on counting rainfall exceedances above a threshold in "tiles" making up a unit "cube" in two spatial dimensions plus a time dimension. They suggest that if the ARF is insensitive to climate, season, etc., then it can be robustly estimated from just one or very few space-time data sets. However, when the models are fitted to empirically derived ARFs they need the same amount of data as these methods (while not necessarily spanning as large range of spatial and temporal scales). By being based on a counting exercise, these methods superficially have similarities with empirical methods, but the elaborate multifractal framework encompassing the counts sets them apart from these. The concept of exceedances was also used in the crossing properties method (Bacchi and Ranzi 1996), but again the theoretical framework here is different.

Reflecting the scaling properties of rainfall in space and time, de Michele et al. (2001) present a simple model that is calibrated using empirically derived ARFs for Milan, Italy (empirical ARFs are derived as the mean of annual maximum areal rainfalls divided by the mean of annual maximum point rainfalls).

$$\mathrm{ARF}_{\mathrm{de\,Michele}} = \left[1 + \varpi\left(\frac{A^{*z}}{D}\right)^{b}\right]^{-\upsilon/b},$$

where A^* is the (catchment) area minus the area of the raingauge, D is the duration of the rainfall, and ϖ, z, b and υ are fitted parameters. Because only eight years of data were available, the authors did not attempt to develop a model that incorporates the return period of the rainfall. The model was fitted to durations between 20 minutes and 6 hours, and areas between 0.25 and 300 km^2. A plot of expected rainfall intensities versus A^* shows that the model fits the data well for the 1- and 3-hour durations, but slightly less well for the 20 minute and 6-hour durations. The fit also becomes worse for increasing sizes of area. The systematic manner in which the model results deviate from the observed data is a worrying feature, suggesting that the scaling regime might be rather limited. However, the authors also fit a model to UK ARFs presented in the Flood Studies Report (NERC 1975), with durations ranging from 1 minute to 25 days and areas from 1 to 18,000 km^2. Although the explained variance of the fitted model is very high, at R^2=0.96, it is not clear from the scatter plot of modelled and observed ARFs which points relate to particular durations and areas, and hence it is difficult to judge whether or not there are any systematic biases.

Also using ARF values presented for the UK in NERC (1975), Veneziano and Langousis (2005) deduce that ARFs are scale-invariant with regard to area and duration for (roughly) areas, A, between 1000 and 10,000 km^2 and durations, D, between 15 minutes and several hours. Specifically, the ARF is constant for $D \propto \sqrt{A}$. Outside these spatial and temporal ranges, rainfall increasingly deviates from perfect multifractality, consistent with local intensity fluctuations being smaller than required for scale invariance. Veneziano and Langousis (2005) argue further that under perfect multifractality ARFs show asymptotic scaling behaviour with return period. However, they note that this may not apply in reality, or may occur for return periods that are too large to be of practical interest.

Storm movement

In contrast to previous methods which were empirically or statistically based, Bengtsson and Niemczynowicz (1986) take a simplified conceptual physics approach to ARF estimation by moving an idealised storm across an area. The resulting ARFs should therefore be more similar to ARFs derived using storm-centred rather than fixed-area approaches. The data requirements are limited as the method is applicable to small areas and is representative of frequent events. It makes assumptions about the shape and movement of convective rainfalls resulting in a fairly simple ARF calculation.

The method is intended for urban catchments, up to about 30 km^2 in size, and for short durations up to 40 minutes. It is referred to as the moving storm derived areal reduction factor, M-ARF, and was developed using 12 recording raingauges in the city of Lund, Sweden. The method is based on the movement of convective storms, and ARFs are calculated from rainfall observations at a fixed point (point hyetograph) and storm speed.

The assumption is made that the shape of the hyetograph and its velocity of movement do not change during the storm's passage over the area. Since urban areas are limited in areal extent, rainfall intensities are not expected to change drastically. Further, the lateral rainfall intensity (transverse to the storm's direction of movement) is assumed to decay exponentially, such that

$$i = i_c e^{-ky}$$

where i is the rainfall intensity at a lateral distance y from the centre, which has rainfall intensity i_c, and k is a distribution coefficient. The storm speed is derived using a regression relationship with concurrent wind velocity at 600 mb height, which is generally available from nearby airports. The areal rainfall is calculated by integrating the rainfall field as it moves across the catchment. This is divided by the point rainfall to obtain the ARF estimate. The areal reduction factor turns out not to depend much on which raingauge hyetograph is used, but average ARFs derived from hyetographs at any three raingauges give a very stable estimate. The results are reported to agree well with empirical ARF estimates of 0.5 years return period. Where hyetographs are not available, synthetic storms such as a block rainfall derived from an intensity-duration-frequency curve may be simulated to move across the catchment. The authors note that ARFs have been found to depend on return period, and suggest that for design storms this can be taken into account by varying the storm speed and/or the lateral rainfall intensity distribution.

The method assumes a laterally decaying rainfall intensity that is appropriate only for small convective storms, and the main area of application is in urban hydrology. Hence, it is not suitable for application to large catchment areas, or for long durations.

Radar data

Traditional ARF estimation has been carried out using dense networks of raingauges. More recently radar data have become available and several authors have investigated its use as an alternative to using ground observations (e.g. Durrans et al. 2002; Allen and DeGaetano 2005b; Lombardo et al. 2006). Radar data provide a much improved spatial coverage compared with even the more dense raingauge networks, resulting in good indications of the spatial patterns of rainfall. However, radar records are short, particularly for the finest spatial resolutions. The quantitative measurements are also poor compared with raingauge data, although this might be overcome by using raingauge-calibrated radar data.

Durrans et al. (2002) evaluated the potential of radar-rainfall data for development of geographically-fixed depth-area relationships that vary with return period (compare Eq. (2)). They used data on a (roughly) 4 km grid covering a rectangular area from eastern Colorado to western Arkansas, and from northeastern Texas to central Kansas, United States. Durations of 1, 2 and 4 hours were investigated during a 7.5-year study period, May 1993 to September 2000. This study found the following issues: the short period of record is a limitation for the application of frequency analysis to obtain the point and areal rainfalls of a particular return period when calculating the ARF. Heterogeneities in the radar data occurring because of continual improvements to the data processing algorithm is another difficulty. The sampling variation due to short records and heterogeneities can give rise to unexpected results, such as ARFs greater than 1 for some averaging areas and return periods. This mainly occurred because of edge effects in the spatial smoothing algorithm. Record lengths and poor homogeneity should improve with the passage of time.

Durrans et al. (2002) also report a concern about possible biases in the radar estimates of extreme rainfall. Precipitation estimates for the 100-year return period were found to be at least 20-35% smaller than gauge-based estimates published in Hershfield (1961) and Frederick et al. (1977). This may be due to short radar records and natural climate variability. The ARFs are less affected, because the biases cancel out to some extent when calculating the area to point rainfall ratio. Hence, composite ARFs developed using radar data (averaging ARFs calculated for several sub-areas, and avoiding edge areas) are considered reasonably consistent with earlier gauge-based studies presented in U. S. Weather Bureau (1958). However, they do not decrease with area as rapidly as gauge-based estimates do. There is not a pronounced difference between the curves for different return periods, but the ARFs for larger return periods recede somewhat quicker than for smaller return periods. Radar data are expected to become more reliable as the technique develops in the future.

In contrast to the study by Durrans et al. (2002), Allen and DeGaetano (2005b) found that ARFs derived from radar data decay at a faster rate with increasing area than ARFs calculated from raingauge data. This more recent study uses five years, 1996-2000, of daily radar data on a 2 km grid over two areas in the eastern United States (New Jersey and North Carolina). The ARFs are calculated according to Eq. (3). For a basin size of 20,000 km^2 the difference between ARFs from radar and gauge data ranges from 11 to 32%. Between-station variance of same-day extreme precipitation, as well as the coefficient of variation, tends to be larger for the radar-derived areal extreme events, favouring a smaller radar areal precipitation. Smaller radar ARFs are also favoured because, on average, a larger percentage of gauges have coincident annual maxima than do the radar pixels that correspond to these gauges. The authors conclude that the accuracy of the calibrated radar data for extreme events is suspect. When radar areal precipitation amounts were calculated and compared with gauged areal precipitation, the results varied from region to region as to which were the largest.

Discussion and recommendations

The relationship between the point and areal rainfalls has been found to vary with, for example, predominant weather type, season, return period and estimation method. This section discusses the methods reviewed in detail earlier in the paper. A summary of the key characteristics of each method and its advantanges and limitations is given in Table 1. Two distinct groups of methods can be recognised; the generally data intensive and computationally laborious traditional empirical methods and the often more elegantly formulated and recently developed analytical methods.

The analytical methods (correlation by Rodriguez-Iturbe and Mejía 1974, Sivapalan and Blöschl 1998 and Omolayo 1989; crossing properties by Bacchi and Ranzi 1996; scaling methods by de Michele et al. 2001 and Veneziano and Langousis 2005; and storm movement by Bengtsson and Niemczynowicz 1986) attempt to put areal reduction factor (ARF) estimation on a sounder scientific basis. However, they are generally based on assumptions that are not entirely true descriptions of the real rainfall process, which is a cause for concern and uncertainty regarding the results. This concern is compounded by the often limited amount of actual rainfall data that so far has been used to verify them. However, with further verification, some of these methods may prove to provide perfectly adequate ARF estimates with a much smaller amount of computational effort and data requirements than the traditional methods. Results from methods based on scaling relationships seem to agree with empirical estimates within a limited scaling regime, but similarly to methods developed for short durations and small areas (e.g. the empirical small-scale study and the analytical crossing properties and storm movement methods), they may not be appropriate for application to a comprehensive set of temporal and space scales.

A number of both the empirical and analytical methods, such as storm-centred methods, some correlation-based methods and the annual maxima-centred method, may not result in probabilistically correct areal rainfall estimates. That is, when multiplying the ARF with a T-year point rainfall, the resulting areal rainfall may not necessarily have the same T-year return period. However, the question is how large the discrepancy is, as, for any method, a small difference may be acceptable when considered in conjunction with a method's other advantages. Until the magnitudes of the discrepancies have been assessed, it seems prudent not to recommend these methods for use with rainfall frequency estimates. Instead, a fixed-area approach can be used to obtain probabilistically correct areal rainfall estimates. But for any method it should be borne in mind that the results will not be better than the underlying data. A fixed-area approach may not give probabilistically correct results, for example, if there are biases in the areal rainfall estimates, say, because of underrepresentation of gauges in upland parts of a catchment.

The use of radar data is at present problematic. Differences can be expected between successive time periods for the same area because of heterogeneities in the data, as resolution and radar data processing improves with time. Other problems are short records and possible biases in the ARF estimates. Although this type of data holds much promise for the future, it seems too early to apply it for ARF estimation at national levels.

There is no quick and unambiguously correct way of updating the current ARF estimates, which in most parts of the world have probably been derived using traditional empirical fixed-area methods. Although being data-intensive and laborious, these traditional methods still have advantages over the newer analytical methods, mainly because of the limitations of the latter as discussed above (i.e. assumptions not strictly met by the real rainfall process; limited range in space and time; not probabilistically correct). Some empirical fixed-area methods have the advantage also over empirical storm-centred approaches in that they can provide probabilistically correct ARFs.

With modern database systems for data storage and powerful computers for data processing, the application of empirical fixed-area methods should not be problematic. For example, in the UK more than 30 years of rainfall data have been collected since the ARF estimates in the Flood Studies Report (NERC 1975) were presented. The UK has a relatively dense network of daily gauges, and over the past decades the digital rainfall records from the network of gauges with a sub-daily resolution have become more plentiful. Data availability in many other countries has probably followed a similar pattern, and this increase in available data would provide improved ARF estimates if new studies were undertaken. For example, a version of Bell's fixed-area method (Bell 1976), which varies with return period, may be suggested perhaps in conjunction with a regionalisation scheme taking into account the differing rainfall characteristics of different climatic regions. Bell's method offers specific advantages over other empirical methods. In particular, compared to the US Weather Bureau (1957-1958), NERC (1975) and the Desbordes et al. (1984) methods, it has the advantage of incorporating return period, and it has the additional advantage over the last of these in that it encompasses a wider range of time and space scales. It is simpler than the National Weather Service method (Myers and Zehr 1980). Finally, it is intuitively more probabilistically correct than the annual-maxima centred method (Asquith and Famiglietti 2000).

Acknowledgements

This review was undertaken as part of the Defra-funded project “Reservoir Safety – Long Return Period Rainfall” (WS194/2/39).

References

Allen, R. J. and DeGaetano, A. T. (2005a) Areal reduction factors for two eastern United States regions with high rain-gauge density. J. Hydrol. Eng., 10 (4), 327-335.

Allen, R. J. and DeGaetano, A. T. (2005b) Considerations for the use of radar-derived precipitation estimates in determining return intervals for extreme areal precipitation amounts. J. Hydrol., 315, 203-219.

Asquith, W. H. and Famiglietti, J. S. (2000) Precipitation areal-reduction factor estimation using an annual-maxima centred approach. J. Hydrol., 230 (1-2), 55-69.

ARR (2001) Design rainfall considerations, Book 2 of Volume 1 of Australian Rainfall and Runoff (ed. Pilgrim, D. H.). Engineers Australia, Barton, Australia, 49 pp.

Bacchi, B. and Ranzi, R. (1996) On the derivation of the areal reduction factor of storms. Atmos. Res., 42, 123-135.

Bell, F. C. (1976) The areal reduction factor in rainfall frequency estimation. Institute of Hydrology Report No 35, Natural Environment Research Council, UK.

Bengtsson, L. and Niemczynowicz, J. (1986) Areal reduction factors from rain movement. Nordic Hydrol., 17 (2), 65-82.

Chow, V. T. (1951) A general formula for hydrologic frequency analysis. Trans. Am. Geophys. Union, 32, 231-237.

Chow, V. T. (1964) Statistical and probability analysis of hydrologic data. In Chow, V. T. (ed) Handbook of Hydrology, pp. 8.1-8.42. McGraw-Hill, New York.

Clark, C. and Rakhecha, P. R. (2002) Areal PMP distribution of one-day to three-day duration over India. Meteorol. Appl., 9, 399-406.

Davis, A., Marshak, A., Wiscombe, W and Cahalan, R. (1994) Multifractal characterizations of nonstationarity and intermittency in geophysical fields: Observed, retrieved, or simulated. J. Geophys. Res., 99, 8055-8072.

de Michele, C., Kottegoda, N. T. and Rosso, R. (2001) The derivation of areal reduction factor of storm rainfall from its scaling properties. Water Resour. Res., 37 (12), 3247-3252.

Desbordes, M., Raous, P. and Trévisiol, Y. (1984) Areal reduction factors on short time and space intervals. Wat. Sci. Tech., 16, 189-198.

Durrans, S. R., Julian, L. T. and Yekta, M. (2002) Estimation of depth-area relationships using radar-rainfall data. J. Hydrol. Eng., 7 (5), 356-367.

Einfalt, T., Johann, G. and Pfister, A. (1998) On the spatial validity of heavy point rainfall measurement. Wat. Sci. Tech., 37 (11), 21-28.

Frederick, R. H., Myers, V. A. and Auciello, E. P. (1977) Five- to 60-minute precipitation frequency for the eastern and central Unived States. NOAA Technical Memorandum NWS HYDRO-35. Office of Hydrology, National Weather Service, Silver Spring, Md.

Grebner, D. and Roesch, T. (1997) Regional dependence and application of DAD relationships. Proc. FRIEND'97 – Regional hydrology: Concepts and models for sustainable water resource management. Postojna, Slovenia, Sept.-Oct. 1997. IAHS Publ., 246, 223-230.

Gumbel, E. J. (1958) Statistics of extremes. Columbia University Press, New York.

Hershfield, D. M. (1961) Rainfall frequency atlas of the Unites States for durations from 30 minutes to 24 hours and return periods from 1 to 100 years. Technical Paper No. 40. U. S. Department of Commerce, Washington D. C.

Huff, F. A. (1995) Characteristics and contributing causes of an abnormal frequency of flood-producing rainstorms at Chicago. Water Resour. Bull., 31 (4), 703-714.

Huff, F. A. and Shipp, W. L. (1969) Spatial correlations of storm, monthly, and seasonal precipitation. J. Appl. Meteorol., 8 (4), 542-550.

Lombardo, F., Napoletano, F. and Russo, F. (2006) On the use of radar reflectivity for estimation of the areal reduction factor. Nat. Hazards Earth Syst. Sci., 6, 377-386.

Myers, V. A. and Zehr, R. M. (1980) A methodology for point-to-area rainfall frequency ratios. NOAA Technical Report NWS 24. U. S. Department of Commerce, National Oceanic and Atmospheric Administration, National Weather Service, Washington D. C., USA.

NERC (1975) Flood studies report, Volume II. Natural Environment Research Council, UK.

NERC (1977) Flood studies supplementary report No 1: The areal reduction factor in rainfall frequency estimation. Natural Environment Research Council, UK.

Omolayo, A. S. (1989) Relationships between point and areal rainfall for flood and drought assessments. Ph. D. Thesis, University of New South Wales, Australia.

Omolayo, A. S. (1993). On the transposition of areal reduction factors for rainfall frequency estimation. J. Hydrol., 145 (1-2), 191-205.

Porter, J. W. and Ladson, A. R. (1993) An application of fixed-area rainfall reduction factors. Hydrology and Water Resources Symposium, Newcastle, Australia, 30 June-2 July 1993. IEAust. Nat. Conf. Publ., 93/14, 273-279.

Prudhomme, C and Reed, D. W. (1999) Mapping extreme rainfall in a mountainous region using geostatistical techniques: A case study in Scotland. Int. J. Climatol., 19, 1337-1356.

Ramos, M. H., Creutin, J.-D. and Leblois, E. (2005) Visualization of storm severity. J. Hydrol., 315 (1-4), 295-307.

Rodriguez-Iturbe, I. and Mejía, J. M. (1974) On the transformation of point rainfall to areal rainfall. Water Resour. Res., 10 (4), 729-735.

Seed, A. W., Srikanthan, R. and Menabde, M. (1999) A space and time model for design storm rainfall. J. Geophys. Res., 104 (D24), 31623-31630.

Siriwardena, L. and Weinmann, P. E. (1996) Derivation of areal reduction factors for design rainfalls in Victoria. Report 96/4, Cooperative Research Centre for Catchment Hydrology, Australia.

Sivapalan, M. and Blöschl, G. (1998) Transformation of point rainfall to areal rainfall: Intensity-duration-frequency curves. J. Hydrol., 204 (1-4), 150-167.

Skaugen, T. (1997) Classification of rainfall into small- and large-scale events by statistical pattern recognition. J. Hydrol., 200 (1-4), 40-57.

Srikanthan, R. (1995) A review of the methods for estimating areal reduction factors for design rainfall. Report 95/3, Cooperative Research Centre for Catchment Hydrology, Australia.

Stewart, E. J. (1989) Areal reduction factors for design storm construction: Joint use of raingauge and radar data. In: New directions for surface water modeling, Proc. Baltimore Symposium, May 1989. IAHS Publ., 181, 31-40.

U. S. Weather Bureau (1957-1958) Rainfall intensity-frequency regime Parts 1 and 2, Technical Paper No. 29. U.S. Department of Commerce, Washington D.C., US.

Veneziano, D. and Langousis, A. (2005) The areal reduction factor: A multifractal analysis. Water Resour. Res., 41, W07008, 15pp, DOI:10.1029/2004WR003765.

Veneziano, D., Langousis, A. and Furcolo, P. (2006) Multifractality and rainfall extremes: A review. Water Resour. Res., 42 (supplement), W06D15, 18 pp, DOI: 10.1029/2005WR004716.

Yoo, C., Kim, K., Kim, H. S. and Park, M. J. (2007) Estimation of areal reduction factors using a mixed gamma distribution. J. Hydrol., 335 (3-4), 271-284.

Zehr, R. M. and Myers, V. A. (1984) Depth-area ratios in the semi-arid southwest United States. NOAA Technical Memorandum NWS HYDRO-40. Office of Hydrology, National Weather Service, NOAA, Silver Spring, MD, USA.

Word count: 8776 (excluding references and figure and table captions)

Figure caption

Figure 1. Areal reduction factors for precipitation in the United Kingdom presented in the Flood Studies Report (diagram derived from tabulated values in NERC (1975)).

Table caption

Table 1. Summary of methods for areal reduction factor (ratio of areal to point rainfall) estimation.

Table 1. Summary of methods for areal reduction factor (ratio of areal to point rainfall) estimation.

Type	Method name and/or reference	Characteristics	Comments
General empirical methods	Fixed-area (e.g. Omolayo 1993)	Geographically fixed area, such as a catchment; the point rainfall used is representative for the whole catchment	The ARF can be (but does not have to be) defined as an areal rainfall of a particular return period divided by the point rainfall of the same return period resulting in a probabilistically correct ARF estimate; data intensive
	Storm-centred (e.g. Omolayo 1993)	The area changes for each storm, and is outlined by a selected isohyet; the point rainfall used is the highest within each storm.	Typically used for PMP estimates, which are not associated with a particular return period
Specific empirical methods	US Weather Bureau (1957-1958)	Fixed-area; relates mean annual max areal rain to mean annual max point rain (for all stations and years)	The most commonly used method in the US; intuitive to apply but laborious; does not take into account return period
	Small-scale study – "fixed-area" (Desbordes et al. 1984)	Largest point rainfall of a given duration at each gauge for each event in the area is noted and the concurrent Thiessen-weighted areal rainfall is divided by it; ARFs are then averaged to get a final estimate; events selected based on discharge	Applicable to small space and time scales; unclear how results compare with more standard methods, but makes good use of data collected during a short experiment; may not be representative of extreme rainfalls as area small and period of record short; does not take into account return period; intermediate ARFs may be > 1
	Small-scale study – "storm-centred" (Desbordes et al. 1984)	A subset of the above, using only the largest point rainfall in the study area (but not necessarily for the entire storm area) for each event	All intermediate ARFs ≤ 1, but otherwise similar comments to above; ARFs recede more quickly than for the above method
	NERC (1975)	Fixed area; assumes that "an average of ratios" can	A simplification of the US Weather Bureau method;

		approximate "a ratio of averages"	unorthodox assumption makes computations convenient; does not take return period into account
	Bell (1976)	Fixed area; fits frequency distributions to point and Thiessen-weighted areal annual max rainfalls and calculates ARFs for specified return periods	These ARFs recede more quickly than those of the NERC method; takes return period into account; following Bell's method Stewart (1989) introduced a standardisation taking into account local variation in the mean annual max rainfall
	National Weather Service (Myers and Zehr 1980)	Based on frequency analysis of annual max rainfall at pairs of stations and the distance between them; uses small five-station networks	Some of the theory developed is rather involved; takes return period into account; does not require large, dense networks of concurrent observations
	Annual-maxima centred method (Asquith and Famiglietti 2000)	Concurrent rainfalls surrounding a point annual max are used for calculation of ratios between gauge-pairs (the max is always one of the pair), which are then plotted against distance; a curve is fitted and ARFs integrated; stratification on return period at annual max gauge	Takes return period into account, but may not be "probabilistically correct"; labour-saving as it does not require spatial averaging of rainfall, explicit calculation of correlation coefficient, or definition of representative area; requires a dense network of gauges
Spatial correlation structure	Rodriguez-Iturbe and Mejía (1974)	Relates the ARF to the correlation between two gauges separated by a "characteristic correlation distance"; assumes a particular spatial correlation structure and Gaussian point precipitation	Straight-forward, but requires estimation of mean distance between two random points in the area; does not take return period into account; assumption of Gaussian point rainfall not likely met
	Sivapalan and Blöschl (1998)	Extension to the Rodriquez-Iturbe and Mejía method; point rainfall is assumed to follow an exponential distribution	Expression of ARF is more complex than for the above method, depending on area, "spatial correlation length", duration and return period; method is verified using

			individual storm events rather than a traditional fixed-area approach
	Omolayo (1989)	The ARF is calculated using the average spatial correlation and the number of gauges in the area	Depends on return period; if Gaussian rainfall is assumed rather than log-normal, the ARF reduces to a measure similar to that of Rodriguez-Iturbe and Mejía
Crossing properties	Bacchi and Ranzi (1996)	Applies a peak-over-threshold approach to the spatial rainfall field; relies on assumptions of independence in the rainfall field in the two spatial directions and in time, in addition to distributional assumptions	Suitable for small areas and short durations; takes return period into account; relies on many assumptions about the rainfall field that may not be met
Scaling relationships	de Michele et al. (2001), Veneziano and Langousis (2005)	Based on scale-invariant (scaling) behaviour of rainfall; statistical properties can be characterised over a range of scales in terms of a few parameters	The range of spatial and temporal scales for which the derived relationships are valid is in practice limited; could take return period into account
Storm movement	Bengtsson and Niemczynowicz (1986)	Approach based on the movement of convective storms; uses point hyetograph and storm speed; assumes exponential decay transverse to the direction of movement	Applicable to small areas and short durations; example used agrees with empirically derived ARF estimate of 0.5 years return period
Radar data	Durrans et al. (2002), Allen and DeGaetano (2005b), Lombardo et al. (2006)	Various methods described above can be applied to this type of data	Improved spatial coverage compared with raingauge networks, but short inhomogeneous records and poor quantitative rainfall measurements cause uncertainties in the ARF estimates