Measurements of the backscattering characteristics of suspensions having a broad particle size distribution

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Abstract: *Acoustic backscatter systems (ABS) can be used to non-intrusively measure profiles of both the concentration and particle size of suspended sediments in the marine environment. Inversion of ABS measurements into sediment size and concentration requires knowledge of two scattering parameters, namely the total normalised scattering cross-section, χ, and the form function, f. χ quantifies the acoustical scattering by a given particle over all angles, relative to its cross sectional area, and represents attenuation due to particle scattering losses. f describes the backscattering characteristics of a particle relative to its geometrical size. In recent years, a number of studies have presented measurements of f and χ for populations of sediments sieved over narrow size ranges, thereby essentially providing values for nominally a single particle size in suspension. In the present study, we extend these works by looking at the impact that a broad particle size distribution has on the form of f and χ. Here we model and measure the average form function for a broad size distribution ($\sigma = \pm 0.35a_0$, where σ is the standard deviation about the mean particle radius, a_0) of suspended glass spheres, whose scattering characteristics are well documented. The model is in close agreement with the provisional measurements, and suggests that for populations of suspended glass spheres with broad size distributions, the form function increases by about 40% in the Rayleigh regime ($\lambda >> 2\pi a_0$, where λ is the wavelength of the sound in water), whilst decreasing by a factor of around 25% in the geometric regime ($\lambda << 2\pi a_0$), relative to that obtained for populations with a nominally single size in suspension. The output from this work has direct implications for the calculation of particle size and concentration profiles, obtained from acoustic backscatter data collected on suspensions of marine sediments at sea.*

Keywords: *Acoustic, scattering, cross, section, form, function, suspended, particle, sediment, size, distribution.*

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1. Introduction

Over the past three decades, acoustics has been increasingly used as a measurement tool in sediment transport studies[1,2,3,4]. Acoustics offers a number of advantages over traditional measurement techniques, since it enables non-intrusive, co-located, simultaneous profile measurements to be made of the bed features, the suspended sediment concentration and size, and the three orthogonal components of flow. The high spatial and temporal resolutions achievable with acoustical measurements permits the study of sediment transport processes at centimetric resolution down to the bed, over both inter-tidal and inter-wave timescales[4].

The retrieval of sediment concentration and size from acoustic data is based upon the principle that the amplitude of the backscattered sound is dependent upon the size and concentration of sediment in suspension[2]. Early studies carried out detailed laboratory calibration measurements for different concentrations and sizes of suspended sediment, to develop empirical algorithms to retrieve these parameters from acoustic backscatter data collected during field deployments[1]. Such algorithms were of limited accuracy however, and more recently, profiles of suspended sediment concentration and size have been derived by conducting multi-frequency inversions of the amplitude of the backscattered acoustic signal[2,3,5]. Inversion requires knowledge of the variation of the sediment scattering characteristics with frequency. The sediment scattering characteristics required are the normalised total scattering cross section, χ, which describes the attenuation of the acoustic signal due to sediment scattering, and the form function, f, which describes the backscattering characteristics of a particle relative to its geometrical size. For irregularly shaped particles, no full theoretical descriptions of f and χ are available, and therefore these parameters have been measured on a study by study basis[6,7,8], often employing a heuristic representation to describe the variation of f and χ with the dimensionless parameter x (= ka, where $k = 2\pi/\lambda$, with λ the wavelength of the acoustic sound in water, and a is the radius of the sediments). Most reported measurements of f and χ have been made for populations of sediments sieved over narrow size ranges, so called ¼ φ size fractions, where the sediment diameter (in mm), $d=2^{-\varphi}$. Sieving over ¼ φ size intervals generates narrow particle size distributions (PSD), for which the standard deviation (σ) about the mean size (a_0) is nominally $\pm 0.09a_0$. Even for such narrow ¼ φ PSDs, the form of f and χ for suspensions of glass spheres has been observed to change significantly from that expected for a single size of sphere, with the resonant structure observed for a single size being absent from the f and χ of the ¼ φ PSD[6]. Furthermore, measurements of *in-situ* sediment size distributions in coastal and estuarine environments frequently encounter variability in the range of sizes encompassed by the PSD, with both broader, and multi-modal size distributions observed[9,10,11]. Modelling results and measurements of χ for suspensions of glass spheres with broader PSDs ($\sigma = \pm 0.20a_0$)[12], along with those for ¼ φ PSDs, suggest that the form of χ continues to change as the standard deviation of the PSD increases. More recently, the effects of broader Rayleigh and bi-modal PSDs on the form of f and χ were evaluated through modelling, though no measurements of f or χ were available to validate the model results[13]. The present study therefore aims to extend these earlier works, by ascertaining what effect broad normal size distributions ($\sigma = \pm 0.35a_0$) have on both f and χ. The approach taken, is to model f and χ for suspensions of glass spheres, and to compare the modelling results with ABS derived measurements obtained for suspensions of glass spheres in the laboratory.

2. Modelling χ and *f*

For a population of particles, the average normalised total scattering cross section, χ_0, and the average form function, f_0, are theoretically:

$$\chi_0 = \frac{\int_0^\infty aP(a).da \int_0^\infty a^2 \chi(ka) P(a).da}{\int_0^\infty a^3 P(a).da} \tag{1a}$$

$$f_0 = \left[\frac{\int_0^\infty aP(a).da \int_0^\infty a^2 [f(ka)]^2 P(a).da}{\int_0^\infty a^3 P(a).da} \right]^{1/2} \tag{1b}$$

where *a* is a particle radius, *P*(*a*) is the probability distribution of the particles, and *k* is the wavenumber. Thus, for a given probability distribution, and providing the form of $\chi(ka)$ and $f(ka)$ are known, χ_0 and f_0 can be evaluated using Equation 1. In this study, a normal probability distribution was employed, with:

$$P(a) = \frac{1}{\sigma\sqrt{2\pi}} e^{(-(a-a_0)^2/2\sigma^2)} \tag{2}$$

Defining the product of *k* and *a* as the non-dimensionless parameter, *x*, the form of $\chi(x)$ and $f(x)$ employed in Equation 1 was taken for a suspension of glass spheres to be[14]:

$$\chi(x) = \frac{0.24(1 - 0.3e^{-((x-5.5)/1.5)^2})x^4}{1 + 1.2x^{1.8} + 0.24x^4} \tag{3a}$$

$$f(x) = \frac{(1 - 0.37e^{-((x-1.5)/0.5)^2})(1 + 0.28e^{-((x-2.0)/2.8)^2})(1 - 0.3e^{-((x-5.5)/0.5)^2})x^2}{1 + 0.95x^2} \tag{3b}$$

Thus, the variation of χ_0 and f_0 with x_0 (=ka_0) was calculated using Equations 2 and 3 in Equation 1, for a range of frequencies covering a range of x_0.

3. Measuring χ and *f*

Assuming that the phase of the backscattered sound received by the ABS is randomly distributed, the root mean square of the backscattered voltage can be shown to be:

$$V_{RMS} = \frac{K_t M^{1/2} f_0}{r\psi\sqrt{\rho_S a_0}} e^{-2r\alpha} \tag{4}$$

where r is the range from the transducer, M the mass concentration of suspended sediment, ρ_S the density of the sediment grains, ψ accounts for the departure from spherical spreading in the transducer near field[15], and K_t is a system constant. K_t incorporates the electronic response, the transmit and receive sensitivities, and the directivity response (beam pattern) of the transducer, and is specific to a given ABS system. The normalised total scattering cross section is contained within the total attenuation term, α, given by:

$$\alpha = \alpha_W + \frac{3\chi_0 M}{4\rho_S a_0} \tag{5}$$

where α_W is the absorption by water, and all other terms are as previously defined. Rearranging Equation 4, and taking the natural log transformation yields a linear function of $Log_e(V_{RMS} r\psi)$ with range r from the transducer:

$$Log_e(V_{RMS} r\psi) = Log_e\left(K_t M^{1/2} f_0 / \sqrt{\rho_S a_0}\right) - 2r\alpha \tag{6}$$

Thus, by measuring the variation of V_{RMS} with r, for a homogenous suspension of glass spheres, a profile mean value of χ_0 can be calculated from the slope of Equation 6, using Equation 5. Whilst Equation 6 could also be used to retrieve the form function from the intercept, small measurement errors throughout the ABS profile can lead to uncertainties in the calculation of f_0 in this way. Consequently, the approach adopted in this study was to model χ_0 using Equation 1a, and to calculate f_0 from the measured V_{RMS} by directly re-arranging Equation 4 for each ABS bin.

4. Results

Fig. 1 shows the variation of χ_0 with x_0, as predicted by the model for normal probability distributions with σ equal to $\pm 0.09 a_0$ (¼ φ narrow distribution, dotted line) and $\pm 0.35\ a_0$ (broad distribution, dashed line). Also shown in Fig. 1, are the values of χ_0 obtained from the ABS measurements using Equation 6, for the narrow (crosses) and broad (open circles) particle distributions.

Similarly, Fig. 2 shows the variation of f_0 with x_0, as predicted by the model for the narrow (dotted line) and broad (dashed line) distributions, along with the profile mean values of f_0 obtained from the ABS measurements, with the symbols as defined for Fig. 1.

5. Discussion

The measured values of χ_0 were in close agreement to those predicted by the model, for both the narrow and broad size distributions. Fig. 1 shows that the effect of increasing the standard deviation of the size distribution is to increase χ_0 below x ≈ 2, whilst decreasing it above this value. The close agreement between the modelled and measured values of χ_0 justifies the use of the modelled χ_0 to obtain f_0 from the ABS measurements, and Fig. 1 suggests this would not introduce any significant source of error, or biasing. The measured values of f_0 were also in close agreement with the modelled values, and Fig. 2 shows the spread of the particle size distribution significantly and measurably changes the form function for suspensions of glass spheres. The behaviour of the broad size distribution form function, relative to that for the narrow size distribution, showed considerable variability over the range of x_0 studied. At $x_0 = 0.35$, our lowest observation to date, the form function for the broad size distribution was observed to be ≈ 35 % greater than the form function for the narrow size distribution (see Fig. 2), whilst larger differences of up to 100% were predicted at $x_0 = 0.1$. In contrast, above $x_0 \approx 2$, the modelled form function for the broad size distribution was up to 25 % smaller then that observed for the narrow size distribution.

Physically, the observed variation in the form function is associated with enhanced backscattering below x ≈ 2, with generally diminished backscattering above this point. Since changes in the backscattering amplitude are interpreted in terms of changes in sediment concentration and size, the results of this study therefore suggest that knowledge of the size *distribution* of suspended particles is likely a key input for the inversion of ABS data collected in the marine environment.

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Figures

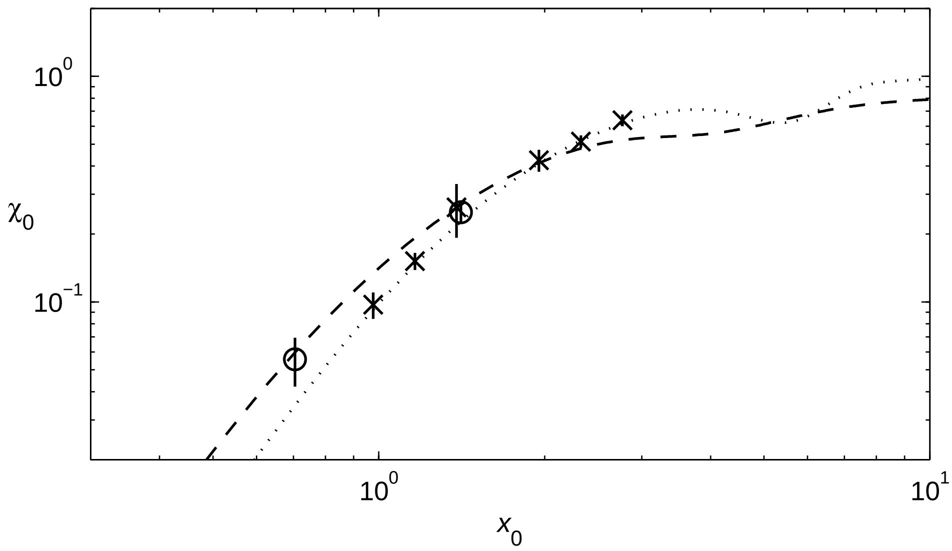

Fig. 1: Modelled variation of χ_0 with x_0 for narrow (dotted line) and broad (dashed line) particle distributions. ABS derived measured values of χ_0 are shown for the narrow (crosses) and broad (open circles) distributions. Error bars denote one standard deviation about the mean value.

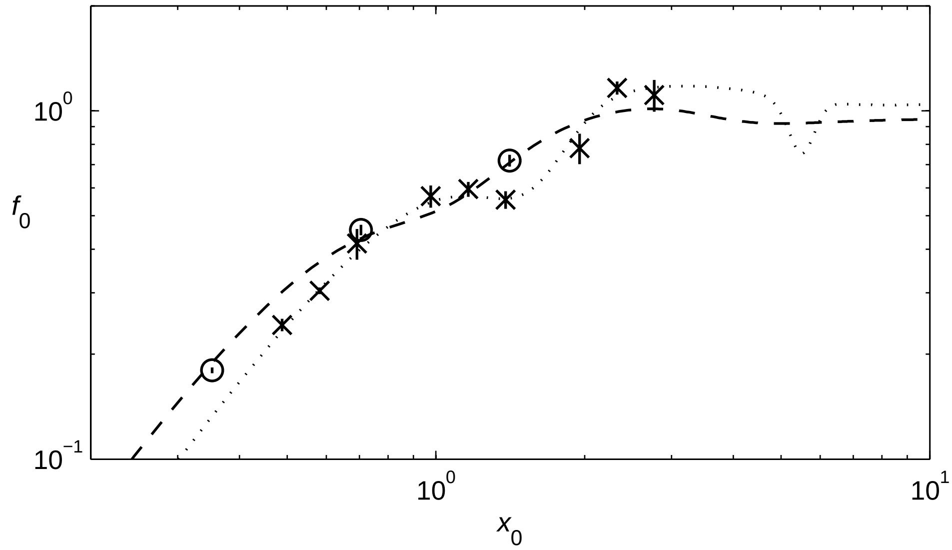

Fig. 2: Modelled variation of f_0 with x_0 for narrow (dotted line) and broad (dashed line) particle distributions. ABS profile mean values of f_0 are shown for the narrow (crosses) and broad (open circles) distributions. Error bars denote one standard deviation about the mean value.