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Influence of open boundary conditions and sill height  
upon seiche motion in a gulf

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## Abstract

A cross sectional model of an idealised constant depth gulf with a sill at its entrance, connected to a deep ocean, is used to examine the barotropic and baroclinic response of the region to wind forcing. The role of the oceanic boundary condition is also considered. Calculations show that in the case of a tall sill, where the pycnocline intersects the sill, the baroclinic response of the gulf is similar to that of a lake, and internal waves cannot radiate energy out of the gulf. The barotropic response shows free surface oscillations, with nodes located close to the centre of the oceanic basin and entrance to the gulf, with associated barotropic resonant periods. As the sill height is reduced, baroclinic wave energy is radiated from the gulf into the ocean, and the form of the baroclinic response changes from a standing wave (tall sill) as in a lake to a progressive wave (no sill). The location of sea surface elevation nodes, and resonant periods changes as the sill height is reduced. Calculations of the barotropic resonant periods with and without stratification could not determine if they were influenced by the presence of stratification, although published analytical theory suggests when energy is lost from the gulf by internal wave radiation, they should be. This inability to detect changes in barotropic resonant period due to stratification effects is due to the small change in resonant frequency produced by baroclinic effects, as shown by analytical results, and the broad peak nature of the computed resonant frequency. In the case of a closed off shore boundary (an off shore island) there is a stronger and narrower energy peak at the resonant frequency than when a barotropic radiation condition is applied. However the influence of stratification upon the resonant frequency could not be accurately determined. Although the off shore boundary was well removed from the gulf,

to such an extent that any baroclinic waves reflected from it could not reach the gulf within the integration period, it did however slightly influence the gulf baroclinic response due to its influence on the barotropic response.

## 1. INTRODUCTION

Although extensive analytical and numerical studies have been performed on the processes generating and influencing the barotropic and baroclinic seiche motion in lakes, the comparable problem in gulfs and fjords has received less attention. In particular the role of sill height and off shelf boundary in influencing the radiation of energy from a gulf and hence determining the intensity of the seiche within the gulf and its period has until recently (Cushman-Roisin et al., 2005, Arneborg and Liljebladh, 2001) received little attention. In the case of the analytical solution of the seiche motion in a narrow lake, a common approach is to solve the linear hydrodynamic equations in slice form, namely  $x$  and  $z$  space, with  $x$  the horizontal and  $z$  the vertical coordinate, subject to no flow through the lake sides. Such a model formulation and boundary conditions can be readily incorporated within a numerical model, and a numerical solution in close agreement with the analytical can be determined (eg. Hall and Davies 2005) and subsequent problems eg. non-linear or non-hydrostatic effects (eg. Berntsen and Bergh 2009) which are not amenable to analytical solution can then be examined with the numerical model.

In the case of the seiche problem in a homogenous or stratified constant depth gulf, the linear hydrodynamic equations in slice form can be solved analytically subject to no flow through the land boundary at the head of the gulf, and radiation of energy into an infinitely deep ocean of infinite extent at the mouth of the gulf (eg. Cushman-Roisin et al.

2005). In theory an identical problem can be solved numerically, although as will be shown here in any numerical exercise aimed at extending the gulf problem to a physically more realistic situation in which there is a sill at the entrance to the gulf, and the gulf is connected to the ocean by a shelf slope, there are problems in including an infinite ocean.

The primary objective of this study is to examine the important parameters, namely sill height and off shelf boundary that influence seiche motion in gulfs, using a process type model of wind forced motion in a gulf with idealized topography (Fig 1). The effects of other parameters namely sill width and stratification are beyond the scope of this study. Obviously in the limit that the sill depth  $h_s$  goes to zero, then there is no water exchange between gulf and ocean, and the gulf response is that of a lake. A secondary objective is to determine to what extent in the context of a gulf and sill, stratification within the gulf influences the barotropic response of the gulf to wind forcing. As shown by Cushman-Roisin et al. (2005), in the case of a lake, the barotropic response is not significantly influenced by baroclinic effects, whereas in a gulf the baroclinic mode can radiate energy into an infinite ocean and thereby influence the barotropic response, by creating stronger coupling between the barotropic and baroclinic motion. Consequently in the numerical model, as the sill depth  $h_s$  decreases, namely sill height increases, then baroclinic energy radiation to the ocean will reduce and eventually reach zero when the sill height is the same as the water depth. In this case there is no flow at the sill, and the gulf becomes a lake. In addition to examining how the sill height, and ocean beyond influence the gulf's barotropic response it is useful to examine how internal wave generation within the gulf is influenced by the sill.

Although the problem of how to formulate the open boundary at the entrance to a semi-enclosed coastal embayment (namely a gulf) and the importance of such a formulation, has been examined by a number of authors e.g. de Young et al. (1993), Davidson et al. (2001), Zhai et al. (2008) to the authors' knowledge the influence of sill height and oceanic boundary upon gulf barotropic resonance and internal wave generation has not previously been examined.

Before proceeding to discuss the details of the numerical model and domain used in the present calculations it is useful to briefly review some aspects of the basic theory controlling resonance in lakes and gulfs in the absence of sills. In the case of a constant depth lake of length  $\ell$ , water depth  $h$ , and two layer stratification with upper layer density  $\rho_1$ , thickness  $h_1$  and lower layer  $\rho_2$ , thickness  $h_2$ , neglecting rotation, the inviscid linear hydrodynamic equations can be readily solved subject to no flow on either land boundary (namely  $u = 0$ ). This solution yields a fundamental barotropic period  $T_b = 2\ell/(gh)^{1/2}$ , with  $g$  accelerated due to gravity, and a baroclinic mode (internal)  $T_i = 2\ell/(g'h')^{1/2}$  where  $g' = \left(\frac{\rho_2 - \rho_1}{\rho}\right)g$  and  $h' = \left(\frac{h_1 h_2}{h_1 + h_2}\right)$  with  $\rho$  mean density. In this case in the linear solution the surface elevation and internal displacement seiche have a node in the centre of the lake with maximum displacement against the coastline.

Since energy cannot be radiated out through the land boundary, and in a two layer linear model energy cannot cascade to shorter wavelengths, then as shown by Heaps and Ramsbottom (1966), in such a case damping and energy loss only occur through frictional effects. However, in more complex models containing non-linear terms and allowing for vertical mixing (e.g. Hall and Davies 2005) the situation is more complex.

In these models vertical mixing and non-linear interaction between the waves leads to seiche damping.

In the case of a seiche in a constant depth gulf of lateral extent  $\ell$ , the equations are solved subject to no flow at the coastal boundary of the gulf, and an assumed infinite undisturbed ocean beyond the gulf. Consequently for the solution of the gulf only problem an elevation node at the open boundary can be assumed. This gives rise to fundamental seiche periods  $T_b = 4\ell/(gh)^{1/2}$  and  $T_i = 4\ell/(g'h')^{1/2}$  in essence the problem resembles one half of a lake. In this case damping can occur as energy is radiated out of the gulf. As shown in analytical work using a two layer model (Cushman-Roisin et al. (2005)) of a gulf and infinite ocean, in this case the barotropic and baroclinic seiches are weakly coupled, and the period of the barotropic seiche (unlike in a lake) is slightly influenced by stratification. By radiating energy from the gulf into this ocean a significant energy loss occurs within the gulf. In addition topographic features within the gulf and at its entrance (e.g. the sill at the entrance to a fjord) generate internal waves with an associated form drag. Consequently in gulfs these processes, in particular energy loss by radiation, contribute more to seiche energy loss than bottom friction, although this must depend on sill height.

In practice there is not an infinite ocean outside a fjord or gulf, where often there are offshore island chains. In addition the gulf is normally separated from an ocean of finite depth by a continental shelf, the lateral extent and slope of which depends upon geographic location. Also the interior of a gulf or fjord is separated from the exterior by a sill which is usually much shallower than the gulf and could influence the energy

radiating from the gulf. To date the role of the sill or the form of the off shore boundary in determining energy radiation from a fjord has not been studied in detail.

In this paper an idealized cross section (slice) model (described in the next section) of a gulf or fjord with associated sill, continental shelf slope and ocean beyond is used to examine the processes influencing seiche motion in the gulf. Initially the depth and lateral extent of the ocean and the assumed boundary condition at the edge of the ocean are considered. In particular a no-flow condition corresponding to blocking of energy radiation by an offshore island, or a free surface radiation condition used to represent an infinite domain are examined. Subsequently, the role of the sill at the entrance to a gulf or fjord is considered. In all calculations the seiche motion is generated by an imposed spatially uniform wind stress of limited duration (see later). This is the most common process (Heaps and Ramsbottom 1966) though to generate motion in lakes and gulfs or fjords which are removed from tidal forcing. This wind stress corresponded to an on-shore wind blowing from ocean to shore (i.e. from left to right in Fig. 1). The numerical model and form of the calculations are described in subsequent sections. Results from this series of calculations are summarized in a final conclusions section.

## 2. NUMERICAL MODEL

Following the recent successful use of the MIT code (see Marshall et al. 1997 for details) for modelling tidally-forced flow in sill regions (Xing and Davies 2006) this model is used in cross sectional form in the present study. The model uses a z-coordinate in the vertical with a finite volume discretisation, and has a non-hydrostatic option. A recent comparison (Berntsen et al. (2006)) with a sigma coordinate non-hydrostatic model developed by Berntsen and Furnes (2005) (see also Heggelund et al. (2004)),

showed that the two models gave similar results. Consequently the MIT code with a “shaved cell” approach at the sea bed appears to accurately represent topographic slopes.

Here the model is applied in two dimensional (x,z) cross sectional form to determine wind forced seiche motion in an idealized fjord. The topography of the region is characterized by a deep oceanic basin (water depths  $h_o = 1650\text{m}$  of length  $\ell_o$ , attached to a sloping continental shelf of length  $\ell_c$ , and a sill of length  $\ell_s$  and sill depth  $h_s$ . This sill is at the entrance to a fjord of length  $\ell_f$  and water depth  $h_f = 80\text{m}$  (see Fig. 1). The coastal (right) side of the sill is closed, while at the oceanic open boundary (left side) of the region, a closed or radiative open boundary condition is assumed.

A fine uniform vertical grid resolution of  $dz = 1\text{m}$  was used in all calculations, although when the sill depth was reduced below 4 m,  $dz$  was reduced to 0.25 m in the top 10m. In the horizontal the grid in the region of the shelf slope and fjord had a grid resolution of 10 m. However in the deep ocean, the grid increased to 1000 m close to the oceanic boundary. By this means high resolution was maintained close to the region of interest, namely the sill and fjord while the oceanic boundary was placed away from the fjord. As previously (Xing and Davies 2006) when the model was used to investigate tidal flow in the sill region of a fjord, the use of a fine grid in the region of interest meant that horizontal and vertical diffusivities ( $K_h$ ,  $K_v$ ) and corresponding viscosities ( $A_h$ ,  $A_v$ ) could be kept to a minimum in the sill region, namely  $K_h = K_v = 10^{-7}\text{m}^2\text{s}^{-1}$ , with  $A_h = 10^{-1}\text{m}^2\text{s}^{-1}$  and  $A_v = 10^{-3}\text{m}^2\text{s}^{-1}$ .

Motion in the region was started ( $t = 0$ , where  $t$  indicates time) from an initial condition where  $u = 0$  and  $w = 0$ , (with  $u$  horizontal and  $w$  vertical components of velocity) by the imposition of a wind stress of duration  $T_w$  and magnitude  $F$  applied at the

sea surface. At the sea bed a quadratic friction law was used with a drag coefficient  $k=0.0025$ . In the stratified calculations the temperature and hence density surfaces were initially horizontal with a surface mixed layer of about 40m (Fig 1). A time step  $dt = 2s$  was used in all calculations, and as a narrow gulf was assumed, rotational effects were neglected everywhere.

### 3. NUMERICAL CALCULATIONS: INFLUENCE OF SILL DEPTH

In an initial calculation the onshore spatially uniform wind stress was increased from zero to 0.5Pa over a 6 hour period, and maintained at this value for a further 6 hours before being “switched off”. Thus the wind stress  $F$  (Pa) was of the form

$$F = 0.5 \sin \left( \frac{\pi t}{2T_0} \right) \text{ for } t \leq T_0$$

and  $F = 0.5$             for  $T_0 < t \leq T_1$

and  $F = 0.0$             for  $t > T_1$

where  $t$  is time with  $T_0 = 6\text{hrs}$  and  $T_1 = 2T_0$  in these calculations, aimed at examining the influence of sill depth  $h_s$  upon motion in the region, the western (left) open boundary was closed.

#### 3.1 Pulse wind forced motion (Calc 1)( $h_s = 4m$ )

In an initial calculation (Calc1), the topography of the region was as shown in Fig 1, with the sill depth  $h_s = 4m$ , typical of a shallow fjord sill. Wind induced circulation in the region was produced by the wind stress described previously. This had the effect of driving surface water over the sill (Fig. 2a), giving rise to an increase in surface elevation at Posn A (Fig. 2a) located next to the fjord land boundary. Associated with the increase in elevation at this location is a vertical downwelling velocity which leads to a downward

displacement of the thermocline (Fig 2a) at the coastal boundary and on the western side of the sill. In addition there is an upward displacement on the right side of the sill. Associated with these regions are downwelling and upwelling vertical velocities ( $w$ ). As time progresses ( $t=12$ hrs) further downwelling occurs at the coastal boundary of the gulf, with upwelling at the sill (Fig 2b). Associated with this is an internal wave train that leaves the coastal boundary region (Fig 2b). Following the switching off of the wind forcing at  $t=12$ hrs, the pycnocline in the gulf exhibits first mode internal seiche motion between the coastal boundary and the right hand side of the sill (Fig 2c). This is identical to that found in a flat bottom lake (Hall and Davies 2005) where upwelling/downwelling of the thermocline occurs at opposite ends of the lake, with a nodal point in the centre. In the present case the righthand side of the sill acts like a wall, and seiche motion takes place between this and the coast, with a nodal point close to the centre of the gulf. In essence a standing internal wave is formed in the gulf.

Time series of the free surface elevation at Posn A (close to the coastal boundary at the head of the gulf) show (Fig 3a, note time origin starts at  $t = 0.5$  days, after the initial spin up period) an initial increase in elevation as the surface wind stress forces water over the edge of the sill and elevations at the head of the gulf rapidly increase. Subsequently following the removal of the wind stress, elevations decrease as barotropic energy is radiated over the sill, with the elevation time series (Fig 3a) exhibiting damped periodic motion with a period of about 4hrs.

Across basin free surface elevation contours (Fig 4) reveal that initially sea surface elevation increases rapidly at the coastal boundary with a small decrease in the ocean. As barotropic energy radiates from the gulf, sea surface elevation decreases in this

region. As time progresses and surface elevations decrease in the gulf, elevation nodal points are evident at about  $x = 400\text{km}$ , a whole basin response,  $x = 45\text{km}$ , a gulf shelf edge response, with some indication as surface elevations decrease to zero on the shelf, of a nodal point on the shelf at about  $x = 25\text{km}$ , that would correspond to a weak “lake like” barotropic seiche on the shelf, between the sill and the coastal boundary of the gulf. From simple theory, with a gulf depth of  $80\text{m}$ , this gives a barotropic wave speed of  $28.0\text{ms}^{-1}$ , and for the stratification used here a first mode baroclinic wave speed of about  $0.3\text{ms}^{-1}$ . Taking the region between the top of the sill and the coastal boundary to exhibit a “weak lake type” response, then with  $\ell=35\text{km}$ , the barotropic lake period would be about  $0.69$  hrs. In the case of the gulf period, if the node was located on top of the sill, this would give a period of about  $1.38$ hrs. However, as shown in Fig. 4, the nodal point occurs further off shore at about  $x = 45\text{km}$ . This has the effect of increasing the gulf length  $\ell$ , and because the sill is now included within this “effective” gulf region, the average water depth is reduced. Taking the effective water depth as  $50\text{m}$ , gives a barotropic wave speed of  $22.1 \text{ms}^{-1}$ , and a “gulf-ocean” resonance period based on  $\ell=45\text{km}$  of about  $2.3$ hrs, although this would increase if the “effective gulf” was longer and shallower. In the case of the “whole basin” responses in the deep ocean region  $h=1650\text{m}$ , giving a barotropic wave speed of  $127.2\text{ms}^{-1}$ , while on the shelf  $h=80\text{m}$ , giving a speed of  $28.0\text{ms}^{-1}$ . Factoring these by the relative lengths of the two regions, namely  $650$  and  $70\text{km}$ , gives an average barotropic speed of about  $118 \text{ms}^{-1}$ . Consequently in terms of a whole basin “lake type response”, this gives a period of about  $3.4$ hrs, although again this depends upon the effective length of the ocean and shelf regions, and water depths taking account of the presence of the sill. In practice the presence of the sill and shelf slope, makes the analysis

more complex than the approach used above. However this simple method gives some insight into the origins of the various periods found in the time series. Power spectra of the time series at Posn A (Fig 3a), reveal (Fig 5a), peaks at about 3.90 hrs (assumed to originate from the whole basin seiche), 2.76hrs (related to the ocean-gulf resonance) and a very weak peak at 0.66hrs corresponding to the shallow water across basin shelf oscillations found in Fig 4.

In a subsequent calculation aimed at determining how the gulf barotropic period is influenced by stratification as shown by Cushman-Roisin et al. (2005), the calculation was repeated without stratification effects. The derived power spectra (not presented) were indistinguishable from that given in Fig 5a. One reason for this is that the gulf spectral peak at 2.76hrs is very small (Fig 5a) and rather broad due to having to perform a power spectral analysis on the limited time series following the removal of the wind stress when the surface elevation is non zero. Also based upon Cushman-Roisin et al. (2005) the change in period produced by stratification effects would be of order about 0.002hrs, and to distinguish this from the homogenous solution spectrum would require a strong narrow peak. In addition as shown by Cushman-Roisin et al. (2005) the barotropic period is modified by baroclinic effects in a gulf much more than in a lake because internal waves can radiate out of a flat bottom gulf which is connected to an infinite ocean. As shown here, when there is a sill and the stratification lies well below the top of the sill, the baroclinic response in the gulf is an internal seiche, namely the same as would occur in a lake. In essence the presence of a sill in the present case leads to a standing internal wave in the gulf, rather than a progressive internal wave which radiates baroclinic energy out of the gulf as in the analytical work of Cushman-Roisin et al.

(2005). In addition the barotropic gulf response to a wind leads to a small broad peak in the spectrum and any small effect upon the gulf frequency from the stratification (of order 0.002hrs) will be difficult to detect.

In order to avoid these problems, and enhance the amplitude and reduce the width of the power spectrum peak at  $T = 2.76$ hrs in subsequent calculations the same wind magnitude and duration was used but the wind stress  $F$  (Pa) was periodic and of the form:

$$F = 0.5 \sin \frac{2\pi t}{T_R} \text{ for } t \leq T_1,$$

$$F = 0.0 \quad \text{for } t > T_1,$$

where  $T_1 = 4.25T_R$ , but  $T_R = 2.7$ hrs approximates the gulf resonant frequency. In addition to increasing the barotropic power input at the 2.76 hr period, as will be shown this gives rise to a significant internal wave that propagates away from the coastal boundary. By this means how the sill influences this propagating internal wave can be examined for a range of sill heights.

### 3.2. Limited duration sinusoidal forced motion (Calc2)( $h_s=4m$ )

In this calculation the model domain and stratification were as previously, however motion was induced by the combined pulse and sinusoidal wind stress as described previously. Time series of the free surface elevation at Posn A (Fig. 3b) shows a periodic variation with periods between 2 hrs and 4 hrs during the first day. The amplitude of this variation decreases with time once the wind forcing is removed, as barotropic energy is radiated from the gulf back into the ocean. In addition as time progresses the solution is progressively dominated by a single period of about 3 hrs.

Time series of the vertical displacement of the thermocline at Posn A (not presented), shows an initial downward displacement, followed by long periods (namely

periods longer than the free surface elevation) oscillations of decreasing amplitude. This reduction in amplitude is because initially the downwelling region occurs in a thin boundary layer adjacent to the coast, with a corresponding upwelling next to the sill (Hall and Davies 2005). This generates internal gravity waves that propagate away from these regions with a speed  $(g' h')^{1/2}$  of order  $0.3\text{ms}^{-1}$  and hence baroclinic energy is lost from the lateral boundary layers into the interior of the gulf, as shown by the distribution of vertical velocity contours at  $t = 6$  hrs (Fig. 6a). Associated with this there appears to be a decrease in baroclinic energy in the lateral boundary layers. Besides producing internal waves within the gulf, internal waves are generated on the western side of the sill and propagate into the ocean. However, as the ocean is very deep and their speed of propagation is slow then within the integration period considered here, namely 72 hours, they are mainly concentrated on the shelf slope and are unaffected by the western oceanic boundary which is well removed from the gulf region. However, in the gulf region internal waves generated at the sill do propagate towards the centre of the gulf as shown by the vertical velocity distribution at  $t = 6$ hrs (Fig 6a). Although there is some internal wave propagation from the sill into the gulf, it is evident that the strongest internal wave propagation is from the coast (Fig 6a). As time progresses the internal waves propagate into the centre of the gulf giving rise to significant internal waves throughout the gulf (see vertical velocities at  $t = 12$ hrs, Fig 6b). Subsequently ( $t = 18$ hrs) there are strong internal waves in the sill region of the gulf (Fig 6c) which persist for some time (Fig 6d) until eventually the internal wave field within the gulf decays through a combination of form drag, radiation and dissipation.

Consider first the time decay of the free surface seiche in the region. Since the propagation speed of the free surface wave is given by  $(gh)^{1/2}$ , then within 36 hrs any barotropic disturbance generated in the shelf edge or coastal regions can propagate to the oceanic boundary. As in the present calculation this boundary is closed, barotropic energy is reflected from it, and a standing free surface elevation wave is produced. Across basin free surface elevation contours at 6 hourly intervals show initially a transition period (Fig. 7) where surface elevation increases at the coastal boundary (right side of domain) with no change in the interior. However, as surface gravity waves propagate into the domain from the coastal boundary, sea surface elevation changes over the whole region, and nodal points (positions of zero elevation) develop in the interior, although the position of these points changes slightly with time.

On the longer time scale (Fig. 7) a standing wave develops with a nodal point situated in the region of the sill and shelf slope, namely at about  $x = 35\text{km}$  (Fig 7). In addition a nodal point develops in the ocean at about  $x = 400\text{km}$ , corresponding to a barotropic seiche node between the closed oceanic boundary and the western end of the gulf.

Power spectra of the free surface elevation at A, based on time series for the whole period, show (Fig. 5b) a peak at 3.92 hrs in essence the whole deep water basin response corresponding to a lake of period  $T_b = 2L_b/C_b$  with  $L_b = 720\text{km}$ , and  $C_b$  a mean speed of propagation (see earlier analysis). A second dominant peak since this is the forcing frequency, is at 2.77hrs, namely a gulf response  $T_g = 4 L_g/C_g$  with a node in the region to the west of the sill, and  $L_g$  of order 100 km, and  $C_g$  a mean speed for the region. In addition there is a small peak at 0.67 hours corresponding to the sill blocking the flow,

in essence a lake response namely  $T_\ell = 2 L_\ell / C_\ell$  with  $L_\ell = 35$  km the lake dimension and  $C_\ell = 28.0$  m based on a water depth of 80 m (see earlier analysis). As previously the power spectra computed from a homogenous calculation (not presented) was indistinguishable from Fig. 5b.

### 3.3 *Reduced Sill Depth (Calc 3) ( $h_s = 1$ m)*

To determine to what extent the sill height influenced the whole basin and gulf response, the previous calculation was repeated (Calc 3) with  $h_s = 1$  m. In this case significantly less water was able to flow over the sill and elevation increase at A was appreciably less (compare time series (b) and (c) in Fig. 3). As previously, after about 1.5 days the elevation signal was dominated by a period of about 3 hrs. Snap shots of the vertical velocity at  $t = 12$  hrs (Fig 8a) show that internal wave propagation from the coastal boundary into the interior of the lake has been reduced compared to previously (Fig. 6b). This gives rise to significantly stronger vertical velocities in the coastal region since the internal wave has not propagated as far away as previously (Calc 2) from its generation point. Although some propagation towards the sill has occurred by  $t = 24$  hrs (Fig 8b) this is significantly smaller than found with the 4m sill depth (Fig 6d). In addition, on the longer term (Fig 8c), calculations showed that more internal wave energy was trapped in the gulf when  $h_s = 1$  m compared to 4m. This is to be expected in that as  $h_s$  goes to zero, the gulf becomes a lake and internal wave energy cannot propagate out of the region. This comparison between  $h_s = 4$  m and 1m, clearly shows that even small differences in  $h_s$  when the across sill water depth is shallow can influence the internal wave field.

Across basin elevation contours (not presented) showed that the nodal point had moved slightly closer to the sill. However, as the magnitude of the free surface elevation response had also reduced, the exact location of the nodal point in the ocean was unclear. This is reflected in the power spectra (Fig 5c) based upon elevation time series at A over the whole period, which showed no peak at the basin wide frequency, with a sharp peak, although reduced energy (due to a decrease in water crossing over the sill) at the gulf period. The gulf period namely  $T_g = 2.94$ hrs had also been increased slightly due to the shift in nodal point changing the length  $L_g$  and effective gulf depth. In addition the peak at about 0.67 hrs increased relative to the background energy, compared to previously, showing that as the sill height increases the local response of the gulf corresponds to that of a lake, which is to be expected in the limit as  $h_s = 0$ .

As previously, power spectra computed without stratification were indistinguishable from those computed with. As discussed earlier, this is to be expected since internal wave energy loss by radiation from the gulf is negligible. As shown by Cushman-Roisin et al. (2005) this must be significant if appreciable coupling between the gulf's barotropic mode and baroclinic radiating waves is to occur. However, it is clear that a small reduction in  $h_s$  has affected the propagation of the baroclinic waves in the gulf on the short time scale, and slightly modified the values of the barotropic seiche periods  $T_g$  and  $T_l$ . Subsequent calculations (not presented) showed that decreasing  $h_s$  produced a further decrease of energy within the gulf, and eventually when  $h_s = 0$ , obviously the gulf response became that of a lake, and a single peak occurred in the power spectrum at about  $t = 0.67$  hours.

#### *3.4 Increase in sill depth (Calc 4) ( $h_s = 40$ m)*

In the previous series of calculations the stratification in the gulf lay below the sill height, and hence any internal waves generated on the righthand side of the sill were partially trapped in the gulf. In the present calculation by reducing the sill depth to the level of the thermocline, namely  $h_s = 40\text{m}$ , internal waves generated at the sill or the eastern end of the gulf could more readily radiate out of the gulf. As shown previously within the time period considered here, the western boundary had no direct influence on the internal waves in the sense of wave reflection into the gulf region, but could affect the barotropic response.

On the short time scale ( $t=6\text{hrs}$ , Fig 9a) comparison of a snapshot of  $w$  velocity components with those derived previously with  $h_s = 4\text{ m}$  (Fig. 6a) showed that only very weak internal waves were generated in the sill region. In addition those generated at the coast were not substantially different to those found when  $h_s = 4\text{m}$  (compare Figs 6a and 9a). This is to be expected since the stratification does not intersect the sill topography, then the main source of internal wave generation is at the coast. On the longer time scale ( $t = 18\text{hrs}$ ), the internal wave pulse produced at the coast by the wind impulse propagates towards the sill, as seen in the vertical velocity distribution (Fig 9b). As the sill is appreciably deeper in this calculation than previously (Calc 2), the internal waves can propagate over the sill and there is no build up of internal waves in the sill region as found in Calc 2 (compare Figs 9b and 6c). The propagation of internal waves over the sill, rather than the trapping found with  $h_s = 4\text{m}$ , is clearly evident at  $t = 24\text{hrs}$  from a comparison of Figs 6d and 9c. Consequently, besides the influence of the sill height upon internal wave generation in the gulf, in the present calculation, internal waves can radiate

out of the gulf as shown in the different characteristics of the vertical velocity induced by the density field.

Time series of the free surface elevation at A (Fig 3d) exhibit similar characteristics to that found with a sill depth of 4m (Fig. 3b). However, the tendency towards a single period is diminished, and even after 1.6 days multiple periods are present (Fig. 3d).

As previously (Calc 2) barotropic energy that radiates from the gulf is reflected from the oceanic boundary and seiche motion is generated in the ocean. Across basin elevation contours (Fig. 10) exhibit a similar behaviour to that found with  $h_s = 4$  m (Calc 2) (compare Figs. 10 and 7). However, even after 1.6 days it is evident from the time series (Fig 3d) that there is no dominant period, and as shown by the across basin contours of surface elevation, the nodal point positions change with time (Fig 10).

Power spectra of the free surface elevation at A show (Fig. 5d) peaks at about 3.20 hours and 1.94 hrs, corresponding to a whole basin and gulf responses, but with slightly changed lengths and mean depths, due to the change in position of the nodal point produced by the increase in sill depth. In addition, unlike previously where the shorter period gulf response dominated (see Fig. 5b), in the present case both peaks have a comparable magnitude (Fig. 5d). As previously calculations were also performed without stratification and power spectra based on the elevation time series at A were computed. Although in this case it is evident that internal wave radiation out of the gulf does occur, no significant differences were found in power spectra computed with and without stratification. This arises because the power spectra were computed using the full time series, whereas the internal wave propagation following the wind pulse is a transient

event, as found in nature following wind forcing, which only occurs for part of the period. Unfortunately if only the part of the time series when internal waves are present is used then the spectral peaks broaden and are not significantly different under homogenous and stratified conditions. As in the previous two calculations changes in sill depth  $h_s$  appreciably change the internal wave response in the gulf to wind forcing.

### 3.5 Removal of Sill ( $h_s = 80m$ ) (Calc 5)

For the case in which the sill is removed (Calc 5) weak internal waves are generated at the shelf edge and can propagate both onto the shelf and back into the ocean as shown by vertical velocity contours,  $t = 6$  hrs (Fig. 11a). In addition as found in earlier calculations, internal wave generation is mainly in the coastal boundary layer region with propagation away from the coast (Fig. 11a). On the short time scale ( $t = 6$ hrs) internal wave propagation away from its coastal generation region is independent of sill height (compare Figs 6a, 9a and 11a). However, on the longer time scale ( $t = 18$ hrs) when the internal waves have reached the sill, their propagation is influenced by the sill, even when  $h_s = 40m$  (compare Figs 9b and 11b). In the case of no sill, as to be expected the internal waves readily propagate into the ocean (Fig 11c). Although even when  $h_s = 40m$ , it is evident from a comparison of Figs 11c and 9c, the sill does not significantly reduce internal wave propagation out of the gulf. In the absence of the sill the majority of internal wave propagation is towards the ocean and by  $t = 36$  hrs (not presented) there is only a small on-shelf response.

Time series of the sea surface elevation response at Posn A as previously (Calc 4) shows that initially the wind excites a number of periods, however after about 1.5 days the time series is dominated by a period of about 3 hrs (Fig. 3e). Contours of cross basin

elevation (Fig.12) show a number of nodal points, the position of which change with time, giving rise to the various frequencies found in the time series. However, no time invariant single nodal point is found in the shelf edge region (Fig 12). In addition in the oceanic region although there appears to be a nodal point at about  $x = 350\text{km}$ , this also varies with time.

Power spectra of free surface elevation at position A show (Fig 5e) a dominant period at about 3.18 hrs, corresponding to the seiche period of the whole domain. In addition two other seiche modes are excited with periods of about 1.79 hrs and 1.35 hrs associated with nodal points located in the shelf edge region, giving rise to seiche motion on the continental shelf. The absence of the sill that occurred in Calc 2, which in essence “anchored” the location of a “gulf” type response with a node located near the sill, explains why the node in the shelf edge region varies with time. In addition although the region is forced by a wind impulse perturbed by a superimposed 2.76hr period wind, it is evident that in the absence of a sill, the expected “gulf like response” of 2.76hrs does not appear. This shows that the short duration sinusoidal wind forcing does not dominate the barotropic resonant period which is primarily influenced by the topography. However, as previously there was no difference in spectra computed with and without stratification.

#### 4. NUMERICAL CALCULATIONS: INFLUENCE OF THE OFF SHELF BOUNDARY

In the previous series of calculations, the off shelf boundary was assumed to be closed. Consequently both barotropic and baroclinic waves were generated and could be reflected from both boundaries. In essence the speed of propagation of the baroclinic waves, and the large oceanic domain of the model meant that baroclinic waves generated

at the off shelf boundary, within the integration period considered, did not reach the gulf region and hence the baroclinic response of this area was not affected by any reflected internal wave. However, in the case of barotropic waves their propagation speed was so fast that reflection could occur at the off shelf boundary, and a whole domain seiche was generated in addition to the local “gulf” and “lake” type seiche motion generated on the shelf. To determine to what extent the off shelf boundary influenced the on shelf response, Calc 2 ( $h_s = 4$  m) was repeated with a progressive wave barotropic radiation condition applied on the western (off shelf) boundary (Calc 6).

The effect of applying a radiation condition on the western boundary is that barotropic waves generated at the coastal boundary at the head of the fjord due to wind forcing are no longer reflected at the off shelf boundary and barotropic energy can radiate out of the domain. This gives rise to a rapid decrease in the sea surface elevation at Position A (Fig. 3f) as energy is radiated from the coastal region out of the model domain.

Comparison of “snap shots” of  $w$  velocity on the short time scale ( $t = 6$ hrs) (Fig 13a) with those generated with a closed off shore boundary (Fig 6a) show no appreciable difference in the on-shelf internal wave field. However, on the longer time scale ( $t = 12$ hrs) there are significant differences on the oceanic side of the sill between the vertical velocities associated with the internal wave field computed with a closed western boundary (Fig 6b) and an open boundary (Fig 13b). In addition there are some small changes on the shelf in the region next to the sill (compare Figs 6b and 13b). On the longer time scale ( $t = 24$ hrs) it is evident that although both calculations show that the internal waves decay on the shelf, this decay is more rapid when a barotropic radiation

condition is applied off shore than when this boundary is closed as in the case of an off shore island. To understand this, it is valuable to compare cross sectional displacements of the free surface elevation computed with the radiation condition (Fig 14) with those derived previously (Fig 7).

In the case of the barotropic radiation condition, after some initial (namely upto  $t = 12$ hrs) whole basin seiche motion with a node at about  $x = 400$ km (Fig 14), sea surface elevations within the ocean rapidly decrease to zero as barotropic energy is radiated through the western boundary. This response on the longer term ( $t > 12$ hrs) is very different than previously (Fig. 7) where the continuous whole basin barotropic seiche motion gives rise to upwelling and downwelling at the shelf edge which produces the strong vertical velocities in this region that are evident in Fig. 6d, but not Fig 13c. In essence the differences that are evident in the internal wave field in the shelf edge and sill region between Figs 6d and 13c are not related to internal wave propagation but are due to the whole basin barotropic seiche motion. This shows that even in the case of a shallow sill the presence of an off shore island will influence the internal wave field close to the sill, because of barotropic seiche motion that can occur in the region. In the present calculation the offshore island, namely the western boundary, is well removed from the sill and hence this effect will be less than that found in nature where off shore islands are within 100km of the entrance to fjords or lochs (e.g. the Hebrides islands off the west coast of Scotland).

From Fig 3f it is evident that superimposed upon the decaying elevation at A, is a sinusoidal motion of period about 3.5 hrs. Contours of free surface elevation across the whole model domain show (Fig. 14) that in essence surface elevation close to the oceanic

boundary is near zero, with a nodal point located at about  $x = 38\text{km}$ . Using this location for the nodal point associated with the gulf, gives a period of order 3.5hrs. The rapid decay of the sea surface elevation that is evident in Figs 3f and 14, means that peaks within the power spectrum will be broad rather than the sharp peaks found previously. Power spectra of the elevation time series at A, shows (Fig. 5f) two peaks, the dominant one, namely gulf like response at about 3.67 hrs, and a smaller peak, namely the lake response at 0.67 hrs.

This calculation clearly shows that the western open boundary has a significant influence upon the location of the nodal point associated with the seiche motion in the gulf. In addition, as to be expected, if a radiation condition is applied on this boundary, then energy is lost from the model domain, and free surface seiching within the gulf is rapidly damped. Although the gulf seiche period is influenced by the western open boundary because it affects the location of the associated node, the lake seiche period is not substantially affected because its node is influenced by local topography, namely sill location, with intensity influenced by sill height.

## 5. CONCLUDING DISCUSSION

A cross sectional model of an idealised constant depth gulf or fjord with a sill at its entrance connected to a constant depth deep ocean was used to examine the barotropic and baroclinic response of the region to wind forcing. Calculations were performed with a range of sill heights and off shelf boundary conditions corresponding to an offshore island (no normal flow through the island) or a radiation condition, namely an ocean of infinite extent. The objective of the calculations was to examine to what extent the sill

height at the entrance to the gulf influenced the barotropic and baroclinic response of the gulf to wind forcing. In addition the extent to which the stratification in the gulf influenced its barotropic response, particularly when internal wave energy was radiated from the gulf (Cushman-Roisin et al., 2005) was also considered. In the case of the initial wind stress of 12 hours duration and a tall sill the baroclinic response in the gulf took the form of a first mode internal seiche between the sill and the coast. In this case little or no internal progressive wave was generated in the gulf. Calculations with a closed off shore boundary showed that the major barotropic period was at approximately  $T_b = 3.9$  hrs corresponding to a whole domain barotropic seiche. A second, shorter seiche period, namely that involving the sill/shelf edge region and head of the gulf was also present. Identical barotropic calculations showed that these barotropic periods could not be distinguished from those computed with stratification. One reason for this is that the spectral peaks at the barotropic resonance frequency were very broad, and the peaks were not particularly strong due to the form of the wind forcing. Consequently any change in their frequency due to the presence of stratification could not be determined. In addition the standing wave nature of the internal wave in the calculation, due to the high sill meant that baroclinic wave energy could not be lost by radiation as in the Cushman-Roisin et al. (2005) theory.

To avoid these problems the wind forcing was modified to include a sinusoidal component at the gulf frequency, although as previously a ramped form of forcing, falling to zero after 12hrs was applied. In this case progressive internal waves were generated at the coastal boundary at the head of the gulf. As time progressed these waves propagated towards the sill. In the case of a high sill, the internal waves could not

propagate over it but were trapped close to the sill. As the sill depth decreased such that the pycnocline intersected the sill then internal waves could radiate energy over the sill. In the limit that there was no sill, internal wave propagation from the gulf into the ocean could readily occur. Despite the fact that wind forcing of a form that contained the gulf period, significantly enhanced the energy in the gulf/ocean resonant peak, and sharpened the peak, together with generating radiating progressive internal waves, no difference in barotropic resonant periods between the stratified and homogenous solutions could be identified. However calculations did show that the sill height significantly influenced the radiation of baroclinic energy by propagating internal waves. In addition the resonant barotropic gulf seiche period was influenced by sill height.

Calculations using a barotropic radiative off shore boundary, showed that the enhanced loss of energy due to this boundary condition lead to a significant broadening of the barotropic spectral response. Consequently differences in resonant frequency due to including stratification effects could not be determined. In addition to influencing the barotropic spectral response, the internal wave response in the sill region was modified. This suggests that the “classic” approach to modelling wind forced internal wave motion, of placing a barotropic radiating boundary condition far away from the region of interest may not be perfect. Although the distance between the open boundary and region of interest may be sufficiently large that internal waves reflected from it cannot propagate back to the region of interest within the integration period, any false barotropic propagation into the region can influence the baroclinic solution.

In essence the calculations presented here show that sill height and off-shore boundary have an important role in determining the nature of the baroclinic response of a

gulf to wind forcing. Consequently it is important to consider the role of the sill and adjacent ocean in any modelling exercise. In addition, although Cushman-Roisin et al. (2005) have clearly shown that when a gulf region can lose energy by internal wave radiation, the barotropic seiche period is changed, this could not be demonstrated here due to a lack of accuracy in computing resonant frequencies. At present it is not clear how to overcome this problem. However, the present calculations, unlike those of Cushman-Roisin et al. (2005) which considered only a constant depth gulf, clearly show that sill height compared to pycnocline depth will control the radiation of internal wave energy out of the gulf, and hence the modification of the resonant barotropic frequency by baroclinic effects. This is to be expected since once the sill reaches the sea surface, the gulf in essence becomes a lake, from which internal wave energy radiation is not possible and hence as shown by Cushman-Roisin et al. (2005) the barotropic seiche period is not substantially affected by stratification.

## REFERENCES

- Arneborg, L. and Liljebladh, B. (2001) The internal seiches in Gullmar Fjord. Part 1: Dynamics. *Journal of Physical Oceanography*, 31, 2549-2566.
- Bergh, J and Berntsen, J. (2009) Numerical studies of wind forced internal waves with a nonhydrostatic model. *Ocean Dynamics* (submitted)
- Berntsen, J. and G.K. Furnes (2005) Internal pressure errors in sigma-coordinate ocean models – sensitivity of the growth of the flow to the time stepping method and possible non-hydrostatic effects. *Continental Shelf Research*, 25, 829-848.
- Berntsen, J., Xing, J., Alendal, G., (2006) Assessment of non-hydrostatic ocean models using laboratory scale problems. *Continental Shelf Research*, 26, 1433-1447.
- Cushman-Roisin, B., Willmott, A.J. and Biggs, N.R.T. (2005) Influence of stratification on decaying surface seiche modes. *Continental Shelf Research*, 25, 227-242.
- Davidson, F., Greatbatch, R. J., de Young, B., (2001). Asymmetry in the response of a stratified coastal embayment to wind forcing. *Journal of Geophysical Research* 106, 7001-7016.
- de Young, B., Otterson, T., Greatbatch, R.J (1993). The local and non-local response of Conception Bay to wind forcing. *Journal of Physical Oceanography* 23, 2636-2649.
- Hall, P. and Davies, A.M. (2005) Effect of coastal boundary resolution and mixing upon internal wave generation and propagation in coastal regions. *Ocean Dynamics*, 55, 248-271.
- Heaps, N.S. and Ramsbottom, A.E. (1966) Wind effects on the water in a narrow two-layered lake. *Philosophical Transactions Royal Society London*, 259, 391-430.

- Heggelund Y, Vikebø, F., Berntsen, J., Furnes, G. (2004) Hydrostatic and non-hydrostatic studies of gravitational adjustment over a slope. *Continental Shelf Research*, 24, 2133-2148.
- Marshall, J., Hill, L. Perelman and A. Adcroft (1997) Hydrostatic, quasi-hydrostatic and nonhydrostatic ocean modelling. *J. Geophys. Res.*, 102, 5733-5752.
- Xing, J. and Davies, A.M. (2006) Influence of stratification and topography upon internal wave spectra in the region of sills. *Geophysical Research Letters*, 33, L23606, doi:10.1029/2006GL028092.
- Zhai, L., Sheng, J., Greatbatch, R. J. (2008) Baroclinic dynamics of wind-driven circulation in a stratified bay: A numerical study using models of varying complexity. *Continental Shelf Research* (in press).

Table 1: Summary of Calculations

<i>Calc</i>	<i>Sill Depth <math>h_s</math> (m)</i>	<i>Western Boundary</i>	<i>Wind Stress</i>
1	4	Closed	Pulse
2	4	Closed	Pulse + Sinusiod
3	1	Closed	Pulse + Sinusiod
4	40	Closed	Pulse + Sinusiod
5	80	Closed	Pulse + Sinusiod
6	4	Radiation	Pulse + Sinusiod

### *Figure Captions*

Fig 1: (a) Model domain and initial temperature field ( $^{\circ}\text{C}$ , contour interval c.i. = 1.0) and (b) model sub-domain in the region of the sill for the case of a high sill (sill depth  $h_s = 4\text{m}$ ).

Fig 2: Snap shot of temperature field ( $^{\circ}\text{C}$ , contour interval c.i. = 1.0) and vertical velocity  $w$  ( $\text{cm s}^{-1}$ , c.i. =  $20 \times 10^{-3}$ ) in the gulf region (Fig 1b) at (a)  $t = 6\text{hrs}$ , (b)  $t = 12\text{hrs}$ , and (c)  $t = 24\text{hrs}$  for the case of an onshore (left to right) spatially uniform impulse wind and sill depth  $h_s = 4\text{m}$  (Calc 1). Also shown (Fig. 2a) is the location of position (A) adjacent to the coastal boundary.

Fig 3: Time series of free surface elevation at the head of the gulf after the wind impulse has stopped from

- (a) Calc 1 (wind impulse,  $h_s = 4\text{m}$ ),
- (b) Calc 2 (sinusoidally perturbed impulse,  $h_s = 4\text{m}$ ),
- (c) Calc 3 (sinusoidally perturbed impulse,  $h_s = 1\text{m}$ ),
- (d) Calc 4 (sinusoidally perturbed impulse,  $h_s = 40\text{m}$ ),
- (e) Calc 5 (sinusoidally perturbed impulse,  $h_s = 80\text{m}$ , no sill),
- (f) Calc 6, as Calc 2, but with an ocean boundary radiation condition.

Fig 4: Sea surface displacements at 6 hourly intervals (a) over the whole domain, and (b) a sub-domain covering shelf edge and gulf, from Calc 1.

Fig 5: Power spectral density of sea surface elevation near the head of the gulf from

- (a) Calc 1 (wind impulse,  $h_s = 4\text{m}$ ),
- (b) Calc 2 (sinusoidally perturbed impulse,  $h_s = 4\text{m}$ ),
- (c) Calc 3 (sinusoidally perturbed impulse,  $h_s = 1\text{m}$ ),

(d) Calc 4 (sinusoidally perturbed impulse,  $h_s = 40\text{m}$ ),

(e) Calc 5 (sinusoidally perturbed impulse,  $h_s = 80\text{m}$ , no sill),

(f) Calc 6, as Calc 2, but with an ocean boundary radiation condition.

Fig 6: Snap shot of vertical velocity  $w$  ( $\text{cm s}^{-1}$ , c.i. =  $2 \times 10^{-3} \text{ cm s}^{-1}$ ) in the gulf region (Fig 1b) at (a)  $t = 6\text{hrs}$ , (b)  $t = 12\text{hrs}$ , (c)  $t = 18\text{hrs}$ , and (d)  $t = 24\text{hrs}$ , for the case of a sinusoidally perturbed wind impulse and sill depth  $h_s = 4\text{m}$  (Calc 2).

Fig 7: As Fig 4 but for Calc 2.

Fig 8: Snap shot of temperature field ( $^{\circ}\text{C}$ , c.i. = 1.0) at  $t = 12\text{hrs}$  and vertical velocity  $w$  ( $\text{cm s}^{-1}$ , c.i. =  $2 \times 10^{-3}$ ) in the gulf region (Fig 1b) at (a)  $t=12\text{hrs}$ , (b)  $t = 24\text{hrs}$ , and (c)  $t = 36\text{hrs}$ , for the case of a sinusoidally perturbed wind impulse and sill depth  $h_s = 1\text{m}$  (Calc 3).

Fig 9: Snap shot of vertical velocity  $w$  ( $\text{cm s}^{-1}$ , c.i. =  $2 \times 10^{-3}$ ) in the gulf region (Fig 1b) at (a)  $t = 6\text{hrs}$ , (b)  $t = 18\text{hrs}$ , and (c)  $t = 24\text{hrs}$ , for the case of a sinusoidally perturbed wind impulse and sill depth  $h_s = 40\text{m}$  (Calc 4).

Fig 10: As Fig 4 but for Calc 4.

Fig 11: Snap shot of temperature field ( $^{\circ}\text{C}$ , c.i. = 1.0) at  $t = 6\text{hrs}$  and vertical velocity  $w$  ( $\text{cm s}^{-1}$ , c.i. =  $2 \times 10^{-3}$ ) in the gulf region (Fig 1b), at (a)  $t = 6\text{hrs}$ , (b)  $t = 18\text{hrs}$  and (c)  $t = 24\text{hrs}$ , for the case of a sinusoidally perturbed wind impulse and sill depth  $h_s = 80\text{m}$  (Calc 5).

Fig 12: Sea surface displacements at 6 hourly intervals (a) over the whole domain, and (b) a sub-domain covering the shelf edge and gulf from Calc 5.

Fig 13: Snap shot of vertical velocity  $w$  ( $\text{cm s}^{-1}$ , c.i. =  $2 \times 10^{-3} \text{ cm s}^{-1}$ ) in the gulf region (Fig 1b) at (a)  $t = 6\text{hrs}$ , (b)  $t = 12\text{hrs}$ , and (c)  $t = 24\text{hrs}$ , for the case of a sinusoidally

perturbed wind impulse and sill depth  $h_s = 80\text{m}$ , with an ocean radiation condition (Calc 6).

Fig 14: Sea surface displacements at 6 hourly intervals (a) over the whole domain, and (b) a sub-domain covering the shelf edge and gulf from Calc 6.

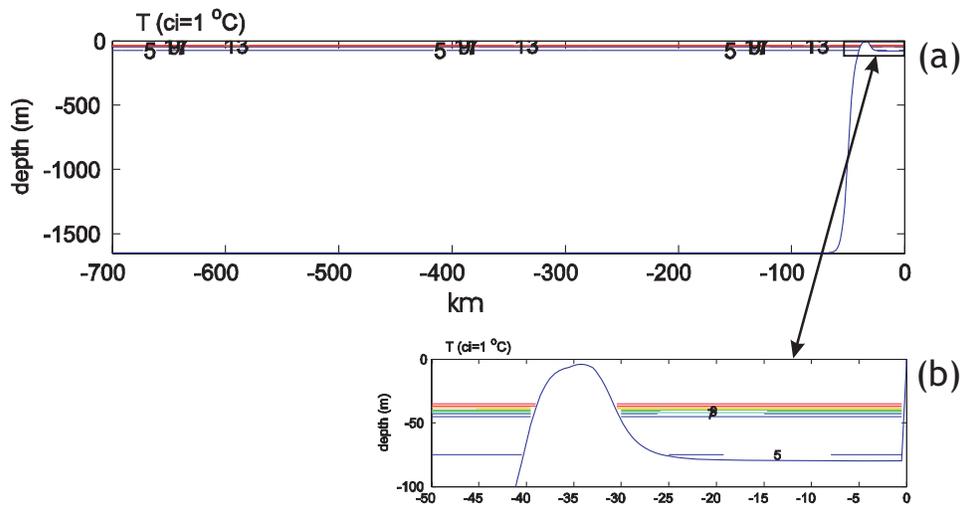


Fig 1

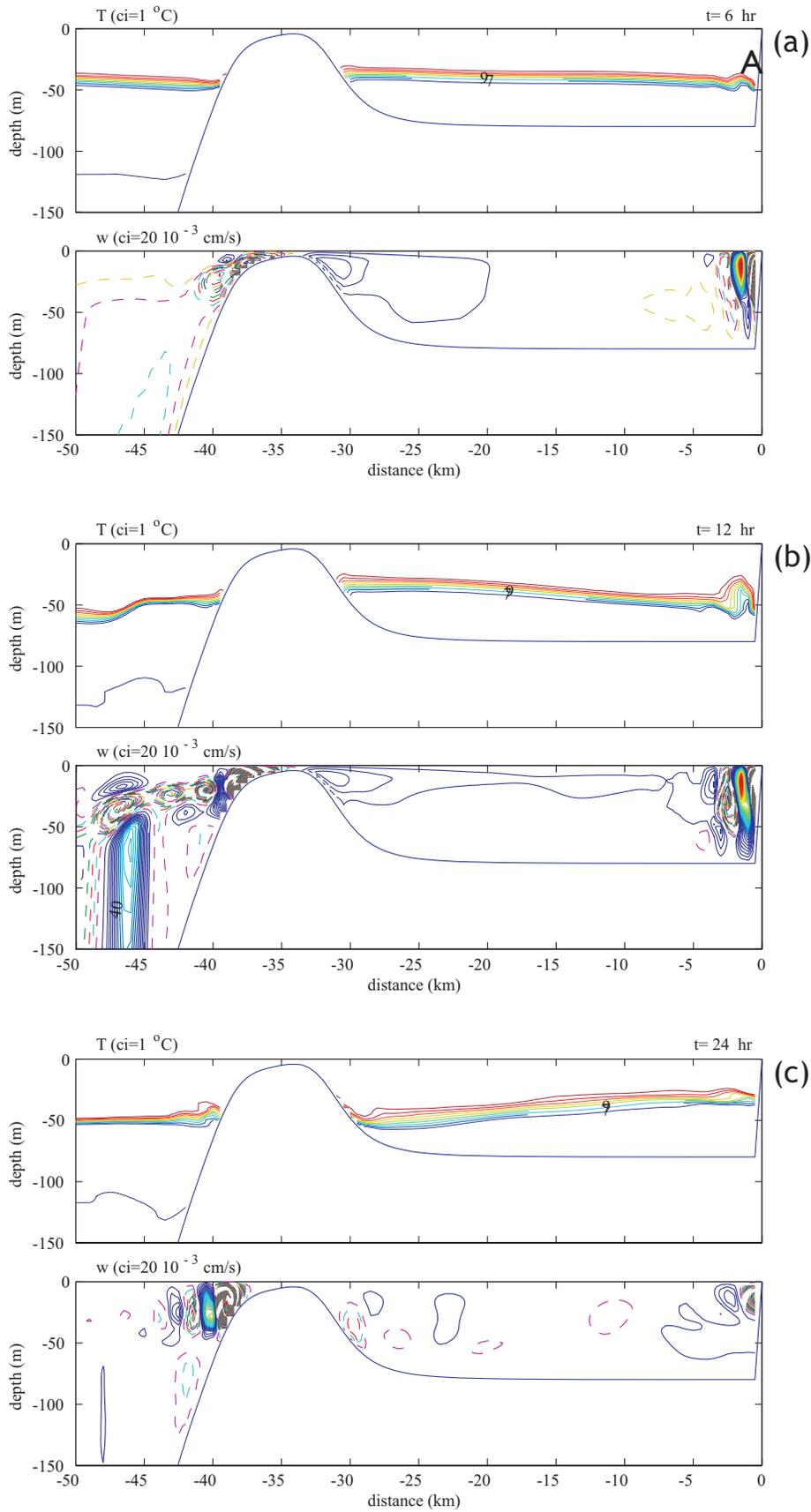


Fig 2

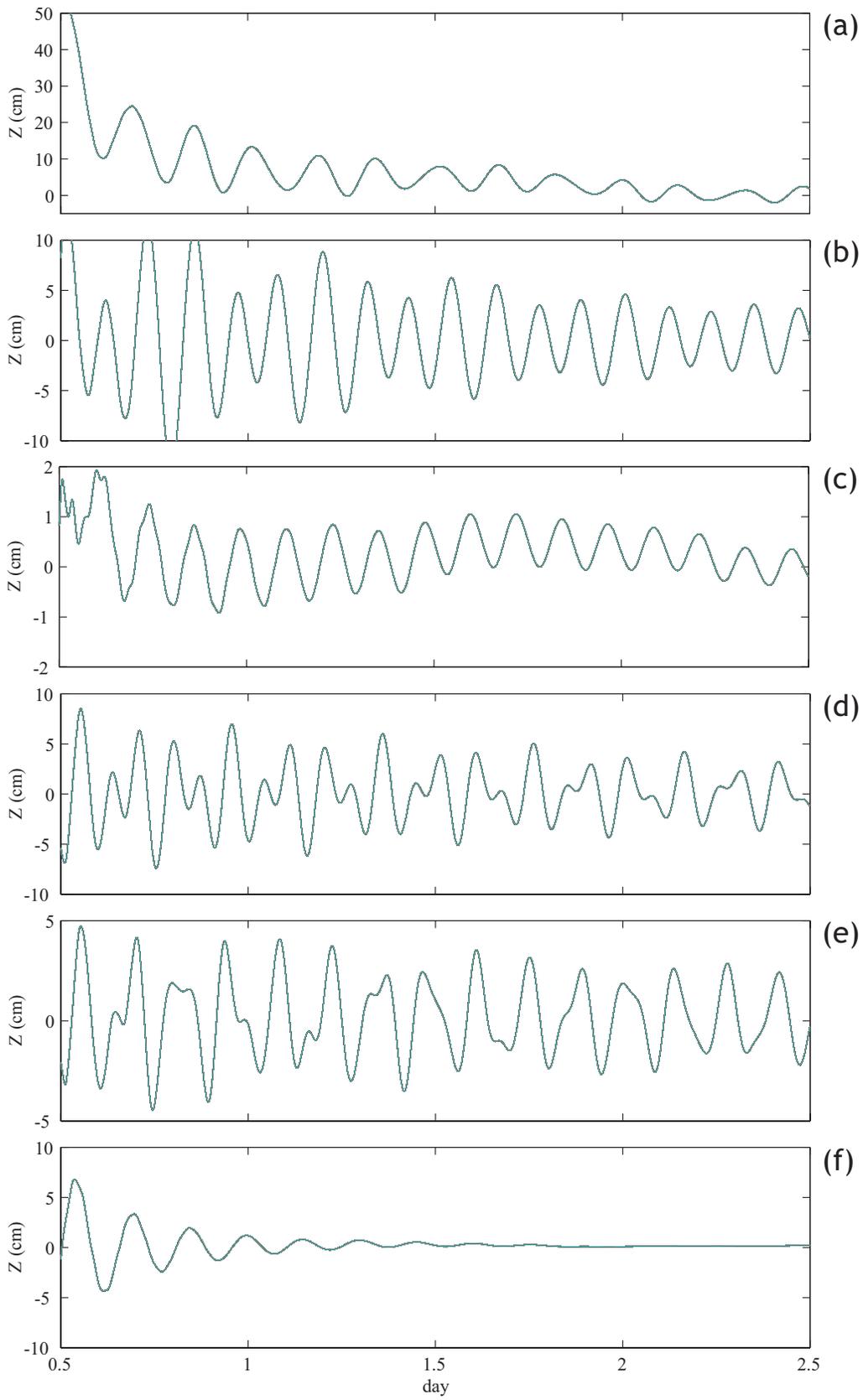


Fig 3

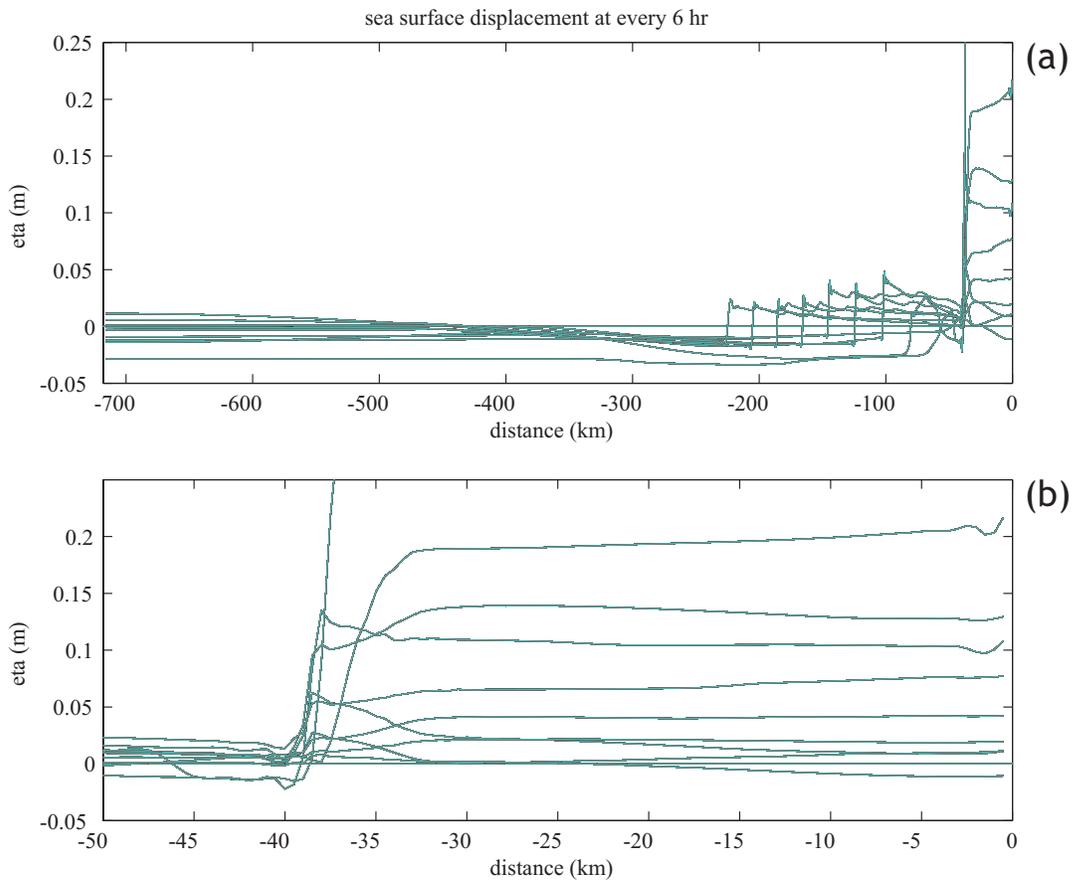


Fig 4

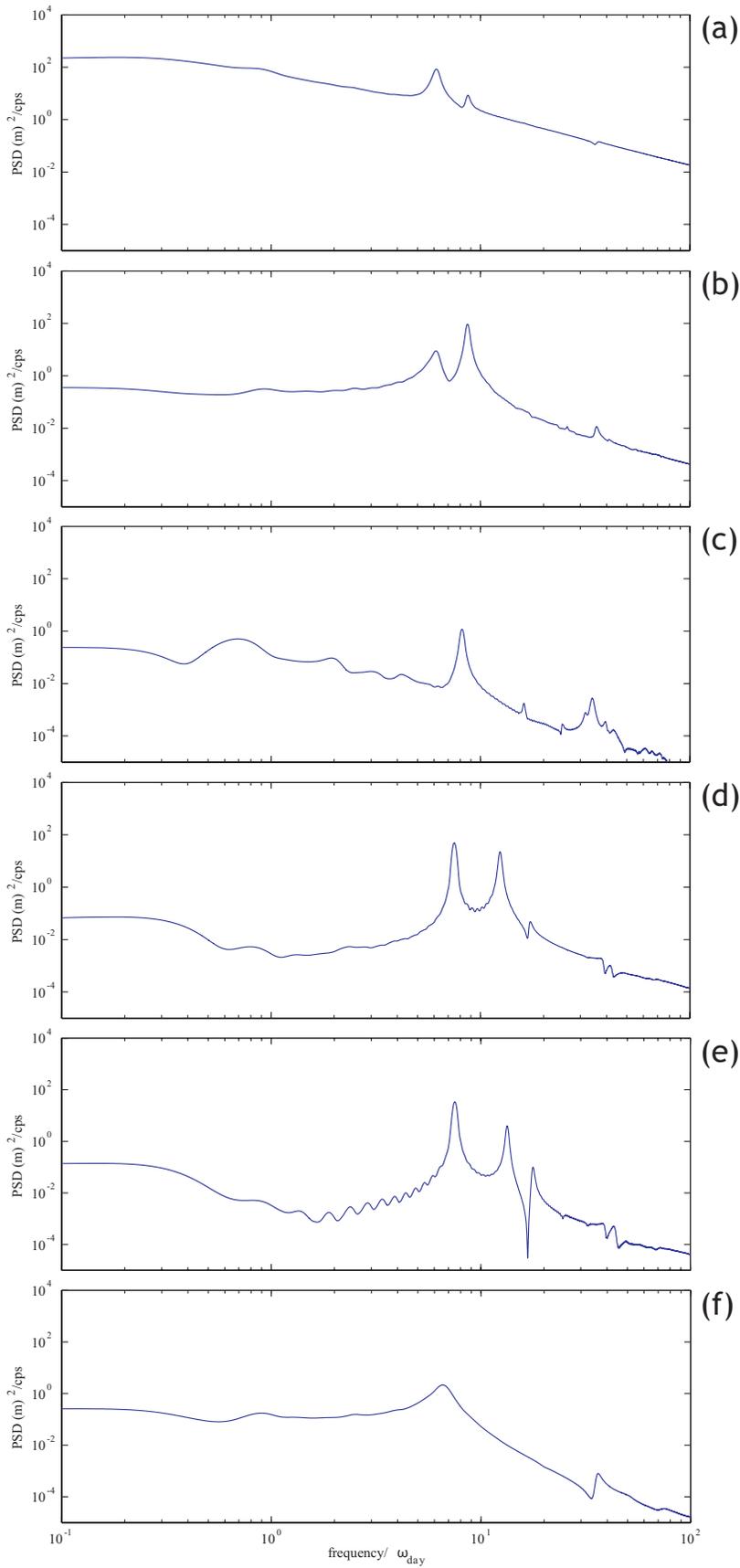


Fig 5

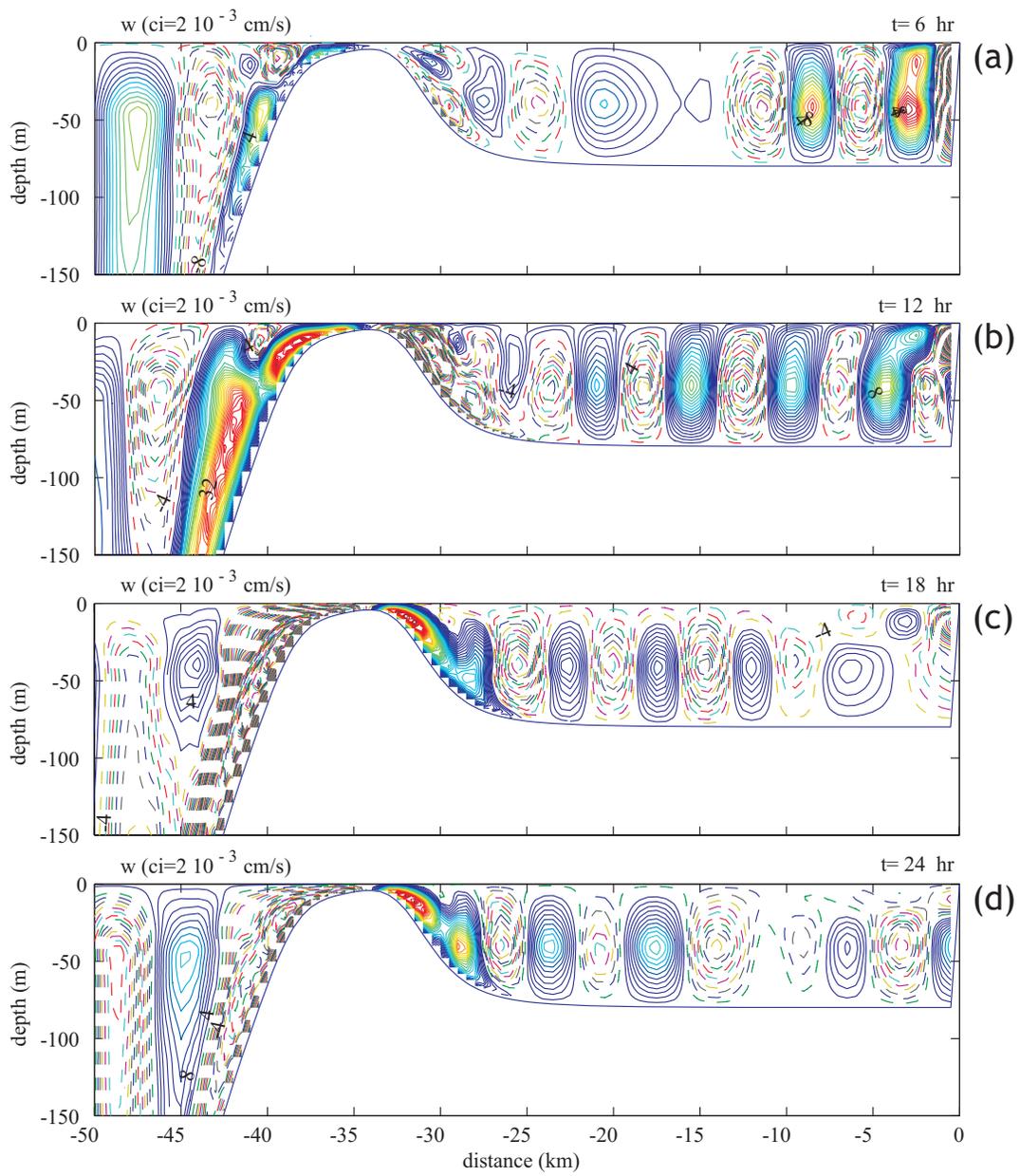


Fig 6

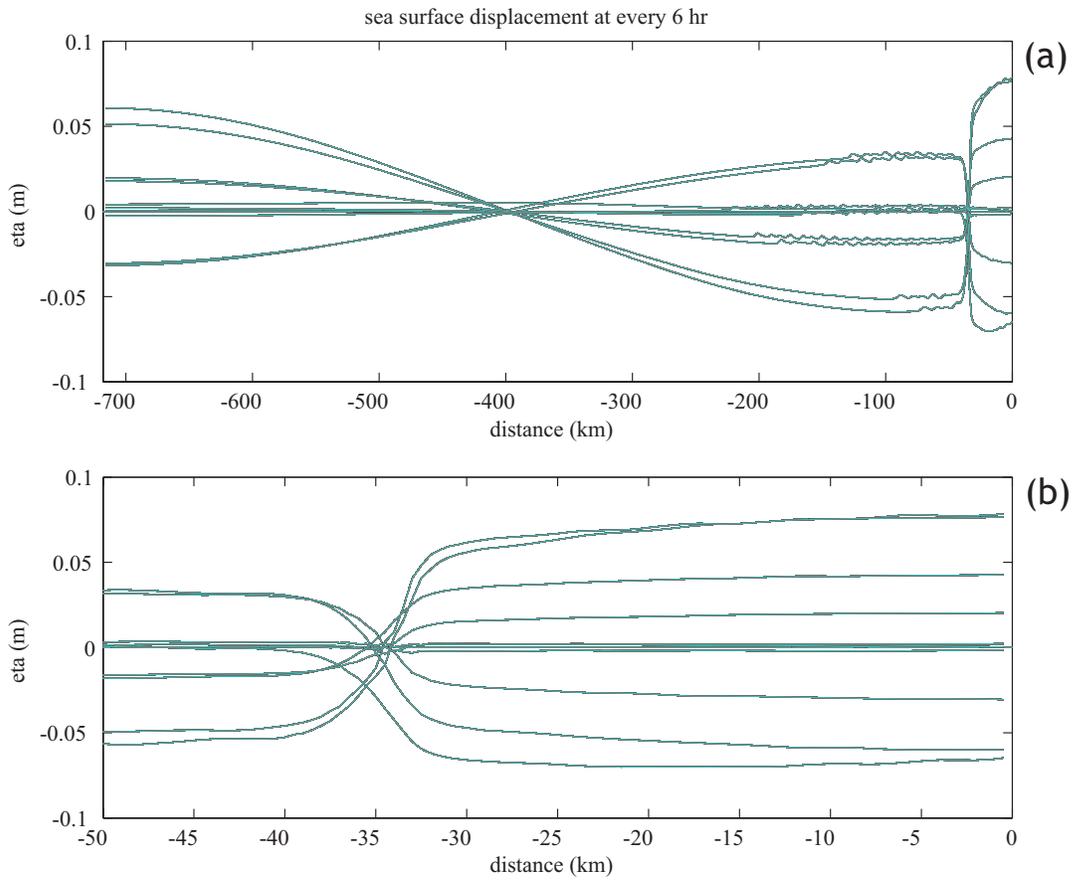


Fig 7

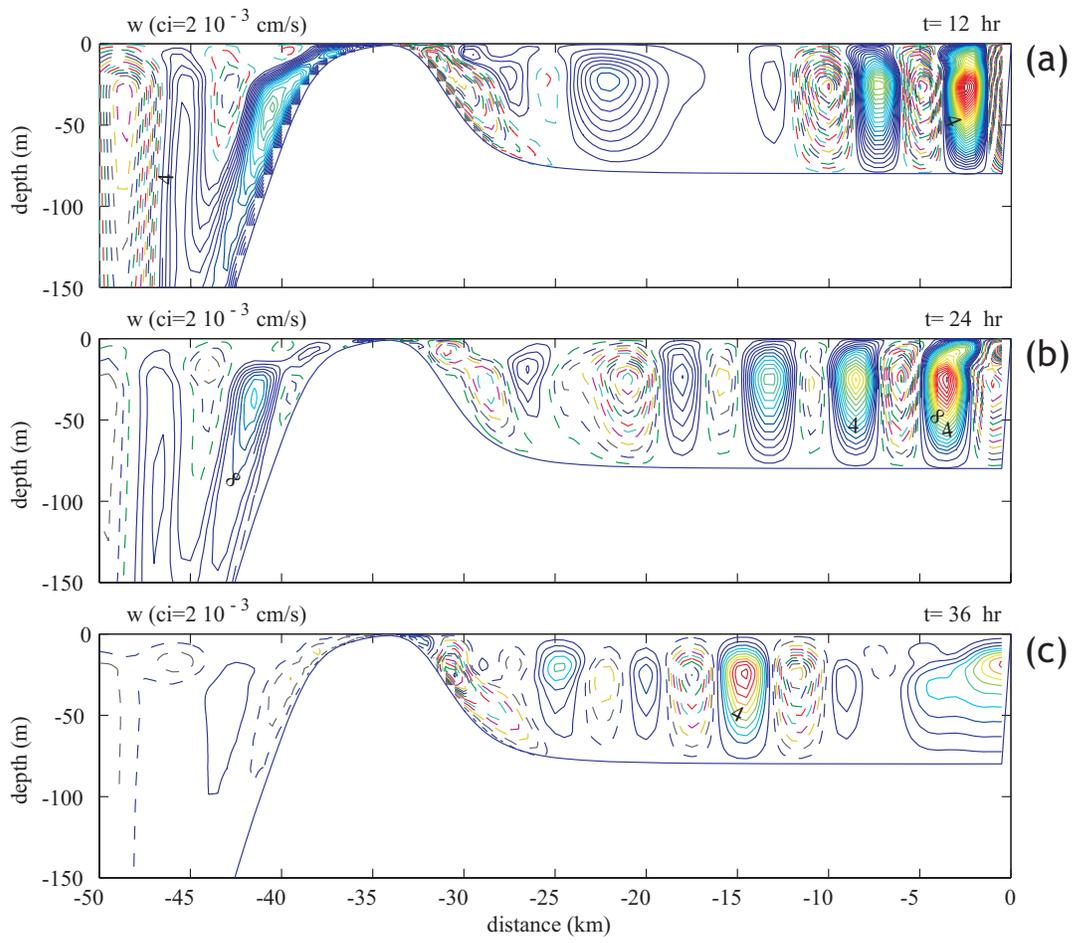


Fig 8

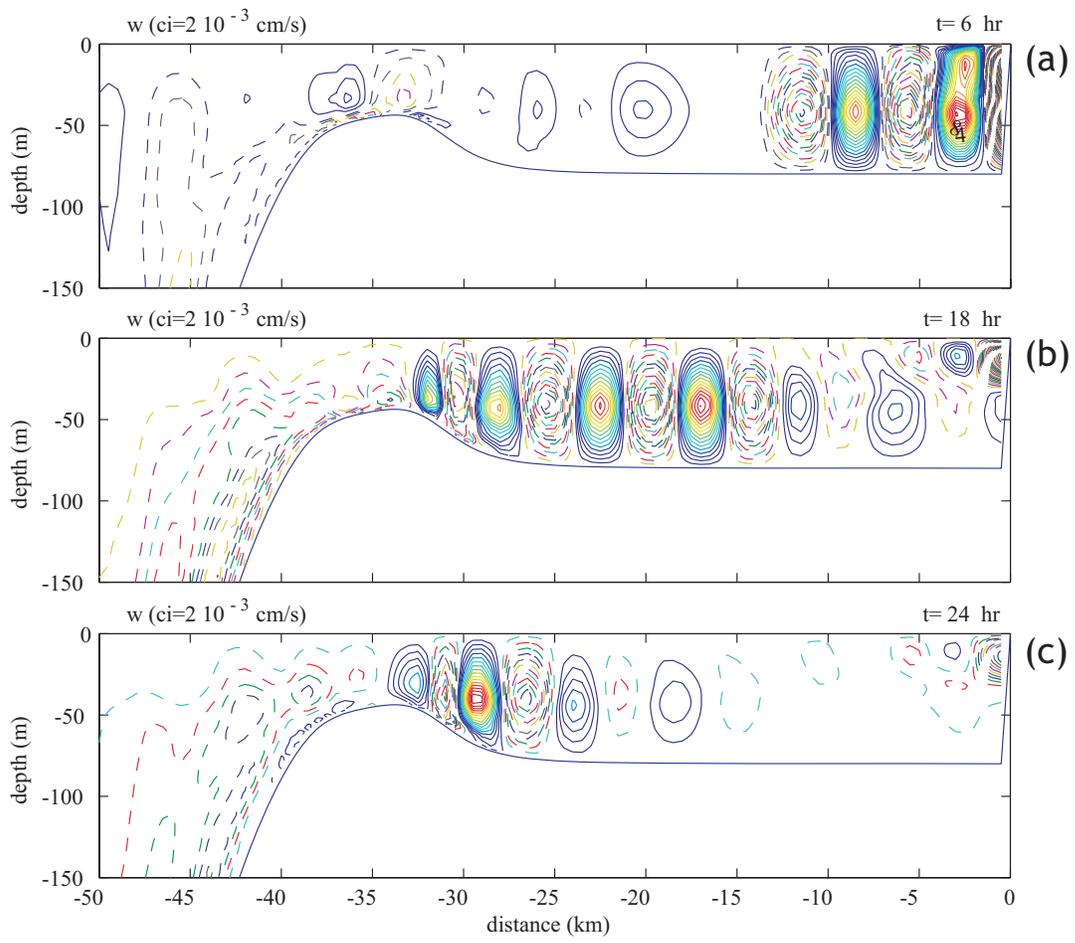


Fig 9

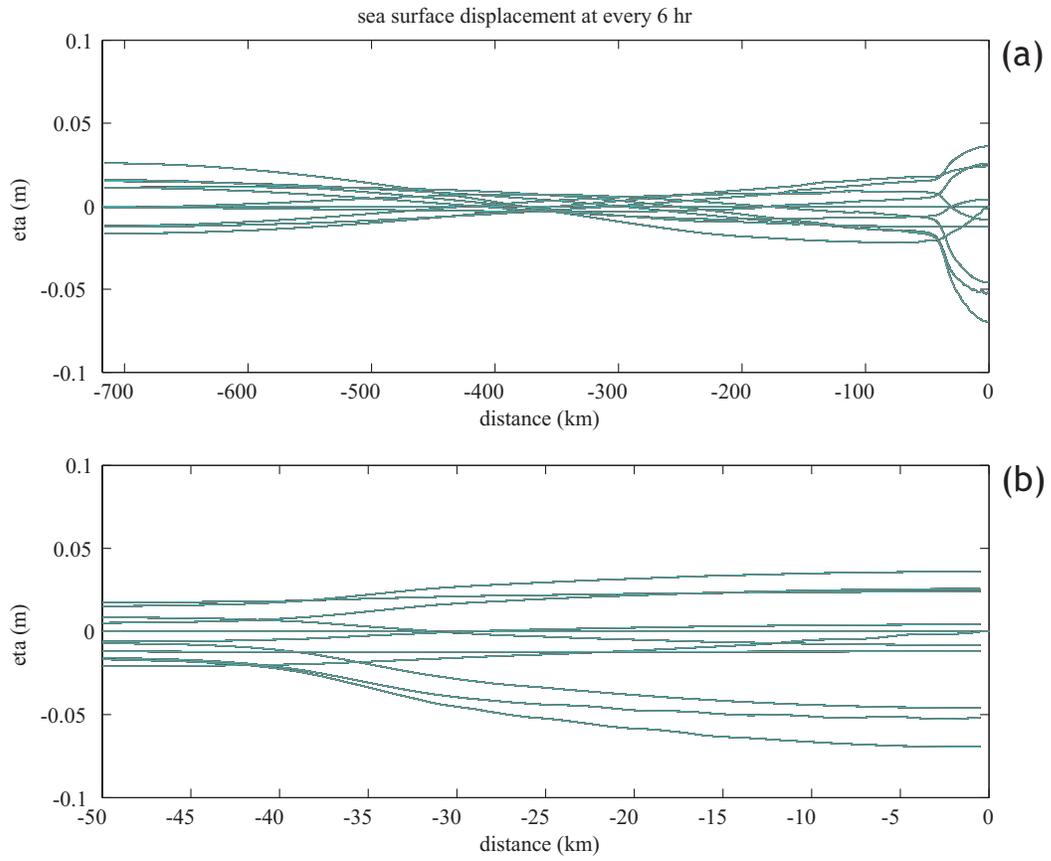


Fig 10

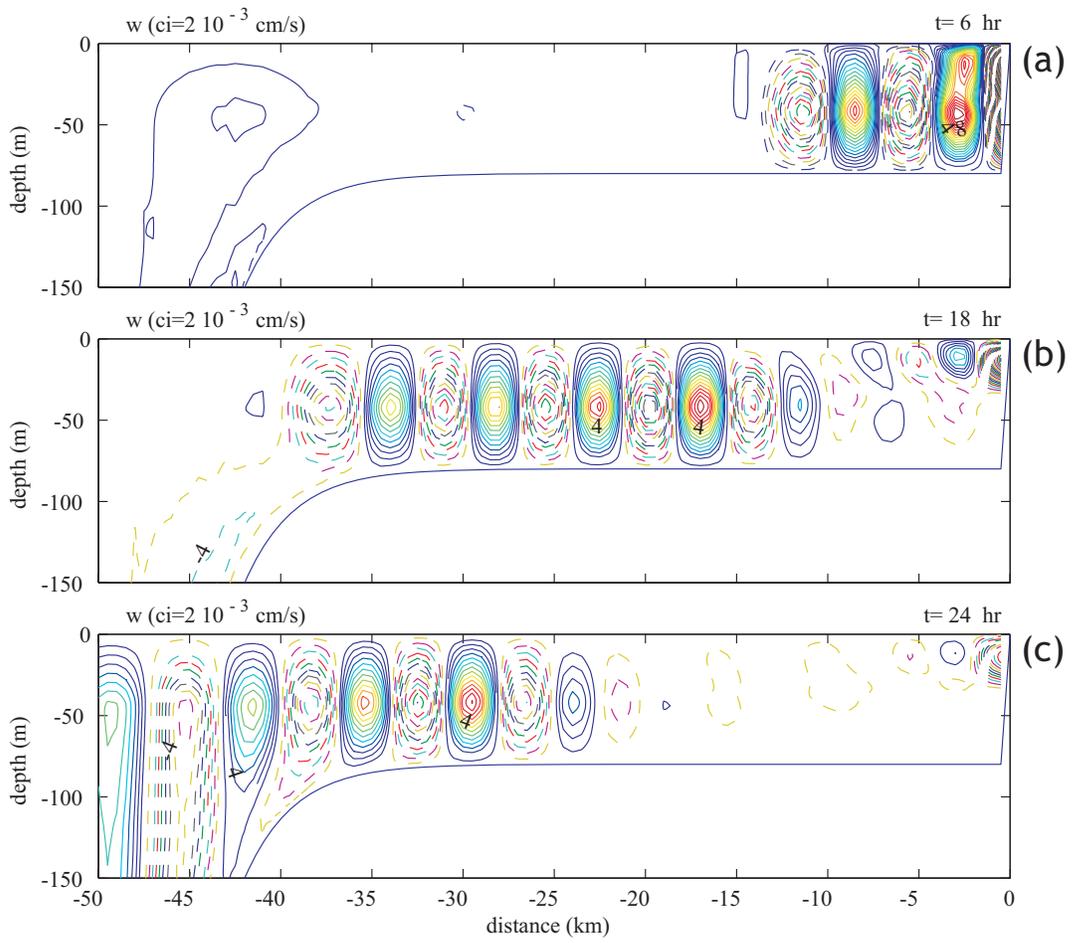


Fig 11

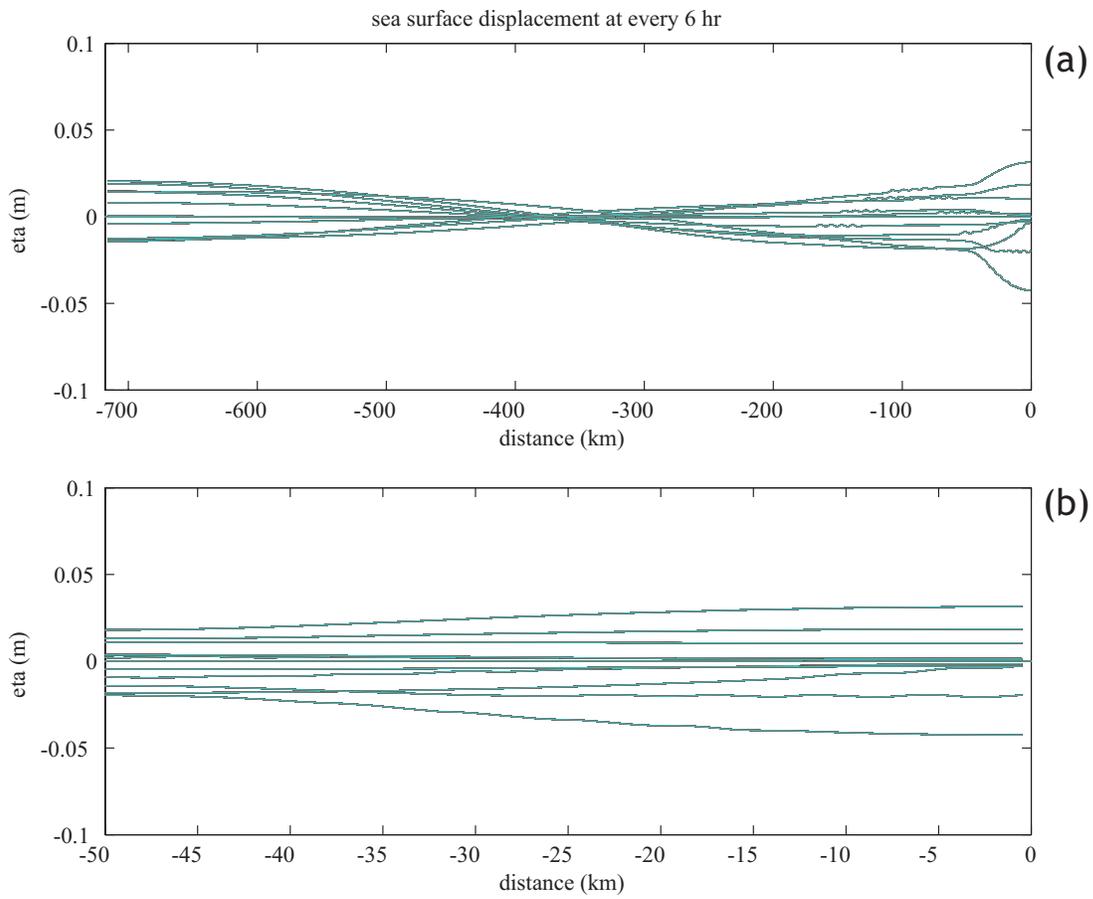


Fig 12

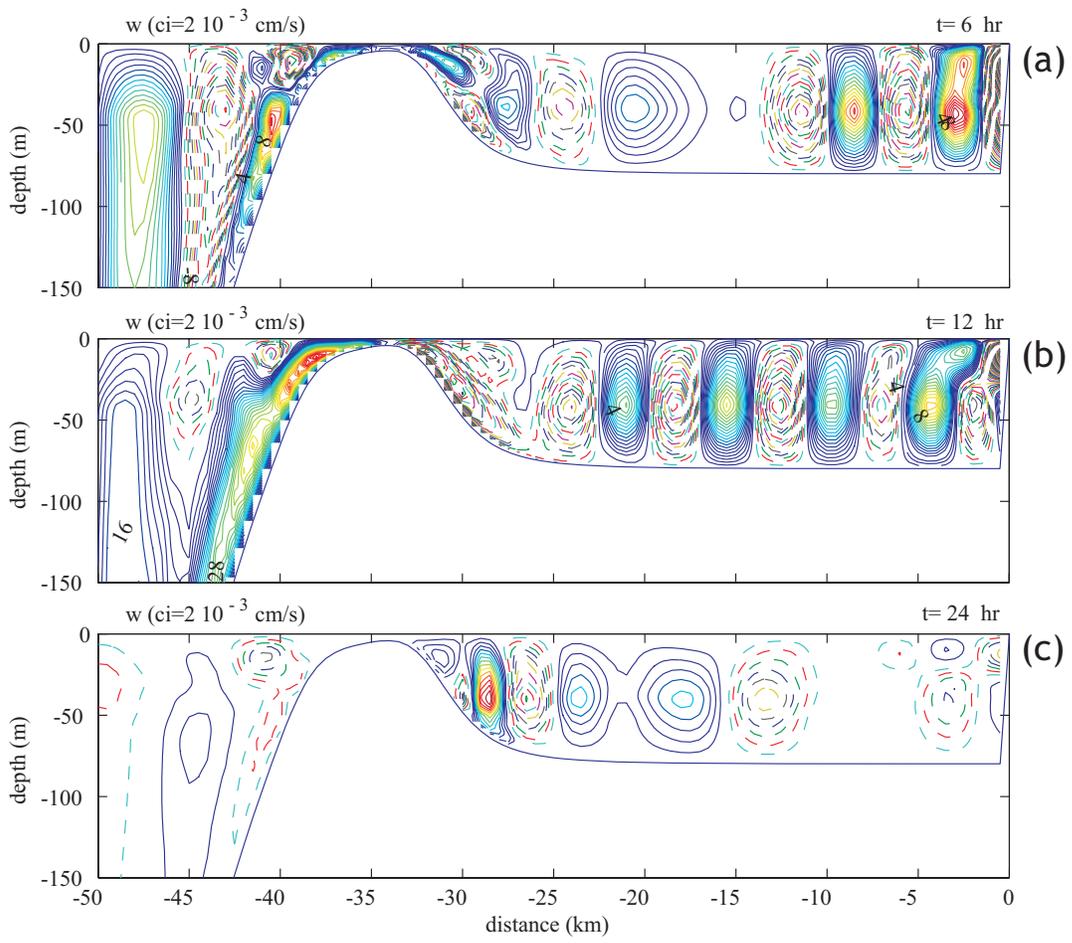


Fig 13

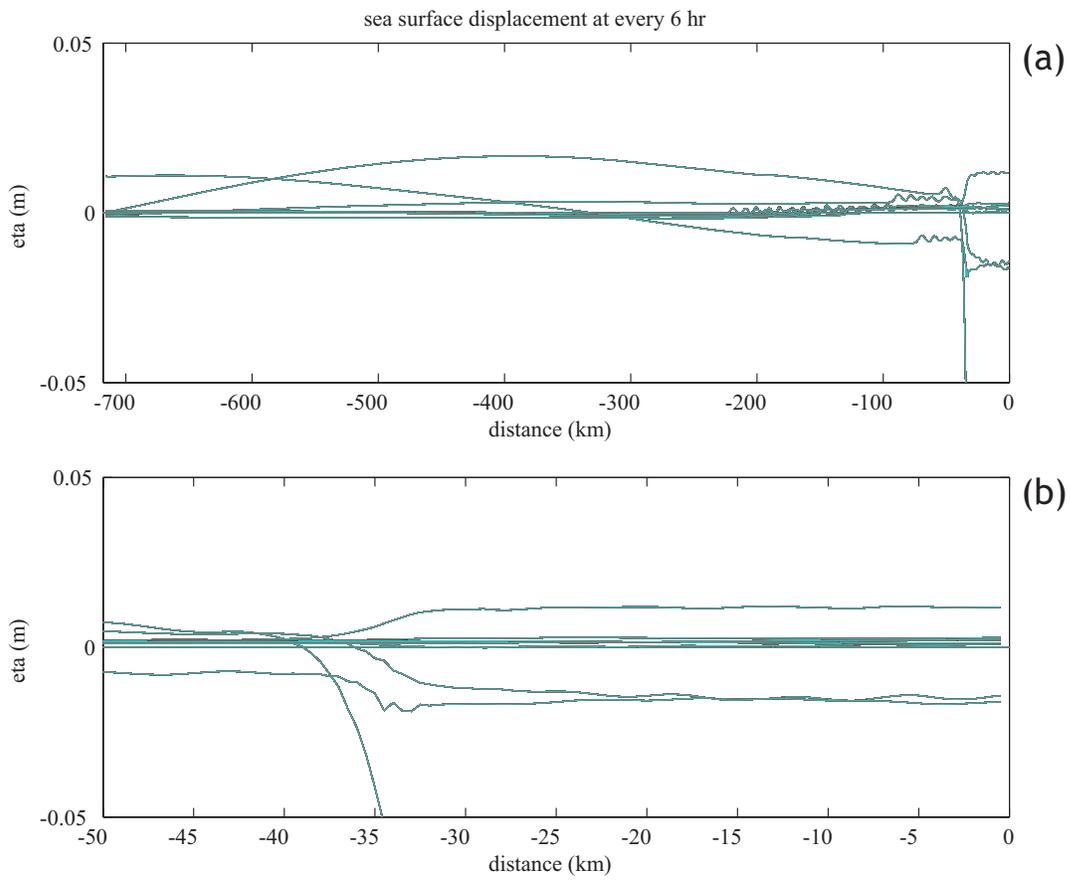


Fig 14