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**Observations and analysis of sediment diffusivity profiles over sandy
rippled beds under waves**

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Abstract

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Acoustic measurements of near-bed sediment diffusivity profiles are reported. The observations were made over two sandy rippled beds, classified as ‘medium’ and ‘fine’ in terms of sand grain size, under slightly asymmetric regular waves. For the medium sand, the ripples that formed had relatively steep slopes, while for the fine sand, the slopes were roughly half that of the medium sand. In the medium sand case, the form of the sediment diffusivity profiles was found to be constant with height above the bed, to a height equal approximately to the equivalent roughness of the bed, k_s , while above this the sediment diffusivity increased linearly with height. For the case of the fine sand there was no constant region; the sediment diffusivity simply increased linearly with height from the bed. To understand the difference between the respective diffusivity profiles, advantage has been taken of the high temporal-spatial resolution available with acoustic systems. Using intra-wave ensemble averaging, detailed images have been built up of the variation in concentration with both the phase of the wave and also height above the bed. These intra-wave observations, combined with measurements of the bed forms and concepts of convective and diffusive entrainment, have been used to elucidate the mixing mechanisms that underlie the form of the diffusivity profiles observed over the two rippled beds. These mechanisms centre on coherent vortex shedding in the case of steeply rippled beds and random turbulent processes above ripples of lower steepness.

I Introduction

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In many marine environments, from river estuaries through to the offshore regime, suspended sediments are a significant component of the total sediment transport and, in numerous cases, are dominant. It is therefore necessary to obtain a description of how the sediments are entrained into the water column and to ascertain the resulting form of the suspended sediment concentration profile. Predictions for the form of the concentration profile differ according to the flow, the seabed sediment and, importantly, any resulting bed forms (Sleath, 1984; Soulsby, 1997; Van Rijn et al., 2001). Most of the formulations used have been underpinned by the classical Fickian concept of gradient diffusion (Coleman, 1970; Glenn and Grant, 1987; Vincent and Green, 1990; Vincent and Osborne, 1995; Ogston and Sternberg, 2002; van der Werf et al., 2006), originating from kinetic molecular theory where random molecular movements induce mixing. In the case of suspended sediments in field situations, it is the turbulent fluctuations in the vertical velocity component that give rise to the upward mixing process. In the simplest case the time averaged vertical turbulent diffusive flux of sediment, q_v , is considered to be balanced by the settling of the suspended sediment under gravity, such that:

$$q_v = w_s C \quad \text{where} \quad q_v = -\epsilon_s \frac{\partial C}{\partial z} \quad (1)$$

64 Here C is the time-averaged sediment concentration at height z above the bed, w_s is
65 the sediment settling velocity, and ϵ_s is the sediment diffusivity. The vertical profile of
66 ϵ_s is frequently linked to the eddy viscosity, ν_t , used to model the transfer of
67 momentum by turbulent eddies. The eddy viscosity, ν_t , represents the product of a
68 turbulent velocity scale and a mixing length scale. Both of these factors therefore
69 affect the sediment diffusivity which is commonly expressed as $\epsilon_s = \beta \nu_t$ where the
70 coefficient β is either assumed to be a constant (equal to unity, or larger or smaller
71 than unity) or is sometimes considered to have a functional dependence upon the
72 sediment in suspension and the flow parameters (Van Rijn, 1984; Whitehouse, 1995;
73 Rose and Thorne, 2001). The vertical profile of ν_t , and hence ϵ_s , in previous
74 applications has been taken to be constant, linear, parabolic, exponential or some
75 combination thereof (Grant and Madsen, 1979; Nowell and Long, 1983; Nezu and
76 Rodi, 1986; Nielsen, 1992; Van Rijn, 1993; Chung and Van Rijn, 2003). These

77 different forms have been associated with various concepts regarding the mixing in
78 the near-bed boundary layer. Hitherto, there has been no consensus on a general form
79 for profiles of the sediment diffusivity or eddy viscosity, though constant (Nielsen,
80 1986; van der Werf et al., 2006) and linear profiles (Ribberink and Al-Salem, 1994;
81 Vincent and Osborne, 1995) with height above the bed have been used in many near-
82 bed sediment studies.

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84 Despite the wide use of gradient diffusion, several studies (Sleath, 1982; Hansen et
85 al., 1991; Ribberink and Al-Salem, 1994; Osborne and Vincent, 1992; Fredsøe et al.,
86 1999; Villard and Osborne, 2002; Thorne et al., 2003) have indicated that this is not
87 always the dominant process generating the suspended sediment concentration profile,
88 particularly for sediment entrainment by waves over rippled beds. These studies have
89 shown that, if the ripples are relatively steep with $\eta_r/\lambda_r \geq 0.12$, where η_r is the ripple
90 height and λ_r is the ripple wavelength, then the mixing close to the bed is dominated
91 by a coherent process involving boundary layer separation on the lee-side of the ripple
92 crest during each wave half-cycle near maximum flow velocity. The resulting lee-
93 wake vortex remains attached to the bed entraining sediment into the flow as it grows
94 in size and strength. At flow reversal the sediment-laden vortex is ejected into the
95 water column, carrying sediment to several ripple heights above the bed. This process
96 is coherent and repeatable, with two main periods of sediment entrainment during the
97 cycle at around the times of flow reversal. The sediment mixing process is thus
98 fundamentally different from that associated with gradient diffusion. Gradient
99 diffusion relies on the ‘mixing length’ being small compared with the vertical extent
100 of the concentration profile as a whole, and the rate of diffusion is proportional to the
101 concentration gradient (Equation (1)). In contrast, the mixing due to vortex
102 entrainment occurs on a (relatively) larger ‘convective’ length scale that is not,
103 therefore, linked so directly to the concentration gradient. Interestingly, the ‘finite
104 mixing length’ approach proposed by of Nielsen and Teakle (2004) offers a novel way
105 of reconciling the two different physical concepts contrasted above. Nielsen (1988,
106 1992) had earlier suggested that in many circumstances, particularly involving rippled
107 beds under waves and also combined waves and currents, both convective and
108 diffusive processes occur together and, in some recent studies (Lee and Hanes, 1996;
109 Lee et al., 2002; Thorne et al., 2002), this approach has been adopted.

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111 The present study represents a contribution towards our understanding of these
112 fundamental mixing processes. Measurements were collected in a large flume, the
113 Deltaflume, Deltares (formerly WL|Delft Hydraulics), the Netherlands, which is 240
114 m long, 5 m wide and 7 m deep. The size of the flume allowed the wave and sediment
115 transport processes to be studied at full scale (Williams et al., 2003;
116 <http://www.wldelft.nl/facil/delta>). Simultaneous, closely co-located observations were
117 made of: suspended sediment concentration, suspended particle size, the flow and the
118 ripples on the sandy beds. The data were obtained beneath regular weakly-
119 asymmetrical surface waves over beds of medium and fine sand. These data are used
120 here to examine the sediment diffusivity profiles over the two sandy beds. To interpret
121 the form of the observed sediment diffusivity, advantage is taken of the high
122 temporal-spatial resolution available with acoustic systems. In particular, intra-wave
123 ensemble averaging, coupled with bed form measurements, have been used to build
124 up detailed images of the variation in concentration with both the phase of the wave
125 and also the height above the bed. These data have been used to highlight the
126 underlying entrainment mechanisms that led to the form of the measured sediment
127 diffusivity profiles presented in this study.

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129 The paper is laid out as follows; in Section II a physical discussion is presented of the
130 key modelling concepts, followed in Section III, by a summary of the observational
131 work and data analysis. In Section IV the measurements are presented and interpreted
132 to explain the different profiles for the sediment diffusivity, obtained above the two
133 sandy beds, in terms of convective and diffusive processes. This is followed in
134 Section V by a discussion on the implications of the observations, with conclusions
135 drawn in Section VI.

136

II Models

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138

139 A modelling framework can be set out for turbulent oscillatory flows above various
140 naturally occurring bed types in terms of the wave Reynolds number, $RE = A_0U_0/\nu$,
141 where A_0 is the orbital amplitude, U_0 is the near-bed velocity amplitude and ν is the
142 kinematic viscosity, and the relative roughness, A_0/k_s , where k_s is the equivalent bed
143 roughness (Davies and Villaret, 1997). Table 1 summarises a simplified framework
144 for oscillatory flows above erodible sandy beds (see also Davies and Thorne, 2008).
145 Essentially, steeply rippled beds having $\eta_r/\lambda_r \geq 0.12$ occur in low energy flows; such
146 ripples tend to be long-crested (two-dimensional) with vigorous, alternate eddy
147 shedding occurring above them. Such ripples are characterised by low values of RE ,
148 A_0/k_s and also of $\hat{\theta} = \hat{\tau}_0 / \{(\rho_s - \rho)gd_{50}\}$ where $\hat{\tau}_0$ is the peak bed shear stress during the
149 wave cycle, ρ_s and ρ are the densities of the sediment and water respectively, and d_{50}
150 is the median grain diameter. For larger values of the respective non-dimensional
151 parameters the ripples are reduced in amplitude and tend to have shorter crest-lengths
152 (2D-3D ‘transitional’ ripples). Ultimately, for high energy flows, ‘dynamically plane’
153 beds occur; here any ripples that are present are of such small steepness ($\eta_r/\lambda_r \leq 0.08$)
154 that the oscillatory flow becomes closely similar *dynamically* to that above a plane
155 bed. [It may be noted that the beds referred to here as ‘dynamically plane’ are
156 commonly denoted also, in the limit of very high mobility, as ‘upper stage plane
157 beds’] The equivalent roughness, k_s , depends upon the grain size for flat sandy beds
158 with, typically, $k_s = 2.5d_{50}$ for ‘lower stage plane beds’, and upon the ripple height and
159 steepness for rippled beds, $k_s \propto \eta_r(\eta_r/\lambda_r)$, with k_s enhanced by a ‘mobile bed’
160 contribution for low ripples and plane beds in high energy flows.

161

162 It was noted by Davies and Villaret (1997) that many of the modelling concepts
163 developed for steady turbulent flow remain valid in oscillatory flow. Above smooth
164 flat beds, at high RE , the turbulent eddy viscosity may be assumed to vary linearly
165 with height throughout the wave boundary layer. However, for rougher beds, also at
166 high RE , data suggest the existence of an outer layer in which the turbulent velocity
167 scale decreases with height and in which, therefore, v_t remains approximately constant
168 (e.g. Trowbridge and Madsen, 1984). The wave boundary layer thickness is
169 overestimated by models that do not include this outer, constant, v_t -layer. Several

170 eddy viscosity models have assumed, either implicitly or explicitly, that v_t is also
171 time-varying (Trowbridge and Madsen, 1984; Fredsøe, 1984; Davies, 1986).

172

173 The models above are all based upon turbulent diffusion as the dominant mechanism
174 of momentum exchange. In contrast, at lower flow stages above very rough and
175 rippled beds, the situation is entirely different. Here momentum transfer is due
176 mainly to eddy shedding from individual roughness elements at times of flow
177 reversal, and so is well organised in space and time. For relatively low values of RE
178 and A_0/k_s (Table 1) Sleath (1991) and Nielsen (1992) suggested that it is reasonable
179 to treat v_t as constant in height and time. For the range $1 \leq A_0/k_s \leq 120$, Sleath (1991)
180 proposed the following expression for v_t by analogy with grid-turbulence
181 experiments:

$$182 \quad v_t = 0.00253A_0^{3/2}k_s^{1/2}\omega \quad (2)$$

183 where the angular frequency $\omega = U_0/A_0$. Subsequently, on the basis of data sets for
184 very rough conditions in the range $A_0/k_s < 16$, Nielsen (1992) proposed the constant
185 eddy viscosity:

$$186 \quad v_t = 0.004A_0\omega k_s \quad (3)$$

187 These two formulae give identical results for $A_0/k_s = 2.5$.

188

189 Although turbulence is strongly related to eddy shedding, it is the coherent vortex
190 shedding mechanism itself that dominates the mixing in the near-bed layer above very
191 rough and rippled beds. Ranasoma and Sleath (1992) demonstrated experimentally
192 that the effect of turbulent Reynolds stresses above steep ripples is negligible in
193 comparison with the momentum transfer associated with coherent vortices. Their
194 measurements showed large time variations in the vertical transfer of momentum
195 corresponding to the release of coherent vortex structures at the ripple crest. This was
196 reflected in the ‘convective eddy viscosity’ coefficient used by Davies and Villaret
197 (1997) who introduced time variation into v_t in order to represent the combined
198 effects on momentum transfer of turbulence and, more importantly, organised eddy
199 shedding at flow reversal.

200

201 The vertical mixing of sediment is necessarily closely related to the vertical transfer of
202 momentum. If the bed is flat, the periodic surface-wave-induced vertical velocity, w_w ,

203 is very small in the near-bed layer, tending to zero at the bed itself. Thus w_w may be
 204 assumed to contribute little to the upward flux of sediment $\overline{w_w C_w}$ near the bed,
 205 where C_w is the periodic component of the suspended concentration and the over-bar
 206 denotes time averaging. Rather higher above the bed, it has been shown by Sheng and
 207 Hay (1995) that this flux remains relatively small, with typically $|\overline{w_w C_w} / w_s C| < 0.2$.
 208 This suggests the validity of the following approximation, related to turbulent
 209 processes only, for the upward sediment flux above a flat bed (c.f. Equation (1)):

$$210 \quad -\overline{C'w'} \approx \varepsilon_s \frac{dC}{dz} \quad (4)$$

211 where the primes here denote, respectively, the random turbulent contributions to the
 212 concentration and the vertical velocity.

213

214 In contrast, above a rippled bed, the sloping sides of the bed forms give rise to locally
 215 significant, periodic, vertical velocity contributions arising from both the (frictionless)
 216 wave action and the (frictional) process of vortex formation. Thus, in a ripple-
 217 averaged sense, the (convective) term $\overline{w_w C_w}$ can contribute significantly to the
 218 upward flux of sediment; in fact, this term can dominate the upward sediment flux in
 219 the bottom part of the wave boundary layer.

220

221 If the sediment diffusivity ε_s is still identified solely with the turbulent flux $\overline{C'w'}$ (c.f.
 222 Equation (4)), then the time-averaged sediment balance in the case of a rippled bed
 223 may be expressed:

$$224 \quad -w_s C + \overline{w_w C_w} - \varepsilon_s \frac{dC}{dz} = 0 \quad (5)$$

225 such that

$$226 \quad \varepsilon_s = \frac{-w_s C + \overline{w_w C_w}}{dC/dz} \quad (6)$$

227

228 In the present paper, however, we effectively absorb the convective transfer
 229 represented by $\overline{w_w C_w}$ into a ‘convective diffusivity’ whereby ε_s is defined simply by

$$230 \quad \varepsilon_s = \frac{-w_s C}{dC/dz} \quad (7)$$

231 The physically interesting and practically significant consequences of this widely used
232 technique are discussed in Section IV where we obtain contrasting vertical profiles for
233 ε_s based on observations made in the Deltaflume above beds of different grain size.
234 These measured ε_s profiles are compared with three established expressions for ε_s ;
235 two of these were specifically formulated for waves propagating above rippled and
236 very rough sand beds, while the third, a linearly increasing diffusivity, is that most
237 commonly used in sediment and flow studies involving plane beds (Grant and
238 Madsen, 1979; Lee and Hanes, 1996; Lee, Friedrichs and Vincent, 2002). These
239 expressions are discussed here in turn.

240

241 Firstly, Nielsen's (1992) sediment diffusivity for rough and rippled beds follows from
242 the eddy viscosity formulation (3) and the relationship $\varepsilon_s = \beta v_t$ wherein the value
243 adopted for β reflects the relatively high efficiency of the eddy shedding process in
244 entraining sediment into suspension. In particular, Nielsen (1992) adopted $\beta = 4$
245 leading to the following expression for the near-bed sediment diffusivity:

$$246 \quad \varepsilon_s = 0.016 k_s U_o \quad (8)$$

247 The physical explanation for the large value of β used by Nielsen and others has not
248 been fully explained by either models or experiments. However, it would appear to
249 be linked to 2D and/or 3D temporal-spatial correlations between the instantaneous
250 velocity and concentration fields, as shown by Magar and Davies (2005) using a
251 particle tracking model.

252

253 Nielsen's (1992) proposed expression for the equivalent roughness k_s in equation (8)
254 was $k_s = \delta \eta_r (\eta_r / \lambda_r)$ where $\delta = 8$. However, as explored by Thorne et al. (2002), this
255 rather low value for δ does not take into account the convective contribution to the
256 upward mixing of momentum and sediment. Here therefore, following Thorne et al.
257 (2002) we have adopted the more commonly used value $\delta = 25$ (Swart, 1974) in
258 equation (8) and determined ε_s using detailed in-situ measurements of the ripple
259 dimensions, η_r and λ_r , made in the Deltaflume.

260

261 The second formulation for ε_s highlighted in Section IV is that of Van Rijn (1993).
262 This was derived empirically for waves alone and involves a three-layer structure for

263 ε_s covering the full water column. Importantly, it represents the sediment diffusivity
 264 in the near-bed layer ($z \leq \zeta_s$) as being constant with height:

$$265 \quad \varepsilon_s = \varepsilon_b \quad z \leq \zeta_s \quad 9(a)$$

$$266 \quad \varepsilon_s = \varepsilon_m \quad z \geq 0.5h \quad 9(b)$$

$$267 \quad \varepsilon_s = \varepsilon_b + (\varepsilon_m - \varepsilon_b) \left[\frac{z - \zeta_s}{0.5h - \zeta_s} \right] \quad \zeta_s < z < 0.5h \quad 9(c)$$

268

269 Here ε_b and ε_m are, respectively, constant values for the sediment diffusivity near the
 270 bed and in the upper half of the water column, with the latter value being the larger; ζ_s
 271 is the thickness of the near-bed mixing layer and h is the water depth. This eddy
 272 diffusivity is constant in the near-bed layer, is linearly increasing with height in the
 273 layer above this, and then remains constant in the upper half of the flow. Van Rijn
 274 suggested a lower layer thickness given by $\zeta_s = 3\eta_r$. In the present paper, we have
 275 adopted the expression $\zeta_s = k_s (=25\eta_r(\eta_r/\lambda_r))$ for ease of comparison with Nielsen's
 276 formulation. It may be noted that Van Rijn's expression is recovered from this for
 277 ripples having a steepness of $\eta_r/\lambda_r = 0.12$. In Section IV, due to variations in the
 278 observed ripple steepness in different experimental runs, this results in $\zeta_s = (3.4 \pm 0.2)\eta_r$
 279 for the medium sand bed and $\zeta_s \approx (1.9 \pm 0.2)\eta_r$ for the fine sand bed. Assuming that ζ_s
 280 = k_s , Van Rijn's formulation can be expressed in the same form as that of Nielsen,
 281 namely:

$$282 \quad \varepsilon_b = \alpha_b k_s U_o \quad (10)$$

283 in which Van Rijn's coefficient $\alpha_b = 0.004D^*$, wherein the dimensionless grain size D^*
 284 = $d_{50}[(s-1)g/v^2]^{1/3}$; s is the relative density ρ_s/ρ , ρ_s is the grain density and ρ is the fluid
 285 density; g is the acceleration due to gravity; and v is the kinematic viscosity. The eddy
 286 diffusivity in the upper layer is given by Van Rijn (1993) as:

$$287 \quad \varepsilon_m = \alpha_m \frac{Hh}{T} \quad (11)$$

288 where H is the wave height, T is the wave period and the empirical coefficient
 289 $\alpha_m = 0.035$. In the present study, where the measurements were confined to the bottom
 290 quarter of the water column, it is only predictions for the near-bed constant and linear
 291 regions that are assessed.

292

293 The final form used for the sediment diffusivity is a simple linear increase in ε_s with
294 height above the bed. This is commonly expressed (Grant and Madsen, 1979; Lee and
295 Hanes, 1996; Lee, Friedrichs and Vincent, 2002) as

296

$$297 \quad \varepsilon_s = \beta \kappa \bar{u}_* z \quad (12)$$

298 where $\kappa=0.4$ is Von Karman's constant. Here we have used the mean magnitude of
299 the friction velocity, \bar{u}_* , in the wave cycle as representative of the turbulent mixing
300 during the wave cycle as a whole (see Davies, 1986):

301

$$302 \quad \bar{u}_* = 0.763(f_w/2)^{0.5} U_o \quad \text{and} \quad f_w = 0.237 \left(\frac{k_s}{A_o} \right)^{0.52} \quad (13)$$

303

304 where f_w is the friction factor formulated by Soulsby (1997). In applying equation
305 (12) to the observations, consideration must be given to the appropriate expression to
306 be used for k_s in the analysis. For a flat (or lower stage plane) bed the Nikuradse
307 roughness value is normally used which, as noted earlier, is commonly expressed as
308 $k_s=2.5d_{50}$. The implications of using this skin-friction expression over a rippled bed
309 are considered in Section IV.

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III Experimental arrangement and data analysis

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316 The study was undertaken as part of a collaborative European experiment, and was
317 conducted in the Deltaflume. Details of the experimental arrangement have been
318 provided in an earlier publication (Thorne et al., 2002) and are therefore only briefly
319 summarised here for completeness. The large size of this flume, 240 m in length, 5 m
320 in width and 7 m deep, allow hydrodynamic and sediment transport phenomena to be
321 studied at full scale. The experiments were conducted beneath weakly-asymmetrical,
322 regular, surface waves with heights, H , and periods, T , in the respective ranges $H=0.6$ -
323 1.1 m and $T=4-6$ s for the medium sand and $H=0.5-1.1$ m and $T=4-5$ s for the fine
324 sand. Therefore the hydrodynamic conditions for the experiments involving the two
325 sands were comparable. The medium sand had $d_{10}=170$ μm , $d_{50}=330$ μm and $d_{90}=700$
326 μm , while the fine sand had $d_{10}=95$ μm , $d_{50}=160$ μm and $d_{90}=300$ μm ; both the sands
327 were therefore reasonably well sorted. The sediments were located in a layer of
328 thickness 0.5 m and length 30 m, approximately halfway along the flume, where the
329 mean water depth was 4.5 m. The measurements were conducted first above the
330 medium sand bed; this was then removed and replaced by the fine sand bed.

331

332 Figure 1 shows the instrumented tripod platform ‘STABLE’ (Sediment Transport And
333 Boundary Layer Equipment) used to collect the measurements. The main instruments
334 on STABLE relevant to the present study were: a multi-frequency acoustic
335 backscatter system, ABS, a pumped sampling system, an acoustic ripple profiler,
336 ARP, and electromagnetic current meters, ECMs. All measurements were
337 synchronised. A study of the impact of STABLE on the processes being measured
338 was shown to be minimal (Williams et al., 2003). Typically an experiment consisted
339 of propagating waves over the bed for about an hour, until the bed-forms came to
340 nominal equilibrium, and then collecting data for a 17 min period.

341

342 High-resolution vertical profiles of the suspended sediments were measured using a
343 triple-frequency ABS (Crawford and Hay, 1993; Thorne et al., 1997; Thorne and
344 Hanes, 2002). The ABS provided 128 backscatter profiles each second, at each of the
345 three frequencies, 1 MHz, 2 MHz and 4 MHz. Each profile consisted of 128 range
346 bins, with a spatial resolution of 0.01 m, thereby covering a range of 1.28 m. Physical

347 samples of the suspension were obtained by pumping through nozzles (Bosman et al.,
348 1987) located at ten heights above the bed between 0.053-1.55 m. The collected
349 samples of the suspension were sieved to provide the mass size distribution with
350 height above the bed. They were used to calibrate and assess the veracity of the
351 acoustic backscatter measurements and provide profiles of w_s . To establish whether
352 ripples were present on the bed, and to monitor their evolution and migration, a
353 specifically designed acoustic ripple profiler, ARP, (Bell et al., 1998; Thorne et al.,
354 2002; Williams et al, 2004) was used. The ARP operated at 2.0 MHz, and provided
355 sub-centimetric measurements of the bed location over a 3m transect along the
356 direction of wave propagation. To measure the flow three ECMs were located at 0.3,
357 0.6 and 0.91 m above the bed. They provided measurements of the along-flume and
358 vertical components of the flow velocity at 8 Hz.

359

360 Measurements of the suspended concentration were collected with the ABS. Using the
361 particle size data obtained from the pumped samples an explicit acoustic inversion
362 (Thorne and Hanes, 2002) was carried out on the recorded 17 min averaged
363 backscatter voltages to convert them to mean concentration profiles. For each
364 experiment three independent concentration profiles were obtained, one for each
365 frequency. Since 13 experiments were carried out above the medium sand and 7 were
366 carried out above the fine sand, this resulted in 39 and 21 mean concentration profiles
367 in the respective cases. Using the bed echoes the concentration profiles were
368 referenced to the undisturbed bed, such that in the plots that follow z is the height
369 above the undisturbed bed, with a vertical sampling interval of 0.01 m. The veracity
370 of the profiles has been assessed previously (Thorne et al., 2002) using the pumped
371 sample measurements and this is not repeated here. However, for the purpose of
372 illustrating the magnitude and form of the concentration profiles for the two sands,
373 examples are provided in Figure 2 for wave conditions $H=0.5$ m and 0.8 m and $T=5$
374 s. The figure shows mean concentrations, averaged over the burst period (17 min,
375 ~ 200 wave cycles), at the three acoustic frequencies for the two sands. The detailed
376 differences between the profiles at the three frequencies are due the accuracy of the
377 system calibration, the model used for the acoustic scattering properties of the
378 suspended sediments and the inversion methodology employed. However, the
379 important factor as far as this study is concerned is that the general profile features are
380 consistent across the three frequencies. For the $H=0.5$ m case it can be seen that the

381 magnitude of the suspended concentration for the fine sand ($x, *, \Delta$) is significantly
 382 greater than for the medium sand ($+, o, \square$). This was also the case for $H=0.8$ m, though
 383 the difference was less. This was a general trend for the two sands, with the difference
 384 in suspended concentration levels decreasing as wave height increased. The form of
 385 the profiles can also be seen to be different, with the relative reduction in
 386 concentration being greater for the fine sand in the first 0.1 m above the bed while,
 387 between 0.1-0.4 m, the medium sand concentration reduces somewhat more rapidly
 388 than the fine. Above 0.4 m the gradients become comparable for the two sands.

389

390 Using the mean concentration profiles, the sediment diffusivities ε_s were calculated
 391 for each experiment using equation (7), with w_s determined from a d_{50s} particle size
 392 profile empirically fitted to the pumped sample data. The expressions used were:

393

$$394 \quad \varepsilon_s = \frac{\frac{w_{sj} + w_{sk}}{2} \frac{C_j + C_k}{2}}{\frac{(C_k - C_j)}{\Delta_{jk}}} \quad (14a)$$

$$395 \quad z = \frac{z_j + z_k}{2} \quad (14b)$$

396 with w_s given by Soulsby (1997) as:

397

$$398 \quad w_s = \frac{v}{d_{50s}} [(10.36^2 + 1.049D_*^3)^{0.5} - 10.36] \quad (14c)$$

399 where Δ_{jk} was the separation between range bins j and k . For the near-bed layer 0.01-
 400 0.21 m above the bed, j and k were taken as adjacent range bins while, between 0.21-
 401 0.43 m, j and k were defined as two range bins apart and, above 0.43 m, as four range
 402 bins apart. This increase in the separation of j and k with height above the bed
 403 smoothed the derivative of the concentration profile and reduced scatter in the
 404 diffusivity profiles.

405

406 The resulting sediment diffusivity profiles were next normalised using four different
 407 non-dimensional scalings and they were also averaged in three different ways. The
 408 aim here was to clarify the trends in ε_s and assess whether the different approaches

409 gave consistent results. The four normalisations used for height z and sediment
 410 diffusivity ε_s were, respectively:

411

$$412 \quad z/h \quad \varepsilon_s/\kappa \bar{u}_* h \quad (15a)$$

413

$$414 \quad z/\delta_w \quad \varepsilon_s/\kappa \bar{u}_* \delta_w \quad (15b)$$

415

$$416 \quad z/\eta_r \quad \varepsilon_s/U_o \eta_r \quad (15c)$$

417

$$418 \quad z/k_s \quad \varepsilon_s/U_o k_s \quad (15d)$$

419

420 The first two of the normalisations have been used by previous authors (eg Sheng and
 421 Hay, 1995) and the latter two were chosen here on the basis of the theoretical
 422 expressions in section II. The scale thickness of the wave boundary layer δ_w has been
 423 taken here as:

$$424 \quad \delta_w = \bar{u}_*/\omega = 0.763(f_w/2)^{0.5} U_o/\omega, \quad (16)$$

425

426 For the normalisations in equations (15b) and (15d), the equivalent bed roughness has
 427 been taken as $k_s=25\eta_r(\eta_r/\lambda_r)$. The three averages used on the normalized ε_s data at
 428 each range bin above the bed were (i) the median which is a relatively robust mean
 429 with regard to outliers; (ii) a trimmed mean value which excluded the 20% highest
 430 and 20% lowest data values; and (iii) a mean based on a simple in-house filter that
 431 rejected outliers. These normalised averages were then smoothed using localised
 432 vertical averaging over intervals that increased in extent with height above the bed, in
 433 order to further reduce the scatter in the resulting ε_s profiles. Range bins 1-20 above
 434 the bed had no averaging applied; range bins 21-40 were averaged over three adjacent
 435 bins; and bins 41-86 were averaged over five adjacent bins.

436

437

IV Sediment diffusivity measurements and interpretation

438

439

440 (i) Medium sand

441 Using equation (14), ϵ_s was calculated above the medium sand bed using the ABS
442 concentration profiles, together with the pumped sample particle size profiles. The
443 values for the suspended sediment size varied from around $d_{50s}=230 \mu\text{m}$ within a
444 centimetre or two of the bed, to about $d_{50s}=170 \mu\text{m}$ at 1 m above the bed. The
445 reduction in particle size, of about a 30% in the bottom metre above the bed, is small
446 compared with the change in concentration and had a second order effect on the
447 variation of ϵ_s with height above the bed. The results for the 39 sediment diffusivity
448 profiles, from the 13 experiments involving the medium sand, are shown in Figure 3.
449 Here it can be seen that the sediment diffusivity is relatively consistent in form in the
450 bottom 0.2 m above the bed, having values around $0.001\text{-}0.003 \text{ m}^2\text{s}^{-1}$. Above 0.2 m
451 the values for the sediment diffusivity increase with height above the bed and the
452 scatter in the data increases. This increase in scatter with height is due both to noisier
453 lower concentration levels at the greater heights and also to the different flow and bed
454 conditions associated with the thirteen different experiments. In an attempt to clarify
455 trends in the data, the normalisations in equation (15) were applied to the respective
456 sediment diffusivity profiles. The normalized data, shown by the small solid dots in
457 Figure 4, have a scatter which is approximately one third that of the data shown in
458 figure 3 and an enhancement in the form of the trends. Although none of the four
459 normalizations collapse all the data on to a single profile, they clearly show a common
460 trend in the sediment diffusivity profile, with a near-bed region that is nominally
461 constant with height above the bed, above which there is a trend of increasing
462 diffusivity with height. It can also be seen that the four different normalizations yield
463 comparable clustering of the data. These normalised data were next averaged and
464 smoothed using the three approaches described at the end of section III. This gave the
465 three averaged results shown in Figures 4(a) to (d), respectively. These averaged
466 profiles clarify significantly the form of the normalized sediment diffusivity with
467 height above the bed. Also, since the different averaging schemes give very
468 comparable results, the veracity of the final trends in the normalised sediment
469 diffusivity profile is considered to be high.

470

471 The final result, obtained using the normalization given by equation (15d), together
472 with averaging over the three means, is the profile shown in Figure 5 represented by
473 the large solid circles. The error bars shown on the final ε_s profile were not derived
474 from the three averages, but were calculated from the whole data set, shown by the
475 small solid dots in Figure 4, at each height above the bed. The data show
476 approximately constant normalised sediment diffusivity in the region below $z/k_s \approx 1.3$.
477 At heights greater than $z/k_s \approx 1.3$, $\varepsilon_s/U_0 k_s$ increases linearly, though above about $z/k_s > 3$,
478 the trend in the data becomes less clear due to increasing scatter, mainly arising from
479 taking the derivative of rather noisy low concentration data at these greater heights
480 above the bed. However, notwithstanding this increase in scatter with height, the data
481 clearly show a normalised sediment diffusivity that is approximately constant for
482 $z/k_s \leq 1.3$ and above which there is a linear increase with height.

483

484 Using equation (8), Nielsen's empirical prediction for the constant normalized
485 sediment diffusivity was calculated. This is shown by the dotted line in Figure 5 and
486 has a value of 0.016. This prediction is somewhat less than the presently inferred,
487 measured value of 0.029. The lower value given by equation (8) could indicate that
488 Nielsen's assumed value of $\beta = 4$ linking the sediment diffusivity to the eddy viscosity
489 should be larger, or that the constant term of 0.004 in equation (3) is somewhat
490 underestimated. In any event, the agreement between Nielsen's predictions and the
491 measurements is not considered to be unreasonable, given the accuracy of the
492 previously available data upon which equation (8) was based.

493

494 Considering next the Van Rijn formulation for the constant sediment diffusivity layer,
495 the value predicted by equations (9a & 10) is 0.028 which is very close to the value
496 obtained here. Given the limited measurements upon which equations (9a & 10) were
497 based, the agreement may be somewhat fortuitous. However, the main feature of a
498 near bed constant diffusivity, with a value close to both Nielsen's and Van Rijn's
499 predictions, does indicate that the present observed magnitude and form for the
500 sediment diffusivity is not unreasonable. Unlike the Nielsen formulation, the Van
501 Rijn one also involves a linearly increasing sediment diffusivity above $z/k_s > 1$. Using
502 equation (9b, 9c & 11) the predicted linear portion of the normalized sediment
503 diffusivity does not result in a single curve for the present normalization. Therefore,

504 rather than showing the calculations for each case, the bounds from the calculations
 505 are given by the two dashed lines. The spread is not large and is associated primarily
 506 with changes in the wave period, together with the assumptions implicit in the
 507 determination of ε_m via Equations (9b) and (11) which cannot be validated here.
 508 Again, given the limited data upon which equations (9-11) were based, the predictions
 509 are considered to be in reasonable agreement with the present data, though
 510 overestimating their value in the linear region. However, simply by increasing the
 511 lower layer thickness ζ_s from k_s to $1.3k_s$ brings the centre line of the linear predictions
 512 much closer to the observations.

513

514 To complete the comparison of predictions with observations, equation (12) has been
 515 evaluated using equation (13), and the result has then been normalised to yield:

516

$$517 \quad \frac{\varepsilon_s}{k_s U_o} = 0.763 \kappa \beta \sqrt{\frac{f_w}{2}} \frac{z}{k_s} \quad (17)$$

518

519 If equation (17) is evaluated using a mean value for f_w , from all the medium sand
 520 experiments, calculated using $k_s=25\eta_r(\eta_r/\lambda_r)$ in equation (13) and with $\beta=1$, the
 521 predictions for the sediment diffusivity ('x' in Figure 5) substantially overestimate the
 522 observed values in the linear region. However, if f_w is calculated using a flat bed
 523 approximation $k_s=2.5d_{50}$ based on the grain size, then equation (17), again with $\beta=1$,
 524 yields the line in Figure 5 represented by the '+' symbols. Evidently this latter
 525 outcome compares very favourably with the data in the linear region, with only a
 526 marginal underestimation of the diffusivity occurring. However, this result could be
 527 coincidental, since, from equations (12) and (13), \bar{u}_* has only a weak power
 528 dependence upon k_s of 0.26. In any event, what is clear is that the use of equation
 529 (12), with an equivalent roughness based on $k_s=25\eta_r(\eta_r/\lambda_r)$, significantly
 530 overestimates the present observations of sediment diffusivity.

531

532 Finally, in order to capture the behaviour of the diffusivity in this case involving the
 533 medium sand, simple expressions have been fitted to the present data set to yield
 534 empirical expressions for the variation of sediment diffusivity with height above the

535 bed. These expressions, which are consistent with those of both Nielsen and Van Rijn
 536 in the bottom layer and with Van Rijn in the linear layer above this, are as follows:

537

$$538 \quad \varepsilon_s = \xi_1 U_o k_s \quad z \leq 1.3k_s \quad (18a)$$

$$539 \quad \varepsilon_s = \xi_2 U_o z - \xi_3 U_o k_s \quad z \geq 1.3k_s \quad (18b)$$

540

541 where $\xi_1=0.029$, $\xi_2=0.028$, $\xi_3=0.007$ and the expression is given by the solid line in
 542 figure 5. Although it is acknowledged that the parameter space of the present study is
 543 relatively limited, it was considered of interest to put the above expressions forward,
 544 since they are compatible with the other formulations and suitable for comparison
 545 with diffusivities based on any new or emerging data sets.

546

547 To elucidate the processes underlying the form of the sediment diffusivity profile,
 548 both the bed forms and also the variation of suspended sediment concentration with
 549 the phase of the wave and height above the bed were examined. This takes advantage
 550 of the bed form measuring capability of acoustics and the high spatial and temporal
 551 resolution of suspension measurements also provided by acoustics. To illustrate the
 552 type of bed forms present on the medium sand, a typical measurement from the ARP
 553 is shown in Figure 6a. The plot shows the development of a transect, over a 17 min
 554 period, for the case of $T=5$ s and $H=0.81$ m. The ripples were well developed with
 555 mean dimensions of $\lambda_r=0.34$ m, $\eta_r=0.047$ m, and therefore slope of $\eta_r/\lambda_r=0.14$. This
 556 was typical for the medium sand, with η_r and λ_r lying respectively in the range 0.04 –
 557 0.06 m and 0.26 – 0.51 m and with $\eta_r/\lambda_r=0.12$ -0.15. Plots of the ripple slopes and
 558 equivalent roughness, given by $k_s=25\eta_r(\eta_r/\lambda_r)$, are shown in Figures 6b and 6c. The
 559 ripple slopes had a mean value of 0.14 which implies that vortex formation and
 560 entrainment should have occurred (Sleath, 1984). Also the roughness of the bed is
 561 quite large, around 0.17 m, indicating that the bed is having a major impact on the
 562 near-bed flow.

563

564 To assess the mechanisms of sediment entrainment directly over the medium sand,
 565 intra-wave processes were investigated. The results are shown in Figure 7; here the
 566 intra-wave height variation of the ripple-averaged suspended sediment concentration,
 567 has been constructed using ensemble wave phase-averaging with an 18° interval over

568 200 wave cycles as the ripple slowly migrated below the ABS over the 17 min
569 recording period. The wave conditions were $H=1.06$ m and $T=5$ s. It can be seen
570 clearly that there are two main entrainment events and that these occur close to flow
571 reversal; they do not coincide with maximum flow. Further analysis of this data
572 (Thorne et al., 2003; Davies and Thorne, 2005) supported the concept that the
573 observations shown in Figure 7 can be interpreted as arising from flow separation on
574 the lee slope of the ripple, with the consequent generation of growing lee slope
575 vortices (Sleath, 1982; Hansen et al., 1991; Vincent et al., 1999; van der Werf et al.
576 2007). The vortices, while attached to the bed, entrain sediment and become sediment
577 laden. Then, near flow reversal, they are lifted up into the water column, carrying
578 sediment away from the bed. The processes are not random, but are repeatable and
579 coherent. Importantly, the layer in which these effects occur may be seen to
580 correspond to several ripple heights in thickness.

581

582 The intra-wave observations in Figure 7 may be related to the sediment diffusivity
583 profile in Figure 5 in the following way. Due to the formation of vortices on the
584 ripple lee slopes, suspended sediments were contained within a relatively fixed
585 mixing region, of height comparable with the ripple height η_r , for most of the wave
586 cycle. Near flow reversal the vortices were lifted up into the water column, retaining
587 their structure to a height of the order of k_s . This is consistent with the detailed flow
588 measurements made by Ranasoma and Sleath (1992) who concluded that vortex
589 shedding effects dominate the dynamics in a near-bed layer of thickness at least one
590 or two ripple heights above the ripple crest level. It is the associated coherence of
591 sediment entrainment and structure that leads to the constant value for the sediment
592 diffusivity within about $z/k_s \leq 1.3$ ($3\eta_r-4\eta_r$ for the medium sand). At heights greater
593 than $z/k_s \approx 1.3$, the coherent structure of the vortices breaks down, with mixing of
594 momentum increasingly becoming dominated by random turbulent processes
595 (Ranasoma and Sleath, 1992). Here, therefore, gradient diffusion dominates and
596 mixing increases due to an increase in the mixing length scale with height above the
597 bed, leading to the linear increase in sediment diffusivity above the vortex layer.

598

599

600 (ii) Fine sand

601 Again using equation (14), ϵ_s was calculated for the fine sand bed, using the ABS
602 concentration profiles together with the pumped sample particle size profiles. The
603 values for the suspended sediment size in this case varied from around $d_{50s}=125 \mu\text{m}$
604 within a centimetre or two of the bed, to about $d_{50s}=90 \mu\text{m}$ at 1 m above the bed. As
605 with the medium sand the change in particle size with height above the bed did not
606 strongly affect the form of the diffusivity profile. The results for the 21 sediment
607 diffusivity profiles, from the 7 experiments involving the fine sand, are shown in
608 Figure 8. Unlike the results for the medium sand, there does not appear to be a region
609 of constant sediment diffusivity just above the bed. In contrast, the sediment
610 diffusivity can be seen to increase from around $0.0002\text{-}0.0006 \text{ m}^2\text{s}^{-1}$ close to the bed,
611 to values in the region of $0.003\text{-}0.01 \text{ m}^2\text{s}^{-1}$ at 0.8 m above the bed. These values for ϵ_s
612 are around one fifth of those for the medium sand near the bed, but are more
613 comparable in magnitude at about 0.8 m above the bed. As with the medium sand, the
614 scatter in the data increases with height above the bed, due to noisier lower
615 concentration levels at greater heights and due also to the different flow and bed
616 conditions associated with the different experiments. Following the same
617 methodology as described earlier, four normalisations and three averaging procedures
618 were applied to the sediment diffusivity profiles. The results are shown in Figure 9.
619 The different normalisations and averages give consistent results, particularly in
620 Figures 9b-9d which show no indication of a constant diffusivity near-bed layer, but
621 instead exhibit a sediment diffusivity that increases linearly with height above the bed.
622 As with the medium sand, the final normalisation, namely equation 15(d), with a
623 mean taken from the three averaging schemes, was used to produce the final result
624 shown in Figure 10. This shows no indication of a near-bed constant sediment
625 diffusivity, associated in the medium sand measurements with vortex formation and
626 entrainment of sediments. Instead, the results show, in the near-bed region, that the
627 normalised sediment diffusivity increases linearly with height above the bed. Because
628 there is no obvious constant near-bed sediment diffusivity, no useful comparison can
629 be made with the formulations of Nielsen (Eq. (8)) or Van Rijn (Eq. (10)). However,
630 it is possible to compare Van Rijn's linearly increasing sediment diffusivity region
631 with the present data. If, in equation (9c), ζ_s and ϵ_b are set to zero, then using linear
632 wave theory in the determination of ϵ_m we have

633

634
$$\frac{\varepsilon_s}{k_s U_o} = \frac{2\alpha_m}{\pi} \sinh(kh) \frac{z}{k_s} \quad (19)$$

635

636

637 where k is the wave number of the surface waves. Using this expression and taking
 638 the mean value of k for all the fine sand experiments, the dashed line in Figure 10 is
 639 obtained. Evidently the resulting, predicted, normalised sediment diffusivity is
 640 comparable with the observed values, though it somewhat overestimates them.
 641 Reducing α_m from 0.035 to 0.022 brings Van Rijn's expression into line with the
 642 observations. Given the limited data set upon which equation (9c) is based, this
 643 adjustment does not seem unreasonable. Secondly, equation (12) expressed in the
 644 form of equation (17) was compared with the data. It is interesting to note that, if
 645 equation (17) is evaluated using $k_s=25\eta_r(\eta_r/\lambda_r)$ in equation (12), with $\beta=1$, as shown
 646 by the 'x' symbols in Figure 10 the predictions again significantly overestimate the
 647 observed values. However, if the flat bed approximation $k_s=2.5d_{50}$ is used, the line in
 648 Figure 10 represented by the '+' symbol is obtained, which can be seen to compare
 649 favourably with the data, with only a minor overestimation occurring. Given both
 650 these fine sand results and also those for the medium sand, it does appear to be the
 651 case that the use of $k_s=25\eta_r(\eta_r/\lambda_r)$, for a rippled bed, overestimates the roughness
 652 length substantially if equation (12) is used to calculate ε_s .

653

654 Finally if, as in the medium sand case, an empirical fit is made to the data, forcing
 655 $\varepsilon_s=0$ at $z=0$, then the following expression results:

656

657
$$\varepsilon_s = \chi_1 U_o z \quad (20)$$

658

659 where $\chi_1=0.017$. This is comparable, though a somewhat smaller gradient than that for
 660 the linearly increasing region of the sediment diffusivity in the medium sand case.

661

662 To explain the form of the sediment diffusivity over the fine sand and its difference
 663 from the medium sand, we have again looked at the bed forms. Figure 11 shows a
 664 typical example of the bed-forms, with associated plots of the ripple slopes and the
 665 equivalent bed roughness. Figure 11a shows the ripple formation for waves with $T=5$

666 s and $H=0.79$ m; these inputs are very comparable with the case shown in Figure 6a
667 for the medium sand. However, for the fine sand the ripples can be seen to be less
668 well developed and less coherent in form, with, in the case shown, $\eta_r=0.019$ m,
669 $\lambda_r=0.27$ m and $\eta_r/\lambda_r=0.07$. This was typical of all the experiments, with η_r and λ_r
670 respectively being in the ranges $0.01 - 0.03$ m and $0.15 - 0.84$ m and, as shown in
671 Figure 11b, with $\eta_r/\lambda_r= 0.06-0.09$. For this range of slopes no significant flow
672 separation or vortex formation is expected to occur (e.g. Sleath 1984). Therefore,
673 although the ripples enhanced the bed roughness somewhat, they acted on the flow
674 dynamically like a plane bed. As seen in Figure 11c, the equivalent roughness of the
675 bed, if based upon $k_s=25\eta_r(\eta_r/\lambda_r)$, would be just over a quarter that of that in the
676 medium sand case, indicating that the impact of the bed on the flow is restricted to a
677 region much closer to the bed than for the medium sand. However, the roughness of a
678 dynamically plane bed is more appropriately defined simply in terms of the sediment
679 grain size, as discussed earlier with reference to Figure 10.

680

681 To assess the impact of ripples of low slope on sediment entrainment, the variation of
682 the suspended sediment with the phase of the wave and the height above the bed of
683 fine sand was examined. As in the case of the medium sand, the result was
684 constructed using ensemble wave phase averaging over 200 wave cycles. An example
685 of the results is shown in Figure 12 for the following wave conditions: $H=0.82$ m and
686 $T=5$ s. The structure of the intra-wave suspended sediments is seen to be quite
687 different from that shown in Figure 7; there are no significant suspension events near
688 flow reversal lifting sediment well up into the water column. High concentrations are
689 confined to a relatively thin layer within a few centimetres of the bed and the variation
690 in the suspended load seems to be only weakly dependent on the phase of the wave,
691 with only marginal increases in suspended concentration levels at maximum flow
692 speed. The results in Figure 12 indicate that the bed is behaving dynamically more
693 like a plane bed, rather than a bed that is inducing vortex formation and entrainment.
694 Therefore, the lack of a constant sediment diffusivity region in the fine sand case is
695 not surprising, since the conditions for vortex entrainment were not present and it is
696 the formation of vortices which are considered to be the underlying process leading to
697 the constant sediment diffusivity region. For the fine sand case it is considered that the
698 dynamics are comparable with the classical flat bed situation and that turbulent

699 processes dominate the near-bed sediment entrainment. In this case the turbulent
700 eddies are considered to grow with height above the bed (Davies and Villaret, 1997),
701 leading to the linear increase in sediment diffusivity measured in this study over the
702 bottom quarter of the water column.

V Discussion

703

704

705 The aim of the present study was to contribute to the detailed understanding of the
706 form of the sediment diffusivity profile above a rippled bed and elucidate the
707 underlying mechanisms that give rise to the ϵ_s -profile. This was stimulated by the
708 contrasting descriptions of sediment entrainment due to the processes of gradient
709 diffusion and vortex shedding, the former being associated with random turbulence
710 and the latter with repeatable coherent structures. The gradient diffusion process,
711 where the ‘mixing length’ is considered small compared with the vertical extent of the
712 concentration profile, is readily represented via the concept of a sediment diffusivity;
713 in contrast, the vortex shedding process cannot be so directly associated conceptually
714 with a diffusion rate dependent upon the concentration gradient. The present work
715 was aimed at examining the relationship between the different processes and their
716 widely used representation via the formulation of a sediment diffusivity profile.

717

718 The occurrence of vortex shedding in the oscillatory boundary layer above ripples
719 depends upon the ripple steepness and, more subtly, on the detailed shape of the ripple
720 crests. Roughly speaking, vortex shedding is expected to occur if η_r/λ_r is greater than
721 about 0.12 and ‘dynamically plane’ bed conditions are expected if the steepness is less
722 than about 0.08. In the Deltaflume experiments reported here, the ripple steepness
723 above the medium and fine sands was consistently close to 0.14 and 0.07,
724 respectively, suggesting that in the medium sand case vortex shedding was occurring
725 while in the fine sand case it was not. This proposition was confirmed by the intra-
726 wave observations described in the previous section, and was translated into the
727 contrasting forms found for the respective diffusivity profiles, namely ‘constant +
728 linear’ for the medium sand and ‘linear’ for the fine sand. The reason why ripples of
729 different steepness were generated by essentially the same wave conditions is beyond
730 the scope of the present paper. The difference between the medium and fine sand sizes
731 may have given rise, for example, to some different combination of bed load and
732 suspended load processes that promoted ripple development in the medium sand case
733 and inhibited it in the fine sand case. Probably, in the latter case, the relatively larger
734 amount of suspended sediment gave rise to settling patterns over the ripple surface
735 that counteracted any tendency for the ripples to grow (see O'Donoghue et al 2006 for

736 more detailed discussions). In any event, the two sand sizes highlighted in this paper
737 exemplified very clearly the consequences of the ripple steepness for the mixing
738 processes in the wave boundary layer, which form a key part of the complex ‘triad of
739 interactions’ between the oscillating flow, the bed forms and the sediment transport
740 processes.

741

742 With regard to the modelling framework introduced in Section II and Table 1, it is
743 interesting to note that the experiments conducted here had Reynolds numbers in the
744 same approximate range; $RE \sim 3.2 \cdot 10^4 - 2.1 \cdot 10^5$ for the medium sand $2.8 \cdot 10^4 - 1.4 \cdot 10^5$
745 for the fine sand. However the relative roughness, A_0/k_s , in the medium and fine sand
746 cases was significantly different. In the medium sand case, with $k_s = 25\eta_r(\eta_r/\lambda_r)$, A_0/k_s
747 lay in the range 1.3-3.1, while for the fine sand, with $k_s = 2.5d_{50}$, A_0/k_s lay in the range
748 470 -1060. The expected ‘bed form characteristics’ in Table 1 are necessarily well
749 matched with the respective A_0/k_s values in the experiments, by the above choice of
750 dynamically based roughness. However Table 1 implies a rather oversimplified link
751 between A_0/k_s and RE, which is not borne out by the present observations, ie RE
752 values were comparable, while k_s differed by more than two orders of magnitude. As
753 explained by Davies and Villaret (1997) the two parameters need to be treated as
754 independent of one another, in a way that depends in practice on the triad of
755 interactions referred to above.

756

757 The relevance of achieving greater understanding of the sediment diffusivity above
758 bed forms is considerable. The prediction of the bed roughness still remains a central
759 obstacle in the accurate prediction of sand transport rates. As illustrated in the present
760 study, misinterpretation of the type of flow and/or misuse of the bed roughness k_s can
761 give rise to completely fallacious diffusivities and, hence, inaccurately predicted
762 concentration profiles. Here we have focussed only upon the ripple- and cycle-
763 averaged concentration profile and its interpretation. In terms of sand transport
764 prediction by waves or by wave+current flows this is simply the first step, since the
765 mean concentration profile can give information about only the ‘current related’
766 component of the transport. As noted by Davies and Thorne (2005) this component
767 may be only a relatively small part of the total transport comprising also the ‘wave-
768 related’ component that depends upon intra-wave processes. They noted further how
769 intra-ripple processes must be invoked in order to understand the mechanisms giving

770 rise to the observation that values of β ($=\varepsilon_s/v_t$) are greater than unity above ripples
771 (Section II). Davies and Thorne (2005) suggested that, in some average sense above
772 a rippled bed, regions of high (or low) suspended concentration are correlated with
773 regions of high (or low) vertical velocity in a way that is different from the correlation
774 that exists between the horizontal and vertical components of velocity field. The
775 former correlation determines the sediment diffusivity ε_s while the latter correlation
776 determines the eddy viscosity v_t . While these complex issues remain as key
777 challenges for future work, the present study is believed to have elucidated a vital part
778 of the phenomenon of sediment dynamics above ripples. The results for the sediment
779 diffusivity ε_s presented here provide simple, critical tests for modelling systems. They
780 also lend strong support to research modelling approaches such as presented by
781 Davies and Thorne (2005) who used a two-layer diffusivity (including a height-
782 constant near-bed layer) to represent quite successfully detailed sediment
783 concentration profiles observed above steep ripples.
784

V1 Conclusions

785

786

787 Acoustic measurements have been presented of sediment diffusivity profiles above
788 sandy rippled beds under regular, weakly asymmetrical, waves. For the two beds
789 investigated, comprising medium and fine sand respectively, different mean
790 suspended sediment concentration profiles were observed. For the medium sand the
791 sediment diffusivity ε_s inferred from the concentration profiles was constant with
792 height above the bed up to a distance of approximately $z \approx 1.3k_s$. Above this ε_s
793 increased linearly with height. In order to explain the form of the diffusivity profile
794 an assessment was made of the ripples on the bed and the variation of the suspended
795 concentration with the phase of the wave. In the case of the medium sand, the
796 steepness of the ripples indicated that flow separation on the lee-side of the ripple
797 crest should be occurring. This was confirmed by the intra-wave suspended sediment
798 measurements, which yielded results consistent with vortex entrainment, with the
799 major inputs of sediment into suspension occurring around flow reversal. The
800 formation of the vortices led to a relatively constant mixing length, resulting in a
801 constant value for ε_s close to the bed. Above this region the vortices appeared to lose
802 their coherence, with gradient diffusion becoming dominant, characterised by the
803 mixing length scale growing and resulting in ε_s increasing with height above the bed.
804 In contrast, for the fine sand, the diffusivity, ε_s , was observed to increase linearly for
805 all heights above the bed, and no ' $\varepsilon_s = \text{constant}$ ' lower layer was present. Analysis of
806 the ripples and the intra-wave suspended sediment showed no evidence of flow
807 separation or vortex formation. In this case it was concluded that the bed was
808 behaving as 'dynamically plane', with turbulent eddies growing in size with height
809 above the bed, leading to the observed linear form for ε_s .

810

811 To compare the observed profiles of sediment diffusivity with previous empirical
812 results, the formulations of Nielsen (1992), Van Rijn (1993) and the standard
813 'constant stress layer' expression were assessed. Nielsen's prediction, for very rough
814 beds, of a constant value of ε_s in the near-bed layer, was confirmed for the medium
815 sand and found to have a value similar to that observed, though somewhat lower. The
816 Nielsen formulation was not applicable to the fine sand observations due to the
817 absence of coherent near-bed mixing processes. The Van Rijn expression for ε_s
818 captured with reasonable accuracy both the constant and also the linear diffusivity

819 regions for the medium sand. Applying the linear component of Van Rijn's
820 formulation to the fine sand gave a result in the outer layer that was comparable with,
821 though an overestimate of, the observed diffusivity. Comparison of the conventional
822 flat bed formulation, $\epsilon_s = \beta \kappa \bar{u}_* z$, with $\beta=1$ and $k_s=25\eta_r(\eta_r/\lambda_r)$ gave substantial
823 overestimates for the linear component of ϵ_s for both of the sands studied. However, if
824 $k_s=2.5d_{50}$ was used in the evaluation of ϵ_s predictions were obtained which were much
825 more comparable with the observations. It appears therefore that, in the medium sand
826 case, where steep ripples were observed, the sediment diffusivity in the outer layer,
827 i.e. above the vortex layer, scales approximately on the grain size associated with an
828 equivalent flat bed. For the low slope ripples in the fine sand the sediment diffusivity
829 behaved, both in form and also magnitude, as expected above a 'dynamically plane'
830 bed. Based on the present observations in the Deltaflume, new empirical formulae
831 have been proposed here for the sediment diffusivity above both steep and also low
832 ripples that may be used in the future by other workers.

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834

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1034 **Tables**

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Bed form characteristics	2D Steep Ripples	2D and 3D Transitional Ripples	Dynamically Plane Bed
Ripple steepness η_r/λ_r	$\eta_r/\lambda_r \geq 0.12$	$0.08 \leq \eta_r/\lambda_r \leq 0.12$	$\eta_r/\lambda_r \leq 0.08$
Relative roughness A_0/k_s	O(1)	O(1-10)	O(100-1000)
Reynolds number RE	O(10^3 - 10^4)	O(10^4 - 10^5)	O(10^6 - 10^7)
Peak Shields Parameter $\hat{\theta}$	$0.05 \leq \hat{\theta} \leq 0.2$	$0.2 \leq \hat{\theta} \leq 0.7$	$\hat{\theta} \geq 0.7$

1040

1041

1042 Table 1. Bed form characteristics related approximately to boundary layer flow

1043 characteristics.

1044 **Figure Captions**

1045

1046 Fig 1 Schematic of the instrumented tripod used for the measurements; STABLE-
1047 Sediment Transport And Boundary Layer Equipment. Shown is the triple frequency
1048 acoustic back scatter system, ABS, operating at 1 MHz, 2 MHz, and 4MHz, the
1049 acoustic ripple (bed) profiler, ARP, the pumped sampling heights and the
1050 electromagnetic current meters, ECMs.

1051

1052 Fig2. Measurements of burst averaged concentration profiles at the three frequencies
1053 for fine ($\times, *, \Delta$) and medium ($\circ, +, \square$) sands for a) $H=0.5$ m and b) $H=0.8$ m, both had
1054 periods of 5s.

1055

1056 Fig 3. All the measurements of the sediment diffusivity with height above the
1057 undisturbed bed level for the medium sand.

1058

1059 Fig 4 Measurements of the normalised sediment diffusivity (\cdot), with three estimates
1060 of the mean; \circ filtered, Δ median, $+$ trimmed, with normalised height above the
1061 medium sand bed.

1062

1063 Fig 5. Comparison of the mean measured normalised sediment diffusivity (\bullet) over
1064 the medium sand bed, with the calculations from equations (8) (\cdots), (9-11) ($----$), (17)
1065 ($\times, +$, see text) and (18) ($---$).

1066

1067 Fig 6. a) Measurements for the medium sand bed of; a) a transect of the bed over time
1068 for an experimental run with $H=0.81$ m and $T=5$ s, b) the ripple slopes and c) the
1069 equivalent bed roughness for all experimental runs.

1070

1071 Fig 7. Measurement of the variation in concentration with the phase of the wave and
1072 height above the bed for the medium sand. a) The wave velocity at 0.31 m above the
1073 bed and b) the suspended sediment concentration. The wave conditions were $H=1.06$
1074 m and $T=5$ s.

1075

1076 Fig 8. All the measurements of the sediment diffusivity with height above the
1077 undisturbed bed level for the fine sand.

1078

1079 Fig 9. Measurements of the normalised sediment diffusivity (\cdot), with three estimates
1080 of the mean; \circ filtered, Δ median, $+$ trimmed with normalised height above the fine
1081 sand bed.

1082

1083 Fig 10. Comparison of the measured normalised sediment diffusivity (\bullet) over the fine
1084 bed, with the predictions from equations (19) ($- -$), (17) ($x, +$ see text) and (20) ($-$)

1085

1086 Fig 11. a) Measurements for the fine sand bed of; a) a transect of the bed over time
1087 for an experimental run with $H=0.79$ m $T=5$ s, b) the ripple slopes and c) the
1088 equivalent bed roughness for all experimental runs.

1089

1090 Fig 12. Measurement of the variation in concentration with the phase of the wave and
1091 height above the fine bed. a) The wave velocity at 0.31 m above the bed and b) the
1092 suspended sediment concentration. The wave conditions were $H=0.82$ m and $T=5$ s.

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