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LINEARITY EXPERIMENTS WITH WORLD2

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R. & D. 73/42

SUMMARY

This paper describes a series of experiments designed to ascertain the effect of linear auxiliary equations on model behaviour. The experiments suggest that the shapes of the curves describing the auxiliary equations have a significant effect on model behaviour.

EXPERIMENTS

E 1. Procedure and results

The non-linear relationships displayed in Figures 1a, 1b and 1c were inspected and tentative linear approximations proposed. The approximations were separately tested on the model and the parameters adjusted to produce a model response which matched the standard run.

The equations resulting from this exercise are displayed in LIST1.

Approximations were not found for N4(NRFM), D3(DRFM), F3(FCM), and D1(DRMM). In the case of D3 it was considered that the curvature of the Forrester curve was such as to make experiment with a linear equation pointless. A satisfactory equation for N4 was not found. At one point it was considered that the reason was the continuing increase in the natural resource usage rate with increasing standard of living (Forrester's curve approaches exponentially a maximum NRFM of 4).

D1 and F3 possess similar forms in the original equations and it was considered that curvature was responsible for the difficulty in finding suitable linear approximations for these relationships.

The model response with 18 linear approximations for the auxiliary equations is reproduced in PLOT2.

Discussion

One would hesitate to use the modified model in this form. It would be instructive to know the manner in which the auxiliary equation independent variables vary during the standard run simulation.

E 2. Procedure and results

Table 1 presents the values of the auxiliary equation independent variables during the standard run. The variables are:-

F1	Food ratio
Z1	Pollution ratio
N2	Natural resource fraction remaining
C5	Capital investment ratio in agriculture
C1	Capital investment ratio
M	Material standard of living
C4	Crowding ratio
Q1	Quality of life from material
Q3	Quality of life from food

The variation of these variables is indicated in the auxiliary equation curves of Figures 2a, 2b and 2c. The broken lines describe the linear auxiliary equations used in the experiment. The equations appear in LIST2.

The response for the modified model is reproduced in PLOT3. Table 2 shows the manner in which the auxiliary equation independent variables change during the simulation.

Discussion

The model response differs significantly from that of WORLD2. The auxiliary equation independent variables are significantly different, particularly Z1, C4, C5 and Q1/Q3, c.f. Table 1 and Table 2.

The relationships plotted in Figures 1a, 1b and 1c are not precisely those used in the BASIC version of WORLD2. The BASIC version auxiliary equations are curves fitted to the Forrester relationships.

A further useful exercise may be plotting the BASIC curves over the limited dynamic range of the independent variables during the standard run, and estimating a new set of linear auxiliary equations.

E 3. Procedure and results

Figures 3a, 3b and 3c show the auxiliary equations used in the BASIC model of WORLD2. The dynamic ranges of the independent variables during the standard run simulation are indicated in these graphs and the broken line represents a linear approximation. Table 3 shows the variations in the auxiliary equation independent variables during the simulation.

Plot 4 is the model response.

Discussion

The model response still differs significantly from the standard run. It does appear as if the model is very sensitive to the unique shape of each of the relationships. One would certainly hesitate to employ any linear approximations in exercises to test the effect of policy changes, since such changes could produce a significant change in the level and range of the auxiliary equation independent variables.

The variation in the value of the auxiliary equation independent variables in most cases is much less than might be expected. This is possibly due to the aggregation of WORLD2.

It is possible that the model currently hides phenomena which would emerge from a simulation which identified the existence of the developed and underdeveloped worlds. Such a model might demonstrate rather more dramatic variations in material standard of living, crowding ratio, food ratio, and pollution ratio.

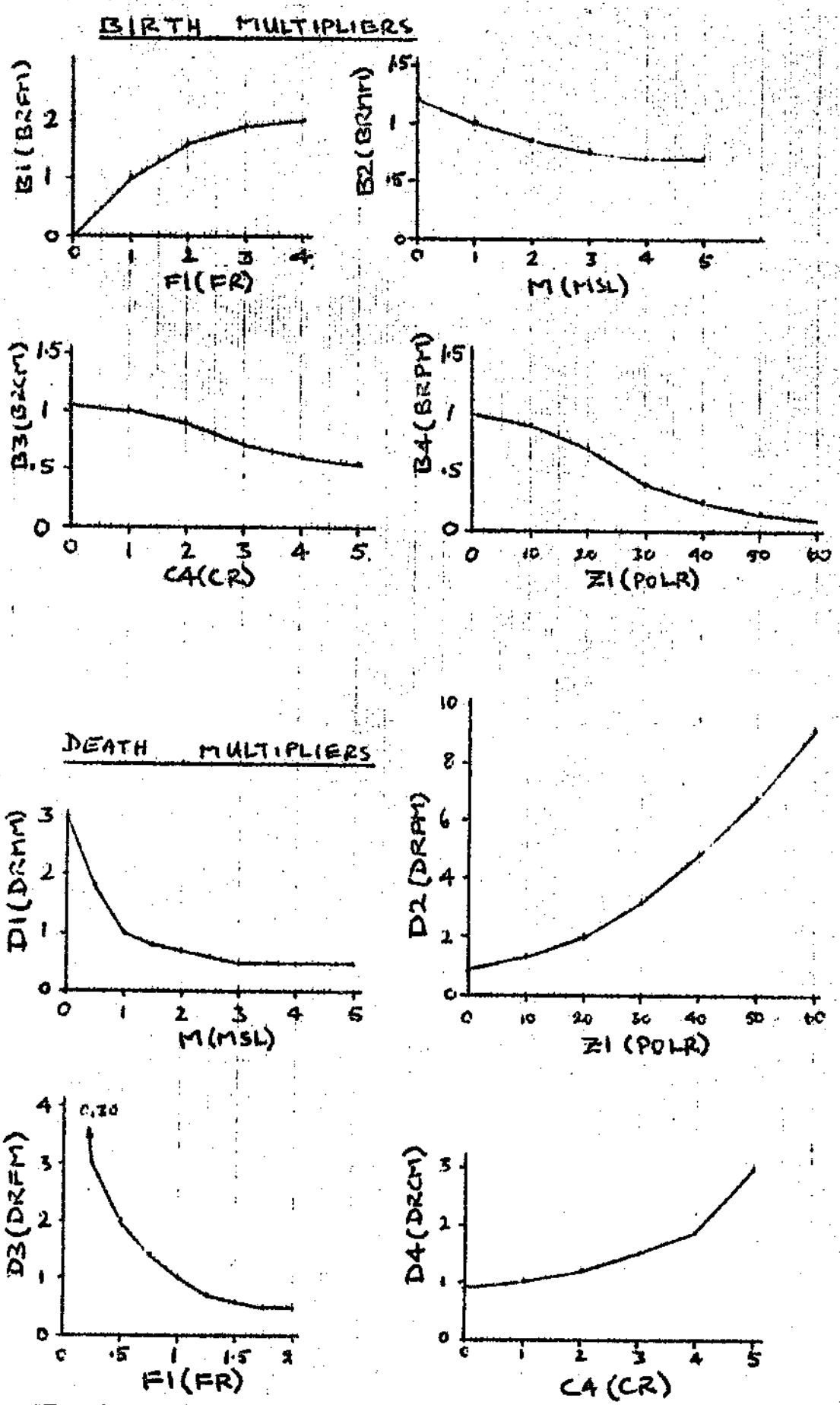


FIG 1a WORLD 2 NON-LINEAR RELATIONSHIPS

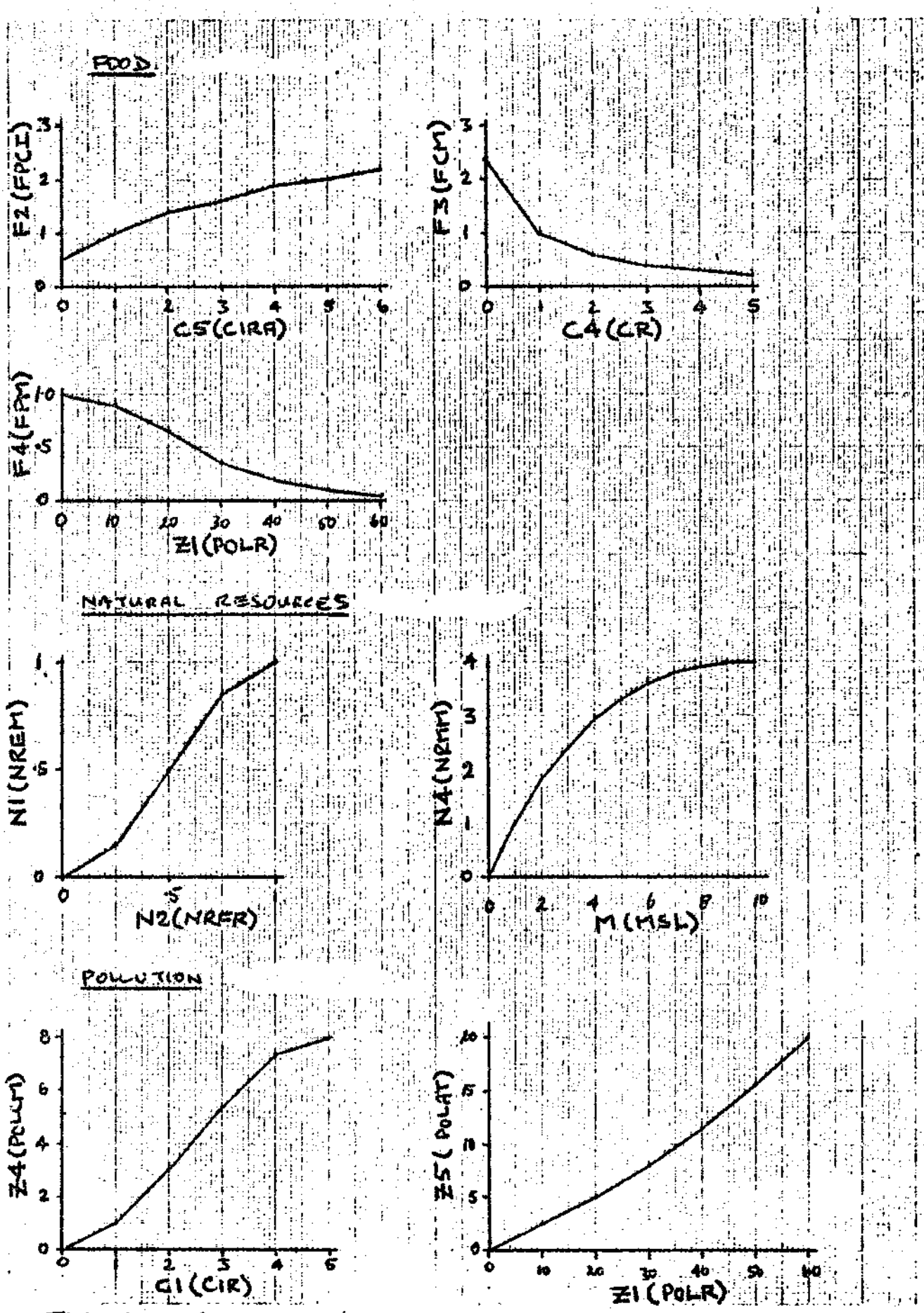


FIG 16. WORLD 2 NON-LINEAR RELATIONSHIPS

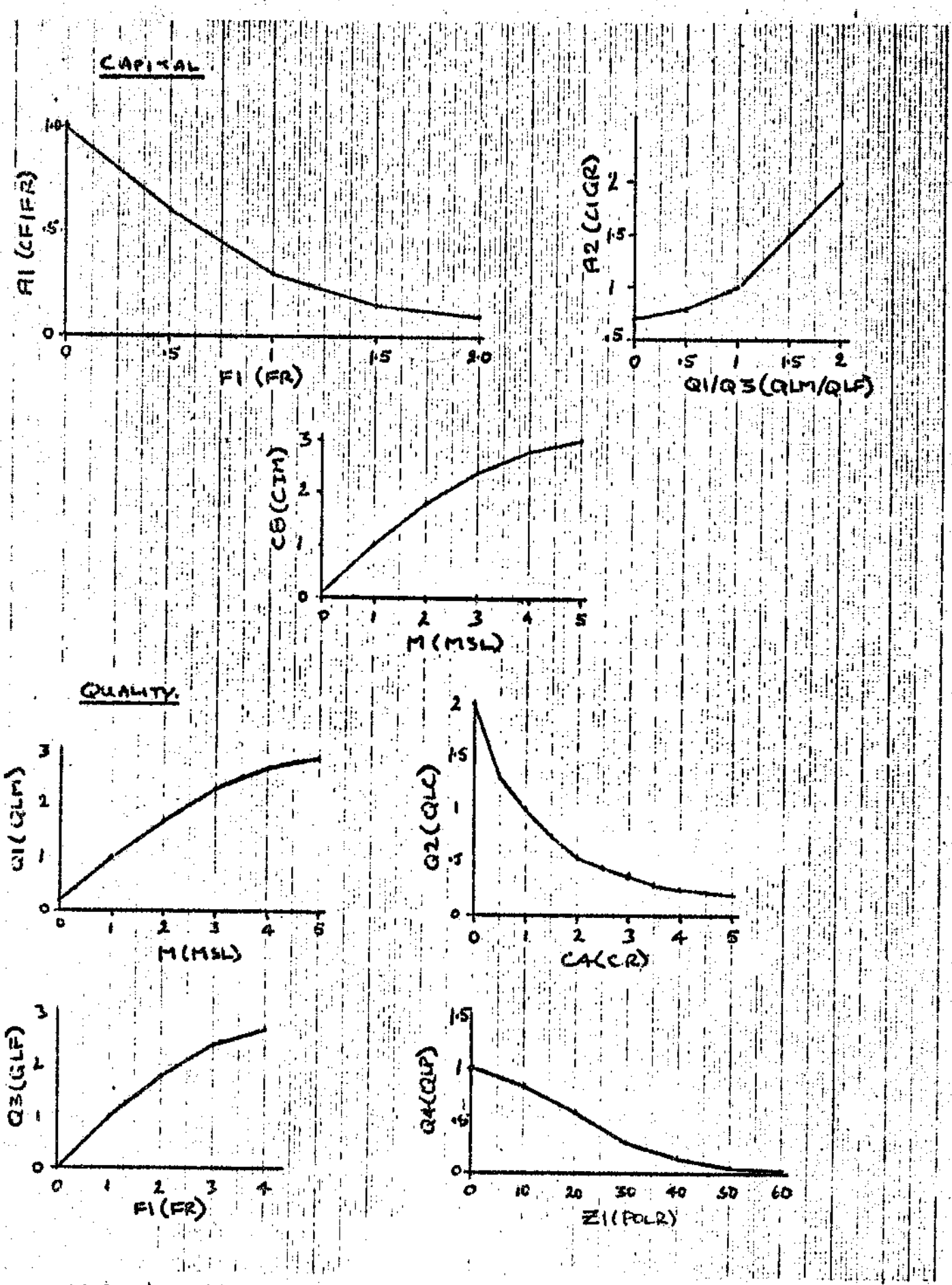


FIG. 1c. WORLD 2 NON-LINEAR RELATIONSHIPS

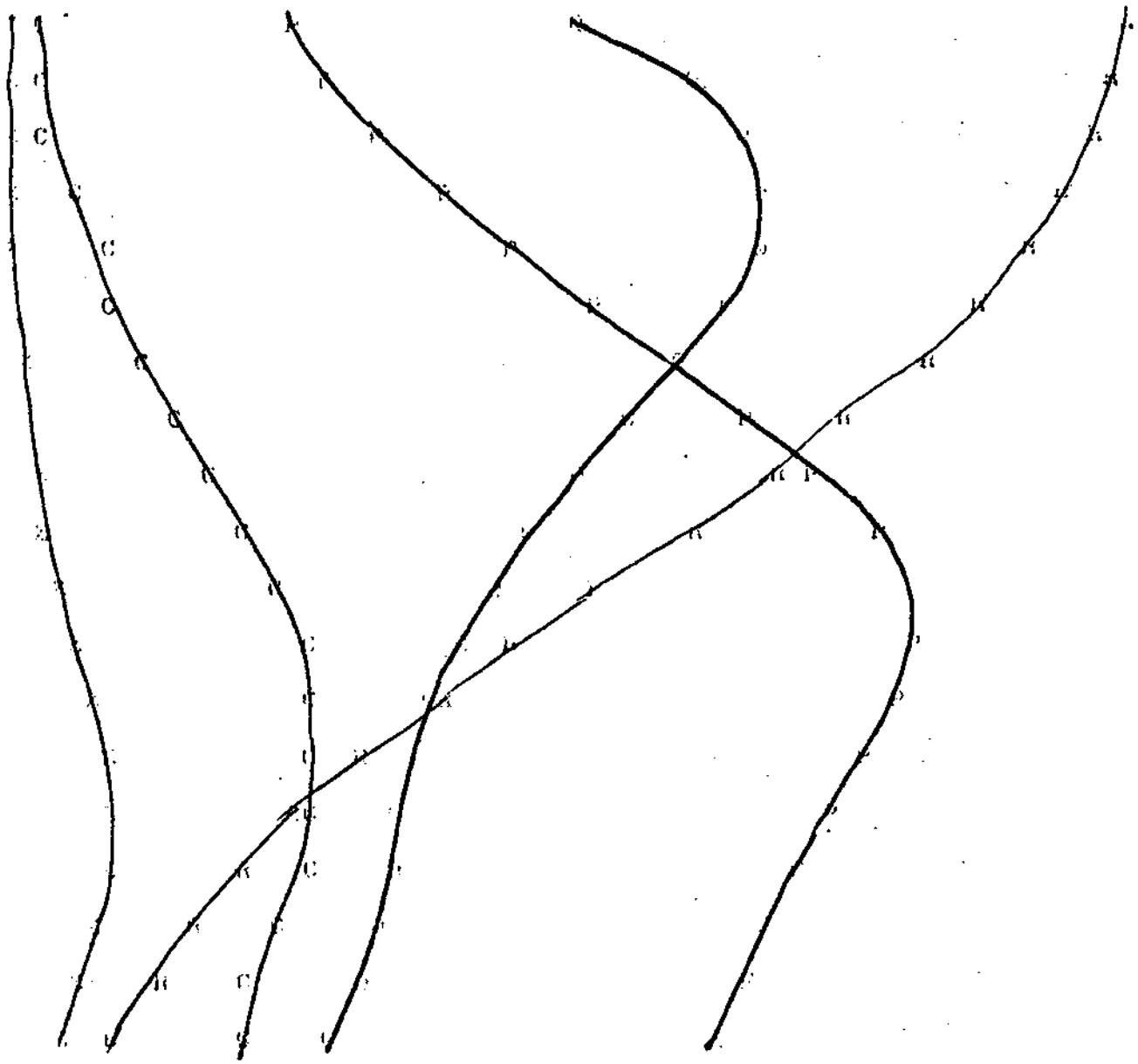
List 1. Main subroutine Experiment 1

```
READY

LIST 1000,1600
1000 REM MAIN SUBROUTINE
1010 LET Z1=Z/3.6E9
1020 LET F4=1.2-.02*Z1
1030 LET C7=C*0.025
1040 LET C1=C/P
1050 LET B4=1.03-.015*Z1
1060 LET C4=P/(L*L0)
1070 LET B3=1.1-.115*C4
1080 LET D4=.85+.105*C4
1090 LET D2=.8+.04*Z1
1100 LET N2=R/R0
1110 LET N1=1.08*N2
1120 LET E=C1*(1-C2)*N1/(1-C3)
1130 LET M=E/1.0
1140 LET B2=1.07-.09*M
1150 LET 04=1.05-.015*Z1
1160 LET 02=1.22-.3*C4
1170 LET Z5=.9+.135*Z1
1180 LET Z3=Z/Z5
1190 LET N4=0.114+M*(0.899-0.052*M)
1200 LET 01=.3+.8*M
1210 LET Z4=1.5*C1
1220 LET Z2=P*1*Z4
1230 LET C8=.28+.74*M
1240 LET C6=P*C8*0.05
1250 LET C5=C1*C2/C3
1260 LET F2=.55+.4*C5
1270 LET F3=2.40-2.158*C4+0.955*C4↑2-0.1955*C4↑3+0.01465*C4↑4
1280 LET F1=F2*F3*F4*F5/F6
1290 LET B1=.94*F1
1300 LET D3=3.8125-3.998*F1+1.19*F1*F1
1310 LET D1=0.5178+2.521*0.2186147↑M
1320 LET D=P*0.028*D1*D2*D3*D4
1330 LET N3=P*N5*N4
1340 LET B=P*0.04*B1*B2*B3*B4
1350 LET 03=.3+.72*F1
1360 LET 0=1*01*02*03*04
1370 LET A1=.7-.39*F1
1380 LET A2=.65+.43*01/03
1390 LET C2=C2+(1/15)*(A1*A2-C2)
1400 LET C=C+(C6-C7)
1410 LET R=R-N3
1420 LET P=P+(B-D)
1430 LET Z=Z+(Z2-Z3)
1490 RETURN
1600 END

READY
```

INITIAL CONDITIONS: 1900

POPULATION	1.6569
NATURAL RESOURCES	9.0000
CAPITAL INVESTMENT	0.4619
POLLUTION	0.0259
LAND AREA	135156

Plot 1. Standard Run

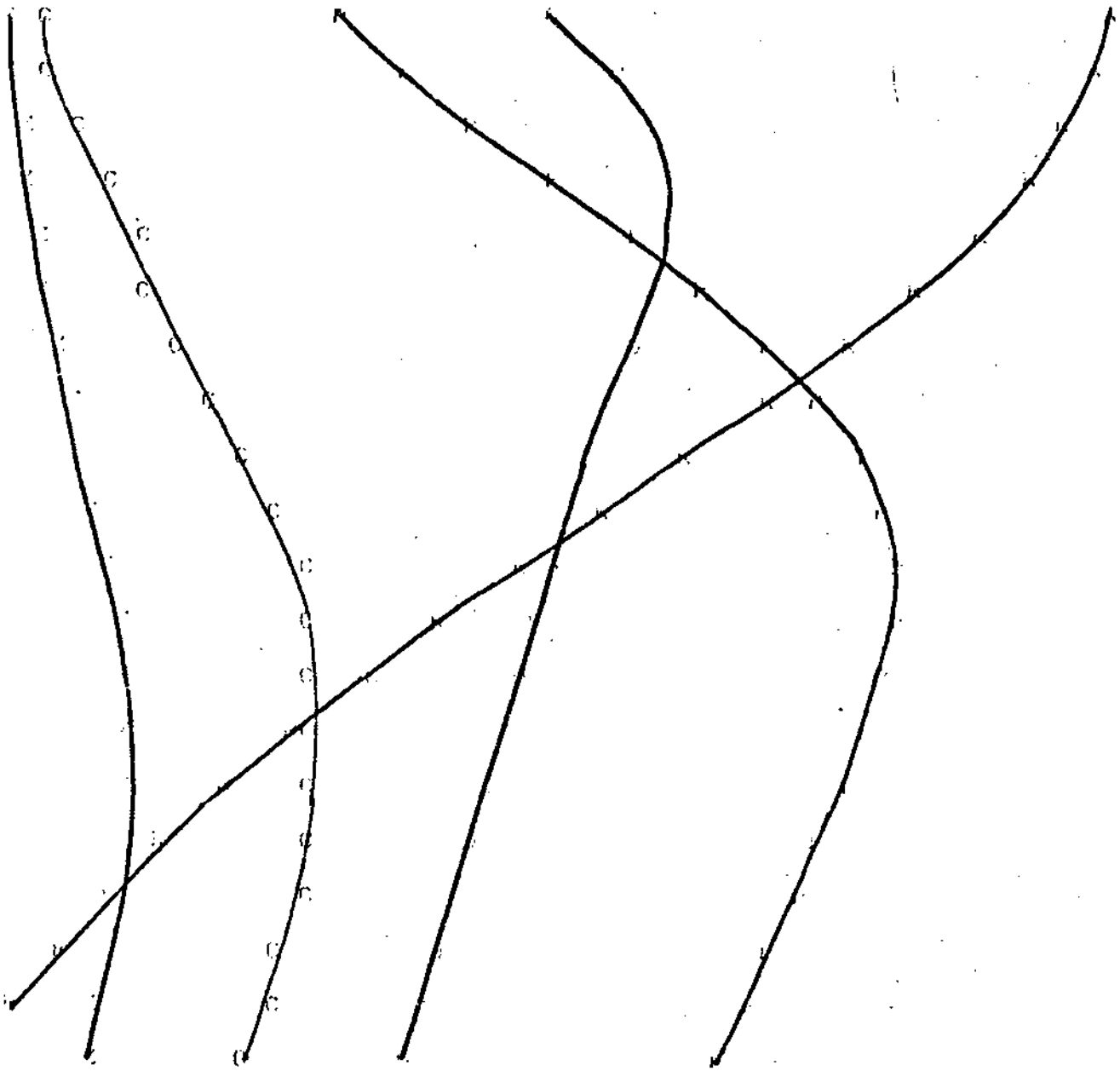


RUN

INITIAL CONDITIONS: 1900

POPULATION	1.65E9
NATURAL RESOURCES	9.0E9
CAPITAL INVESTMENT	0.4E9
POLLUTION	0.2E9
LAND AREA	135E6

Plot 2.



READY

[illegible]
RUN

READY

RUN

INITIAL CONDITIONS:1900

POPULATION	1.65E9
NATURAL RESOURCES	900E9
CAPITAL INVESTMENT	0.4E9
POLLUTION	0.2E9
LAND AREA	135E6

Table 1. Auxiliary independent variables for standard run

Z1	C4	N2	C1	M	C5	F1	01/03
.1	.46	.99	.36	.4	.3	1.06	.51
.1	.52	.98	.49	.5	.4	1.09	.6
.1	.6	.97	.6	.7	.5	1.08	.7
.2	.7	.95	.72	.8	.6	1.06	.8
.4	.82	.93	.84	.9	.7	1.04	.9
.6	.96	.89	.97	1	.9	1.01	.98
.8	1.09	.85	1.09	1	1.1	.99	1.05
1.2	1.23	.81	1.21	1.1	1.3	.97	1.09
1.8	1.34	.76	1.32	1.1	1.5	.96	1.1
2.4	1.44	.7	1.43	1	1.7	.95	1.1
3.3	1.49	.64	1.53	1	1.8	.95	1.07
4.2	1.51	.59	1.62	.9	1.9	.95	1.02
5.2	1.49	.53	1.71	.9	2	.96	.96
6	1.44	.49	1.78	.8	2	.97	.89
6.3	1.38	.44	1.82	.7	1.9	.99	.81
5.8	1.32	.4	1.83	.6	1.8	1.01	.73
4.8	1.27	.37	1.79	.6	1.7	1.02	.65
3.7	1.22	.34	1.71	.5	1.5	1.02	.58
2.8	1.17	.32	1.63	.4	1.4	1.02	.52
2.1	1.12	.3	1.54	.3	1.3	1.03	.47

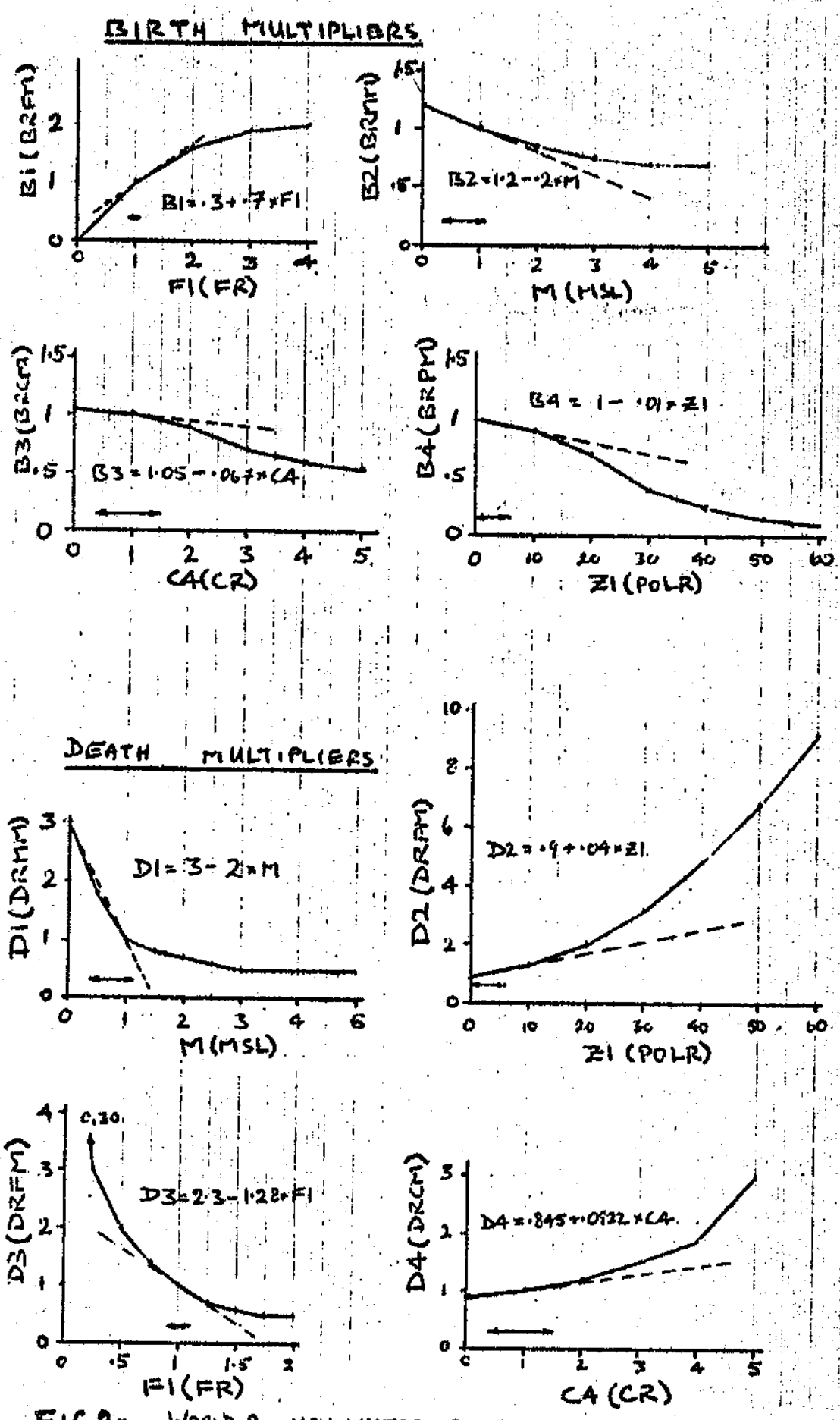


FIG 2a WORLD 2 NON-LINEAR RELATIONSHIPS

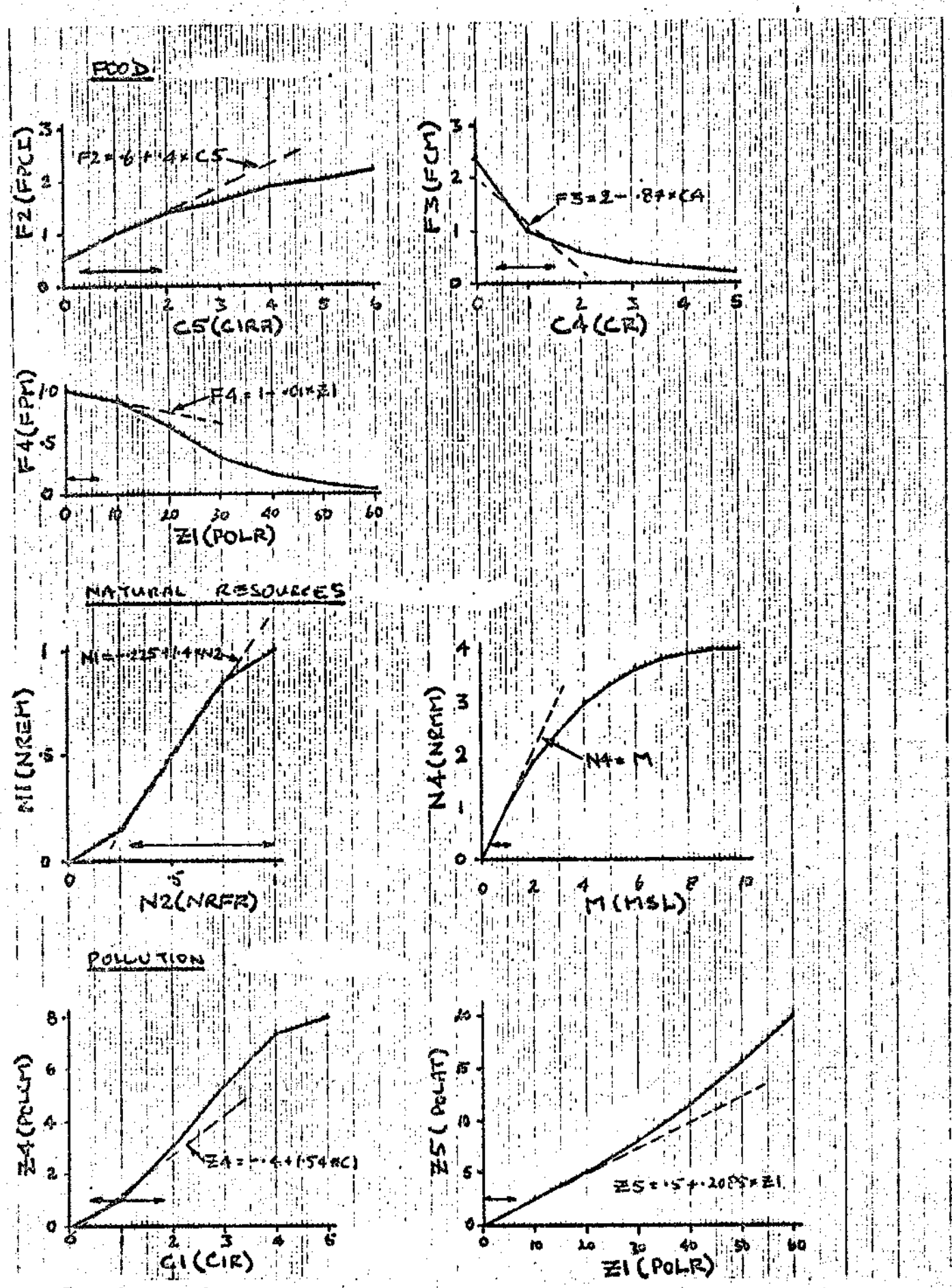


FIG 2b WORLD 2 NON-LINEAR RELATIONSHIPS

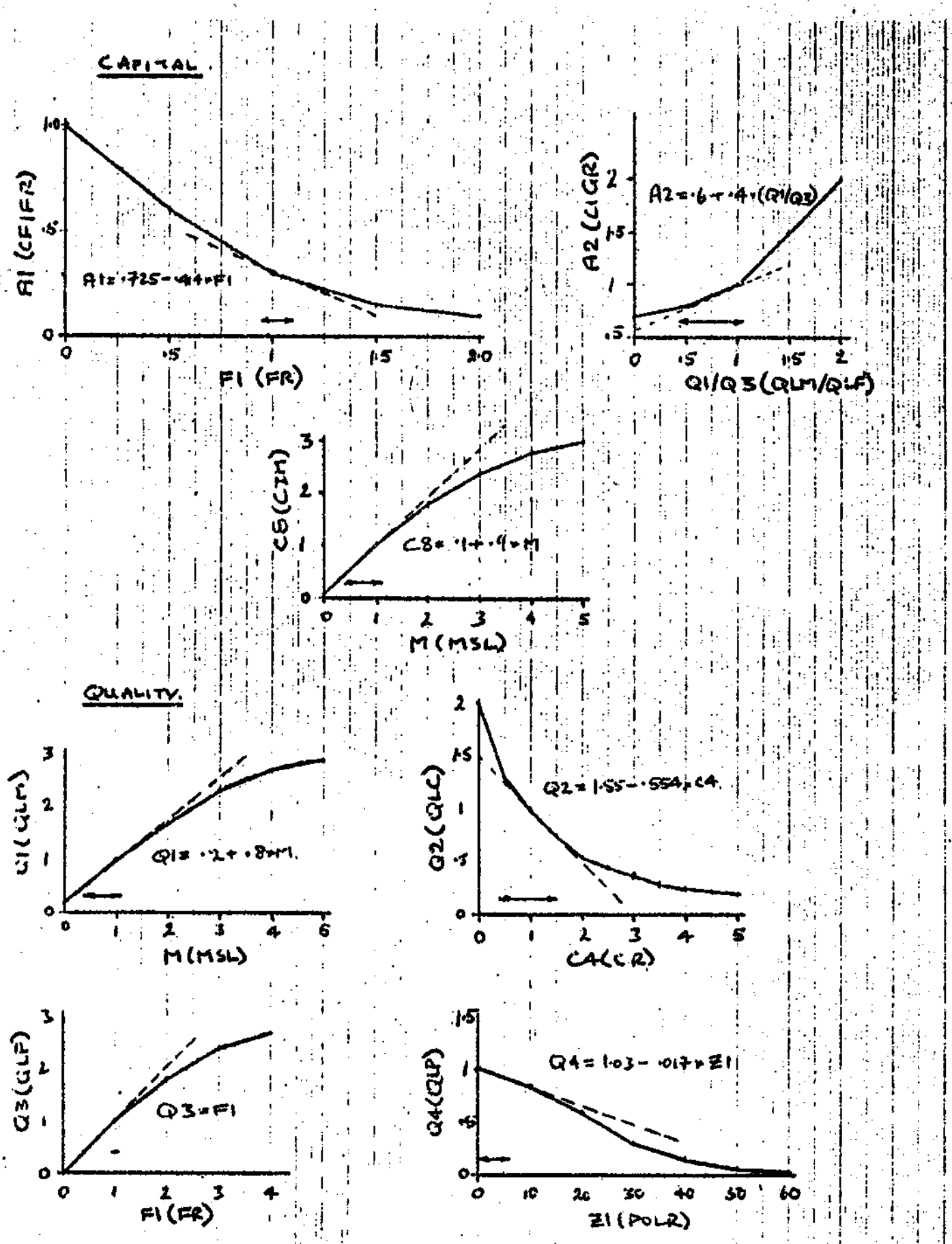


FIG. 2c. WORLD 2 NON-LINEAR RELATIONSHIPS

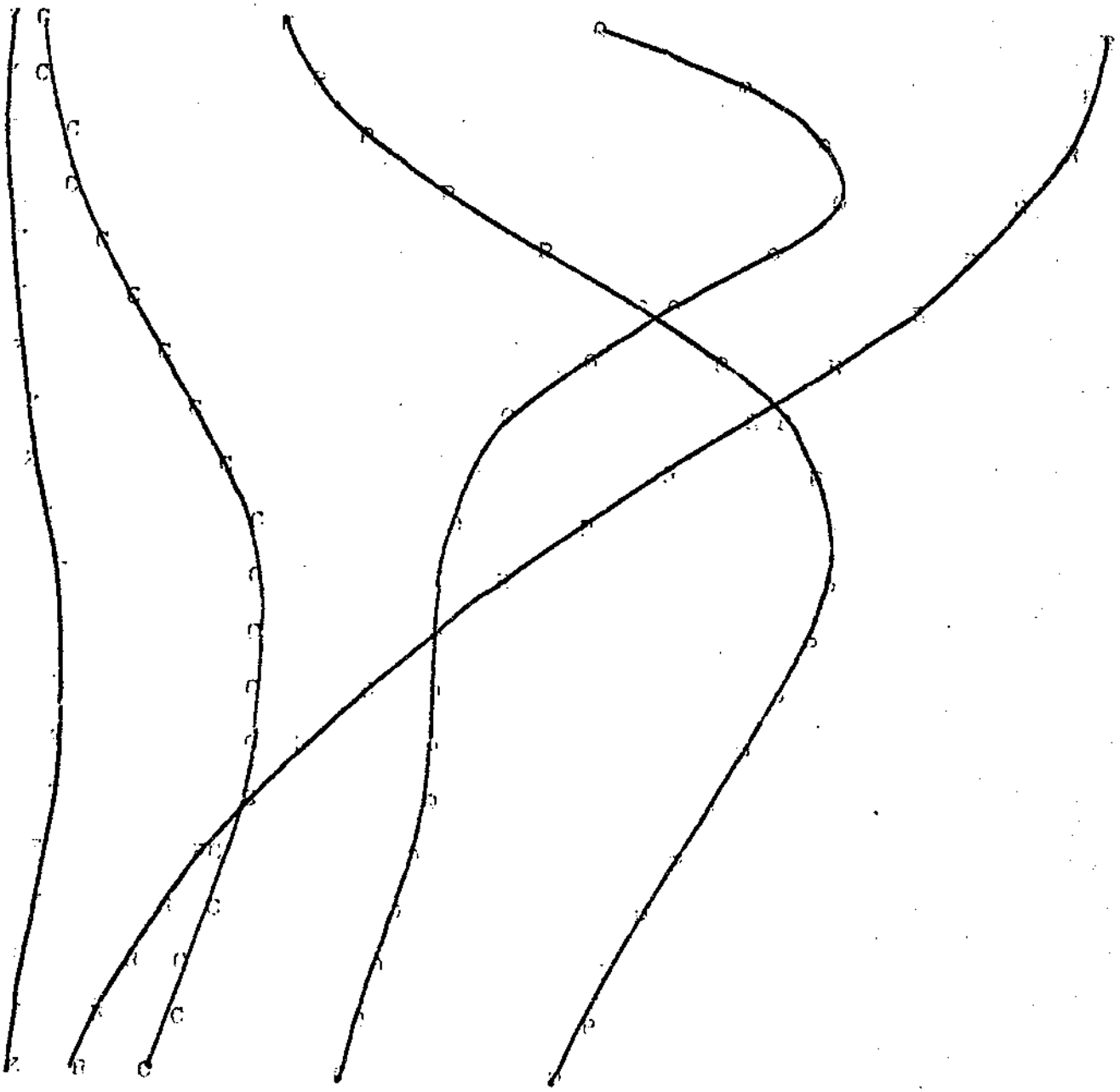
List 2. **Main subroutine Experiment 2**

```
LIST 1020,1380
1020 LET F4=1-Z1*.01
1030 LET C7=C*0.025
1040 LET C1=C/P
1050 LET B4=1-Z1*.01
1060 LET C4=P/(L*L0)
1070 LET B3=1.05-C4*.067
1080 LET D4=.9+C4*.133
1090 LET D8=.9+Z1*.04
1100 LET N2=R/B0
1110 LET N1=-.025+N2*1.2
1120 LET E=C1*(1-C2)*N1/(1-C3)
1130 LET M=E/1.8
1140 LET B2=1.2-M*.2
1150 LET Q4=1.03-Z1*.017
1160 LET Q2=1.55-C4*.554
1170 LET Z5=.5+Z1*.2085
1180 LET Z3=Z/Z5
1190 LET N4=4
1200 LET Q1=.2+M*.8
1210 LET Z4=-.4+C1*1.54
1220 LET Z2=P*1*Z4
1230 LET C8=.1+M*.9
1240 LET C6=P*C2*0.05
1250 LET C5=C1*C2/C3
1260 LET F2=.6+C5*.4
1270 LET F3=2.0-C4*.87
1280 LET F1=F2*F3*F4*F5/F6
1290 LET B1=.3+F1*.7
1300 LET D3=2.3-F1*1.28
1310 LET D1=3.0-M*2.0
1320 LET D=P*0.028*D1*D2*D3*D4
1330 LET M3=P*M5*N4
1340 LET B=P*0.04*B1*B2*B3*B4
1350 LET Q3=F1
1360 LET Q=1*Q1*Q2*Q3*Q4
1370 LET A1=.725-F1*.414
1380 LET A2=.6+(Q1/Q3)*.4
```

INITIAL CONDITIONS:1900

POPULATION	1.65E9
NATURAL RESOURCES	9[illegible]E9
CAPITAL INVESTMENT	0.4E9
POLLUTION	0.2E9
LAND AREA	135E6

Plot 3.



INITIAL CONDITIONS:1900

POPULATION	1.65E9
NATURAL RESOURCES	900E9
CAPITAL INVESTMENT	0.4E9
POLLUTION	0.2E9
LAND AREA	135E6

Table 2. Auxiliary equation independent variables for Experiment 2

Z1	C4	N2	C1	N	C5	F1	01/03
0	.45	.99	.36	.4	.2	1.12	.46
.1	.48	.99	.48	.5	.3	1.15	.54
.1	.54	.97	.59	.6	.4	1.16	.61
.2	.63	.96	.69	.7	.5	1.15	.68
.3	.73	.94	.79	.8	.6	1.13	.74
.5	.84	.91	.88	.8	.7	1.1	.8
.6	.95	.88	.98	.9	.8	1.07	.85
.9	1.05	.85	1.07	.9	.9	1.04	.88
1.1	1.13	.81	1.16	.9	1	1.02	.9
1.4	1.2	.77	1.24	.9	1.2	1.01	.9
1.7	1.24	.72	1.33	.9	1.3	1.01	.89
2	1.26	.68	1.4	.8	1.4	1.01	.86
2.2	1.26	.64	1.46	.8	1.4	1.02	.83
2.4	1.25	.6	1.51	.8	1.4	1.04	.79
2.4	1.23	.56	1.54	.7	1.4	1.06	.74
2.4	1.2	.53	1.56	.7	1.4	1.07	.69
2.3	1.16	.5	1.56	.6	1.3	1.09	.64
2.1	1.12	.47	1.55	.6	1.2	1.1	.6
1.9	1.08	.45	1.52	.5	1.2	1.11	.56
1.7	1.04	.43	1.47	.5	1.1	1.12	.52

READY

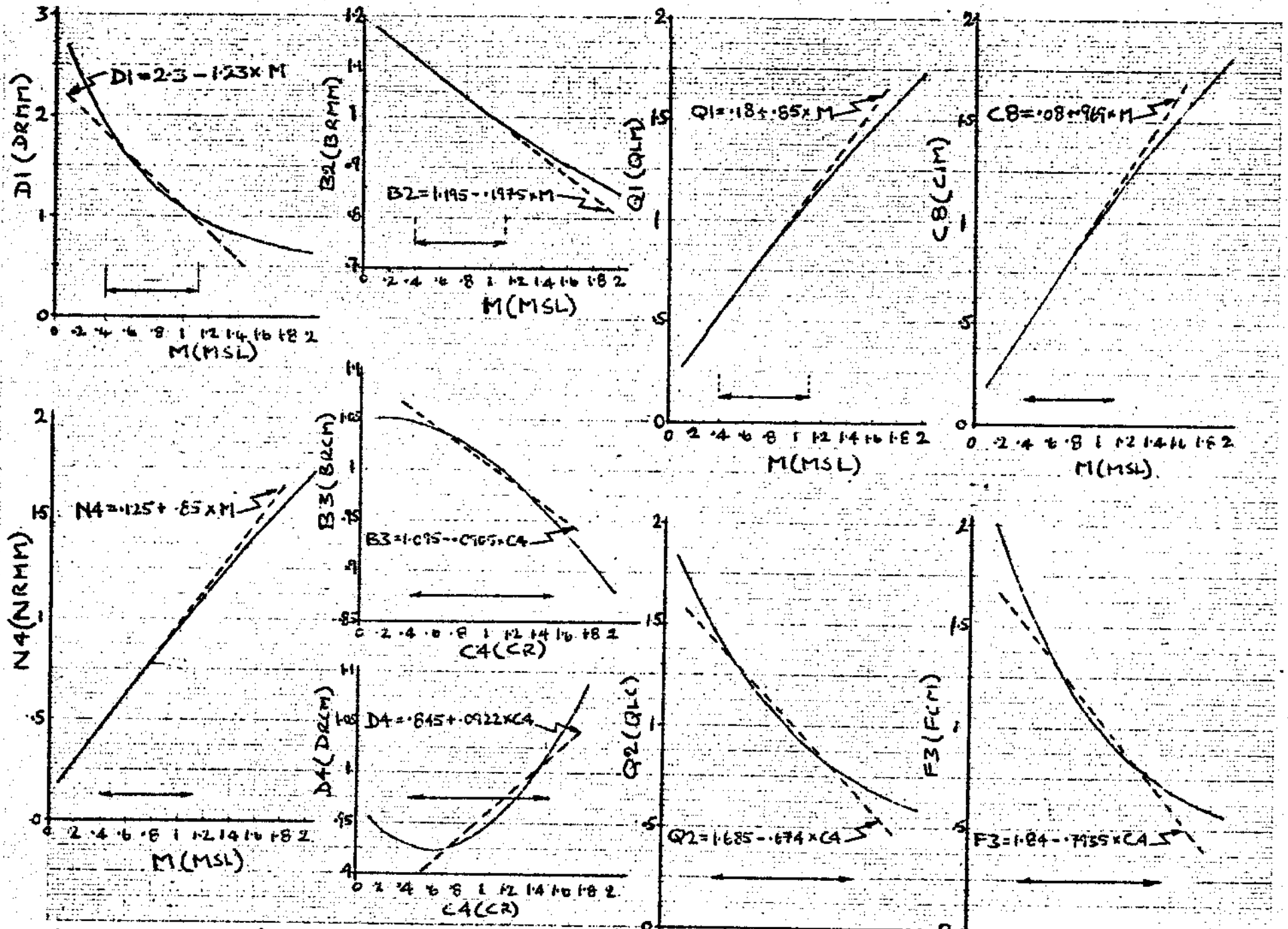


FIG 3a. Auxiliary Equation Fitted Curves

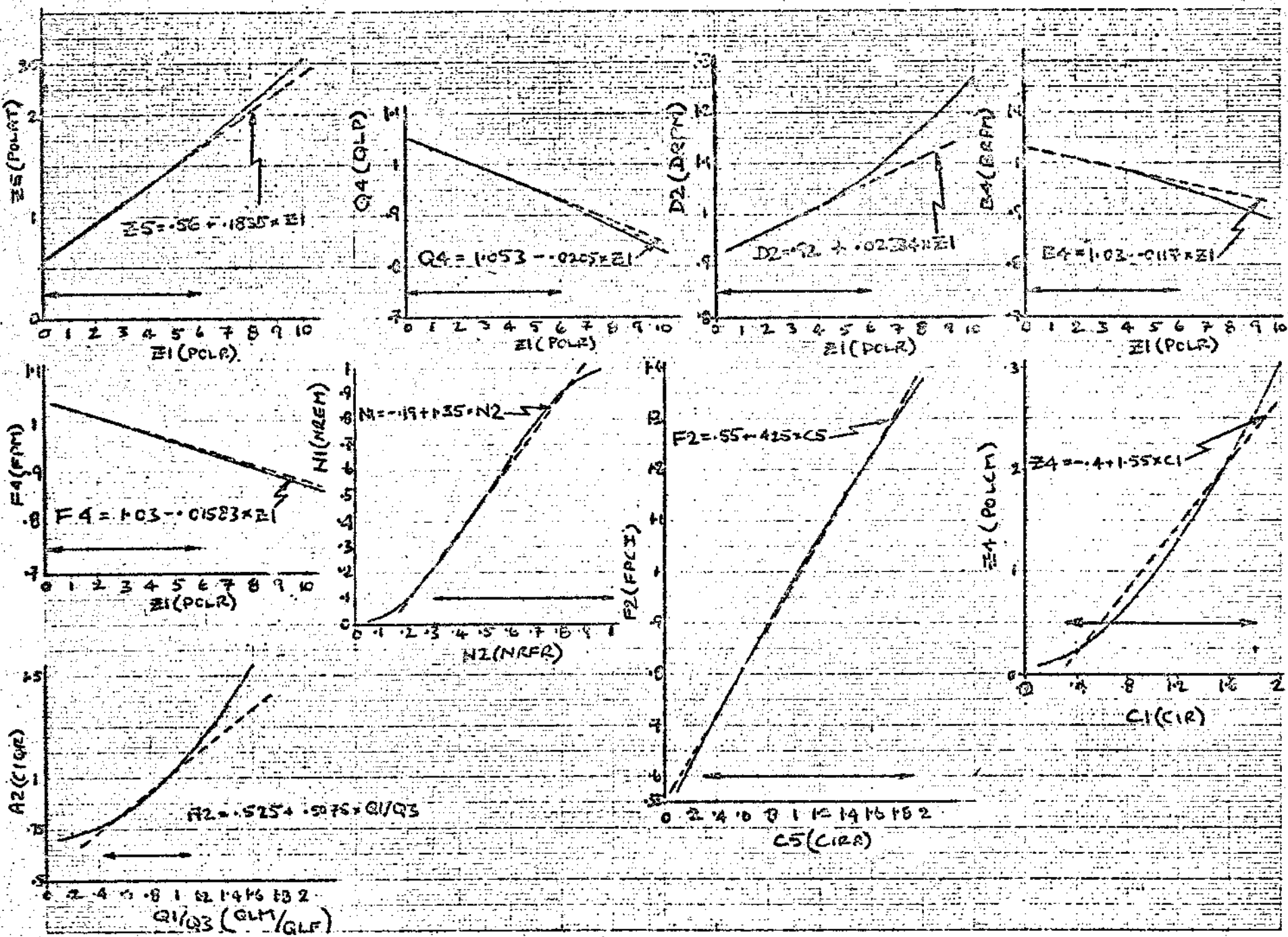


FIG 36 Auxiliary Equation Fits and Curves

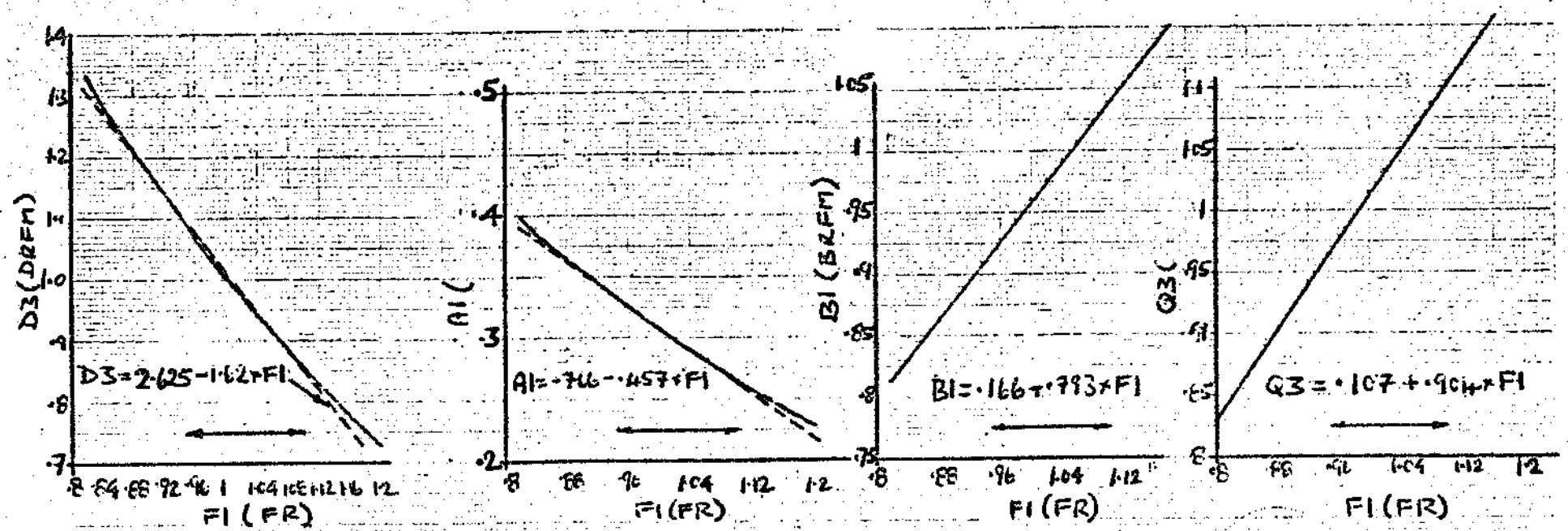


FIG 3c. Auxiliary Equation Fitted Curves

READY

```
LIST 1000,1600
1000 REM MAIN SUBROUTINE
1010 LET Z1=Z/3.6E9
1020 LET F4=1.03-.01583*Z1
1030 LET C7=C*0.025
1040 LET C1=C/P
1050 LET B4=1.03-.01117*Z1
1060 LET C4=P/(L*L0)
1070 LET B3=1.095-.0905*C4
1080 LET D4=.845+.0922*C4
1090 LET D2=.92+.02334*Z1
1100 LET N2=R/R0
1110 LET N1=-.19+1.35*N2
1120 LET E=C1*(1-C2)*N1/(1-C3)
1130 LET M=E/1.0
1140 LET B2=1.195-.1973*M
1150 LET Q4=1.053-.0205*Z1
1160 LET Q2=1.685-.674*C4
1170 LET Z5=.56+.1835*Z1
1180 LET Z3=Z/Z5
1190 LET N4=.125+.85*M
1200 LET Q1=.18+.85*M
1210 LET Z4=-.4+1.55*C1
1220 LET Z2=P*1*Z4
1230 LET C8=.06+.969*M
1240 LET C6=P*C8*0.05
1250 LET C5=C1*C2/C3
1260 LET F2=.55+.425*C5
1270 LET F3=1.84-.7935*C4
1280 LET F1=F2*F3*F4*F5/F6
1290 LET B1=.162+.798*F1
1300 LET D3=2.632-1.62*F1
1310 LET D1=2.3-1.23*M
1320 LET D=P*0.028*D1*D2*D3*D4
1330 LET N3=P*N5*N4
1340 LET B=P*0.04*B1*B2*B3*B4
1350 LET Q3=.104+.909*F1
1360 LET Q=1*Q1*Q2*Q3*Q4
1370 LET A1=.766-.457*F1
1380 LET A2=.525+.5075*(Q1/Q3)
1390 LET C2=C2+(1/15)*(A1*A2-C2)
1400 LET C=C+(C6-C7)
1410 LET R=R-N3
1420 LET P=P+(B-D)
1430 LET Z=Z+(Z2-Z3)
1490 RETURN
1600 END
```

READY

List3. Main subroutine Experiment 3

INITIAL CONDITIONS:1900	
POPULATION	1.65E9
NATURAL RESOURCES	900E9
CAPITAL INVESTMENT	0.4E9
POLLUTION	0.2E9
LAND AREA	135E6

Plot 4.

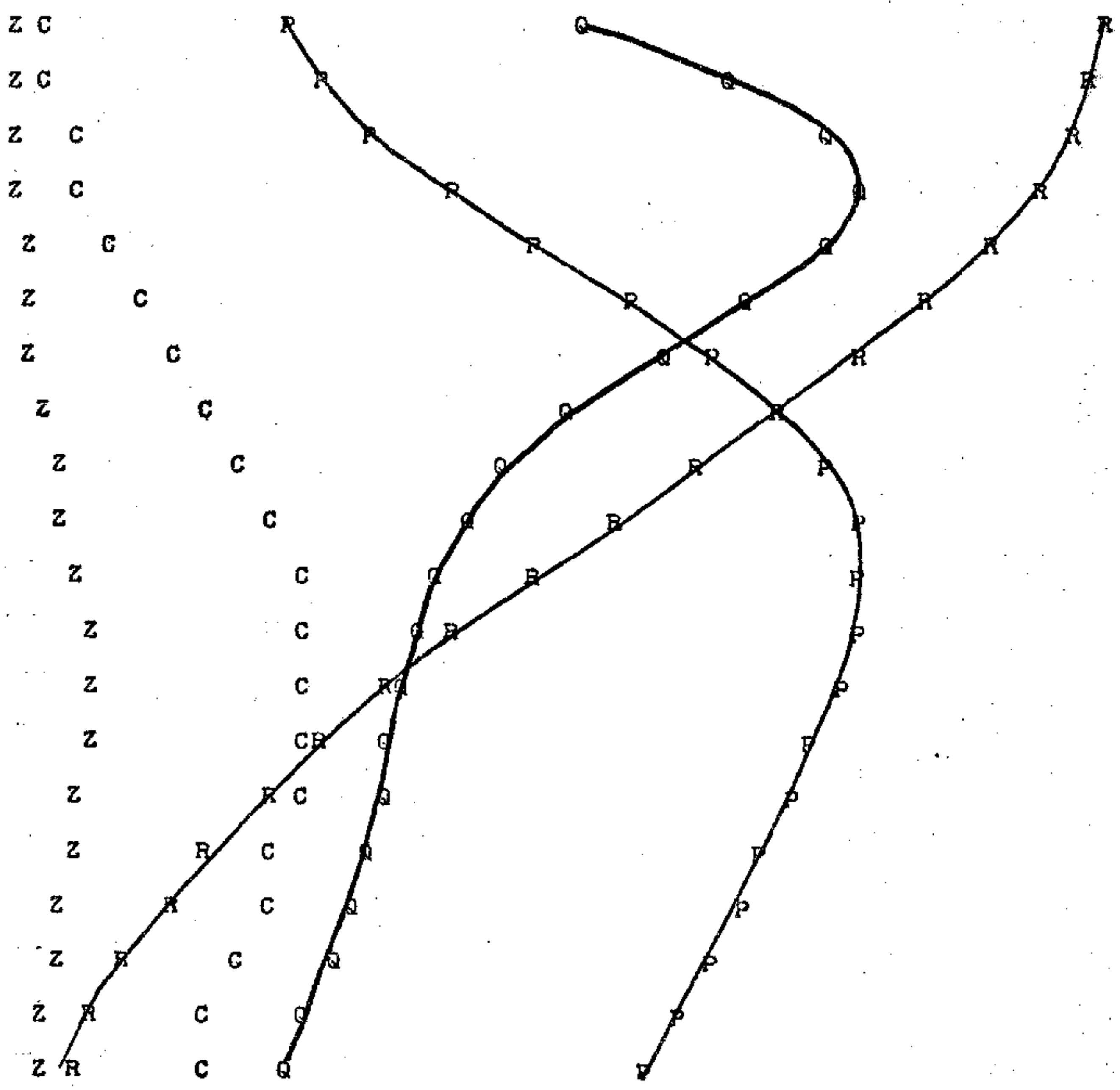


Table 3. Auxiliary equation independent variables for List3 equations

Z1	C4	N2	C1	M	C5	F1	Q1/Q3
0	.47	.99	.38	.5	.3	1.01	.57
.1	.52	.98	.52	.6	.4	1.06	.68
.2	.62	.96	.66	.8	.5	1.08	.79
.4	.74	.94	.8	.9	.7	1.07	.9
.6	.89	.91	.94	1	.8	1.05	.99
.9	1.04	.87	1.07	1.1	1	1.01	1.07
1.4	1.19	.83	1.2	1.1	1.3	.98	1.12
1.9	1.31	.77	1.33	1.1	1.5	.95	1.14
2.6	1.39	.72	1.46	1	1.7	.93	1.13
3.3	1.44	.66	1.57	1	1.9	.93	1.09
4	1.45	.61	1.67	1	2	.94	1.04
4.5	1.44	.55	1.74	.9	2.1	.95	.97
4.8	1.42	.51	1.79	.8	2	.97	.9
4.6	1.38	.46	1.81	.8	2	.98	.82
4.5	1.34	.42	1.8	.7	1.8	.99	.75
3.9	1.3	.39	1.76	.6	1.7	1	.67
3.3	1.25	.36	1.7	.5	1.6	1.01	.6
2.7	1.2	.33	1.62	.4	1.4	1.01	.54
2.2	1.15	.31	1.53	.4	1.3	1.01	.48
1.8	1.1	.29	1.43	.3	1.2	1.01	.44

READY