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ON THE ROLE OF PHYSICALLY-BASED DISTRIBUTED MODELLING IN HYDROLOGY

by

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ABSTRACT

The rationale of hydrological modelling is discussed, and the capabilities of various models are considered. The potential of physically-based distributed modelling in hydrology is noted with reference to specific application areas, and the current status of distributed modelling assessed. The role of the Système Hydrologique Européen (SHE) in developing a general methodology for physically-based distributed catchment modelling is defined.

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"Nowadays model building has become a generously supported indoor sport"

Von Bertalanffy, 1968

1. ON THE RATIONALE OF HYDROLOGICAL MODELLING

Mathematical modelling in hydrology can be justified from two standpoints: (a) the necessity to gain a better understanding of the land phase of the hydrological cycle, and (b) the necessity for models to be used as aids to decision-making in the planning and management of water resources. From the former, research-oriented standpoint, a rigorous approach to river basin modelling would seek to formulate, couple and solve the equations of conservation of mass, energy and momentum describing the movement of water over and through the soil, in stream channels and in aquifers; however, lacking the computational facilities to implement such an approach, research hydrologists have invariably adopted a less rigorous approach where a model has been required to provide simulations of flow showing good agreement with observed flows, achieved through some model fitting process. In the past, this has been achieved with models of a quasi-physical/semi-empirical nature which have not recessarily increased the understanding of the systems being modelled, nor have the model parameters had a sound physical basis. Only in recent years has it proved possible to adopt the more rigorous physics-based approach, thus providing an opportunity to increase our understanding of catchment/river basin behaviour.

Besides their role as research tools, hydrological models have come to play an increasingly important role as decision-making aids in water resources planning and management. For example, in evaluating reservoir yield, streamflow records are rarely long enough to allow reliable estimates of yield to be obtained; longer records of rainfall are frequently available and a rainfall-runoff model can be used to extend the streamflow record, thus providing more information for reservoir yield evaluation. The increasing use of telemetry in the short-term management of water resource systems now means that hydrological models can be employed as real-time flow forecasting tools, while water quality forecasting, also dependent on hydrological modelling, is also a developing area of application.

The models employed in the above applications are typically lumped with either a quasi-physical/semi-empirical or black-box structure, and thus require historical records of streamflow, as well as of meteorological variables, for their calibration. Hence, they all share the common attribute that if the hydrological response of a catchment or river basin changes, for example, due to man's activities, then there is no reliable means available of altering the model parameters to reflect such changes, since the parameters are not physically based.

This report is concerned with physics-based distributed models of catchment/river basin behaviour which, on the one hand, offer the prospect of gaining a better understanding of river basin hydrology and, on the other hand, can occupy important roles in a range of applications to be defined in Sections 3 and 4 of this report. There is an increasing amount of concern worldwide about the effects of development and man-made changes on both the water quantity, water quality and erosion/sediment transport regimes of river basins, and it is in this area, where such models can provide a reliable physical basis for assessing the effects of such changes, that these models promise to be of greatest practical use. Until now, their use has been rather fragmented, with a tendency towards the development of models limited to only one or two processes and specific to a single application. It is hoped in this report to present a first approach towards a general methodology for distributed modelling in hydrology. This will be discussed with particular reference to SHE, the Système Hydrologique Européen. SHE has been developed as a general physically based distributed modelling system by the Danish Hydraulics Institute, SOGREAH (France) and the Institute of Hydrology (UK). The structure of the model that is the basis of SHE is described in Section 5.

"In practice, if the lumped systems models can serve our purpose, then why should we bother to investigate the more difficult distributed system for any other reason that the satisfaction of our intellectual curiosity?"

Ven Te Chow, 1967

2. ON THE CHOICE OF A MATHEMATICAL MODEL

There is a stark contrast between the complexity of hydrological reality and the important and often pressing practicality of making management decisions based on limited knowledge of that reality. Knowledge of the real system will be embodied in a collection of *perceptions of the system* with all its complex web of variability in space and time. These perceptions will be essentially qualitative in nature and will be based on previous personal experience of observed behaviour as well as that recorded in the hydrological literature. We may call this accumulated understanding a perceptual map of the system. There is an important distinction between this personal perceptual map and what actually goes on in the real world, a distinction which implies that the perceptual map is both inherently subjective and capable of continued improvement.

However, decision-making processes require mathematical models that are rigorously defined and capable of producing quantitative estimates of the behaviour of the real world system. The definition of such a model will necessitate considerable simplification of the perceptual map, and in particular an important change of scale. Qualitatively we may distinguish changes in hydrological processes and variables at very small space and time scales, whereas for management decisions we often require bulk estimates (even if in a spatially distributed sense) of the behaviour of the system. It is important to note that the simplication in the definition of a mathematical model and also computational constraints must involve the introduction of further subjectivity. A direct consequence is that:

(i) more than one model or model parameter set may give equally good results;

(ii) *different* models may rate more or less highly in different applications;

(iii) complex models may not necessarily give better results than simpler ones.

There are many ways of classifying hydrological models (see Clarke, 1973) but for present purposes we may distinguish models that treat a catchment as a spatially variable system (distributed models) or only in terms of average quantities over the catchment area (lumped models). Lumped models rely primarily on comparison between observed and simulated catchment outflows for calibration of the model parameters, and may in fact be based solely on the techniques of systems analysis in relating input to output without reference to the internal mechanisms of the catchment (eg the constrained linear systems model of Todini and Wallis, 1977, or the transfer function models of Box and Jenkins, 1970.

There is a large number of lumped models which represent hydrological processes by different functional forms defined intuitively. These models are commonly termed 'conceptual' models in hydrology although all models must be considered as conceptual constructions. In that the parameters of lumped 'conceptual' models must be averages over the catchment area, the derivation of such parameters must then be dependent on curve fitting techiques to match simulated predictions with observations. This requires a period of historical data to be available for model calibration and implies that physical interpretation of parameter values must then be made with care. A classic model of this type is the Stanford Watershed Model (Crawford and Linsley, 1962) and its derivatives. However, the possibilities of subjectivity in the definition of such models are evidenced by the plethora of models of this type (see for instance Fleming, 1975).

A major category of distributed models is that in which the models are physically-based in the sense of being defined in terms of theoretically acceptable continuum equations. Physically-based models are thus descriptive in the sense that the main objective is to represent the behaviour of a physical process, often with the object of gaining a better understanding of that process. Instinctively, it would be expected that physically-based models would provide simulations that are in some way better than other models in that they must reflect more closely our perceptual map of the real world. It must be stressed, however, that the distinctions between types of model drawn here is not clear cut. In particular, the mathematical definition of physically-based models is often such that analytical solutions to the equations cannot be found and resort must be had to approximate numerical solutions based on a finite difference or finite element discretisation of the space and time dimensions. Such models, in this case, also involve a degree of 'lumping' of catchment processes at the model grid scale, a scale that will often be larger (because of computational limitations) than a scale characteristic of the operation of the hydrological processes. Secondly, physically-based models are usually based on process 'laws' that are essentially empirically based (eg Darcy's Law or the uniform flow equations such as the Manning or Darcy-Weisbach equations). There remains therefore considerable scope for subjectivity in the definition of physically-based distributed models. However, the important difference from the lumped 'conceptual' models is that the relationships used may be validated by process measurements in the field, and improved over time as a result of experiments in the same way as other scientific laws. Thus, physically-based distributed models have parameters that are in principle measurable in the field.

There is, in addition, a class of semi-distributed models of varying degrees of theoretical acceptability that bridge the gap between the lumped models and the fully distributed physically-based models. These models use conceptual functional relationships for different hydrological processes applied to a number of relatively homogeneous subareas of the catchment treated as lumped units. They are essentially an extension of lumped 'conceptual' models to the semi-distributed case and are to some extent subject to similar limitations. While some of these models can be applied on the basis of field measured parameters alone (eg Beven and Kirkby, 1979) it is more usual for the models, which remain relatively simple, to be calibrated by optimisation against an observed discharge record (eg

Mein et al, 1974; Knapp et al, 1975; Lee and Delleur, 1976; Solomon and Gupta, 1977; Boyd et al, 1979). These models do, nonetheless, allow additional hydrological, soil and vegetation data to be incorporated into the calibration process.

The above classification of hydrological models has been based mainly on a consideration of model structure. A further classification may be made into deterministic and stochastic models. This depends largely on whether the model parameters and or model outputs are considered to be totally error-free (deterministic) or subject to error (stochastic). Lumped models that are deterministic in concept (eg the unit hydrograph) often have an implicit stochastic structure (see Clarke, 1973). For the case of physically-based models that are deterministic in operation and have parameters that can be measured in the field, the classification will depend on whether these measurements are without error (a deterministic model) or subject to error (a stochastic model). In the case where the required parameters are measured without error, a stochastic classification for the model may still result if, when model predictions are compared with observations, errors are observed. If the errors are due to the fact that the model is not a perfect facsimile of the real world, then it should be classed as stochastic. If, however, the only source of error is due to measurement errors in the model inputs then the source of stochasticity lies essentially outside the model which should then be classified as deterministic. In practice of course it is impossible to separate sources of error, but it will generally be true that most models, including those that are deterministic in operation, will have an implied stochastic component. This strict definition is often relaxed so that models are referred to as deterministic if they are deterministic in operation alone. Underlying the use of deterministic models is an assumption that the resolvable (deterministic) component of a process dominates the unresolvable (stochastic) part to the extent that a deterministic simulation can provide meaningful forecasts of system behaviour.

Given the wide range of hydrological models available, the choice of a model for a particular problem is never a simple one, and will inevitably be based on economic constraints and personal preferences as well as purely hydrological considerations. Data availability is often a crucial factor in such a decision. However, any general criteria for model choice should be based on matching the requirements of a management problem with the complexity of the model used. Modelling hydrological systems is an activity that is, in many applications, governed by a law of diminishing returns. Very simple models can often provide a useful, if not necessarily adequate, representation of the system and the benefits that derive from additional complexity may not warrant the corresponding increase in costs. Thus, if a problem requires a model to extend catchment discharge records from a longer rainfall record, it may be sufficient to use a lumped model and probably would not make sense to use a complex distributed model demanding far greater resources. On the other hand, if it is necessary to predict the effects of localised land use change then it is necessary to use a distributed physically-based model in which the changes in catchment characteristics can be directly reflected in changes in the model parameters.

There are further reasons for choosing physically-based distributed hydrological models in preference to lumped models. In particular, distributed models allow much readily available data to be incorporated directly into the model. For example, one of the most important characteristics determining the hydrologic response of a catchment must be the nature of the catchment topography. Yet, topography is very often completely ignored in the formulation of lumped 'conceptual' models. In addition there is ample evidence that, due to topography and the variability of soils and vegetation, the response of catchments is non-linear and spatially variable, both within and between storm periods (see review by Hewlett, 1974; Freeze, 1974 and Dunne, 1978). Such variability will be reflected in the predictions of a distributed and physically-based model. In addition, physically based models may overcome some of the problems posed by the prediction of extreme events that are not represented in historical rainfall or discharge records. The forecasts of lumped models in cases that are beyond the range of the records used for calibration, must be open to doubt. Providing that the assumptions underlying a physically-based model are not invalidated under extreme conditions, forecasts of extreme response should be theoretically more acceptable. It is expected that the sound theoretical basis of these models will be the principal reason for a more extensive application of distributed models.

"Nearby is the graceful loop of an old dry creek bed. The new creek bed is ditched straight as a ruler; it has been 'uncurled' by the county engineer to hurry the runoff. On the hill in the background are contoured strip crops; they have been 'curled' by the erosion engineer to retard the runoff. The water must be confused by so much advice".

Aldo Leopold - The Sand County Almanac.

3. ON THE NEED FOR PHYSICALLY BASED DISTRIBUTED MODELS IN HYDROLOGY

There are four (related) areas for which physically-based spatially distributed hydrological models can fulfil the needs of practical applications: catchment changes; spatially variable inputs and outputs; the movement of pollutants and sediment through a catchment and forecasting the hydrological response of ungauged catchments. In each case alternative methods exist and these will be discussed below in relation to the distributed modelling approach. In addition, some further benefits of physically-based models are discussed.

3.1 Catchment changes

This category covers all forms of hydrologically effective changes in catchment characteristics, including both natural and man-made changes (but not the external problem of changes in natural inputs to a catchment). Such changes include land use changes due to the effects of forest management, urbanisation, forest clearance for agricultural purposes, the effects of forest fires, etc. Such changes are almost invariably localised and piecemeal, such that catchment change is a distributed problem. The parameters of a physically-based distributed model have direct physical interpretation, and their range can be established reasonably well on the basis of field and laboratory investigations. It follows that knowledge of parameter values can be directly related to catchment characteristics such as soil type and land use and such knowledge should be transferable, within reason, from area to area. Thus it should be possible to predict the effects of catchment changes prior to data becoming available. In addition the localised nature of such catchment changes can be easily incorporated into the spatially distributed model structure.

There are other methods available for predicting catchment changes, in particular using lumped 'conceptual' models. The representation of specific hydrologic processes by functional forms in such models has led to the interpretation of the parameter values as quasi-physical quantities. This has resulted in much unguarded optimism as exemplified by the following quotations:

> 'A model is not only an excellent exploratory tool but also a direct aid to creative thinking. Potentially it can provide answers regarding the sensitivity of the total system to changes in certain parameters and the amount of change needed to achieve a desired result' (Riley et al., 1968, p.27);

> 'Any activity of man which may change the watershed can be measured or predicted and again related to the model parameters

used, and a new flow sequence can be simulated. The planner will then have a quantitative estimate of marginal hydrologic change to use, an estimate of a degree of precision never before possible' (James, 1972, p. 305);

'Other changes such as vegetation manipulation, forest fires, soil conservation techniques etc can be predicted if data for calibration are available. Where adequate rainfall and streamflow exist ordinary calibration techniques will define the amount of the change and the mechanism through which it works' (Hydrocomp, 1971, p. 4-2).

It must be stressed that these authors overstate the case for lumped 'conceptual' models. It is dangerous to interpret the degree of mathematical equivalence with observed catchment behaviour achieved by such models in terms of physical significance, particularly in the analysis of catchment changes. Lumped parameter values should not be taken as having significance outside the model structure (Nash and Sutcliffe, 1970), and the transfer of parametric information over time or space to different model applications must be considered extremely hazardous.

3.2 Spatially variable inputs and outputs

In that lumped catchment models can deal only with quantities averaged over the catchment area, it is necessary to use a distributed catchment model when there is significant spatial variability in catchment inputs and outputs. Particular examples are the movement of rainstorms over a catchment (Surkan, 1974, Urlson et al, 1979) and localised river and groundwater abstractions and recharge. Such spatial variability is handled easily by distributed models, provided that suitable input data are available. It may be that technical developments such as rainfall measurement by radar and other remote sensing techiques will provide such data in a form directly usable by distributed models.

3.3 The movement of pollutants and sediment

Water provides a supply, transport and dispersion mechanism for pollutants and sediment, and since most water quality problems are distributed in nature, distributed models are therefore most suitable to provide the basic hydrological input to predictions of water quality (see for instance, Bresler, 1973; Brazil et al., 1977; Nelson, 1978; Selim, 1978). Again there are alternative methods for making such predictions. Water quality modelling can be approached using systems analysis input/output techniques provided that data are available for calibration of a model. Indeed the complexity of water quality interactions makes this an attractive alternative when such data are available, provided that the system is relatively stationary over both calibration and forecasting periods. Lumped 'conceptual' models have also been used to make quality predictions. An enhanced version of the Stanford Watershed Model has been used to predict the movement of radioactive aerosols through a catchment (Huff and Kruger, 1967). It would seem that this case of non-point pollution, in which the quality variable may be considered as uniform over an area, is much more amenable to lumped modelling than the case of spatially variable inputs. In fact it is difficult to see how localised sources of pollutants or sediment can be successfully handled by spatially averaged models except in the case of one or two recurring sources.

3.4 Prediction of the hydrological responses of ungauged catchments

The possibility of using physically-based distributed models for predicting the responses of ungauged catchments arises directly from the physical significance of the model parameters. Since it is possible to establish reasonable ranges for the parameters on the basis of intensive, short-term field investigations, such models can be used to generate at least approximate hydrological predictions without the benefit of long concurrent records of precipitation and streamflow for calibration. In fact any information on the characteristics and hydrology of a catchment that is readily available can usually be incorporated in the modelling process.

Most alternative methods for making predictions on ungauged catchments rely on the use of statistical regression of the model parameters against catchment characteristics using parameters calibrated on gauged catchments. The regression equations may then be used to predict parameter values for ungauged catchments. Such analyses have been made both for unit hydrograph models (NERC, 1975) and more complex conceptual models (Ross, 1970; James, 1972; Jarboe and Haan, 1974). In view of some of the difficulties of calibrating conceptual models (see for example, Ibbitt, 1972) there are obvious dangers in this approach which necessarily only crudely approximates the causes of variability between catchments by the use of surrogate variables in the regression equations.

3.5 Some further benefits of distributed physically-based models

Distributed models of water flow can provide information for a variety of linked problems, some of which, such as estimating consolidation or subsidence, may be directly coupled to the solution of the flow problem (see for instance, Corapcioglu and Brutsaert, 1977; Narasimhan and Witherspoon, 1978). Other problems may be less directly linked, for example the prediction of pore pressures in slope stability studies (eg Hodge and Freeze, 1977) and the water quality problems discussed above. Other interactions with deterministic hydraulic models may be expected to develop.

The use of physically based models focusses attention on the inadequacy of the hydrological data upon which many water resources management decisions are based. By considering the uncertainties in the parametric data explicitly, estimates of the resultant uncertainty in the predicted hydrology can be made. At present this process is at a relatively crude level in applications of complex catchment models, generally involving a sensitivity analysis within an expected range of estimated parameter values. However, present studies involving one or two processes (see Section 4) show that the available techniques may be expected to become considerably more sophisticated. This type of sensitivity analysis should aid the management process in both providing estimates of the uncertainty in hydrological predictions and in suggesting how additional expenditure in the field determination of parameter values might be used most beneficially. This may become one of the most important uses of physically-based simulation models.

'As Scientists we are intrigued by the possibility of assembling our concepts and bits of knowledge into a neat package to show that we do, after all, understand our science and its complex interrelated phenomena'.

W M Kohler, 1969.

4. ON THE CURRENT STATUS OF DISTRIBUTED MODELLING OF CATCHMENT HYDROLOGY

The development and application of distributed modelling in hydrology has been very fragmented. There have been numerous papers on the theoretical aspects of modelling various hydrologic processes independently using both analytical methods for simple cases and approximate numerical methods of increasing sophistication. There is a much smaller literature on models involving more than one process and on the application of distributed models to real world problems. It is beyond the scope of this report to provide a full review of the development of distributed models and we shall concentrate on some general features of distributed modelling with appropriate illustrations from a small number of specific studies.

First, consider the catchment as a fully distributed system, continuously variable over time and 3 space dimensions. It is possible to write down general partial differential equations for the process of mass and energy transfer within the catchment continuum, together with boundary conditions for those equations, based on physically realistic assumptions compatible with current knowledge of hydrological processes. These equations comprise the general distributed model of catchment hydrology (see for example the discussions of Freeze and Harlan, 1969; and Freeze, 1978). The complete system of non-linear equations is impossible to solve analytically for any case of practical interest and resort must then be made to approximate numerical solutions. Numerical solutions can only be calculated for a finite number of points (nodes) in space and time, and the complexities of the system are such that computing constraints will limit that number of nodes. A considerable amount of research, mostly concentrated on studies of single processes, has been aimed at developing solution methods that are sufficiently accurate but remain efficient in terms of computing requirements and the discretisation of the catchment into nodes.

The earliest solution technique, and still the most commonly used, is the finite difference method pioneered by Richardson (1910). Finite difference techniques were first applied to groundwater flows by Shaw and Southwell (1941), unsaturated flows by Klute (1952) and channel flows by Stoker (1957). Finite difference methods have been considerably improved and a wide range of methods is now applied quite routinely to steady state and transient problems in groundwater and channel routing studies. More recently, however, several other methods have been used in hydrology including finite element methods (Pinder and Gray, 1977), integrated finite difference methods (Narasimhan and Witherspoon, 1977) and the methods of characteristics (Amein, 1966; Wiggert and Wylie, 1976). The application of a 3-dimensional complete catchment model solved by finite difference methods has been demonstrated on limited hypothetical problems by Freeze (1971, 1972a, 1972b).

The stage has now been reached where the solution methods are sufficiently well advanced to give satisfactory results for a wide range of problems, providing that the problem of interest is well posed in terms of the specification of the boundary conditions and data input to the model. Many studies have shown that these distributed models can reproduce the behaviour of hydrological systems as measured in the laboratory (eg Ragan, 1966; Tang and Skaggs, 1977; Haverkamp et al, 1977; Luthin et al., 1975; Muzik, 1974) and in the field Amein and Fang, 1970; Bresler et al., 1979; Feddes et al., 1976; Pikul et al., 1974; Smith and Woolhiser, 1971, although in complex field situations it may be difficult to obtain completely satisfactory simulations (Stephenson and Freeze, 1974).

Now that the basic solution techniques of distributed modelling are well understood there has been a considerable number of studies essentially filling in points of detail. There have, for instance, been several comparisons of numerical solution techniques (eg Price, 1974; Trescott and Larson, 1977; Haverkamp et al., 1977; Hayhoe; 1978). Other works have attempted to increase the generality of the mathematical formulation, often at the expense of additional complexity. Examples are the inclusion of compaction coefficients (Freeze, 1971; Narasimhan and Witherspoon, 1976), non-Darcy type flow in ground water flow models (Volker, 1969), inclusion of hysteresis in unsaturated flow models (Whisler and Klute, 1965; Hoa et al., 1977; Giesal et al., 1973; Lees and Watson, 1975) and alternative flow relationships for upland channel flows (Beven, 1979b). Further studies have examined the effects of using different discretisations (Hayhoe, 1978; Rushton and Tomlinson, 1977), of different methods of calculating the equation coefficients (Appel, 1976; Haverkamp et al., 1979), specifying the initial and boundary conditions (Rushton and Weddesburn, 1973; Morris, 1979; Zaradny, 1978) and stability criteria (eg Young, 1971; Strelkoff, 1970). Methods for reducing the computing requirements of distributed models have also been developed. Some of these methods rely on formulating the solution method to reduce the amount of storage required (eg Prickett and Lonnquist, 1973; Fread, 1971) and others on decoupling the fully 3-dimensional problem into a number of one or two dimensional components, for example the surface flow models of Kibler and Woolhiser (1970); Engman and Rogowski (1974) and Ross et al. (1979); the two dimensional hillslope elements of Beven (1977); the models of multilayer groundwater systems (eg Bredehoft and Pinder, 1970); and the structure of the SHE model described in Section 5 below. This decoupling must introudce some additional inaccuracy, particularly in models such as the kinematic cascade where the complexities of the topography of natural hillslopes are represented by 'equivalent' rectangular planes (see Lane and Woolhiser, 1974). However, in models such as the SHE model (see Figure 1) the loss of accuracy compared with an equivalent 3-dimensional model may be small and a finer discretisation may be handled on current computers, so as to allow application of a complete catchment model to real catchments. It must be added that with all approximate solution methods the need for discretisation may itself generate problems in the application to real world problems, since the grid scale of the model must often necessarily be larger than the scale at which the hydrologic processes operate. The problem of defining parameter values at the grid scale will be considered further in Section 6.

There has also been recent interest in distributed models viewed as stochastic models in the sense defined in Section 2. All the models described above are deterministic in operation and assume that the true values of the parameters and boundary conditions of the model are known without error. This will rarely, if ever, be true in applications to real situations and the effect of allowing the parameter values and boundary conditions to be defined stochastically with a non-zero variance has been the subject of several studies in groundwater flow (eg Bibby and Sunada, 1971; Freeze, 1975; Gelhar, 1974; Sagar and Kisiel, 1972; Tang and Pinder, 1977; Sagar, 1978; Bakr et al., 1978; Dagan, 1979); overland and channel flow (Chiu, 1968; Machado and O'Donnell, 1979); and a complete hillslope model (Freeze 1980). It is also worth noting that field observations, on which parameter estimates are based and comparisons with simulated results made, are also subject to sampling and measurement errors. Some studies have shown that the error variance associated with such measurements may be considerable (see for instance Hills and Reynolds, 1969, and Kiesling et al., 1977) and that single measurements may not give good estimates of the spatial average of a variable. This certainly has a bearing on the specification of parameter values at the grid scale. This problem will be discussed further in Section 6 but two points seem worth making here.

First, estimates of the variance of parameter values have generally been made on the basis of intensive spatial sampling programs (eg the log normal distribution of hydraulic conductivities found by Willardson and Hurst, 1965; Nielsen et al., 1973; Rogowski, 1972). It is commonly assumed in studies of the stochastic nature of physically based distributed models that these measured <u>spatial distributions</u> represent the point sampling distribution <u>of that parameter and</u> that each measurement represents an independent sample from the underlying population. This can only be true if the spatial autocorrelation of the parameter is small relative to the distance between samples. If spatial autocorrelation is significant at the model grid scale, then the spatial <u>pattern</u> of the measurements must be taken into account in the determination of parameter values defined at the grid scale. In this case, the sampling variance associated with the grid square parameter values may be smaller than that based on the assumption of independent measurements from a point sampling distribution (see discussion in Beven, 1981).

A second, related, problem is that point measurements of hydrological parameters are usually based on experiments that are small in scale relative to practical model grid scales (from laboratory cores to small plot experiments). The theory of geostatistics suggests that, for static quantities, as the sampling volume of a quantity (the support volume or measurement scale) increases, the sampling variance will decrease (Journel and Huijbregts, 1978; Delhomme, 1979; Burgess and Webster, 1980). A similar relationship between sampling variance and measurement scale can be expected to hold by analogy for dynamic systems. Certainly, theoretical experiments have shown that the calculated variance of the solution output variables from distributed models arising from the variance in parameter estimates, decreases as the number of space dimensions considered increases (see Bakr et al., 1978; Dagan, 1979), reflecting the compensation for variability that might be expected in the continuous flow system. It should be noted, however, that for *non-linear* flow systems, the nature of this compensation may depend on the flow conditions, such that appropriate grid square parameter values may vary over time. The definition of grid square parameter values, while dependent on the measured point sampling distribution of a parameter, must be considered as a higher order problem, in that grid square values will also reflect model structure, model grid scale and spatial structure in the measurements. This whole question of the interactions between hydrological processes, measurement scale, model structure and model parameters requires much further study.

This review has necessarily been brief and incomplete but has attempted to illustrate the current status of both the techniques and understanding of distributed modelling in catchment hydrology. There have been very few attempts to model complete catchments in a truly distributed manner except in cases where catchment response is dominated by surface flow. The methods of distributed modelling are now sufficiently well developed that data limitations generally outweigh the limitations of technique in applications to the real world. It should then be possible to specify a catchment model for general use to gain experience of applications to practical problems. A first attempt at such a model is the basis of SHE which is described in the next section.

"We must not omit to take notice of what we see with our eyes, that water naturally tends downwards, that it cannot suffer the air to be anywhere beneath it ... that it loves to fill up every concavity into which it runs; that the more you endeavour to force it, the more obstinately it struggles against you, nor is it ever satisfied till it obtains the rest which it desires."

Alberti (1404-1472)

5. A BRIEF INTRODUCTION TO THE SHE MODEL

The SHE model that is the basis of the Système Hydrologique Européen is a physically-based distributed model, deterministic in operation, that incorporates components for all the major hydrological processes. The model has been developed from the non-linear partial differential equations of flow for the processes of overland and channel flow, unsaturated and saturated subsurface flow, solved by finite difference methods. The model is completed by point snow melt, interception and evapotranspiration components. While considerable effort has been made to ensure that the solutions for each component are accurate and efficient, the structure of the model represents a compromise between the restrictions of computing and data requirements of the model and the need to represent the complexity of real catchments. A description of SHE is given by Jonch-Clausen (1979); only a brief summary is provided here.

It is not yet economically viable to produce an operational model that is fully 3-dimensional in space and which will also allow the required accuracy of discretisation in both horizontal and vertical planes. On the assumption that, in the unsaturated zone, vertical flow is, on most slopes, far more important than lateral flow the SHE model has been rationally simplified such that independent one-dimensional unsaturated flow components of variable depth are used to link a two-dimensional saturated subsurface flow component and a two-dimensional surface flow component (Figure 1). It is planned to use up to 2000 grid points in the horizontal and 30 in the vertical to allow adequate definition of a catchment areal unit.

The application of a model of this type requires considerable inputs of parametric and exogenous data, including parameter values that change over time, for example through the growing season of a crop. Such data will not always be readily available and considerable flexibility has been built into the model to allow different modes of operation demanding different levels of data availability. It is stressed that parameter values are in principle measurable in the field and it is hoped that the general availability of a model of this type, will instigate more widespread measurement of the data required, if not on a routine basis, then at least as part of the application of the model to a specific project.

The interception/evapotranspiration component of the model determines the total evapotranspiration and net rainfall from meteorological input data. The interception process is modelled using a variant of the Rutter model (Rutter et al., 1975) and has canopy storage and drainage parameters that can be estimated by experiment (see for instance Rutter et al., 1975; Gash and Morton, 1978). There are several modes for the evapotranspiration component of which the most complex is the full Penman-Monteith equation (Monteith, 1965)

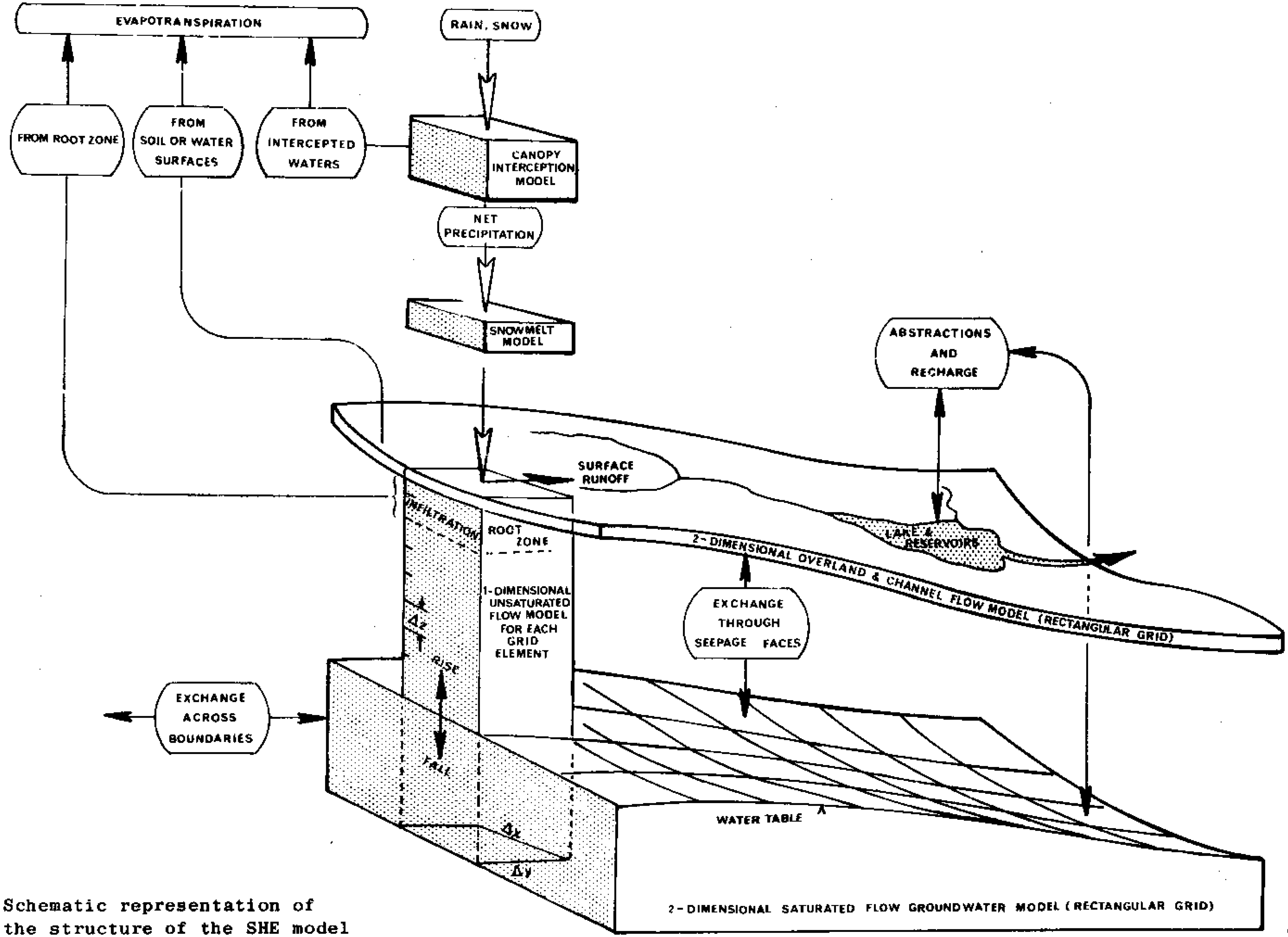

FIGURE 1 Schematic representation of the structure of the SHE model

which requires aerodynamic and canopy resistance parameters (see for instance Szeicz and Long, 1969; Szeicz et al, 1969; Gash and Stewart, 1975; Tan and Black, 1976). The interception/evapotranspiration component interacts directly with the root zone, which is the upper part of the unsaturated zone component. The extraction of moisture for transpiration from the root zone is distributed according to the vertical distribution of roots. Vegetation characteristics and meteorological inputs to the model can vary from grid square to grid square. A full description and sensitivity analysis of this part of the SHE model is given in Beven (1979a).

The overland and channel flow components represent the surface runoff and river flow processes. They are modelled by finite difference solutions to the simplified Saint Venant equations (with inertia terms neglected). In the modelling of overland flow the slope of the water surface is assumed to be parallel to the ground surface (kinematic assumption) but for channel flow, a water surface slope term is included in the formulation so that backwater effects can be modelled (see Preissman and Zaoui, 1979). The overland and channel flow components require parametric data on channel dimension and flow resistance functions for each grid square.

The unsaturated flow component uses a finite difference solution of the Richard's equation in a multi-layered soil including the root zone. An interactive solution technique has been adopted to cope with the important non-linearities of the flow equation. The unsaturated flow component determines rates of infiltration and recharge in the coupling with the surface flow and saturated groundwater flow components. A particular problem associated with the structure of SHE is the link between vertical flow in the unsaturated zone and horizontal flow in the saturated zone. A satisfactory approach to modelling this transition has been developed (Abbott et al., 1979). The unsaturated zone solution requires information on the soil moisture and hydraulic conductivity characteristics, but these may vary for each soil layer and with soil type in different grid squares. The combined unsaturated zone/evapotranspiration components have been tested by Jensen and Jonch-Clausen (1981).

The saturated groundwater flow component is, in the first version of the SHE model, restricted to a single layer, unconfined aquifer with direct links to any surface water bodies. The model is based on an alternating direction implicit finite difference solution to the Boussinesq equation. This formulation must restrict application of the model at present, but it is envisaged that this component will be extended to cope with multi-layered, confined/unconfined aquifers in the future. The saturated zone model requires data on the extent and depth of the aquifer together with conductivity and storage coefficient parameters for each grid square.

The snow melt component can also be used in several modes dependent upon data availability. The simpler modes are based on Degree-day and Energy Balance predictions of melt rates, with routing through the snowpack based on the empirical model of Anderson (1968). The most complex form is represented by the first attempt to model both the energy and mass flux within a snowpack taking changes in the structure of the pack into account. Finite difference methods are used to obtain simultaneous solutions to the flow of heat and water in the pack (see Morris and Godfrey, 1979; Morris 1982). Snowmelt runoff may be generated either at the surface or at the base of the pack.

Factors affecting the accumulation and melting of snow will be reflected in the difference in meteorological inputs and parameter values between grid squares.

It is stressed that all the model components, while being physically-based as far as possible, rely on empirical equations or simplifying assumptions that are approximations to the complexities observed in the field. It is important to recognise that, while such approximations must be limited in validity to some extent, they are consistent with present quantifiable knowledge of the physical processes of catchment hydrology, and may be expected to be improved over time. The modular form of the structure of the SHE model facilitates such changes. Perhaps the most important part of the SHE model is the model 'frame', known as the Chef d'Orchestre, that links the model components and orders the solution of the components in time. It is the flexibility of the frame that allows a general methodology of distributed modelling to be outlined, as is discussed in Section 6.

"The deterministic simulation approach espoused here is potentially superior to any empirical model that requires less data. If the technique can be shown to have practical value, it will encourage increased measurement of the necessary data."
Allan Freeze, 1971

6. TOWARDS A GENERAL METHODOLOGY FOR THE DISTRIBUTED MODELLING OF CATCHMENT HYDROLOGY

The formulation of the SHE model outlined in the previous section incorporates elements defined on the basis of the available (quantifiable) physical knowledge of the catchment system. At this stage it has been necessary to make some simplifying assumptions such as the exclusion of hysteresis effects and lateral unsaturated flows, but providing that the model is not used outside the range of conditions for which its assumptions remain reasonably valid, it is likely that the effects of such simplifications will be small in relation to the limitations imposed by the data requirements of the model. Thus the SHE model should provide a suitable basis for a first approach to a general methodology for distributed modelling (Beven et al, 1980). However the physical acceptability of the model is only one aspect of a methodology for operational applications. In general,

(1) the model must be physically relevant to the system that it represents;

(2) the model should incorporate the information from as much readily available hydrological and other catchment data as possible;

(3) the model should be able to simulate a wide range of problems, both transient and steady state, for single processes and combinations of processes up to the level of a complete catchment system;

(4) the model should be able to cope adequately with the difference in response times of different processes and with the time variant rates of change due to different meteorological conditions;

(5) the model should be able to cope adequately with non-stationarity in the model parameters due, for example, to seasonal effects on the vegetation of the catchment;

(6) the model should be able to cope with dynamic changes in the component structure of the model for example when the unsaturated zone disappears as the water table rises to the surface in variable source areas.

The first two criteria should be satisfied by any distributed model for which both the underlying theory and the characteristics of the solution techniques are acceptable in a given application. It has been suggested that the SHE model will satisfy these criteria over a wide range of hydrological and geographical conditions. In addition a considerable part of the effort expended in the development of SHE has been concerned with satisfying the last four criteria in the specification of the model 'frame' (the Chef d'Orchestre).

The model frame is used to control the input of the catchment specification and the nature of the solution that is required, reconciling the dynamic time step requirements for the individual components and then ordering the solutions for the components with the correct transfer of valid internal boundary data between components. The detailed specification of the frame necessitates the achievement of a satisfactory compromise between maintaining scientific rigour and numerical efficiency. It is obviously necessary that the transfer of internal boundary data be handled so as to minimise inaccuracies in the maintenance of mass balance, and that the control of time steps be handled to maintain stability in the component solutions. The first version of the frame is already operational with some restrictions on the types of solutions that can be handled, but is expected to be revised and extended over time as experience in different situations is incorporated.

Given the availability of a model like SHE with a frame suitable for general application, there are several stages in the application to a specific project, as follows:

(1) Problem definition
(2) Parameter definition
(3) Model calibration/validation
(4) Sensitivity analysis
(5) Scenario evaluation

6.1 Problem definition

Perhaps at this stage, it is unnecessary to raise the question of whether a distributed model is really required to solve a particular management problem with its additional complexity, data requirements and computational expense. However, while there will be projects for which the choice will be obvious (see discussion in Sections 2 and 3), in many situations it will be much less clear cut. In such situations it is necessary that the theoretical advantages of distributed models be demonstrated in applications to field problems so as to overcome the economic advantage of alternative lumped models. At present it can be expected that the use of distributed models will be confined to projects specifically requiring distributed simulations as discussed in Section 3.

In these cases it is necessary to further decide whether transient or steady state simulations for single processes or combinations of processes are required. This will be in part decided by the nature of the problem and the availability of data and resources. It is obvious that a single process, steady state solution will be much cheaper but will not always provide all the information required. The aim of a model frame, such as that embodied in SHE, must be to allow flexibility in the choice of such options, so that it will certainly be possible to proceed from a simple analysis to more complex formulations as appropriate.

A final important part of problem definition is the specification of appropriate model boundary conditions. This may involve considerable effort, particularly in the case of subsurface flow systems where boundaries may not be clearly defined, or where attention is focussed on a small part of the catchment with non-hydrological boundaries.

6.2 Parameter definition

The definition of parameter values is of crucial importance to the application of distributed modelling, particularly when it is intended that such models should be used deterministically. Distributed models require the specification of several parameters and parametric functions at each individual grid square (see Table 1) leading to large numbers over a catchment area. Temporal changes in parameter values may further increase this number. The problems of defining all the individual parameter values by separate measurement are immense and it is therefore necessary to explore the possibility of reducing the expense of parameter estimation. Fortunately, the physical basis of those parameter values allows a degree of hope that this may be possible, in that it should be possible to transfer parameters measured in one location to be representative of similar areas elsewhere. Thus not only may it be possible to define parameters on the basis of vegetation or soil type within a catchment (obviously requiring a much smaller overall number of parameter values) but it should also be possible to transfer information from studies outside the catchment of interest. In addition, the

TABLE 1 Typical parameters required for each grid element in a distributed model of catchment hydrology *(as taken from the SHE model)*

Vegetation	Aerodynamic resistance parameter canopy resistance parameter (may be related to soil moisture in root zone)
Topography	Mean altitude Surface slope
Unsaturated zone	Root distribution with depth Hydraulic conductivity/soil moisture relationship for each soil layer Capillary tension/soil moisture relationship for each soil layer
Saturated zone	Conductivity in direction of each axis Storage coefficient (may be related to unsaturated zone solution)
Overland and channel flow	Surface roughness or channel dimensions and flow coefficients as appropriate
Snow melt	Snow surface roughness Parameters of hydraulic conductivity/snow water content/snow density/grain size relationship Parameters of snow water tension/snow water content/snow density/grain size relationship Parameters of effective thermal conductivity/snow density/grain size Parameters of equilibrium temperature/snow water tension relationship

application of remote sensing in hydrology will have beneficial interactions with distributed modelling in terms of providing spatially distributed data and data handling procedures (see for instance Jackson et al 1976; Bauer et al, 1978; Schmugge, 1978; Jordan et al, 1978; Jackson and McCuen, 1979).

However, it cannot be denied that a sampling problem remains, as demonstrated by the discussion of parameters as stochastic variables in Section 4. This has several implications. First, where more than one estimate of a parameter value exists, the arithmetic average may not be the most appropriate estimate to use in the simulation (see for instance Freeze, 1975; Dagan, 1979). Secondly, even where a parameter is easily measured within all grid squares, for example topography, there may be no simple way of defining the most appropriate value of that parameter at the grid scale, except to assume that some characteristic average value will suffice. The consequence is that the parameters should be viewed as defined by some (unknown) distribution rather than as fixed values truly characteristic of an area. The definition of parameter values should be associated with at least a realistic range of possible values, and the sensitivity of simulations to variations within that range should be explored. Ultimately, it may be possible to reformulate the model equations to characterise the system at the grid scale.

Given an appreciation of these problems of parameter definition it is important that the techniques of measurement used should, as far as possible, correspond to the structure and scale of the model being used. Thus, measurements in the field are more appropriate than laboratory methods, and coarse scale measurements (such as pumping tests or tube well methods for saturated conductivity values) are better than point measurements (for example core samples taken back to the laboratory). The aim should be as far as possible to integrate the detailed characteristics of hydrological processes and obtain parameter values characteristic of the overall response of a process. It is likely then that a closer match between measured values and appropriate grid scale values will be obtained, although the sampling problem (in both space and time) will never be eliminated. Vandenberg (1977) explicitly considers the effects of parameter variability on the determination of transmissibility parameters by pumping test.

The development of tracer techniques may be useful in this respect. As one example, the use of tracers in upland channels has shown how the complete response of a sequence of pools, riffles and falls can be integrated and used to define parameter values for channel routing (Beven et al., 1979) and lead to new insights into the nature of the bulk characteristics of flow, as they change with discharge (Pilgrim 1976; Newson and Harrison, 1978). Similar methods can be used to study overland flow on complex vegetated surfaces (Newson and Harrison, 1978). Tracers have also been used for some time as a method for determining groundwater flow parameters though without complete agreement as to the interpretation of results (see for instance Kreft and Zuber, 1978).

A number of field methods are available for measuring the unsaturated flow properties of soils (see for example Hillel et al., 1972; Jeppson et al., 1975, Royer and Vachaud, 1975). The parameters of the interception and evapotranspiration component can be obtained from net rainfall (see for instance Calder and Rosier, 1977) and micrometeorological measurements (Stewart, 1977).

The measurement of parameter values in the field assumes a particular importance for distributed physically-based modelling. Every measurement serves to help define a correct range for a given parameter value and with continued accumulation of information may allow specific data deficiencies to be shown up. Each measurement also serves to some extent to examine critically the physical basis of the model being applied. While it may be of little importance to the application of lumped models that the modeller should have studied the characteristics of a catchment other than just its rainfall/discharge record, it is far more important in the application of physically-based models.

6.3 Model calibration/validation

The term model calibration is here used specifically to mean the selective improvement of initial parameter estimates by a comparison between observed and simulated hydrological variables. Model validation implies the acceptance of a model as an accurate simulator of the real world system, which will also depend on a comparison of observed and simulated behaviour. In the application of a distributed model there may not always be the requisite historical data available for model calibration or validation. A program of data collection should usually therefore be implemented at the same time as the measurement of parameter values, although it is worth noting that measurement and spatial sampling problems also pertain to the collection of data to represent observed behaviour, which can itself, therefore, only be estimated with some error.

The allowance of the possibility of calibration of the parameters of distributed models implies that the estimated values may vary from the 'true' model values and that these models are therefore stochastic. However, the number of parameter values required by distributed models poses a difficult calibration problem. Several writers have pointed out that the information content of a rainfall/discharge record alone must limit the number of parameters that can be satisfactorily estimated in the application of lumped models. It is true that the calibration of a distributed model can take much more information into account in the calibration process, but it is not known how far this may overcome the problem of the number of parameters to be estimated.

The calibration or inverse problem has been the subject of considerable study in groundwater modelling. Early solutions relied on trial and error methods to reduce the difference between observed and calculated behaviour. Subsequently more rigorous mathematical methods have been developed (see for instance Garay et al., 1976; Guvanasen and Volker, 1978; Smith and Piper, 1978) including methods that try to incorporate subjective information on the nature of the system (Neuman, 1973). Methods have also been proposed for the identification of resistance functions in rivers by comparison of observed and computed downstream discharges (Fread, 1975; Becker and Yeh, 1973).

It is clear, however, that the calibration problem increases in difficulty with increasing complexity of the system, since due to the integrative nature of hydrological systems, the parameters may exhibit inter-dependence such that changing different parameter values may produce a similar effect. One study that simulated a complex hillslope system at a well instrumented site (Stephenson and Freeze, 1974) used trial and error calibration and concluded:

> "We recognise that our calibration is less than perfect, but it is probably representative of what can be attained when a fully deterministic mathematical model is applied to a field site with a fairly complex, but as always imperfect, set of field measurements" (Stephenson and Freeze, 1974, p 293).

Faced with this situation the success of the application of physically-based distributed models must depend on defining parameter values by a priori estimation of field measurement alone within a sufficiently close range to the true model value so that the calibration process assumes less significance and observed data can be used to identify specific deficiencies in the model.

6.4 Sensitivity analysis

The need for an analysis of the sensitivity of model simulations to changes in parameter values takes on an increased importance in the light of the parameter definition and model calibration problems discussed above. The methods of stochastic distributed modelling discussed in Section 4 are directly applicable and can give confidence limits around the predicted values of variables due to variance in the estimates of parameter values. However, these methods, and more standard methods of sensitivity analysis (see for instance Vemuri et al., 1969; McCuen, 1973, 1974; Saxton, 1975; Colman and De Coursey, 1976; Beven, 1979a) have only been applied to single process systems and extension to more complex systems may not be easy. The use of approximate sensitivity methods, in examining the effects of changes in certain parameter values within their specified range, remains very important for coupled systems.

The importance of sensitivity analysis in the application of distributed models has been discussed in Section 3.5. It is re-emphasised that this may prove to be one of the greatest benefits of physically-based modelling for which estimates of parameter uncertainty can be obtained independently of the model structure. Sensitivity analysis allows estimates of the reliability of deterministic predictions to be made which, at a later stage in the management decision process, should be incorporated into the evaluation of risk associated with project designs.

6.5 Planning scenario evaluation

We shall now assume that a model has been specified and been successfuly validated according to some criterion of utility. Such a model can then be used in a planning rôle to investigate, for example, the effects of alternative development scenarios on the hydrological regime of a river basin. At this stage the physical basis of distributed models and the possibility of specifying parameters a priori, at least within some range, assumes particular significance since the effects of various changes to the catchment or management strategies for the hydrologic system can be evaluated by changing the model parameter values. The utility of such predictions must be evaluated by comparing the amount of predicted change with the sensitivity of the predictions to the estimated range of given parameter values.

"To prophesy is extremely difficult - especially with respect to the future."

Chinese proverb.

7. FUTURE PROSPECTS

It is likely that the future development of improved distributed models will remain fragmented, with research aimed at improving the formulation and solution technique of models of individual processes and the coupling between models of component processes. However, the foundations of distributed mdoelling are now sufficiently firmly laid to enable general distributed models, such as the SHE model described above, to be made available for more routine application. Successful exploitation of these models will depend on a number of factors but the main scientific criteria to be satisfied are seen as follows.

(1) The development of techniques to enable both the efficient estimation of parameters to within some hydrologically meaningful level of accuracy, and the use of available data for calibration or tuning of the model.

(2) The development of a sufficiently flexible frame structure to enable a wide range of problems to be tackled easily.

(3) The addition of water quality (including sediment), components to the hydrological model to extend the range of potential problems that can be tackled by SHE into an area that often requires distributed predictions. (Such additions are envisaged as a second stage of the SHE project).

(4) The successul validation of the models (including the validation of water quality components where appropriate) on a number of real world problems, including problems posed in a water resource management context where data may be expected to be limited, at least prior to the application.

It should be noted that when the model is used specifically for scenario evaluation at the design phase of a management project, full validation can only take place following the implementation of decisions taken on the basis of model simulations. This last point is, therefore, a particularly stringent criterion of success, since it will test the theoretical concepts on which these physically-based models rest.

The economics of distributed modelling in the future will depend on improvements in modelling and parameter estimation techiques, the availability of improved computing techiques such as the more widespread introduction of parallel processors, and improved methods for handling large quantities of spatial data in model specification, calibration and validation. However, it seems likely at this stage that future developments, rather than leading to cheaper applications, may enable the present generation of relatively crude distributed models to attain a higher level of sophistication. Indeed the very availability of the spatially distributed data required by this type of model may require a critical re-examination of the theoretical constructs on which present models are based. This will be particularly important in the area of sensitivity analysis which is seen as one of the most important functions of this type of model. The limitations of the hydrological data on which many management decisions are based must be recognized, and should be taken into account in the decision-making process. Present physically-based models allow the effects of data uncertainty to be explored using crude parameter perturbation methods. In the future, improved model formulations and improved knowledge of the spatial variability of hydrological processes and parameters may allow this to be done in a more rigorous and satisfactory way.

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