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THE AREAL REDUCTION FACTOR IN RAINFALL FREQUENCY ESTIMATION

by

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ABSTRACT

This report details the work carried out to re-evaluate and check the areal reduction factors contained in the Flood Studies Report. Different definitions and methods are described and discussed. Following this areal and point rainfalls for various durations and United Kingdom locations are calculated which permit a direct estimate of ARF as the ratio of areal to point rainfall of the same return period. Good general agreement was found with the Flood Studies values at moderate return periods but a tendency to overestimate slightly at long return periods. The evidence for locational differences in ARF was inconclusive. Suggestions are made for further research and data requirements. The report includes a full account of the theory and the data handling procedures for which computer programs are given.

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1. INTRODUCTION

Volume II of the Flood Studies Report (Natural Environment Research Council, 1975) presents data for the estimation of rainfall depths corresponding to given durations and return periods for any point in the United Kingdom. Since engineering and hydrological applications of these data are usually concerned with volumes or average depths of rainfall over various areas rather than with depths at particular points, the Report also provides for conversion of point to areal values using "areal reduction factors". This conversion procedure may be expressed simply by:

$$R_a = ARF \times R_p \qquad \ldots\ldots (1)$$

where R_a = average rainfall depth over the area for the given duration and return period,
R_p = mean of point rainfall values within the same area for the same duration and return period,
ARF = areal reduction factor, varying with the duration and size of area.

The recommended values of ARF are tabulated in the Report for areas up to 30,000 km^2 and durations up to 25 days. A subset of these data is also published in graphical form and is reproduced in the present study in Figure 1.

It is assumed that ARF is approximately constant for all return periods and all parts of the United Kingdom but the validity of this assumption is not completely certain, as suggested in discussion at the 1975 Flood Studies Conference (proceedings published by Institution of Civil Engineers, May 1975). In the same discussion the method used for deriving the recommended values of ARF was questioned and also certain doubts were raised about the physical interpretation of ARF in rainfall frequency estimation. These expressions of concern have prompted further studies of the topic, some of which are reported in the present study.

2. INTERPRETATION AND DERIVATION OF ARF

The concept of an areal reduction factor in catchment rainfall studies has been widely accepted and applied in various countries of the world for several decades (see, for example, Linsley, Kohler and Paulhus,

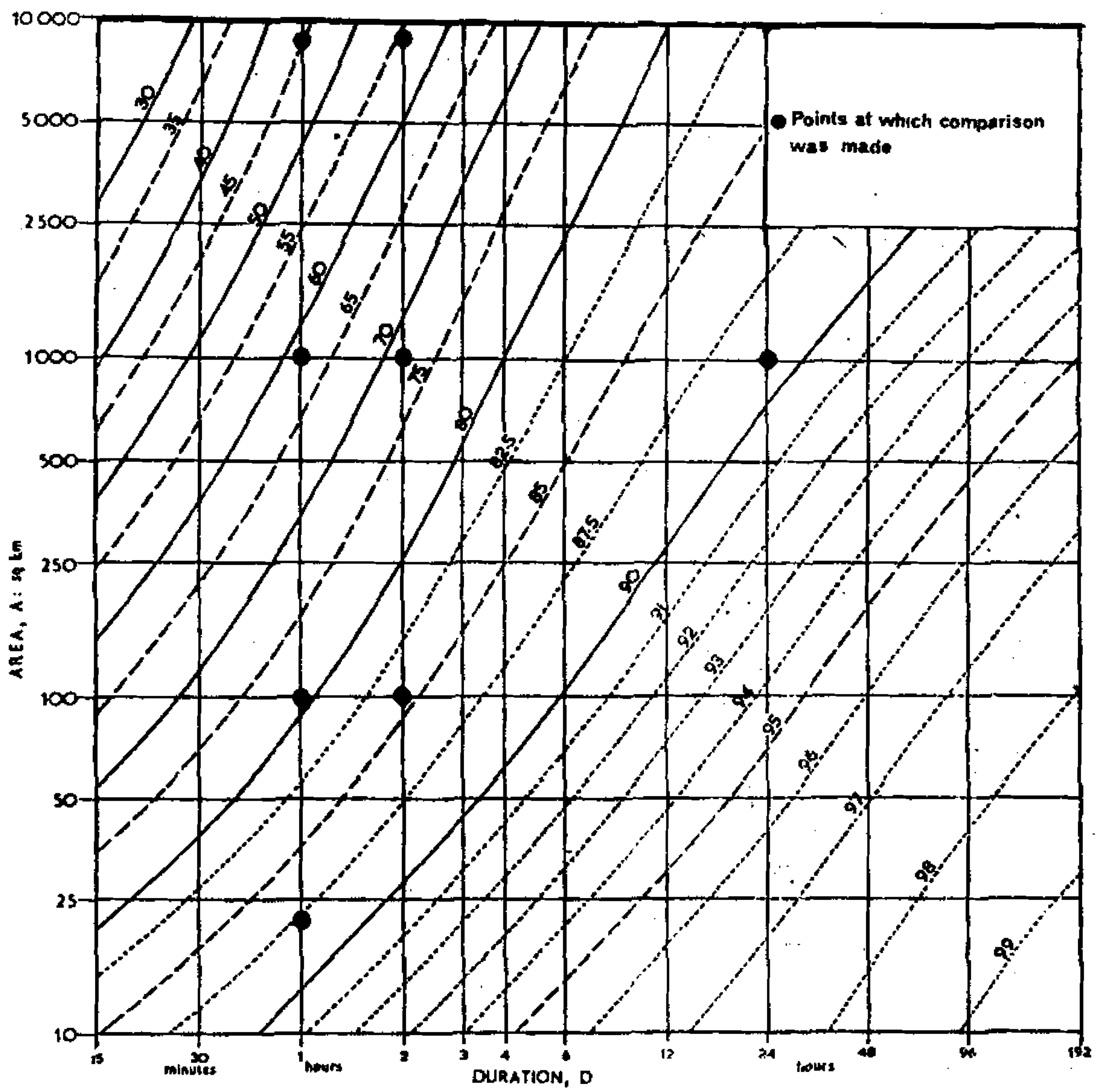


FIGURE 1 Areal reduction factor from Flood Studies Report

1949; Roche, 1963). In the United Kingdom prior to the Flood Studies Report, values of ARF were derived by Holland (1967) for a small range of areas and durations. Some detailed studies of both practical and theoretical aspects of the topic have been made in the United States, for example by the United States Weather Bureau (1960), Smith (1974) and Rodriguez-Iturbe and Mejia (1974).

As explained by Hershfield (1962), two types of ARF are recognised in the United States, viz storm-centred and fixed-area. Storm-centred values are used mainly for converting point estimates of probable maximum precipitation (PMP) to areal estimates, but are not recommended for frequency estimates, ie. when a definite return period is involved (United States Weather Bureau, 1960). For the latter purposes fixed-area ARFs are used, and therefore these are the type of ARF implied by Equation (1) and presented in the Flood Studies Report. The basic differences between the two types of ARF may be seen in their different methods of derivation as described below.

2.1 Storm-centred ARFs

Storm-centred ARFs are calculated for individual rainfall events from the ratio R_1/R_2 where:

R_1 = maximum areal rainfall within the storm zone for the given area and duration,
R_2 = maximum point rainfall within the same storm for the same duration.

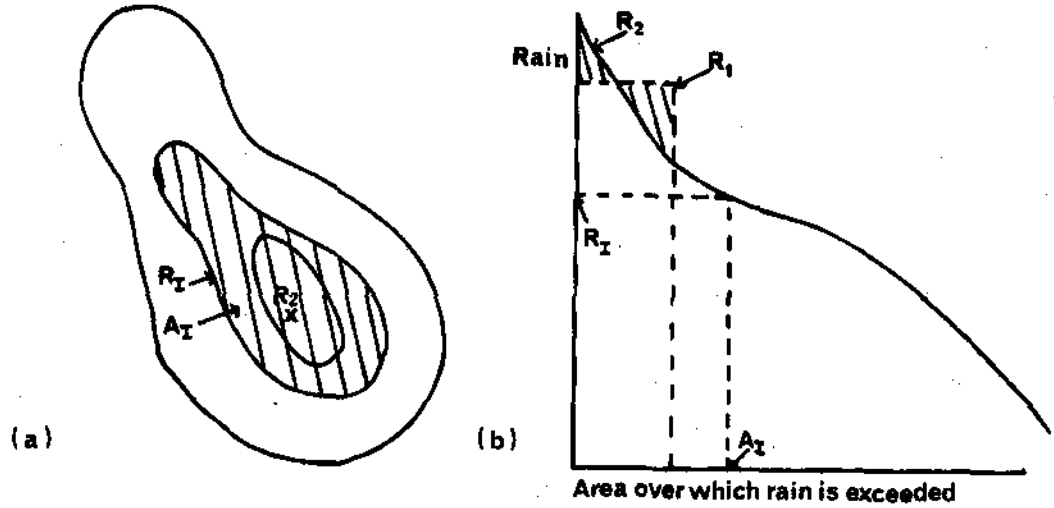
In general, the location of the given area for calculation of R_1 is determined by the isohyetal pattern and varies from storm to storm. The area is usually selected so that it is centred around the highest rainfall, R_2, its boundaries corresponding with a particular isohyet as shown in Figure 2 which illustrates the procedure.

Average values of storm-centred ARF have been derived on a regional basis by the U.S. Weather Bureau from major storm data for North America (Hershfield, 1962). An indication of the range of these values for a duration of 24 hours is given in Figure 3.

2.2 Fixed-area ARFs

The fixed-area ARFs used in the United States were derived originally from 10 to 15 years of data in a number of areas with high densities of rainfall stations (U.S. Weather Bureau, 1958; Rodriguez-Iturbe and Mejia, 1974). In the method of derivation, which is illustrated in Figure 4, it was assumed that the required ARFs were independent of return period and equal to the ratio R_3/R_4 where:

R_3 = mean of annual maximum areal values for the given duration and area,



1. For each duration and storm event plot isohyetal map (a)
2. Planimeter area within each isohyet to draw rain area curve (b)
3. *Interpolate to area of interest and calculate average rain with* the isohyet R_1
   This is given as the average rain ordinate between the origin and area of interest.
4. Calculate event $ARF = R_1/R_2$. Average over many events to give $\overline{R_1/R_2}$

FIGURE 2 Derivation of storm centred ARF

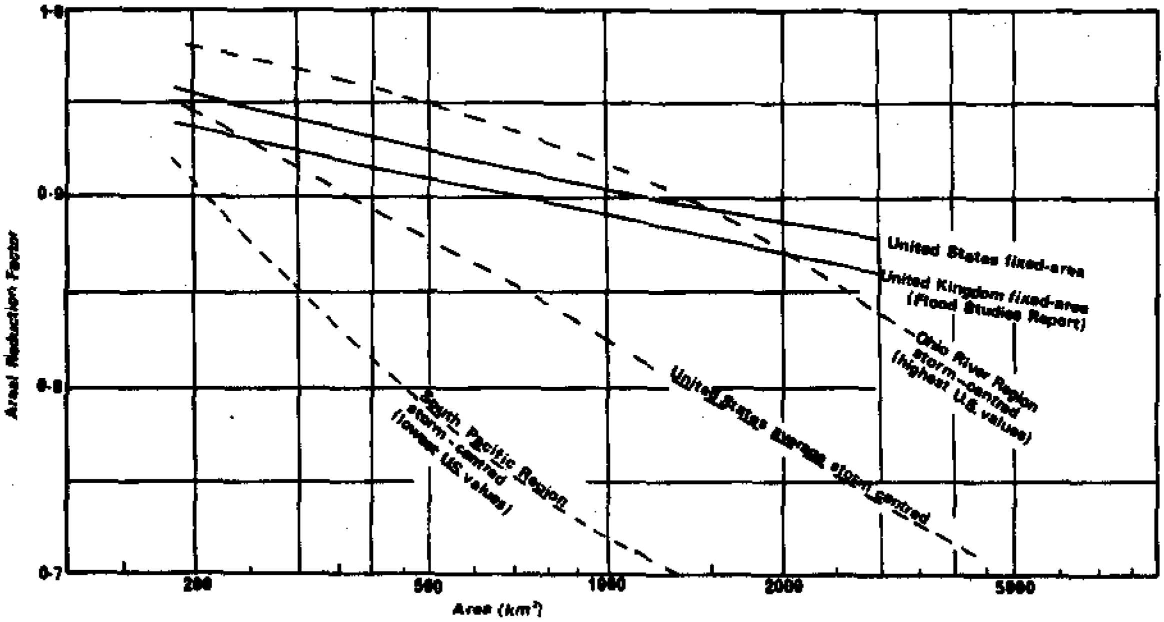
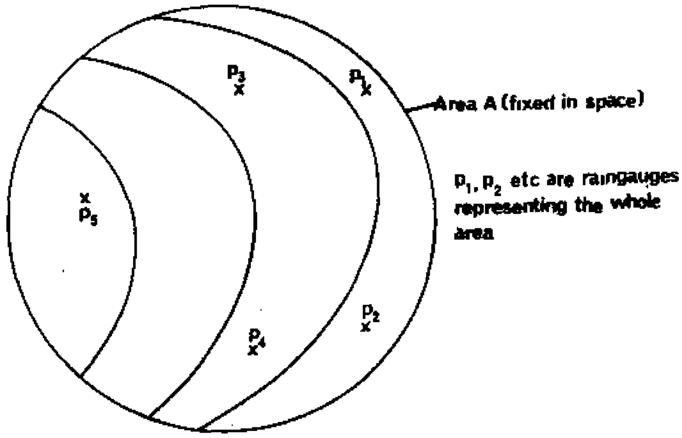


FIGURE 3 Comparison of UK and US Areal Reduction Factors



1. For each day of record calculate the average areal rainfall r_a. Use either isohyets or $r_a = w_1 r_{p1} + w_2 r_{p2}$ + etc where w is a Thiessen weight.
2. Extract the annual maximum values of r_a, $=R_a(1)$ in year 1, $R_a(2)$ in year 2 etc. up to $R_a(N)$ in year N.
3. Mean annual areal rainfall maximum $R_3 = \sum_1 Ra(i)/N$
4. From the records of raingauge p_1 extract the annual maximum values, $Rp_1(1)$ in year 1, $Rp_1(2)$ in year 2 up to $Rp_1(N)$ in year N.
5. Mean annual rainfall maximum $\bar{R}p_1 = \sum_1 Rp_1(i)/N$.
6. Repeat for p_2, p_3 etc and calculate $\bar{R}p_2$, $\bar{R}p_3$. Note that there is no requirement for the days on which the maxima occur to coincide.
7. Calculate areal average of the point mean annual rainfalls, $R_4 = w_1\bar{R}p_1 + w_2\bar{R}p_2$ + etc.
8. ARF = R_3/R_4
9. Repeat for other durations.

FIGURE 4 Derivation of Fixed Area ARF

R_4 = mean of annual maximum point values for the same duration, and for a number of points within the same area.

As regional variations in the ratio were generally less than five per cent, the same set of values was adopted for the whole of the United States. In most cases these are higher than the storm-centred ARFs, as exemplified by the comparison of 24-hour values shown in Figure 3.

Also shown in Figure 3 are the 24-hour fixed-area ARFs for the United Kingdom as given in the Flood Studies Report, and which differ only slightly from the United States values. In the derivation of the Flood Studies ARFs, which is illustrated in Figure 5, it was assumed that they are equal to the means of many sample values of the ratio R_5/R_6 where:

R_5 = rainfall at any point within the given area during the period of the annual maximum areal rainfall,

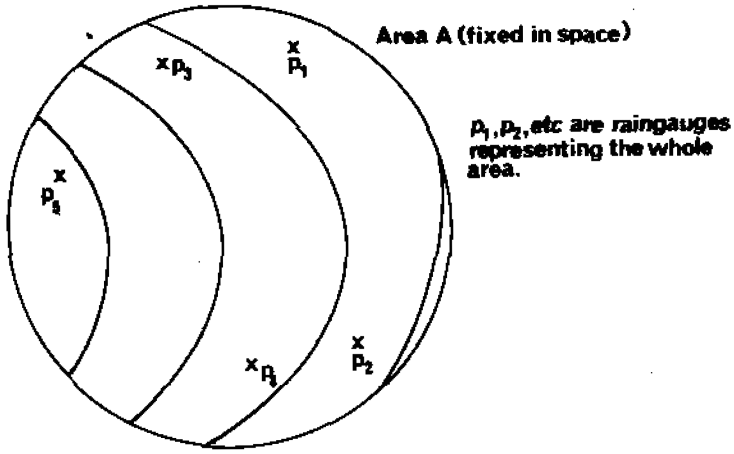
R_6 = annual maximum rainfall for the same point and for the same duration and year as R_5.

As there was no significant correlation between the ratios and R_6 it was assumed that ARF is independent of return period. An absence of distinct regional variations and close similarity to the United States values both supported the additional assumption that ARF does not vary much with geographical location.

Fixed-area ARFs are not directly related to the ratios of area to point rainfall in any individual recorded storm nor in any hypothetical design storm. Their conceptual significance is therefore not immediately obvious, being more statistical than physical. Perhaps the most lucid practical interpretation is in terms of point and areal frequency curves as shown in Figure 6. Here, fixed-area ARF is simply the ratio between areal and point rainfall with the same return period. Attempts to interpret it directly in terms of the characteristics of particular storms commonly result in its confusion with storm-centred ARFs and with other parameters largely irrelevant to rainfall frequency estimation. Misconceptions of this type are clearly evident in some of the previously mentioned criticisms of the Flood Studies Report at the Flood Studies Conference (Institution of Civil Engineers, 1975).

Nevertheless, the derivation methods adopted by both the U.S. Weather Bureau and the United Kingdom Flood Studies group provide only indirect estimates of fixed-area ARF, the validity of which appears to depend on some assumptions not thoroughly tested. A less equivocal method would be to derive the values directly from the appropriate areal and point frequency distributions as suggested by Figure 6. Any tendency for ARF to vary with return period should be clearly revealed by this method, whereas the other methods tend to obscure such variations because of their pooling of the data.

Possible reasons why ARFs have not been derived from frequency curves in past studies are (a) the considerable computational effort required, and (b) the expectation of large sampling errors due to the relative brevity of most records suitable for estimates of areal rainfall. There is no doubt that rainfall frequency estimates from brief records are inaccurate but this does not necessarily result in large sampling errors in ARF because of the high degree of positive correlation between point and areal rainfall. Thus, if the observed point rainfall from a sample of data for a given return period tended to be, for example, higher than the population value, then the corresponding areal rainfall would also tend to be higher and therefore the ratio between the two should still be close to the correct value. Direct derivation of ARF from frequency curves has been adopted in the present study and it will be shown in Section 5 that the associated sampling errors are not excessive.



1. For each day of interest within the record calculate the average areal rainfall, r_a.
2. Extract the annual maximum value and note the day on which it occurred.
3. Note the point rainfall values for that same day, $=R_5(p_1)$, $R_5(p_2)$, $R_5(p_3)$ etc.
4. For the same year extract the maximum point rainfalls at each point, $=R_6(p_1)$, $R_6(p_2)$ etc. In some cases R_5 and R_6 will coincide, while in others R_6 will exceed R_5.
5. For each point and year calculate R_5/R_6. ARF is grand average over points and years.
6. *Repeat for other durations.*

FIGURE 5 Derivation of Flood Studies Report ARF

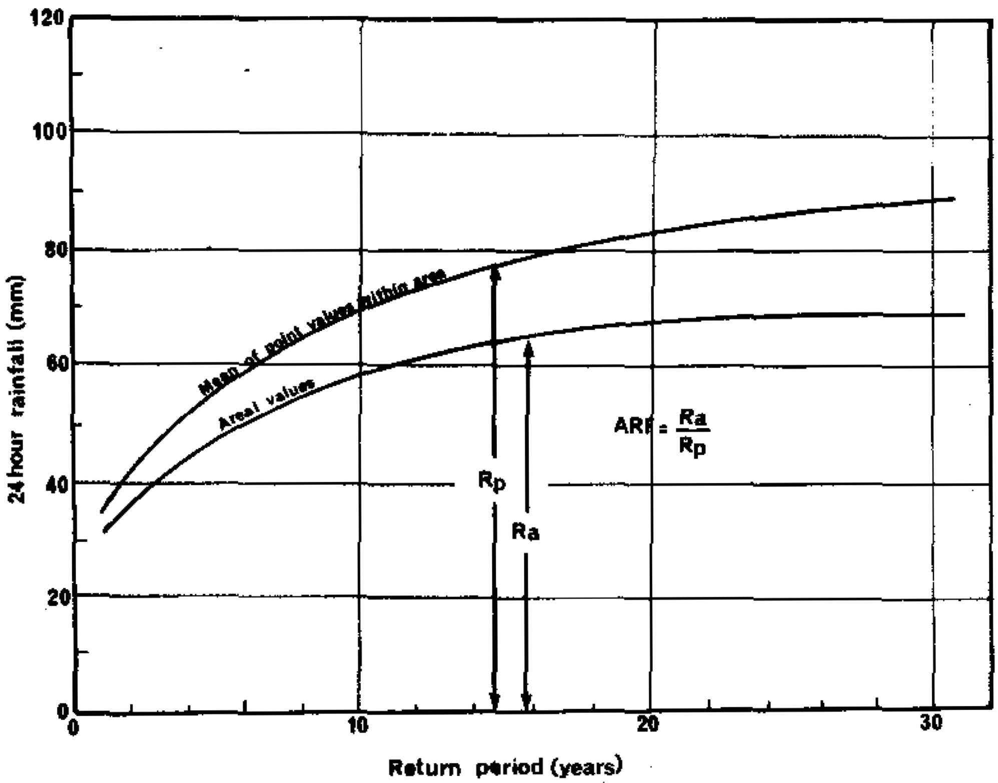


FIGURE 6 Interpretation of Areal Reduction Factor

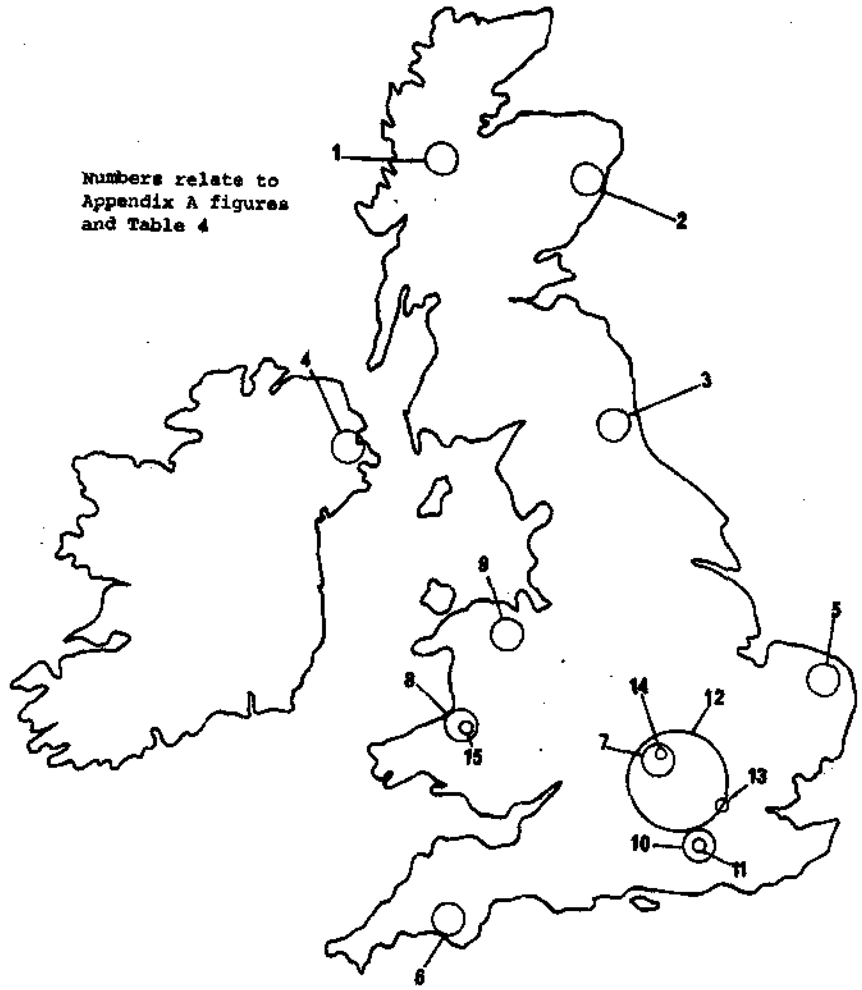


FIGURE 7 Location of areas selected for analysis of areal rainfall

3. ESTIMATION OF 24-HOUR ARF FOR SAMPLE AREAS OF 1000 KM^2

It was decided to test the 24-hour ARF values of the Flood Studies Report with samples of data from a set of records designated *British Rainfall Data* held by the Institute of Hydrology on magnetic tape. These records had been assembled by the Meteorological Office and consisted of daily rainfall observations at some 14000 stations in the United Kingdom for the 14-year period 1961 to 1974.

For the analysis of areal rainfalls using the above data, nine circular areas of 1000 km^2 were selected at the locations indicated by the numbers 1 to 9 in Figure 7. These were selected so that (a) the general range of meteorological conditions in the United Kingdom was sampled, and (b) each area contained at least 12 rainfall stations with reasonably complete records. The objective was to derive frequency curves of areal and average (or representative) point rainfall for each circular area to permit estimation of 24-hour ARFs as shown in Figure 6.

3.1 Determination of Areal Rainfall Frequency Curves

Before areal rainfall frequency curves could be derived it was necessary to calculate mean rainfall depths within the circular areas (representing total volumes of rainfall on each area) for every day of the 14-year record. Statistical analyses of these daily areal values were then made in essentially the same way as point rainfalls are usually analysed to derive point frequency curves.

Several methods of calculating mean areal rainfall depths were considered, including Thiessen polygons and various trend surface analyses (Rainbird, 1967; Mandeville and Rodda, 1970; Lee, Lynn and Shaw, 1974). The adopted method consisted of dividing the stations in each sample area into between three and seven groups so that equal Theissen polygons could be constructed around the group centres of gravity. A modified Thiessen weighting of $1/nN$ was assigned to each station, where n = number of stations in group (varying because of incomplete records at some stations), and N = number of groups in sample area (constant for entire period of records). This method of calculating areal rainfalls was found to have certain computational advantages over the normal Thiessen method and gave virtually the same estimates.

The daily areal rainfalls from the above calculations were ranked to give the 20 highest values for each sample area. Any of these values in the same storm period (defined as a period of consecutive days with rain) were considered non-independent and therefore only the maximum daily rainfall in each storm period was used. Following the recommendations of Alexander (1970), the rainfalls selected by this procedure were regarded as a partial duration or peaks-over-threshold (POT) series with an exponential distribution. The corresponding frequency curves may be expressed by:

$$T = T_o \exp\left[(x - x_o)/B\right] \qquad \text{..... (2)}$$

or the equivalent:

$$x = B \log_e (T/T_o) + x_o \qquad \text{..... (3)}$$

where: x = rainfall,
T = return period of x, ie. the average period in years between values equal to, or greater than x,
x_o = base or selected minimum value of x,
T_o = return period of x_o

B = slope parameter of distribution (theoretically equal to the standard deviation of the population).

For all sample areas x_o was selected to have a return period (T_o) of 1.5 years because small systematic departures from the exponential distribution were observed for shorter return periods. Also, rainfalls with shorter return periods are usually of less practical interest. Derivation of the areal rainfall frequency curves was therefore a matter of evaluating the parameters x_o and B for each sample area. This was done by the maximum likelihood method described in Vol I of the Flood Studies Report which uses the equations:

$$\hat{B} = \frac{N}{N-1} (\bar{x} - x_1) \qquad \ldots\ldots (4)$$

$$\hat{x}_o = \bar{x} - \frac{\hat{B}}{N} \qquad \ldots\ldots (5)$$

where: $\hat{x}_o$ and $\hat{B}$ are maximum likelihood estimates of x_o and B respectively,

N = size of sample (=9 for 14 years of data and T_o = 1.5 years),

$\bar{x}$ = average of sample values,

x_1 = lowest value in sample

Substitution of the appropriate values of x_o, B and T in Equation (3) thus provided estimates of areal rainfall for any specified return period. In particular, the 2, 5, 10 and 20-year rainfalls were estimated in this manner and their values are listed in Table 1.

Frequency curves corresponding to Equation (3) and the derived values of x_o and B are plotted graphically in Appendix A for each area. On the same diagrams are tabulated the ranked rainfalls which are also plotted graphically according to the Gringorten formula, as recommended in Vol I of the Flood Studies Report. These points provide visual checks on the accuracy of the frequency calculations and they also verify that the exponential distribution is appropriate for the ranges of values considered.

Table 1 Areal rainfalls from frequency curves for areas 1 to 9

SAMPLE AREA	2-YEAR R.P.		5-YEAR R.P.		10-YEAR R.P.		20-YEAR R.P.		NOTES
	RAINFALL	S.E.	RAINFALL	S.E.	RAINFALL	S.E.	RAINFALL	S.E.	
1	50	4	61	7	69	10	77	13	Rainfalls are in mm.
2	34	3	42	5	49	7	55	9	
3	31	2	37	4	42	6	47	8	S.E. = Standard error
4	35	4	45	7	53	9	62	13	(calculated from equation (8))
5	27	2	34	4	38	6	43	7	
6	41	4	52	7	60	9	68	12	
7	32	3	41	5	47	7	52	9	
8	52	5	65	8	76	11	86	15	
9	38	2	44	3	48	4	51	6	

3.2 Determination of Average Point Rainfall Frequency Curves

For the derivation of average point frequency curves it was necessary to obtain the 20 highest daily rainfalls for every station with 12 or more years of data. As in the case of areal rainfalls, only the maximum value from each storm period was included to ensure independence.

Statistical analyses of the ranked rainfalls could have been made to derive separate frequency curves for each station in a sample area, and an appropriately weighted average of these curves would have provided the required average point frequency curve for the area. However, a computationally simpler but numerically equivalent procedure was adopted. This procedure made use of the fact that the same theoretical return period and plotting position applies to all values of the same rank. The weighted average point value was calculated for each rank and each area, the weightings being determined by the same modified Thiessen method used for estimating areal rainfalls. Small adjustments were made to the rainfall values of stations with less than 14 years of data to interpolate them to the standard set of plotting positions.

The ranked average point rainfalls for each area were given a similar statistical treatment to the ranked areal rainfalls. Exponential distributions were assumed and their parameters were estimated by Equations (4) and (5), as described in Section 3.1. The resulting 2, 5, 10 and 20-year values of point rainfall for each area are listed in Table 2, and the corresponding frequency curves are plotted on the same diagrams as the areal rainfall frequency curves in Appendix A.

Computer programmes were formulated for extracting the daily rainfalls from the British Rainfall Data magnetic tape, and for performing the above calculations for both areal and point rainfall. Copies of these are given in Appendix II.

Table 2 Point rainfalls from frequency curves for areas 1 to 9

SAMPLE AREA	2-YEAR R.P.		5-YEAR R.P.		10-YEAR R.P.		20-YEAR R.P.		NOTES
	RAINFALL	S.E.	RAINFALL	S.E.	RAINFALL	S.E.	RAINFALL	S.E.	
1	52	4	65	5	75	6	85	7	Rainfalls are in mm.
2	36	3	46	3	55	4	61	5	S.E. = Standard error
3	35	3	42	3	48	4	53	5	(calculated from equation (8))
4	38	3	50	4	59	5	68	6	
5	29	2	38	3	46	4	52	4	
6	45	3	59	4	70	6	80	7	
7	34	3	46	3	54	4	63	5	
8	61	5	77	6	90	7	103	9	
9	43	3	52	4	59	5	65	5	

3.3 Estimation of ARF

The required values of ARF were calculated directly from corresponding rainfalls in Tables 1 and 2, ie. from areal and average point rainfalls having the same return period. The resulting ARFs range from 0.80 to 0.95 with a mean of 0.88 as shown in Table 3. The Flood Studies ARF for the same duration and area is 0.89 (see Table 4), indicating a reasonable agreement.

Table 3 ARF calculated from values in Tables 1 and 2

SAMPLE AREA	2-YEAR R.P.		5-YEAR R.P.		10-YEAR R.P.		20-YEAR R.P.		NOTES
	ARF	S.E.	ARF	S.E.	ARF	S.E.	ARF	S.E.	
1	.95	.04	.93	.06	.92	.08	.91	.10	
2	.95	.04	.92	.05	.90	.07	.89	.10	S.E. = Standard error
3	.89	.04	.89	.06	.89	.08	.89	.09	(calculated from equation (9))
4	.90	.06	.90	.08	.90	.10	.90	.13	
5	.95	.05	.88	.05	.86	.07	.82	.09	
6	.90	.04	.88	.07	.86	.09	.85	.11	*mean S.E. was calculated from:
7	.93	.04	.87	.06	.86	.08	.82	.10	$\sqrt{\frac{\Sigma S.E.^2}{9}}$
8	.86	.04	.85	.06	.84	.08	.84	.11	
9	.89	.04	.84	.05	.81	.05	.80	.05	
MEAN	.91	*.04	.88	*.06	.87	*.08	.86	*.10	

EXPECTED ARF FROM F.S.R. = .89 FOR ALL SAMPLE AREAS

Table 4 ARF from Flood Studies Report

REF No. (FIG 7)	LOCATION	AREA km^2	DURATION hrs	No OF STATIONS	YEARS OF RECORD	ARF in F.S.R.
1	Scottish Highlands	1000	24	24	14	0.89
2	Aberdeen	1000	24	14	14	0.89
3	Newcastle-Hexham	1000	24	19	14	0.89
4	Belfast	1000	24	35	14	0.89
5	Norwich	1000	24	31	14	0.89
6	Plymouth	1000	24	31	14	0.89
7	Grendon Underwood	1000	24	37	14	0.89
8	Plynlimon	1000	24	25	14	0.89
9	River Dee	1000	24	31	14	0.89
10	Surrey	1000	1 & 2	8	12	0.61, 0.72
11	Surrey	100	1 & 2	3	12	0.79, 0.84
12	Chilterns	8000	1 & 2	14	9	0.46, 0.57
13	Greenwich	100	2	9	8	0.84
14	Grendon Underwood	20	1	3	12	0.88
15	Plynlimon	20	1	3	7	0.88

4. ESTIMATION OF 1-HOUR AND 2-HOUR ARFs FOR VARIOUS AREAS

It would have been desirable to evaluate the 1-hour and 2-hour ARFs from as wide a range of geographical locations as was used for the 24-hour ARFs. Unfortunately this was not possible because the only closely spaced networks of recording raingauges with suitable lengths of record appeared to be in southern England and in Wales.

Annual maximum 1-hour and 2-hour rainfalls were obtained from the Meteorological Office for a number of stations centred around the Chilterns and in Surrey with records varying in length from 9 to 12 years. Although some of these data were also used in the derivation of the original Flood Studies ARFs it was still considered desirable to include them in the present study because the completely different method of derivation would not necessarily result in the same values of ARF. The Chilterns and Surrey data permitted the estimation of average point and areal frequency curves for areas of 100, 1000 and 8000 km^2, the locations of which are shown in Figure 7.

In the estimation of the Chilterns and Surrey frequency curves, the procedure for obtaining ranked areal and average point rainfalls was the same as that for the 24-hour rainfalls. However, it was necessary to assume a different form of frequency distribution, viz the extreme value type I or Gumbel distribution, since the ranked values were annual maxima rather than partial duration series. Maximum likelihood estimates of the distribution parameters were made with the equations given in Section I.1.3.4 of the Flood Studies Report which are more complex than Equations (4) and (5). Average point and areal rainfalls and corresponding ARFs were then calculated for return periods of 2.54, 5.52, 10.51 and 20.5 years respectively, based on a theoretical relationship between the two series suggested by Langbein (1949). The resulting rainfalls and ARFs were therefore consistent with the 2, 5, 10 and 20-year values derived for other areas. They are listed in Tables 5, 6 and 7.

A considerable amount of short-duration rainfall data has been collected by the Greater London Council and should prove valuable for future studies of rainfall in the London area. Little use was made of this information in the present study because (a) it was still being processed and insufficient time was available for extraction of the raw data, and (b) the area was close to the Chilterns and Surrey districts for which data had already been obtained as described previously. However, the Council made available a number of isohyetal maps and other records which provided reasonable estimates of all important 2-hour rainfalls in a 100 km^2 area near Greenwich for the 8-year period to June, 1976. Average point and areal frequency curves were derived from these data by the procedure described in Section 3, except that areal rainfalls were estimated by the isohyetal method (planimetric measurement of areas between isohyets) rather than by the modified Thiessen method. Again, the resulting 2, 5, 10, and 20-year rainfalls and corresponding ARFs are listed in Tables 5, 6 and 7.

Table 5 Areal rainfalls from frequency curves for areas 10 to 15

SAMPLE AREA	DURATION (HRS)	2-YEAR R.P.		5-YEAR R.P.		10-YEAR R.P.		20-YEAR R.P.		NOTES
		RAINFALL	S.E.	RAINFALL	S.E.	RAINFALL	S.E.	RAINFALL	S.E.	
10	1	10	1	12	2	13	4	14	2	
10	2	15	1	17	2	18	3	20	3	
11	1	12	1	14	1	15	2	17	3	See Tables 1 and 2
11	2	17	1	20	2	22	3	25	4	
12	1	6	1	8	2	10	3	11	4	
12	2	11	1	13	3	16	3	18	4	
13	2	12	3	20	6	26	8	32	10	
14	1	8	1	11	2	13	2	15	3	
15	1	13	2	16	3	19	4	22	5	

Table 6 Point rainfalls from frequency curves for areas 10 to 15

SAMPLE AREA	DURATION (HRS)	2-YEAR R.P.		5-YEAR R.P.		10-YEAR R.P.		20-YEAR R.P.		NOTES
		RAINFALL	S.E.	RAINFALL	S.E.	RAINFALL	S.E.	RAINFALL	S.E.	
10	1	15	2	20	4	23	5	22	6	
10	2	20	2	26	4	30	6	34	7	
11	1	16	2	22	4	26	5	30	7	See Tables 1 and 2
11	2	21	2	26	4	29	5	33	6	
12	1	11	3	16	5	21	7	25	9	
12	2	17	3	23	5	27	7	31	8	
13	2	18	4	22	7	29	9	36	12	
14	1	9	1	12	2	15	3	18	4	
15	1	15	2	19	3	22	5	26	6	

Table 7 ARF calculated from values in Tables 5 and 6

SAMPLE AREA	DURATION (HRS)	2-YEAR R.P.		5-YEAR R.P.		10-YEAR R.P.		20-YEAR R.P.		MEAN* ARF	F.S.R. ARF
		ARF	S.E.	ARF	S.E.	ARF	S.E.	ARF	S.E.		
10	1	.66	.06	.58	.07	.54	.03	.51	.08	.57	.61
10	2	.73	.06	.66	.07	.62	.08	.59	.09	.65	.72
11	1	.75	.08	.64	.09	.59	.09	.56	.10	.64	.79
11	2	.81	.05	.78	.07	.76	.08	.74	.09	.77	.84
12	1	.53	.10	.48	.11	.45	.11	.44	.12	.48	.46
12	2	.63	.07	.59	.10	.58	.11	.57	.11	.59	.57
13	2	.95	.10	.92	.12	.91	.13	.91	.13	.92	.84
14	1	.98	.10	.89	.12	.85	.13	.82	.14	.89	.88
15	1	.89	.09	.86	.11	.86	.13	.85	.15	.87	.88

* MEAN ARF = MEAN OF 2, 5, 10 and 20 YEAR VALUES

The remaining rainfall and ARF values in Tables 5, 6 and 7 were calculated from the Grendon Underwood and Plynlimon experimental catchment data collected by the Institute of Hydrology. In these cases the two 20 km^2 circular areas were positioned around the rain gauges so that equal *Thiessen weightings applied to all gauges*. Otherwise the computations were essentially the same as for the 24-hour ARFs as described in Section 3.

Graphical plots of all the above 1-hour and 2-hour frequency curves are included in Appendix A. Tables on the same diagram list the ranked areal and point rainfalls and other relevant information.

5. ESTIMATION OF SAMPLING ERRORS

Tables 1 to 3 and 5 to 7 list estimated standard errors for the calculated rainfall and ARF values. In the determination of these errors it was assumed that the major source of uncertainty is due to limited sample sizes, ie. to the necessarily finite records used for estimating point and areal rainfall frequencies. Other errors such as those due to spatial sampling and measurement inaccuracies were regarded as either insignificant or mutually compensating.

The tabulated standard errors were estimated by the principles described in Section 1.4 of the Flood Studies Report from which it may be shown that the sampling distributions of parameters B and x_o in the exponential distribution have variances given by:

$$\text{var } \hat{B} = \frac{B^2 (N + 1)}{(N - 1)^2} \qquad \text{..... (6)}$$

and

$$\text{var } \hat{x}_o = \frac{B^2 (N^3 - 2N^2 + 2N + 1)}{N^2 (N - 1)^2} \qquad \text{..... (7)}$$

But the form of the exponential distribution is such that

$$\hat{x} = \hat{x}_o + B\log_e(T/T_o)$$

$$\therefore \text{var } \hat{x} = \text{var } \hat{x}_o + (\log_e(T/T_o))^2 \text{ var } \hat{B} \quad \text{(assuming } \hat{x}_o \text{ and } \hat{B} \text{ are independent)}$$

$$\therefore \text{var } \hat{x} = B^2 \left[\frac{N^2 - N+2+ (N-1) (\log_e(T/T_o))^2}{N(N-1)^2} \right] \qquad \text{..... (8)}$$

where $\hat{x}$ = estimated point or areal rainfall corresponding to a particular return period;

other symbols are the same as for Equations (4) and (5). Equation (8) was used to estimate the variances of point and areal rainfalls, and the square roots of these variances are the standard errors listed in Tables 1, 2, 5 and 6.

Reverting to the symbols R_p for the average point rainfall and R_a for corresponding areal rainfall, the variances of ARF may be expressed in terms of the rainfall variances as follows:

$$\text{var ARF} = \text{var}\left(\frac{R_a}{R_p}\right) = \left(\frac{R_a}{R_p}\right)^2 \left(\frac{\text{var } R_a}{R_a^2} + \frac{\text{var } R_p}{R_p^2} - \frac{2 \text{ cov } (R_a, R_p)}{R_a \; R_p}\right) \qquad \ldots\ldots (9)$$

(see Kendall and Stuart, 1961)

The covariance of R_a and R_p may be calculated from their correlation coefficient r by:

$$\text{cov } (R_a, R_p) = r\sqrt{\text{var } R_a \text{ var } R_b} \qquad \ldots\ldots (10)$$

Appropriate values of r for Equation (10) could not be estimated directly from the paired point and areal rainfalls in Tables 1, 2, 5 and 6. This would have been invalid because the rainfall sampling distributions vary with location as well as with return period, duration and area, and no method of grouping could allow for all these sources of variability. In an attempt to overcome this difficulty the tabulated rainfalls were "standardised" by the conversion:

$$y = \frac{x - x_e}{x_e}$$

where x = original point or areal rainfall,
x_e = estimate of "true" or population value of x for the given return period, as obtained from the frequency data in Vol II of the Flood Studies Report,
y = "standardised" point or areal rainfall.

In the estimation of x_e allowances were made for the fact that most of the rainfalls in Tables 1, 2, 5 and 6 are for fixed-interval durations (rainfall days or clock hours) while the Flood Studies data are for unrestricted durations, as explained in Sections II.3.2 and II.3.3 of the Flood Studies Report.

After standardisation the rainfalls were grouped according to return period and duration with nine pairs of values in each group. Correlation coefficients were then calculated for each group, as listed in Table 8, and these were used in Equations (9) and (10) for estimating the variances of ARF. The square roots of the variances are the standard errors shown in Tables 3 and 7.

Table 8 Correlations between standardised point and areal rainfalls

R.P.	2-YEAR		5-YEAR		10-YEAR		20-YEAR		Computed from 9 pairs of standardised values in each group.
DURN.	24 HRS	1 & 2 HRS	24 HR	1 & 2 HR	24 HR	1 & 2 HR	24 HR	1 & 2 HR	
r	.87	.90	.89	.75	.83	.70	.76	.68	

6. DOES ARF VARY WITH LOCATION?

To answer this question in statistical terms two appropriate null hypotheses may be formulated, namely (a) that ARF does not vary with location, and (b) that there is no significant difference between the ARFs derived in this study and the corresponding ARFs of the Flood Studies Report. Since standard errors have been estimated these hypotheses may be formally tested by the usual procedures with the assumption that the rainfalls of each of the selected areas represent an independent sample. It is obvious that neither hypotheses would be rejected by such tests because the largest differences between comparable ARFs in Tables 3 and 7 are generally of the same magnitudes as the standard errors. One is led to conclude that the observed variability in ARF between different locations may be fully explained by sampling errors and there is no significant difference between the values derived in this study and those of the Flood Studies Report.

However, several reservations should be expressed concerning these conclusions. Firstly, the testing of the hypotheses is not strictly valid if there is significant correlation between rainfall frequencies in different sample areas, ie. if they are not independent. Unfortunately, some degree of correlation may be expected because of the extensive spatial coverage of meteorological conditions associated with exceptional rainfalls. Secondly, failure to reject the hypotheses on the evidence of calculated sampling errors does not preclude their possible rejection on other evidence. In other words, there could still be some variations due to locational factors although their magnitudes should not exceed the calculated sampling errors in Tables 3 and 7.

Further investigation of this issue included the calculation of correlation coefficients between the average 24-hour ARFs for each area and the following locational factors:

Rainfall magnitude as expressed by the 5-year value from Vol II of the Flood Studies Report,

Ratio of 60-minute, 5-year rainfall to 2-day, 5-year rainfall (index of local convective activity),

Latitude,

Longitude.

The only significant correlation was found with latitude, the coefficient being 0.69 which just reached the 95 per cent level of significance (using the equal tails test described in Crow *et al*, 1960). Therefore, there may be a trend towards higher values of 24-hour ARF in more northerly latitudes but no definite conclusions should be drawn concerning the magnitude of such a trend, except that it is probably smaller than the sampling errors listed in Table 3.

A similar analysis was not carried out for the 1-hour and 2-hour ARFs because of the disparities in the sizes of the areas and the inadequate range of locations.

7. DOES ARF VARY WITH RETURN PERIOD?

Tables 3 and 7 suggest a consistent trend towards lower values of ARF with longer return periods. Two methods of testing the significance of this were used, namely (a) the non-parametric sign test, and (b) an adaptation of the t test for comparing means of samples from populations with different variances (p. 60, Crow *et al*, 1960).

Application of the sign test to the 24-hour ARFs in Table 3 showed the differences between grouped values for any pair of return periods to be significant at the 95 per cent level. When applied to the short-duration ARFs in Table 6, this test showed even more significant differences at the 99 per cent level. The more efficient t test also indicated significant differences between the 2-year and 20-year 24-hour ARFs at the 95 per cent level. The latter test was not used with any other values because of doubts concerning independence of samples and homogeneity of the populations represented.

The data therefore provide reasonable evidence that ARFs decrease with increasing return periods. The differences between 2-year and 20-year values are apparently of the order of 2 per cent to 5 per cent for 24-hour ARFs and 5 per cent to 15 per cent for 1-hour and 2-hour ARFs.

In general, the values of ARF in the Flood Studies Report correspond to return periods of 5 to 10 years and tend to be conservative for the longer return periods commonly adopted for engineering design purposes.

A theoretical estimate of the limiting value of ARF (ie. with an infinitely long return period) is given by the ratio B_a/B_p where

B_a = parameter B in Equation (3) for areal rainfalls

B_p = parameter B in Equation (3) for corresponding average point rainfalls.

This is readily demonstrated by expressing ARF in terms of Equation (3), viz:

$$ARF = \frac{x_o \text{ (area)} + B_a T}{x_o \text{ (point)} + B_p T} \rightarrow \frac{B_a}{B_p} \text{ as } T \rightarrow \infty$$

Table 9 shows limiting values of ARF for 24 hours and 1000 km^2 calculated by means of the above ratio. These range from 0.57 to 0.90 with a mean of 0.77 which is about 12 per cent lower than the Flood Studies ARF and about 6 per cent lower than the corresponding average storm-centred ARF used in the United States for PMP estimates (see Figure 3).

Also shown in Table 9 are limiting values of 1-hour and 2-hour ARFs calculated in the same way. They are very variable, some being as much as 25 per cent lower than the corresponding Flood Studies ARFs. Comparable United States storm-centred ARFs for these durations do not appear to be available.

Table 9 Limiting ARFs calculated from B_a/B_p

SAMPLE AREA	DURATION	20-YR ARF	B_a	B_p	LIMITING ARF
1	1 day	.91	12.0	14.4	.83
2	1 day	.89	8.8	11.0	.80
3	1 day	.89	7.1	7.9	.89
4	1 day	.90	11.8	13.2	.90
5	1 day	.82	6.6	10.1	.65
6	1 day	.85	11.6	15.2	.76
7	1 day	.82	8.8	12.8	.69
8	1 day	.84	14.6	18.1	.81
9	1 day	.80	5.4	9.5	.57
10	1 hr	.51	1.90	5.43	.35
10	2 hrs	.59	2.40	6.10	.39
11	1 hr	.56	2.10	6.00	.35
11	2 hrs	.74	3.30	5.30	.62
12	1 hr	.44	2.40	6.27	.38
12	2 hrs	.57	3.13	6.07	.52
13	2 hrs	.91	9.12	10.32	.88
14	1 hr	.82	2.60	4.00	.65
15	1 hr	.85	3.90	4.75	.82

8. FURTHER RESEARCH NEEDS

Although it may require a substantial amount of processing and computation there are ample daily rainfall data available in the United Kingdom to extend the above analyses of 24-hour ARFs to:

(a) longer return periods,
(b) longer durations such as 2, 3 or more days,
(c) areas other than 1000 km^2
(d) non-circular areas.

Information on all of these would be useful in the further testing and refining of the Flood Studies ARFs for relatively long durations.

Unfortunately, hydrological needs more often involve shorter durations and the data situation for these is less satisfactory. A thorough investigation of short-duration ARFs would probably require at least 15 years of recording raingauge data from dense networks of stations representing the general range of meteorological conditions in the United Kingdom. It may be some years before such data become available, and their analyses could be a formidible task, even with the latest processing and computing techniques.

On the other hand, extreme rainfall data for durations of 1 or 2 hours show surprising consistencies over diverse and extensive areas (see Bell, 1969). This is apparently because such extremes are mainly due to local convection (ie. thunderstorm activity) which has similar space-time characteristics under a wide range of geographical conditions. The Flood Studies assumption that ARF is essentially constant throughout the United Kingdom therefore seems quite reasonable for short durations, and probably should be accepted until adequate data are available to show otherwise. Whether the Flood Studies ARFs for short durations should be adjusted to allow for the effects of return period is another matter, however, since the results of the present study suggest that such effects could be appreciable for long return periods and maximum rainfalls. Perhaps further light will be shed on this particular aspect when the analyses of the Greater London Council data are complete.

The question arises as to whether storm-centred ARFs (which may be derived from relatively short periods of data) provide more satisfactory estimates for very long return periods and maximum rainfalls than the fixed-area ARFs of the Flood Studies Report. It should be pointed out that the rationale for using storm-centred ARFs for maximum rainfall events is not completely convincing. Is there any reason why the maximum areal rainfall could not have a more uniform spatial distribution (and therefore higher ARF) than the "average" storm event? If not, the United States practice of using average storm-centred ARFs for PMP may be rather doubtful. Similar comments are relevant to possible applications of theoretical limiting values of ARF such as those estimated from the ratio B_a/B_p in Section 7. Before definite recommendations can be made about these values, further investigations are needed into such matters as the suitability of the exponential frequency distribution for the total range of extreme rainfalls, and the influence of the assumed distribution on the calculations.

Some of the above issues might be clarified by appropriate research with theoretical models linking point and areal rainfall, such as that of Rodriguez-Iturbe and Mejia (1974). Evaluations of the parameters of these models could be made without dense networks of stations, although they would require data from a few closely spaced recording rain gauges in each meteorological region. The possible use of such models with the data of this study was one of the reasons for adopting the exponential frequency distribution as given by Equation (3). The relatively simple mathematical form of this distribution makes it more tractable in theoretical analyses than extreme value, log-normal and other commonly used distributions.

9. SUMMARY AND CONCLUSIONS

Several of the criticisms of the areal rainfall section in the Flood Studies Report suggest that there is some misunderstanding of the areal reduction factor (ARF) in rainfall frequency estimation. In particular, the fixed-area ARFs of the Flood Studies Report have been confused with storm-centred ARFs of the type derived in the United States for purposes other than rainfall frequency estimation. The conceptual significance of the fixed-area ARF is more statistical than physical and it is probably best interpreted in terms of the areal and average point frequency curves, being simply the ratio of areal to point rainfall with the same return period.

The least equivocal method of deriving values of ARF appears to be directly from frequency curves of areal and average point rainfall, and this method has been used in the present study to check the values given by the Flood Studies Report. Nine circular areas of 1000 km^2 were selected for the derivation of 24-hour ARFs from a common 14-year period of data. Six areas of varying size with 7 to 12 years of data were selected for the derivation of 1-hour and 2-hour values. Inaccuracies were expected due to the relatively brief records but estimates of the sampling errors in ARF showed that these were not excessive.

The main results of the analyses, as listed in Tables 3 and 7, show derived values of ARF that have reasonable agreement with the corresponding values of the Flood Studies Report. Although the variability between locations can be explained completely by sampling errors there may be a slight tendency for 24-hour ARFs to increase with latitude. The maximum discrepancy due to this effect is probably less than 3 per cent.

A statistically significant trend towards lower ARFs with longer return periods was found for both 24-hour and short-duration values. This suggests that the ARFs of the Flood Studies Report probably give conservatively high estimates of areal rainfall for the return periods commonly used for engineering design purposes (say, 10 to 100 yrs). The resulting bias may be of the order of 5 per cent for 24-hour durations and 10 per cent or more for 1-hour and 2-hour durations, but there is considerable uncertainty about the short-duration values because of the lack of suitable data for investigating these.

Although it is possible to make estimates of theoretical limiting values of ARF for maximum rainfalls and very long return periods, such estimates vary widely and definite recommendations concerning their use should not be made without further research. It may be worth while, also, to carry out research into the application of theoretical models linking point and areal rainfall, such as that of Rodriguez-Iturbe and Mejia (1974). These models might clarify some of the doubtful issues and possibly reduce the problems of brief records, inadequate spatial coverage and tedious data extraction.

Additional testing of the Flood Studies ARFs along the lines developed in this paper could be made with currently available daily rainfall data. This should involve longer records, durations greater than 24 hours, non-circular areas, and areas other than 1000 km^2. At the present time suitable data from recording raingauges is generally of insufficient length for comparable 1-hour and 2-hour analyses. Nevertheless, further information on short-duration ARFs might be provided in the near future from the Greater London Council data.

Although the evidence in this paper suggests possible variations in ARF that are not allowed for in the Flood Studies Report, these variations do not appear to be large when compared with inaccuracies due to sampling and other factors in practical applications of rainfall frequency estimates. When adequate periods of recording raingauge data become available from closely spaced stations in more northerly parts of the United Kingdom it should be possible to determine values of ARF with reasonable precision and confidence. This may eventually result in replacement of the present Flood Studies method of estimating areal rainfalls with a necessarily more elaborate method to allow for location and return period as well as for area and duration. With the present data situation, however, such refinement seems to have little justification as it would greatly increase the required computational effort for little overall improvement in accuracy.

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Appendix A: PLOTTED FREQUENCY CURVES USED FOR DERIVATION OF ARF

Notes on Symbols in Figures A1 to A18

1. Number in brackets after location refers to map reference number in Figure 7 and Table 4 of main text.

2. In the sections headed "duration", (F) refers to fixed periods, ie. between standard times such as 9 am to 9 am or between fixed clock-hours. (U) refers to unrestricted periods.

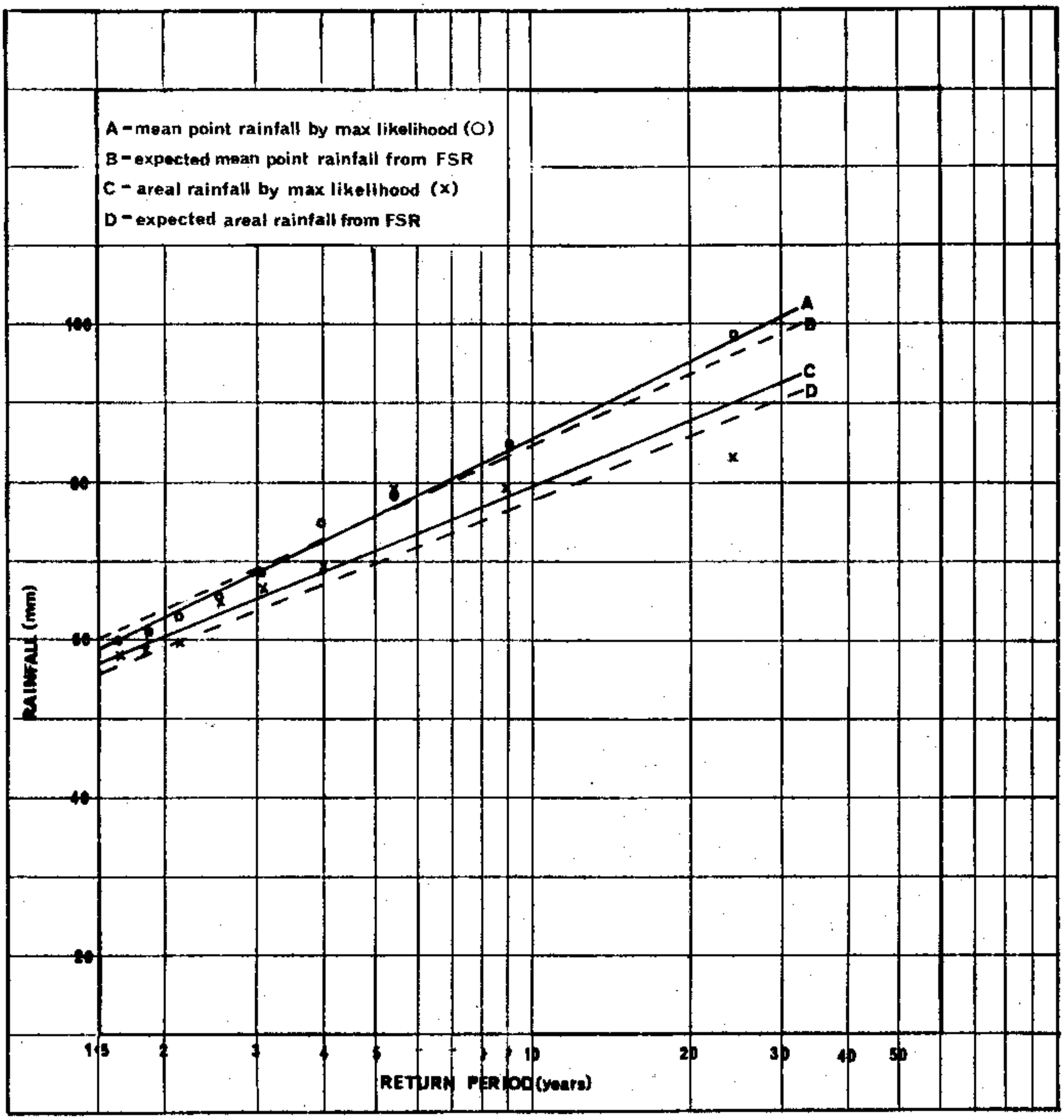
3. Number in brackets after number of rainfall stations indicates actual number of stations used to determine average point frequency curve if this differs from the number of stations used to determine areal rainfalls.

4. Frequency Curves marked B and D were calculated directly from the data given in Vol II of Flood Studies Report. Annual maxima return periods were converted to POT return periods using the relationship given by Langbein (1949).

Computation of Areal Reduction Factor.

LOCATION SCOTTISH HIGHLANDS (1)	AREA: 1000 km²
	DURATION: 1 DAY (F)
Number of rainfall stations: 24(10)	Period of data: Jan 1961 – Dec 1974 (incl)

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	88·4	74·2	68·3	64·5	58·1	55·4	52·3	50·7	49·8
AREAL RAINFALL (mm)	72·6	69·4	68·8	58·6	56·2	55·2	49·5	49·1	47·6
PLOTTING POSITION (yrs)	25·2	9·05	55·2	3·97	3·10	2·54	2·15	1·87	1·65



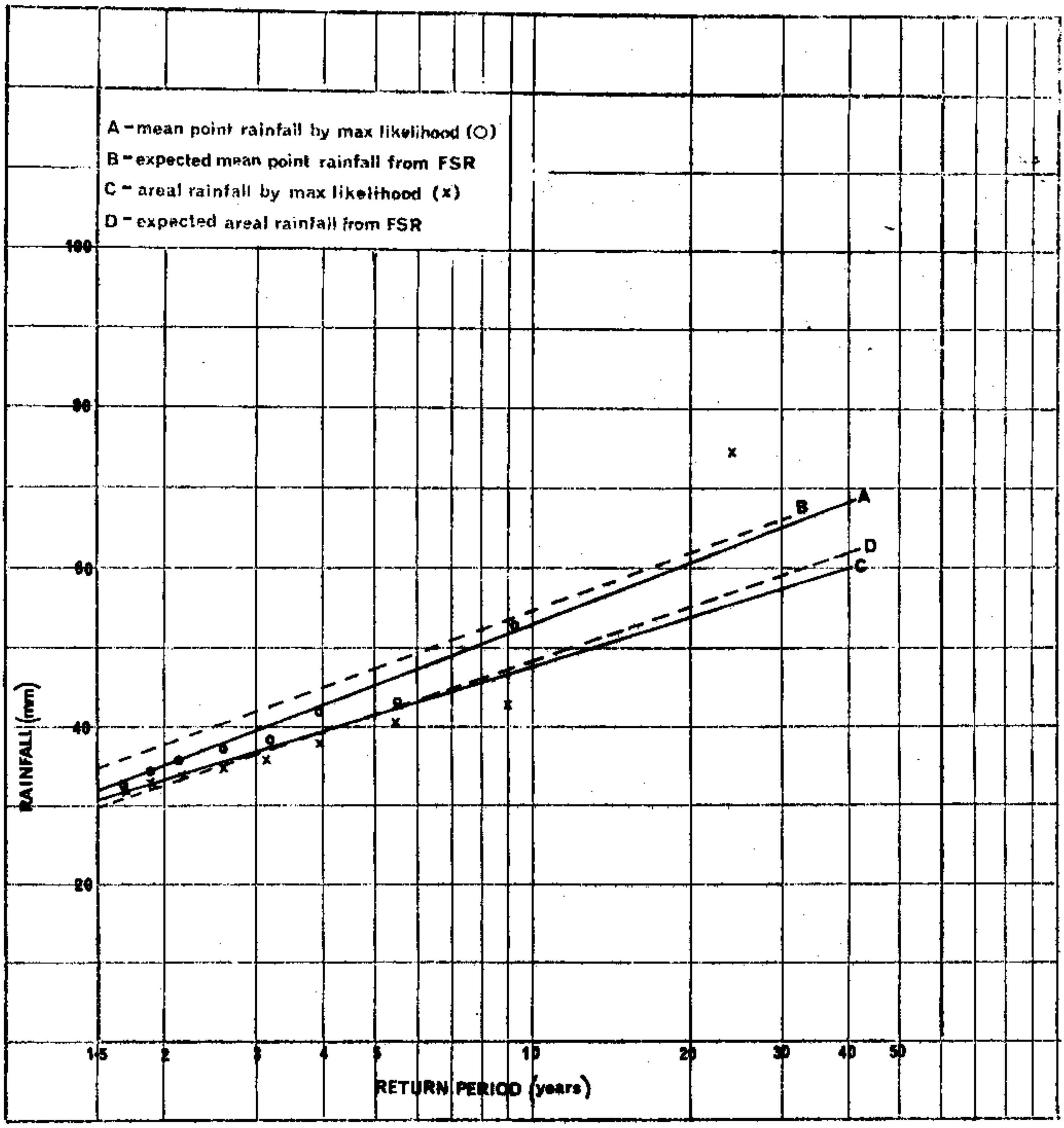
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·95	0·93	0·92	0·91
ARF EXPECTED FROM FSR	0·90			

Fig. A1 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION **ABERDEEN (2)**	AREA: **1000km²**	DURATION: **1 DAY(F)**
Number of rainfall stations: **14 (8)**	Period of data: **Jan.1961 – Dec.1974 (incl)**	

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	74·7	53·5	43·8	42·4	39·1	38·0	36·9	35·2	33·7
AREAL RAINFALL (mm)	74·7	42·3	40·6	38·0	36·5	35·5	34·1	33·7	32·2
PLOTTING POSITION (yrs)	25·2	9·05	5·52	3·87	3·10	2·54	2·15	1·87	1·65



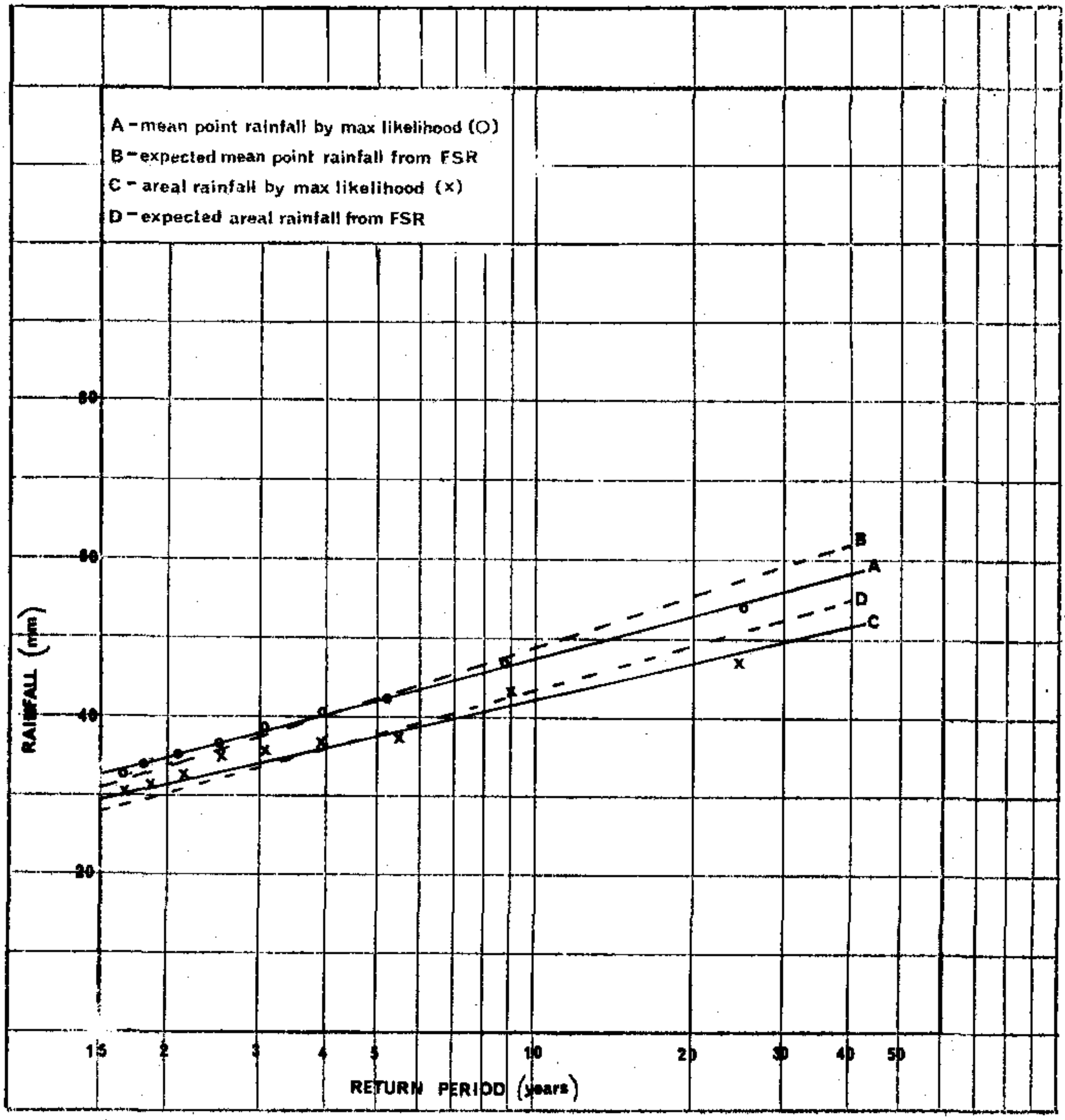
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·95	0·92	0·90	0·89
ARF EXPECTED FROM FSR	0·89			

Fig.A2 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION NEWCASTLE-HEXHAM (3)	AREA: 1000 km²	DURATION: 1 DAY (F)
Number of rainfall stations: 19 (13)	Period of data: Jan 1961 – Dec 1974 (incl)	

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	54·1	47·6	42·8	40·4	38·8	36·8	35·3	34·6	33·4
AREAL RAINFALL (mm)	47·3	42·3	37·2	36·7	35·5	34·3	32·0	30·4	29·6
PLOTTING POSITION (yrs)	25·2	9·05	5·52	3·97	3·10	2·54	2·15	1·87	1·65



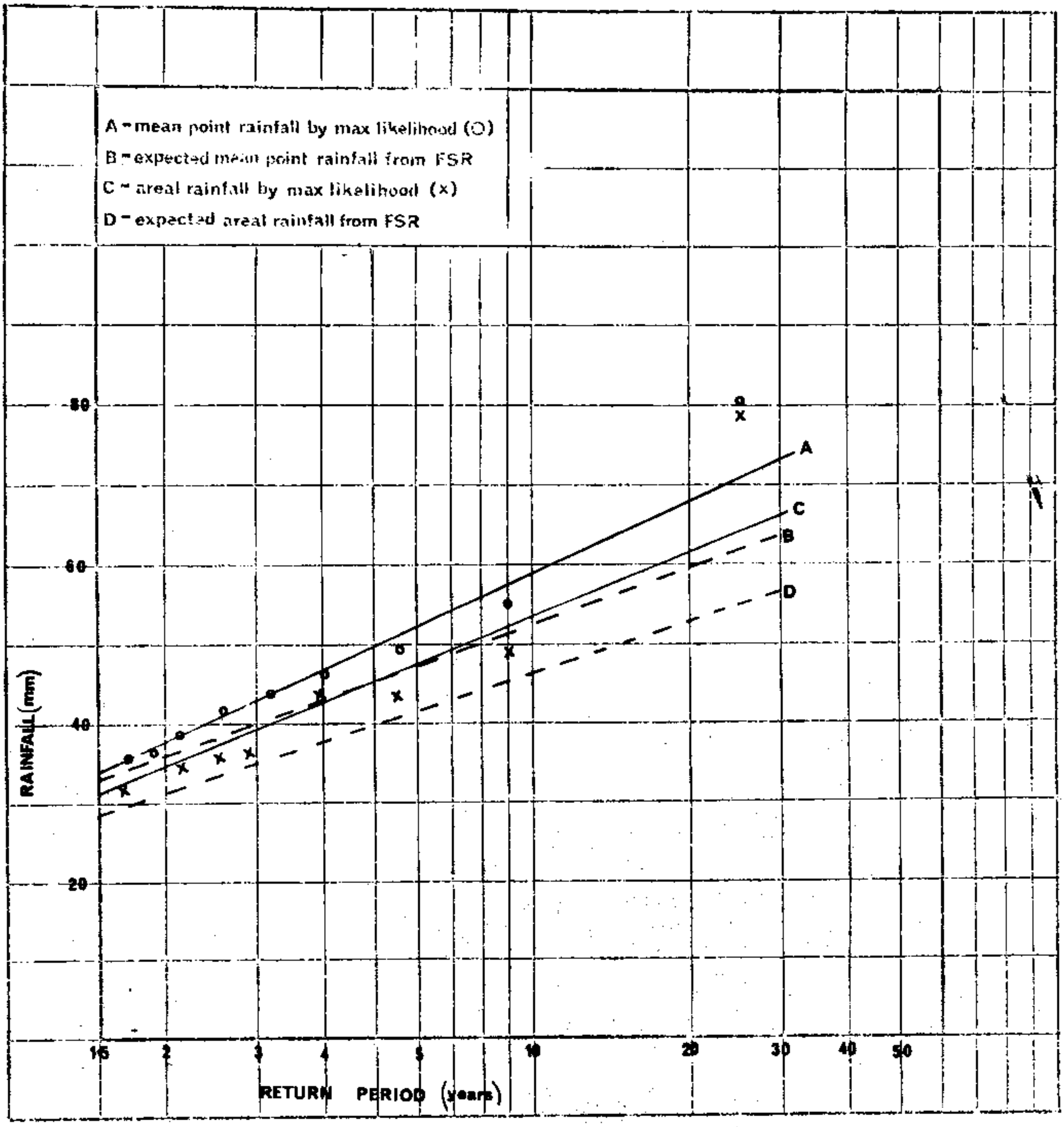
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·89	0·89	0·89	0·89
ARF EXPECTED FROM FSR	0·89			

Fig. A3 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION **BELFAST** (4)	AREA: **1000km²**	DURATION: **1 DAY** (F)
Number of rainfall stations: **35** (20)	Period of data: **Jan 1961 – Dec 1974**	

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	[illegible]
MEAN POINT RAINFALL (mm)	81·3	55·5	49·7	46·2	43·9	41·8	38·4	36·7	35·7
AREAL RAINFALL (mm)	78·7	49·2	43·3	43·3	41·6	35·1	33·9	32·5	31.5
PLOTTING POSITION (yrs)	25·2	9·05	5·52	3·97	3·10	2·54	2·15	1·87	1·65



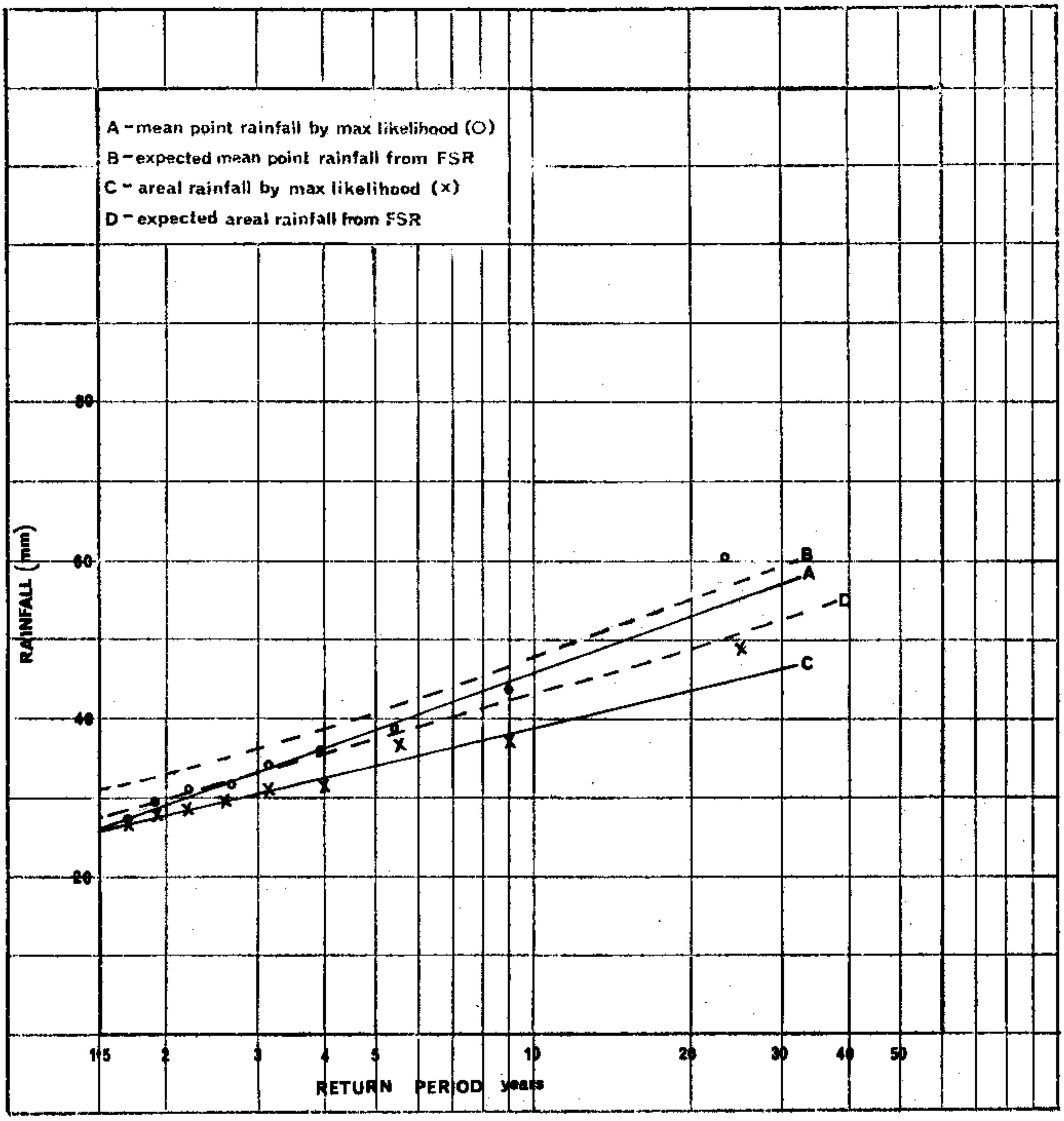
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUANCY ANALYSIS	0·90	0·90	0·90	0·90
ARF EXPECTED FROM FSR	0·89			

Fig. A4 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION **NORWICH (5)**	AREA: **1000 km²**
DURATION: **1 DAY (F)**	
Number of rainfall stations: **31 (14)**	Period of data: **Jan 1961 Dec 1974 (incl)**

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	60·1	43·6	38·8	35·7	33·8	31·6	30·5	29·3	26·9
AREAL RAINFALL (mm)	48·3	36·9	35·9	30·6	30·2	29·3	28·0	27·4	26·2
PLOTTING POSITION (yrs)	25·2	9·05	5·52	3·97	3·10	2·54	2·15	1·87	1·65

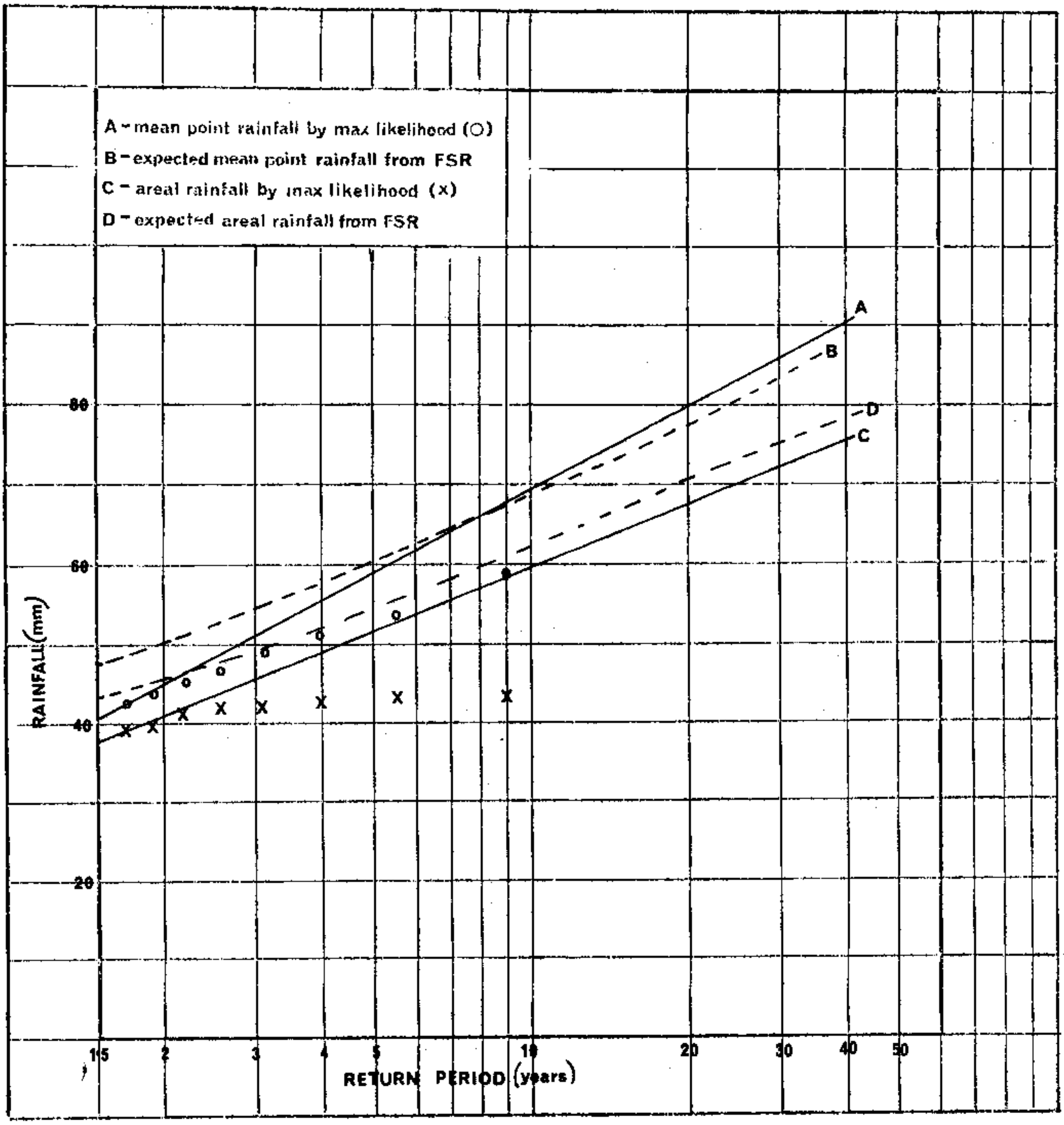


RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·95	0·88	0·86	0·82
ARF EXPECTED FROM FSR	0·89			

Fig. A5 Frequency Curves.

LOCATION PLYMOUTH (6)	AREA: 1060km²	DURATION: 1 DAY (F)
Number of rainfall stations: 31 (23)	Period of data: Dec 1961 — Jan 1974 (incl)	

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	119·0	58·5	53·7	51·0	48·8	46·5	45·2	43·8	42·1
AREAL RAINFALL (mm)	116·0	43·1	42·6	42·3	41·9	41·8	41·0	39·9	38·8
PLOTTING POSITION (yrs)	25·2	9·05	5·52	3·97	3·10	2·54	2·15	1·87	1·65



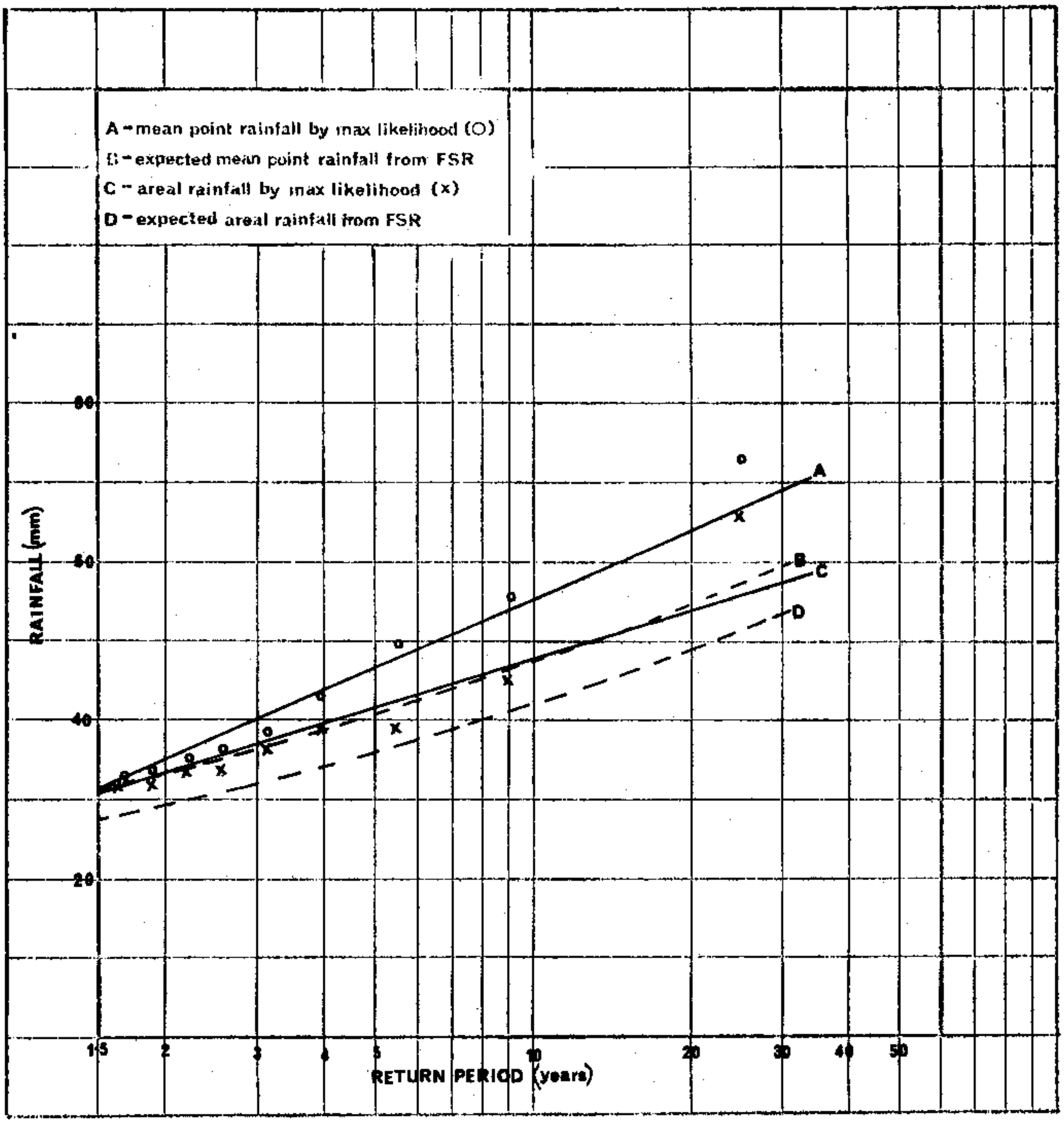
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUANCY ANALYSIS	0·90	0·88	0·86	0·85
ARF EXPECTED FROM FSR	0·89			

Fig.A6 Frequency Curves.

Computation of Areal Reduction Factor

LOCATION **GRENDON-UNDERWOOD** (7)	AREA: 1000 km²
	DURATION: 1 DAY (F)
Number of rainfall stations: 37 (18)	Period of data: Jan 1961 - Dec 1974 (incl)

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	72·7	55·2	49·1	42·2	37·9	35·4	34·0	32·6	32·0
AREAL RAINFALL (mm)	65·8	44·2	37·8	37·5	35·4	33·0	32·6	30·7	30·6
PLOTTING POSITION (yrs)	25·2	9·05	5·52	3·97	3·10	2·54	2·15	1·87	1·65



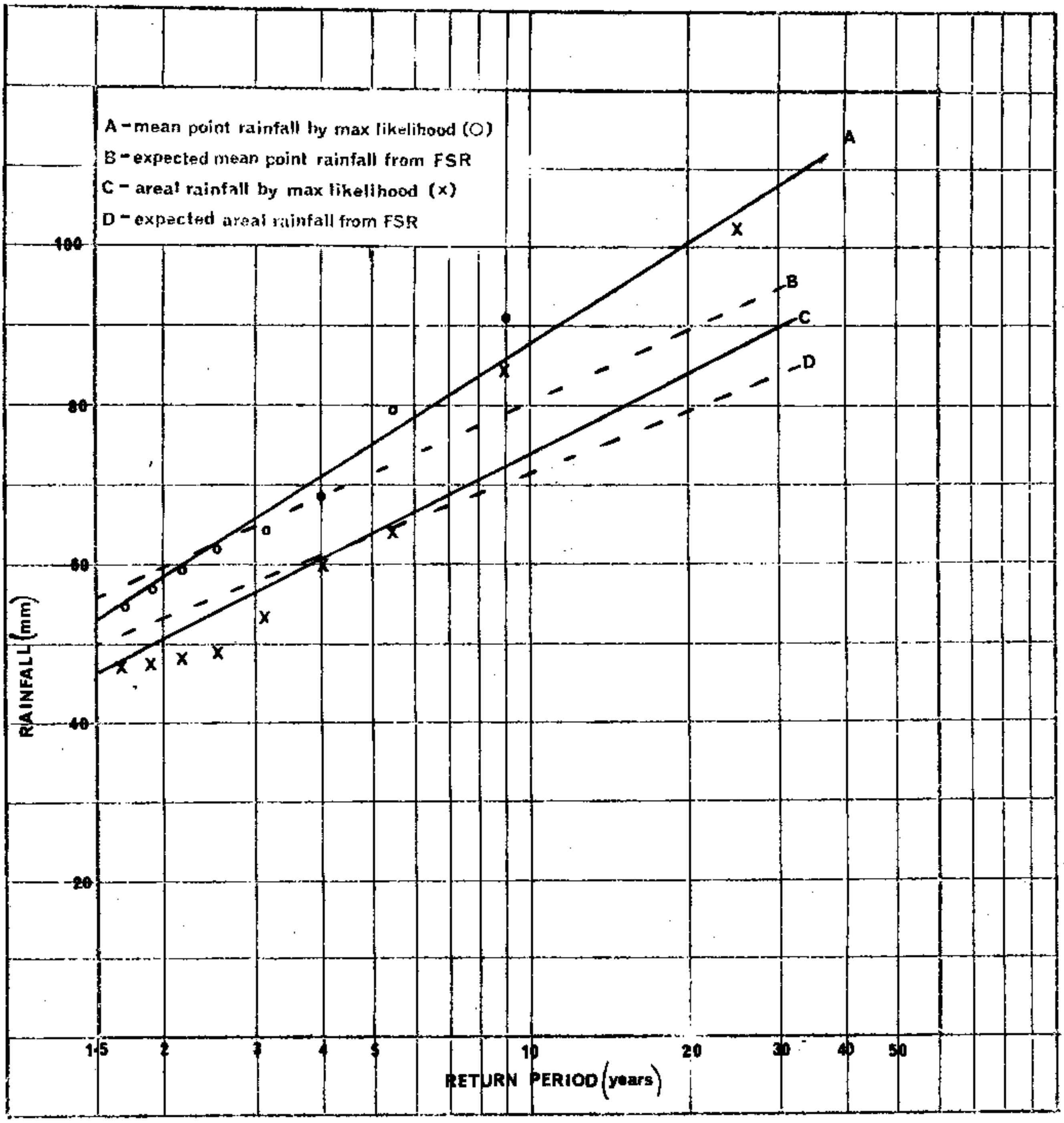
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·93	0·87	0·86	0·82
ARF EXPECTED FROM FSR	0·89			

Fig. A7 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION PLYNLIMON (8)	AREA: 1000km^2	DURATION: 1 DAY (F)
Number of rainfall stations: 25 (12)	Period of data: Jan 1961 – Dec 1974 (incl)	

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	117·4	91·3	81·9	70·5	66·4	64·0	61·3	59·3	57·5
AREAL RAINFALL (mm)	101·2	86·6	66·1	60·7	55·4	49·9	49·6	49.3	48·7
PLOTTING POSITION (yrs)	25·2	9·05	5·52	3·97	3·10	2·54	2·15	1·87	1·65



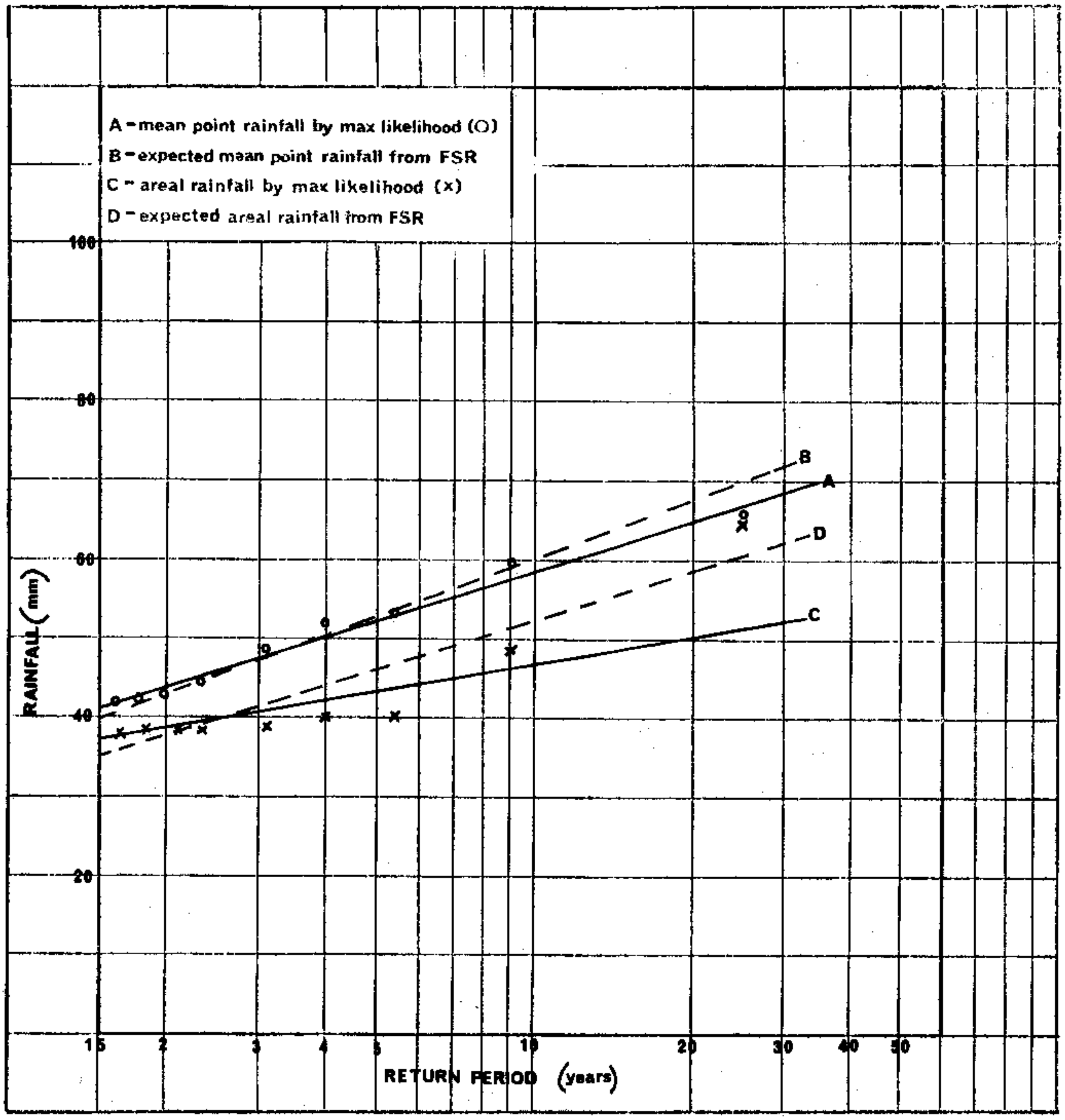
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·86	0·85	0·84	0·84
ARF EXPECTED FROM FSR	0·89			

Fig.A8 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION **RIVER DEE** (9)	AREA: 1000 km²
	DURATION: 1 DAY (F)
Number of rainfall stations: 31 (13)	Period of data: Jan 1961 - Dec 1974 (incl)

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	66·5	59·7	53·9	51·2	48·2	44·9	43·4	42·4	41·8
AREAL RAINFALL (mm)	64·9	48·7	39·5	39·5	38·9	38·7	38·4	38·2	37·8
PLOTTING POSITION (yrs)	25·2	9·05	5·52	3.97	3·10	2·54	2·15	1.87	1·65



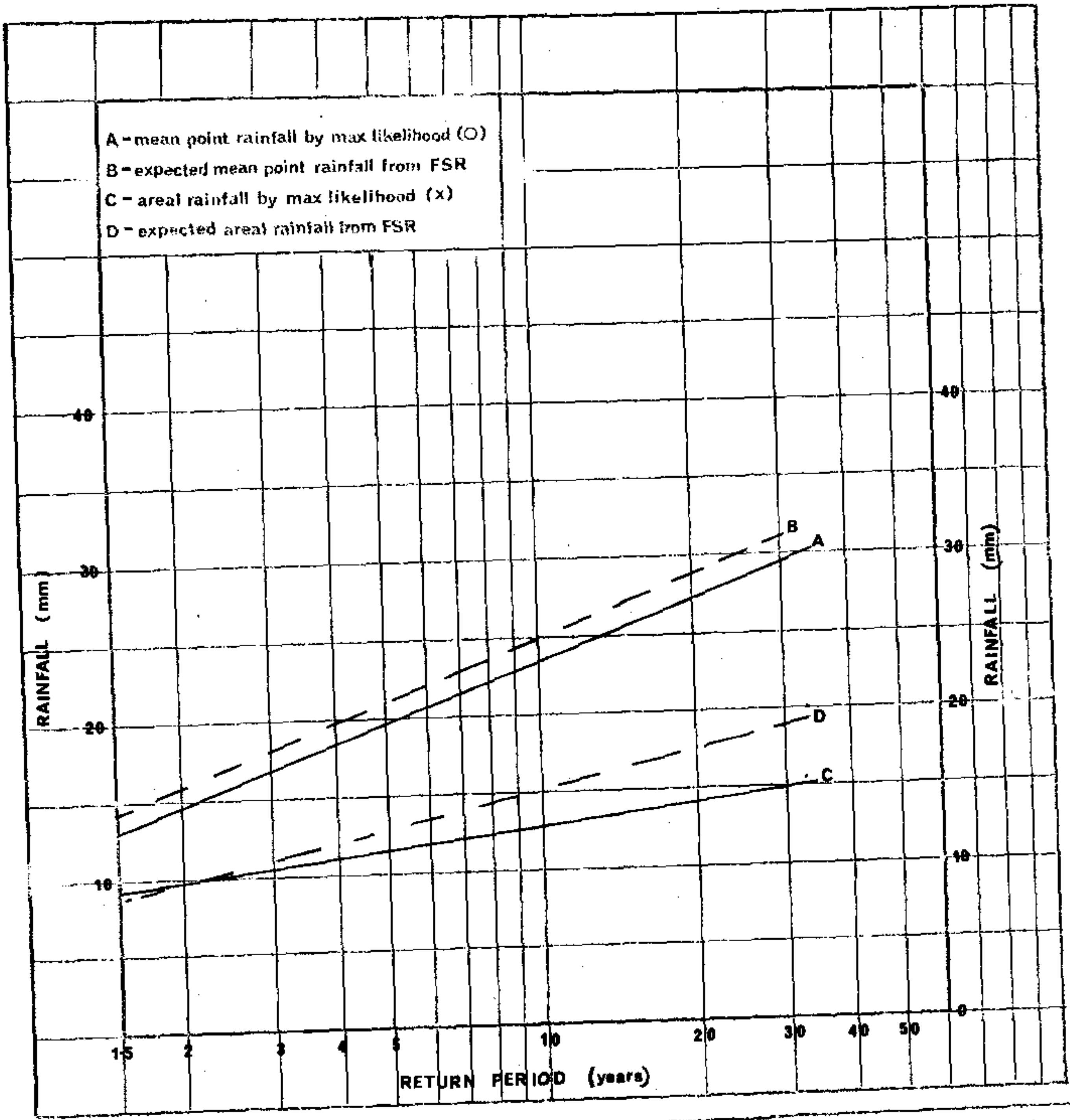
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·89	0·84	0·81	0·80
ARF EXPECTED FROM FSR	0·89			

Fig. A9 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION SURREY (10)	AREA: 1000 km^2	DURATION: 1 HOUR (U)
Number of rainfall stations: 8 (7)	Period of data: 1958 – 1969 (incl.)	

RANK OF RAINFALL EVENT *	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	28·4	20·8	17·8	16·3	14·7	14·0	12·2		
AREAL RAINFALL (mm)	15·7	10·7	10·2	10·2	9·65	9·65	8·89		
PLOTTING POSITION (yrs) *	21·1	7·26	4·21	2·87	2·12	1·63	1·29		



RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·66	0·58	0·54	0·51
ARF EXPECTED FROM FSR	0·61			

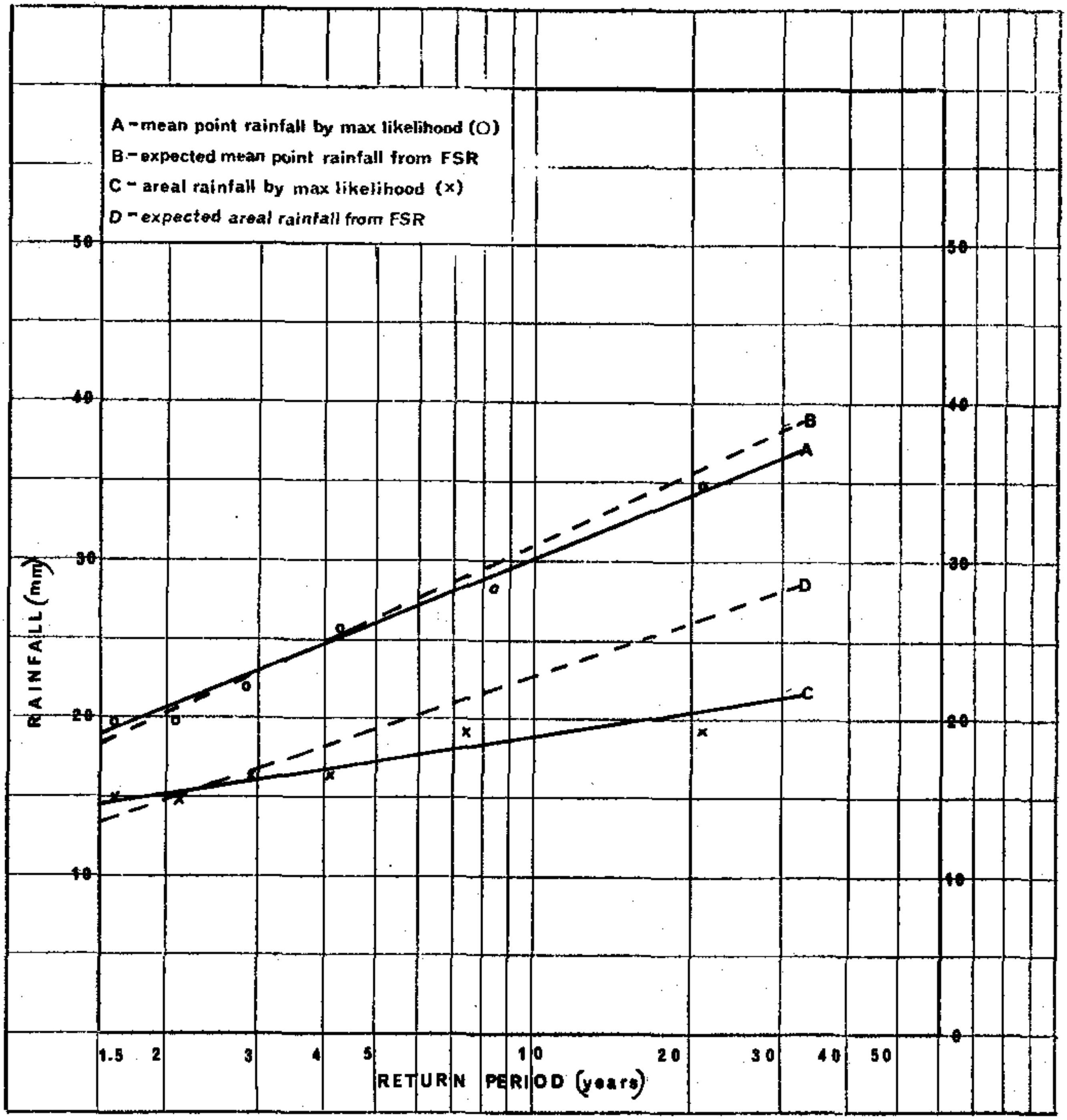
* Ranked values are annual series but plotting positions have been converted to POT series

Fig. A10 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION SURREY (10)	AREA: 1000 km^2	DURATION: 2 HOURS (u)
Number of rainfall stations: 8 (5)	Period of data: 1958–1969 (incl)	

RANK OF RAINFALL EVENT *	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	34.9	26.5	25.4	21.5	19.7	19.5	17.0		
AREAL RAINFALL (mm)	19.1	19.0	16.0	15.8	14.7	14.6	14.2		
PLOTTING POSITION (yrs) *	21.1	7.26	4.21	2.87	2.12	1.63	1.29		



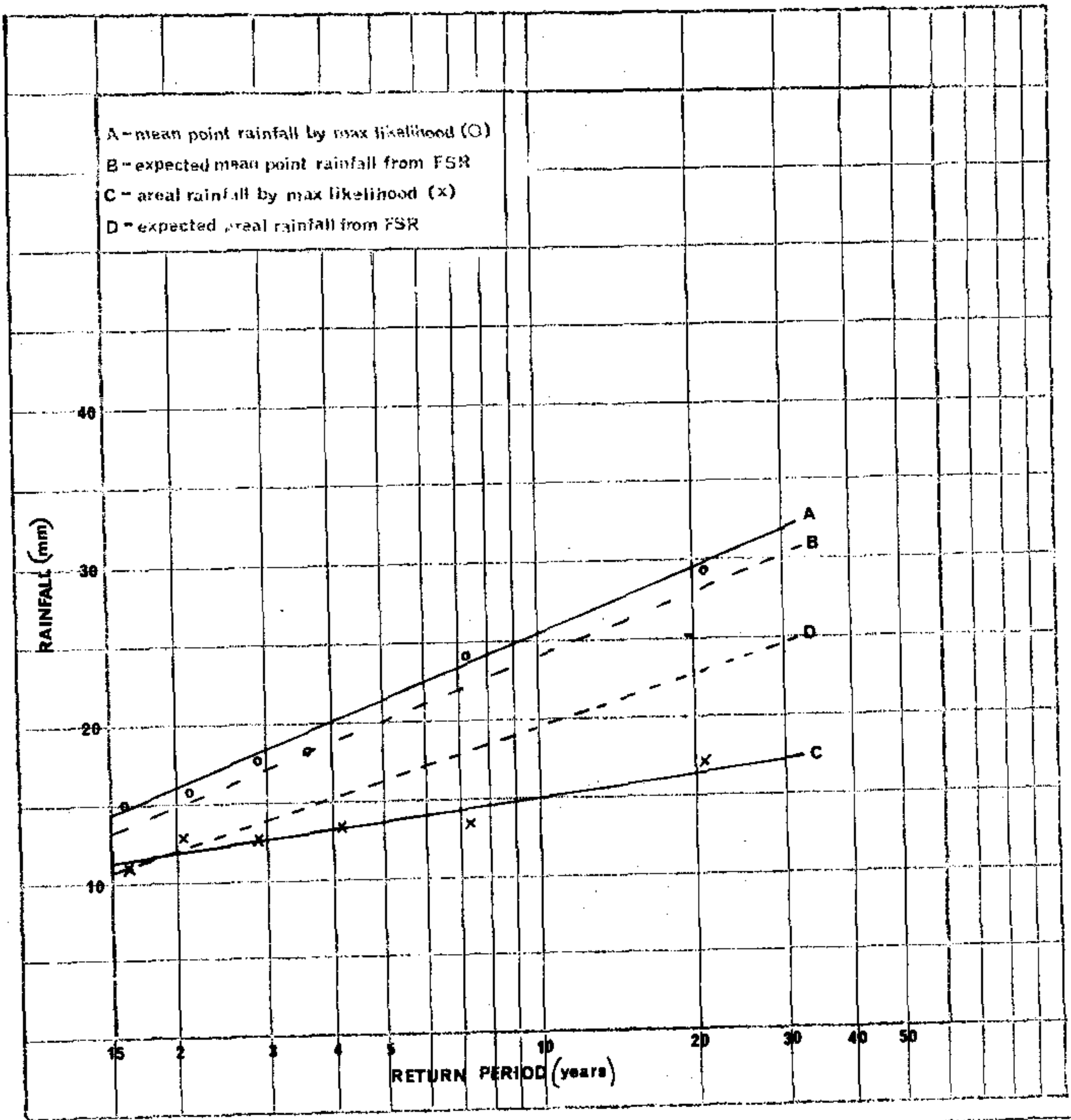
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0.73	0.66	0.62	0.59
ARF EXPECTED FROM FSR	0.72			

* Ranked values are annual series but plotting positions have been converted to POT series

Fig. A 11 Frequency Curves.

LOCATION SURREY (11)	AREA: 100 km²	DURATION: 1 HOUR(U)
Number of rainfall stations: 3	Period of data: 1958 – 1969 (incl)	

RANK OF RAINFALL EVENT *	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	29·8	24·1	18·9	17·7	15·9	15·0	14·5		
AREAL RAINFALL (mm)	17·1	13·6	13·1	12·7	12·6	10·8	10·7		
PLOTTING POSITION (yrs) *	21·1	7·26	4·21	2·87	2·12	1·63	1·29		



RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·75	0·64	0·59	0·58
ARF EXPECTED FROM FSR	0·79			

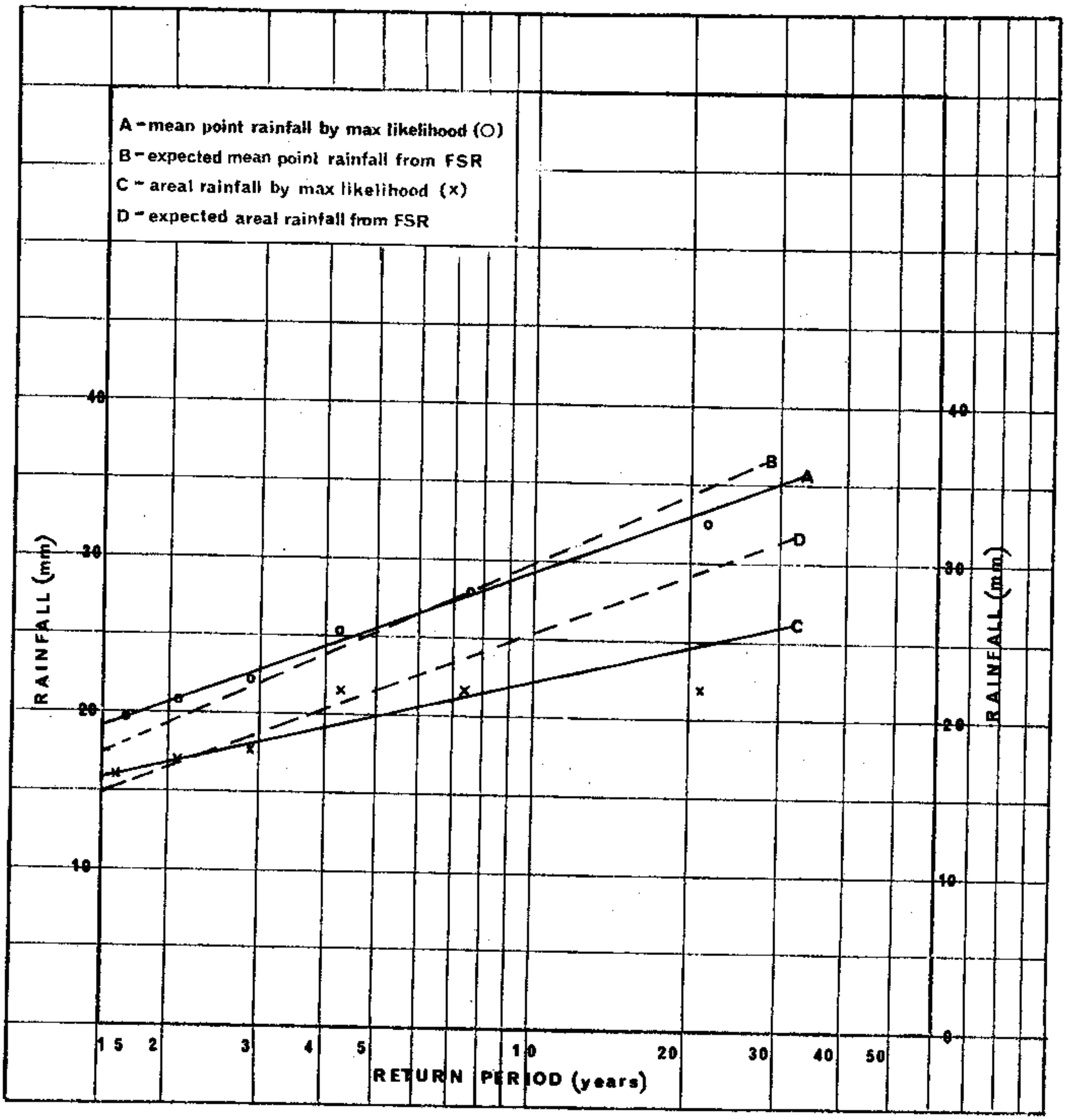
* Ranked values are annual series but plotting positions have been converted to POT series.

Fig. A12 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION SURREY (11)	AREA: 100 km².	DURATION: 2 HOURS (U)
Number of rainfall stations: 3	Period of data: 1958 - 1969 (incl)	

RANK OF RAINFALL EVENT *	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	32.0	27.4	25.3	22.4	21.0	19.7	18.8		
AREAL RAINFALL (mm)	22.0	21.5	21.2	17.3	16.9	16.1	15.6		
PLOTTING POSITION (yrs) *	21.1	7.26	4.21	2.87	2.12	1.63	1.29		



RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0.81	0.78	0.76	0.74
ARF EXPECTED FROM FSR	0.84			

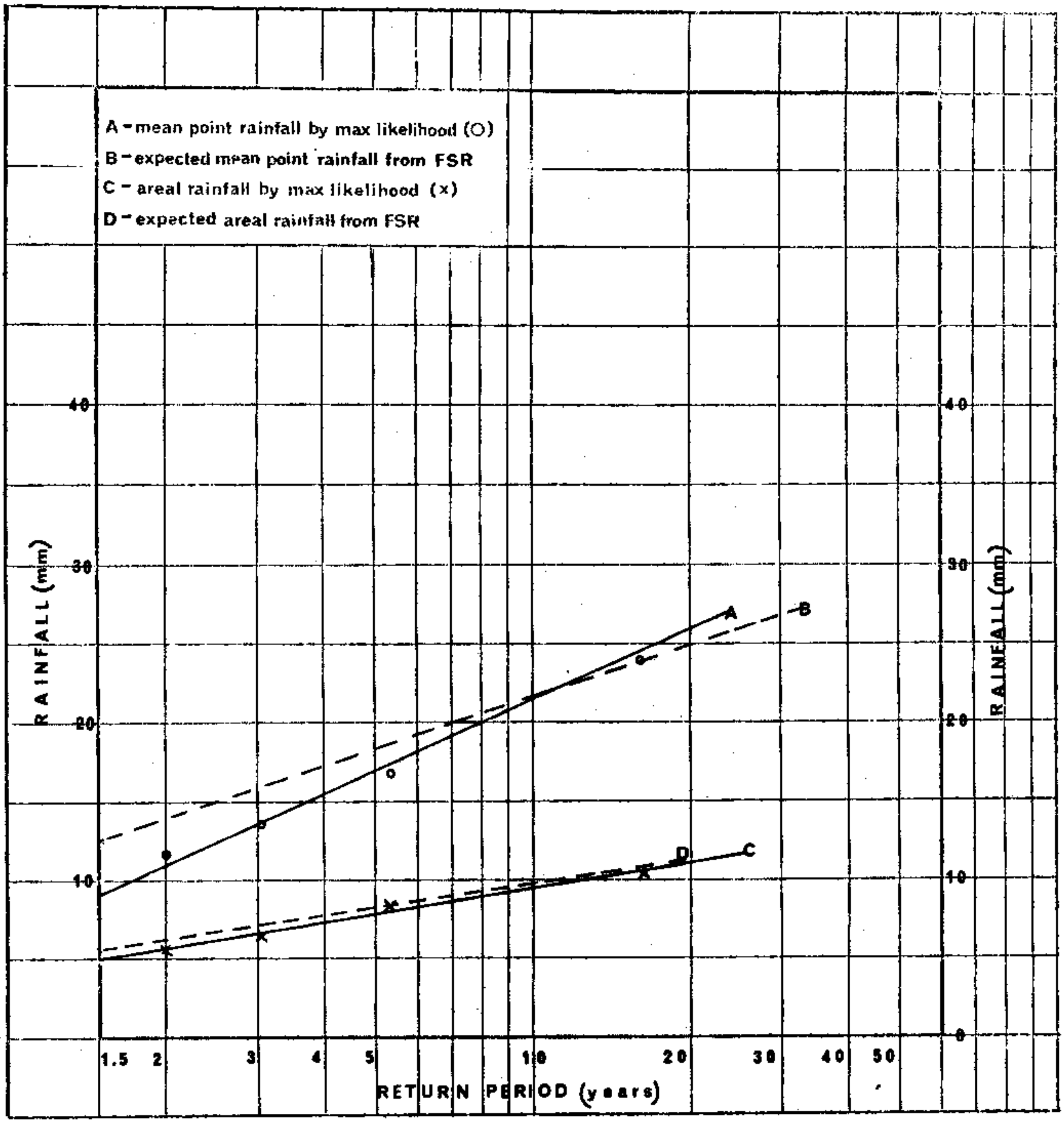
* Ranked values are annual series but plotting positions have been converted to POT series

Fig. A 13 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION **CHILTERNS** (12)	AREA: [illegible] km²	DURATION: 1 HOUR (F)
Number of rainfall stations: 14	Period of data: 1961 - 1969 (incl)	

RANK OF RAINFALL EVENT ✱	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	23.1	16.2	13.2	11.3	11.1	10.1			
AREAL RAINFALL (mm)	10.3	8.2	6.4	5.9	5.7	5.3			
PLOTTING POSITION (yrs) ✱	15.8	5.33	3.04	2.02	1.44	1.06			



RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0.53	0.48	0.45	0.44
ARF EXPECTED FROM FSR	0.46			

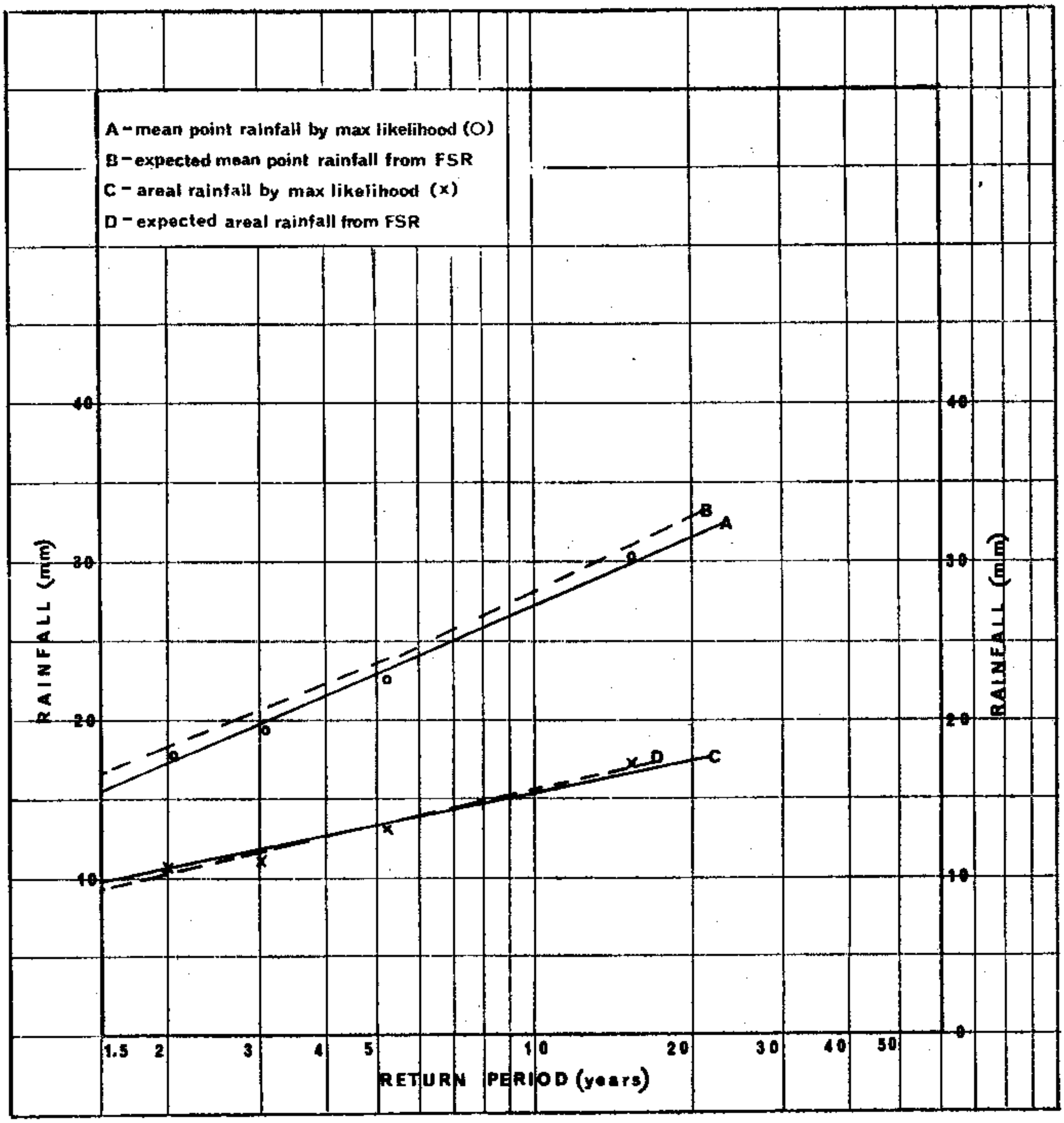
✱ Ranked values are annual series but plotting positions have been converted to POT series

Fig. A14 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION **CHILTERNS** (12)	AREA: 8000 km² / DURATION: 2 HOURS (F)
Number of rainfall stations: 14	Period of data: 1961 - 1969 (incl)

RANK OF RAINFALL EVENT *	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	30.1	22.1	19.1	17.7	15.5	14.7			
AREAL RAINFALL (mm)	17.6	13.2	11.3	10.9	9.2	7.8			
PLOTTING POSITION (yrs) *	15.6	5.33	3.04	2.02	1.44	1.06			



RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0.63	0.59	0.58	0.57
ARF EXPECTED FROM FSR	0.57			

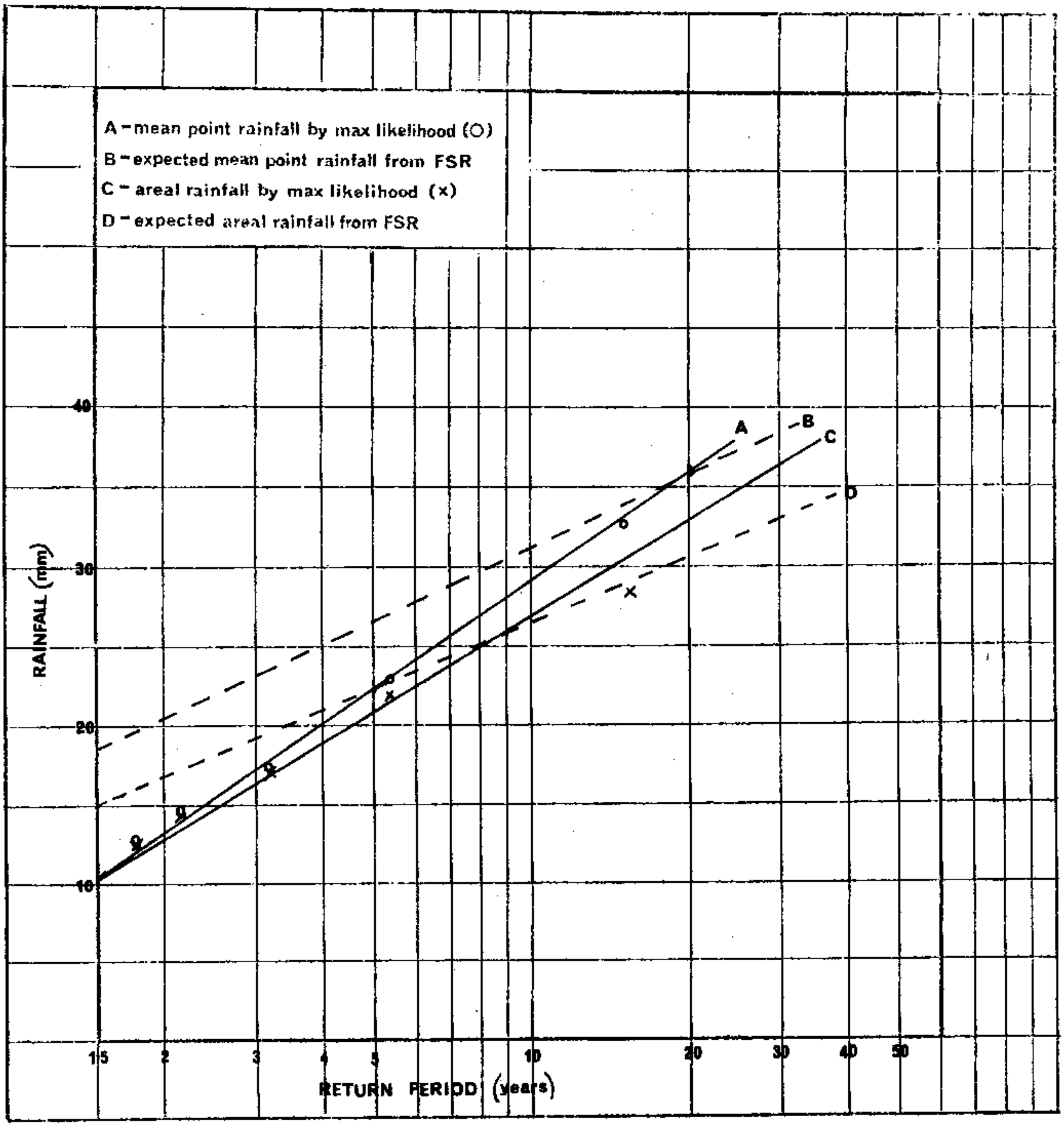
* Ranked values are annual series but plotting positions have been converted to POT series

Fig. A 15 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION **GREENWICH G.L.C. (13)**	AREA: **100 km²**	DURATION: **2 HOURS (U)**
Number of rainfall stations: **9**	Period of data: **July 1968 – June 1976 (incl)**	

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	32·6	22·4	16·7	14·9	12·8	9·6			
AREAL RAINFALL (mm)	27·6	21·0	16·6	14·8	12·7	9·4			
PLOTTING POSITION (yrs)	14·5	5·21	3·17	2·28	1·78	1·46			



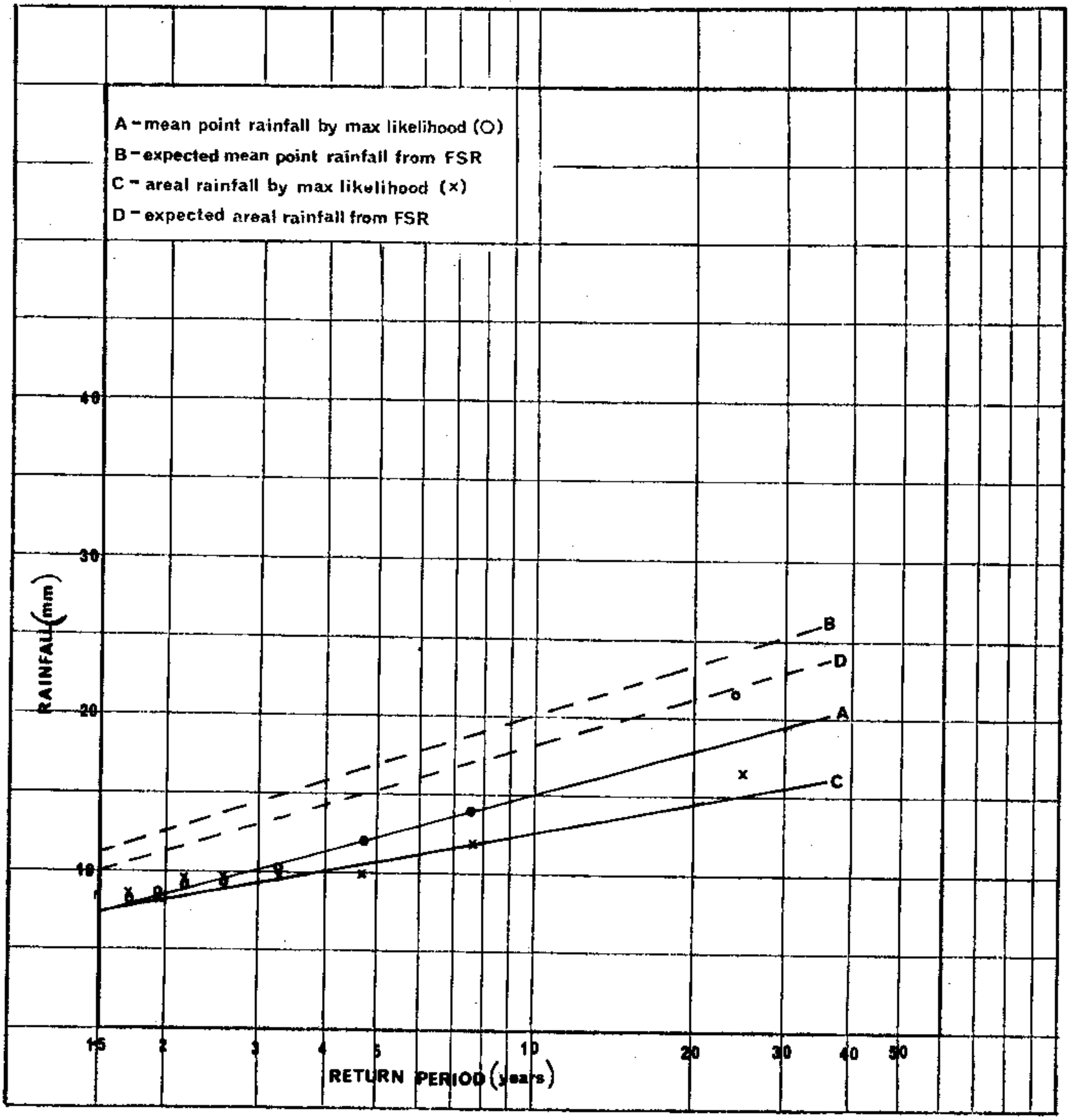
RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·95	0·92	0·91	0·91
ARF EXPECTED FROM FSR	0·84			

Fig. A16 Frequency Curves.

Computation of Areal Reduction Factor.

LOCATION GRENDON-UNDERWOOD (14)	AREA: 20 km²	DURATION: 1 HOUR (F)
Number of rainfall stations: (3)	Period of data: Jan 1964 - Dec 1975 (incl)	

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	20·5	14·0	12·0	10·1	9·8	9·1	8·5	8·0	
AREAL RAINFALL (mm)	16·9	12·1	9·9	9·8	9·8	9·2	8·3	8·2	
PLOTTING POSITION (yrs)	21·6	7·77	4·73	3·40	2·66	2·18	1·85	1·60	

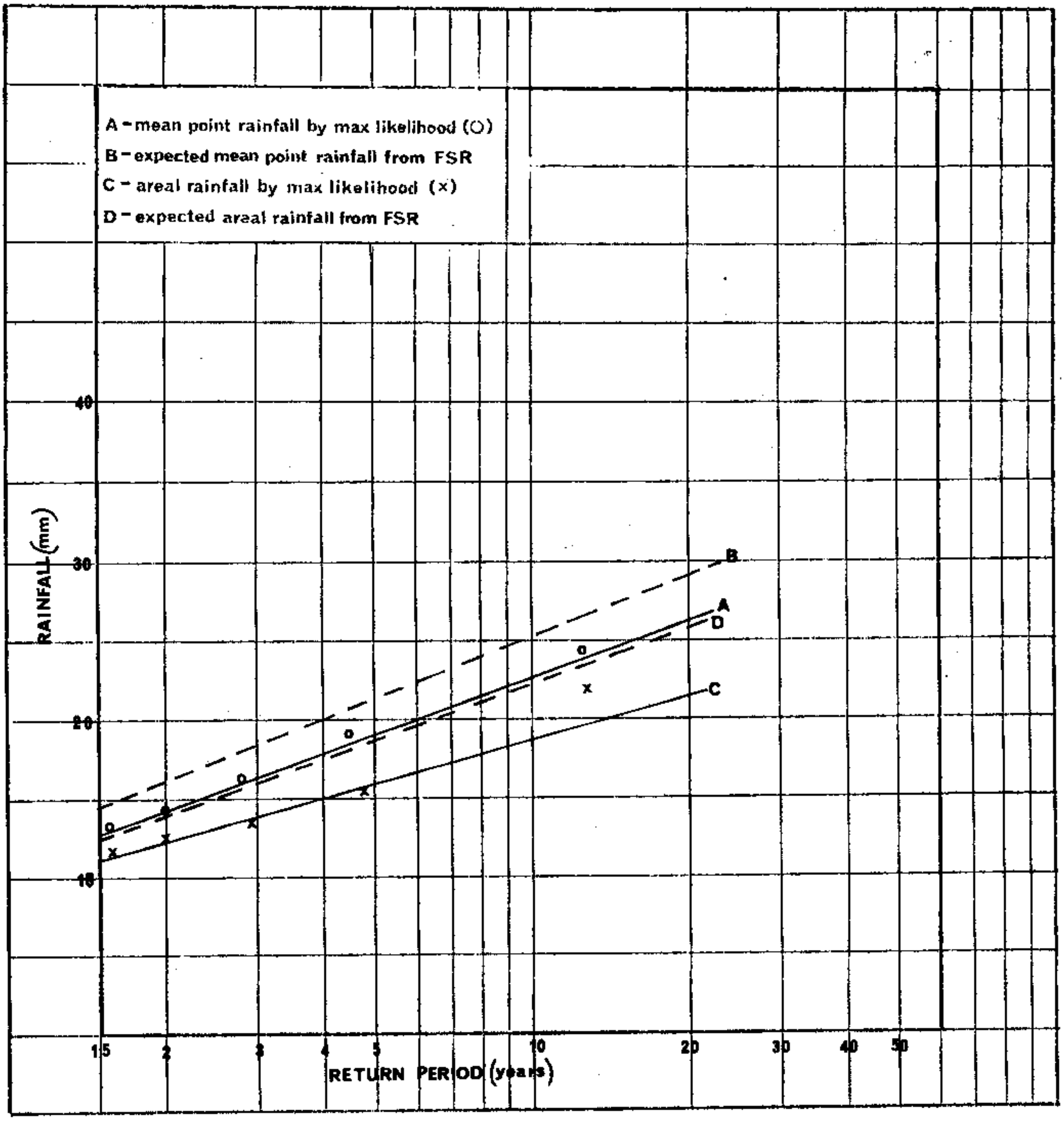


RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·98	0·89	0·85	0·82
ARF EXPECTED FROM FSR	0·88			

Fig. A 17 Frequency Curves.

LOCATION PLYNLIMON (15)	AREA: 20 km²	DURATION: 1 HOUR (F)
Number of rainfall stations: (3)	Period of data: Jan 1968 – Dec 1975	

RANK OF RAINFALL EVENT	1	2	3	4	5	6	7	8	9
MEAN POINT RAINFALL (mm)	24·1	19·0	16·3	14·6	13·8				
AREAL RAINFALL (mm)	21·9	15·5	13·8	13·0	12·2				
PLOTTING POSITION (yrs)	12·7	4·56	12·2	2·08	1·56				



RETURN PERIOD (yrs)	2	5	10	20
ARF FROM FREQUENCY ANALYSIS	0·89	0·86	0·86	0·85
ARF EXPECTED FROM FSR	0·88			

Fig. A18 Frequency Curves.

Appendix B: COMPUTER PROGRAMMES

PROGRAMME 1

Extracts one year of daily rainfall values from magnetic tape for all sample areas. Selects highest values for each station. Calculates mean areal rainfalls and selects highest daily areal values for each sample area. Performs other analyses appropriate for development of theoretical models to link point and areal rainfall.

```
      COMMON/FCOM/IARRAY(1288),NLREC(10,50),
     1              JGAGE(10,50),NGAGES(10),NTODO,NYP,NAREAS
      DIMENSION RAIN(400),NRAIN(400),SAVE(50,3),REDMAX(3),IDTMAX(3),
     1JSAVE(50),BMEAN(10,50),AMAXR1(50),AMAXR2(50),IGROUP(10,30),
     2  NGRAIN(15,366),NGROUP(10),IDATE1(50),IDATE2(50),GRAIN(15,366)
     3 ,NINGRP(50),ASAVE(50),BSAVE(50,3),KSAVE(50),W(50),EFAREA(50,3)
      LOGICAL IMOFF
      NMAX=1288
      NAREAS=9
      NCOUT=7
      NPRINT=6
      NCRD=5
      NCOUT1=9
      NCIN=3
      CALL SSWTCH(1,IDIAG)
      IDIAG=2-IDIAG
      NTODO=0
      MING=9999999
C---  ----READ IN GAUGES AND GROUPINGS AREA BY AREA
      DO 20 I=1,NAREAS
      READ(NCRD,1000)NGAGES(I),(IGROUP(I,j),J=1,30)
      DO 5 J=1,30,2
      IF(IGROUP(I,J))10,10,
    5 CONTINUE
      GO TO 700
   10 NGROUP(I)=(J-1)/2
      NGR2=NGROUP(I)*2
      NG=NGAGES(I)
      NTODO=NTODO+NG
      READ(NCRD,1000)(JGAGE(I,J),J=1,NG)
      DO 12 J=1,NG
      READ(NCIN,1001)NDS,BMEAN(I,J)
      IF(NDS.EQ.0) BMEAN(I,J)=-1.
   12 CONTINUE
      DO 15 J=1,NG
      MING=MIND(JGAGE(I,J),MING)
   15 CONTINUE
      WRITE(NPRINT,2010)I,NGAGES(I),(IGROUP(I,J),J=1,NGR2)
      WRITE(NPRINT,2011)(JGAGE(I,J),J=1,NG)
   20 CONTINUE
      CALL FIND(MING,IFLAG)
      IF(IFLAG)710,,
```

```
      NDSYR=365
      IF(MOD(NYR,4).EQ.0) NDSYR=366
      ITIMES=0
   25 NOLDBL=0
----------LOOP OVER AREAS
      DO 500 K=1,NAREAS
      WRITE(NPRINT,2001) K,NYR
      DO 30 J=1,50
      AMAXR1(J)=-1.
      AMAXR2(J)=-1.
   30 CONTINUE
      KK=K
      NLOOP=NDSYR
      NGRPD=0
      IGO=1
      NG=NGAGES(K)
      NGR=NGROUP(K)
      NGR2=NGR*2
      DO 50 J=1,NG
      IF(NLREC(K,J)),,50
      WRITE(NPRINT,2012)JGAGE(K,J)
   50 CONTINUE
   60 DO 63 J=1,400
      RAIN(J)=0.
      NRAIN(J)=0
   63 CONTINUE
      DO 64 J=1,15
      DO 64 I=1,366
      GRAIN(J,I)=0.
      NGRAIN(J,I)=0
   64 CONTINUE
C----------NO. OF GAUGES IN AREA
      NG=NGAGES(K)
      IF(IDIAG.EQ.1.AND.K.EQ.4)WRITE(NPRINT,9006) (NLREC(K,J),J=1,NG)
C----------LOOP OVER GAUGES
      NOUTG=D
      DO 200 J=1,50

      NINGRP(J)=0
      JSAVE(J)=J
      DO 69 I=1,3
      EFAREA(J,I)=-1.
      SAVE(J,I)=99999.
   69 CONTINUE
C----------GET DATA FROM RAINFALL FILE
      IF(NLREC(K,J))200,200,
      NOUTG=NOUTG+1
      NBLK=NLREC(K,J)/3
      JSTN=NLREC(K,J)-NBLK*3
      IF(NOLDBL-NBLK),75,
      CALL RDRAIN(NBLK,NMAX,IARRAY(1),IFLAG)
   71 IF(IFLAG)720,,
   75 NOLDBL=NBLK
      NOP=IARRAY(1)
```

```
      NIR=IARRAY(3)
C----------CHECK RIGHT STATION
      IADD=JSTN*NIR+NOP
      ISTN=IRRAY(IADD+1)*1000+IARRAY(IADD+2)
      IF(JGAGE(K,J)-ISTN)730,,730
C----------HERE FOR CORRECT STATION,IGO=1,FIND 2 MAX POINT FALLS
C----------(NLOOP=366)
C----------IGO=2 TO FIND MAX 3 DAILY AREAL FALLS(NLOOP=3)
      INR=IADD+11
      IMOFF=.FALSE.
      NDSOFF=0
      DO 100 I=1,NLOOP
      GO TO (76,87),IGO
   76 IF(IRRAY(INR).GE.0)GO TO 80
C------FIND CONSECUTIVE DAYS WHEN GAUGE NOT OPERATING
      IF(IMOFF) GO TO 77
      IMOFF=.TRUE.
      IBEG=1
   77 NDSOFF=NDS OFF+1
      IEND=I
      IF(I.EQ.NLOOP) GO TO 80
      GO TO 98
C----------FIND 2 MAX FALLS
   80 IF(IMOFF) WRITE(NPRINT,2000)JGAGE(K,J),NDSOFF,IBEG,IEND,NYR
      ARINR=IARRAY(INR)
      IF(ITIMES.EQ.1)ARINR=ARINR/BMEAN(K,J)
      IF(I.EQ.NLOOP) GO TO 100
      IMOFF=.FALSE.
      NDSOFF=0
      IF(AMAXR1(J).GT.ARINR)GO TO 85
      AMAXR2(J)=AMAXR1(J)
      IDATE2(J)=IDATE1(J)
      AMAXR1(J)=ARINR
      IDATE1(J)=I
      GO TO 90
   85 IF(AMAXR2(J).GT.ARINR)GO TO 90
      AMAXR2(J)=ARINR
      IDATE2(J)=I
      GO TO 90
C----------SAVE RAINFALL FOR ALL GAUGES IN AREA FOR 3 MAX AREAL FALLS.
   87 II=INR+IDTMAX(I)-1
      IF(IARRAY(II).GE.0) GO TO 88
      NLREC(K,J)=-1
      GO TO 100
   88 SAVE(J,I)=IARRAY(II)
      IF(ITIMES.EQ.1)SAVE(J,I)=SAVE(J,I)/BMEAN(K,J)
      GO TO 100
C----------IS GAUGE IN A GROUP?
   90 DO 95 JJ=1,NGR2,2
      IF(J.GE.IGROUP(K,JJ).AND.J.LE.IGROUP(K,JJ+1))GO TO 96
   95 CONTINUE
C----------TOTAL RAIN FOR ALL NON-GROUPED GAUGES FOR I-TH DAY OF
C----------THE YEAR
      RAIN(I)=RAIN(I)+ARINR
```

```
      NRAIN(I)=NRAIN(I)+1
      GO TO 98
C-----------TOTAL RAIN FOR GROUPS OF GAUGES FOR I-TH DAY OF YEAR
   96 JJ=(JJ+1)/2
      GRAIN(JJ,I)=GRAIN(JJ,I)+ARINR
      NGRAIN(JJ,I)=NGRAIN(JJ,I)+1
   98 INR=INR+1
  100 CONTINUE
      GO TO (105,110),IGO
  105 IF(AMAXR1(J))200,,
      IF(IDIAG.EQ.1.AND.K.EQ.4)WRITE(NPRINT,9004)NLOOP,(RAIN(I),I=1,
     1NLOOP)
      GO TO 200
  110 IF(IDIAG.EQ.1.AND.K.EQ.4)WRITE(NPRINT,9007)(SAVE(J,I)
     1,I=1,NLOOP)
  200 CONTINUE
      GO TO(205,280),IGO
C-----------HERE TO GET MEAN OF GROUP AND INCLUDE IT IN CALCULATION OF
C-----------AREAL RAINFALL
  205 DO 220 I=1,NLOOP
      DO 210 J=1,NGR
      IF(NGRAIN(J,I))210,210,
      RAIN(I)=RAIN(I)+GRAIN(J,I)/NGRAIN(J,I)
      NRAIN(I)=NRAIN(I)+1
  210 CONTINUE
  220 CONTINUE
      IF(IDIAG.EQ.1.AND.K.EQ.4) GO TO 221
      GO TO 228
  221 DO 223 J=1,NGR
      WRITE(NPRINT,9005)(GRAIN(J,I),NGRAIN(J,I),I=1,NLOOP)
  223 CONTINUE
      IF(IDIAG.EQ.1.AND.K.EQ.4)WRITE(NPRINT,9004)NLOOP,(RAIN(I),I=
     11,NLOOP)
C-----------CALCULATE AREAL REDUCTION FACTORS FOR 2 MAX. FALLS
  228 WRITE(NCOUT,2018)K,NG,NYR
      DO 230 J=1,NG
      IF(NLREC(K,J))229,229,
      IDT=IDATE1(J)
      REDF1 RAIN(IDT)/NRAIN(IDT)/AMAXR1(J)
      IDT=IDATE2(J)
      REDF2=RAIN(IDT)/NRAIN(IDT)/AMAXR2(J)
      IF(AMAXR1(J))229,,
      WRITE(NPRINT,2015)JGAGE(K,J),AMAXR1(J),IDATE1(J),
     1REDF1,AMAXR2(J),IDATE2(J),REDF2
  229 WRITE(NCOUT,2002)JGAGE(K,J),AMAXR1(J),IDATE1(J),REDF1,
     1AMAXR2(J),IDATE2(J),REDF2
  230 CONTINUE
C-----------FIND 3 HIGHEST DAILY AREAL RAINFALLS FOR YEAR
      DO 235 I=1,3
      REDMAX(I)=-1.
  235 CONTINUE
      DO 250 I=1,NLOOP
      IF(NRAIN(I))250,250,
      ARAIN=RAIN(I)/NRAIN(I)
```

```
      IF(REDMAX(1).GT.ARAIN)GO TO 238
      REDMAX(3)=REDMAX(2)
      IDTMAX(3)=IDTMAX(2)
      REDMAX(2)=REDMAX(1)
      IDTMAX(2)=IDTMAX(1)
      REDMAX(1)=ARAIN
      IDTMAX(1)=I
      GO TO 250
  238 IF(REDMAX(2).GT.ARAIN) GO TO 240
      REDMAX(3)=REDMAX(2)
      IDTMAX(3)=IDTMAX(2)
      REDMAX(2)=ARAIN
      IDTMAX(2)=I
      GO TO 250
  240 IF(REDMAX(3).GT.ARAIN)GO TO 250
      REDMAX(3)=ARAIN
      IDTMAX(3)=I
  250 CONTINUE
      WRITE(NPRINT,2014)(REDMAX(I),IDTMAX(I),I=1,3)
      WRITE(NCOUT1,2004)(REDMAX(I),IDTMAX(I),I=1,3)
      WRITE(NCOUT,2004)(REDMAX(I),IDTMAX(I),I=1,3)
      NLOOP=3
      IGO=IGO+1
      GO TO 60
C----------FIND MEDIAN POINT RAINFALL FOR 3 MAX. EVENTS
  280 DO 400 I=1,NLOOP
  284 II=0
      DO 290 J=1,NG
      IF(NLREC(K,J))290,290,
      DO 285 JJ=1,NGR2,2
      IF(J.GE.IGROUP(K,JJ).AND.J.LE.IGROUP(K,JJ+1))GO TO 287
  285 CONTINUE
      II=II+1
      RAIN(II)=SAVE(J,I)
      GO TO 290
  287 JJ=(JJ+1)/2
      GRAIN(JJ,I)=GRAIN(JJ,I)+SAVE(J,I)
      NGRAIN(JJ,I)=NGRAIN(JJ,I)+1
  290 CONTINUE
C----------INCLUDE GROUPED FALLS
      DO 295 JJ=1,NGR
      IF(NGRAIN(JJ,I))295,295,
      II=II+1
      RAIN(II)=GRAIN(JJ,I)/NGRAIN(JJ,I)
  295 CONTINUE
      IF(IDIAG.EQ.1.AND.K.EQ.4) WRITE(NPRINT,9004)II,(RAIN(J),J=1,II)
C----------RANK THE II RAINFALL VALUES HELD IN RAIN
     IORD=II-1
  296 IFLIP=0
      DO 298 J=1,IORD
      IF(RAIN(J).LT.RAIN(J+1))GO TO 298
      HOLD=RAIN(J)
      RAIN(J)=RAIN(J+1)
      RAIN(J+1)=HOLD
```

```
      IFLIP=1
  298 CONTINUE
      IF(IFLIP.EQ.0)GO TO 300
      IORD=IORD-1
      IF(IORD.GT.0)GO TO 296
  300 MIDPT1=(II+1)/2
      MIDPT2=(II+2)/2
      APRAIN=(RAIN(MIDPT1)+RAIN(MIDPT2))/2.
C----------FIND STANDARD DEVIATION OF GROUP MEANS
      SSQR=0.
      TOT=0.
      DO 310 J=1,II
      SSQR=SSQR+RAIN(J)*RAIN(J)
      TOT=TOT+RAIN(J)
  310 CONTINUE
      VAR=1./(II-1)*(SSQR-TOT*TOT/II)
      STDEV=SQRT(VAR)
C----------RANK RAINFALL FOR ALL STATIONS,NOT GROUP MEANS
      DO 313 J=1,50
      ASAVE(J)+SAVE(J,I)
      KSAVE(J)=JSAVE(J)
  313 CONTINUE
      IORD=NG-1
  314 IFLIP=0
      DO 320 J=1,IORD
      IF(ASAVE(J).LT.ASAVE(J+1))GO TO 320
      HOLD=ASAVE(J)
      ASAVE(J)=ASAVE(J+1)
      ASAVE(J+1)=HOLD
      IHOLD=KSAVE(J)
      KSAVE(J)=KSAVE(J+1)
      KSAVE(J+1)=IHOLD
      IFLIP=1
  320 CONTINUE
      IF(IFLIP.EQ.0)GO TO 330
      IORD=IORD-1
      IF(IORD.GT.0)GO TO 314
C----------CALCULATE EFFECTIVE CUMULATIVE AREAS FOR RANKED RAINFALL
  330 IF(I.NE.1) GO TO 342
      NTOTG=0
      NUMGPS=0
      IF(IDIAG.EQ.1.AND.K.EQ.4)WRITE(NPRINT,9008)(ASAVE(J),KSAVE(J)
     1    ,J=1,50)
      DO 340 J=1,NG
      IF(NLREC(K,J))340,340,
      NTOTG=NTOTG+1
      DO 335 JJ=1,NGR2,2
      IF(J.GE.IGROUP(K,JJ).AND.J.LE.IGROUP(K,JJ+1))
     1  GO TO 337
  335 CONTINUE
      NUMGPS=NUMGPS+1
      GO TO 340
  337 JJ2=(JJ+1)/2
      IF(NINGRP(JJ2).EQ.0) NUMGPS=NUMGPS+1
```

```
      NINGRP(JJ2)=NINGRP(JJ2)+1
  340 CONTINUE
  342 WRITE(NPRINT,2005)NUMGPS,APRAIN,STDEV,(ASAVE(J),J=1,NTOTG)
      WRITE(NCOUT1,2017)NTOTG,APRAIN,STDEV,(ASAVE(J),J=1,NTOTG)
      IF(IDIAG.EQ.1.AND.K.EQ.4)WRITE(NPRINT,9009)
     1(NINGRP(J),J=1,50)
      JX=0
      TOTSV=0.
      WTOT=0.
      DO 360 J=1,NG
      JS=KSAVE(J)
      IF(NLREC(K,JS))360,360,
      JX=JX+1
      EFAREA(1,I)=1000.
      DO 345 JJ=1,NGR2,2
      IF(JS.GE.IGROUP(K,JJ).AND.JS.LE.IGROUP(K,JJ+1))
     1 GO TO 355
  345 CONTINUE
      NGSTNS=1
      GO TO 356
  355 JJ2=(JJ+1)/2
      NGSTNS+NINGRP(JJ2)
  356 W=1000./NUMGPS/NGSTNS
      WTOT=WTOT+W
      EFAREA(JX+1,I)=1000.-WTOT
      TOTSV=TOTSV+ASAVE(J)*W
      BSAVE(JX,I)=ASAVE(J)
  360 CONTINUE
C----------OUTPUT EFFECTIVE AREAS
      WRITE(NPRINT,2006)(EFAREA(J,I),J=1,NTOTG)
      WRITE(NCOUT1,2016)NTOTG,(EFAREA(J,I),J=1,NTOTG
      AMEAN=TOTSV/WTOT
      WRITE(NPRINT,2003)AMEAN
  400 CONTINUE
      ITHT=ITIMES+1
      CALL FPLOT(EFAREA,BSAVE,NG,KK,NTOTG,ITHT)
  500 CONTINUE
      ITIMES=ITIMES+1
      GO TO (25,600),ITIMES
  600 CALL PLOT(30.,0.,999)
      GO TO 99
  700 WRITE(NPRINT,2007)I
      GO TO 99
  710 WRITE(NPRINT,2008)IFLAG
      GO TO 99
  720 WRITE(NPRINT,2008)IFLAG
      GO TO 99
  730 WRITE(NPRINT,2009)ISTN,JGAGE(K,J)
      GO TO 99
   99 STOP
 1000 FORMAT()
 1001 FORMAT(10X,I10,F10.5)
 2000 FORMAT(' GAUGE 'I6,' NOT OPERATING FOR',I4,' DAYS FROM',
     1I4,' TO',I4,'TH DAYS OF',I5)
```

```
2001 FORMAT(1H1,/'        AREA',I3,' YEAR',I5/)
2002 FORMAT(I10,F10.5,I10,F10.5,F10.5,I10,F10.5)
2003 FORMAT(/' MEAN FROM ABOVE=',F10.5)
2004 FORMAT(3(F10.5,I10))
2005 FORMAT(///' FOR',I5,' GROUPS, MEDIAN POINT RAINFALL=',F10.5,
    1' STANDARD DEVIATION=',F10.4//'  RANKED POINT RAINFALL'/
    2(10F10.3))
2006 FORMAT(//'     EFFECTIVE CUMULATIVE AREA'/(10F10.3))
2007 FORMAT(' TOO MANY GROUPS FOR',I3,'TH AREA')
2008 FORMAT(' ERROR,IFLAG=',I10)
2009 FORMAT(' GAUGE ',I10,'INSTEAD OF',I10)
2010 FORMAT(' NO OF GAUGES IN',I3,'TH AREA IS',I5,' IN GROUPS'/
    116(2I3,2X))
2011 FORMAT(' GAUGES'/(10I8))
2012 FORMAT(' GAUGE',I7,' IS NOT AVAILABLE')
2014 FORMAT(//' MAX 3 AREAL VALUES AND THEIR DATES',3(F8.2,I4))
2015 FORMAT(' GAUGE',I7,' MAX 2 FALLS,DATES,AND REDUCTION FACTORS',
    12X,F10.5,I5,F10.5,2X,F10.5,I5,F10.5)
2016 FORMAT(I10/(10F10.3))
2017 FORMAT(I10,2F10.5/(10F10.3))
2018 FORMAT(3I10)
9002 FORMAT(' ARRAY'/(1X,20I5))
9003 FORMAT(' AMAXR1,AMAXR2,DATE1,DATE2',4I10)
9004 FORMAT(' NLOOP,RAIN',I10/(1X,20F5.0))
9005 FORMAT(' GRAIN,NGRAIN'/(1X,10(F8.2,I2)))
9006 FORMAT(' NLREC'/(10I10))
9007 FORMAT(' SAVE',10F10.1)
9008 FORMAT(' SAVE,JSAVE',/(1X,10(F6.1,I4)))
9009 FORMAT(' NINGRP'/(1X,20I5))
     END
```

PROGRAMME 2

Performs statistical analyses of data from Programme 1 for determination of areal and average point frequency curves.

```
      DIMENSION MAXR(550,20),PACK(550,20),NYRSON(275),IDATE(20),
     1          REDF(21),ISORTM(60),SORIP(60),TOTR(275),
     2          REDMAX(3,9,20),IDTMAX(3,9,20),JGAGE(275),NG(9)
     3         ,IYEAR(20),ARTAL(9),NARTAL(8),SSQTAL(9),
     4          ART5(9),NART5(9),SSQT5(9),ART16(9),NART16(9),
     5          SSQT16(9)
      NCRD=5
      NCOUT=7
      NPRINT=6
      NTHYR=0
      CALL SSWTCH(1,IDIAG)
      IDIAG=2-IDIAG
   10 READ(NCRD,1008,END=35)KAREA,NGS,NYR
      IF(KAREA.GT.1) GO TO 15
      J=1
      NTHYR=NTHYR+1
   15 NG(KAREA)=NGS
      DO 30 I=1,NGS
      J2=(J+1)/2
      READ(NCRD,1000)JGAGE(J2),MAXR(J,NTHYR),IDATE1,REDF1,
     1               MAXR(J+1,NTHYR),IDATE2,REDF2
      PACK(J,NTHYR)=IDATE1*100+REDF1+NTHYR*100000
      PACK(J+1,NTHYR)=IDATE2*100+REDF2+NTHYR*100000
      J=J+2
   30 CONTINUE
      READ(NCRD,1007)(REDMAX(I,KAREA,NTHYR),IDTMAX(I,KAREA,NTHYR)
     1,I=1,3)
      GO TO 10
C---------HERE WITH ALL DATA READ IN,DATE AND REDF PACKED INTO PACK.
C         FIND HOW MANY YEARS EACH STATION OPERATED AND MEAN OF ANNUAL
C         MAXIMA
   35 DO 40 J=1,275
      NYRSON(J)=0
      TOTR(J)=0.
   40 CONTINUE
      DO 45 J=1,9
      ARTAL(J)=0.
      NARTAL(J)=0.
      SSQTAL(J)=0.
      ART5(J)=0.
      NART5(J)=0.
      SSQT5(J)=0.
      ART16(J)=0.
      NART16(J)=0.
      SSQT16(J)=0.
   45 CONTINUE
   50 DO 80 K=1,NTHYR
      J=0
      DO 70 I-1,534,2
```

```
      J=J+I
      IF(MAXR(I,K))70,,
      TOTR(J)=TOTR(J)+MAXR(I,K)
      NYRSON(J)=NYRSON(J)+1
   70 CONTINUE
   80 CONTINUE
      DO 100 J=1,267
      IF(NYRSON(J))90,90,
      AMEAN=TOTR(J)/NYRSON(J)
      WRITE(NPRINT,2000)JGAGE(J),NYRSON(J),AMEAN
   90 WRITE(NCOUT,2001)JGAGE(J),NYRSON(J),AMEAN
  100 CONTINUE
      IGJ=0
      KAREA=1
C----------RANK MAXIMA FOR EACH STATION
      DO 180  I=1,534,2
      NGS=NG(KAREA)
      TOTRAL=0.
      NTOTAL=0.
      SSQRAL=0.
      TOTR12=0.
      NTOT12=0
      SSQR12=0.
      TOTR5=0.
      NTOT5=0.
      SSQRS=0.
      TOTR16=0.
      NTOT16=0
      SSQR16=0.
      I2=(I+1)/2
      IGJ=IGJ+1
      IF(IGJ.LE.NG(KAREA)) GO TO 105
      IGJ=0
      KAREA=KAREA+1
  105 IF(NYRSON(I2))180,180,
C----------PUT INTO 1-DIMENSIONAL ARRAYS FOR SORTING
      KK=1
      DO 110 K=1,NTHYR
      IF(MAXR(I,K))110,,
      ISORTM(KK)=MAXR(I,K)
      SORTP(KK)=PACK(I,K)
      ISORTM(KK+1)=MAXR(I+1,K)
      SORTP(KK+1)=PACK(I+1,K)
      KK=KK+2
  110 CONTINUE
      KK=KK-1
C----------SORT ON ISORTM CARRYING SORTP
      LOOP=KK-1
      IFLIP=0
  115 DO 120 K=1,LOOP
      IF(ISORTM(K).GT.ISORTM(K+1))GO TO 120
      IHOLD=ISORTM(K)
      ISORTM(K)=ISORTM(K+1)
      ISORTM(K+1)=IHOLD
```

```
      HOLD=SORTP(K)
      SORTP(K)=SORTP(K+1)
      SORTP(K+1)=HOLD
      IFLIP=1
  120 CONTINUE
      LOOP=LOOP-1
      IF(LOOP)130,130,
      IF(IFLIP),,115
C----------HERE WITH SORTED MAXIMA
  130 IF(NYRSON(12).GE.12) GO TO 135
      WRITE(NPRINT,2012)JGAGE(I2)
      GO TO 137
  135 WRITE(NPRINT,2013)JGAGE(I2)
  137 N20=KK
      IF(N20.GT.20) N20=20
      WRITE(NPRINT,2002)(ISORTM(K),K=1,N20)
      N5=0
      N16=0
      DO 150 K=1,N20
      IDATE(K)=SORTP(K)/100.
      REDF(K)=SORTP(K)-IDATE(K)*100
      IYEAR(K)=IDATE(K)/1000
      IDATE(K)=IDATE(K)-IYEAR(K)*1000
      IYEAR(K)=IYEAR(K)+1959
      IF(NYRSON(I2).LT.12) GO TO 150
      TOTRAL=TOTRAL+REDF(K)
      SSQRAL=SSQRAL+REDF(K)*REDF(K)
      IF(K.GT.5)GO TO 140
      N5=N5+1
      TOTR5=TOTR5+REDF(K)
      SSQR5=SSQR5+REDF(K)*REDF(K)
      GO TO 150
  140 IF(K.LT.16)GO TO 150
      IF(K.GT.20)GO TO 150
      N16=N16+1
      TOTR16=TOTR16+REDF(K)
      SSQR16=SSQR16+REDF(K)*REDF(K)
  150 CONTINUE
      WRITE(NPRINT,2004)(REDF(K),K=1,N20)
      WRITE(NPRINT,2003)(IDATE(K),K=1,N20)
      WRITE(NPRINT,2014)(IYEAR(K),K=1,N20)
C----------FIND MEANS AND STANDARD DEVIATIONS
      IF(NYRSON(I2).LT.12) GO TO 180
      AMEAN=TOTRAL/N20
      ARTAL(KAREA)=ARTAL(KAREA)+TOTRAL
      NARTAL(KAREA)=NARTAL(KAREA)+N20
      SSQTAL(KAREA)=SSQTAL(KAREA)+SSQRAL
      VAR=1./(N20-1)*(SSQRAL-TOTRAL*TOTRAL/N20)
      STDEV=SQRT(VAR)
      WRITE(NPRINT,2005)N20,AMEAN,STDEV
      IF(N5)180,180
      AMEAN=TOTR5/N5
      ART5(KAREA)=ART5(KAREA)+TOTR5
      NART5(KAREA)=NART5(KAREA)+N5
```

```
      SSQT5(KAREA)=SSQT5(KAREA)+SSQR5
      VAR=1./(N5-1)*(SSQR5-TOTR5*TOTR5/N5)
      STDEV=SQRT(VAR)
      WRITE(NPRINT,2006)N5,AMEAN,STDEV
      IF(N16)180,180,
      AMEAN=TOTR16/N16
      ART16(KAREA)=ART16(KAREA)+TOTR16
      NART16(KAREA)=NART16(KAREA)+N16
      SSQT16(KAREA)=SSQT16(KAREA)+SSQR16
      VAR=1./(N16-1)*(SSQR16-TOTR16*TOTR16/N16)
      STDEV=SQRT(VAR)
      NX16=N16+15
      WRITE(NPRINT,2007)NX16,AMEAN,STDEV
  180 CONTINUE
C----------LOOP OVER AREAS
      J=0
      DO 300 KAREA=1,9
      NGS=NG(KAREA)
      NGS12=0
      NTOT=0
      TOT=0
      DO 190 I=1,NGS
      J=J+1
      IF(NYRSON(J).LT.12)GO TO 190
      J2=(J-1)*2+1
      DO 185 N=1,NTHYR
      IF(MAXR(J2,N))185,,
      TOT=TOT+MAXR(J2,N)
      NTOT=NTOT+1
  185 CONTINUE
      NGS12=NGS12+1
  190 CONTINUE
      AMEAN=TOT/NTOT
      WRITE(NPRINT,2008)KAREA,NGS12,AMEAN
      TOT=0
      DO 200 N=1,NTHYR
      TOT=TOT+REDMAX(1,KAREA,N)
  200 CONTINUE
      AMEAN=TOT/NTHYR
      WRITE(NPRINT,2009)AMEAN
      WRITE(NPRINT,2011)
C----------PUT REDMAX INTO SINGLE ARRAY FOR SORTING
      K=1
      DO 210 N=1,NTHYR
      SORTP(K)=REDMAX(1,KAREA,N)
      SORTP(K+1)=REDMAX(2,KAREA,N)
      SORTP(K+2)=REDMAX(3,KAREA,N)
      ISORTM(K)=IDTMAX(1,KAREA,N)+N*1000
      ISORTM(K+1)=IDTMAX(2,KAREA,N)+N*1000
      ISORTM(K+2)=IDTMAX(3,KAREA,N)+N*1000
      K=K+3
  210 CONTINUE
C----------SORT ON ISORTM CARRYING SORTP
      LOOP=K-2
```

```
  215 IFLIP=0
  220 DO 230 N=1,LOOP
      IF(SORTP(N).GT.SORTP(N+1)) GO TO 230
      IHOLD=SORTP(N)
      SORTP(N)=SORTP(N+1)
      SORTP(N+1)=IHOLD
      HOLD=ISORTM(N)
      ISORTM(N)=ISORTM(N+1)
      ISORTM(N+1)=HOLD
      IFLIP=1
  230 CONTINUE
      LOOP=LOOP-1
      IF(LOOP)240,240,
      IF(IFLIP),,215
  240 DO 245 N=1,20
      IYEAR(N)=ISORTM(N)/1000
      IDATE(N)=ISORTM(N)-IYEAR(N)*1000
      IYEAR(N)=IYEAR(N)+1959
  245 CONTINUE
      WRITE(NPRINT,2017)(SORTP(N),IDATE(N),IYEAR(N),N=1,20)
C-----------FIND MEANS AND STANDARD DEVNS FOR AREAS
      AMEAN=ARTAL(KAREA)/NARTAL(KAREA)
      VAR=1./(NARTAL(KAREA)-1)*(SSQTAL(KAREA)-ARTAL(KAREA)*ARTAL(KAREA)
     1/NARTAL(KAREA))
      STDEV=SQRT(VAR)
      WRITE(NPRINT,2005)NARTAL(KAREA),AMEAN,STDEV
      AMEAN=ART5(KAREA)/NART5(KAREA)
      VAR=1./(NART5(KAREA)-1)*(SSQT5(KAREA)-ART5(KAREA)*ART5(KAREA)
     1/NART5(KAREA))
      STDEV=SQRT(VAR)
      NX5=NART5(KAREA)/NGS12
      WRITE(NPRINT,2006)NX5,AMEAN,STDEV
      AMEAN=ART16(KAREA)/NART16(KAREA)
      VAR=1./(NART16(KAREA)-1)*(SSQT16(KAREA)-ART16(KAREA)*ART16(KAREA)
     1/NART16(KAREA)).
      STDEV=SQRT(VAR)
      NX16=NART16(KAREA)/NGS12+15
      WRITE(NPRINT,2007)NX16,AMEAN,STDEV
  300 CONTINUE
 3110 FORMAT(3I1),F10.5,2I10,F10.5)
 1007 FORMAT(3(F10.5,I10))
 1008 FORMAT(3I10)
 1010 FORMAT()
 2000 FORMAT(' GAUGE',I7,' OPERATED FOR',I3,' YEARS,MEAN OF ANNUAL',
     1' MAXIMA=',F10.3)
 2001 FORMAT(2I10,F10.6)
 2002 FORMAT(12X,'VALUES',20I5)
 2003 FORMAT(13X,'DATES',20I5)
 2004 FORMAT(' REDUCTION FACTORS',20F5.2)
 2005 FORMAT(18X,'MEAN REDFAC OF',I4,' HIGHEST FALLS=',F10.3,
     1' STANDARD DEVN.=',F10.5)
 2006 FORMAT(20X,'MEAN REDFAC OF',I4,' HIGHEST FALLS=',F10.3,
     1' STANDARD DEVN.=',F10.5)
 2007 FORMAT (9X,'MEAN REDFAC OF 16-',I3,' HIGHEST FALLS=',F10.3,
```

```
     1' STANDARD DEVN.=',F10.5)
2008 FORMAT(///' FOR AREA',I3,',',I3,' GAUGES WERE OPERATING FOR 12 OR'
     1,' MORE YEARS'/' MEAN OF THE ANNUAL MAXIMA IS',F10.5)
2009 FORMAT(' THE MEAN OF AREAL ANNUAL MAXIMA IS',
     1F10.5)
2011 FORMAT(/' RANKED HIGHEST FALLS  DATE YEAR'/)
2012 FORMAT(//' GAUGE',I7)
2013 FORMAT(//' GAUGE',I7,' ** 12 YEAR GAUGE **')
2014 FORMAT(13X,' YEAR',20I5)
2017 FORMAT(2X,F10.5,12X,I3,I7)
9000 FORMAT(' REDMAX ETC',3(F10.5,I10),2I10)
      STOP
      END
```