Relating the diffusive salt flux just below the ocean surface to boundary freshwater and salt fluxes

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ABSTRACT

We detail the physical means whereby boundary transfers of freshwater and salt induce diffusive fluxes of salinity. Our considerations focus on the kinematic balance between the diffusive fluxes of salt and freshwater, with this balance imposed by mass conservation for an element of seawater. The flux balance leads to a specific form for the diffusive salt flux immediately below the ocean surface and, in the Boussinesq approximation, to a specific form for the salinity flux. This note clarifies conceptual and formulational ambiguities in the literature concerning the surface boundary condition for the salinity equation and for the contribution of freshwater to the buoyancy budget.

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1. Introduction

In high latitude regions, substantial quantities of salt are exchanged between liquid seawater and sea ice during the process of sea ice melting and formation. In contrast, aeolian processes exchange only very limited quantities of salt with the atmosphere over scales larger than a few meters. So for purposes of ocean circulation studies, away from ice covered regions, the flux of salt across the ocean surface is insignificant (e.g., Beron-Vera et al. 1999). Ocean salinity and buoyancy changes from air–sea fluxes thus arise from the exchange of freshwater (FW) rather than the exchange of salt.

For dynamical purposes, seawater can be approximated by a two-component fluid comprised of freshwater and dissolved salt, where this 'salt' represents the total mass of various solutes, each with in reality slightly different behaviour (e.g., see Section 2.2 of Olbers et al. (2012)). We conventionally measure the seawater matter content in terms of salt concentration (salinity) rather than freshwater concentration. As discussed here, the impact of a boundary freshwater flux on what is generally termed the surface ocean salinity, but which is more properly the ocean surface boundary-layer bulk salinity, appears as a vertical diffusive salt flux just below the ocean surface. In the following, we continue to follow normal oceanographic convention and use the term 'surface salinity' to denote the bulk boundary-layer salinity rather than the actual 'skin' salinity value, which may differ by as much as 0.4 g kg^{-1} (Wurl et al. 2019). The purpose of this note is to clarify a conceptual and formulational discrepancy in the literature regarding this vertical boundary flux. We do so by making use of the kinematic constraint placed on the diffusive (molecular and turbulent) transport of salt and freshwater within the ocean. This constraint arises from the convention of working with a barycentric (center of mass) velocity which in turn leads

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to a zero diffusive flux of seawater mass, and so the requirement that any diffusive salt flux be balanced by an equal and opposing diffusive freshwater flux.

a. Two fluxes used in the literature

In the absence of freshwater or salt fluxes from melting or freezing ice the first form of the vertical diffusive salt flux just below the ocean surface is given by Phillips (1977), Eqs (2.5.1) and (2.7.1) of Gill (1982), Eq. (7) of Huang (1993), Eq. (9) of Beron-Vera et al. (1999), Eq. (11.56) of Griffies (2004) and, most recently and rigorously, by Warren (2009) as:

$$\mathscr{S}_{\text{in}} = S(E-P), \tag{1}$$

where $\mathscr{S}_{\text{in}}$ is the surface diffusive mass flux of salt (mass per time per area) just below the surface, S is the local surface ocean salinity (mass of salt per mass of seawater) (IOC et al. 2010), expressed as a fraction (kg kg^{-1}) rather than per mille (g kg^{-1}), and $E-P$ is the net oceanic freshwater mass loss (mass per time per area) from precipitation P and evaporation E. Note that here and in the following, the calligraphic $\mathscr{S}$ (and for freshwater fluxes $\mathscr{F}$) denote specifically the diffusive components of the salt flux just below the surface, *not* the total salt mass flux. The second flux is given on p209 of Stern (1975), in section 4 of Schmitt et al. (1989), in most detail by (see her Eq. 3) Steinhorn (1991) and on p122 of Huang (2010):

$$\mathscr{S}'_{\text{in}} = (E-P)S/(1-S) = \mathscr{S}_{\text{in}}/(1-S). \tag{2}$$

As we show in this note, equation (2) is a *pure salt flux* whereas (1) is a *balanced diffusive salt flux*, which we term here a *salinity flux*. The balanced salt flux (1) represents a vertical diffusive salt flux balancing an opposing diffusive freshwater flux, with this balance required to maintain the kinematic constraint of zero net diffusive flux of seawater mass. The balanced salt flux (1) is the natural means to specify salinity changes and the consequent density changes and surface

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buoyancy forcing. In contrast, calculating salinity and density changes from the pure salt flux (2) is less straightforward. Notably, Schmitt et al. (1989), Speer and Tziperman (1992) and Large and Nurser (2001) have used an incorrect formulation for the buoyancy flux based on the pure salt flux $\mathscr{S}'_{\text{in}}$ in (2), in which they mistakenly used this salt flux to compute the buoyancy flux.

b. Purpose of this note

The purpose of this note is to emphasize how the balanced diffusive flux of salt just below the ocean surface boundary (1) results from the kinematic constraint placed on diffusive transport of salt and freshwater. Namely, since the mass of seawater in a fluid element is constant, the diffusive salt flux is balanced by an equal and opposite diffusive freshwater flux. Just below the ocean surface, this kinematic constraint leads to a specific form for the diffusive salt flux induced by the boundary flux of freshwater (and salt when sea ice melts or forms). In the Boussinesq approximation, this then leads to a specific form for the diffusive *salinity* flux just below the surface. This kinematic framing of the surface salinity boundary condition clarifies and corrects a variety of treatments given in the literature.

The salinity of sea ice is roughly 5 parts per thousand, though it is quite variable (Hunke et al. 2011). Hence, where there is freezing and melting of sea ice, there can be significant fluxes of saline water (and hence salt) into and out of the liquid ocean. We therefore consider the effects of mass fluxes of salt as well as freshwater throughout the rest of this note.

c. Remainder of this note

In Section 2 we discuss a slab model that illustrates the distinction between a pure salt flux and a balanced salt flux. Then in Section 3 we consider the continuum mass budgets for salt and freshwater within the ocean, and in so doing detail why the salt and freshwater diffusive fluxes are

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balanced. In Section 4 we derive the general diffusive salt flux boundary condition (1) associated with an air-sea freshwater flux, as well as sea ice melt and formation. We conclude this note in Section 5.

2. Bucket slab model

Consider a homogeneous bucket containing seawater of mass $\mathbb{M}$ made up of salt mass $\mathbb{S}$ and freshwater mass $\mathbb{F}$, with uniform salinity $S = \mathbb{S}/\mathbb{M}$. We examine the change in salinity of the bucket arising from the transfer of salt and/or freshwater across the bucket surface. Let $\mathrm{d}\mathbb{S}$ be the change in salt mass, $\mathrm{d}\mathbb{F}$ the change in freshwater mass, and $\mathrm{d}\mathbb{M} = \mathrm{d}\mathbb{S} + \mathrm{d}\mathbb{F}$ be the total mass change (salt plus freshwater). The associated salinity change (assuming homogenization of seawater in the bucket) is thus given by

$$\mathrm{d}S = S_{\text{new}} - S, \tag{3}$$

where

$$S_{\text{new}} = \frac{\mathbb{S} + \mathrm{d}\mathbb{S}}{\mathbb{M} + \mathrm{d}\mathbb{M}}. \tag{4}$$

In the following we consider various means to represent salinity changes associated with salt, freshwater, and mass changes.

Note that the equations set out in this section are directly applicable to the 1-D salinity budget of the uppermost (surface) layer of an ocean model; in that case all masses such as $\mathbb{S}$, $\mathbb{F}$, $\mathrm{d}\mathbb{S}$ etc. should be regarded as masses per unit horizontal area.

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a. Inputs of seawater and salinity

For the first thought experiment (see Fig. 1a), add a mass of 'seawater' $\mathrm{d}\mathbb{M}_{\mathrm{seawater}}$ with the same salinity as the water already in the bucket; viz.

$$\mathrm{d}\mathbb{S} = S\,\mathrm{d}\mathbb{M}_{\mathrm{seawater}} \tag{5}$$

$$\mathrm{d}\mathbb{F} = (1-S)\,\mathrm{d}\mathbb{M}_{\mathrm{seawater}} \tag{6}$$

$$\mathrm{d}\mathbb{S} + \mathrm{d}\mathbb{F} = \mathrm{d}\mathbb{M}_{\mathrm{seawater}}. \tag{7}$$

In this case the total amount of salt in the bucket changes but the salinity remains unchanged, with

$$S_{\mathrm{new}} = \frac{S\mathbb{M} + S\,\mathrm{d}\mathbb{M}_{\mathrm{seawater}}}{\mathbb{M} + \mathrm{d}\mathbb{M}_{\mathrm{seawater}}} = S, \tag{8}$$

i.e.

$$\mathrm{d}S_{\mathrm{seawater}} = 0. \tag{9}$$

Now consider a balanced salt input (Fig. 1b), whereby we add a mass of salt

$$\mathrm{d}\mathbb{S} = \mathrm{d}\mathbb{S}_{\mathrm{bal}} \tag{10}$$

but simultaneously remove an equal mass of freshwater

$$\mathrm{d}\mathbb{F} = -\mathrm{d}\mathbb{S}_{\mathrm{bal}} \tag{11}$$

so that there is zero net mass input to the bucket:

$$\mathrm{d}\mathbb{M} = \mathrm{d}\mathbb{S} + \mathrm{d}\mathbb{F} = 0. \tag{12}$$

We thus replace freshwater in the bucket by salt while keeping the total mass unchanged. In this case the new salinity of the bucket is given by

$$S_{\mathrm{new}} = \frac{\mathbb{S} + \mathrm{d}\mathbb{S}_{\mathrm{bal}}}{\mathbb{M}} = S + \frac{\mathrm{d}\mathbb{S}_{\mathrm{bal}}}{\mathbb{M}}, \tag{13}$$

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and the salinity change is

$$\mathrm{d}S_{\text{salinity}} = \frac{\mathrm{d}\mathbb{S}_{\text{bal}}}{\mathbb{M}}. \tag{14}$$

As we will argue in Sections 3 and 4, this balanced salt input provides the most natural way to formulate the boundary forcing of salinity and hence density. It is most natural since seawater fluid mechanics is formulated in terms of constant-mass fluid elements, thus corresponding to the constant mass bucket.

b. Representing arbitrary salt & freshwater inputs as balanced salt & seawater inputs

The expressions (5), (6) (10) and (11) allow us to represent arbitrary inputs of salt $\mathrm{d}\mathbb{S}$ and freshwater $\mathrm{d}\mathbb{F}$ as inputs of seawater (which changes mass but not salinity) and balanced salt (which changes salinity but not mass)

$$\begin{pmatrix} \mathrm{d}\mathbb{S} \\ \mathrm{d}\mathbb{F} \end{pmatrix} = \mathrm{d}\mathbb{M}_{\text{seawater}} \begin{pmatrix} S \\ 1-S \end{pmatrix} + \mathrm{d}\mathbb{S}_{\text{bal}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{15}$$

Since a balanced salt input does not alter the mass (i.e., adding the two rows of equation (15)) we have

$$\mathrm{d}\mathbb{M}_{\text{seawater}} = \mathrm{d}\mathbb{M} = \mathrm{d}\mathbb{S} + \mathrm{d}\mathbb{F}. \tag{16}$$

Upon rearranging the first row of equation (15), we see that the salt mass input as a balanced salt input is the difference between the total salt input and the salt that is contained in the added seawater, thus giving an expression for the balanced salt input:

$$\mathrm{d}\mathbb{S}_{\text{bal}} = \mathrm{d}\mathbb{S} - S\,\mathrm{d}\mathbb{M}_{\text{seawater}}. \tag{17}$$

Or, re-expressing $\mathrm{d}\mathbb{M}_{\text{seawater}}$ using (16) the balanced salt input can be expressed purely in terms of $\mathrm{d}\mathbb{S}$ and $\mathrm{d}\mathbb{F}$ as:

$$\mathrm{d}\mathbb{S}_{\text{bal}} = (1-S)\,\mathrm{d}\mathbb{S} - S\,\mathrm{d}\mathbb{F}. \tag{18}$$

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These equations (16), (17), and (18) for the sea water and balanced salt inputs hold for completely general $\mathrm{d}\mathbb{S}$ and $\mathrm{d}\mathbb{F}$ that may have opposite signs. However, there are interesting cases where $\mathrm{d}\mathbb{S}$ and $\mathrm{d}\mathbb{F}$ have the same sign, such as happens when ice melt of some salinity S_{melt} (note that the salinity of the ice melt may differ from that of the ice salinity) passes into the ocean, or ice of salinity S_{ice} is formed by freezing. In the case of ice-melt where $\mathrm{d}\mathbb{S}$, $\mathrm{d}\mathbb{F}$ and $\mathrm{d}\mathbb{M}$ are all positive, we can write

$$\mathrm{d}\mathbb{S} = S_{\mathrm{melt}}\,\mathrm{d}\mathbb{M}; \quad \mathrm{d}\mathbb{F} = (1 - S_{\mathrm{melt}})\,\mathrm{d}\mathbb{M}, \tag{19}$$

in which case we can write equation (17) as

$$\mathrm{d}\mathbb{S}_{\mathrm{bal}} = (S_{\mathrm{melt}} - S)\,\mathrm{d}\mathbb{M}_{\mathrm{seawater}}. \tag{20}$$

We thus interpret the salt mass input via balanced salt influx as the difference between the salt mass contained in the added water from the salt mass contained in seawater with equal mass. Correspondingly, the equal and opposing freshwater input associated with this salinity input represents the extra freshwater contained in the meltwater versus that contained within the seawater:

$$-\mathrm{d}\mathbb{S}_{\mathrm{bal}} = -(S_{\mathrm{melt}} - S)\,\mathrm{d}\mathbb{M}_{\mathrm{seawater}}. \tag{21}$$

Where there is instead freezing, with $\mathrm{d}\mathbb{S}$, $\mathrm{d}\mathbb{F}$ and $\mathrm{d}\mathbb{M}$ all negative, the above Eqs. (19)–(21) still hold, but with S_{melt} replaced by S_{ice}.

c. Representing pure salt & pure freshwater inputs as balanced salt/freshwater & seawater inputs

We now consider the case of pure freshwater input, where $\mathrm{d}\mathbb{S} = 0$ and $\mathrm{d}\mathbb{F} \neq 0$ (e.g., evaporation and precipitation). Mathematically this case is revealed by setting $\mathrm{d}\mathbb{S} = 0$ in equation (15). As indicated by the schematic in Fig. 1c, a pure freshwater input can be represented as an input of seawater mass $\mathrm{d}\mathbb{M}_{\mathrm{seawater}} = \mathrm{d}\mathbb{F}$, plus a negative (out of the bucket) mass of salt, $-S\,\mathrm{d}\mathbb{F}$, that cancels

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the salt mass $S\,\mathrm{d}\mathbb{F}$ added to the bucket via the seawater. The consequent change in bucket salinity, $\mathrm{d}S_{\text{pure FW}} = S_{\text{new}} - S$, is given by

$$\mathrm{d}S_{\text{pure FW}} = \frac{-S\,\mathrm{d}\mathbb{F}}{\mathbb{M}+\mathrm{d}\mathbb{F}} = \frac{-S\,\mathrm{d}\mathbb{F}}{\mathbb{M}}[1+\mathscr{O}(\mathrm{d}\mathbb{F}/\mathbb{M})]. \tag{22}$$

Now consider the case of pure salt input with $\mathrm{d}\mathbb{S} = \mathrm{d}\mathbb{S}_{\text{pure salt}} > 0$ and $\mathrm{d}\mathbb{F} = 0$. Mathematically this case is revealed by setting $\mathrm{d}\mathbb{F} = 0$ in equation (15). As indicated by the schematic in Fig. 1d, we can represent this salt input as the sum of a seawater input of mass $\mathrm{d}\mathbb{M}_{\text{seawater}} = \mathrm{d}\mathbb{S}$ plus a balanced salt input with mass $\mathrm{d}\mathbb{S}_{\text{bal}} = (1-S)\,\mathrm{d}\mathbb{S}$. The salinity change for this thought experiment is given by

$$\mathrm{d}S_{\text{pure salt}} = \frac{(1-S)\,\mathrm{d}\mathbb{S}}{\mathbb{M}+\mathrm{d}\mathbb{S}} = \frac{(1-S)\,\mathrm{d}\mathbb{S}}{\mathbb{M}}[1+\mathscr{O}(\mathrm{d}\mathbb{S}/\mathbb{M})]. \tag{23}$$

Comparing to equation (14), we see that the salinity change due to a pure salt input is diluted relative to the salinity change arising from a balanced salt input, $\mathrm{d}\mathbb{S} = \mathrm{d}\mathbb{S}_{\text{bal}}$. There are two terms contributing to the dilution:

(i) The salt $S\,\mathrm{d}\mathbb{S} = S\,\mathrm{d}\mathbb{M}_{\text{seawater}}$ contained in the added seawater $\mathrm{d}\mathbb{M}_{\text{seawater}} = \mathrm{d}\mathbb{S}$ before constructing the massless salinity input.

(ii) The dilution caused by the increase in the total mass in the bucket from $\mathbb{M}$ to $\mathbb{M}+\mathrm{d}\mathbb{S}$, which only contributes at $\mathscr{O}(\mathrm{d}\mathbb{S}/\mathbb{M})^2$.

d. Representing salt & freshwater inputs as pure salt & seawater inputs

Arbitrary inputs of salt and freshwater can alternatively be represented as inputs of seawater (which changes mass but not salinity) and salt (which changes salinity and mass but not freshwater content):

$$\begin{pmatrix} \mathrm{d}\mathbb{S} \\ \mathrm{d}\mathbb{F} \end{pmatrix} = \mathrm{d}\mathbb{M}'_{\text{seawater}} \begin{pmatrix} S \\ 1-S \end{pmatrix} + \mathrm{d}\mathbb{S}_{\text{pure salt}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{24}$$

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with now

$$\mathrm{d}\mathbb{M}'_{\text{seawater}} = \mathrm{d}\mathbb{F}/(1-S), \tag{25}$$

$$\mathrm{d}\mathbb{S}_{\text{pure salt}} = \mathrm{d}\mathbb{S} - S\,\mathrm{d}\mathbb{M}'_{\text{seawater}}. \tag{26}$$

This representation (see Fig. 1e) decomposes a pure freshwater input into a seawater input $\mathrm{d}\mathbb{M}'_{\text{seawater}} = \mathrm{d}\mathbb{F}/(1-S)$ (which is larger than the $\mathrm{d}\mathbb{M}_{\text{seawater}}$ defined in Section 2c as it provides all the freshwater input) plus a negative (out of the water) salt input $-S\,\mathrm{d}\mathbb{F}/(1-S)$ balancing the salt $S\,\mathrm{d}\mathbb{F}/(1-S)$ added via the seawater. The salinity change is the same as that given by the balanced decomposition (22), since the pure salt flux is less effective in driving salinity change by a factor $1-S$ (equation (23)), and so the $1/(1-S)$ factor cancels out.

e. The Boussinesq bucket

The discussion has thus far focused on mass conservation (both total and for FW and salt separately), as applied to a non-Boussinesq fluid. When describing ocean dynamics, it is often more convenient to make the Boussinesq approximation (e.g., Griffies and Greatbatch (2012)). For a Boussinesq fluid, the 'mass-density' used to calculate mass fluxes, tracer content, and momentum is assumed to take a constant value ρ_0. Mass input is thus simply proportional to volume input, and so volume is conserved in the absence of mass input. The density ('buoyancy-mass density') calculated from the equation of state is only used by Boussinesq models to calculate buoyancy and therefore pressure. Changes in volume associated with expansion or contraction of constant-mass elements in a non-Boussinesq fluid become changes in 'buoyancy-mass' associated with changes in the 'buoyancy-mass density' of constant-volume elements in a Boussinesq fluid.

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Suppose that the water in the Boussinesq bucket has volume $\mathbb{V}_0$ with constant density ρ_0, and again initially contains mass $\mathbb{M}$ made up of FW mass $\mathbb{F}$ and salt mass $\mathbb{S}$:

$$\mathbb{M} = \rho_0 \mathbb{V}_0; \qquad \mathbb{S} = \rho_0 S \mathbb{V}_0; \qquad \mathbb{F} = \rho_0 (1 - S) \mathbb{V}_0. \tag{27}$$

Then we can reproduce our previous results if we choose volume changes proportional to the salt and FW mass inputs:

$$\mathrm{d}\mathbb{V}_0 = \rho_0^{-1}\, \mathrm{d}\mathbb{M} = \rho_0^{-1} (\mathrm{d}\mathbb{F} + \mathrm{d}\mathbb{S}), \tag{28}$$

together with a balanced salt flux given from equation (17) as:

$$\mathrm{d}\mathbb{S}_{\mathrm{bal}} = \mathrm{d}\mathbb{S} - S \rho_0\, \mathrm{d}\mathbb{V}_0. \tag{29}$$

It is normal procedure in models to add volume according to (28) when freshwater is input, but not always when salt is input: it is counter-intuitive for salt to have volume, so it is sometimes assumed that addition of salt makes no difference to the volume. But of course the total mass is proportional to the volume in the Boussinesq approximation, so increasing the salinity but keeping the volume constant implies replacement of FW by salt; i.e. a massless balanced salt input rather than a pure salt input.

It is important to note that we use $\mathrm{d}\mathbb{V}_0$, the mass input divided by the Boussinesq density ρ_0, not the *actual* volume added $\mathrm{d}\mathbb{V}$, which depends on temperature and salinity, as well as $\mathrm{d}\mathbb{M}$.

So far we have framed the discussion in this paper in terms of inputs of salt mass and freshwater mass which are well defined extensive quantities (like heat, or enthalpy). In the Boussinesq approximation, however, because the reference density is uniform, it can be useful to consider the volume-integrated salinity (in the same way as it can be sometimes useful when both density and specific heat are uniform to consider volume-integrated temperature). We thus define the volume-integral of the salinity $S^{‰}$ as normally defined in units of per mille, i.e. g kg^{-1}, related to the

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fractional salinity S by $S^{‰} = 1000\,S$ as:

$$\mathbb{S}\mathrm{al} = 1000\rho_0^{-1}\,\mathbb{S}, \tag{30a}$$

the ‘salinity input’ as:

$$\mathrm{d}\mathbb{S}\mathrm{al} = 1000\rho_0^{-1}\,\mathrm{d}\mathbb{S}, \tag{30b}$$

and the ‘balanced salinity input’ as:

$$\mathrm{d}\mathbb{S}\mathrm{al}_{\mathrm{bal}} = \mathrm{d}\mathbb{S}\mathrm{al} - S^{‰}\,\mathrm{d}\mathbb{V}_0. \tag{30c}$$

3. Continuum considerations

We here consider how salinity is forced by salt and freshwater fluxes within the ocean as revealed through the continuum mass budgets for seawater, salt, and freshwater. When formulating the continuum mass budgets, we consider a constant mass fluid element and examine the kinematic constraints imposed by mass conservation. The constant mass seawater element corresponds to the constant mass bucket ($\mathrm{d}\mathbb{M} = 0$) considered in the previous thought experiments. We follow standard treatments for multi-component fluids, such as that given in Section II.2 of DeGroot and Mazur (1984), page 228 of Landau and Lifshitz (1987), chapter 1, Section 9 of Salmon (1998), Beron-Vera et al. (1999), and Section 2.2 ofOlbers et al. (2012).

a. Relating balances of salt, freshwater and total mass

Consider the ocean as a two-component fluid continuum, with separate differential equations for the evolution of salt density $\rho_S = \rho\,S$ and freshwater density $\rho_F = \rho\,F$ where $F = (1 - S)$ is the

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freshwater fraction:

$$\frac{\partial \rho_S}{\partial t} + \nabla \cdot (\rho_S \mathbf{u}_S) = 0 \qquad \text{salt} \tag{31}$$

$$\frac{\partial \rho_F}{\partial t} + \nabla \cdot (\rho_F \mathbf{u}_F) = 0 \qquad \text{freshwater.} \tag{32}$$

These two components are moved around by velocities $\mathbf{u}_S$ and $\mathbf{u}_F$, representing the mean velocities of salt and freshwater molecules, and defined as the total fluxes of salt and FW, divided by their respective densities. Note that these velocities include both 'diffusive' and 'advective' contributions, so may be substantially divergent even for a Boussinesq fluid. See, for example, Section 2.2 of Olbers et al. (2012).

The total mass flux is the sum of the salt and FW fluxes, and then the mass-weighted or 'barycentric' velocity $\mathbf{u}$ is defined as the total mass flux divided by the total density, so is a density weighted mean of the salt and freshwater velocities

$$\rho \mathbf{u} = \rho_S \mathbf{u}_S + \rho_F \mathbf{u}_F, \tag{33}$$

or

$$\mathbf{u} = S \mathbf{u}_S + F \mathbf{u}_F. \tag{34}$$

Summing (31) and (32) and using (33) gives the differential total mass balance as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad \text{seawater.} \tag{35}$$

Split the salt and freshwater fluxes into components with salt and FW moving with the barycentric velocity (the advective flux) and the remainder (the molecular diffusive fluxes) associated with differing directions of flow of salt and FW:

$$\rho_S \mathbf{u}_S = \rho_S \mathbf{u} + \mathbf{J}_S^{\text{mol}}, \tag{36}$$

$$\rho_F \mathbf{u}_F = \rho_F \mathbf{u} + \mathbf{J}_F^{\text{mol}}. \tag{37}$$

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Then the molecular diffusive fluxes of salt and FW, $\mathbf{J}_S^{\mathrm{mol}}$ $\mathbf{J}_F^{\mathrm{mol}}$ represent exchanges of salt and FW and sum to zero (so to give a zero total mass flux)

$$\mathbf{J}_S^{\mathrm{mol}} + \mathbf{J}_F^{\mathrm{mol}} = 0. \tag{38}$$

This identity can be seen by summing (36) and (37) and then applying the definition of the barycentric velocity (33). The fluxes are generally parameterized as downgradient diffusive fluxes

$$\mathbf{J}_S^{\mathrm{mol}} = -\rho\kappa\nabla S \qquad \text{and} \qquad \mathbf{J}_F^{\mathrm{mol}} = -\rho\kappa\nabla F, \tag{39}$$

where $\kappa > 0$ is the kinematic diffusivity for salt in seawater (Gill 1982). Hence, these fluxes vanish in regions of zero concentration gradients. Note that the fundamental derivation of (38) is consistent with the result from summing the explicit expressions for the diffusive fluxes: $\mathbf{J}_S^{\mathrm{mol}} + \mathbf{J}_F^{\mathrm{mol}} = -\rho\kappa\nabla(S+F) = 0$, which follows trivially since $S+F=1$. Or, reversing the argument, since the gradients of salinity and freshwater are equal and opposite, $\nabla S = -\nabla F$, the cancellation of the fluxes (38) confirms that the diffusivities for salt and freshwater are identical, as assumed above in the standard form (39).

Substituting (36) and (37) into (31) and (32) gives the standard advective-diffusive conservation equations for salt and freshwater:

$$\frac{\partial(\rho S)}{\partial t} + \nabla\cdot(\rho\,\mathbf{u}S) = -\nabla\cdot\mathbf{J}_S^{\mathrm{mol}} \qquad \text{salt} \tag{40}$$

$$\frac{\partial(\rho F)}{\partial t} + \nabla\cdot(\rho\,\mathbf{u}F) = -\nabla\cdot\mathbf{J}_F^{\mathrm{mol}} \qquad \text{freshwater,} \tag{41}$$

which can be written in terms of the material time derivative as

$$\rho\,\frac{\mathrm{D}S}{\mathrm{D}t} = -\nabla\cdot\mathbf{J}_S^{\mathrm{mol}} \qquad \text{salt} \tag{42}$$

$$\rho\,\frac{\mathrm{D}F}{\mathrm{D}t} = -\nabla\cdot\mathbf{J}_F^{\mathrm{mol}} \qquad \text{freshwater,} \tag{43}$$

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where the material time operator is computed using the barycentric velocity

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla. \tag{44}$$

Hence it is the diffusive flux $\mathbf{J}_S^{\mathrm{mol}}$ rather than the total salt flux $\rho_S\,\mathbf{u}_S$ that changes the salinity of fluid elements; the advective component $\rho_S\,\mathbf{u}$ is associated with the barycentric velocity and fluxes of seawater mass.

In summary, the diffusive fluxes represent the exchange of salt mass with freshwater mass, and by definition produce no net mass flux when summed, so do not appear in the seawater mass continuity equation (35). That is, a diffusive flux of salt is exactly compensated by an equal and opposite flux of freshwater so that there is identically zero diffusive flux of seawater mass. Moreover, it is the diffusive fluxes that modify the salinity and hence the density.

Note that, because $\mathbf{u}$ is by definition the total barycentric (density-weighted) velocity, there is no 'density diffusion' in the non-Boussinesq continuity equation for total seawater density (35). Instead, specific volume changes, driven by changes in salinity driven by diffusive fluxes of salt and freshwater (or indeed changes in temperature driven by diffusion of heat), are associated with divergence in the barycentric velocity. In the Boussinesq approximation, however, the 'buoyancy density' evolves in response to changes in temperature and salinity but is decoupled from the (incompressible) flow.

b. Kinematic balance of turbulent fluxes

We here show that the flux balance (38) is maintained in the presence of turbulent fluctuations. For that purpose, we perform an eddy/mean decomposition making use of the density-weighted averages of McDougall et al. (2002)

$$\overline{\mathbf{m}} = \overline{\mathbf{u}\rho} \qquad \overline{S}^{\rho} = \overline{\rho S}/\overline{\rho} \qquad \overline{F}^{\rho} = \overline{\rho F}/\overline{\rho} \tag{45}$$

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along with the corresponding fluctuations

$$\mathbf{m}' = \mathbf{m} - \overline{\mathbf{m}} \qquad S' = S - \overline{S}^{\rho} \qquad F' = F - \overline{F}^{\rho}. \tag{46}$$

Taking the mean of equations (35)–(41) and applying this decomposition then leads to the mean mass balances

$$\frac{\partial \overline{\rho}}{\partial t} + \nabla \cdot (\overline{\mathbf{u}}^{\rho}\,\overline{\rho}) = 0 \tag{47}$$

$$\frac{\partial (\overline{\rho}\,\overline{S}^{\rho})}{\partial t} + \nabla \cdot (\overline{\mathbf{u}}^{\rho}\,\overline{\rho}\,\overline{S}^{\rho}) = -\nabla \cdot (\overline{S'\,\mathbf{m}'}) - \nabla \cdot \mathbf{J}_S^{\text{mol}} \tag{48}$$

$$\frac{\partial (\overline{\rho}\,\overline{F}^{\rho})}{\partial t} + \nabla \cdot (\overline{\mathbf{u}}^{\rho}\,\overline{\rho}\,\overline{F}^{\rho}) = -\nabla \cdot (\overline{F'\,\mathbf{m}'}) - \nabla \cdot \mathbf{J}_F^{\text{mol}}. \tag{49}$$

We have introduced the density weighted velocity $\overline{\mathbf{u}}^{\rho} = \overline{\mathbf{m}}/\overline{\rho}$, a generalization to turbulent flow of the barycentric velocity $\mathbf{u}$ for molecular motions used in equations (35)–(41). McDougall et al. (2002) argue that $\overline{\mathbf{u}}^{\rho}$ is the natural definition of the mean velocity for a non-Boussinesq fluid. The relation $S+F=1$ holds also for the mean,

$$\overline{S}^{\rho} + \overline{F}^{\rho} = \overline{(S+F)\,\rho}/\overline{\rho} = 1, \tag{50}$$

so that the fluctuations satisfy $S' + F' = 0$. Hence, the turbulent fluxes of salt and freshwater are correspondingly balanced

$$\mathbf{J}_S^{\text{turb}} + \mathbf{J}_F^{\text{turb}} = \overline{\mathbf{m}'\,S'} + \overline{\mathbf{m}'\,F'} = \overline{\mathbf{m}'\,(S'+F')} = 0. \tag{51}$$

This relation (together with (38)) then ensures that the sum of the mean salt budget and mean freshwater budget equals the mean mass budget; i.e., (48) + (49) = (47).

Analogously to (44), we can define a material derivative in terms of the density weighted mean velocity $\overline{\mathbf{u}}^{\rho}$:

$$\frac{\overline{\mathrm{D}}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \overline{\mathbf{u}}^{\rho} \cdot \nabla, \tag{52}$$

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and set out (48) and (49) in terms of this mean advection:

$$\overline{\rho}\,\frac{\overline{\mathrm{D}S}^{\rho}}{\mathrm{D}t} = -\nabla\cdot\mathbf{J}_S \qquad \text{salt} \tag{53}$$

$$\overline{\rho}\,\frac{\overline{\mathrm{D}F}^{\rho}}{\mathrm{D}t} = -\nabla\cdot\mathbf{J}_F \qquad \text{freshwater,} \tag{54}$$

where the total diffusive fluxes:

$$\mathbf{J}_S = \mathbf{J}_S^{\mathrm{mol}} + \mathbf{J}_S^{\mathrm{turb}}, \tag{55}$$

$$\mathbf{J}_T = \mathbf{J}_F^{\mathrm{mol}} + \mathbf{J}_F^{\mathrm{turb}}, \tag{56}$$

sum to zero by (38) and (51).

Molecular processes are important in carrying the diffusive flux within the surface skin layer, but below this the turbulent fluxes dominate. In the rest of the paper (apart from the Boussinesq subsection immediately below) we shall drop the explicit averaging operator and simply consider the total diffusive fluxes of salt and freshwater, with the understanding that in different parts of the water column they are expressed in different ways

The form of the equations for the material derivative of salinity, (42) and (53), together with the flux balance in equations (38) and (51) suggests that a salt flux balanced by an opposing freshwater flux is the correct flux to force the salinity equation. A pure, unbalanced salt flux carries mass and so would modify the fluid velocity $\mathbf{u}$ (or $\overline{\mathbf{u}}^{\rho}$) that is by definition barycentric. In Section 4 we see how this result impacts on the boundary condition for the salinity equation.

c. Boussinesq fluid

In this case the analysis of sections 3a and 3b goes through as before, except that the total mass density ρ_0 is now constant, so $\rho_S = \rho_0 S$, and $\rho_F = \rho_0 F = \rho_0(1-S)$. Fluxes of salt and FW mass

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now take the form:

$$\rho_S\,\mathbf{u}_S = \rho_0 S\,\mathbf{u}_S = \rho_0 S\,\mathbf{u} + \mathbf{J}_S^{\mathrm{mol}} \tag{57}$$

$$\rho_F\,\mathbf{u}_S = \rho_0 F\,\mathbf{u}_S = \rho_0 F\,\mathbf{u} + \mathbf{J}_F^{\mathrm{mol}}, \tag{58}$$

where the molecular diffusive fluxes are $\mathbf{J}_S^{\mathrm{mol}} = -\rho_0\kappa\nabla S$ and $\mathbf{J}_F^{\mathrm{mol}} = -\rho_0\kappa\nabla F$. As for the non-Boussinesq case, we have the flux balance for molecular fluxes

$$\mathbf{J}_S^{\mathrm{mol}} + \mathbf{J}_F^{\mathrm{mol}} = 0 \tag{59}$$

as well as for turbulent fluxes

$$\mathbf{J}_S^{\mathrm{turb}} + \mathbf{J}_F^{\mathrm{turb}} = \rho_0\overline{\mathbf{u}'S'} + \overline{\mathbf{u}'F'} = \rho_0\overline{\mathbf{u}'\,(S'+F')} = 0, \tag{60}$$

and so also for the total diffusive flux:

$$\mathbf{J}_S + \mathbf{J}_F = 0. \tag{61}$$

The mass budgets (47)–(49) reduce to their Boussinesq form

$$\rho_0\nabla\cdot\overline{\mathbf{u}} = 0 \tag{62}$$

$$\rho_0\,\frac{\overline{\mathrm{D}S}}{\mathrm{D}t} = \rho_0\frac{\partial\overline{S}}{\partial t} + \nabla\cdot(\rho_0\overline{\mathbf{u}}\,\overline{S}) = -\nabla\cdot\mathbf{J}_S \tag{63}$$

$$\rho_0\,\frac{\overline{\mathrm{D}F}}{\mathrm{D}t} = \rho_0\frac{\partial\overline{F}}{\partial t} + \nabla\cdot(\rho_0\overline{\mathbf{u}}\,\overline{F}) = -\nabla\cdot\mathbf{J}_F, \tag{64}$$

where averages no longer need be density-weighted. Here we have retained the ρ_0 factor for consistency with Sections 3a and 3b and to emphasize that these are still fluxes of salt and FW *mass*.

However, if we wish to instead simply consider salinity (now assumed in its conventional units of g kg^{-1}), we then have:

$$\frac{\partial\overline{S}^{‰}}{\partial t} + \nabla\cdot(\overline{\mathbf{u}}\,\overline{S}^{‰}) = -\nabla\cdot\mathbf{J}_{\mathrm{salinity}} \tag{65}$$

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where the total diffusive Boussinesq salinity flux is related to the balanced total diffusive salt flux by:

$$\mathbf{J}_{\text{salinity}} = 1000\rho_0^{-1}\mathbf{J}_S. \tag{66}$$

4. Decomposing surface freshwater fluxes into seawater and balanced salt/freshwater fluxes

a. Formulating the kinematic surface boundary conditions

The vertical position of a point on the ocean free surface is $z = \eta(x,y,t)$. Rewriting this boundary as $\sigma(x,y,z,t) \equiv z - \eta = 0$ allows us to write the outward normal at the free surface as

$$\hat{\mathbf{n}} = \nabla\sigma/|\nabla\sigma| = (\hat{\mathbf{z}} - \nabla\eta)/|\nabla\sigma| \equiv \mathbf{N}/|\nabla\sigma|, \tag{67}$$

where $\mathbf{N} = \nabla\sigma$ is a shorthand. The upwards total mass flux across the free surface per unit area of the sloping free surface is then:

$$\rho\,(\mathbf{u} - \mathbf{u}_\eta)\cdot\hat{\mathbf{n}}, \tag{68}$$

where $\mathbf{u}$ is the barycentric velocity and $\mathbf{u}_\eta$ is the velocity of a point attached to the free surface with constant $\sigma = 0$ so that

$$\frac{\partial\sigma}{\partial t} + \mathbf{u}_\eta\cdot\nabla\sigma = 0. \tag{69}$$

We can link this mass flux (68) to the precipitation, evaporation etc. which are typically given as mass fluxes per unit *horizontal* area. Since each unit of free surface area intercepts a horizontal area $|\nabla\sigma|^{-1}$ (i.e. $\cos(\theta)$ where θ is the angle of the sloping free surface to the horizontal) the flux (68) needs to be multiplied by $|\nabla\sigma|$ (i.e. $\hat{\mathbf{n}}$ replaced by $\mathbf{N}$) to give the flux per unit horizontal area.

The kinematic boundary condition for the upwards flux of total mass per unit horizontal area is then (see Section 2.2.2 of Olbers et al. (2012))

$$\rho\,(\mathbf{u} - \mathbf{u}_\eta)\cdot\mathbf{N} = E - P - M_F - M_S, \tag{70}$$

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where M_F and M_S are the FW and salt mass fluxes into the ocean associated with ice melting and freezing and, for completeness, aeolian deposition of salts, although this is relatively unimportant. Strictly speaking, river runoff is a lateral rather than a surface flux, but it can be apportioned in a similar manner into advective-seawater and diffusive parts, and is indeed often specified in ocean models as a surface flux per unit horizontal area.

Rather than using the barycentric velocity, $\mathbf{u} = S\,\mathbf{u}_S + F\,\mathbf{u}_F$, we can follow Beron-Vera et al. (1999) and Huang (2010) and decompose the kinematic boundary condition (70) into its salt and freshwater components

$$\rho\, S\,(\mathbf{u}_S - \mathbf{u}_\eta)\cdot\mathbf{N} = -M_S \tag{71a}$$

$$\rho\, F\,(\mathbf{u}_F - \mathbf{u}_\eta)\cdot\mathbf{N} = E - P - M_F. \tag{71b}$$

In regions where there is no boundary salt flux, $M_S = 0$, then the free surface acts as a material surface for salt (Beron-Vera et al. 1999), in which case

$$\rho\, S\,(\mathbf{u}_S - \mathbf{u}_\eta)\cdot\mathbf{N} = 0. \tag{72}$$

More generally, the kinematic salt flux boundary condition (71a) can be re-arranged into a kinematic boundary condition for the diffusive fluxes:

$$-M_S = \rho\, S\,(\mathbf{u}_S - \mathbf{u} + \mathbf{u} - \mathbf{u}_\eta)\cdot\mathbf{N} \tag{73a}$$

$$= \mathbf{J}_S\cdot\mathbf{N} + \rho\, S\,(\mathbf{u} - \mathbf{u}_\eta)\cdot\mathbf{N} \tag{73b}$$

$$= \mathbf{J}_S\cdot\mathbf{N} + S\,[E - P - M_F - M_S]. \tag{73c}$$

For the second equality (73b) we split (as in equation (36)) the total salt mass flux into a diffusive flux and an advective component carried by a mass flux with salinity S; this mass flux is the 'seawater flux' of the bucket decomposition (15). The surface kinematic boundary condition (70) sets

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this (upwards) ‘sea-water mass flux’ as

$$M_{\text{seawater}} = E - P - M_F - M_S, \tag{74}$$

yielding the third expression (73c). Collecting the M_S terms on both sides of equation (73c) reveals that the diffusive salt flux has a component up across the free surface given by

$$\mathscr{S}_{\text{out}} = \mathbf{J}_S \cdot \mathbf{N} = S\,(P - E + M_F) - (1 - S)\,M_S. \tag{75}$$

We can similarly re-arrange the FW flux boundary condition (71b) to give

$$E - P - M_F = \mathbf{J}_F \cdot \mathbf{N} + \rho\,F\,(\mathbf{u} - \mathbf{u}_\eta) \cdot \mathbf{N} \tag{76a}$$

$$= \mathbf{J}_F \cdot \mathbf{N} + F\,[E - P - M_F - M_S], \tag{76b}$$

thus rendering an expression for the diffusive FW flux

$$\mathscr{F}_{\text{out}} = \mathbf{J}_F \cdot \mathbf{N} = (1 - F)\,(E - P - M_F) + F\,M_S = -\mathscr{S}_{\text{out}} \tag{77}$$

that exactly balances the diffusive salt flux (75). Given this balance between salt and FW fluxes, and according to our convention in Section 1a, we refer to the RHS of (75) and (77) as a balanced diffusive salt flux $S\,(P - E + M_F) - (1 - S)\,M_S$, which is calculated from salt and FW fluxes exactly as for the bucket in Eq. (18). It is this balanced diffusive salt flux that should be used as the surface boundary condition for the salt and freshwater conservation equations (40) and (41) (or the turbulence-averaged versions (48) and (49)) and hence for calculating derived properties such as buoyancy.

b. Interpreting the kinematic boundary conditions

We interpret the boundary condition (75) by noting that the diffusive mixing of salt within the ocean is required to mediate the incorporation or removal of a boundary freshwater flux into the

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ocean. Since it is the mass of a fluid element that is constant, any transfer of freshwater into that element must be compensated by a removal of salt (77), and vice versa. Through the act of salt diffusion in one direction, freshwater diffuses in the opposite. That is the physical content of the boundary conditions (75) and (77).

For example, suppose pure freshwater is removed from the ocean at a rate, $E - P > 0$; $M_s = 0$. Part of this freshwater flux leaves the ocean (moves upwards) as the freshwater component of an advective sink of seawater, with (76b) mass flux $\mathscr{M}_{\text{seawater}} = E - P$, salinity S, and FW concentration F (see Fig. 2a). Since $F < 1$, the advective flux is always less than the total freshwater sink, and so the balance diffuses upwards as a diffusive FW flux

$$\mathscr{F}_{\text{out}} = \mathbf{J}_F \cdot \mathbf{N} = S(E - P) > 0. \tag{78}$$

This upwards FW diffusive flux is balanced by an equal and opposite diffusive downwards flux of salt just below the surface

$$\mathscr{S}_{\text{in}} = -\mathbf{J}_S \cdot \mathbf{N} = \mathbf{J}_F \cdot \mathbf{N} = \mathscr{F}_{\text{out}} = S(E - P) > 0. \tag{79}$$

This breakdown into seawater and balanced salinity fluxes is also evident in the slab model of section 2; see Fig. 1c and section 2c.

Correspondingly, for a thought experiment without diffusive mixing (e.g., a perfect fluid), boundary freshwater is not incorporated into or removed from the ambient ocean fluid. For such a perfect fluid, there is a fundamental asymmetry between precipitation and evaporation. In the case of precipitation the surface salinity remains equal to zero, and so $F = 1$ in (76b), and no diffusive flux is required to maintain the balance (76b). Instead, the pure freshwater forms a thickening, unmixed lens sitting on top of the seawater. Where there is net evaporation in the perfect fluid, however, the decomposition into pure salt is appropriate, as there is no diffusion and the freshwater that is evaporated can only come from an advective flux $\mathscr{M}'_{\text{seawater}} = (P - E)/(1 - S)$.

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Pure salt would simply build up on the surface at the rate given by (2) and in section 4c below: $\mathscr{S}'_{\text{in}} = S(P-E)/(1-S)$.

This discussion of net precipitation into a perfect fluid emphasizes the sensitivity of the split into 'seawater' (advective) and 'diffusive' fluxes to the choice of reference salinity S. In the slab (bucket) case discussed in section 2, where we assume the fluid will always remain well-mixed, the reference salinity is clearly the pre-existing salinity of the slab or bucket, but the choice of reference salinity is less clear in the continuum case described here. In practice the mixed-layer salinity is generally chosen on the assumption that fluid in the mixed-layer is reasonably well-mixed.

c. The surface layer salt flux

We now summarize the argument of Steinhorn (1991) leading to the vertical boundary flux in equation (2). Imagine again an upward net freshwater mass flux $E-P>0$. Steinhorn (1991) conjectures (see Fig. 2b) that this freshwater flux is supplied by an upward vertical flux of seawater, $\mathscr{M}'_{\text{seawater}}$, within the ocean surface layer so that

$$F\,\mathscr{M}'_{\text{seawater}} = E-P. \tag{80}$$

With this formulation, the seawater mass flux just below the ocean surface layer is larger in magnitude than the freshwater flux out of the ocean

$$\mathscr{M}'_{\text{seawater}} = (E-P)F^{-1} > E-P. \tag{81}$$

Along with freshwater, this seawater mass flux carries a salt flux $(P-E)\,S/(1-S)$ upwards towards the surface. However, since salt does not cross the air-sea interface, Steinhorn (1991) infers a downward compensating salt flux with magnitude $(E-P)\,S/(1-S)$, thus leading to the expres-

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sion (2),

$$\mathscr{S}'_{\text{in}} = (E-P)\,S/(1-S)$$

for the surface boundary condition.

The error in Steinhorn's argument is that it ignores the kinematic balance (38) between diffusive salt and freshwater fluxes. Maintaining this balance requires a downward diffusive flux of freshwater in the surface layer when there is an upward diffusive flux of salt, as discussed in the text surrounding equation (75).

d. Boussinesq fluxes

For a Boussinesq ocean, the diffusive salt-mass and freshwater-mass fluxes are still given by (75) and (77), and the seawater mass flux given by (74). However the natural requirements of the Boussinesq model are the seawater volume outflux per unit area (upwards velocity through the sea-surface):

$$w_{0\,\text{seawater}} = \rho_0^{-1}(E-P-M_S-M_F), \tag{82}$$

and the diffusive upwards flux of salinity expressed as per mille (g kg^{-1}):

$$\begin{aligned}\mathscr{S}^{‰}_{\text{out}} &= \mathbf{J}_{\text{salinity}}\cdot\mathbf{N} = 1000\rho_0^{-1}\mathbf{J}_S\cdot\mathbf{N}\\ &= 1000\rho_0^{-1}[S(P-E+M_F)-(1-S)M_S.]\end{aligned} \tag{83}$$

Where precipitation P and evaporation E are given as velocities rather than mass fluxes, we suggest both for Boussinesq and non-Boussinesq applications that they always be converted to mass fluxes by multiplying by the density of pure water at the sea surface temperature (SST) and atmospheric pressure. Similarly, volume fluxes of ice-melt should be converted to mass fluxes using the density at the appropriate salinity and temperature and then split into salt and FW mass

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fluxes according to the salinity of the ice-melt. In Boussinesq applications volume fluxes and per mille salinity fluxes should always be calculated from mass fluxes by dividing by ρ_0.

The suggestion made e.g. in Olbers et al. (2012) that

$$\rho_w(T, p_a) = (1 - S)\rho(T, S, p_a), \tag{84}$$

(where T is SST and p_a atmospheric pressure) is incorrect, because the haline contraction coefficient $\rho^{-1}\partial\rho/\partial S \approx 0.8 < 1$ (where S is expressed as a fractional salinity).

5. Closing comments

Salinity is the ratio of salt mass to seawater mass in an element of seawater. In the presence of air-sea freshwater fluxes, ocean salinity changes in the surface boundary layer are affected by the vertical *balanced* diffusive salt flux boundary condition according to equation (1), which in turn leads to changes in ocean buoyancy. The alternative expression in equation (2) is a surface layer 'unbalanced' salt flux that is *not* balanced by an opposing freshwater flux and is not appropriate for computing surface ocean buoyancy forcing. For purposes of forcing a Boussinesq ocean model, a diffusive salinity flux can be constructed from the balanced salt flux (1) according to (66) and (83).

We encountered the ambiguity in the literature between expressions (1) and (2) while pursuing watermass analysis (e.g., Large and Nurser 2001; Groeskamp et al. 2019). The differences between expressions (1) and (2) are small relative to uncertainties in measured freshwater fluxes. So most practitioners of watermass analysis ignore the distinction. Even so, we emphasized in this note the conceptual distinction for the two boundary fluxes. In brief, expression (1) respects the kinematic constraints on how matter (salt and freshwater) is exchanged between seawater elements whereas expression (2) does not.

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The distinction between the 'balanced' and 'unbalanced' salt flux is only noticeable because salt makes up a significant ($\approx 3.5\%$) fraction of seawater mass, so the factor $(1-S)^{-1} \approx 1.036$. Fluxes of heat already carry no mass and so require no decomposition into seawater mass fluxes and massless diffusive fluxes. For material tracers that have much lower mass fractions λ than salt, i.e. $\lambda << 1$ (e.g. CFCs), the difference between the 'balanced' and unbalanced diffusive fluxes becomes insignificant as the factor $(1-\lambda)^{-1} \to 1$.

We finally note that in considering regional and global budgets of freshwater and salt, similar ideas appear in the split of lateral fluxes of salt and FW into components associated with (i) the salt and freshwater carried in the transports of water with section-mean salinity (the advective flux carried by the section-mean barycentric velocity, analogous to the advective flux carried by the local-mean salinity and barycentric velocity in equations (48) and (49)) and (ii) the 'eddy' fluxes associated with correlations of deviations from section mean salinity and velocity analogous to the turbulent diffusive fluxes in equations (48) and (49). See Wijffels et al. (1992) and Bacon et al. (2015) for examples.

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LIST OF TABLES

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TABLE 1. List of variables used in section 2

Variable	Symbol	Units
Absolute (fractional) salinity	S	$\mathrm{kg\,kg^{-1}}$
Absolute fractional salinity of ice melt	S_{melt}	$\mathrm{kg\,kg^{-1}}$
Absolute fractional salinity of freezing ice	S_{ice}	$\mathrm{kg\,kg^{-1}}$
Total mass	$\mathbb{M}$	kg
Salt mass	$\mathbb{S}$	kg
Freshwater mass	$\mathbb{F}$	kg
Boussinesq mass density	ρ_0	$\mathrm{kg\,m^{-3}}$
Volume of Boussinesq fluid	$\mathbb{V}_0$	$\mathrm{m^3}$
Increment in Boussinesq volume	$\mathrm{d}\mathbb{V}_0$	$\mathrm{m^3}$
Absolute salinity (per mille)	$S^{‰}$	$\mathrm{g\,kg^{-1}}$
Volume integrated salinity	$\mathbb{S}\mathrm{al}$	$\mathrm{g\,kg^{-1}\,m^3}$
Increment of volume integrated salinity	$\mathrm{d}\mathbb{S}\mathrm{al}$	$\mathrm{g\,kg^{-1}\,m^3}$
Increment of volume integrated salinity at constant volume	$\mathrm{d}\mathbb{S}\mathrm{al}_{\mathrm{bal}}$	$\mathrm{g\,kg^{-1}\,m^3}$
Increment of total mass	$\mathrm{d}\mathbb{M}$	kg
Increment of salt mass	$\mathrm{d}\mathbb{S}$	kg
Increment of freshwater mass	$\mathrm{d}\mathbb{F}$	kg
Increment of mass of water with same salinity as in bucket	$\mathrm{d}\mathbb{M}_{\mathrm{seawater}}$	kg
Increment of salt balanced by loss of same mass of freshwater	$\mathrm{d}\mathbb{S}_{\mathrm{bal}}$	kg
Pure increment of salt with no associated freshwater input	$\mathrm{d}\mathbb{S}_{\mathrm{pure\ salt}}$	kg
Increment of mass of water with		

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TABLE 2. List of continuum variables used in Section 3

Variable	Symbol	Units
Total mass, salt and FW density	ρ, ρ_S, ρ_F	$\mathrm{kg\,m^{-3}}$
Boussinesq reference density	ρ_0	$\mathrm{kg\,m^{-3}}$
Barycentric velocity	$\mathbf{u}$	$\mathrm{m\,s^{-1}}$
Salt and FW velocity	$\mathbf{u}_S$, $\mathbf{u}_F$	$\mathrm{m\,s^{-1}}$
Molecular diffusive flux of salt	$\mathbf{J}_S^{\mathrm{mol}}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Molecular diffusive flux of FW	$\mathbf{J}_F^{\mathrm{mol}}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Turbulent diffusive flux of salt	$\mathbf{J}_S^{\mathrm{turb}}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Turbulent diffusive flux of FW	$\mathbf{J}_F^{\mathrm{turb}}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Total diffusive flux of salt	$\mathbf{J}_S$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Total diffusive flux of FW	$\mathbf{J}_F$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Total diffusive flux of salinity	$\mathbf{J}_{\mathrm{salinity}}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Molecular diffusivity of salt	κ	$\mathrm{m^2\,s^{-1}}$
Total mass flux per unit area	$\mathbf{m}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Mean density	$\overline{\rho}$	$\mathrm{kg\,m^{-3}}$
Density-weighted mean velocity	$\overline{\mathbf{u}}^{\rho}$	$\mathrm{m\,s^{-1}}$
Density-weighted mean salinity	$\overline{S}^{\rho}$	$\mathrm{kg\,kg^{-1}}$
Density-weighted mean FW	$\overline{F}^{\rho}$	$\mathrm{kg\,kg^{-1}}$

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TABLE 3. List of near-surface flux variables

Variable	Symbol	Units
Evaporation	E	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Precipitation	P	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Diffusive downwards balanced salt flux	$\mathscr{S}_{\mathrm{in}}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Downwards pure salt flux	$\mathscr{S}'_{\mathrm{in}}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Sea surface height (SSH)	η	m
Distance above SSH	σ	m
Upward unit normal through sea surface	$\hat{\mathbf{n}}$	None
$\hat{\mathbf{n}}\times$ real (sloping) surface area $\div$ horizontal surface area	$\mathbf{N}$	None
Velocity following sea surface	$\mathbf{u}_\eta$	$\mathrm{m\,s^{-1}}$
Salt flux into ocean from ice melt and/or runoff	M_S	$\mathrm{kg\,m^{-2}\,s^{-1}}$
FW flux into ocean from ice melt and/or runoff	M_F	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Diffusive upwards FW flux	$\mathscr{F}_{\mathrm{out}}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Upwards near-surface sea- -water flux associated with diffusive salt flux	$\mathscr{M}_{\mathrm{seawater}}$	$\mathrm{kg\,m^{-2}\,s^{-1}}$
Upwards near-surface sea- -water flux associated with pure salt flux	$\mathscr{M}'_{\mathrm{seawater}}$	$\mathrm{kg\,m^{2}\,s^{-1}}$
Density of pure water	ρ_w	$\mathrm{kg\,m^{-3}}$
Boussinesq seawater loss per unit area	$w_{0\ \mathrm{seawater}}$	$\mathrm{m\,s^{-1}}$

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LIST OF FIGURES

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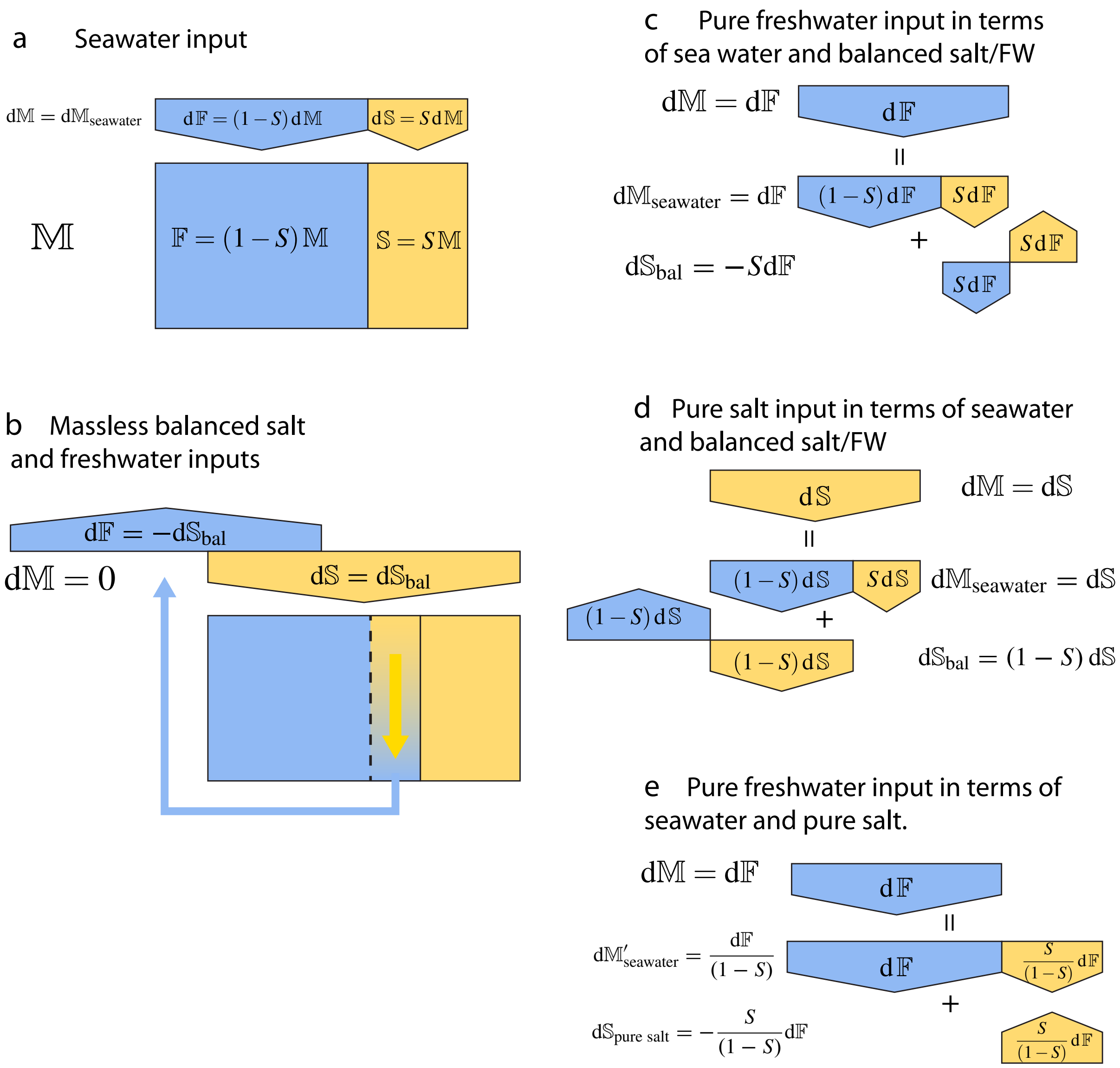

FIG. 1. Bucket science. (a) The addition of a mass of seawater $d\mathbb{M}$ with the same salinity as the pre-existing bucket salinity $S = \mathbb{S}/\mathbb{M}$. (b) A massless salinity input with input of salt $d\mathbb{S}$ balanced by freshwater loss $d\mathbb{F} = -d\mathbb{S}$. (c) Decomposition of a pure freshwater input into seawater and salinity inputs. (d) Decomposition of pure salt input into seawater and salinity inputs. (e) Decomposition of pure freshwater input into seawater and pure salt inputs.

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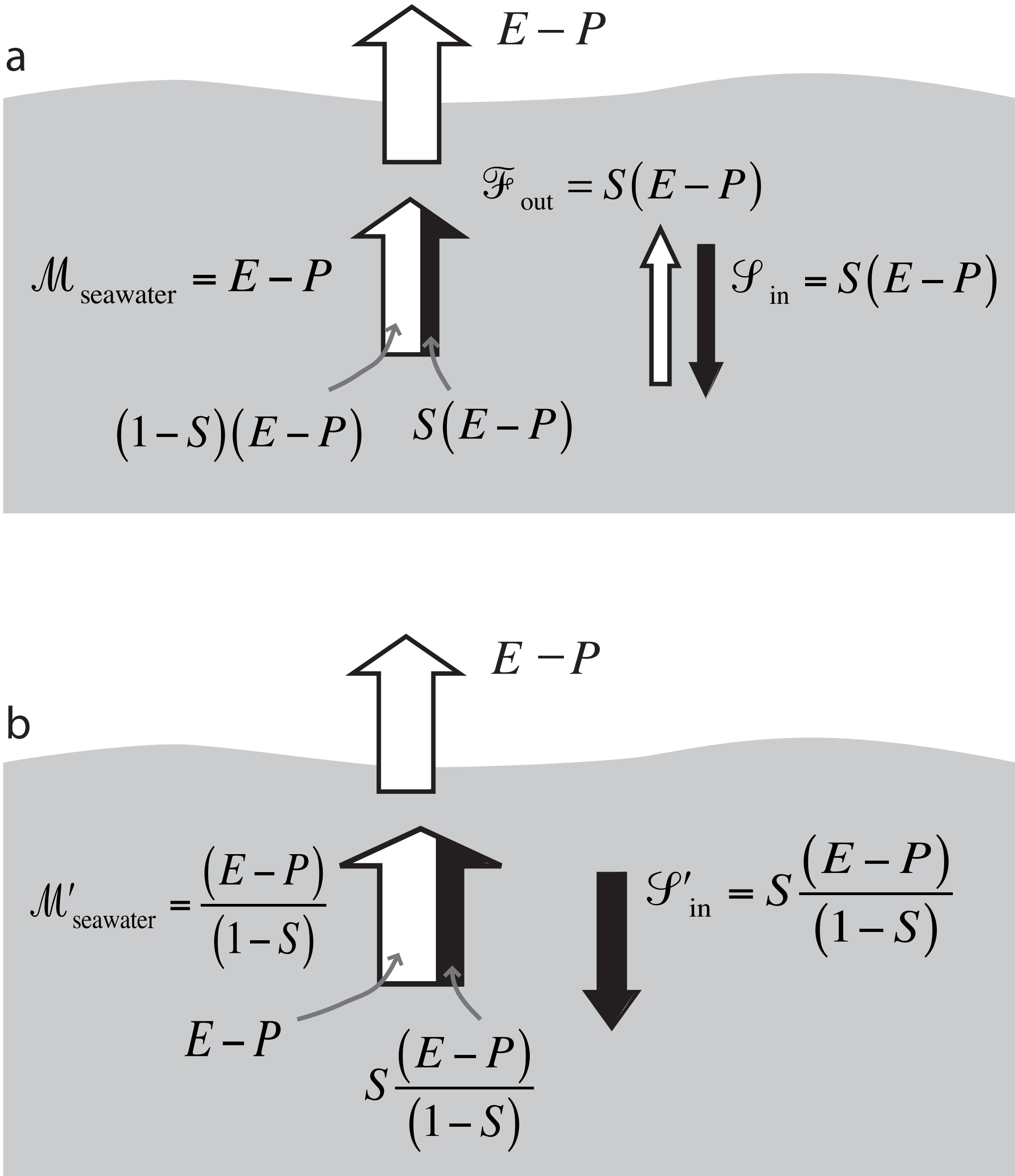

FIG. 2. Schematic of the two conceptual perspectives on the fluxes of salt and freshwater in the ocean surface layer (denoted by the gray shaded region). Panel a: The decomposition of $E-P$ as a seawater flux $\mathscr{M}_{\text{seawater}}$ and a salt flux $\mathscr{S}_{\text{in}}$ balanced by an equal and opposite freshwater flux $\mathscr{F}_{\text{bal}}$. Widths of the arrows represent the strength of the associated mass fluxes. Panel b: The decomposition of outward freshwater flux $E-P>0$ as a seawater flux $\mathscr{M}'_{\text{seawater}}$ and a pure, unbalanced, salt flux $\mathscr{S}'_{\text{in}}$.

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