A form of potential vorticity equation for depth-integrated flow with a free surface.

Chris W. Hughes *

Proudman Oceanographic Laboratory, Liverpool, U.K.

19 September 2007

**Corresponding author address:* Chris W. Hughes, Proudman Oceanographic Laboratory, 6 Brownlow St., Liverpool L3 5DA, U.K.

E-mail: cwh@pol.ac.uk

Abstract

A form of linear, barotropic potential vorticity equation is derived for an ocean with a free surface, in which only one scalar variable appears (ocean bottom pressure, or subsurface pressure). Unlike quasigeostrophic or rigid lid derivations, the only approximation made (apart from linearization) is that changes in the circulation must be slow compared with the inertial frequency. Effects of stratification are included, but only parametrically in the sense that density is treated as a given quantity or forcing term rather than a variable.

1. Introduction

In recent years, a number of large-scale barotropic modes have been identified in the ocean. Particularly visible at mid- to high-latitudes in the Southern Ocean and the North Pacific, these modes dominate the large-scale sea level variability at these latitudes for periods longer than a few days and shorter than annual. Fukumori et al. (1998) give a good general overview of the dynamics of these modes, which have been examined in more detail by, among others, Hughes et al. (1999), Ponte (1999), Webb and de Cuevas (2002a), Webb and de Cuevas (2002b), Fu (2003), and Weijer and Gille (2005). The modes discussed are typically basin scale, with single-signed sea level signals occupying of order 90° of longitude in the Southern Ocean. At these frequencies and length scales, the theory of Gill and Niiler (1973) suggests that the influence of stratification can be ignored. This is consistent with the model diagnostics of Fukumori et al. (1998), and is confirmed by the success of the barotropic models of Tierney et al. (2000), Hirose et al. (2001), and Carrére and Lyard (2003), among others, in explaining observed sea level signals.

At subinertial frequencies, theoretical understanding of these motions is aided by approximations which filter out inertio-gravity waves, making it possible to construct a linear potential vorticity equation in one variable, which can be interpreted as describing a combination of topographic Sverdrup balance and topographic Rossby waves. It is relatively easy to construct such an equation if the Coriolis parameter f can be taken to be constant (clearly not the case for modes with the length scales discussed here), or if the rigid lid approximation can be made. Unfortunately, with such large scales, assuming a rigid lid is also a poor approximation. The term in the potential vorticity equation which relates the the free surface is comparable to other terms if the wavelength of the mode considered is close to $2\pi R_0$, where R_0 is the barotropic

Rossby radius $R_0 = \sqrt{gH}/f$, with g the acceleration due to gravity, and H the ocean depth. This intuitive scaling will be rigorously justified later. Taking an ocean depth of 4000m gives $\sqrt{gH} = 200\mathrm{m\,s^{-1}}$. Substituting a numerical value for the earth's rotation rate, and expressing R_0 in degrees of longitude gives $R_0 = (24.7/\sin(2\phi))^\circ$, where ϕ is latitude. Between latitudes of 30° and 60° (north or south), the term $\sin(2\phi)$ is between 0.866 and 1. This means that free surface effects must be accounted for when the half-wavelength approaches $\pi R_0 \approx 85^\circ$ of longitude at mid-latitudes. This is precisely the length scale of the observed modes, meaning that free surface effects are important.

The purpose of this note is to derive a linear barotropic potential vorticity equation which will be valid for these large-scale subinertial modes. The modes occur in regions of strong topography, so we cannot apply a quasigeostrophic approximation which is incapable of representing flows which cross a depth range which is a significant fraction of the total ocean depth. In fact, apart from linearizing the momentum equations, the only approximation that must be made is that the evolution of the flow be assumed slow compared to an inertial period. For completeness, we allow forcing by mass sources and atmospheric pressure, and include the effect of stratification in a parametric manner, although the main value of the equation we derive is for situations in which the stratification can be considered constant. In order to link the final result with some previously-used equations, however, we first consider the simpler rigid-lid case.

2. The rigid lid case

If we start with the linear momentum equation in terms of horizontal velocity $\mathbf{u}$ and density ρ:

$$\rho \mathbf{u}_t + f\mathbf{k} \times (\rho \mathbf{u}) = \mathbf{G}, \tag{1}$$

where $\mathbf{k}$ is a unit vector in the local vertical (upwards), and $\mathbf{G}$ is the horizontal force per unit volume, then an integral from the sea floor ($z = -H$) to the surface ($z = \eta$) gives the depth-integrated equation

$$\mathbf{U}_t + f\mathbf{k} \times \mathbf{U} = \mathbf{F}, \tag{2}$$

where $\mathbf{U} = \int_{-H}^{\eta} \rho \mathbf{u} \, dz$ is depth-integrated mass transport, and $\mathbf{F} = \boldsymbol{\tau} - \int_{-H}^{\eta} \nabla p \, dz$. Here, $\boldsymbol{\tau}$ is meant to represent the surface wind stress, but can be taken to represent (wind-bottom) stress, or indeed any other depth-integrated force. Pressure is p, and ∇ represents the horizontal gradient operator.

For the rigid lid case, we can set $\eta = 0$. The pressure contribution to $\mathbf{F}$ then becomes

$$\int_{-H}^{0} \nabla p \, dz = \int_{-H}^{0} \nabla (p - p_b) \, dz + H \nabla p_b, \tag{3}$$

where ∇ is the horizontal gradient operator, and the difference between pressure p and bottom pressure p_b can be calculated using hydrostatic balance as $p - p_b = -g \int_{-H}^{z} \rho \, dz'$. This means that the first term on the right had side of (3) can be written as

$$-g \int_{-H}^{0} \nabla \int_{-H}^{z} \rho \, dz' \, dz = -g \nabla \int_{-H}^{0} \int_{-H}^{z} \rho \, dz' \, dz = \nabla \int_{-H}^{0} \rho g z \, dz = \nabla E. \tag{4}$$

This allows us to write

$$\mathbf{F} = \boldsymbol{\tau} - \nabla E - H \nabla p_b. \tag{5}$$

In addition, mass conservation in the rigid lid case ($\nabla \cdot \mathbf{U} = 0$) allows us to introduce a mass transport streamfunction Ψ defined by $\mathbf{U} = \mathbf{k} \times \nabla\Psi$, so that (2) becomes

$$\mathbf{k} \times \nabla\Psi_t - f\nabla\Psi = \boldsymbol{\tau} - \nabla E - H\nabla p_b. \tag{6}$$

In the case of constant density, the pressure and density terms are more clearly combined as $\nabla E + H\nabla p_b = H\nabla p_0$, where p_0 is pressure at the rigid lid ($\eta = 0$).

As discussed by Hughes and Killworth (1995), it is possible to eliminate three different terms of this equation by taking the curl after dividing by 1, H, or f. The first option gives the barotropic vorticity equation:

$$\nabla^2\Psi_t + J(\Psi, f) = \nabla \times \boldsymbol{\tau} + J(p_b, H) \tag{7}$$

(since we are focusing on two-dimensional equations, the operator $\nabla\times$ is taken as shorthand for the scalar given by $\mathbf{k} \cdot \nabla\times$). Dividing by H first gives the barotropic potential vorticity equation:

$$\nabla \cdot \left(\frac{\nabla\Psi_t}{H}\right) + J(\Psi, f/H) = \nabla \times \left(\frac{\boldsymbol{\tau}}{H}\right) + J(E, 1/H). \tag{8}$$

Dividing by f first gives a third equation:

$$\nabla \cdot \left(\frac{\nabla\Psi_t}{f}\right) = \nabla \times \left(\frac{\boldsymbol{\tau}}{f}\right) + J(E, 1/f) + J(p_b, H/f). \tag{9}$$

Unlike the other two equations, no variable is eliminated by this manipulation (except in the steady case).

There is no clear, exact way to incorporate the effect of a free surface into any of these equations. A free surface will introduce sea level or bottom pressure from the mass continuity equation, but the closest to a formulation in terms of bottom pressure is (9), which retains the streamfunction in the tendency term. This is understandable, since the presence of a free surface permits a new class of waves: inertio-gravity waves. If we are to derive a free surface form of barotropic vorticity or potential vorticity equation, it must be in an approximation which removes these waves. A different approach is needed.

3. A free surface derivation

Starting again with the depth-integrated linear momentum equation (2), we also need the mass continuity equation which can be written as

$$\nabla \cdot \mathbf{U} = Q - M_t, \tag{10}$$

where Q is the mass source per unit area, and M is the ocean mass per unit area. This can be rewritten using hydrostatic balance in terms of the difference between bottom pressure and surface atmospheric pressure: $M = (p_b - p_a)/g$. The aim is now to eliminate $\mathbf{U}$ from these equations, leaving equations in pressure only. We start by deriving two equations for $\nabla \times \mathbf{U}$ by taking the divergence of (2), and the curl of (2)/f, and substituting for $\nabla \cdot \mathbf{U}$ from (14). This gives

$$Q_t - M_{tt} - f\nabla \times \mathbf{U} + \mathbf{U} \times \nabla f = \nabla \cdot \mathbf{F}, \tag{11}$$

and

$$\frac{1}{f}\nabla \times \mathbf{U}_t + \frac{\mathbf{U}_t \times \nabla f}{f^2} + Q - M_t = \nabla \times \left(\frac{\mathbf{F}}{f}\right). \tag{12}$$

We can now eliminate $\nabla \times \mathbf{U}$ by taking the time derivative of (11) plus f^2 times (12):

$$(\partial_{tt} + f^2)(Q - M_t) + 2\mathbf{U}_t \times \nabla f = \nabla \cdot \mathbf{F}_t + f^2\nabla \times \left(\frac{\mathbf{F}}{f}\right). \tag{13}$$

If f is uniform, the second term is zero and we have achieved our aim. When f varies, however, we need another equation for the zonal component of $\mathbf{U}$. We get this by eliminating the meridional component from (2) by taking $(2)_t \times \nabla f + f(2) \cdot \nabla f$, to obtain

$$(\partial_{tt} + f^2)\mathbf{U} \times \nabla f = \mathbf{F}_t \times \nabla f + f\mathbf{F} \cdot \nabla f. \tag{14}$$

We can now eliminate $\mathbf{U}$ from (13) and (14) by taking $(\partial_{tt} + f^2)(13) - 2(14)_t$. After some gathering of terms, this gives

$$(\partial_{tt} + f^2)^2 (Q - M_t) = f^4 \nabla \times \left(\frac{\mathbf{F}}{f}\right) + f^4 \nabla \cdot \left(\frac{\mathbf{F}_t}{f^2}\right) + \nabla \times (f\mathbf{F}_{tt}) + \nabla \cdot \mathbf{F}_{ttt}. \tag{15}$$

This is the exact equation, with $\mathbf{U}$ eliminated, which we were seeking. The only approximation which has been made so far is linearization of the momentum equation. Together with $\mathbf{F} = \boldsymbol{\tau} - \int_{-H}^{\eta} \nabla p \, dz$ and $M = (p_b - p_a)/g$, it is an equation expressed purely in terms of pressures. However, being fifth order in time it is probably of little practical use in itself. Instead, (15) can be used to derive consistent approximations (LeBlond and Mysak (1978) derive an exactly equivalent equation, their Eq. 20.13a, expressed in terms of the inverse of the operator $(\partial_{tt} + f^2)$, and with no external forcing).

A complete analysis of the relative orders of magnitude of each of the terms would depend on the length scales, ocean depth, structure of the wind stress, and many other details. Our concern here is to leave the equation as general as possible, making approximations only in terms of the frequency. Accordingly, we will make an expansion in the term $\delta = \omega/f$, where δ is assumed to be much less than one. Dropping terms of order δ^2 and smaller is then equivalent to dropping terms differentiated more than once with respect to time, and has the effect of filtering out inertio-gravity waves. Doing this, and dividing by f^4, reduces (15) to

$$Q - M_t = \nabla \times \left(\frac{\mathbf{F}}{f}\right) + \nabla \cdot \left(\frac{\mathbf{F}_t}{f^2}\right). \tag{16}$$

Again, LeBlond and Mysak (1978) perform the same expansion, but they also make a substitution in which a term of order δ is dropped. This results in the factor $1/f^2$ occurring outside the divergence term. Following their method through, but retaining the order δ term again results in an equation equivalent to (16) but with no external forcing.

In order to make the terms on the right hand side more explicit (to separate out rapid pressure changes connected to the external mode, pressure changes due to stratification, and externally-applied stresses), we extend the results of Section 2 to write $\mathbf{F} = \boldsymbol{\tau} - \nabla E - H\nabla p_b - \int_0^\eta \nabla p \, dz$. As long as η is much smaller than the depth over which horizontal pressure gradients are substantial, the last term is much smaller than $\nabla E + H\nabla p_b$ and can be neglected. However, for completeness, we will give an almost exact form.

Assuming only that $z = 0$ is always below the sea surface, but within a mixed layer such that $\rho = \rho_s(x, y)$ independent of depth above $z = 0$, a simple integral of the pressure gradient above $z = 0$ leads to

$$\mathbf{F} = \boldsymbol{\tau} - \nabla E - H\nabla p_b - g\nabla\left(\frac{\rho_s\eta^2}{2}\right) - \eta\nabla p_a, \tag{17}$$

which can also be written as

$$\mathbf{F} = \boldsymbol{\tau} - \nabla E - H\nabla p_b + \frac{g\eta^2}{2}\nabla\rho_s - \eta\nabla p_0, \tag{18}$$

where p_0 is the pressure at $z = 0$ (sub-surface pressure). As in the rigid lid case, the vertical integral of the pressure gradient simplifies in the homogeneous case ($\rho =$ constant), to give

$$\mathbf{F} = \boldsymbol{\tau} - h\nabla p_0, \tag{19}$$

where $h = H + \eta$ is the total ocean depth.

Ignoring the small free surface terms in (17) or (18), substitution into (16) gives

$$\frac{p_{bt}}{g} - \nabla\cdot\left(\frac{H\nabla p_{bt}}{f^2}\right) + J(p_b, H/f) = Q + \frac{p_{at}}{g} - \nabla\times\left(\frac{\boldsymbol{\tau}}{f}\right) - J(E, 1/f) + \nabla\cdot\left(\frac{\nabla E_t - \boldsymbol{\tau}_t}{f^2}\right). \tag{20}$$

Alternatively, for the homogeneous case, if we note that $p_{bt} = p_{0t}$, then substitution of (19) into (16) gives

$$\frac{p_{0t}}{g} - \nabla \cdot \left(\frac{(h\nabla p_0)_t}{f^2} \right) + J(p_0, h/f) = Q + \frac{p_{at}}{g} - \nabla \times \left(\frac{\boldsymbol{\tau}}{f} \right) - \nabla \cdot \left(\frac{\boldsymbol{\tau}_t}{f^2} \right), \qquad (21)$$

where $h = H + \eta = H + (p_0 - p_a)/\rho g$ is the total ocean depth, and p_0 is again the pressure at some reference level $z = 0$ not far below the surface. Substituting $p_0 = p_a + \rho g \eta$, (21) can also be written in terms of sea level η as

$$\eta_t - \nabla \cdot \left(\frac{g(h\nabla \eta')_t}{f^2} \right) + J(\eta', gh/f) = \frac{Q}{\rho} - \nabla \times \left(\frac{\boldsymbol{\tau}}{\rho f} \right) - \nabla \cdot \left(\frac{\boldsymbol{\tau}_t}{\rho f^2} \right), \qquad (22)$$

where $\eta' = \eta + p_a/(\rho g)$ represents the inverse-barometer corrected sea level.

For the large-scale, relatively high frequency modes we are considering, the role of stratification is small and (21) or (22) is the appropriate equation (for which a reasonable simplifying approximation is to replace h with H). The more general equation (20) is included to make it clear how this links to the rigid lid case and to other equations in the literature.

4. Interpretation and comparison with other equations

The forcing terms on the right hand side of (21) can clearly be seen to be related to mass sources. There is the oceanic mass source Q, a measure of locally increasing atmospheric mass, and the divergence of the Ekman flux. The final term is an order δ correction to the Ekman flux divergence to account for the finite time needed for the Ekman layer to come into equilibrium with changing wind stress. We have already noted how this equation relates to that derived by LeBlond and Mysak (1978). It is interesting to compare (20) and (21) to the rigid-lid equations and to other equations which have been invoked in discussions of the large-scale modes of interest here.

The rigid-lid equation (9) is clearly the closest to (20). With hindsight it is simple to derive a version of (9), correct to order δ, in which the streamfunction is eliminated. The left hand side of (9) can be calculated by taking $-\nabla\cdot((6)_t/f^2)$, in which the term derived from $\mathbf{k}\times\nabla\Psi_t$ in (6) can be dropped as it is of order δ^2. Substituting into (9) then gives (20), except that the terms in p_{bt} and p_{at} are missing (equivalent to taking the limit $g \to \infty$), together with the oceanic mass source Q, as appropriate for a rigid-lid limit.

We can now justify the scaling given in the introduction by looking at the order δ time-dependent terms in (21). It is clear that the size of the free surface term p_{0t}/g relative to the rigid-lid term $-\nabla \cdot (H\nabla p_{0t}/f^2)$ depends on the length scales over which p_0 varies. Indeed, substituting $p_0 = a \exp i(kx + ly - \omega t)$, and assuming spatial variations in p_0 dominate over those in H/f^2, the ratio of the rigid-lid term to the free surface term is $(k^2 + l^2)gH/f^2 = (k^2 + l^2)R_0^2 = (2\pi R_0)^2/\lambda^2$, where λ is the wavelength considered. Thus, the free surface term is as important as the rigid-lid term when $\lambda = 2\pi R_0$, and more important for longer wavelengths, as stated in the introduction.

A number of alternative potential vorticity equations have already been invoked in theoretical discussions of these large-scale modes. For example, Fu (2003) quotes the following equation (for a homogeneous fluid):

$$\frac{\partial}{\partial t}\left(\nabla^2\eta - \frac{f^2}{gH}\eta\right) + HJ\left(\eta, \frac{f}{H}\right) = \frac{f}{\rho g}\nabla\times\left(\frac{\boldsymbol{\tau}}{H}\right). \tag{23}$$

A systematic derivation of this equation requires the quasigeostrophic approximation, which is not capable of describing flows which cross a large range of depths. In fact, of the terms on the left hand side, the only one which is wrong when large topography is included is $\nabla^2\eta_t$, the other (planetary geostrophic) terms being correct even with finite changes of f and H. The equation is quoted as derived from Cummins (1991), but can be traced further back to Willebrand et al.

(1980), where it takes the form (including a nonlinear term)

$$\left(\nabla^2 - \frac{f^2}{gH}\right)\Psi_t + \frac{1}{\rho H}J(\Psi, \nabla^2\Psi) + HJ(\Psi, f/H) = \nabla \times \boldsymbol{\tau} - \frac{f p_{a_t}}{g}, \quad (24)$$

(The factor $1/\rho H$ corrects an error in Willebrand et al. (1980)). In addition to quasigeostrophy, (24) assumes in the nonlinear term that $\mathbf{u}$ is independent of depth. When the Ekman flux is limited to near-surface and near-bottom layers, or where stratification is important for the mean flow, this is no longer true, and the nonlinear term no longer takes a simple form. The apparent differences between (23) and (24) result from the use of different variables, and from rather arbitrary choices as to whether factors of f and $1/H$ occur inside or outside the spatial derivative in the stress and ∇^2 terms: these are equivalent to quasigeostrophic accuracy. In (21), we improve on these equations by allowing first order changes in f and H throughout, which results in a clarification of where these factors should appear for best accuracy.

A second equation recently quoted by Vivier et al. (2005) is

$$\nabla \cdot \left(\frac{\nabla\Psi_t}{H}\right) - \rho\frac{f\eta_t}{H} + J(\Psi, f/H) = \nabla \times \left(\frac{\boldsymbol{\tau}}{H}\right), \quad (25)$$

which they use in a model formulation, with an additional geostrophic approximation to calculate η in terms of Ψ (a frictional term has here been absorbed into the stress term on the right hand side). Ignoring the free surface term in η, this is equivalent to a homogeneous ocean version of the rigid-lid equation (8). It therefore has better than quasigeostrophic accuracy when the free surface term is negligible. However, the derivation of (25) in Appendix A of Vivier et al. (2005) neglects a term of order δ which results from the depth-integrated flow across f/H contours due to the divergent part of the flow (this additional term is negligible in the quasi-geostrophic limit), so that (25) is not uniformly accurate to order δ, even before the geostrophic substitution is used for η. Our formulation in terms of pressure rather than streamfunction allows us to retain order δ accuracy, and to produce an equation in a single variable.

A third equation, given by Ponte (1999), is expressed in terms of bottom pressure p_b as

$$\frac{p_{bt}}{g} + J(p_b, H/f) + J(E, 1/f) = \nabla \times \left(\frac{\boldsymbol{\tau}}{f}\right) + Q \tag{26}$$

plus "other terms of order ω/f". Comparing with (20) (and ignoring forcing by atmospheric pressure) we see that this is correct, apart from a sign error in the wind stress term, but it appears somewhat arbitrary, given that one term (the first) of order $\delta = \omega/f$ has been retained but others discarded. The missing terms are supplied in (20). The scaling argument given above shows that the term retained in (26) can only be more important than the missing terms at wavelengths greater than $\lambda = 2\pi R_0$. This is the planetary geostrophic limit, which can be considered as the opposite extreme to the rigid-lid case. It may be valuable in reduced gravity models, in which the free surface is below an infinite layer of constant density water, and the corresponding Rossby radius is much smaller. Otherwise it can only be applied to the largest possible scales of the ocean.

5. Conclusions

For subinertial motions, (20) represents the most general form of a linear barotropic potential vorticity equation in a single scalar variable (actually, the small pressure gradient forces acting over a narrow surface layer have been neglected in (20), which can be generalized further by substituting (17) or (18) into (16)). In interpreting this as an equation in a single variable, however, we are implicitly assuming that the stratification (terms in E) does not respond significantly to the flow on the timescales of the motion. For oscillating motions in which baroclinic coupling is important, (20) is not very helpful.

When the stratification can be considered constant, it is simpler to describe time-dependent motions using (21) or (22), which is the equivalent equation for a homegeneous ocean. When the free surface height changes by a small fraction of the ocean depth, it is appropriate to replace the total ocean depth h in this equation with the depth H measured relative to a constant near-surface level. This equation includes forcing due to wind stress, atmospheric pressure, and mass sources. It can represent motions on any length scale (subject to validity of the linear momentum approximation and hydrostatic balance), on a planet with varying f and with topography occupying a significant fraction of the maximum depth. The main limitation, required to filter out inertio-gravity waves, is that the frequency of motions must be low compared to f. In practice, there is also a low frequency limit on validity, determined by the timescale on which the flow produces changes in stratification. This is dependent on amplitude, length scale and background stratification, but is in practice at least a few months for midlatitude motions at scales measured in thousands of kilometers. This order of accuracy is higher than that given by other equations currently available in the literature.

Acknowledgments.

Thanks to John Huthnance and Andrew Willmott for useful feedback on a draft of this note, and to two anonymous referees for their input. This work was funded by the U.K. Natural Environment Research Council, and is a contribution to the Proudman Oceanographic Laboratory's "Sea level, bottom pressure and space geodesy" programme.

References

Carrére, L. and F. Lyard, 2003: Modeling the barotropic response of the global ocean to atmospheric wind and pressure forcing - comparisons with observations. *Geophys. Res. Letters*, **30(6)**, 1275, 8–1–8–4.

Cummins, P. F., 1991: The barotropic response of the subpolar north pacific to stochastic wind forcing. *J. Geophys. Res.*, **96**, 8869–8880.

Fu, L. L., 2003: Wind forced intraseasonal sea level variability of the extratropical oceans. *J. Phys. Oceanogr.*, **33**, 436–449.

Fukumori, I., R. Raghunath, and L. L. Fu, 1998: Nature of global large-scale sea level variability in relation to atmospheric forcing. *J. Geophys. Res.*, **103**, 5493–5512.

Gill, A. E. and P. P. Niiler, 1973: The theory of the seasonal variability in the ocean. *Deep-Sea Res.*, **20**, 141–177.

Hirose, N., I. Fukumori, V. Zlotnicki, and R. M. Ponte, 2001: Modeling the high-frequency barotropic response of the ocean to atmospheric disturbances: Sensitivity to forcing, topography and friction. *J. Geophys. Res.*, **106(C12)**, 30,987–30,995.

Hughes, C. W. and P. D. Killworth, 1995: Effects of bottom topography in the large-scale circulation of the southern ocean. *J. Phys. Oceanogr.*, **25**, 2485–2497.

Hughes, C. W., M. P. Meredith, and K. Heywood, 1999: Wind-driven transport fluctuations through drake passage: A southern mode. *J. Phys. Oceanogr.*, **29**, 1971–1992.

LeBlond, P. H. and L. A. Mysak, 1978: *Waves in the Ocean*. Elsevier, Amsterdam and New York.

Ponte, R., 1999: A preliminary model study of the large-scale seasonal cycle in bottom pressure over the global ocean. *J. Geophys. Res.*, **104**, 1289–1300.

Tierney, C., J. Wahr, F. Bryan, and V. Zlotnicki, 2000: Short-period ocean circulation: implications for satellite altimetry. *Geophys. Res. Letters*, **27(9)**, 1255–1258.

Vivier, F., K. A. Kelly, and M. Harismendy, 2005: Causes of large-scale sea level variations in the southern ocean: Analyses of sea level and a barotropic model. *J. Geophys. Res.*, **110**, C09 014.

Webb, D. J. and B. A. de Cuevas, 2002a: An ocean resonance in the indian sector of the southern ocean. *Geophys. Res. Letters*, **29(14)**, 1664, 9–1–9–3.

———, 2002b: An ocean resonance in the southeast pacific. *Geophys. Res. Letters*, **29(8)**, 93–1–93–3.

Weijer, W. and S. T. Gille, 2005: Adjustment of the southern ocean to wind forcing on synoptic time scales. *J. Phys. Oceanogr.*, **35**, 2076–2089.

Willebrand, J., S. G. M. Philander, and R. C. Pacanowski, 1980: The oceanic response to large-scale atmospheric disturbances. *J. Phys. Oceanogr.*, **10**, 411–429.