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THE EFFECT OF FLOOD TIMING ON THE BENEFIT OF FLOOD ALLEVIATION: DEVELOPMENT OF A METHODOLOGY AND ITS APPLICATION TO MAIDENHEAD

Report to Middlesex Polytechnic
Flood Hazard Research Centre

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1. Introduction

The present value of the future benefit of a flood alleviation scheme is conventionally determined by assuming that the average benefit is achieved each year. In practice, the benefit enjoyed depends on the exact timing of flood occurrences, and although on average the present value after, say, 50 years that calculated assuming the average benefit is achieved each year, the present value of benefit in one 50 year period may be very different to that in another. Since the future pattern of flooding at a site is not known, computer simulation techniques must be used to estimate the probability that a desired benefit is achieved within a fixed time period.

A computer program has been written and applied to data from Maidenhead. Under some limiting assumptions, it has been found that although the present value of benefits as conventionally estimated is less than currently estimated scheme costs, there is a 34% chance that the benefit will actually exceed costs over the next 50 years. There is a 21% chance that costs will be covered within 25 years, but there is also a 59% probability that costs will never be covered. For comparative purposes, if costs are such that the benefit-cost- ratio is exactly 1, there is still only a 44% chance that the scheme would actually achieve a benefit greater than costs. Some sensitivity analyses have been performed to assist with the understanding of the robustness of the estimates.

2. Background

Flood alleviation schemes are assessed by comparing the costs of the scheme with the benefits of that scheme, where the benefits are the losses the scheme prevents. Benefits are conventionally calculated by deriving a notional 'average annual benefit', which can be seen as the benefit achieved over a very long period *distributed equally between years (although in practice average annual* benefit is evaluated 'theoretically' using a damage-probability relationship rather than simple averaging). The current worth of the future stream of benefits is then determined by discounting each annual benefit to present values, assuming, for example, that the average benefit is attained each year for the next 50 years.

However, the "average" damage and benefit does not occur each year: in some years there are no floods, and hence no benefit from the alleviation scheme, but in others there are floods of greatly varying magnitudes. In general, the sooner a flood occurs, the greater the benefit of protecting against that flood. Because of discounting, the current value of protecting against a big flood that happens next year is much greater than the value of the benefit from protecting against the same flood occurring in 50 years time. The scheme planner can therefore ask two questions:

1) what benefit will actually have been attained in 50 years time?

2) if the current value of the benefit attained by then has not reached

some critical target value - such as scheme costs - how long would it be before it will?

The answer to these questions depends on the future timing of floods, which cannot of course be predicted.

It is therefore possible to appraise a flood alleviation investment as having returns which will depend on the future pattern of flooding - which is unknown - and this investment can be seen as very similar to a commercial investment made under conditions of commercial uncertainty. An investor will be prepared to invest in a venture if the investment is expected to be recouped. The decision to invest will be based on either a best guess of the probability that the desired returns will be attained in a specified period, or on an estimate of the time it will take to recover the investment (and will be influenced by the investor's degree of risk aversion).

Although the hydrologist does not know the future pattern of flooding at any site, enough is usually known of flood characteristics to make estimates of conceivable future patterns. The number of such patterns will be infinitely large (since floods can take on a continuum of magnitudes), and categorisation of floods and benefit into discrete classes still gives a very large number of alternative scenarios: even if benefit fell into one of only two classes - none and some, for example - there would be 2^{50} (or 1.1×10^{15}) different, equally likely, possible future 50 year time series.

The most practical approach to modelling the future is therefore based on computer simulation experiments. With such experiments it is possible to generate many synthetic time series and build up distributions of the current value of the benefit attained after a fixed period N years and the time it takes for the current value of benefit to reach a predetermined fixed figure (such as the current value of scheme costs). It is then possible to estimate the probability that the benefit after N years is greater than - or alternatively less than - certain key values, and the probability that the desired benefit is achieved within specified time periods.

3. Methodology

A computer program has been written to perform the analyses, and has the general structure shown in Figure 3.1. In essence, the program generates a time series of floods, determines the losses that are incurred each year with and without the proposed scheme, and converts the annual benefit to present values using

$$\text{Present value } PV_I = \frac{\text{Benefit}_I}{(1 + r)^I} \qquad \text{. . (1)}$$

where 'Benefit_I' is the benefit in year I and r is the discount rate. A running total of the present value of benefit attained is kept, and both the sum after N years and the number of years necessary for the sum to reach a specified target figure C are recorded.

Figure 3.1 Structure of benefit estimation program

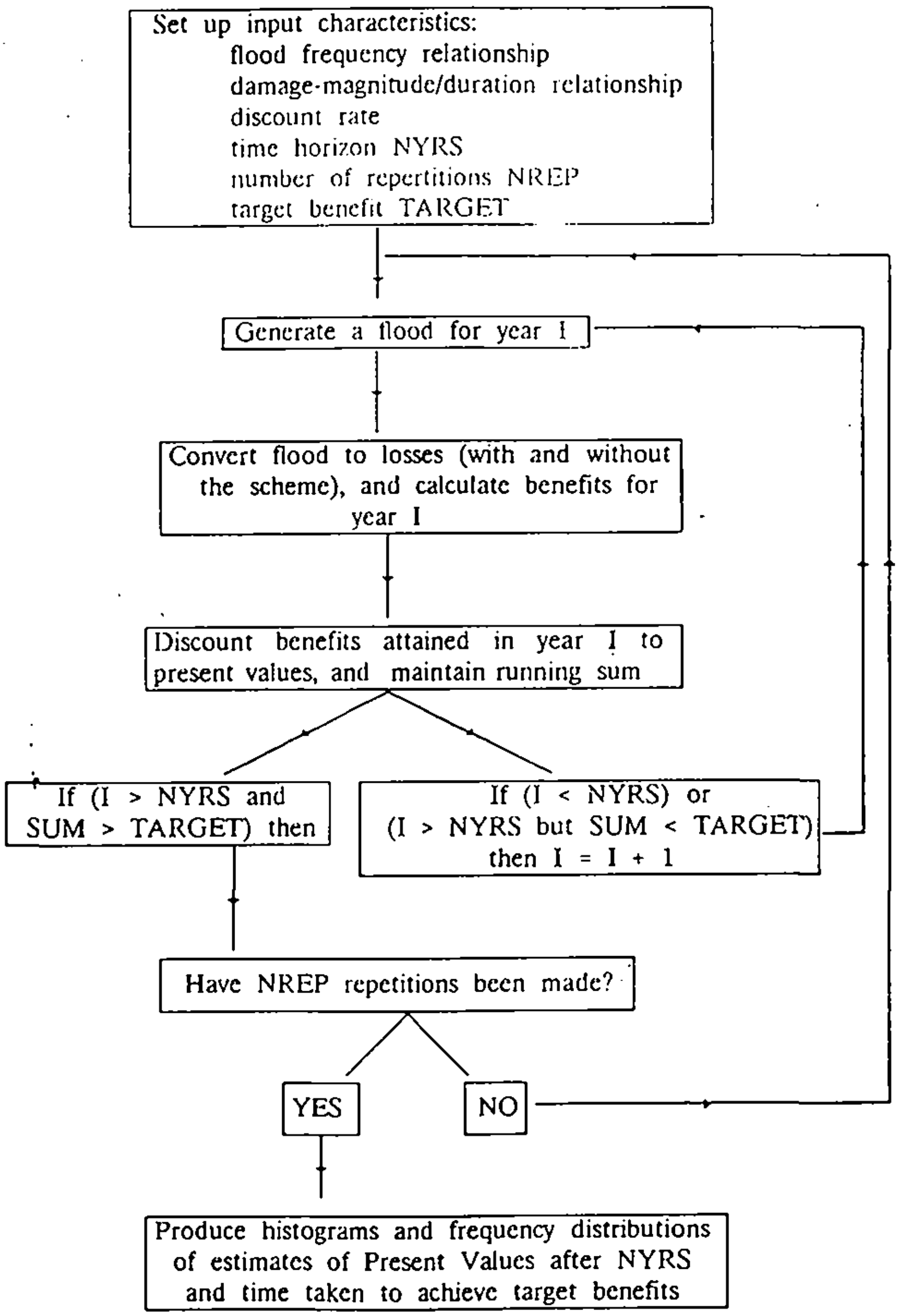


Figure 3.2 illustrates the procedure using 4 different synthetic time series of length 50. After many repetitions frequency distributions of the present value of benefit after N years and the time needed for the present value to reach

C can be built up. From these distributions it is possible to determine the probability that benefit in the next N years will be greater than some specified figure (or example the current value of scheme costs), or the probability that the desired benefit will be attained within a specified time period.

It is important to note that the mean of all the possible estimates (the 'sampling distribution') of the present value of benefit after N years is equal to the present value of the average annual benefit, discounted over N years. This is shown below:

$$E(PV \text{ in N years}) = E(PV_1 + PV_2 + \dots + PV_N) \qquad ..(2)$$

$$= E(PV_1) + E(PV_2) + \dots + E(PV_N) \qquad ..(3)$$

$$= \frac{E(ben_1)}{(1+r)} + \frac{E(ben_2)}{(1+r)^2} + \dots \frac{E(ben_N)}{(1+r)^N} \qquad ..(4)$$

where PV_I is the present value in year I and ben_I is the actual benefit in year I. Since the expected damage in each year is constant (assuming no change in damage potential over time), equation (4) can be rewritten as

$$E(PV \text{ in N years}) = E(ben) \left\{ \frac{1}{1+r} + \frac{1}{(1+r)^2} + \dots + \frac{1}{(1+r)^N} \right\} \qquad ..(5)$$

$$= E(ben) \left\{ 1 - \left(\frac{1}{1+r}\right)^N \right\} / r \qquad ..(6)$$

The bracket in equation (6) is the conversion used to determine present value from average annual benefit.

The conventional approach therefore gives an unbiased estimate of the present value of future benefit (assuming the flood frequency relationship is perfectly known), even though it does not provide any information on the probability of attaining this benefit.

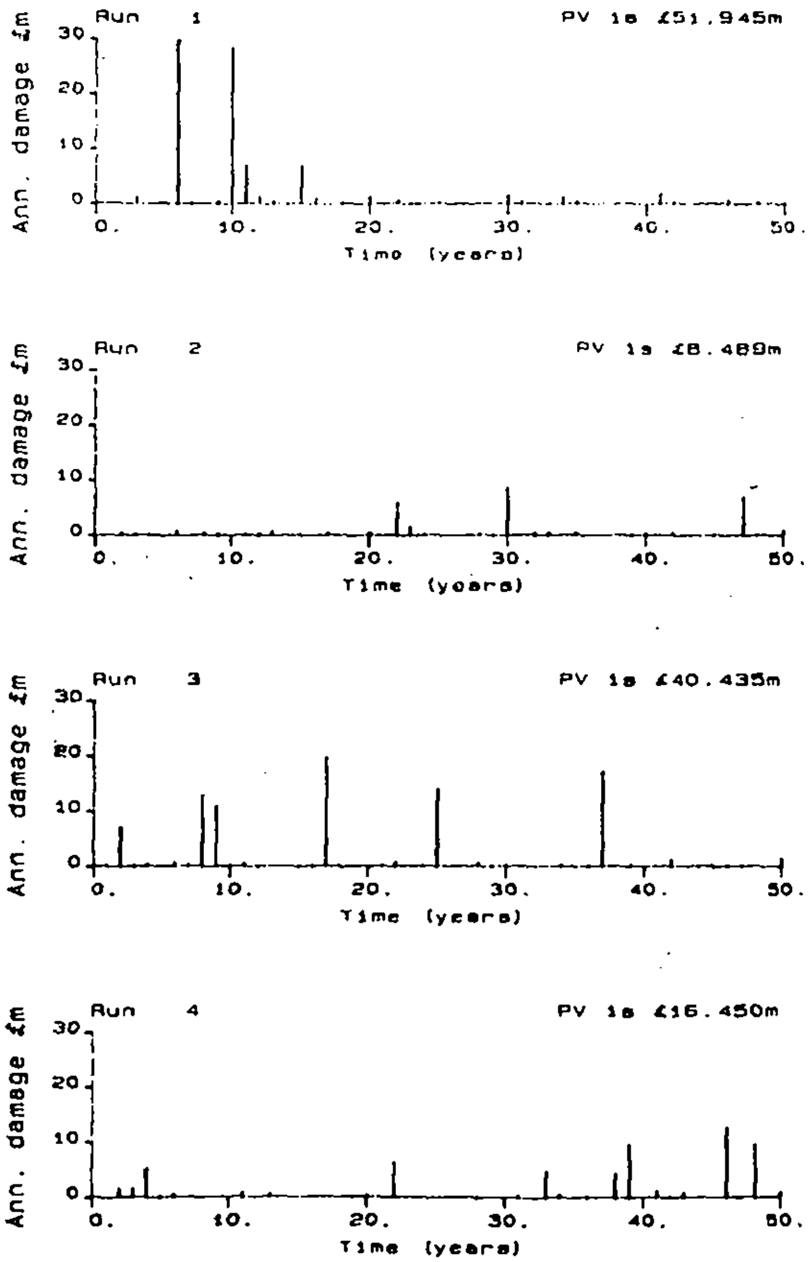
4. Application to Maidenhead

4.1 CONDITIONS AND ASSUMPTIONS

The computer model was applied to Maidenhead, with the following conditions and assumptions:

1) The flood frequency relationship was taken from Institute of Hydrology (1988), and is based on probability distributions fitted to data from several gauging stations on the Thames. It is assumed that the

Figure 3.2 Four synthetic 50 year time series



estimated flood frequency relationship describes perfectly the true frequency relationship at Maidenhead.

2) The relationship between flood magnitude and flood damage as defined by Middlesex Polytechnic, and as given in Table 4.1, was used. This relationship includes only damage to property: indirect and transport losses are excluded.

Table 4.1 Damage-flood magnitude relationship used in study

Return period (years)	Discharge (m^3 sec^{-1})	Damage (£)
2	231	0
5	320	542,882
9	360	1,568,686
25	440	11,012,424
56	500	19,799,397
101	550	28,947,680
204	610	38,145,128

Average annual damage: £ 1,325,358

Present value: £24.28 million
discount rate = 5%
time period = 50 years

3) A discount rate of 0.05 and time horizons of 25, 50 and 75 years were used.

4) The desired benefits were fixed to be the present value of the scheme costs, £26.68 millions.

5) It is assumed that all costs occur as a lump sum at the beginning of the specified time horizon: there are no further costs. It is further assumed that the scheme is implemented 'instantaneously'.

6) The scheme prevents all losses up to a discharge of 610 m^3 sec^{-1} (with a return period of approximately 200 years), with no protection thereafter. This is consistent with the assumption used to calculate the discounted scheme benefit to be £24.28 millions (Table 4.1).

7) There is no change in flood risk over time, and exposure to loss remains constant.

8) Only one flood can cause damage in any one year. This is acceptable because, whilst some damage is incurred in floods which may occur several times a year, significant damage begins only in much rarer events.

9) The simulation experiments involved 1000 repetitions.

Each of these assumptions can be relaxed as more information becomes available.

4.2 RESULTS: PRESENT VALUE OF BENEFITS AFTER 50 YEARS

The conventional estimate of the present value of benefit after 50 years is £24.28 millions (Table 4.1), which is calculated from the average annual damage and is equivalent to the mean of all possible estimates of the present value of benefit over 50 years (Section 3). Over the 1000 synthetic 50 year time series, the present value of benefit ranged from just 14% of this value to approximately 400%. Figure 4.1 shows the histogram of the 1000 estimates of present value, and indicates the skew in the estimates: the mean is significantly influenced by a few large values. The cumulative frequency distribution is more useful, however, and Figure 4.2 shows the probability of attaining a benefit in 50 years less than any given value. There is thus a 66% chance that benefit will be less than the scheme costs of £26.68 millions, but also therefore, conversely, a 34% probability that the scheme costs will be recovered as flood damages saved over the next 50 years.

Figure 4.2 also shows the distribution of present values after 25 and 75 years. As the time horizon increases, the distribution of present values changes less: there is only a slightly higher probability of achieving a benefit of £26.68 millions in 75 years than in 50 years, and this is because damages occurring at such distances into the future have low present values.

To place these values in context it is interesting to calculate the probability of a scheme which is deemed to be only just cost effective (with a benefit-cost ratio of 1) actually yielding a benefit greater than costs in the next 50 years. Figure 4.2 shows that there is a 56% chance that benefits in the next 50 years will be less than the mean benefit of £24.48 millions, and therefore there is only a 44% chance that a scheme assumed to be just efficient will actually be so.

4.3 RESULTS: TIME NECESSARY TO ACHIEVE DESIRED BENEFITS

The desired benefit is fixed to be the present value of scheme costs (£26.68 millions). In 59% of the 1000 synthetic time series the present value of benefit never totalled £26.68 million. Figure 4.3, which shows the cumulative frequency distribution of 'time to profit' indicates that there is approximately a 34% chance that the desired benefit will be attained within 50 years (consistent with the result given in 4.2). There is a slightly higher chance (40%) that the desired benefit will be achieved within 100 years, and a 21% probability that the benefit will be reached within 25 years.

Figure 4.1 *Histogram of the present value of scheme benefit after 50 years*

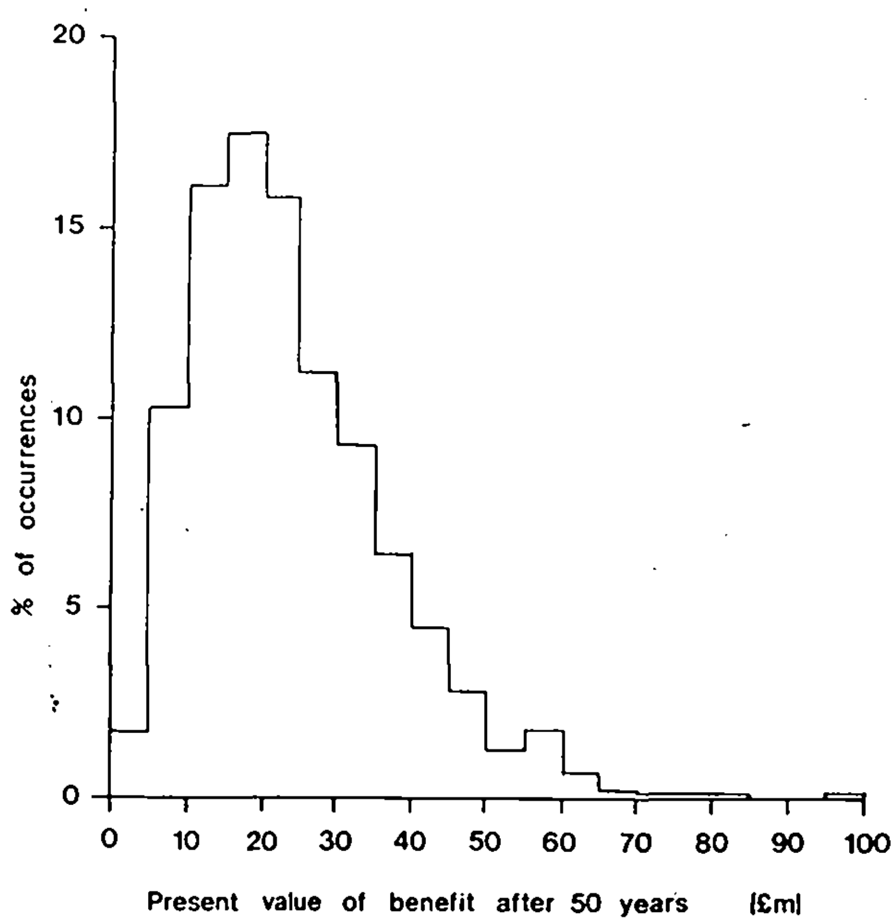


Figure 4.2 Distribution of present value of scheme benefit after 25, 50 and 75 years

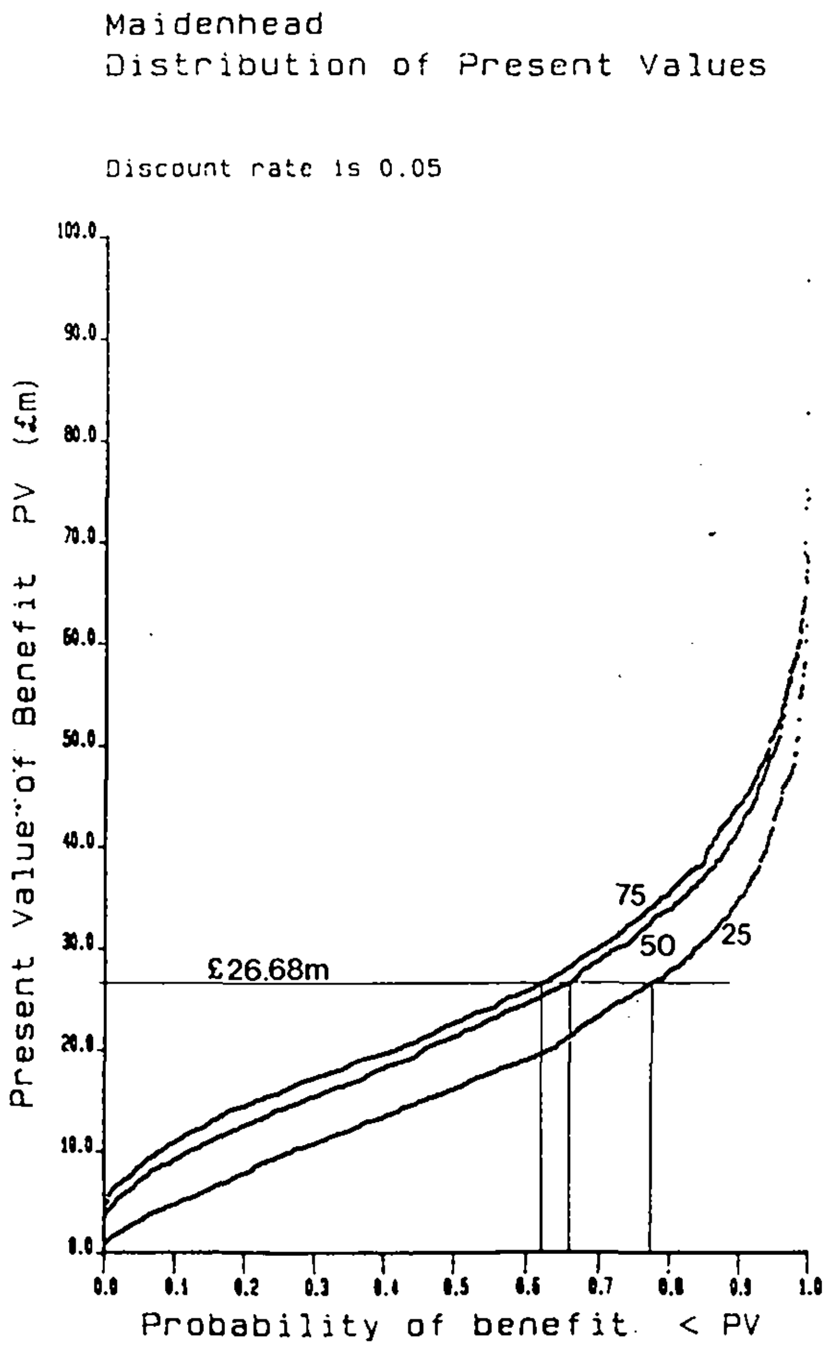
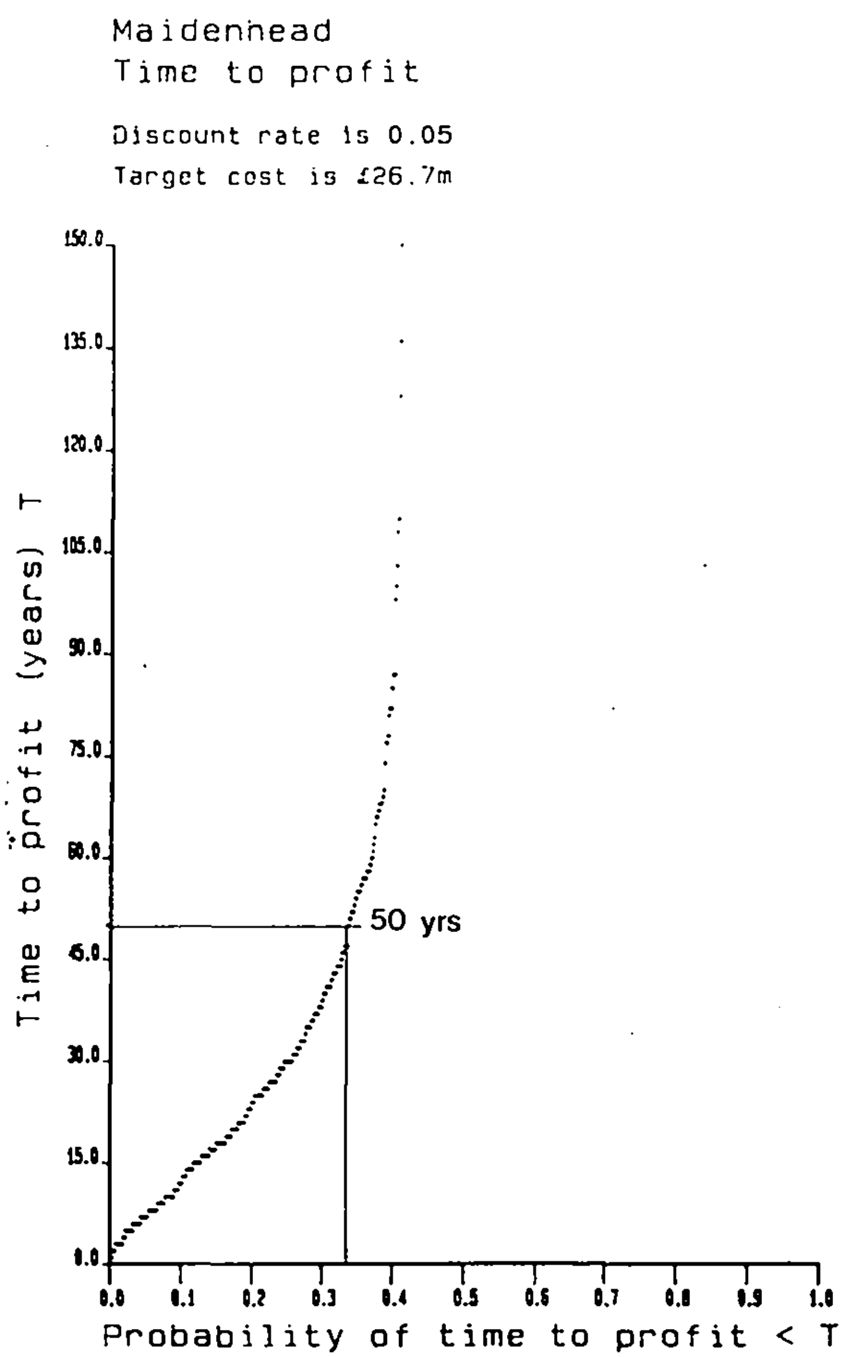


Figure 4.3 *Distribution of time needed to reach benefit of £26.68 millions*



5. Sensitivity Analysis

5.1 THE SIZE OF THE DESIRED BENEFIT

The time taken to achieve the desired benefit obviously depends on the magnitude of this benefit, but the effect of changes in the target is less immediately clear. Accordingly, simulations were run using the Maidenhead damage function with desired benefits of £20 millions, £25 millions, £30 millions and £35 millions. The distributions of the resultant 'time to profit' are shown in Figure 5.1, and the probabilities of achieving the target benefit by 25 and 50 years - or never - are given in Table 5.1. For example, the probability of achieving the desired benefits within 50 years varies between 18% and 50% as the desired benefit reduces from £35 millions to £20 millions. If the desired target is changed by a given percentage the probability of achieving that value within a specified time period changes by a greater amount, and a decrease in desired benefit has a greater effect than an increase.

Table 5.1 Probability of achieving different desired benefits in 25 or 50 years or never

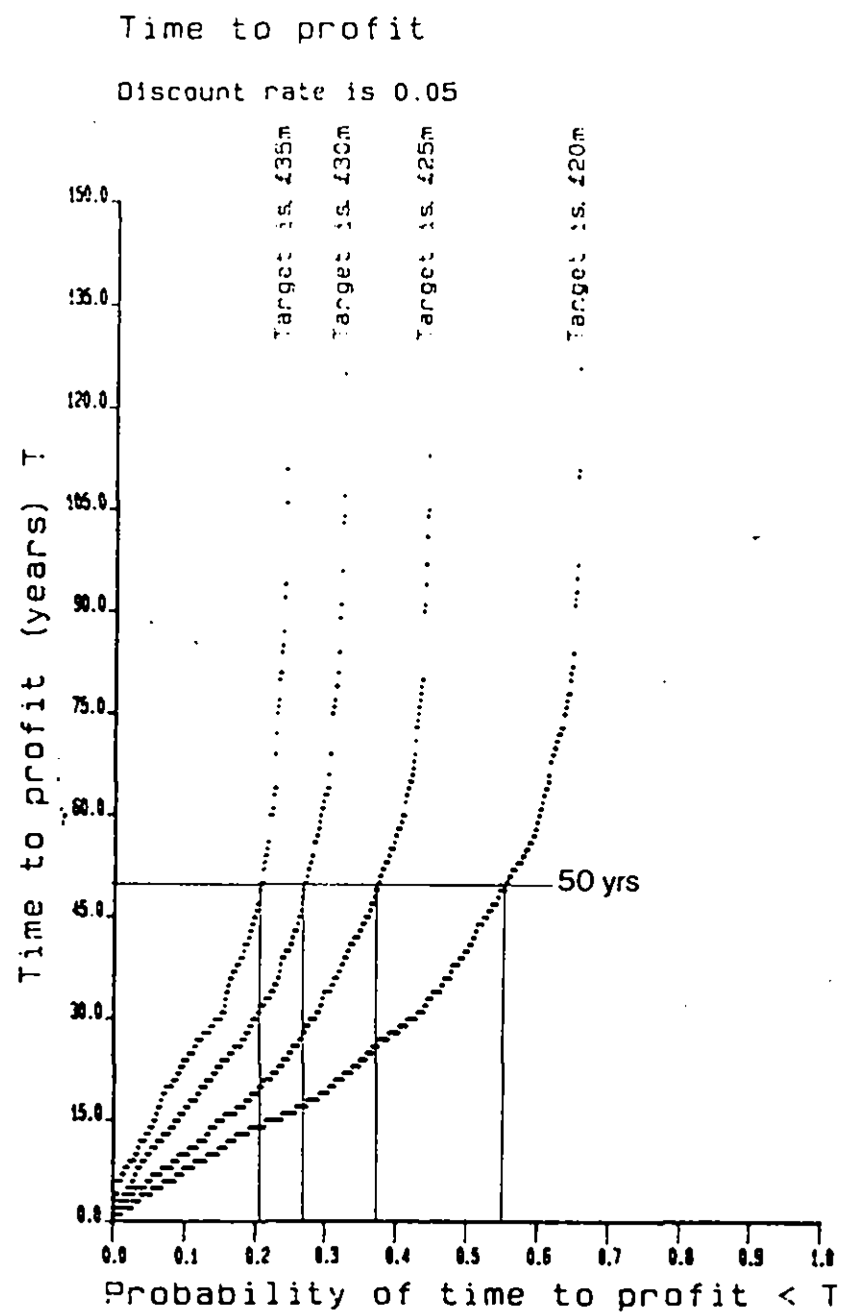
Desired benefit	% chance of achieving benefit in 25 years	% chance of achieving benefit in 50 years	% chance of never achieving benefit
£20m	36	50	34
£25m	25	33	56
£26.68m	21	34	59
£30m	17	24	68
£35m	11	18	76

It is possible to interpolate in Figure 5.1 to determine the probabilities of achieving other values of benefit within a given time period.

5.2 THE EFFECTS OF RESIDUAL DAMAGES

It has been assumed so far that the scheme provides complete protection to 610 m^3sec^{-1}, or a return period of approximately 200 years. The real design standard of any scheme at Maidenhead is likely to be somewhat less than this, and an attempt was therefore made to assess the consequences of lowering the threshold at which residual damages begin.

Figure 5.1 *Distribution of times needed to achieve different desired benefit*



The computer program was run for several different standards of protection, assuming that there was complete protection against events less than the design standard and partial protection against larger events, as indicated in Figure 5.2. This is recognised to be an approximation, and the analysis could be further refined once more detailed information on the magnitude of residual damage becomes available. Figure 5.3 shows the distributions of the present values of benefit over 50 years for protection to the 56 year (the 1947 flood) and 100 year events, in comparison with the distribution assuming complete protection to 610 m^3sec^{-1}. It is clear that allowing for residual damages has an effect on the estimated risk of not achieving specified target benefits within 50 years: for example, the probability of the benefit after 50 years being less than scheme costs increases from 66% to approximately 74% if it is assumed that the scheme only gives complete protection to the 56 year flood rather than 610 m^3sec^{-1}. When a scheme design standard of 100 years is considered, the probability of getting a benefit less than scheme costs is, however, little changed at 68%.

5.3 THE CHARACTERISTICS OF THE FLOOD GENERATING PROCESS

The form of the flood frequency relationship has been assumed to be known with certainty, but in reality it is an estimate from one of many conceivable samples of flood data on the River Thames. The sensitivities of the shape of the distributions of present values and 'time to profit' to the underlying flood frequency relationship were assessed by both increasing and decreasing the discharge estimates for given return periods by 5%. The new 'perturbed' frequency curves are shown in Figure 5.4, and are within the defined confidence limits (Institute of Hydrology, 1988). It must be emphasised, however, that it is not possible to assign a probability or confidence value to the modified curves: it is sufficient to note that the modified curves could feasibly represent the true underlying form of the flood data at Maidenhead.

Figure 5.5 shows the distribution of the present value of benefit after 50 years for the original frequency relationship and the relationships with a 5% increase or decrease in discharge at each return period. Increasing the discharge at a given return period - and hence increasing the frequency of a given discharge - has a greater effect on the probability of attaining a specified benefit than reducing the estimated frequency of fixed discharges. For example, if discharges are increased by 5% the probability of getting a benefit less than £26.68 millions in 50 years decreases from 66% to 44%: if discharges are reduced by 5% the probability of not achieving this benefit rises to 77%. The same effect is illustrated in Figure 5.6, which shows the probability of the desired benefit (£26.68 millions) being achieved in a given number of years: the estimated economic viability of the scheme is therefore strongly influenced by the estimated characteristics of the flood generating process.

Figure 5.2 Form of damage-probability functions used when considering residual damages above scheme design standard

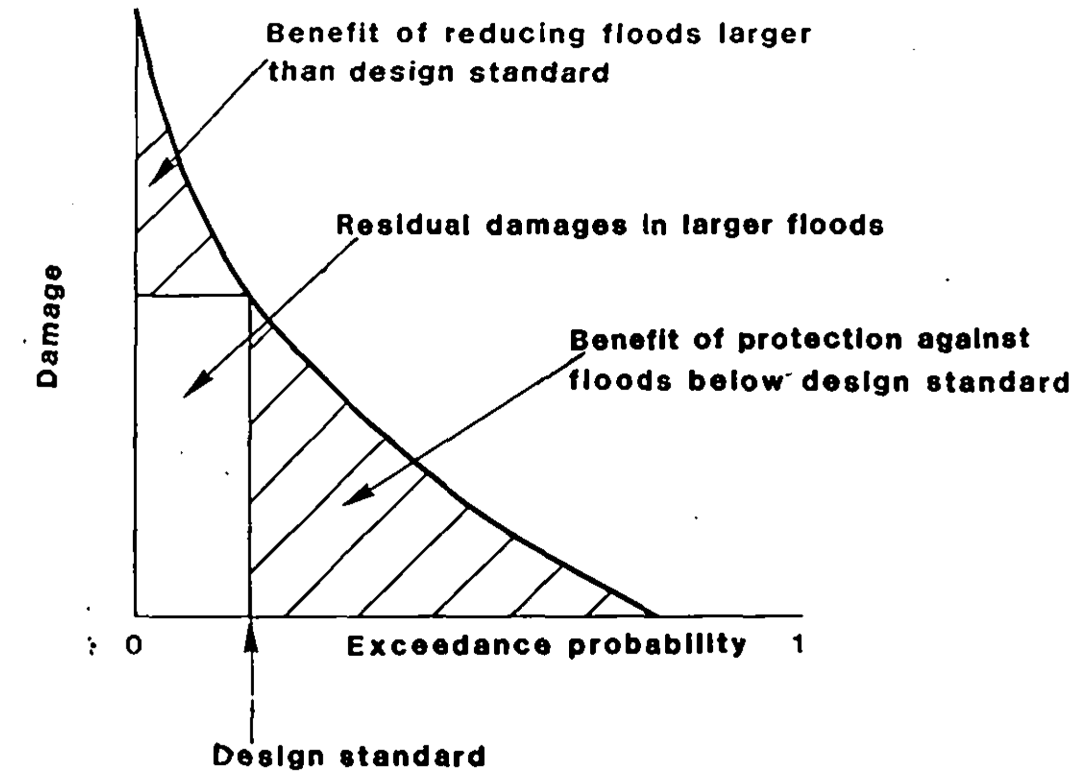


Figure 5.3 Distribution of present value of benefit with different standards of protection

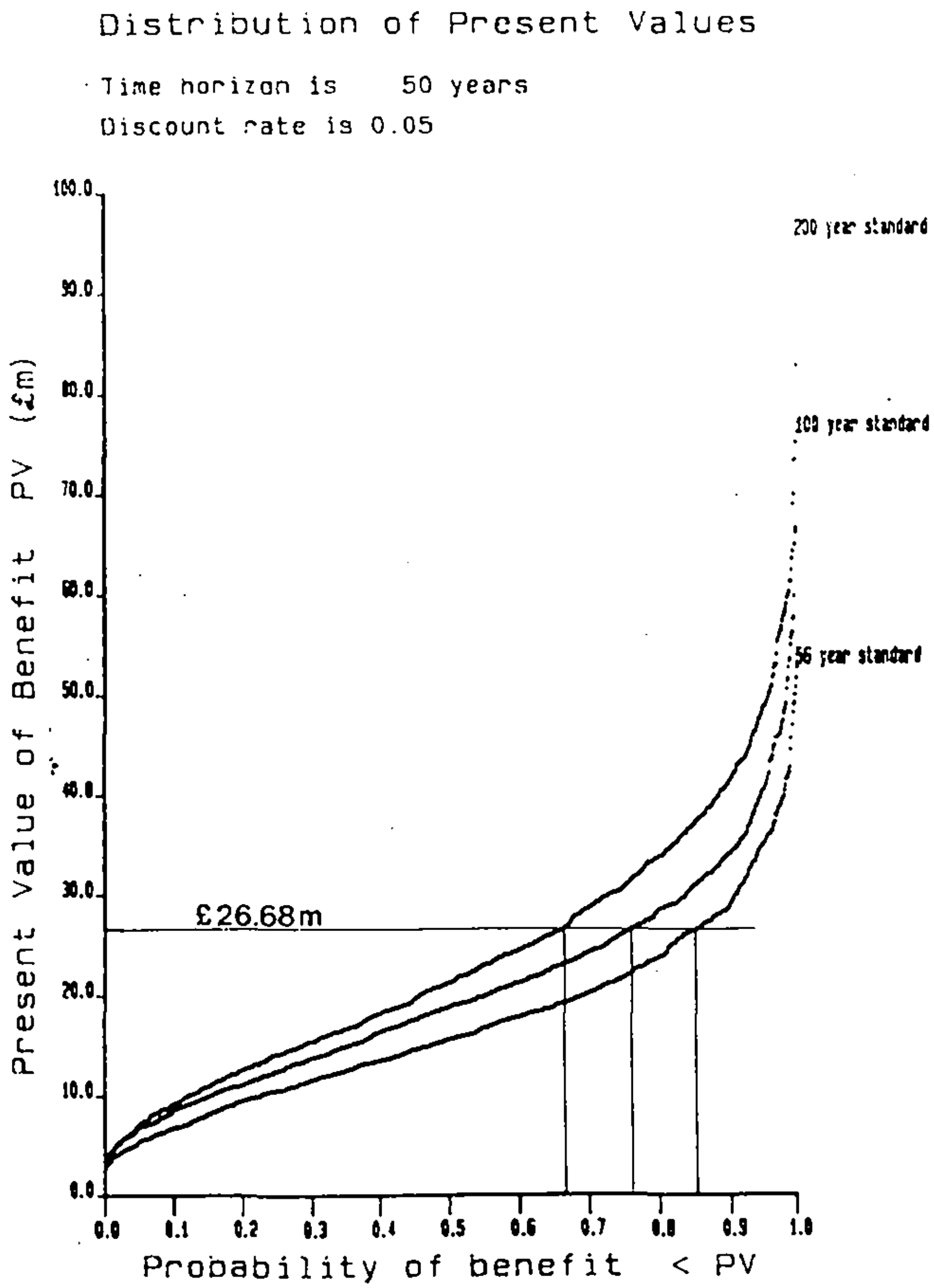


Figure 5.4 Perturbed frequency curves used in sensitivity analysis

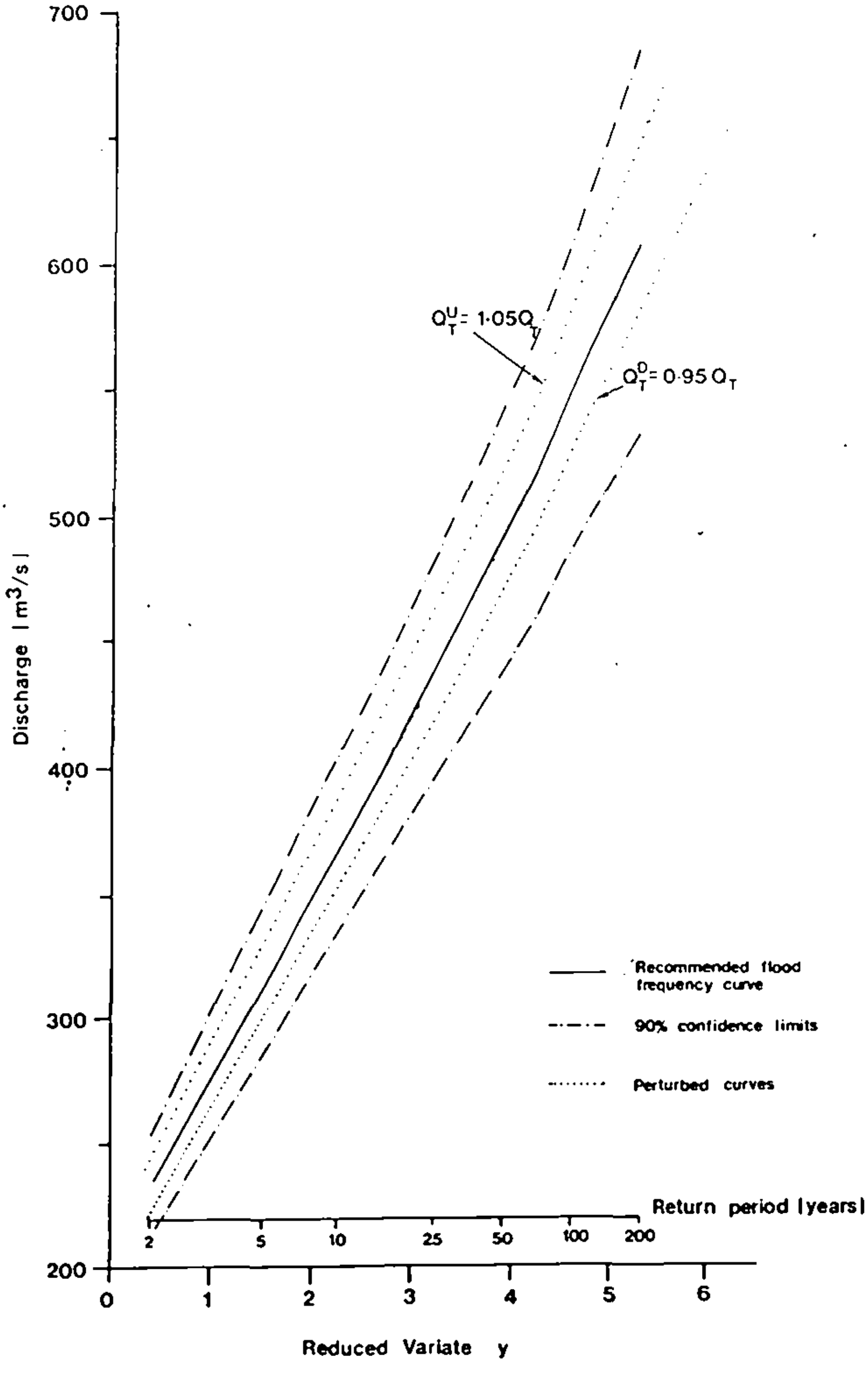


Figure 5.5 Distribution of present value of benefit with different frequency curves

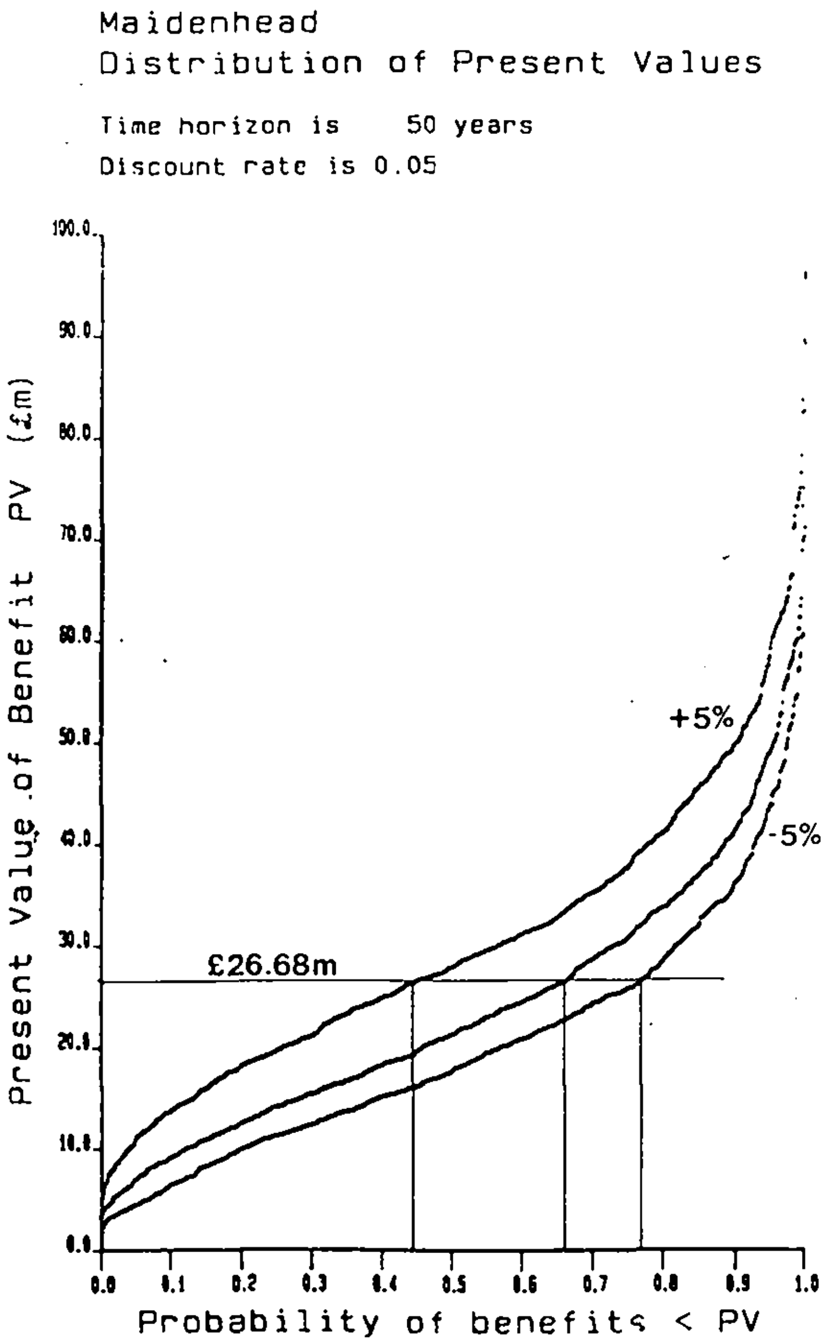
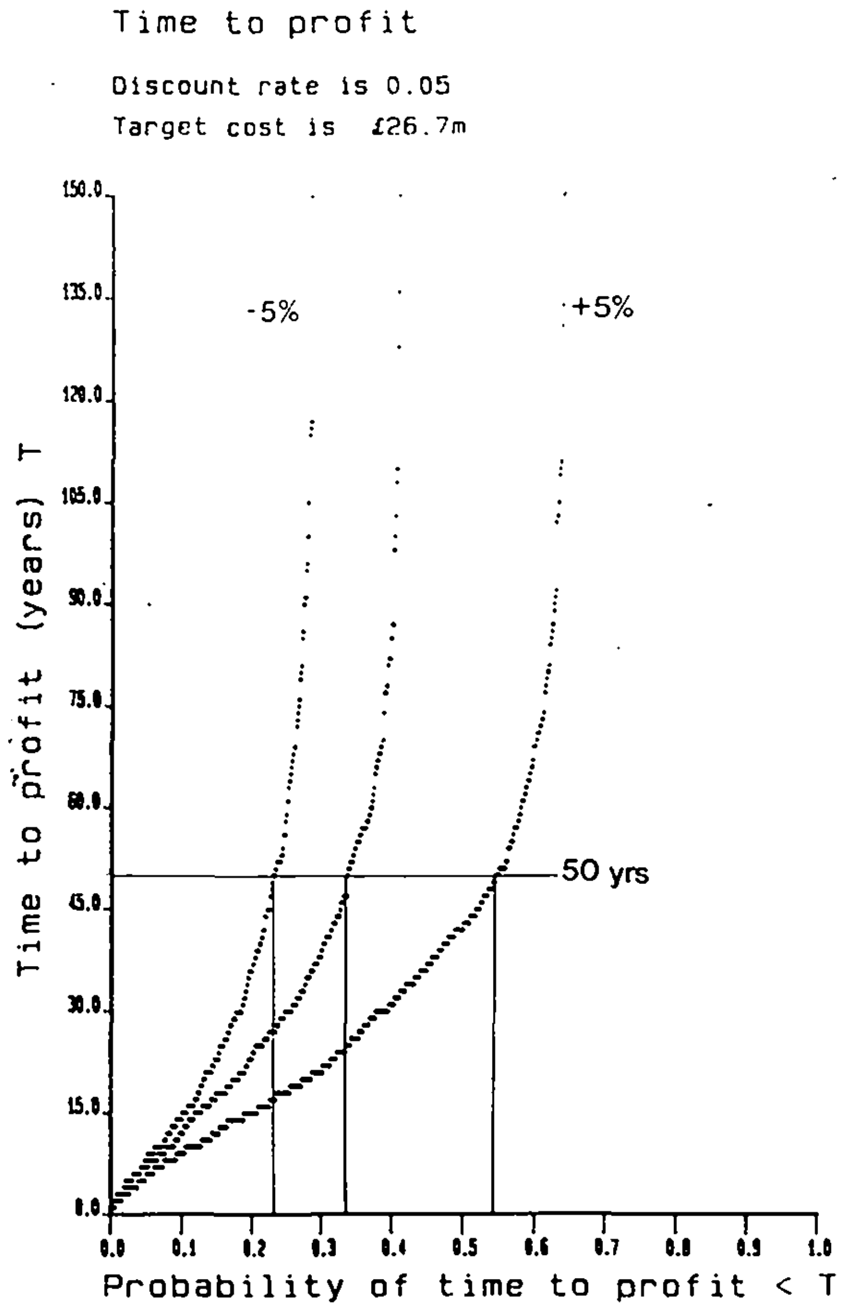


Figure 5.6 Distribution of times needed to reach target benefit of £26.68 millions with different frequency curves



6. Conclusions

This report presents the results of an investigation into the effect of the timing of future floods on the present value of flood alleviation scheme benefit, and in particular on the probability that realised benefit will be greater than specified desired values or that the time taken to achieve a desired benefit will be less than a defined number of years. A general methodology has been developed.

The procedure was applied to data from Maidenhead, with assumptions as specified in Section 4.1. The estimated present value of scheme benefit of £24.28 millions represents the mean benefit achieved over all possible future patterns of flooding in the next 50 years and is less than currently estimated scheme costs of £26.68 millions. The computer simulation experiments, however, showed that in 34% of the possible patterns of future flood timing the actual benefit enjoyed would exceed costs: in other words, there is a 34% probability that the present value of benefit over the next 50 years will actually exceed scheme costs.

The experiments also show that there is a 21% chance that actual benefit will exceed scheme costs within 25 years, but a 59% chance that benefits will never exceed costs. It is also interesting to note, for comparative purposes, that there is only a 44% chance that a scheme which is just efficient (a benefit-cost ratio of exactly 1) would actually give a benefit greater than costs in 50 years.

The estimated time necessary to achieve desired of benefit is sensitive to the actual value of the target, and a given percentage change in target results in a greater percentage change in the probability of that benefit being achieved within a specified time. Altering the frequency of particular events also significantly influences the estimated probability of achieving the desired benefit, with increases in frequency having the greatest effect.

Further studies are needed to consider the following issues:

(1) use of a more realistic damage-probability function which caters for residual damages beyond scheme design standards;

(2) incorporation of expenditure on maintenance during scheme life;

(3) consideration of the effect of uncertainties in the flood frequency relationship on estimates of the probability of achieving desired benefit in a specified time period. This would be done by repeating the simulation experiments with different parent flood frequency generators, and thus building up a distribution of "probability of achieving desired benefit".

The last issue requires many more simulation experiments and would in practice be very time consuming, but the first two issues are much more readily addressed.

References

Institute of Hydrology (1988) *Maidenhead, Windsor and Eton Flood Study*. Stage 2 Hydrology Report. IH, Wallingford