Geostrophic Adjustment Problems in a Polar Basin

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Abstract

The geostrophic adjustment of a homogeneous fluid in a circular basin with idealized topography is addressed using a numerical ocean circulation model and analytical process models. When the basin is rotating uniformly, the adjustment takes place via excitation of boundary propagating waves and when topography is present, via topographic Rossby waves. In the numerically derived solution, the waves are damped because of bottom friction, and a quasi-steady geostrophically balanced state emerges that subsequently spins-down on a long time scale. On the *f*-plane, numerical quasi-steady state solutions are attained well before the system's mechanical energy is entirely dissipated by friction. It is demonstrated that the adjusted states emerging in a circular basin with a step escarpment or a top hat ridge, centred on a line of symmetry, are equivalent to that in a uniform depth semicircular basin, for a given initial condition. These quasi-steady solutions agree well with linear analytical solutions for the latter case in the inviscid limit.

On the polar plane, the high latitude equivalent to the β-plane, no quasi-steady adjusted state emerges from the adjustment process. At intermediate time scales, after the fast Poincaré andKelvin waves are damped by friction, the solutions take the form of steady-state adjusted solutions on the f-plane. At longer time scales, planetary waves control the flow evolution. An interesting property of planetary waves on a polar plane is a nearly zero eastward group velocity for the waves with a radial mode higher than two and the resulting formation of eddy-like small-scale barotropic structures that remain trapped near the western side of topographic features.

Keywords geostrophic adjustment, polar circulation, Kelvin waves, vorticity waves.

1 Introduction

In this paper, we consider how a homogeneous fluid, initially not in geostrophic balance, adjusts to that balance in a circular basin in the presence of an idealized topography. We first consider the case of a uniformly rotating basin, followed by examples in which the latitudinal dependence of the vertical component of the earth's angular rotation is retained. The Nucleus for European Modelling of the Ocean(NEMO)ocean modelling framework (Madec et al., 1998; Madec, 2008) is used to determine the adjustment solutions numerically. Linear, inviscid analytical solutions are also derived to validate the numerical solutions and to add further insight into the adjustment process.

The "classical" geostrophic adjustment problem considers a horizontally unbalanced, uniformly rotating barotropic fluid which is initially at rest relative to the rotating frame of reference in a horizontally unbounded domain. In the initial state, a step in the fluid surface exists which is maintained by a vertical barrier. Upon removal of the barrier, the fluid adjusts to a steady geostrophic state, in which the pressure gradient is balanced by the Coriolis force, by Poincaré waves propagating to infinity (Gill, 1976; Gill., 1982). There are numerous extensions of this classical adjustment problem that address the effects of stratification (Grimshaw et al., 1998), non-linearity (Ou, 1984, 1986; Hermann et al., 1989), the presence of topography (Johnson, 1985; Gill et al., 1986 Willmott and Johnson, 1995), a variety of configurations for the initial unbalanced states (Killworth, 1992) and adjustment in a closed basin (Stocker and Imberger, 2003).

In a closed, uniformly rotating basin of uniform depth, the homogeneous fluid will evolve towards a balanced state through the propagation of Poincaré and Kelvin-type boundary waves (Antenucci and Imberger, 2001). With topography present, topographic Rossby waves will also play a role in the adjustment process. Therefore, in the inviscid limit, no steady geostrophically balanced state will emerge because there is no mechanism to damp or evacuate the waves. In numerical simulations or laboratory experiments, bottom and lateral friction are present, which damp the waves excited during the adjustment process. Further, the adjusted states are nearly in geostrophic balance and quasi-steady, and they all spin down on a long time scale, set primarily by the magnitude of the bottom and lateral friction.

Geostrophic adjustment problems in a closed domain, such as the circular basin considered in this study, have received attention in the refereed literature. For example, the hydrodynamics and energetics of the geostrophic adjustment of a two-layer fluid, initiated by a discontinuity in the interface of two layers, was examined by Wake et al. (2004, 2005) in laboratory experiments in a circular, uniformly rotating tank with either constant depth or ridge topography. They observed the composition of baroclinic Kelvin and Poincaré waves with an emergent geostrophic, double-gyre, quasi-steady state solution, which slowly decayed. The steady-state, analytical solution and frequencies of the dominant waves were found in the linear approximation for the case of a circular basin with a flat bottom.

In this paper, we consider the adjustment problem in a circular basin centred at the pole which either (i) rotates uniformly or (ii) retains the latitudinal variation of the earth's angular rotation in the polar-plane, the so-called γ-approximation (the high latitude equivalent of the mid-latitude β-plane approximation). In case (ii), the analogue of mid-latitude planetary Rossby waves will be excited during the adjustment process. It will be shown in case (ii) that no quasi-steady state, adjusted state is possible except when the contours of the initial surface height anomaly do not cross the planetary potential vorticity contours (i.e., axi-symmetric adjustment).

Free waves in a circular basin on the polar plane, where the Coriolis parameter f decreases quadratically with distance from the pole, were first considered by Le Blond (1964). LeBlond (1964) derived the gravest , or fundamental, eigenfunction of a circular basin and found the approximate analytical expression for the dispersion relation of waves. Later, Haurwitz (1975) and Bringer and Stevens (1980) used cylindrical coordinates to examine freely propagating waves in a high-latitude atmosphere. Harlander (2005) took the further step of deriving the equation for free waves on a δ-plane, which combines both polar (γ) and β-effects. Harlander (2005) studied ray propagation on the polar and δ-planes and showed how to obtain solutions analytically. In the simplest case of free waves in a circular basin on the polar plane, all these solutions give the same result.

We focus on the following questions in this paper:

- How do sharp topographic features and the form of the initial surface elevations affect the geostrophic adjustment?
- How is mechanical energy partitioned between the wave and quasi-steady components of the flow?

The paper is structured as follows: Section 2 formulates the problems to be addressed and describes the numerical model used in the experiments. Results are presented in Section 3. Sections 3a and 3b discuss numerical and analytical solutions, respectively, for a uniformly rotating circular basin with simple topography. Section 3c then considers how the adjustment is altered when the basin is located on a polar plane. Finally, Section 4 considers a polar basin more closely resembling the Arctic basin, followed by a summary of the results obtained throughout the entire paper.

2 Formulation of the problem and the choice of numerical model

a *Set Up of the Problem.*

Consider the problem of the geostrophic adjustment of a barotropic ocean in a circular basin with idealized bathymetry. The pole is located at the centre of the circular basin. We introduce a spherical polar coordinate system (ϕ, θ, a), where θ is the co-latitude, ϕ is thelongitude and a is the radius of the earth. Here we adopt the well-known thin-shell approximation of replacing the radial distance with a, reflecting the fact that the oceans are a shallow layer on the surface of the earth. For analytical convenience, we will also work with a local Cartesian coordinate frame Oxy where the origin lies on the polar axis and $x = r\cos\phi, y = r\sin\phi$, where $r = a\sin\theta$. In the case of the Arctic, the characteristic lateral extent of the basin corresponds to $0 \le \theta \le \pi/12$. Figure 1 shows the spherical and local Cartesian frames introduced above.

In this study, we will address the role that idealized topography plays in the geostrophic adjustment of a prescribed initial, unbalanced, potential vorticity anomaly. Let $H_0(\phi, \theta)$ denote the depth of the fluid measured from the undisturbed surface. Guided by the physical characteristics of the Arctic basin, the depth of the deepest region of the basin is taken to be 3000 m.

Four idealized topographies and basins are considered:

(a) a top-hat ridge of height 2 km and width 100 km centred on a diagonal;

(b) a step escarpment of height 1 km coincident with a diameters;

(c) a semicircular basin of uniform depth; and

(d) a linear sloping bottom occupying one-half of the circular basin, with a uniform depth shallow region in the other half of the basin.

In all cases, the bathymetry contours form a family of straight parallel lines. We first consider the geostrophic adjustment of a homogeneous fluid in the presence of topography on a uniformly rotating (f-plane) circular basin. In this case, the adjustment takes place through the excitation of gravity waves, boundary trapped Kelvin-type waves, super-inertial Poincaré waves and sub-inertial topographic Rossby waves. The study then addresses the equivalent problem on a polar-plane. In addition to the waves that are supported on the f-plane, the polar-plane also supports planetary Rossby waves, the analogue of planetary waves in a uniform depth ocean on a mid-latitude β -plane.

Throughout this study we will assume that the ocean is initially at rest, and we prescribe an initial surface elevation η_0, taking one of two forms. The first form is

$$\eta_0(\phi,\theta) = -(1/2)\hat{\eta}\,\mathrm{sgn}(\phi), \qquad (1a)$$

where $\hat{\eta}$ is a constant defining the initial amplitude of the step elevation. The second form of η_0 corresponds to a flat-top circular cylinder centred on the pole. This distribution is most simply expressed using the Cartesian coordinate frame shown in Fig. 1b:

$$\eta_0(r,\theta) = \hat{\eta} H(\alpha R - r), \qquad (1b)$$

where H denotes the Heaviside function, $0 < \alpha < 1$ is a constant , $R = \pi a/12$ is the radius of the basin, and r is the polar distance from the origin. Clearly Eq. (1b) describes a cylinder of radius αR and height $\hat{\eta}$.

The initial linearized potential vorticity anomaly associated with η_0 is given by

$$Q = -\frac{f\eta_0}{{H_0}^2} \quad , \qquad (2)$$

where f is the Coriolis parameter.

b *Numerical Model*

The majority of geostrophic adjustment solutions are calculated using a barotropic numerical ocean model, augmented by linear analytical solutions in certain cases to facilitate understanding of the numerical solutions. The analytical solutions also provide a consistency check on the validity and overall performance of the numerical model. In this study, we use the numerical ocean circulation model NEMO (Madec et al.,1998), which is a non-linear primitive equation, three-dimensional model. NEMO is used operationally by several meteorological agencies (e.g., the UK Met Office and Météo France). In the experiments reported in this paper, we use the NEMO model with two options for the calculation of barotropic pressure. To reproduce the earlier stage of adjustment we employ a free surface non-linear explicit algorithm to resolve fast waves associated with the propagation of the initial hydraulic jump. As this algorithm needs a very small time step (typically 2 s), in most of the numerical experiments we used a filtered non-linear free surface algorithm, which is stable with a relatively large time step (see Table 1) but damps fast waves. We performed selected numerical experiments with both schemes, which established that the long-time behaviour of the solutions is essentially identical. The vertical viscosity was set to be constant throughout the study. A quadratic law is adopted for the dependence of the bottom shear stresses on velocity. In this study we use a biharmonic operator to prescribe a lateral viscosity. In most experiments, the model domain is a circular basin, initially defined on a sphere with the centre lying on the equator, which is then rotated so that the domain centre lies on the pole. Table 1 provides the values for the grid size, model time step and other model parameters adopted in this study. In this study, the numerical model is set up to simulate inviscid dynamics as closely as possible. Thus, the vertical and lateral mixing coefficients were taken as small as computationally possible while still suppressing numerical instabilities.

3 Results

a *Geostrophic Adjustment f-Plane Solutions.*

In this sub-section, the NEMO model is used to determine all the solutions. We first consider the adjustment in a circular basin from an initial step in the surface elevation given by Eq. (1a) in the presence of a topographic step escarpment, which is oriented to be orthogonal to the initial surface elevation escarpment. This simulation was performed with a time-explicit free surface algorithm for the barotropic pressure to resolve all the waves responsible for the subsequent flow evolution. Figure 2 shows contour plots of the surface elevation at various times and contour plots of the time-averaged elevations. Figure 3 shows surface elevations and the velocity at various times in a the cross-sections A-B and C-D, marked on Fig. 2a, which are located at the deep and shallower parts of the basin,respectively.

The adjustment is characterized as follows.

(i) Propagation of the initial step in the surface elevation as a hydraulic jump in a direction perpendicular to the initial line of surface discontinuity (see Figs 2a and 3a). During this early phase of adjustment the effects of the earth's rotation are unimportant. When the hydraulic jumps reach the edge of the basin, wave reflection and scattering takes place. Wave scattering occurs because of the curvature of the boundary wall of the basin. The reflection and scattering process takes place multiple times (see Figs 3b and 3d).

(ii) On reaching the boundary of the basin, a fraction of wave energy is scattered into boundary trapped Kelvin-type waves with near-inertial periods and these waves propagate cyclonically around the basin (see Fig. 2b). After multiple reflections and scattering of gravity waves at the basin walls, most of the energy resides in the near-inertial Kelvin-type waves.

(iii) Figure 2e shows that after 12 hours the presence of the topography escarpment in time-averaged solutions becomes apparent in the time-averaged contour plot of the surface elevation. Sub-inertial topographic Rossby waves dictate the longer time scale adjustment. After three days, a quasi-steady geostrophically balanced four-gyre structure emerges in the time-averaged solutions.

We now examine the earliest stages of the adjustment in more detail. Figures 3a to 3d show plots of the surface elevation at various times along the vertical sections A-B (deep basin) and C-D (shallow basin). Along section C-D the phase speed of the wave is 139.6 m s^{-1}, and is in excellent agreement with the speed of long non-dispersive gravity waves $(gH_0)^{1/2}$ (H_0=2000 m), namely 140 m s^{-1}. A similar conclusion is valid for the wave propagating along section A-B (H_0=3000 m), with the numerically derived and analytical wave speeds corresponding to 170.6 m $s^{-1}$and 171 m s^{-1}, respectively.

The effects of non-linearity associated with the surface elevation are evident in the steep wavefronts shown in Figs 3a and 3c. The nature of the waves excited during the early stage of the adjustment can also be deduced from the Hovmőller plots along section C-D shown in Fig. 4. In the early stage of the adjustment, the surface elevation anomaly changes sign each time the gravity waves are reflected from the boundary of the basin (see Fig. 4b). This is caused by water convergence at the landing edge of the front, as shown in Fig. 3e. Figure 4a shows that after four days no radially propagating gravity waves are present and Kelvin-type waves with a period of approximately 13 hours dominate the solution. Figure 4c shows the calculations using the time-filtered version of the model that removes high-frequency waves.

In this paper we retain the terminology of "Kelvin-type wave" for the waves that have mixed properties of Kelvin waves and the lowest mode of cyclonically propagating Poincaré waves (following Stoker and Imberger, 2003). Traditionally, waves with sub-inertial frequencies $\omega < f$ have been called Kelvin waves and with super-inertial frequencies, $\omega > f$, Poincaré waves. The existence of Kelvin waves in the circular flat bottom basin (Lamb, 1932) is determined by the value of the Burger number $S = L / R$ for each azimuthal wavenumber N such that $S^{-2} \leq (N+1)N$, where $L = (gH_0)^{1/2} / f$ is the external Rossby radius of deformation and R is the radius of the basin. For $1/\sqrt{6} \leq S \leq 1/\sqrt{2}$ one single Kelvin wave exists, while for $S > 1/\sqrt{2}$ there is none. Stoker and Imberger (2003) have shown that energetic properties associated with the lowest mode waves change smoothly across the boundary $\omega = f$, and the direction of rotation remains cyclonic; they also retained the notation of Kelvin-type waves for the lowest mode cyclonically propagating wave for the case $S > 1/\sqrt{2}$. In a circular basin with step escarpment topography and depths H_0=3000 m and H_0=2000 m, the Burger number takes the value of $S = 0.708 > 1/\sqrt{2}$ and $S = 0.57 < 1/\sqrt{2}$ in the deep and shallow parts of the basin, respectively. To quantify the amplitude of wave we show a contour plot of A in Figs 4d and 4e

$$A = \sqrt{2}\left\{ T^{-1}\int_0^T \eta^2 dt - \left\langle T^{-1}\int_0^T \eta dt \right\rangle^2 \right\}^{1/2} .$$

The time period for the averaging is T=5 days, and the time interval spans nearly 10 periods of the Kelvin-type waves. Also, we plot in Figs 4d and 4e snapshots of the wave component of currents at two times that are in anti-phase for waves with a period of 13 hours. From Figs 4d and 4e we observe that this wave has properties of both Kelvin and Poincaré waves: wave sea surface elevations are higher near the solid boundary, currents have both an along-shore boundary trapped component, relevant for the Kelvin wave, and a cross-centre component, specific to the lowest mode of the Poincaré wave.

During a subsequent adjustment the solution comprises Kelvin-type waves, topographic Rossby waves and a geostrophically balanced quasi-steady four-gyre system. Because friction is present in the numerical solutions, all constituents decay with time. To quantify how the wave component of the flow changes with time, we plot a time series of max(A) in Fig. 5a.

We observe that the waves in this particular experiment decay at day 160, and the initial amplitude of the waves exceeds the initial surface elevation. To quantify how the quasi-steady flow spins down to a state of rest, the time series of $|\Delta\eta| \equiv |\eta_{max} - \eta_{min}|/2$ at both sides of the basin, averaged over 10 Kelvin wave periods are plotted in Fig. 5b. Here

$$\eta_{max} = \max\left\{T^{-1}\int_0^T \eta dt\right\} T^{-1}\int_0^T \eta dt$$
$$\eta_{min} = \min\left\{T^{-1}\int_0^T \eta dt\right\} T^{-1}\int_0^T \eta dt$$

.

In practice, the locations of η_{max} and η_{min} occur at the centre of the cyclonic and anticyclonic gyres, respectively. The rate of decay of the quasi-steady four-gyre system is about 5% per year in this experiment (see Fig. 5b), which is much smaller than the rate of decay of the wave component, and increases with an increasing bottom drag coefficient (not shown).

As mentioned above, the calculation of steep non-linear gravity waves associated with an initial hydraulic jump using NEMO requires a very small time step, one that is 10 times smaller than that demanded by the Courant constraint. In the numerical solution with a filtered free-surface mechanism, fast gravity waves are damped numerically and first stage of the adjustment described earlier in this section (stage i) ,is absent. Instead, the initial stage of the adjustment takes the form of a diffusive front, which can be seen in the profiles of the surface elevations plotted at various time in Fig. 3f. When the surface height anomaly reaches the wall, Kelvin-type waves emerge (see Fig. 4c). However, the Kelvin-type waves are much weaker than in Fig. 4a and mostly decay by day 4. The strength of the quasi-steady four-gyre system and its rate of decay are almost identical in the time-filtered and unfiltered models (Fig. 5b). Thus, as the key focus of this paper is the longer time scale adjustment, hereafter we use the free-surface filtered algorithm version of the NEMO model.

With dissipation present, the four-gyre system corresponds to the adjusted quasi-steady state limit. In the quasi-steady state limit, no fluid can cross the escarpment. Notice that the gyres are nearly symmetric about the escarpment with higher amplitudes at the shallow part of the basin and are antisymmetric about the line of initial discontinuity in the surface elevation.

In a geostrophically adjusted state, the impact of a steep escarpment on the flow is identical to that of a vertical wall because no fluid can cross isobaths. Therefore, we would expect the geostrophically adjusted solutions in Fig. 2f to be qualitatively the same if the step escarpment is replaced with a ridge. Further, we would expect the adjusted solution shown in Fig. 2f to be qualitatively the same if the circular basin is replaced with a semicircular basin of uniform depth and the initial surface elevation coincides with the symmetry line of the basin. Figure 6 confirms these conjectures.

Figure 7 shows the adjusted solutions that emerge when a cylindrical surface elevation Eq. (1b) is initially prescribed. The solutions shown in Fig. 7 again confirm the equivalence of the adjusted solutions in the three basins. With the escarpment topography, Figs 3f, 5b and 7b show that the strength of the circulation is inversely proportional to the water depth.

Up to now, we have considered the geostrophic adjustment problem in a circular basin with either a top-hat ridge or a step escarpment. Are there any new features in the adjustment problem if topography without a depth discontinuity is introduced? This question is addressed in the following experiments. We consider a circular basin with linear sloping topography in one half of the basin and uniform depth in the other half of basin (case (d) in Section 2) which is plotted in Fig. 8. Initially, a surface elevation with a step discontinuity along the diameter perpendicular to the bathymetry contours is imposed (identical to the initial condition associated with the adjustment shown in Fig. 2). Figure 8 shows the contours of the surface deviation. We observe, that after a "rapid Kelvin wave adjustment" two gyres emerge, similar to those discussed by Wake et al. (2004). However, during this adjustment phase, the flow is essentially decoupled from the topography. On the longer topographic Rossby wave adjustment time scale, the flow evolves to create a four-gyre system, similar to that shown in Fig. 6.

Figure 9 shows a time series of the surface elevations at two locations marked A and B in Fig. 8d. Point B lies in the uniform depth region of the basin. At B, the time series in Fig. 9 reveal that topographic Rossby waves with periods of 40–50 days are superimposed on the quasi-steady "adjusted gyre amplitude" of 0.04 m. Contrast this behaviour with the time series at point A lying over the slope. Here, the Rossby waves continue to propagate and the shorter wave periods (40–50 days) are modulated by longer period (200 days) waves. The latter waves propagate energy in the opposite direction to the shorter period waves. Thus, over the slope, an observer sees a two-gyre system that alternates in sign with time. No quasi-steady double gyre system emerges over the slope and, indeed, Fig. 9 shows that the surface elevation oscillating about the zero amplitude is level.

b *Analytical Solutions of a Linear Problem in a Semicircular Domain on an f-Plane.*

Figures 3 and 6 demonstrate the equivalence of the adjusted solutions in a circular basin (with either a ridge or step escarpment topography) and a semicircular basin with uniform depth. Using the linearized shallow water equations, an analytical solution can be derived for the geostrophically balanced state that emerges in numerically determined solutions. This analytical solution provides a useful independent assessment of NEMO model performance. A plan view of the semicircular basin with the right-handed Cartesian coordinate reference frame used in the subsequent analysis is shown in Fig. 1b.

Let us estimate the dimensionless parameters that characterize the ageostrophic terms in the full non-linear problem with friction. The main constituents of the numerical solutions are Kelvin waves and the geostrophically balanced quasi-steady component, which have a length scale equivalent to the Rossby radius L and different velocity scales. The geostrophic component of the solution is determined by the initial potential vorticity, given by Eq. (2). The upper limit of the geostrophic velocity scale can be estimated from the assumption that all of the initial potential vorticity anomaly is transferred to relative vorticity:

$$Q \approx \frac{\nabla \times \mathbf{u}}{H_0} \quad . \quad .$$

Thus, the initial potential vorticity anomaly scale is $Q \approx U_g/(H_0 L)$. Using (1) we obtain $U_g \approx QHL = Lf\eta_0 / H_0 \approx 0.03\,\mathrm{m\,s^{-1}}$ for typical parameter values used in this study.

The dominant component of currents generated by Kelvin waves is alongshore and is in geostrophical balance (Gill, 1982). For the semi-infinite basin,

$$fU_{KW} \approx \frac{g\hat{\eta}}{L},$$

where U_{KW} and $\hat{\eta}$ are typical scales for the alongshore Kelvin wave velocity component and the surface elevation respectively. The appropriate scale for L is the Rossby radius of deformation and therefore

$$U_{KW} \approx \frac{g^{1/2}\hat{\eta}}{H_0^{1/2}} \approx 0.07ms^{-1},$$

using typical parameter values, used in this study.

In the closed basin the effects of solid boundaries must be taken into account. However, these estimates are in agreement with the results of the numerical solution. The Rossby number $\mathrm{R_O} = U(Lf)^{-1} \approx 2\times10^{-4}$ to 4×10^{-4} for this problem is relatively small, justifying the neglect of the non-linear advection terms in the governing equations. The input of the dissipation terms is estimated by the dimensionless numbers $C_d U(fH_0)^{-1} \approx 10^{-5}$ to 10^{-4} for vertical and $A_B f^{-1} L^{-4} \approx 3\times10^{-12}$ for lateral viscosity, respectively, where C_d is the bottom drag coefficient and A_B is the lateral viscosity for the biharmonic operator (see Table 1) employed in the numerical experiments. Thus, all the diffusive and advective terms can be neglected in the subsequent analysis.

Three forms for η_0 will be considered. First, we consider

$$\eta_0(x,y) = -(1/2)\hat{\eta}\,\mathrm{sgn}(y), \qquad (3)$$

corresponding to a step lying along the x-axis of amplitude $\hat{\eta}$. When $\hat{\eta} > 0$, shallow (deep) fluid initially occupies the region $y > 0$ $(y < 0)$. To assess the sensitivity of the steady-state solution to η_0, we also consider a linear sloping initial surface elevation

$$\eta_0(x,y) = -\frac{3\pi}{8}\hat{\eta}\frac{y}{R} \ . \qquad (4)$$

Equation (4) is chosen to ensure that the domain-averaged value of the linearized potential vorticity anomaly is identical to that associated with Eq. (3). We will also calculate the steady-state geostrophically balanced solution that emerges from an initial top-hat, semicircular cylinder surface elevation, with centre at O, located symmetrically about the *x*-axis (see Fig. 1b).

Following Gill et al. (1986) and Willmott and Johnson (1995), the steady-state geostrophically balanced solution attained after releasing the fluid from rest with an initial surface elevation $\eta_0(x, y)$ can be determined without solving the full initial value problem. Instead, neglecting the viscous and non-linear terms in the equations, we invoke the conservation of potential vorticity, which is of course determined from the initial conditions, to calculate the final adjusted solutions directly. The adjustment of an ocean at rest to an initial perturbation in the sea surface elevation η_0 is governed by the following equation (Gill,1976):

$$\eta_{tt} - gH_0\nabla^2\eta + f^2\eta = f^2\eta_0, \quad . \qquad (5a)$$

Let η_s denote the steady-state component of the solution of Eq. (5a). Then η_s satisfies

$$\nabla^2\eta_s - L^{-2}\eta_s = -L^{-2}\eta_0 \quad . \qquad (5b),$$

For analytical convenience, Eq. (5b) is non-dimensionalized. Using primes to denote dimensionless quantities we define

$$r' = \frac{r}{L}, \quad \eta_s' = \frac{\eta_s}{\hat{\eta}}, \quad \eta_0' = \frac{\eta_0}{\hat{\eta}}.$$

On dropping the primes, the dimensionless form of Eq. (5b) becomes

$$\nabla^2\eta_s - \eta_s = -\eta_0 \quad . \qquad (6)$$

On the boundary of the basin, the normal component of velocity vanishes which requires that

$$\eta_{s\theta} = 0 \quad \text{on} \quad r = S^{-1}, \quad |\phi| < \pi/2 \ , \qquad (7a)$$

$$\eta_{sr} = 0 \quad \text{on} \quad |\phi| = \pi/2, \quad 0 \le r \le S^{-1}, \tag{7b}$$

where $S = L/R$ is the Burger number.

The method of solution for Eq. (6), subject to Eqs (7a) and (7b), uses standard techniques (see, for example, Boyce and DiPrima (1992)). The dimensionless steady-state solutions associated with initial conditions Eq. (3) and Eq. (4) are, respectively:

$$\eta_S = -\frac{2}{\pi}\sum_{n=0}^{n=\infty}\frac{1}{2n+1} f_{2n+1}(r)\sin[2(2n+1)\phi], \tag{8}$$

and

$$\eta_S = \frac{3}{2} S \sum_{n=1}^{n=\infty}\frac{(-1)^n n}{(2n-1)(2n+1)} f_n(r)\sin(2n\phi), \tag{9}$$

where

$$\begin{aligned} f_n(r) = {} & \frac{K_n(S^{-1}) I_n(r)}{I_n(S^{-1})}\int_0^{S^{-1}} \xi I_n(\xi)\psi(\xi)d\xi - \\ & - I_n(r)\int_r^{S^{-1}} \xi K_n(\xi)\psi(\xi)d\xi - K_n(r)\int_0^r \xi I_n(\xi)\psi(\xi)d\xi. \end{aligned} \tag{10}$$

In Eq. (10) I_n and K_n denote the modified Bessel functions of the first and second kind, respectively, and

$$\psi(\xi) = \begin{cases} 1, & \text{in Eq. (8)} \\ \xi, & \text{in Eq. (9)} \end{cases}.$$

When the initial surface elevation takes the form of a right semicircular cylinder, the solution in a semicircular basin can be found from the solution in the circular basin and initial conditions that are antisymmetric relative to the *y*-axis. Let

$$\eta_0 = \alpha^2 + \eta_{\text{asym}}, \tag{11}$$

where

$$\eta_{\text{asym}} = \left[H(r - \alpha S^{-1}) - \alpha^2\right]\text{sgn}(\pi/2 - |\phi|) \tag{12a}$$

$$\psi(r) = (H(r - \alpha S^{-1}) - \alpha^2), \qquad 0 < r < S^{-1}. \tag{12b}$$

Here, the semicircular mean value α^2 was subtracted from the initial condition. Because of its antisymmetric form, such a solution satisfies the condition

$$\eta_{\mathrm{asym}} = 0, \quad \text{on} \quad y = 0 \tag{13}$$

and the associated steady-state, dimensionless solution is given by

$$\eta_s = \alpha^2 - \frac{4}{\pi}\sum_{n=0}^{n=\infty}\frac{1}{2n+1} f_{2n+1}(r)\sin[(2n+1)\phi] \quad . \tag{14}$$

Figures 10a and 10e show contour plots of η given by Eqs (8) and (9), calculated by summing over the first three non-zero modes, respectively. Figures 10b to 10d and 10f to 10h show the contribution of each mode, respectively, to solutions of Eqs (8) and (9). The value S=0.708 is used in each case. Although the solutions in Figs 10a and 10e are qualitatively identical, the latter has weaker amplitude. Clearly, the gravest mode is dominant, and the higher modes shift the location of the gyre's centres towards the symmetry axis $\phi = 0$ (see Fig. 10).

Contours of η given by a steady-state solution Eq. (14) are shown in Fig. 11 for $\alpha = 0.5$, where the solution is computed by the summation of the first three modes and the corresponding components of the solution.

Equations (8), (9) and (14) are somewhat complicated to compute, requiring the numerical evaluation of the integrals in Eq. (10) to calculate each term in the summations. Interestingly, these solutions are found to simplify dramatically in the asymptotic case $S >> 1$, where $S = L/R$. This "large S limit" corresponds to the "small (or deep since $L \to \infty$ as $H \to \infty$) basin limit" or weak rotation limit. Clearly, this limit is more relevant to regional seas and lakes (see estimate of S for the Great Lakes in Csanady (1967)), than to the Arctic Ocean, when $S \sim O(1)$. However, we found that the asymptotic solutions in this limit also provide a reasonably accurate estimate for the case when $S \sim O(1)$. In other words, simplified small-basin limit solutions can be used to approximate the adjusted solution in a basin of lateral extent comparable to the Arctic Ocean.

In dimensionless coordinates, the circular basin spans $0 \le r \le S^{-1}$ and therefore in the limit $S >> 1$ we can employ the asymptotic representations

$$I_n(r) \approx \frac{r^n}{\Gamma(n+1)2^n},$$

$$K_n(r) \approx \Gamma(n)2^{n-1}r^{-n},$$

for a fixed n, valid for small arguments (Abramowitz and Stegun, 1964), where Γ denotes the gamma function. We find that Eqs (8), (9) and (14) can, respectively, be approximated in this "small-basin" limit by

$$\eta_S = \frac{2S^{-2}}{\pi}\sum_{n=0}^{n=\infty}\frac{(\varsigma^{2n+1} - \varsigma^2)}{(2n-1)(2n+1)(2n+3)}\sin[2(2n+1)\phi], \qquad (15)$$

$$\eta_S = \frac{3S^{-2}}{2}\sum_{n=1}^{n=\infty} (-1)^n \sin(2n\phi) \times \begin{cases} \dfrac{n(\varsigma^n - \varsigma^3)}{(n+3)(n-3)(2n-1)(2n+1)}, & n \neq 3 \\ \dfrac{\varsigma^n}{2(n+3)} + \ln(\varsigma)\varsigma^3, & n = 3 \end{cases}, \quad (16)$$

$$\eta_s \approx \alpha^2 + \frac{4S^{-2}}{\pi}\sum_{n=0}^{n=\infty}\frac{1}{2n+1}\sin[(2n+1)\phi] + \begin{cases} \dfrac{(1-\alpha^2)\varsigma^{2n+1}}{(2n+1)}\left[\dfrac{\alpha^{2n+3}}{(2n+3)} - \dfrac{\alpha^{1-2n}}{(1-2n)}\right] + \dfrac{(\varsigma^2 + \varsigma^{2n+1})}{(2n+3)(1-2n)}, & \varsigma < \alpha \\ \dfrac{(1-\alpha^2)\alpha^{2n+3}}{(2n+1)(2n+3)}\left[\varsigma^{2n+1} - \varsigma^{-(2n+1)}\right] + \dfrac{(\varsigma^{2n+1} - \varsigma^2)}{(2n+3)(1-2n)}, & \varsigma \geq \alpha \end{cases}, \quad (17)$$

where $\varsigma = Sr$, varies from 0 to 1. The convergence of the series in Eqs (15) to (17) is relatively fast, with the n^{th} term of the series ~n^{-3}, for large n. The series can be truncated at the fourth term if η_s is to be calculated with a precision O(10^{-2}).

In the analytical solutions shown in Figs 10 and 11, S=0.708, which certainly does not satisfy the "large S limit". However, the asymptotic Eqs (15) to (17) are found to provide a reasonable approximation to the solution even when S~O(1). For example, Fig. 12a shows the "amplitudes" of steady-state solutions, which we define as

$$A_s(\eta_s) = 0.5\Delta\eta_s S^2 = 0.5(\eta_s^{\max} - \eta_s^{\min})S^2, \quad (18)$$

plotted as a function of S for the exact Eqs (8), (9) and (3.14). In all cases amplitude A_s asymptotes to a constant, albeit different, value of η^*, as S increases. Also plotted in Fig. 12a is the amplitude A_s for the "small basin" approximation Eq. (15). The maximum deviation between Eqs (15) and (8) is 20% for $0.6<S<1.4$ and very close to the “small basin” limit at $S>1.2$ (lakes, regional seas). Thus, the computationally efficient solution Eq. (15) provides a reasonable approximation to the exact solution, even when $S=0.708$, the representative value of this parameter for the Arctic Ocean. The amplitude, Eq. (16), is also shown on Fig. 12a, associated with the quasi-steady solutions plotted in Fig. 4b (the black diamond), and Fig. 11 ($\alpha=0.5$) (grey circle). The numerical solution has a smaller amplitude than the analytical solution for the initial step escarpment in the surface elevation, whereas it is larger than the analytical solution derived from the linear initial surface elevation.

Figure 13a shows a contour plot of the solution for Eq. (8) and Fig. 13b shows the equivalent plot for Eq. (15). Although the two plots are qualitatively the same, we observe that the amplitudes are not identical, as revealed by the contours near the centres of the gyres . However, when Eq. (15) is multiplied by the factor $A_s(0.708)/\eta^*$, where $* = \lim_{S\to\infty} A_S$, (A_s is plotted as a function of S for Eq. (8) in Fig. 12a) and the resulting field is contoured, we obtain Fig. 13c. The two solutions contoured in Figs 13a and 13c are indistinguishable, as expected.

The reasonable agreement between the numerical and analytical solutions allows us to use the latter to estimate the partitioning of energy between the geostrophic and fluctuating (wave) components of the flow in a manner similar to Stoker and Imberger (2003) and Wake et al. (2004). The energetics of the geostrophic component, as a function of S, calculated from Eqs (8) to (13) are plotted in Figs 12b and 12c. It is easy to demonstrate that, in a semicircular basin (or in a circular basin with a ridge or escarpment in topography), the energetics of the geostrophic component of the flow coincide with that in a circular basin with a flat bottom. Thus, the patterns of the former are similar to those presented in Wake et al. (2004) for an initial step height discontinuity at the interface between fluid layers and for an initial linear gradient of the density interface (Stoker and Imberger, 2003). For the case of a semicircular cylinder initial surface elevation, the expressions for the initial potential energy (IPE), and potential (PE) and kinetic (KE) components of the steady-state solutions are given by

$$\mathrm{IPE} = \frac{\pi}{4}\left(\frac{\alpha}{S}\right)^2 (1-\alpha^2) \quad , \tag{19a}$$

$$\mathrm{PE} = \frac{4}{\pi}\sum_{n=0}^{n=\infty}\frac{1}{(2n+1)^2}\int_0^{S^{-1}} f_{2n+1}^2(r)\,r\,dr \,, \tag{19b}$$

$$\mathrm{KE} = -\mathrm{PE} + \frac{4}{\pi}\sum_{n=0}^{n=\infty}\frac{1}{(2n+1)^2}\int_0^{S^{-1}} f_{2n+1}(r)[H(r-\alpha/S)-\alpha^2]\,r\,dr \,. \tag{19c}$$

According to the asymptotic solutions, in the large S limit, the fraction of energy converted to kinetic geostrophic energy decays with the parameter S as S^{-2} at large S (small/deep basin limit) and as S^{-4} for potential geostrophic energy (PE). In the small S limit, the ratio of geostrophic kinetic energy (KE) to available potential energy asymptotically approaches the infinite domain limit of 1/3, as expected from the classical result (Gill, 1982). At small Burger numbers, potential energy exceeds the kinetic energy, while for large S most of the energy of the quasi-steady state solution is concentrated in the kinetic energy (Fig. 12c).

c *Geostrophic Adjustment on the Sphere*

In this section we consider a circular basin in which the Coriolis parameter is allowed to vary with latitude according to

$$f = 2\Omega\cos(\theta) \,. \tag{20}$$

Near the pole the Coriolis parameter can be approximated by

$$f \approx 2\Omega(1-\theta^2/2) \,, \tag{21}$$

which is the "polar-plane" approximation. Referring to the coordinate system shown in Fig. 1a, we observe that $r/a = \sin(\theta) \approx \theta$, near the pole. Thus, with respect to the polar coordinate frame shown in Fig. 1b

$$f = 2\Omega[1 - \frac{1}{2}(r/a)^2]. \qquad (22)$$

Figure 14 shows contours of the surface elevations at *t*=3 days, 18 days and 720 days that emerge from the initial step escarpment elevation Eq. (1a), calculated using NEMO on a sphere. After three days, a double-gyre system emerges, equivalent to that discussed by Wake et al. (2004), which is established by the propagation of coastal trapped waves, circling the basin about 10 times during this period. On the f-plane, this double-gyre system would correspond to the final adjusted steady-state solution in the absence of dissipation. However, on the sphere, contours of planetary potential vorticity correspond to concentric circles, and their radial gradient supports the analogue of mid-latitude planetary Rossby waves. The fluid in the double gyres that are established after five days crosses the isolines of planetary potential vorticity, thereby generating planetary waves. Thus, the double gyres rotate clockwise (equivalent to westward propagation of planetary waves at mid-latitudes) essentially without change of form, which can be seen by comparing Figs 14a and 14c.

The time series of the surface elevation at four locations, marked A to D in Fig. 14a, are shown in Fig. 15. The time series are dominated by the gravest Rossby wave mode, with a period in the range of 120–125 days, although the asymmetry of the oscillations reveals the presence of higher modes. Figure 15b shows a running time average of the time series over 125 days which reveals that the higher modes have periods of 400–600 days.

We now consider the above solution from a quantitative viewpoint. The approximate solution for the dispersion relation for planetary waves on the polar-plane has been derived by LeBlond (1964) and is given by

$$\omega_{k,n} = |k| / (M + \varepsilon^{-1} \beta_{k,n}^2) \qquad \text{(23a)}$$

where

$$\varepsilon = \left(\frac{R}{a} \right)^2 \quad , \quad M = \frac{(2\Omega a)^2}{gH} \qquad \text{(23b)}$$

In (23a) $\omega_{k,n}$ is the dimensionless frequency (scaled by 2Ω), k<0 is the azimuthal wavenumber and $\beta_{k,n}$, is the n^{th} root of the Bessel function of the first kind $J_{|k|}$. Clearly, the group speed cannot be determined analytically. However, we observe from Fig. 16a, the dependence of $\beta_{k,n}$ on wavenumber which, for a given range of k, is very close to linear (with a regression coefficient $\mathbb{R}$ satisfying $\mathbb{R}^2$R~0.9999, the coefficients for the linear regressions based on $1 \le |k| \le 5$ and $1 \le |k| \le 9$ for lower and higher ranges of k, are shown in Table 2). Therefore, an approximate form for $\beta_{k,n}$ is given by

$$\beta_{n,k} = b_n - a_n k \quad , \qquad \text{(24a)}$$

where b_n and a_n are constants. Thus, the dispersion relation Eq. (23a) becomes

$$\omega_{k,n} = -k / (M + \varepsilon^{-1} (b_n - a_n k)^2) \qquad \text{(24b)}$$

Using Eq. (24b) it is clear, that the phase speed of the waves, $c_{p,}$ is always negative and propagates clockwise, which is equivalent to westward propagation in the mid-latitudes, and

$$c_p = \omega_{k,n} / k = -1/(M + \varepsilon^{-1}(c_n - a_n k)^2 \ ) \ . \qquad (25)$$

The group velocity of the waves is given by

$$c_g = \frac{\partial \omega'_{k,n}}{\partial k} = \frac{\varepsilon^{-1}((a_n k)^2 - {c_n}^2) - M}{(M + \varepsilon^{-1}(c_n - a_n k)^2)^2} \ . \qquad (26)$$

Notice that c_g changes sign, from negative to positive with increasing $|k|$. Equation (26) reveals that long waves transport energy westward and short waves transmit energy eastward, analogous to their mid-latitude counterparts.

For specific values of the parameters $\varepsilon = 0.067$ and $M = 29.33$, corresponding to the NEMO numerical simulations, the dimensional periods and dimensionless frequencies, phase and group speeds are plotted in Fig. 16. The gravest-mode long-wave period *(n*=1*, k*=-1*)* is 125 days (see Fig. 16c), which is in excellent agreement with the numerical results (see Fig. 15a).

We now anticipate that if the initial surface elevation is axi-symmetric, such as the top-hat cylinder Eq. (1b), and the flow remains axi-symmetric throughout the adjustment, no planetary waves will be generated; because after the rapid adjustment associated with fast gravity radial wave propagation, shown in Fig. 17a, isolines of the surface elevation continue to be axi-symmetric. Thus, fluid flows along, rather than across, the gradient of planetary potential vorticity and no planetary waves are therefore generated. Figures 17b to 17c show contours of surface elevation at t=3 days and 720 days that emerge from the initial top-hat circular cylinder elevation Eq. (1b), and clearly no planetary waves are present.

Now consider the geostrophic adjustment in a circular basin on a sphere in the presence of an escarpment in the bottom topography. Figure 18 shows contours of the surface elevation and, for comparison, adjustment in a semicircular basin of uniform depth when the initial surface elevation takes the form of an escarpment. Recall that we found that, on the f-plane, the geostrophic adjusted solutions in the presence of either a ridge or escarpment topography are equivalent to those in a uniform depth semicircular domain. This result also carries over to the polar-plane adjustment problem.

Hereafter, we will analyze in detail the adjustment in a uniform depth semicircular domain since it replicates the behaviour of the solutions in a circular basin with either a ridge or escarpment bottom topography. Figure 19 shows contours of η and kinetic energy (KE) in a uniform depth, semicircular basin when the initial surface elevation takes the form of a step escarpment. During the first three days, the adjustment mirrors the f-plane solution, with two emerging gyres. Later, westward propagating, long Rossby waves (i.e., phase and group velocities both directed anticyclonically) are generated. The waves then reach the western boundary, marked on Fig. 18, where they reflect as "short waves". The cycle then repeats, although the wave amplitude is continuously reduced because of bottom and lateral friction.

The long waves in Fig. 19 are characterized by a radial mode *n*=1 and azimuthal wavenumber *k*=-2. From the dispersion relation Eq. (24b), we find that the wave period is 125 days (see Fig. 16c). The corresponding group velocity in the azimuthal direction for these waves Eq. (26) shows that the travel time for energy to propagate from the eastern to the western boundary $T_{k,n} \approx \pi/(2\Omega(c_g)_{k,n})$ is 139 days, which is in good agreement with the numerical results in Fig. 18. On reflection at the western boundary, the short waves are characterized by $n=1$, with $|k|$ varying between 4 and 6. We see from Fig. 16 that the wave periods and the time taken for energy to return to the eastern boundary are 105–120 days and 732 days, respectively, which is consistent with the plots in Fig. 18 and time series of sea surface elevation and kinetic energy plotted in Figs 20a and 20b.

The geostrophic adjustment for the case when the initial surface elevation takes the form of a semicircular cylinder is shown in Fig. 21. Early in the adjustment, the surface elevation mimics the f-plane solution (see Fig. 6a). Later, the cyclonic and anticyclonic gyres propagate westward (anticyclonically), creating a dipole structure adjacent to the western boundary (see Fig. 21 (c) at t=150 days). Reflection from the western boundary takes place over a long time scale. Short waves or vortices, with radial wavenumber $|k| = 4–8$ are visible for t=450 days. Although the surface elevation amplitude at this time is small (a few millimetres), the barotropic velocity field attains speeds of the order of 1 cm s^{-1}. Contours of eddy kinetic energy reveal that, even after three years of integration, eastward propagating (counter-clockwise) eddies are still present.

A quantitative interpretation of this adjustment in terms of planetary wave dynamics follows. The long waves are characterized by $|k|$=1, n=2, and, from Fig. 16c, we see that the wave period is about 350 days. The associated group speed is shown in Fig. 16e. From this group speed, we calculated the time taken for energy to reach the “western” boundary, which is 155 days, in agreement with the numerical solution (Fig. 21c). The relative short waves, that propagate energy eastward, are characterized by $n=2$ and radial wavenumbers $|k| > 6$ (Figs 21d and 21e). Figure 16 shows that these waves are characterized by periods of 300 days and an extremely long energy propagation time (about 30 years), which makes them appear almost stationary, taking the form of eddies with slowly decaying amplitudes.

4 Discussion and summary

Numerical and analytical solutions have been derived for the geostrophic adjustment of a homogeneous fluid in a circular basin with idealized topography, namely a step escarpment and a top-hat ridge. In all these cases it is demonstrated that the adjusted solutions are equivalent to that in a flat-bottom semicircular basin, which is also studied in this paper. The adjustment problems considered in this study fall into two categories: (i) a uniformly rotating basin and (ii) a polar-plane basin.

In all the adjustment problems, the fluid is initially at rest with respect to the rotating coordinate frame, and the surface elevation of the fluid is displaced from its equilibrium position. The surface displacement takes one of two forms: (i) a step escarpment or (ii) a right circular cylinder centred on the rotation axis of the earth.

When the basin is rotating uniformly, the adjustment takes place through the excitation of fast gravity waves, boundary trapped Kelvin-type waves and when topography is present, topographic Rossby waves. In the numerical simulations, dissipation damps the waves, and a quasi-steady geostrophically balanced state emerges, which, in turn, spins down on a long time scale to a final state of rest. The steady-state solutions for the semicircular basin problem are found by analytical methods. Computationally efficient asymptotic solutions are derived from these analytical solutions in the small basin or equivalently deep basin limit. It is demonstrated that these asymptotic solutions give reasonable results in cases when the small basin limit is not strictly satisfied, such as, for example, the Arctic Ocean basin. In all the examples considered in this paper, the quasi-steady state consists of an even number of gyres, the structure of which is determined by the form of the initial surface elevation and topography. We also calculate the partitioning of energy between the wave and quasi-steady components.

On the polar-plane, the initial adjustment mimics that of the uniformly rotating case; at intermediate time scales of 3 to 20 days, circulation patterns develop that are very similar to those simulated on the f-plane. The fluid, however, undergoes further wave adjustment because of excitation of the planetary (Rossby) waves generated when fluid crosses planetary potential vorticity contours. Because Kelvin waves and polar-plane Rossby waves are separated by a spectral gap, and because the initially adjusted solution on a polarplane mimics the f-plane steady-state solutions, the fraction of initial potential energy that is transferred to Rossby polar waves can be estimated using analytical solutions on the f-plane plotted in Figs 12b to 12c.

As in the f-plane case, the adjustment in a basin with step escarpment or ridge topography is similar to the adjustment in a semicircular basin. Short wavelength planetary waves are generated when the long waves are reflected at the mid-latitude equivalent of the western boundary. The short waves with a radial mode greater than two have an extremely small group speed, leading to a time scale in excess of 30 years for energy to travel from the western to the eastern boundary. The short waves manifest themselves as long-lived barotropic vortices.

One question that naturally arises is whether any of the features identified in the geostrophic adjustment problems described above carry across to a more realistic representation of the Arctic Ocean. This question is addressed in the numerical solutions shown in Figs 22 and 23. The geometry of the basin is identical in both cases, namely, an irregularly shaped domain of uniform depth, 3000 m, the perimeter of which coincides with the 500 m isobath in the Arctic Ocean. A top-hat ridge of width 100 km and height 2000 m spans the deep basin and is representative of the Lomonosov Ridge in the Arctic Ocean. The solutions in Fig. 22 are calculated for the uniformly rotating basin, while those in Fig. 23 are calculated on the polar-plane. Initially, the surface elevation takes the form of a flat-top circular cylinder of radius 800 km and height 0.4 m, centred on a pole and could, for example, have been produced by Ekman pumping associated with an atmospheric cyclonic circulation.

Contours of surface elevation on day 5 and day 365 are plotted in Figs 22a and 22b, respectively, and agree qualitatively with those in Fig. 5. The steady-state solution adjusts to the convoluted shape of the coastline; therefore, higher mode structures emerge near the coast.

On the polar-plane (Fig. 23), the early stages of the adjustment mimic those shown in Fig. 22. However, westward propagating planetary waves are eventually generated which, at later times, reflect at the western boundary and transfer energy into slowly propagating short waves which are subsequently damped by bottom friction. Qualitatively, the adjustment process shown in Fig. 23 mirrors that in the idealized basin shown in Fig. 20.

The problems discussed in this study facilitate an understanding of the rich interplay of rotating flows and topography at high latitudes. A worthwhile extension of this study is to consider the role played by stratification using, for example, a two-layer model.

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Table 1. Parameter values used in the NEMO modelling system. E/F denotes explicit and filtered surface pressure algorithm experiments.

Horizontal Resolution	Vertical Resolution	Time Step E/F	Biharmonic Horizontal Viscosity	Bottom Drag Coefficient
0.1^ox0.1^o	10 levels	2 s / 15min	$-3x10^8$ m^4 s^{-1}	10^{-4}

Table 2. Coefficients for liner regression (24a).

	a(n=1:5)	*c(n=1:5)*	*a(n=1:9)*	*c(n=1:9)*
k=-1	1.235	2.630	1.177	2.798
k=-2	1.329	5.255	1.264	5.920
k=-3	1.379	8.840	1.315	9.020
k=-4	1.412	11.96	1.350	12.12

Figure captions

Fig. 1 (a) Schematic of the polar basin; (b) a polar projection of the basin showing the local Cartesian coordinate frame, where OA= $r= a \sin \theta$.

Fig. 2 Contour plots of surface elevation η associated with the geostrophic adjustment of a homogeneous fluid in a circular basin with a topographic escarpment on an f-plane. (a)–(c): η are contoured at times shown, (d)–(f) are time means for time interval shown. Contour interval is 5 cm and 1 cm on panels (a)–(d) and (e)–(f), respectively. Solid contours correspond to positive elevations and the shaded area with dashed contours corresponds to negative elevations. The upper left panel shows the cross-section of bottom topography.

Fig. 3 (a)–(d) Surface elevation for the wave-resolving numerical simulation at various times at cross-sections A-B and C-D, at deep and shallow parts of the basin, shown in Fig. 2a; (e) the same but for velocity in the *y*-direction at cross-section C-D; (f) as in (c) but for the numerical simulation, filtering fast waves.

Fig. 4 Hovmőller diagrams for η : (a) for wave-resolving simulations; (b) as i (a) zoomed; (c) for wave-filtering simulations. (d) and (e) Wave amplitude $A=\sqrt{2}\max\left\{T^{-1}\int_0^T \eta^2 dt-\left\langle T^{-1}\int_0^T \eta dt\right\rangle^2\right\}^{1/2}$ averaged over 5 to10 days and 10 to15 of the simulation (contours). Wave component of currents $\mathrm{u}_\mathrm{w}=\mathrm{u}-T^{-1}\int_0^T \mathrm{u}dt$

, where the period of averaging was T=13 hours, at 5 days 22 hours and 6 days 4 hours, roughly in antiphase with the Kelvin-type wave (arrows).

Fig. 5 (a) Decay in intensity of the wave amplitude max(A) of the surface elevation (defined as shown in the plot) for the wave-resolving simulation (black line) and the wave-filtering simulation (grey line); (b) $|\Delta\eta|$, averaged over 5 days plotted against time for the first 600 days of the integration at locations D and E shown in Fig. 2f (1) denotes the wave-resolving simulation and (2) the wave-filtering simulation.

Fig. 6 Quasi-steady solutions of η on the f-plane: (a) a circular basin with a ridge; (b) uniform depth semicircular basin with the initial step being the sea surface height elevation. Contour interval is 1 cm. The upper panel shows vertical cross-sections of the bottom topography,

Fig. 7 Quasi-steady solutions on the f-plane: (a) a circular basin with top-hat ridge; (b) a circular basin with step escarpment; (c) a semicircular basin with uniform depth. Initially the surface deviation takes the form of a right circular cylinder, shown in the left panel. Contour interval is 5 mm.

Fig. 8 Contour plots of η in a circular basin with topography shown in the upper icon (topography case (d) in Section 2) on the f-plane. The initial unbalanced surface elevation is given by Eq. (1a). Contour interval is 1 cm.

Fig. 9 Time series of η at locations A and B shown on Fig. 7d.

Fig. 10 Contour plots of η given by: (a), (e) Eqs (8) and (9), respectively, summing over the first three non-zero modes only; (b–d) and (f–h) are the contributions from modes n=1, 2 and 3 respectively. The contour interval is 1 cm for (a), (b), (e), (f) and 2 mm for (c), (d), (g) and (h).

Fig. 11 Contour plot of η given by Eq. (14) for α=0.5, contour interval is 2 mm.

Fig. 12 (a) Dependence of the amplitude function associated with Eqs (8), (9) and (14) on S. Also plotted, the amplitude associated with the asymptotic solution Eq. (15) and dimensionless amplitudes of numerical solutions shown in Fig. 11 (α=0.5) and Fig. 6b; (b) the ratio of total geostrophic energy (KE+PE) at the adjusted steady state to initial potential energy and the ratio of kinetic energy to available potential energy (IPE-PE) for different initial conditions as a function of S; (c) the ratio of kinetic energy to potential energy in the adjusted steady states as a function of S.

Fig. 13 (a) A plot of η given by Eq. (8) for S=0.708; (b) by the asymptotic solution Eq. (15); (c) the asymptotic solution Eq. (15) multiplied by $A(0.72)/\eta^*$, where A refers to the Eq. (8) and $\eta * = \lim_{S\to\infty} A_S$.

Fig. 14 Contour plots of η at the times associated with the adjustment on a sphere of a fluid in a flat bottom circular basin. The initial surface elevation is given by Eq. (1a). The contour interval is 1 cm.

Fig. 15 (a) Time series of sea surface elevation at the locations shown in Fig. 16a; (b) 125 days of running averages of the time series shown in (a).

Fig. 16 (a) Dependence of the roots of the Bessel function on the wavenumber and linear fits. Plots of the dispersion relations derived from the LeBlond (1964) solution for free Rossby waves on a polar plane. (b) Dimensionless frequency, normalized by the Coriolis frequency; (c) dimensional periods; (d) phase velocity; (e) group velocity. On the plots, *k* and *n* denote the azimuthal wavelength and radial mode numbers, respectively.

Fig. 17 As in Fig. 14 but for a right circular cylinder initial sea surface elevation: (a) plots of surface elevation at various times along a diameter revealing the "rapid" gravity wave adjustment; (b) and (c) contour plots of η. The simulation was made with a time-split algorithm for a barotropic pressure gradient, resolving fast waves.

Fig. 18 As in Fig. 14 but for a circular basin with step topography and a semicircular basin. The contour interval is 5 mm.

Fig. 19 Adjustment to initial step sea surface elevation in a semicircular basin on a sphere. Sea surface elevations (upper panels) and kinetic energy (lower panels) at the times shown. Contour interval for sea surface elevation is 1 cm.

Fig. 20 Time series of (a) sea surface elevation and (b) kinetic energy at the locations shown in Fig. 19a.

Fig. 21 Adjustment to initial semicircular cylinder sea surface elevation in a semicircular basin of uniform depth on a sphere. Sea surface elevations (a–e) and kinetic energy (f–j) on the dates shown.

Fig. 22 Quasi-steady sea surface elevations that result from the initial cylinder surface heights in the deep Arctic basin on an *f*-plane.

Fig. 23 (a–c) Sea surface elevations that resultsfrom the initial cylinder surface heights in the deep Arctic Ocean on a sphere at dates shown. (d) kinetic energy at dates shown.