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INTERNAL REPORT

BIAS IN DILUTION GAUGING RESULTS DUE TO SYSTEMATIC VARIATION OF CONCENTRATION ACROSS THE STREAM

SUBSURFACE SECTION - REPORT NO.42

K. GILMAN

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NATURAL ENVIRONMENT RESEARCH COUNCIL

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Bias in dilution gauging results due to systematic variation of concentration across the stream

JANUARY 1972

Institute of Hydrology
Howbery Park
WALLINGFORD
Berkshire

BIAS IN DILUTION GAUGING RESULTS DUE TO SYSTEMATIC VARIATION OF CONCENTRATION ACROSS THE STREAM

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ABSTRACT

This report is an extension of work carried out by P.H. Hosegood at W.R.A. in a desk study of bias in dilution gauging due to incomplete mixing. Linear and quadratic distributions of concentration and streamflow are considered, and the variation of bias with the distribtuion parameters is investigated. The adjustment of results for this bias permits the assignment of closer confidence limits on the revised results.

A worked example is given in Appendix I, and Appendix II explains the "DISTRIBUTION" messages of the DIFLO computer program for dilution gauging results.

CONTENTS

BIAS IN DILUTION GAUGING RESULTS DUE TO SYSTEMATIC VARIATION OF CONCENTRATION ACROSS THE STREAM

INTRODUCTION

In the analysis of dilution gauging results, the concentration of tracer at the downstream sampling point is usually treated as a random variable, that is its arithmetic or its *harmonic mean* is taken as an estimate of its true value, and its standard error is evaluated by the statistical methods used for random variables. (Gilman 1971).

It is often found, especially in gaugings of sluggish streams where mixing processes are inefficient, that the distribution of tracer is not random, but indicates a systematic variation across the stream, that is the concentration is a function of distance across the stream. If the flow per unit width across the stream were uniform, the use of unweighted means would be justified, but this is usually not the case. To give an accurate flow figure, the concentration must be weighted with the flow distribution (Gilman 1971, Appendix II). This presupposes a reliable estimate of this flow distribution, which could be made by current metering.

An alternative approach is to take the harmonic mean of the sample concentrations, and then to use an estimated flow distribution across the stream to assess the systematic error, or bias, involved in calculating the flow figure from this mean. This has the advantage that some of the necessary data, including the biased flow figure, are already calculated by the computer programs of the DIFLO suite, which assume a random distribution of tracer across the stream width. It is suggested that, if the coefficient of variation of this distribution exceed 2.5% the procedure outlined in this report should be followed in order that a more accurate flow figure be obtained.

THEORY AND DEFINITIONS

1.1 Basic techniques

Hosegood calculated the bias in the estimate of concentration for several distributions of concentration and flow and for two sampling procedures, at three points and at five points across the stream. For the sake of simplicity in calculation and a wider application, the continuous case will be considered here instead of the discrete case. This makes it possible to use integration instead of summation, and to characterise the transverse distributions by one or two parameters. As much as possible of the calculation will be done by simple algebra, borrowing some techniques from linear algebra, and the results will be applied in the final stage to the practical situation of discrete sampling, particularly to the analysis of output from the DIFLO program.

1.2 Distribution functions

The transverse variation of flow and concentration may be characterised by a mean value and a distribution function, where the distribution function has a mean of one.

For example in a stream of width L, let the value of the concentration of tracer at a distance y from the bank be:

$$C(y) = p_2y^2 + p_1y + p_o \text{ for } 0<y<L \qquad (1)$$

The mean value $\bar{C}$ is given by

$$\bar{C} = \frac{1}{L} \int_0^L (p_2y^2 + p_1y + p_o)dy \qquad (2)$$

The distribution function may be obtained by setting $x = \frac{y}{L}$ so that x goes from 0 to 1, and assigning parameters q_2, q_1 and q_o so that the mean of the distribution function is one.

Thus $$C(x) = q_2x^2 + q_1x + q_o \qquad (3)$$

where $$q_2 = p_2L^2/\bar{C},\ q_1 = p_1L/\bar{C},\ q_o = p_o/\bar{C}$$

It should be noted that although the distribution functions defined here have some properties in common with probability distributions, they are distributions in a spatial sense, and must not be confused with probability distribution functions (pdf).

In the work that follows, only the spatial distribution functions will be used, that is all streams will be of width one, and all concentrations and flows will have means of one. This involves no loss of generality, and the transition back to dimensional quantities will be made in the final section.

1.3 The coefficient of variation of a spatial distribution function

For each distribution, it is possible to define a coefficient of variation analogous to that of a random variation (this is a close approximation to the quantity which, for the discrete case, is calculated by the DIFLO program and printed out as "COEFF. OF VARIATION OF SAMPLE GROUP"). The coefficient of variation is defined as

$$Cv(c) = \left[\int_0^1 (c - 1)^2 \, dx \right]^{\frac{1}{2}} \quad \text{for any distribution function } c(x) \qquad (4)$$

An alternative form of (4) may be obtained by expansion and substitution, using the fact that

$$\int_0^1 c \, dx = 1 \text{ (the mean of c is 1)}$$

this gives

$$C_v(c) = \left[\int_0^1 c^2 \, dx - 1 \right]^{\frac{1}{2}} \qquad (5)$$

1.4 Linear and bilinear functionals

This section makes use of certain concepts from the theory of linear algebra, the precise definitions of which are outside the scope of this report. The definitions used therefore will be restricted to the immediate application and should not be considered as general definitions.

A <u>mapping</u> is an operation which associates with one object another unique object. For example, a function is a mapping which associates with a variable a unique quantity termed the value of the function at that point.

A <u>functional</u> is a specialised mapping which maps a function on to a point on the real axis. For example, integration over a particular range of a function will yield a result which is a real number.

Two notations can be used to represent a functional: the operator notation, which is normally used to represent integration, or the functional notation. For example, an integral could be represented as

$$\int_a^b f(x) \, dx \quad \text{or as } I(f)$$

provided that the functional I was properly defined elsewhere.

A functional F is <u>linear</u> if and only if it satisfies the condition

$$F(\lambda f + \mu g) = \lambda F(f) + \mu F(g) \tag{6}$$

where f and g are functions, λ and μ are scalars (real or complex numbers). Two of the functionals used in this report will map pairs of functions on to real numbers. A functional $F(f, g)$ of this type is bilinear if and only if it satisfies condition (6) in both its variables, i.e.

$$F(\lambda f + \mu h, g) = \lambda F(f, g) + \mu F(h, g) \tag{7}$$

and a similar relation for the second variable.

An example of a functional already encountered is Cv(c), defined in §2.3 Unfortunately Cv(c) is non-linear, which could make it difficult to handle, but it may be written in terms of a linear functional to be defined below.

CALCULATION OF BIAS

2.1 Bias in using unweighted means

This basic equation of dilution gauging may be stated as

$$Q = qc/C \tag{8}$$

where qc is the rate of injection of tracer, and C is the concentration of tracer at the sampling point. If C is estimated from the analysis of a set of n samples the equation used is actually

$$Q = qc/Ch \tag{9}$$

where Ch is the harmonic mean of the samples, given by

$$Ch = n/\left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots\right) \tag{10}$$

Equation (8) is derived from the continuity equation

$$QC = qc \tag{11}$$

which for the case of a systematic variation in flow and concentration may be quoted in its general form

$$\int_0^L VC\ dy = qc \tag{12}$$

where V is the flow per unit width and C is the concentration at distance y from the bank.

At this point the harmonic mean must be discarded, because it cannot easily be carried over into the continuous case. It is simpler to evaluate the bias in using the arithmetic mean, and to consider separately the contribution to this bias from the use of the harmonic mean. Equation (9) may be rewritten

$$Q = qc/\bar{C} \tag{13}$$

where $\bar{C}$ is the arithmetic mean, given by

$$\bar{C} = (C_1 + C_2 + \ldots\ldots)/n \tag{14}$$

To evaluate the bias in using this mean, first of all the two estimates of Q, using $\bar{C}$ and the weighted mean, must be calculated. From equation (12)

$$\int_0^1 V(xL)\ C\ (xL)\cdot L\ dx = qc \tag{15}$$

from §1.2, $V(xL) = \bar{V}\ v(x)$ (16)

So $$L\bar{V}\bar{C} \int_0^1 vc\ dx = qc \tag{17}$$

But $$L\bar{V} = Q \tag{18}$$

so $$Q = qc/(\bar{C} \int_0^1 vc\ dx) \tag{19}$$

is the estimate of Q made from the weighted mean.

The absolute bias is the difference of these two estimates, and the relative bias is the ratio of the absolute bias to the value of Q given by the weighted mean.

Thus
$$Bi = \left({}^{1}/\bar{C} \quad \frac{1}{\bar{C}\int vc\ dx}\right) / \left(\frac{1}{\bar{C}\int vc\ dx}\right)$$
$$Bi = \int_{0}^{1} vc\ dx - 1 \qquad (20)$$

The bias Bi above is another functional Bi (v, c), but it is not bilinear.

If we define the functional
$$A\ (v,\ c) = \int_{0}^{1} vc\ dx \qquad (21)$$
it is obvious, from the linear nature of integration, that A is bilinear. (20) may be rewritten

$$Bi = A - 1 \qquad (22)$$

Also the coefficient of variation, Cv(c), defined in §1.3 can be written in terms of A:

$$Cv(c) = \left[A(c,c) - 1\right]^{\frac{1}{2}} \qquad (23)$$

2.2 Properties of the bias functional A(v,c)

Three important properties of A(v,c) may be proved simply:

i) $A(1,\ c) \equiv 1$

where c is any distribution function as defined above

ii) $A(v,c) \equiv A(c,\ v)$

iii) $A(x^m,\ x^n) \equiv \dfrac{1}{m + n + 1}$ (24)

where m, n are any real numbers such that $m + n \neq -1$

The bilinearity of A means that polynomials in x may be dealt with as follows:

$$A(p_2x^2 + p_1x + p_o,c) = p_2A(x^2,c) + p_1A(x,c) + p_o \qquad (25)$$

2.3 Values of the bias functional for simple distributions;

Two distribution types will be considered: linear and quadratic. A linear function would normally have two parameters, but one is defined by the requirement that the mean is unity. Similarly, a parabolic or quadratic distribution function has only two parameters. The parameters chosen were the gradient of the linear function, and the coefficient of x^2 and the position of the turning point of the quadratic function. The distributions are sketched and their equations and parameter values are given in fig. 1.

For the two distribution types, values of A were calculated, using the properties (24) and (25).

The following results were obtained:

i) for a linear distribution of both flow and concentration, with parameters a_1 and a_2 respectively,

$$A\ (v,c) = 1 + \frac{a_1\ a_2}{12} \tag{26}$$

ii) for a linear distribution of concentration and a quadratic distribution of flow,

$$A(v,c) = 1 + \frac{a_1\ a_2}{12} - \frac{a_1\ a_2\ m_1}{6} \tag{27}$$

where a_1 is the coefficient of x^2 in v

m_1 is the x-coordinate of the turning point of v

a_2 is the gradient of c

iii) for a quadratic distribution of both flow and concentration,

$$A(v,c) = 1 + a_1\ a_2\ (\frac{1}{180} + \frac{1}{3}\ (m_1 - \tfrac{1}{2})(m_2 - \tfrac{1}{2})) \tag{28}$$

where a_1, a_2 are the coefficients of x^2 in v and c respectively

m_1, m_2 are the x-coordinates of the turning points of v and c respectively.

Thus the general form of the bias Bi(v,c) for these distributions is

$$Bi(v,c) = \frac{a_1\ a_2}{k}$$

where k is determined by the type of the distributions. (and the parameters m in the case of the quadratic distribution). Values of k are given in Table 1.

2.4 Bias due to use of arithmetic mean

The harmonic mean of a set of values C is always less than the arithmetic mean, by an amount which is dependent upon the scatter of the values. This amount can be shown to be approximately equal to

$$\frac{\sigma^2\,(C)}{\bar{C}}$$

where $\sigma^2(C)$ is the variance (the mean of squares of deviations from the mean) of C. The error in taking the difference equal to this amount is given by

$$-\frac{2\,\sigma^4\,(C)}{\bar{C}^{\,3}}$$

which is very small.

Thus the bias in using the arithmetic mean is negative (a larger value for C gives a smaller value for Q) and its relative magnitude is

$$\frac{\sigma^2(C)}{\bar{C}^{\,2}}$$

which is the square of the coefficient of variation of C, defined in §1.3

APPLICATION

3.1 Using the bias estimates

The output from a DIFLO program takes the following form:

SAMPLE GROUP 8
COEFF. OF VARIATION OF SAMPLE GROUP 8 IS 5.1%
DISTRIBUTION - MAXIMUM AT 0.4
FLOW 135.9 L/S PLUS OR MINUS 6.93 L/S AT 95% CONFIDENCE LEVEL (SUBJECT TO BIAS CORRECTION).

The coefficient of variation is calculated from the reciprocals of the sample concentrations which are a discrete set, and it is the best estimate available of the coefficient of variation of the continuous distribution actually present in the river, Cv(c). From the values of $C_v(c)$ and m_2 given in the output (0.051 and 0.4 in the above example) the value of the parameter a_2 may be obtained.

From (23), (26) and (28)

$$a_2 = \pm C_v(c)\sqrt{12} \text{ if c is linear} \quad (30)$$

$$a_2 = \pm C_v(c)\left[\frac{1}{180} + \frac{1}{3}(m_2 - \tfrac{1}{2})^2\right]^{-\frac{1}{2}} \quad (31)$$

Values of the expression by which $C_v(c)$ is multiplied in (31) are given in Table 2.

The value of a_1 is more difficult to establish. A current metering of the stream will supply estimates of the flow through sections of the stream whose centres are the sampling points (see Fig. 2). These estimates must then be fitted with a quadratic or linear function by numerical methods, as described by Lyon (1970) to obtain values of a_1 and m_1.

Table 1 may then be used to give a value of k, which is used in formula (29) to give the bias Bi. Bi is the bias invoked by using the arithmetic mean of C rather than the weighted mean; the true value of the bias is

Bi + Mi

where Mi is the bias invoked by using the arithmetic mean rather than the harmonic mean. Thus if Bi is positive, the true value of bias will be less than Bi (Mi being negative); if Bi is negative, the true value will be greater in magnitude than Bi.

Mi is given by

$$Mi = -C_v^2(C) \quad \text{(from §2.4)}$$

$$Mi = -\frac{a_2^2}{12} \quad \text{if c is linear}$$

$$Mi = -a_2^2\left[\frac{1}{180} + \frac{1}{3}(m_2 - \tfrac{1}{2})^2\right] \quad \text{if c is quadratic (32)}$$

Values of the expression in brackets are given in Table 3.

3.2 Quoting the final result

If C is taken as a random variable, the systematic variation can swamp all other errors due to injection and analysis, and produce a set of unrealistic confidence limits. By considering C as a function of distance across the stream, these confidence limits can be reduced considerably, but the process of estimating bias can contain significant error.

The parameter a_2, calculated from the sample concentrations, is subject to error, as the concentration at each point is a random variable. Its scatter could be estimated by taking several samples at each point across the stream, and this would provide better values of a_2 and m_2, but the errors in these parameters are probably smaller than those in the values of a_1 and m_1, calculated from the results of current metering. Leaving aside the question of the accuracy of current metering, it is unlikely that the flow distribution is as simple as has been assumed here. In practice, the flow distribution will not be determined for every gauging, but will be assumed constant over a range of flows. This is another potential source of error.

It seems wise, therefore, to assign a standard error to the bias estimation process of at least 2.5%. This standard error would be combined with the errors from all other sources (except, of course, the sample group error caused by systematic variation) to give confidence limits to the flow figure of around ±5%. This will be incorporated into the DIFLO program.

An upper limit must also be set to avoid misuse of the technique. A coefficient of variation in excess of 20% indicates bad mixing of a serious degree, and results with this magnitude of error should be discarded. A point to remember is that although inefficient mixing can exist in a steady-state situation, it is more usually a result of failure to achieve plateau. This means that the quantity of tracer flowing through the sample cross-section is less than qc, and the flow figure is subject to a bias from this source.

Both the upper limit and the standard error are arbitary, and subject to revision, and all results calculated by the bias estimation procedure must be treated with a certain amount of caution. It is suggested that the coefficient of variation be adopted as an index of mixing, and should be clearly stated with all results obtained by this method.

3.3 Combining results of gaugings at the same point

In normal practice, several sets of samples will be taken at each sampling cross section (these are denoted by the term "sample group" in the DIFLO program) and it will be desirable to combine these results to give a single value for the flow at this cross-section.

Given a set of results from n sample groups, each with its standard error, the first step is to test for a significant difference between results. The presence of such a difference could stem from a genuine difference in the flow through the section (at two different times, for instance) or from a false value of the standard error in a result. When testing the difference between two results one of the variables used in the calculation is the number of samples used in the determination of the results. For the case of dilution gauging results, it is proposed that the effective number of samples be set equal to the number of samples in a group when bias estimation has been used, and to infinity when the results have a coefficient of variation of less than 2.5%

The significance test used is Student's t test, which is used as follows. Let two results have values Q_1 and Q_2, and standard errors S_1 and S_2. Then the standard error of $Q_1 - Q_2$ is

$$S = \sqrt{(S_1^2 + S_2^2)} \tag{33}$$

and we now compare $Q_1 - Q_2$ with tS, where t is obtained from Table 4. If $Q_1 - Q_2$ is greater than tS, the results are significantly different at the 5% level, and cannot be combined.

If there is no significant difference between the results, taken in pairs, they may be combined to give a better estimate of the flow. The method of combination is not the arithmetic mean, which would take too much account of the less reliable results, but a weighted mean, where the weighting of each result is dependent upon the standard error of the result. The weighted mean is given by

$$Q_m = \frac{\lambda_1 Q_1 + \lambda_2 Q_2 + \ldots}{\lambda_1 + \lambda_2 + \ldots} \tag{34}$$

where $\lambda r = \dfrac{1}{Sr^2}$

and Sr is the standard error of the result Qr. The standard errors used in this section are absolute standard errors, obtained by multiplying the coefficient of variation of the Qr by Qr.

The standard error of the combined result is Sm, where

$$\frac{1}{Sm^2} = \frac{1}{S_1{}^2} + \frac{1}{S_2{}^2} + \qquad = \lambda_1 + \lambda_2 + \ldots \qquad (35)$$

This standard error may be doubled to give the 95% confidence limit on the result.

CONCLUSIONS

Stream discharges calculated from dilution gauging results can be adjusted to allow for bias due to incomplete mixing resulting in a non-uniform distribution of tracer across the stream. The method is based on the assumption that the concentrations of samples across the stream may be weighted according to a transverse distribution of flow which has a simple form and remains sensibly constant for a range of discharges. This flow distribution may be determined by current metering at the sampling section.

Results calculated by the bias estimation procedure can be quoted with much closer confidence limits than if the concentration were treated as a random variable.

TABLE 1

Values of k for calculation of $Bi = \frac{a_1 a_2}{k}$

a) Linear/linear k = 12

b) Quadratic/linear

m_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
k	12	15	20	30	60	∞	-60	-30	-20	-15	-12

c) Quadratic/quadratic

m_1 =	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
m_2 = 0	11.3	13.9	18.0	25.7	45.0	180	- 90.0	- 36.0	- 22.5	- 16.4	-12.9
0.1		17.0	22.0	31.0	52.9	180	-129	- 47.4	- 29.0	- 20.9	-16.4
0.2			28.1	39.1	64.3	180	-225	- 69.2	- 40.9	- 29.0	-22.5
0.3				52.9	81.8	180	-900	-129	- 69.2	- 47.4	-36.0
0.4					113	180	450	-900	-225	-129	-90.0
0.5						180	180	180	180	180	180
0.6							113	81.8	64.3	52.9	45.0
0.7								52.9	39.1	31.0	25.7
0.8									28.1	22.0	18.0
0.9										17.0	13.9
1.0											11.3

Table c is symmetric, for example the value of k for $m_1 = 0.4$ and $m_2 = 0.9$ is -129, the same as the value for $m_1 = 0.9$, $m_2 = 0.4$

TABLE 2

Values of $\alpha = \left[\frac{1}{180} + \frac{1}{3}(m_2 - \frac{1}{2})^2\right]^{-\frac{1}{2}}$

m_2 =	0	0.1	0.2	0.3	0.4	0,5	0.6	0.7	0.8	0.9	1.0
α	3.360	4.133	5.303	7.341	10.81	13.42	10.81	7.341	5.303	4.133	3.360

TABLE 3

Values of $\beta = \frac{1}{180} + \frac{1}{3}(m_2 - \frac{1}{2})^2$

m_2 =	0	0.1	0.2	0.3	0.4	0.5
β	0.0885	0.0586	0.0356	0:0186	0.00856	0.00556

Table 3 is symmetric: the value of β for $m_2 = 1 - \lambda$ is the same as that for $m_2 = \lambda$

TABLE 4

Student's t for 5% significance level.

effective n	3	4	6	8	10	∞
	4.30	3.18	2.57	2.36	2.26	1.96

REFERENCES

Hilman, K. Calculation of stream discharge from chemical dilution gauging results. Institute of Hydrology, Subsurface Section Rept. 37, 1971.

Lyon, A.J. Dealing with data. Pergamon (Oxford) 1970

APPENDIX I - A WORKED EXAMPLE

Gauging of R. Nailbourne

i) Results of current metering carried out 10.1.72 at 0.6 of depth.

	Distance from bank y m.	Depth h.m.	Velocity U m/s	Flow/unit width $v = Uh$ m3/s/m
Rt.bank	0			
	0.2	0.09	0.363	0.0327
	0.6	0.10	0.248	0.0248
	1.0	0.08	0.241	0.0193
	1.4	0.105	0.242	0.0254
	1.8	0.095	0.215	0.0204
	2.2	0.08	0.409	0.0327
	2.6	0.07	0.211	0.0148
	3.0	0.05	0.277	0.0139
	3.4	0.05	0.223	0.0112
	3.8	0.05	0.167	0.0084
Lt.bank	4.0			

The equation of the parabola fitted by the orthogonal polynomial method (Lyon 1970) is

$$v = -.003505\ (y - 2)^2 - .00728\ (y - 2) + .02614$$

and the mean of v is 0.02036. From equation (3), the distribution function parameter

$$a_1 = -.003505 \times 4^2/0.02036 = -2.754$$

The position of the maximum of v is obtained by differentiation of the equation for v. v reaches a maximum at $y = 0.34$, so for the distribution function

$$m_1 = \frac{0.34}{4} = 0.085$$

There is little loss of accuracy in assuming $m_1 = 0.1$

ii) The following is a table of concentration peak heights at six points across the stream, assumed equally spaced, and reading from right bank to left.

38.5	39.5	35.5	34.5	33.5	33

The DIFLO output for these results reads as follows:

SAMPLE GROUP 7

COEFF. OF VARIATION OF SAMPLE GROUP 7 is 6.7%
DISTRIBUTION - MONOTONE L.H.
FLOW 108.0 L/S PLUS OR MINUS 5.68 L/S AT 95% CONFIDENCE LEVEL
(SUBJECT TO BIAS CORRECTION)

The "coeff. of variation of sample group 7" is a good approximation to Cv(C). The message "distribution - monotone L.H." means that the distribution function is approximately a straight line with negative gradient (see Appendix II). The magnitude of the distribution parameter a_2 is given by equation (30)

$$|a_2| = 0.067 \sqrt{12} = 0.232$$

The sign of a_2 is negative

$$a_2 = -0.232$$

iii) The bias in the flow figure

From equation (32)

$$Mi = \frac{-a_2^2}{12} = -0.0045$$

and from (29) $Bi = \frac{a_1 a_2}{k}$ where k is given by Table 1

For $m_1 = 0.1$, $k = 15$. So

$$Bi = \frac{-2.754 \times -.232}{15} = 0.041$$

The net value of the bias

$$Bi + Mi = 0.037$$

Thus the result quoted, 108.0 L/S, is biased by +3.7%.

The true figure is 108 (1 - 0.037) = 104.0 L/S

The confidence limit is 5.68 (1 - 0.037) = 5.47 L/S. Because of the inaccuracy of the bias estimation process, the confidence limit should be quoted to two significant figures only, viz. 5.5 L/S

Corrected flow figure 104.0 L/S plus or minus 5.5 L/S

APPENDIX II - DISTRIBUTION TYPES IN DIFLO OUTPUT

If the coefficient of variation of a sample group exceeds 1% DIFLO will display a message starting "DISTRIBUTION - ". The distribution types are explained below:

DISTRIBUTION - MONOTONE R.H.

The pattern analysis routine has found that the concentration exhibits an increase from right bank to left bank. MONOTONE distributions are treated as linear.

DISTRIBUTION - MONOTONE L.H.

As above, but a decrease from right bank to left

DISTRIBUTION - MAXIMUM AT

A quadratic distribution with a maximum at the stated fraction of the width from the right bank.

DISTRIBUTION - MINIMUM AT

Similar to the above, but this message should be extremely rare.

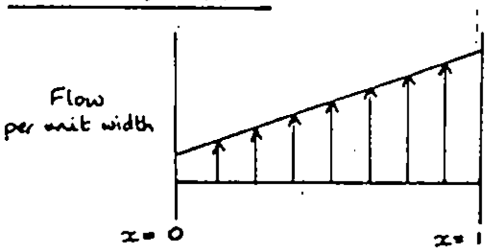
DISTRIBUTION UNIFORM

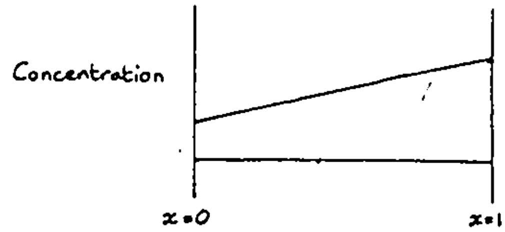
The pattern analysis routine has found a zero trend in concentration. This means that the points are very scattered, and the results should be discarded, as mixing is obviously poor.

FIG 1

SPATIAL DISTRIBUTION FUNCTIONS

Linear distribution





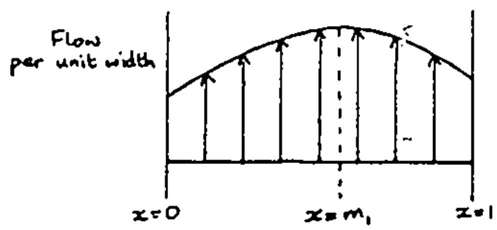
$$v = a_1 x + 1 - \frac{a_1}{2}$$

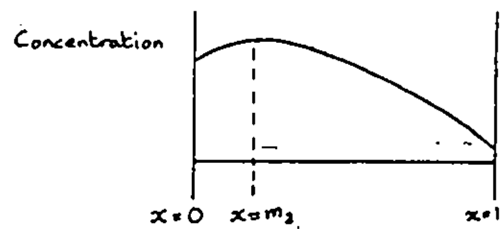
If $|a_1| \leq 2$, v is non-negative

$$c = a_2 x + 1 - \frac{a_2}{2}$$

If $|a_2| \leq 2$, c is non-negative

Quadratic distribution





$$v = a_1(x-m_1)^2 + 1 + a_1 m_1 - a_1 m_1^2 - \frac{a_1}{3}$$

$$c = a_2(x-m_2)^2 + 1 + a_2 m_2 - a_2 m_2^2 - \frac{a_2}{3}$$

The normal situation is a maximum in both flow and concentration at some point in the stream – this means a_1 and a_2 are negative.

v and c are non-negative at all points in the stream if

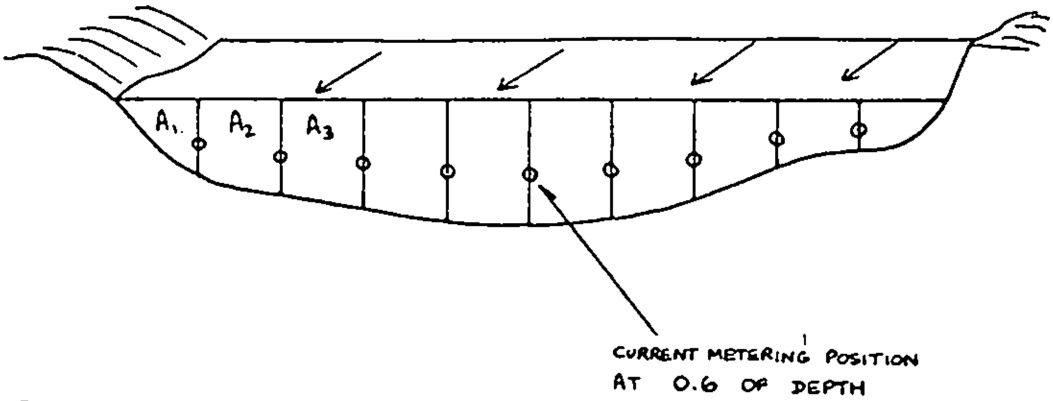
either $a_1 > \frac{-3}{3m_1 - 1}$ or $a_1 > \frac{-3}{2 - 3m_1}$

and $a_2 > \frac{-3}{3m_2 - 1}$ or $a_2 > \frac{-3}{2 - 3m_2}$

e.g. if $m_1 = 0.5$, a_1 must be greater than -6

FIG. 2

CURRENT METERING TO ESTABLISH THE FLOW DISTRIBUTION



To determine discharge

Measured v = v_1 v_2 v_3 v_4 v_5 v_6 v_8 v_9

Flow through sections $q_n = \frac{1}{2}(v_n + v_{n-1})A_n$ $(v_0 = v_{10} = 0)$
where A_n is cross-sectional area of nth section

Total flow $Q = \Sigma q_n$.

To determine distribution function

Measure the depth at stations 1 to 9, h_n
Flow per unit width $V_n = v_n h_n$
Fit curve to V_n to obtain a_1, m_1

FIG. A1
FLOW AND CONCENTRATION DISTRIBUTIONS FOR THE WORKED EXAMPLE.