IHACRES: An update

Background

More than two years ago, at an informal meeting at IH (Naden, 1989), IGL presented some results of work undertaken with Dr Tony Jakeman (AJJ), Centre for Resource and Environmental Studies (CRES), Australian National University, whilst he (AJJ) was a Visiting Scientist at IH during 1988. This work was undertaken in consultation with Dr Paul Whitehead.

The major product of that period was the development of a time series analysis (transfer function) approach to rainfall - streamflow modelling and its application, with good results, using hourly data, to two small upland catchments in Wales (Jakeman, Littlewood and Whitehead, 1990). A novel aspect of this rainfall - streamflow modelling methodology, known as IHACRES (Identification of unit Hydrographs And Component flows from Rainfall, Evaporation and Streamflow data), is that it allows identification of separate impulse response functions for different streamflow components considered to act in a parallel and/or series configuration. Each 'pathway' or 'storage' has a characteristic response time. Where two parallel flow components are identified as the best configuration the sum of the two impulse response functions is the unit hydrograph for total streamflow, and the hydrograph can be separated into its quick and slow flow components. Additional attractive features of the methodology are that neither the selection of individual 'suitable' storm events, nor the prior separation of baseflow, are necessary - these are common steps in many other methods of unit hydrograph identification, e.g. the method used for systematic UK floods studies given in the Floods Study Report (NERC, 1975).

Hydrograph separation into quick and slow flow components allows computation of a Slow Flow Index (SFI) which is analogous to, but arguably more objective than, the BaseFlow Index, BFI, (Institute of Hydrology, 1980). IHACRES and BFI hydrograph separations are reassuringly similar for most of the Surface Water Archive stations investigated so far (see later for a list of these) but it is quite possible that for some catchments there may be important differences between SFI and published values (NERC, 1988) of BFI.

During the last two to three years the IHACRES program has been developed further (moving it from mainframe to PC). Successful application of IHACRES to a range of catchment types (e.g. areas from 1.6 ha to 2344 km² in different climate zones), using hourly or daily data as appropriate, has demonstrated that the rainfall - streamflow model in IHACRES is generic. The main purpose of this note is to summarise progress with IHACRES for those who may not be aware of this more
recent work or may not have had time to read the published papers (some are quite recent and two are in press). Other purposes of this note are to (a) offer an opportunity for colleagues working in areas where IHACRES may be applicable to make an initial appraisal of the technique and (b) provide information to those with a long-established interest in unit hydrograph theory and application. Examples of IHACRES applications are given.

Development of the PC version of IHACRES has been well advanced by AJJ and Ms Heather Symons at CRES (Jakeman, Littlewood and Symons, 1991). IGL visited CRES during January and February to assist with this programme of work, and collaboration to this end continues. For several reasons, however, it may be some time before IHACRES PC is available within IH (and hopefully on a commercial basis eventually) to the same extent as other rainfall - streamflow models (e.g. HYRROM), and this may have to be led by demand.

Although the PC package is still being developed it has already been used on an individual-case basis for project work (e.g. Littlewood and Jakeman, 1991; Jakeman, Littlewood and Whitehead, in press). Several areas of possible research with which IHACRES might assist are currently being discussed. Anyone who wishes to see IHACRES working, or to discuss possible collaborative applications of it in their area of work, should contact IGL. Enquiries will be particularly welcome from those willing to feed back useful comments to assist in its further development as a user friendly package.

The model and estimation of its parameters

The following gives an outline description of the model and some details of the parameter estimation technique employed in IHACRES (see Jakeman, Littlewood and Whitehead (1990, 1991) for further details).

The structure of the model is very simple and can be considered in two parts. The first part is an optional rainfall - excess rainfall module comprising one or both of two steps: rainfall $r_k$ is adjusted by the single-parameter ($t_w$) model given by (1) to account for seasonal variation in evaporative ‘losses’; excess rainfall ($u_k$) is calculated on the basis of a single-parameter ($r_k$) catchment wetness index ($s_k$) which itself is calculated from rainfall (adjusted by (1) if necessary), as shown by (2) - (3).

$$r_k^* = (1-t_w/t_a)r_k$$

(1)

$$s_k = s_{k-1} + r_k^*(r_k^*-s_{k-1})$$

(2)

$$u_k = \text{const.} \ r_k^* s_k$$

(3)
The value of const. in (3) assures equality between the volumes of excess rainfall and streamflow over the period of model calibration. In the second part, excess rainfall is converted to streamflow \( Q_j \) by a transfer function, the structure of which is determined as an integral part of the overall IHACRES modelling procedure. For nearly all catchments investigated so far, the optimal structure of the transfer function is second-order (four parameters) as given by (4).

\[
Q_k = \left[ \frac{b_2 + b_1 z^{-1}}{1 + a_2 z^{-1} + a_1 z^{-2}} \right] u_k
\]  

where \( z^{-1} u_k = u_{k-1} \)

Solving a transfer function recursively with an input \( u_k \) of unity at time \( k = 1 \) (and zero at all other times) gives a system impulse response function (or, in this case, a unit hydrograph). Thus a unit hydrograph for total streamflow can be simply derived from (4). The second-order transfer function given by (4) can be resolved into two first-order transfer functions (each with two parameters) given by (5) and (6), one of which is for quick flow (sub- or superscript q) and the other is for slow flow (s). Characteristic decay times for the quick and slow flow components are given by (7) and relative throughput volumes by (8) - (9).

\[
Q_k^q = \left[ \frac{\beta_q}{1 + \alpha_q z^{-1}} \right] u_k
\]  

\[
Q_k^s = \left[ \frac{\beta_s}{1 + \alpha_s z^{-1}} \right] u_k
\]  

\[ \tau_{q,s} = -\Delta \ln \left( -\alpha_{q,s} \right) \]  

where \( \Delta \) is the data time interval.

\[ V_{q,s} = \beta_{q,s} / \left( (1 + \alpha_{q,s}) g \right) \]  

\[ g = \beta_q / (1 + \alpha_q) + \beta_s / (1 + \alpha_s) \]

By applying the quick and slow first-order transfer functions independently to excess rainfall \( u_k \) the modelled streamflow can be resolved 'continuously' into its quick and slow flow components. Where the model-fit is good it is reasonable to suppose that the modelled components are good approximations of the actual variations of quick and slow flow.
The four parameters in the second-order transfer function (4) are estimated by a Simple Refined Instrumental Variable (SRIV) method. The one, or two, parameters in the optional rainfall - excess rainfall module (1) - (3) are estimated by trial-and-error facilitated by repeated runs of IHACRES, during which the SRIV technique is applied each time to estimate the transfer function parameters in (4). A 'best' model is selected on the basis mainly of a trade-off between two statistics (the coefficient of determination $R^2$ and an average relative parameter error) but supplemented in detailed studies by close scrutiny of model-fit plots for different periods of calibration and validation (simulation). If this procedure does not lend itself well to the brief description given here it does become clear after some practice with the IHACRES package using set examples. Some 'difficult' cases may require a level of insight which comes from a more detailed knowledge of the modelling approach and frequent use of IHACRES on a wide range of catchment types.

The model is, therefore, simple structurally and requires no physiographic description of the catchment (except basin area). The only data requirements are good time series of rainfall and streamflow, and mean monthly temperatures when modelling over periods which include seasonal variation in evaporative losses. In some cases the rainfall - excess rainfall model given by (1)-(3) is too simple to be able to cope with small runoff events in summer or during periods of wetting-up at the end of summer seasons.

However, as has been shown (Jakeman, Littlewood and Whitehead, 1990;1991; in press: Littlewood and Jakeman, 1991), the crude 'lumped' conceptualisation of the excess rainfall model (1)-(3), coupled with the 'black box' excess rainfall - streamflow model (4) within the IHACRES framework, produces good model-fits with a minimal number of parameters. IHACRES analyses of several catchments in the UK, USA, Australia and New Zealand indicate that for most 'natural' catchments (a) there is an essentially time-invariant linear relationship between excess rainfall and runoff at catchment scale and (b) streamflow can be separated dynamically into dominant quick and slow flow components. Work is in hand, or planned, to test these findings more rigorously over a wider range of catchment types and record lengths, and to assess at catchment scale other rainfall - excess rainfall models (including physics-based excess precipitation models which deal with the complexities of vegetation canopies and snowmelt).

The concept of 'pathways' or 'storages' acting in parallel and/or series to produce streamflow from excess rainfall, and the theory that such a structure can be represented by impulse functions of linear systems, has been around for some time (e.g. Chow, 1964). Advances which have been made in IHACRES are that it provides a robust method for identifying a best configuration of such streamflow-generating elements and for estimating the model parameters (and their covariances), most importantly for quick and slow flow components. Other specialised time series analysis packages for rainfall-runoff modelling, e.g. CAPTAIN (Venn and Day, 1977) and commercially available MicroCAPTAIN (Young and Benner, 1989), sometimes do not identify the slow flow component which IHACRES can identify (see Jakeman, Littlewood and Whitehead (1990) and references cited therein for further details of the differences between the parameter estimation techniques involved). They may not, therefore, always identify the dynamics of the quick flow component as successfully as IHACRES. However, when the observed baseflow is a very small component of the hydrograph, even IHACRES can fail to
identify a configuration having two components (quick and slow) as being better than a configuration having one component. Jakeman, Littlewood and Whitehead (in press) discuss an example of this for the IH Monachyle catchment at Balquhidder.

**IHACRES examples**

Examples of IHACRES model-fits and hydrograph separations are given by Jakeman, Littlewood and Whitehead (1990; 1991; in press) and Littlewood and Jakeman (1991). These include using hourly data for the 1.6 ha Maimai catchment in New Zealand and daily data for the 894 km² Teifi catchment in West Wales. The published examples demonstrate the validity of IHACRES models in simulation mode (i.e. when the only inputs are rainfall, mean monthly temperatures — if used, the model parameters and an initial flow value) on periods of record not used for model calibration.

**Gwy at Plynlimon**

Figure 1 shows the rainfall and model-fit ($R^2 = 0.98$) for a 144 hour period in February 1990 for the 3.9 km² IH Gwy catchment at Plynlimon. Figure 2 shows the corresponding hydrograph separation. The model (units: mm and hours) is

\[
s_t = s_{t-1} + \frac{1}{3} (r_t - s_{t-1})
\]

\[
u_k = \text{const. } r_k s_t
\]

\[
Q_t = \left[ \frac{.3066 - .2862z^{-1}}{1 - 1.3935z^{-1} + .4145z^{-2}} \right] .u_t
\]

\[
= \left[ \frac{.2894}{1 - .4303z^{-1}} \right] .u_t + \left[ \frac{.0171}{1 - .9633z^{-1}} \right] .u_t
\]

Quick slow

from which the quick and slow flow characteristic response times are 1.2 hours and 27 hours respectively, and the relative quick and slow flow volumetric throughputs are .52 and .48 respectively. This example demonstrates that IHACRES is able to identify from short periods of record a unit hydrograph which, when convoluted with effective rainfall, gives a good estimate of the observed hydrograph (cf. Floods Study Report method, (NERC, 1975)).
Teifi at Glan Teifi (062001) and at Llanfair (062002)

Figures 3 and 4 (from Jakeman, Littlewood and Symons 1991) show calibration (R²=0.89) and validation model-fits respectively using data for different periods of about three years for the 894 km² Teifi at Glan Teifi (Surface Water Archive, SWA, catchment 062001) in west Wales. The data employed are daily mean flows from the SWA and daily areal rainfall calculated by the "triangle method" (Jones, 1983) using the catches of between 13 and 17 raingauges in and around the catchment. The model (units: °C, mm.day⁻¹ and cumecs) is

\[
\begin{align*}
  r_t^* &= (1 - t_t/40)r_t \quad (14) \\
  s_t &= s_{t-1} + 1/15 (r_t^* - s_{t-1}) \quad (15) \\
  u_t &= r_t^* s_t \quad (16) \\
  Q_t &= \left[ \frac{2.2289 - 2.1664z^{-1}}{1 - 1.6733z^{-1} + .6793z^{-2}} \right] . u_t \quad (17) \\
  &= \left[ \frac{2.1620}{1 - .6928z^{-1}} \right] . u_t + \left[ \frac{.0668}{1 - .9806z^{-1}} \right] . u_t \quad (18)
\end{align*}
\]

from which the quick and slow flow characteristic response times are 2.7 days and 51 days respectively, and the relative quick and slow flow volumetric throughputs are .67 and .33 respectively.

The parameters in the numerator of a transfer function can be considered to incorporate a scaling factor dependent on catchment area. To test this idea, the above model was applied in simulation mode to the 510 km² sub-catchment of the Teifi gauged at Llanfair (SWA station 062002) but with the numerator parameters scaled by the ratio of the catchment areas. The model applied was, therefore, the same as above but using Llanfair areal rainfall for r_t and (19) as the transfer function.

\[
\begin{align*}
  Q_t &= \left[ \frac{1.2721 - 1.2364z^{-1}}{1 - 1.6733z^{-1} + .6793z^{-2}} \right] . u_t \quad (19)
\end{align*}
\]

The model-fit shown in Fig. 5 indicates that IHACRES may provide a method for transferring information in hydrographs from gauged to ungauged sites on the same river. The physical configuration of the natural drainage network in the Teifi (a long
basin with a roughly uniform pattern of tributaries entering the main river at about right angles) may lend itself to this method of information transfer more than other catchments - this remains to be investigated. Another possibility is that this method could be applied across catchment divides in hydrologically homogenous regions.

It is known, however, that simple scaling of transfer function numerator parameters will not be sufficient for transferring information to ungauged catchments in general - the model-fit in Fig. 5 is not particularly good. The smaller Llanfair catchment can be expected to have shorter quick and slow flow characteristic response times than for the Glan Teifi catchment. However, the simulation shown in Fig. 5 uses the same response times (i.e. the same transfer function denominator parameters) as were determined for Glan Teifi. Future work will seek relationships between response times (and other dynamic response characteristics - DRCs) and physical catchment descriptors (PCDs) to improve information transfer to ungauged catchments (Jakeman et al., in press).

Other catchments

Other SWA catchments to which IHACRES has been applied include the Lune at Caton (072004, 983 km²), Kenwyn at Truro (048005, 19.1 km²), Eden at Temple Sowerby (076005, 616 km²), Colne at Denham (039010, 743 km²) and Ettrick Water at Brokopherig (021017, 37.5 km²). Work is continuing on these and other SWA catchments. Catchments in Australia, USA and New Zealand have been IHACRED. Two of the most recent applications (by AJJ and USGS colleagues) is to a 500 m² hillslope plot in China using 6 minute interval data, and to a 2344 km² basin in the US using daily data.

Ian Littlewood/Tony Jakeman
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References


Bibliography
