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Catchment response and the flood frequency curve

April 1988

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Phase I: A simple hourly simulation model of point rainfall for Southern Britain.

Report to MAFF

Abstract

This paper describes a stochastic rainfall model which has been developed to generate synthetic sequences of hourly rainfalls at a point. The model has been calibrated using up to 30 years of rainfall data for each of five sites in Southern Britain. These rainfall data series were divided into wet and dry spells; analysis of the durations of these spells suggests that they may be represented by exponential and pareto distributions respectively. The total volume of rainfall in wet spells was adequately fitted by a conditional gamma distribution. Random sampling from a beta distribution, defining the average shape of all rainfall profiles, is used in the model to obtain the rainfall profile for a given wet spell. The model has a total of 22 parameters some of which are specific to winter or summer and vary at each site, whilst some are constant through the year and over all of southern Britain. Results obtained from the model compare favourably with observed monthly and annual rainfall totals and with annual maximum frequency distributions of 1, 2, 6, 12, 24 and 48 hours duration at Farnborough in Hampshire.

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Definition of the model parameters

1 T_{ES} time of year for start of summer event season

2 T_{EW} time of year for start of winter event season

3 M_{ES} scale parameter for the duration of summer events

4 M_{EW} scale parameter for the duration of winter events

5 S_E scalar of relationship between event duration and scale parameter for all evebt depths

6 C_E constant of relationship between event duration and scale parameter for all event depths

7 $S_{\gamma S}$ scalar of relationships between event duration and shape parameter for the summer event depths

8 $S_{\gamma W}$ scalar of relationship between event duration and shape parameter for the winter event depths

9 C_γ constant of relationship between event duration and shape parameter for all event depths

10 α_S scale parameter for average summer event profiles

11 β_S shape parameter for average summer event profiles

12 α_W scale parameter for average winter event profiles

13 β_W shape parameter for average winter event profiles

14 SVAR standardised variance of distribution of remaining proportions in event profiles

15 E_ρ exponent of relationships between duration and

16 S_ρ scalar of relationship between duration and

17 T_{DS} time of year for start of winter dry period season

18 T_{DW} time of year for start of winter dry period season

19 a_{DS} scale parameter for the duration of summer dry periods

20 k_{DS} shape parameter for the duration of summer dry periods

21 a_{DW} scale parameter for the duration of winter dry periods

22 k_{DW} shape parameter for the duration of winter dry periods

Other notation

$Bt_{(\alpha_t,\ \beta_t)}$	beta distribution with parameters α and β
d	event duration
$\overline{PR}$	mean proportion of remaining rainfall in any hour
p	total rainfall in an event
t	time, from the start of an event
$\chi^2_{(18,0.05)}$	location at which the upper tail area of the chi squared distribution is 5% given 18 degrees of freedom
T_i	return period of ith ranked annual maximum
ρ	correlation between hourly rain falls within an event

1. Background to study

Traditional approaches to regional flood frequency analysis have involved analysing observed sequences of annual floods from a number of gauging stations each draining a catchment with different physical characteristics. By averaging the observed flood frequency curves within groups of physically similar catchments (such as small, steep and wet catchments or large, flat and dry catchments) it is possible to identify the flood frequency distribution which results from various combinations of physical characteristics. However, it is difficult to determine how the individual characteristics contribute to the shape of the flood frequency curve.

An alternative approach is to simulate the response of a catchment with a predetermined set of physical characteristics using a computer model. For modelling purposes the hydrological response system can be sub-divided into two parts: (1) a meteorological input, viewed as a stochastic variable, and (2) a rainfall-runoff transformation process, which is essentially deterministic. The rainfall model is used to generate long sequences of synthetic rainfall totals which retain the statistical properties of observed sequences. These rainfall sequences are transformed into long synthetic records of river flows using the rainfall-runoff model. The flow series can then be analysed using conventional statistical methods. Different climatic and meteorological conditions can be simulated by varying the parameters of the rainfall model. By varying the parameters of the rainfall-runoff model, different physical conditions of the catchment can be simulated. In this way the effects on the hydrological response of changes to a single catchment characteristic, such as soil capacity or slope, can be modelled.

A particular application of this methodology seeks to identify the relative importance of different physical characteristics of a drainage basin in shaping its flood frequency curve. Specifically, rainfall model has been developed and used to generate 1000 years of hourly synthetic rainfall totals. Annual flood peaks have been extracted from the flow sequence output from the rainfall-runoff model and these have been used to define a flood frequency curve. In successive simulation runs different values for the parameters which control the modelling of soil moisture storage and runoff production have been used to investigate the influence of catchment morphology and soil properties. The effects of lake and floodplain storage on the flood frequency curve have also being studied by routing the synthetic flow sequence through an appropriate model. In this way the relative importance of a variety of catchment characteristics in shaping the flood frequency curve can be evaluated.

2. Model specification

This report describes the development of a computer model which is capable of generating unlimited sequences of synthetic hourly rainfall totals at a point which preserve the statistics of observed rainfall series. The main statistics to be preserved are the depth frequency relationships for 1, 2, 4, 6, 12, 24 and 48 hour durations and the mean and standard deviations of monthly and annual rainfall totals.

3. Rainfall characterisation

The way in which rainfall is characterised in a rainfall model depends upon the structure of the rainfall data series being modelled. The structure of a rainfall series depends, in turn, upon the time interval over which the total rainfall depth is measured, or has been aggregated. If the interval is very short, of the order of a second or less, the rainfall will depend on the number of raindrops and their size and thus the rate is likely to vary considerable between intervals. Rainfall rates recorded on a chart by a tilting syphon raingauge, which has a resolution of around one minute, tend to exhibit some serial dependence. As rainfall depths are accumulated over intervals of longer duration, from minutes to hours, the degree of dependence between the ordinates changes. If the interval is much less than the average length of an event the dependence is high due to serial correlation within an event. However, as the interval increases in length it will contain some complete events and some partial events, therefore dependence is likely to decrease. Hence daily rainfall totals exhibit less serial dependence than hourly falls. Each rainfall total may be considered as a discrete random observation, with some small serial dependence over several previous values. For example, the probability that a certain depth of rain will fall tomorrow is a random variable whose likely value is conditional upon how wet it was today and, to a lesser extent how wet it was yesterday, and the day before, and so on. Marcov chains are used for this type of modelling (see for example Todorovic and Woolhiser, 1975 and Haan et al, 1976). Unfortunately when extended to shorter time periods, such as an hour, because the rainfall depth in any hour is conditional on falls over many previous hours, a large number of model parameters need to be optimised (see for example Pettison, 1967). However, as the rainfall depths are accumulated over still longer durations the degree of dependence may increase. This occurrs when the interval is long enough to contain a large sample of different magnitude events. For example, annual rainfall totals may exhibit smooth trends and cyclic fluctuations in harness with sun-spot activity or long-term climatic change. This type of behaviour can be modelled by a polynomial curve or by harmonic analysis.

Ideally a model would be formulated in continuous time and would be appropriate at all levels of aggreation. Model parameters could be fixed using data of various aggregations including minute, hourly, daily and yearly. It would then be possible to partially calibrate the model using any available local data. This methodology, although conceptually reasonable, is very difficult to apply in practice. Consequently rainfall models tend to be based on the structure exhibited by one chosen rainfall duration. The volumes over other intervals are then obtained by aggregating or disaggregating these totals.

Hourly rainfall poses particular problems in modelling, since successive values exhibit properties of random variation but with some serial dependence. Many models are a compromise between the two extremes of independent random variables and deterministic functions. Small clusters, or groups, of rainfall values may be considered to have some deterministic structure, such as those making up a rainfall event, burst or cell, with these groups treated as random variables. Cox & Isham (1987) developed a model based on the concept that the fundamental rainfall unit is a cell of variable duration but with constant intensity. The cells start at variable times and thus may overlap such that the total rainfall profile may exhibit the characteristic castlated appearance of observed hyetographs. A variable number of cells makes up a storm, the length of which is also modelled as a random variable. Dry periods are not modelled explicitly, but make up the spaces between rainfall cells. However, to model the variability observed in profiles of hourly rainfall data from Denver they introduced an element of random noise. Although the parameters have a physical interpretation, such as the duration of a rain cell, they express themselves indirectly in the observed rainfall hyetograph; any given rainfall pattern could have been produced by several different combinations of parameters. Futhermore the model had only been applied to data from summer convective rainfall in the USA and may not be applicable to British meterological conditions.

An alternative model structure is to generate alternate wet and dry periods of random duration. The wet periods are assigned a random total depth, which is conditional on the duration, and is distributed through the event using a fixed profile. Beven (1987) made the most basic assumption that the rainfall was evenly distributed throughout the storm, thus yielding a rectangular profile, whilst a triangular profile was adopted for all storms by Grayman and Eagleson (1966). To make the concept of a triangular profile more realistic, observed rainfall sequences investigated by Marien & Vandewiele (1986) were divided up into storm profiles, or parts of storm profiles, which took the form of a triangle. This makes the assumption of a triangular profile a more reasonable assumption by definition. Acreman (1987) treated the profile as a further random element, in which individual storm profiles are analogous to sample histograms from a population density function which describes the average of all profiles. In this model the total storm rainfall is divided into blocks of 0.5 mm which are distributed within the profile with their probability of occurrence at any time during the event governed by a normal distribution. This is particularly appropriate for data collected from a tipping bucket raingauge which yields rainfall depths in discrete amounts depending on the size of the bucket.

Different models try to characterise rainfall in different ways. Provided that the model generates rainfall values which preserve the statistical characteristics of the observed data sequence required by the particular application, the precise structure is perhaps not important.

4. Data

Prior to the research for the Flood Studies Report (NERC, 1975), long

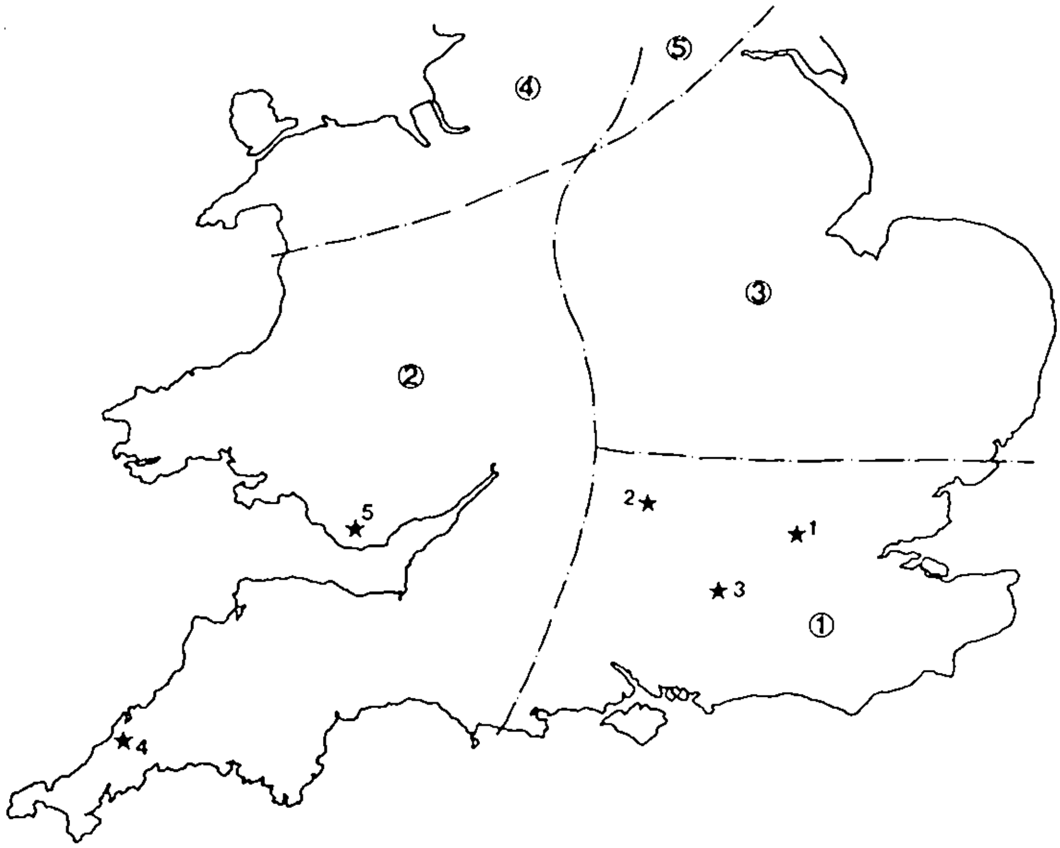

Fig. 4.1 Precipitation regions of England and Wales (after Wigley et al, 1984).

sequences of short duration rainfall totals were not widely available in a convenient form for computational analysis. In 1973 the Meteorological Office and the Oxford University Nuclear Physics Laboratory collaborated on a project to digitise recording rainfall charts automatically. For this work they used a flying spot scanner, known as the Precision Encoder and Pattern Recognition (PEPR) machine. More than one million hyetograms were selected by the Meteorological Office for digitising (Folland and Colgate, 1978). The resulting data are point rainfall totals at a resolution of 1/100th mm at one minute intervals. Data from five sites in Southern Britain were obtained by the Institute of Hydrology in 1975. (Table 4.1) Data are not complete for the whole period of each record, but the gaps form a small proportion of the total record length. The locations of the sites are shown in Figure 4.1 together with the precipitation regions of England and Wales produced by the Climatic Research Unit (Wigley et al, 1984). For the purposes of this study the data were aggregated to produce hourly falls with a resolution of 1/10th mm.

Table 4.1 Summary of rainfall data used in this study

No.	Name	Period of record	Grid Ref.	Met. Off. No.
1	Hampstead	1941-1975	5262 1863	246690
2	Abingdon	1944-1964	4482 1990	260990
		1964-1975	4479 1991	260991
3	Farnborough	1941-1971	4867 1544	271432
4	St. Mawgan	1956-1967	1873 642	383478
5	Rhoose	1958-1975	3066 1678	492325

Table 6.1 Basic statistics of rainfall data

		Hampstead		Abingdon		Farnborough		St. Mawgan		Roose	
		win	sum	win	sum	win	sum	win	sum	win	sum
event depth	n	4789	4284	5340	4542	5140	4473	2396	2363	2810	3104
	μ	2.00	2.22	1.81	1.96	2.09	2.03	2.62	2.28	2.55	2.44
	σ	3.90	5.20	3.53	4.12	4.21	4.41	5.01	5.02	4.43	4.82
	g	4.08	10.89	4.04	5.06	4.64	5.86	3.74	8.59	3.30	4.52
event duration	μ	4.71	4.21	4.25	3.84	4.34	3.75	4.91	4.53	4.98	4.44
	σ	4.39	3.87	3.98	3.35	4.16	3.31	4.57	4.10	4.46	3.94
	g	2.74	3.09	2.89	2.44	2.83	2.81	2.15	2.10	2.07	2.34
dry period duration	n	3184	5659	3794	6055	3700	5881	2764	1973	3994	1872
	μ	20.43	25.35	20.19	25.35	20.03	25.69	14.63	20.14	19.17	23.65
	σ	38.08	54.99	36.74	55.92	36.83	57.07	36.60	42.47	46.09	51.89
	g	4.76	5.16	4.57	5.64	4.33	5.04	7.16	5.06	6.05	5.89

n = number of periods
μ = sample estimate of mean
σ = sample estimate of standard deviation
g = sample estimate of skewness

5. Model definitions

This model treats point hourly rainfall sequences as comprising of alternating wet periods, termed events, when some rainfall has been recorded in each hour and dry periods consisting of one or more consecutive hours when no rain has been recorded. Events are characterised by their duration, total rainfall depth and profile, ie how the depth is distributed in time over the duration of the event. Dry periods are defined in terms of their duration alone. The definition of a rainfall event allows for considerable subjectivity.

The simplest definition is that adopted by Yen and Chow (1980) that a rainstorm is a period of continuous non-zero rainfalls in each time interval. Thus one dry interval defines the division between two events. However, two or more bursts of rainfall separated only by one dry hour are often considered as one single event. Thus some rainfall models use different criteria to define the end of an event. For example, when working with rainfalls of one minute duration Kidd and Packman (1981) considered that an event had ended only when the intensity of rainfall fell below 1 mm hr for 15 minutes. Criteria based on meteorological independence would seem intuitively sensible, but may depend on the synoptic situation. For example, during frontal rainfall a longer dry period may be required to designate events as indepedent than during anticyclonic, showery weather. From a catchment response standpoint, the independence threshold may be chosen to be the critical duration of rainfall to which the catchment responds; small urbanised catchments would require a short dry period threshold, whilst a larger catchment underlain by chalk would be more suited to a long threshold. In a study by Roa (1974) the threshold dry period length was chosen by identifying the minimum number of dry hours required between two wet hours such that there was not significant serial correlation. Thus the critical lag for hourly data was found to be 15 hours. However, rainfall is usually low in the first and last hours of an event, therefore this statistic simply reflected the high dependence of these hours and does not indicate the dependence between successive events as a whole. A more meaningful statistic would be the partial correlation calculated between all the data in each pair of events. A single index of the information held in a whole event is, however, not obvious. Restrepo and Eagleson (1982) utilised the idea that if the arrival times of independent rainfall events can be modelled by a Poisson process, the dry periods between events should be exponentially distributed. They then chose the critical dry period duration such that the exponential hypothesis was best satisfied. However, the independence assumption in the Poisson model relates to arrival rates and not to the dependence of storm duration or depths.

Provided that realistic rainfall sequences can be generated the separation criteria are unimportant i.e. events do not need to be independent provided that any dependence is built into the model. In the model developed here, events are defined as continuous sequences of wet hours; thus a single dry hour is considered as the start of a dry, inter-event period. In this way all dry and wet hours are modelled explicitly.

6. Basic rainfall statistics

For each of the five sites, the observed rainfall sequences were divided into wet and dry periods. A further sub-division was undertaken on the basis of season (Tables 7.1 and 10.1). Various other sub-divisions were considered but rejected. Table 6.1 gives some summary statistics derived from the available data. The statistics appear to be intuitively realistic. For example, at all sites dry periods are, on average, longer in the summer than the winter and, correspondingly, wet periods are shorter and the depth of rainfall in an event is more variable in the summer than in the winter. With regard to regional variations, events are, on average, of longer duration in the west (region 2), being in the range 4.44 to 4.98 hours, than those in the east (region 1) which range from 3.75 to 4.71 hours. Correspondingly, the average length of a dry spell is shorter in the west. In both regions the range is quite small, suggesting some regional homogeneity. Average event depths are higher in the west, 2.28 - 2.62 mm, against 1.81-2.22 mm in the east.

The wet and dry periods were examined for serial correlation. It was speculated that perhaps a long dry spell would usually be followed by an event with a large rainfall depth or high average intensity. The matrices in Tables 6.2 and 6.3 resulted from the analysis of the data from Farnborough. All lag-one correlations are close to zero, but, given the skewed distribution of the data, it is not easy to assess whether or not they are significantly different from zero. The highest correlations are 0.084 and -0.079. The former suggests that during the summer there is a tendancy for the longer dry spells to be followed by events with larger rainfall depth and shorter dry periods by events with smaller depth. The latter negative correlation implies that winter events with a large rainfall depth tend to be followed by short dry periods and vica versa. All lag-one correlations for event depths are less than 0.03. Intuitively, it was felt that large rainfall events sometimes cluster, ie as a front passes there may be several heavy rainfall storms in succession. Further analysis provided no justification for this in the observed data. Consequently, due to the near zero values, the dry and wet periods were assumed to be independent of each other in the model.

A further aspect of correlation is that between the depth, duration and intensity within each event. Table 6.3 shows that correlation between depth and duration is highly significant, as expected, with higher values in the winter. Again these results seem intuitively reasonable since, for a given duration event, greater variability in possible depths would be expected in the summer. This correlation structure is taken into account in the model by making the possible values for the depth conditional on the duration.

7. Modelling event durations

The duration of an event is an integer, *d*, defining a continuous sequence wet hours bounded on either side by at least one dry hour. To illustrate how the

Table 6.2 Lag-one correlation coefficients for raifnall characteristics at Farnborough

			ith wet or dry period							
			wet duriation		depth		avg intensity		dry duration	
			win	sum	win	sum	win	sum	win	sum
wet	dur	i+1th	0.023	0.034	-0.021	0.011	-0.035	0.017	0.005	0.059
wet	dep	i+1th	0.004	0.029	-0.026	0.028	-0.032	0.008	0.014	0.084
wet	int	i+1th	0.029	0.012	-0.011	0.027	0.009	0.026	0.007	-0.013
dry	dur	i+1th	-0.058	-0.033	-0.079	-0.039	0.003	0.014	0.012	0.019

Table 6.3 Cross-correlation coefficients between depth, duration and sverage intensity within individual events

Cross-correlations

	duration		depth		avg. intensity	
	win	sum	win	sum	win	sum
duration	1.000	1.000				
depth	0.761	0.683	1.000	1.000		
avg intensity	0.254	0.268	0.679	0.677	1.000	1.000

Table 7.1 Definition of summer and winter seasons for event durations

	summer	winter
Hampstead	April-September	October-March
Abingdon	April-September	October-March
Farnborough	April-Septmeber	October-March
St. Mawgan	March-September	October-February
Rhoose	March-September	October-February

typical duration of an observed event may vary through the year, Figure 7.1 shows the variation in mean durations, M_E, at Rhoose for each month in which the event starts. A broadly similar pattern is exhibited at each of the other four sites investigated, with higher mean event durations occurring in the winter than summer. Although the mean changes relatively smoothly through the year, a two season model was felt to be adequate. The optimum seasonal division varied between the sites as shown in Table 7.1. The first parameter of the model is the time of year for the start of the summer events season, T_{ES}, whilst the second is the start of the winter event season, T_{EW}.

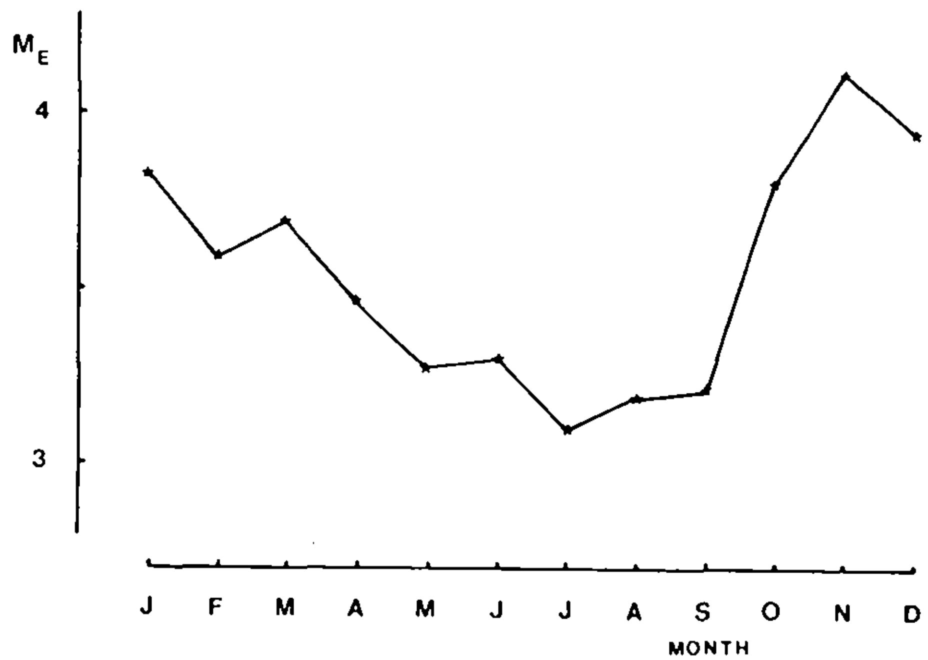

Fig. 7.1 Monthly variation in mean event durations at Rhoose.

The histogram of summer event durations at Abingdon is shown in Figure 7.2. Clearly long events are less frequent than short events with the probability of occurrence reducing in a systematic fashion with increasing duration. An obvious candidate to model this behaviour is the exponential distribution, which has a single parameter, M_E, whose density function is

$$F(d) = 1 - e^{(-d/M_E)} \qquad (7.1)$$

This distribution has been used by Grayman and Eagleson (1969) to model event durations in Massachusetts and by Beven (1987) in Wales. The exponential distribution has a fixed skewness of 2.0. Sample estimates of skewness for the event durations at the five sites all exceed, though are close to, this value. Alternatives to the single parameter exponential distribution include the generalised pareto distribution

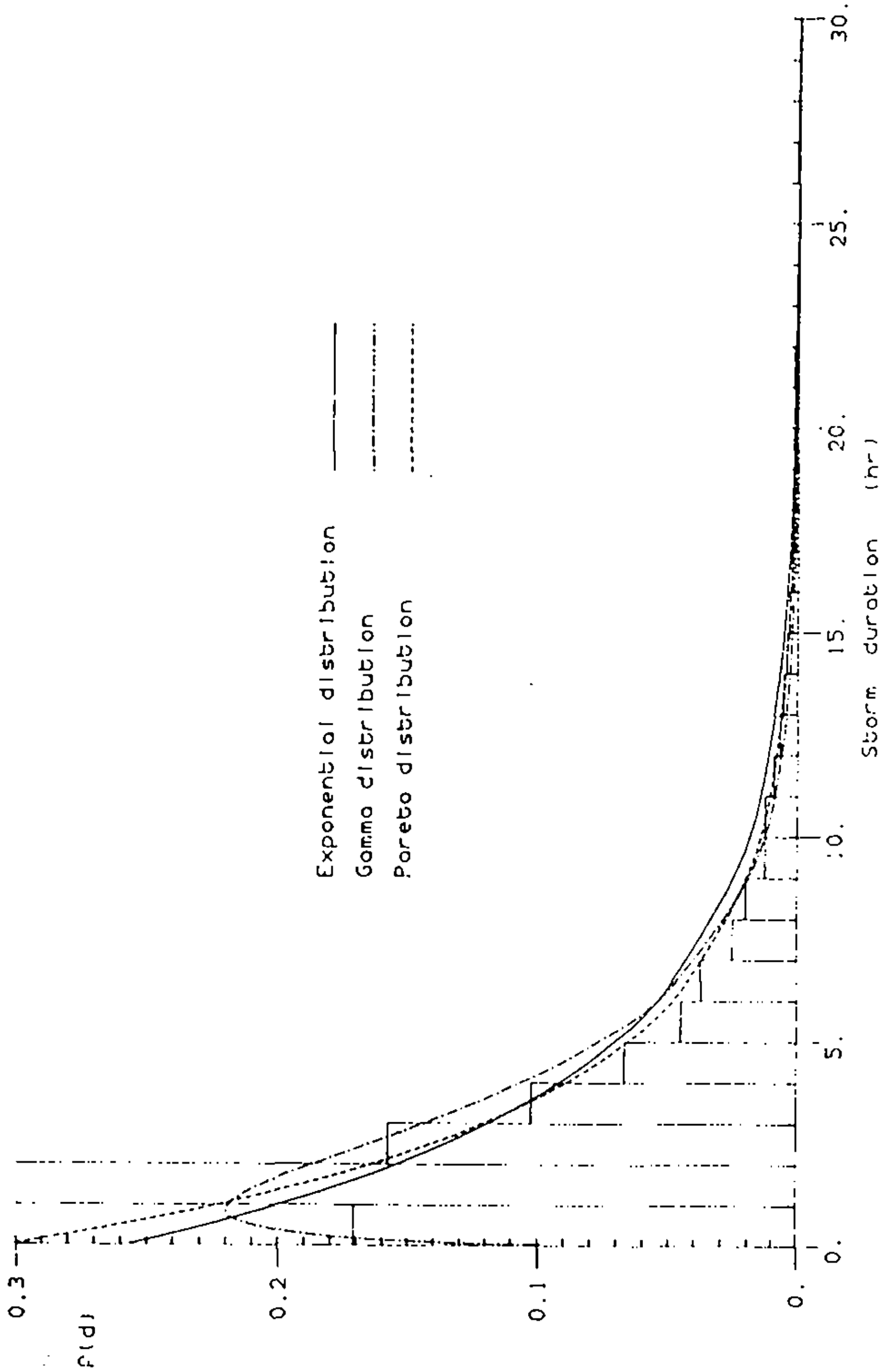

Fig. 7.2 Histogram of summer event durations at Abingdon together with fitted exponential, gamma and generalised pareto distributions.

$$F(d) = \frac{1}{a}\left\{1 - \frac{Kd}{a}\right\}^{1/k-1} \tag{7.2}$$

This distribution has been used to model the frequency distributions of peaks-over-a-threshold rainfall series n the Netherlands (van Montfort & Witter, 1986). The observed histogram suggests that the modal value of the event durations is two hours, whereas the modal value of both the exponential and generalised pareto distributions is zero, or, in this particular discrete application, is one. The gamma distribution

$$F(d) = \frac{d}{\xi}^{\gamma-1} \frac{e^{(-d/\xi)}}{\xi\,\Gamma(\gamma)} \tag{7.3}$$

When parameter γ is greater than one the modal value is $\xi(C-1)$. Both the gamma and generalised pareto distributions may possess higher values of skewness.

Because only positive integer values are acceptable for event durations a *discrete distribution is required for modelling. To achieve this numbers* generated by the model from a continuous distribution are rounded up to the nearest whole number adding, on average, about 0.5 to each observation. Consequently 0.5 was subtracted from each observed event duration prior to fitting each distribution. Probability density functions for each of the three distributions described above, with parameters estimated by the method of maximum likelihood, are shown in Figure 7.2 and the corresponding parameter values are given in Table 7.2. The observed data were divided into 20 classes and compared with the expected number given by integrating the density function between the class limits. These observed and expected values are shown in Table 7.3 for winter events at St. Mawgan. The χ^2 goodness of fit statistic

$$\chi^2 = \sum_{i=1}^{n} (EXD_i - OBS_i)^2 / EXD_i \tag{7.4}$$

where OBS_i is the number opf events observed in the ith class and EXP_i is the number that would be expected if the data were drawn from the distribution under examination. This statistic can be used to test whether the observed data can be assumed to be a random sample from that distribution. χ^2 values for each gauge site are given in Table 7.4. Given that $\chi^2_{(17,0.05)}$ is approximately 27.5, it is unlikely that the sample data came from any of these distributions. A very high percentage of the contribution to the χ^2 statistics comes from the first two classes. The form of the gamma distribution is potentially more appropriate for modelling durations of one and two hours. However, the optimum parameters imply a modal value of one, although the predicted frequency of two hour events is very similar. These results suggest that the exponential and generalised pareto distributions, having approximately equal values for the χ^2 statisitic, are better suited to modelling the tail behaviour than the gamma distribution.

Table 7.2. Parameter values for the exponential, generlaised pareto and gamma distributions fitted to event durations by the method of maximum likelihood

		exponential	gamma		generalised pareto	
		M_E	ξ	γ	a	k
Hampstead	sum	3.710	1.298	2.859	3.609	-0.027
	win	4.215	1.240	3.400	4.089	-0.030
Abingdon	sum	3.336	1.367	2.441	3.323	-0.004
	win	3.753	1.281	2.929	3.591	-0.043
Farnborough	sum	3.752	1.363	2.385	3.214	-0.016
	win	3.844	1.224	3.142	3.606	-0.062
St. Mawgan	sum	4.015	1.237	3.261	3.941	-0.018
	win	4.369	1.158	3.805	4.224	-0.033
Rhoose	sum	3.935	1.289	3.061	3.923	-0.003
	win	4.415	1.239	3.617	4.442	0.006

Table 7.3 Observed and predicted frequencies of winter event durations for St. Mawgan

Class	Observed	Exponential	Gamma	Generalised Pareto
1	387	486.4	411.9	500.8
2	595	387.4	393.0	393.2
3	299	309.0	328.5	309.2
4	217	246.2	266.6	243.8
5	169	196.3	213.4	192.5
6	125	156.4	269.4	152.3
7	95	124.7	133.8	120.7
8	102	99.4	105.2	95.9
9	86	79.2	82.5	76.3
10	50	63.1	64.6	60.9
11	57	50.3	50.5	48.6
12	47	40.1	39.4	38.9
13	29	32.0	30.7	31.2
14	28.	25.5	23.9	25.0
15	24	20.3	18.6	20.1
16	19	16.2	14.4	16.2
17	14	12.9	11.2	13.1
18-20	24	25.0	20.7	26.1
21-25	16	17.4	13.0	19.5
26-	13	8.2	5.0	11.6
	χ^2	162.3	174.0	155.5

Table 7.4. χ^2 statistics for testing the goodness of fit of various distributions to event durations

		exponential	gamma	generalised pareto
Hampstead	sum	292.4	337.1	279.9
	win	330.9	332.8	320.7
Abingdon	sum	481.0	451.2	479.6
	win	522.2	585.9	489.3
Farnborough	sum	404.8	388.1	401.8
	win	412.0	472.9	367.4
St. Mawgan	sum	171.5	169.5	169.0
	win	162.3	174.0	155.5
Rhoose	sum	263.4	241.9	263.5
	win	188.8	179.5	189.4

Table 7.5. Likelihood ratio test results

		maximum log likelihoods			
		exp. (A)	pareto (B)	2. (A-B)	decision Ho
Hampstead	win	-11678.65	-11675.88	5.54	reject
	sum	- 9900.01	- 9897.62	4.78	reject
Abingdon	win	-12402.70	-12395.96	13.48	reject
	sum	-10014.64	-10014.59	0.10	accept
Farnborough	win	-12058.97	-12049.07	19.80	reject
	sum	9747.35	- 9746.91	0.88	accept
St. Mawgan	win	- 5949.92	- 5948.32	3.20	accept
	sum	5658.60	- 5658.29	0.62	accept
Rhoose	win	- 7024.20	- 7024.18	0.04	accept
	sum	- 7364.01	- 7364.01	0.00	accept

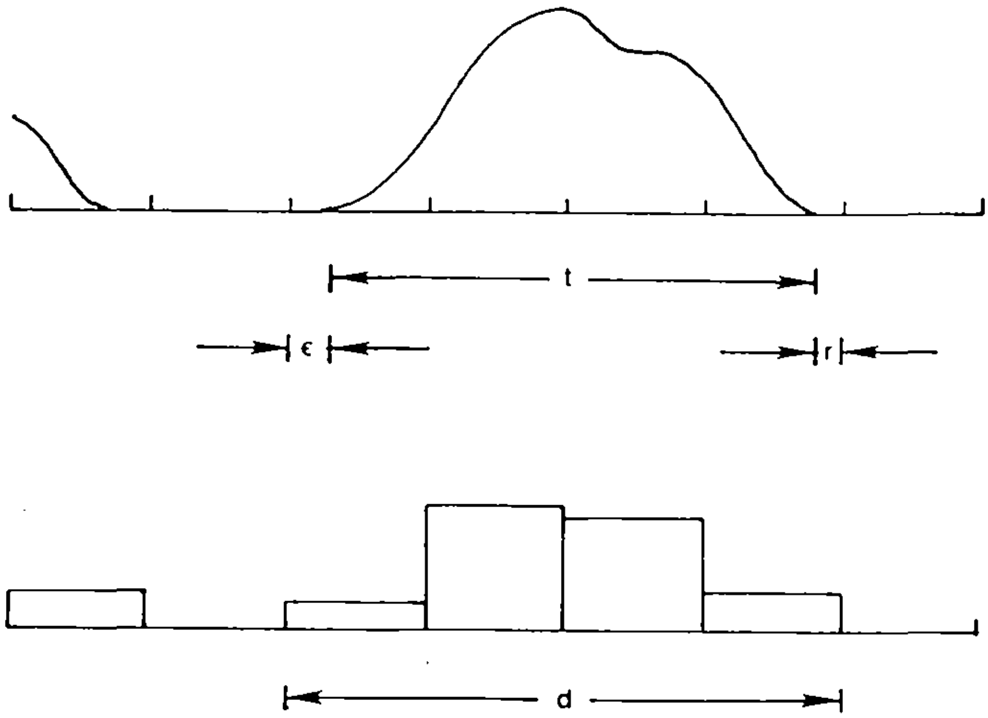

Fig. 7.3 Effect of aggregating rainfall into hourly depths.

Since the exponential distribution is a special case of the generalised pareto distribution, an alternative test of their relative goodness-of-fit is provided by the likelihood ratio statistic, LR

$$LR = -2 \log (LL1 / LL2) \tag{7.5}$$

where LL1 and LL2 are the logarithms of the maximum likelihood for the exponential and generalised pareto distributions respectively. LR is distributed as χ^2 with degrees of freedom equal to the the difference in the number of parameters estimated, in this case 1. Table 7.5 gives the maximum log-likelihoods for the two distributions at the five sites together with the critical χ^2 values. Using this test, only for winter events at Abingdon and Farnborough is the goodness of fit of the generalised pareto distributions significantly better that that of the exponential distribution. Given the large sample of data, using the modified likelihood-ratio test suggested by Hosking and Wallis (1987) where

$$LR' = 1 - 67 / 66 (6n) LR \tag{7.6}$$

has no effect on the results. A potential drawback with the generalised pareto distribution is that when $k > 0$ the distribution has an upper bound at a/k. However, only winter events at Rhoose exhibited a positive k value and this was not significantly different from zero.

The major deficiency in the exponential model is that the frequency of one hour events is over-estimated and that of two hour events is underestimated (infact the frequencies are reversed). Consequently the modal value is not modelled correctly. The form of the observed histogram is influenced by the aggregation of the rainfall into hourly depths. Rain may, or may not, have been falling for all of each hour. Figure 7.3 demonstrates this effect. If we assume that rainfall is continuous for time, t, once it has started, the observed duration of the event, d, will be

$$d = \epsilon + t + r \qquad (7.7)$$

where ϵ is the time from the start of the present clock hour and r is the time from the end of the event until the beginning of the next clock hour. Rather than modelling the duration of an event by a distribution fitted directly to the observed data, it can be modelled as a combination of processes involving the *true* duration of events that would be observed in continuous time, combined with a uniform distribution, giving the discrete density function

$$\begin{aligned} f(d) &= 1 - M_E(1 - e^{-1/M_E}) && d=1 \\ &= M_E\,(e^{1/M_E} - 1)^2\, e^{-d/M_E} && d \geq 2 \end{aligned} \qquad (7.8)$$

Table 7.6 shows the observed number of events in each duration class together with the number predicted using Equation 7.8. Although the fit is reasonably good, this model was out-performed by the use of the exponential distribution when modelling the depth-duration-frequency relationships at Farnborough (section 12). The distribution of depths of one hour duration events is very restricted. At Farnborough, 97% of winter events contained only 0.5 or 1.0 mm. Furthermore, 90% of two hour winter events were of 0.5 or 1.0 mm. These statistics suggest that, for the overall model performance, it is not critical to differentiate between one and two hour events; it is sufficient to ensure that their combined frequency is modelled adequately.

In conclusion, the exponential distribution was felt to be adequate to describe the event durations at all five sites for both seasons. Consequently the mean duration over the two season is sufficient to define the distribution of all events. The third and fourth parameters of the model are the estimates of the mean summer event duration and the mean winter event duration, M_{ES} and M_{EW}.

Table 7.6 Observed frequencies of winter event durations at Farnborough compared with those predicted using the exponential/uniform distribution.

Class	Observed	Exponential/Uniform
1	866	659.8
2	1363	1098.7
3	807	829.3
4	492	625.9
5	369	472.4
6	260	356.6
7	213	269.1
8	158	203.1
9	107	153.3
10	95	115.7
11	85	87.3
12	54	65.9
13	60	49.8
14	46	37.6
15	34	28.3
16	24	21.4
17	17	16.2
18-20	38	28.3
21-25	29	16.1
26-	23	5.2

8. Modelling event depths

Once the duration of an event has been derived, the second characteristic of the event which requires modelling is the total depth of rainfall, p. Total rainfall depth can be treated as a random variable and modelled by fitting a statistical distribution to the sample of observed depths. Because of the strong correlation evident between event depths and durations, the model takes the distribution of depths to be conditional upon the duration. The same methodology was followed by Grayman and Eagleson (1976). Separately for the two seasons, the events were divided into classes according to their duration, 1, 2, 3, ... 11, 12, 13-14, 15-18 and 19- hours. A histogram of event depths was drawn up for each of these duration classes. Two possible candidates for modelling these data are the gamma distribution (4) and the lognormal distribution

$$p = f(d) = \frac{1}{d\,\delta_L (2\pi)^{1/2}} e^{\frac{-\left[\log(d/\mu_2)\right]^2}{2\,\delta_L^2}} \qquad (8.1)$$

where μ_L and δ_L are the mean and standard deviation of the natural logarithms of *d* respectively. These two distributions were fitted to the sample data set by the method of maximum likelihood. An example is shown graphically in Figure 8.1. Table 8.1 gives the observed number in each interval of the histogram of 7 hour events at Hampstead. Also given is the number predicted for each interval by the fitted gamma and lognormal distributions. It can be seen that these data are well fitted by the gamma distribution. This distribution was superior in its description of the data for all durations.

Although the gamma distribution fitted well the rainfall depth data for each duration, the parameters fitted were different in each case. Figure 8.2 shows the parameter, ξ, plotted against event duration, together with a fitted curve. A straight line was considered to be sufficient to describe the variation in ξ with event duration for both seasons and having the form

$$\hat{\xi} = S_E\, d \qquad (8.2)$$

with S_E = 0.17 for Farnborough. The fifth parameter of the model, S_E, is the slope of this line. A straight line function leads to under-prediction of ξ for events of 1-4 hours duration. Therefore a constant value, C_E, of 0.65 was adopted for Farnborough for durations of four hours or less. C_E is the sixth parameter of the model.

Figure 8.3 shows an equivalent graph for the parameter γ. A curvilinear relationship was fitted by eye with a different scalar for winter

$$\gamma_W = S_{\gamma W}^{d} - C_\gamma \qquad (8.3)$$

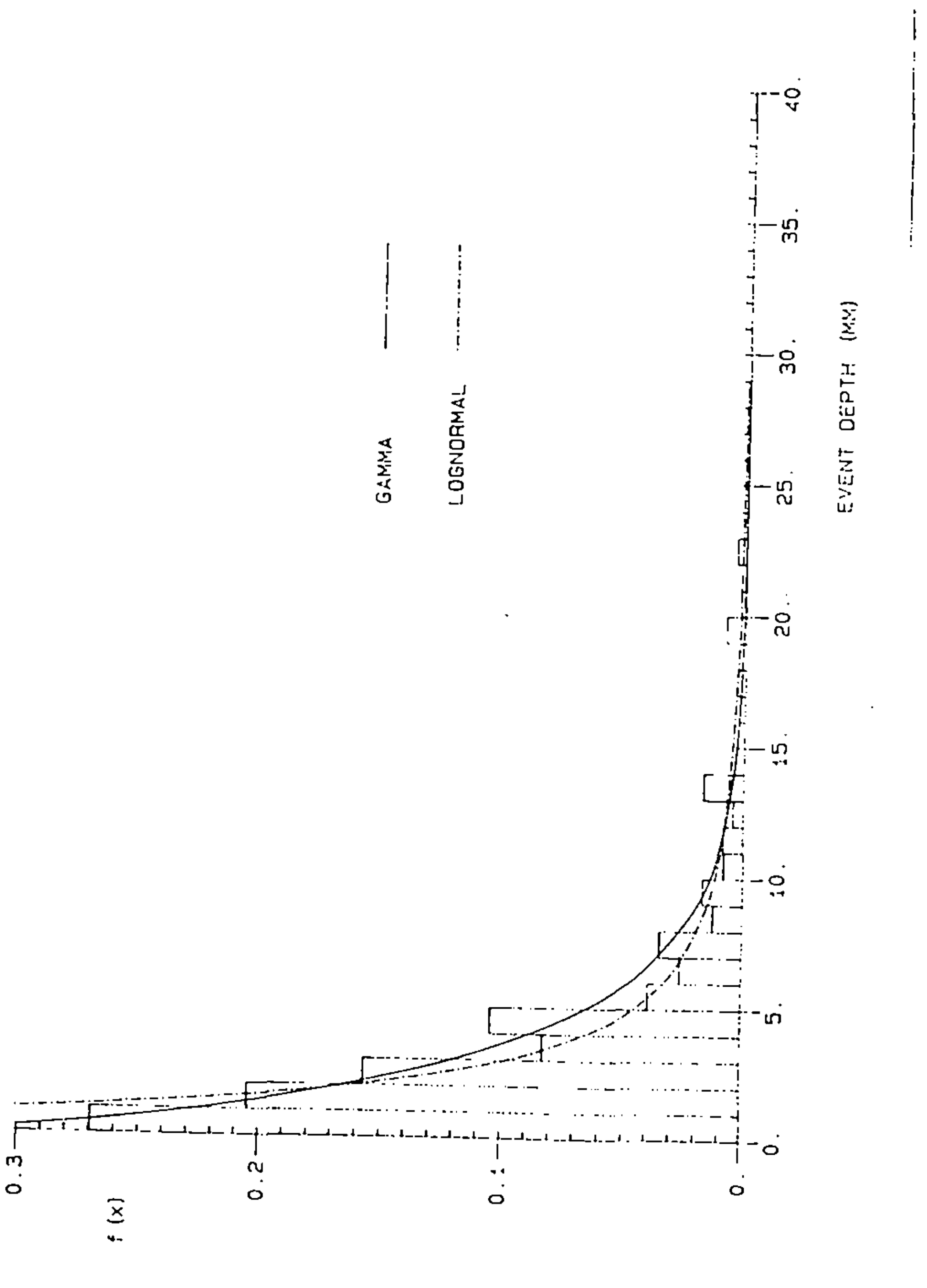

Fig. 8.1 Histogram of summer event depths of 6 hours duration at Farnborough together with fitted gamma and lognormal distributions.

Table 8.1 Observed and predicted frequencies of winter event depths of 7 hours duration at Hampstead

Class	Observed	Gamma	Log normal
- 0.5	33	24.63	23.39
- 1.0	23	28.47	38.07
- 1.5	22	26.65	31.78
- 2.0	22	23.74	24.74
- 2.5	17	20.66	19.21
- 3.0	22	17.73	15.08
- 3.5	17	15.08	12.00
- 4.0	10	12.74	9.68
- 4.5	13	10.71	7.90
- 5.0	13	8.97	6.52
- 5.5	9	7.50	5.44
- 6.0	5	6.24	4.58
- 6.5	5	5.19	3.88
- 7.0	2	4.31	3.32
- 7.5	3	3.57	2.85
- 8.0	2	2.96	2.47
- 9.0	7	4.46	4.02
- 10.0	2	3.04	3.11
- 12.0	4	3.46	4.38
12.1 -	2	2.89	10.61
	χ^2 = 12.07		

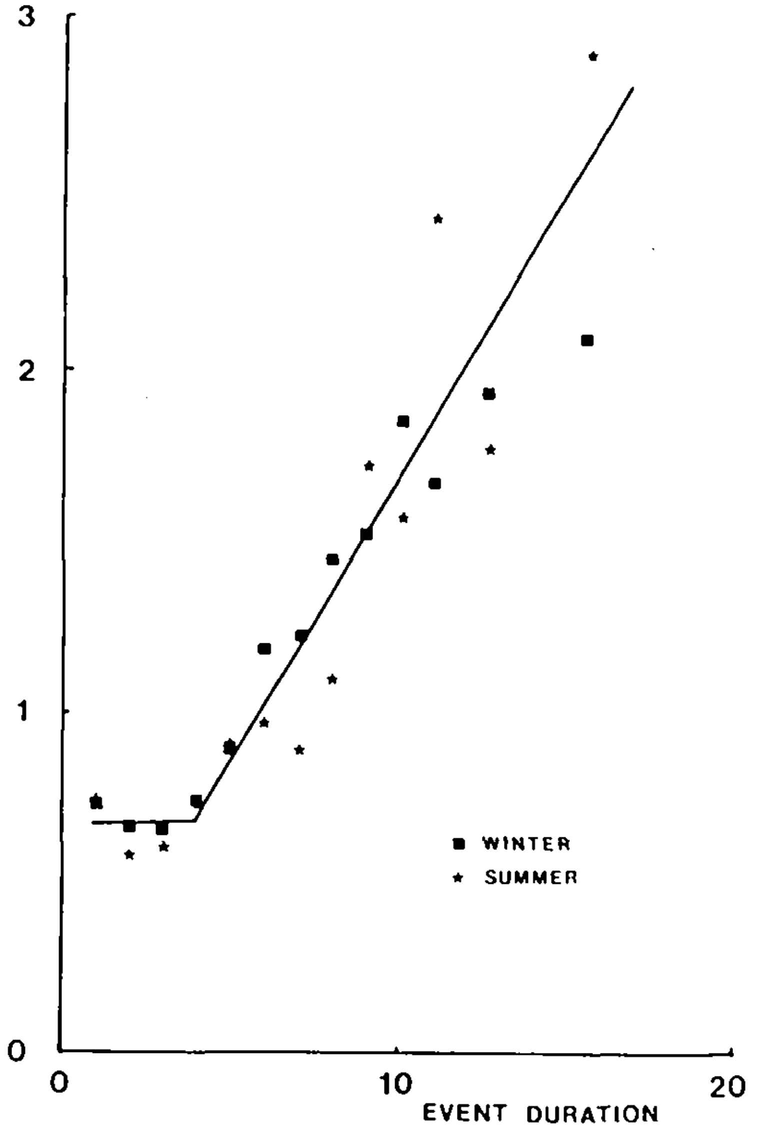

Fig. 8.2. Relationship between event durations and scale parameter of gamma distribution defining associated event depths.

and summer

$$\gamma_S = S_{\gamma S}{}^d - C_{\gamma S} \qquad (8.4)$$

The two scalars, $S_{\gamma W}$ and $S_{\gamma S}$ constitute the seventh and eigth model parameters and the constant term, C_γ ($C_{\gamma S}$ and $C_{\gamma W}$ 0.75 both for Farnborough), is the nineth. The model generates an event depth as a random number from a gamma distribution whose parameters are given as a function of the event duration which has been defined previously.

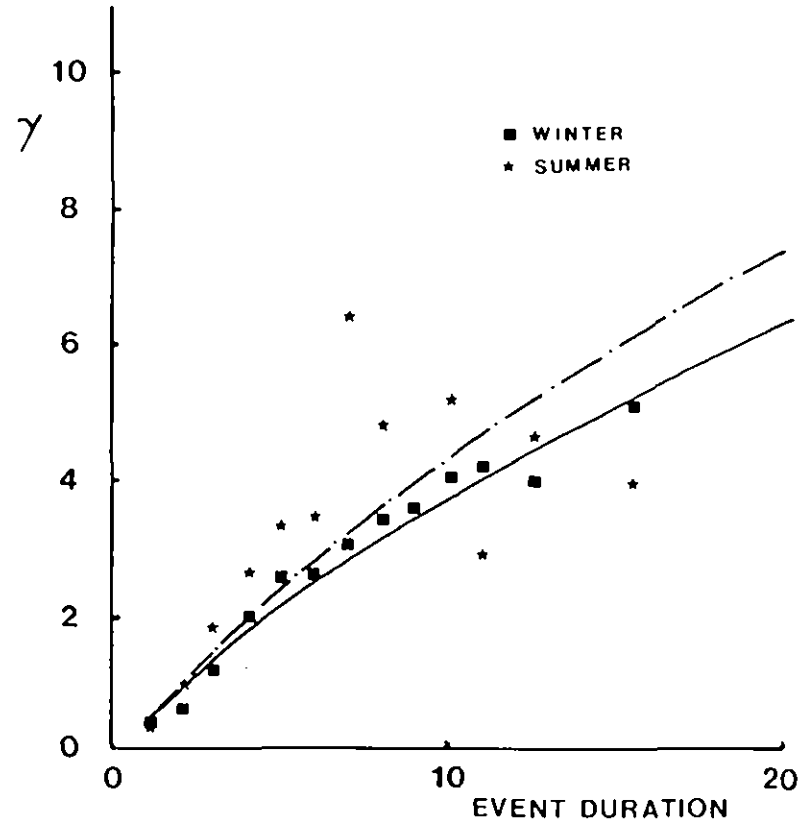

Fig. 8.3 Relationship between event durations and shape parameter of gamma distribution defining associated event depths.

9. Modelling event profiles

The event profile defines the way in which the total event rainfall depth is distributed in time over the duration of the event, in effect the relative proportions of the total event rainfall depth which falls in each of the hours. To examine the nature of the rainfall profiles, hourly totals for all events of 4, 6, 8, 10, 12, 14 and 16 hour duration from Farnborough were investigated. Each of the observed event profiles exhibited a castlated appearance and no two profiles were exactly similar, with the trivial exception of events of one and two hours duration. This suggests that using a single smooth profile for all events would be unreasonable. However, the average of all profiles for each duration (separated for summer and winter) can be modelled adequately by a smooth shape. Individual hyetographs appear to vary randomly from this shape. They can therefore be envisaged as sample histograms from a population probability density function which describes the average profile. Figure 9.1 and

9.2 show the average profiles calculated from all events of 4, 8, 12 and 16 hours duration at Farnborough for the summer and winter respectively. The hourly depths, expressed as a proportion of the total rainfall *(p)*, are modelled as a function, *(f(t))* of the time through the event *(t)*, which is itself expressed as a proportion of the total event duration *(d)*. Also shown is a beta distribution

$$f(t) = t^{\alpha_t - 1} (1 - t)^{\beta_t - 1} \,/\, \beta t(\alpha_t, \beta_t) \qquad (9.1)$$

where $Bt(\alpha_t, \beta_t)$ is the *beta* function with parameters α_t and β_t given by

$$Bt(\alpha_t, \beta_t) = \int_o^1 u^{\alpha_t - 1} \, (1 - u)^{\beta_t - 1} \qquad (9.2)$$

The best fit parameter values imply that the profiles are slightly negatively skewed. Note that all four observed profiles are similar in shape in both the summer and the winter cases and adequately fitted by the beta distribution. This suggests that the average standardised profile is constant for all storm durations. Yen and Chow (1980) also found that the effect of rainstorm duration on the shape of the non dimensional hyetograph was not significant.

Average standardised profiles for 6, 10 and 14 hours (not shown) were similar in shape. Average profiles resulting from sub-dividing events according to their total rainfall depth and according to intensity showed that the standardised profile shape does not vary significantly with event magnitude. Therefore two standardised profiles (one for summer and one for winter) are sufficient to define the standardised profiles for all events. The four parameters, α_S, β_S, α_W and β_W, are the tenth, eleventh, twelth and thirteenth parameters of the rainfall model.

Clearly the rainfall depth in each hour will vary from this average form in a different way for each individual event profile. The variability of the profiles in the observed events can be modelled by fitting a distribution to the sample of observed standardised depths in each hour separately for the events of each duration. The mean value of each distribution will, of course, lie on the average standardised profile curve. There are two problems which arise with this methodology. Firstly, if each profile is composed of hourly depth proportions randomly chosen from a suite of distributions there is no guarantee that the cumulative total will be equal to 1.0 at the end of the event. Secondly, if the sampling is independent for each hour the individual profiles may not retain their smoothish observed shapes which results from dependence between falls in successive hours. To overcome the first problem all proportions can be expressed in a cumulative form as a proportion of the total rainfall still to fall in the remaining hours of the event. Figure 9.3 shows the mean proportion, $\overline{PR}$ of the remaining rainfall which falls in each hour of winter events of 4, 8, 12 and 16 hours duration.

The form of this relationship changes with event duration. However, the

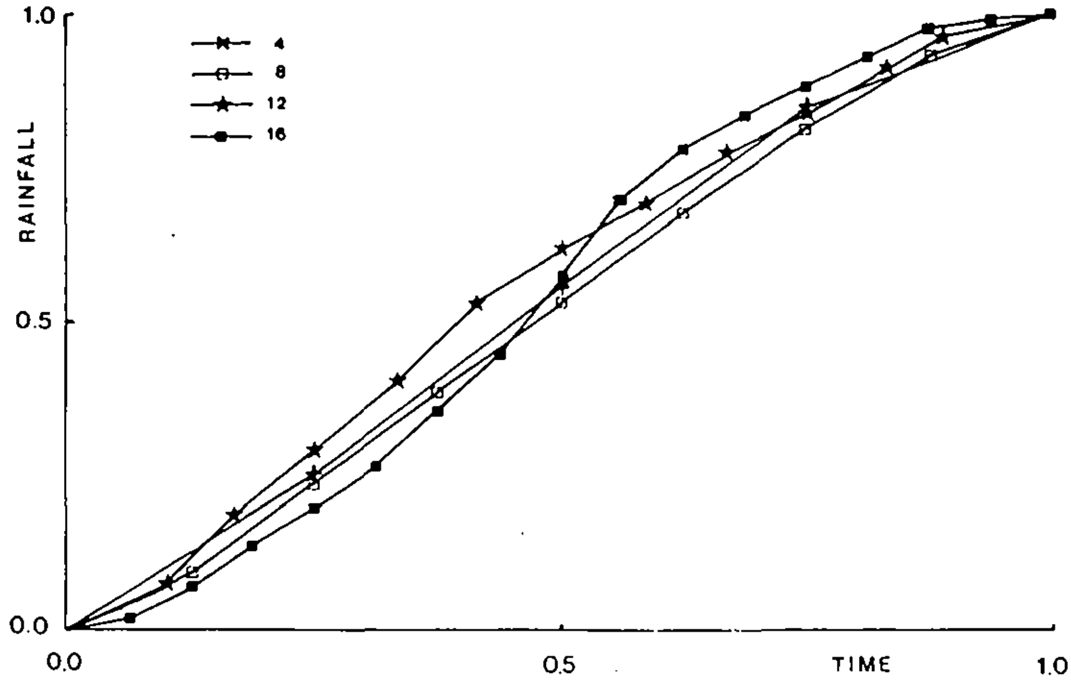

Fig. 9.1 Average summer event profiles of 4, 8, 12 and 16 hours durations at Farnborough togethr with a fitted beta distribution

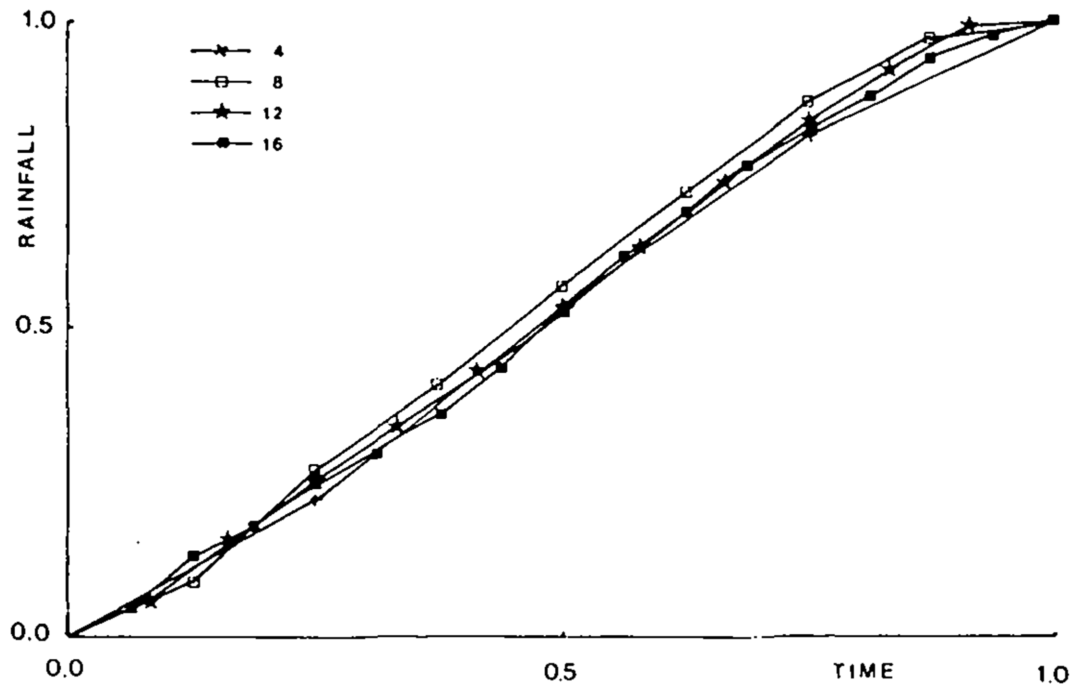

Fig. 9.2 Average winter event profiles of 4, 8, 12 and 16 hours durations at Farnborough together with a fitted beta distribution.

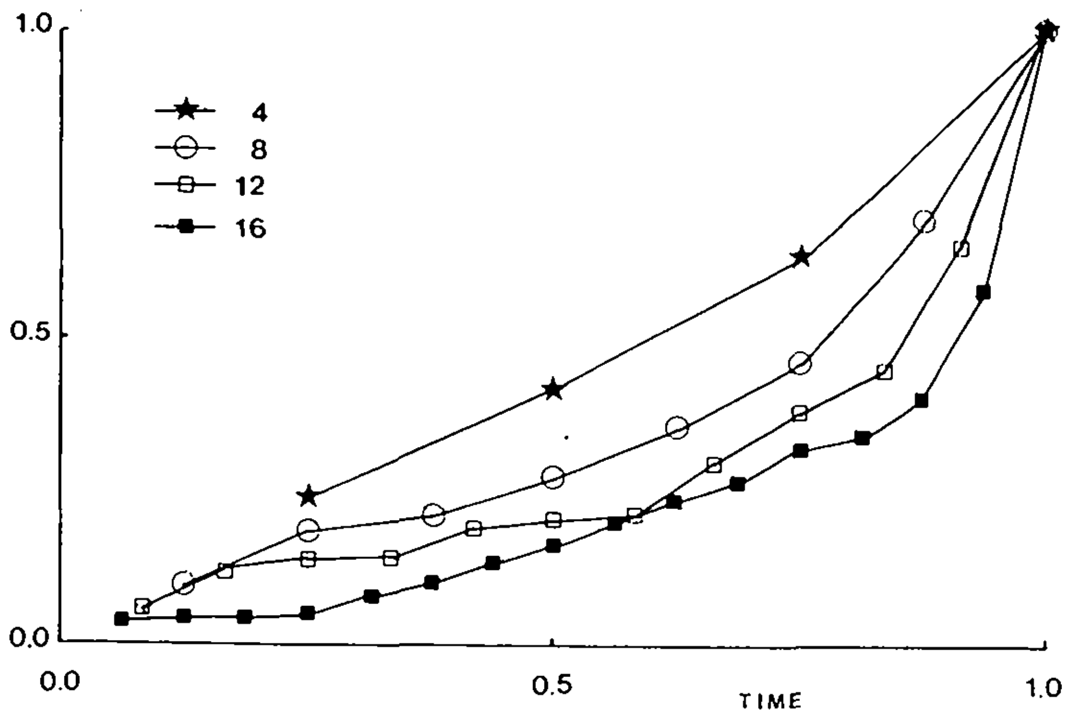

Fig. 9.3 Mean proportion of remaining depth which falls in each hour of winter events of 4, 8, 12 and 16 hours duration.

relationship for each duration can be approximated by accounting the remaining proportions through the event derived directly from the average evetn profile (Figures 9.1 & 9.2). The resulting relationships are shown as curves in Figure 9.3.

An obvious consequence of this methodology is that the last hour of each event will contain all (1.0) of the remaining rainfall. Figures 9.4a to 9.4f show histograms for each hour (except the eighth) of all observed eight hour events together with a beta distribution fitted to the data by the method of moments.

The variance is also required to enable the two parameters of the beta distribution to be defined.

$$\alpha = \overline{PR} \left\{ \left[\overline{PR} \, (1 - \overline{PR}) \, / S^2 \right] - 1 \right\} \tag{9.3}$$

$$\beta = (1 - \overline{PR}) \left\{ \left[\overline{PR} \, (1 - \overline{PR}) \, / \, S^2 \right] - 1 \right\} \tag{9.4}$$

where $\overline{PR}$ and S^2 are the mean and variance of the distribution and α and β are the location and scale parameters of the beta distribution respectively.

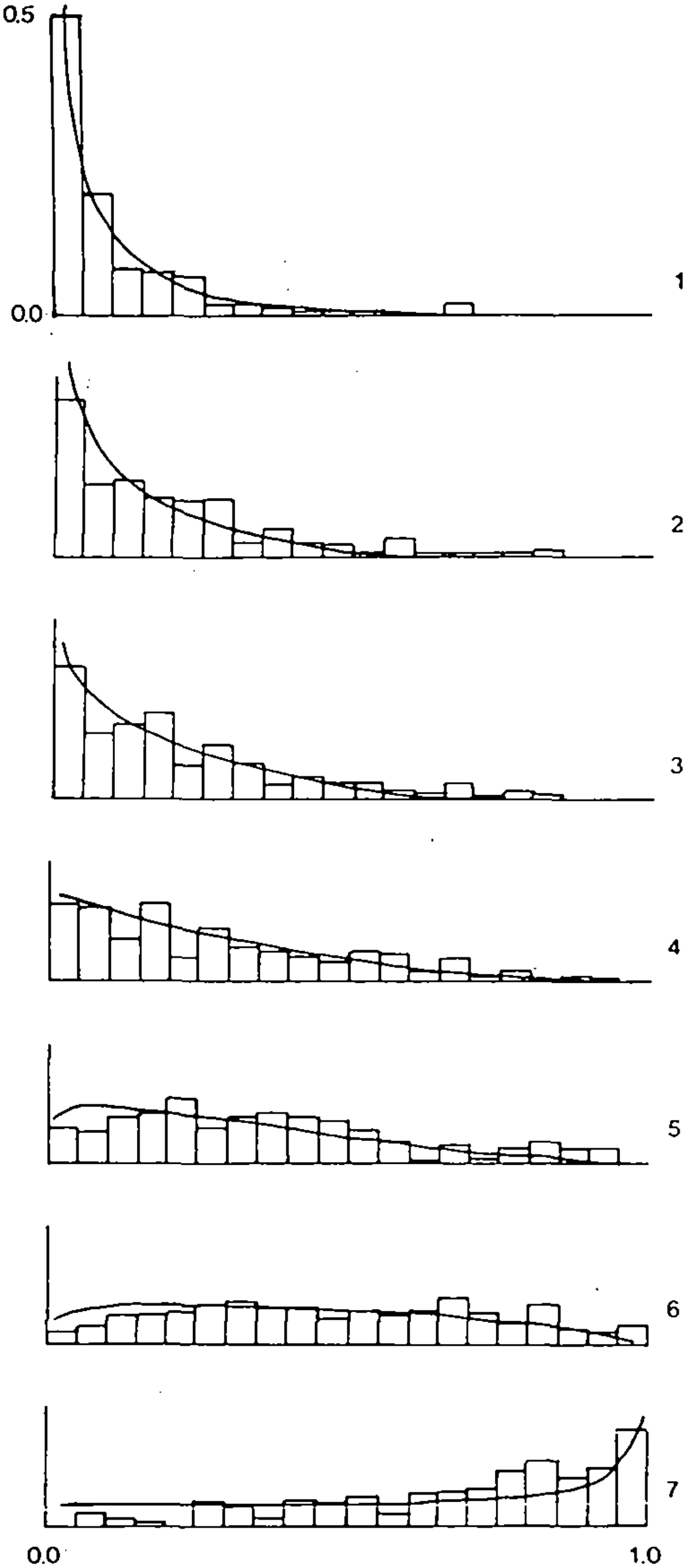

Fig. 9.4 Distribution of the proportion of remaining depth which falls in each hour of winter events of hours duration (except the eighth).

Figure 9.5 shows the standardised variance *SVAR* $(S^2/\overline{PR})$ for each hour of summer events of 4, 8, 12 and 16 hour duration. Similar results were found for winter events. Consequently, a constant value of 0.15 was chosen to model all events. This is the twelth parameter of the model.

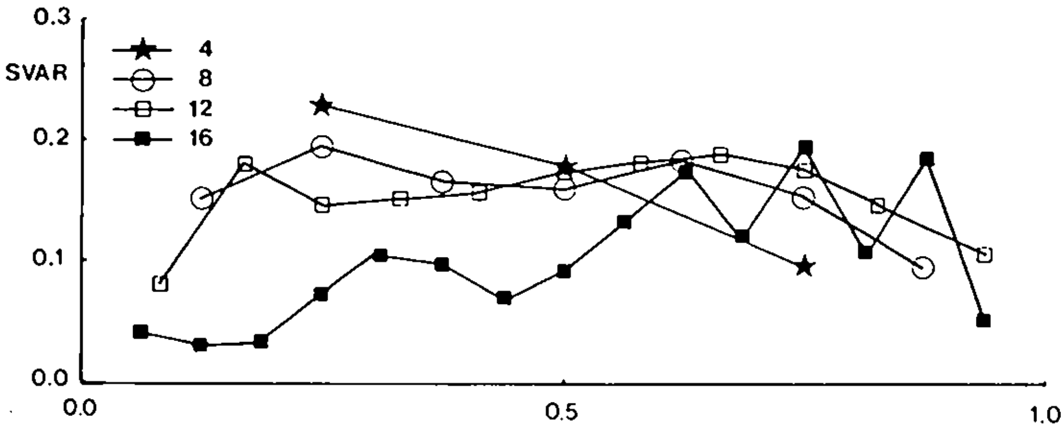

Fig. 9.5 Standardised variance of the proportion of remaining depth which falls in each hour of summer events of 4, 8, 12 and 16 hours duration.

To model the observed dependence between successive hourly falls within events, which is exhibited as the smoothness of the profile, the correlation structure of all events was examined. To keep the model simple only the lag one correlation were analysed. Correlated normally distributed random numbers $(x1,\ x2)$ may be generated using the formula

$$x_1 = x_2\ \rho + \sqrt{(1 - \rho^2)}\ x1 \qquad (9.5)$$

Values of ρ were chosen by examining the correlation between normally distributed random numbers which, when transformed to beta distributed random numbers, reproduce this observed correlation structure in generated rainfall events. Table 9.2 shows the lower triangle of the correlation matrix between the standard normal numbers required to generate the observed hourly rainfalls within all eight hour winter events. Table 9.3 shows the leading diagonal of this matrix all summer and winter events of 4, 8, 12 and 16 hours duration. It was noted that the average correlation varies with duration, but not significantly with season. Figure 9.6 shows a graph of the relationship between the durations and their respective lag-one correlations. This relationship takes the form

$$\rho = S_\rho \, d^{E_\rho} \tag{9.6}$$

The fifteenth and sixteenth model parameters are the scalar, S_ρ and the exponent, E_ρ of this relationship.

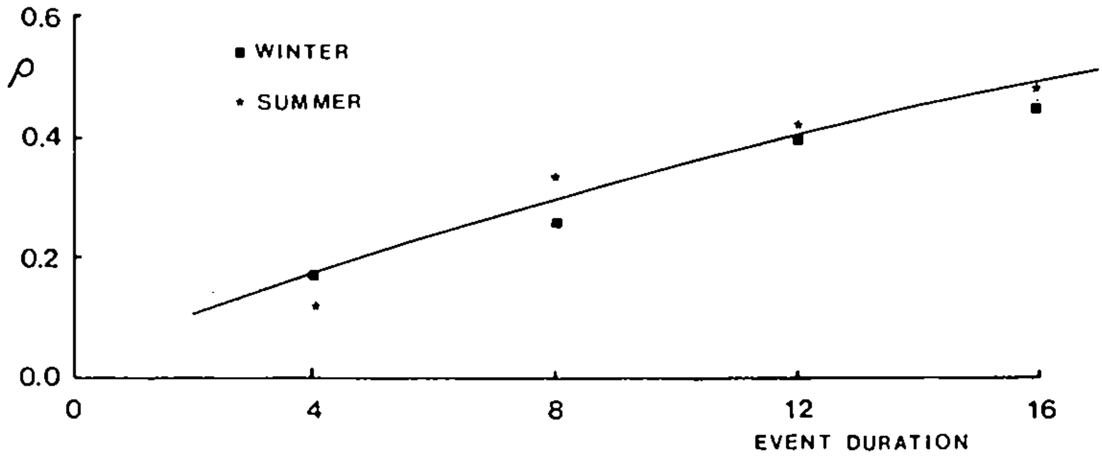

Fig. 9.6. Relationship between event duration and lag-one correlation of hourly rainfall depths.

The mean and variance of each hourly rainfall, expressed as a percentage of the total event depth, are shown in Table 9.1 for observed storms of 8 and 16 hours duration. Also shown are the same statistics resulting from 1000 simulated storms for each duration. The reduction in the peak intensity results from the method of correlation between the hourly falls. The expected value of the correlated beta variables is not equal to input values. Otherwise the simulation results compare favourably with those observed.

Table 9.1 Mean and variance of each hourly rainfall within observed events of 8 and 16 hours duration expressed as a proportion of the total depth.

8 hours	Summer				Winter			
	Mean		Var		Mean		VAR	
	obs	sym	obs	sym	obs	sym	obs	sym
	0.01	0.05	0.014	0.008	0.09	0.10	0.016	0.015
	0.14	0.13	0.025	0.019	0.17	0.13	0.026	0.018
	0.15	0.17	0.020	0.022	0.14	0.14	0.015	0.017
	0.14	0.18	0.020	0.024	0.16	0.16	0.020	0.018
	0.15	0.18	0.017	0.021	0.15	0.15	0.017	0.016
	0.13	0.15	0.018	0.016	0.14	0.13	0.018	0.013
	0.13	0.09	0.016	0.013	0.10	0.11	0.017	0.012
	0.06	0.04	0.012	0.008	0.03	0.08	0.003	0.012
16 hours								
	0.02	0.02	0.000	0.001	0.05	0.04	0.008	0.007
	0.05	0.04	0.001	0.003	0.08	0.06	0.010	0.005
	0.07	0.06	0.002	0.007	0.05	0.07	0.002	0.007
	0.06	0.07	0.004	0.010	0.06	0.07	0.004	0.009
	0.07	0.08	0.005	0.008	0.07	0.08	0.005	0.006
	0.09	0.09	0.005	0.011	0.06	0.08	0.003	0.009
	0.10	0.09	0.004	0.010	0.08	0.07	0.009	0.008
	0.11	0.09	0.004	0.008	0.09	0.08	0.014	0.006
	0.13	0.08	0.008	0.009	0.08	0.07	0.007	0.007
	0.08	0.09	0.003	0.007	0.07	0.08	0.003	0.006
	0.06	0.08	0.003	0.005	0.08	0.07	0.003	0.004
	0.04	0.07	0.003	0.005	0.06	0.07	0.004	0.004
	0.05	0.06	0.003	0.004	0.05	0.06	0.002	0.005
	0.05	0.05	0.004	0.004	0.07	0.05	0.017	0.005
	0.01	0.03	0.000	0.002	0.04	0.04	0.002	0.003
	0.01	0.01	0.000	0.001	0.01	0.02	0.000	0.003

Table 9.2 Lower triangle of correlation matrix between hourly rainfalls within all eight hour winter events at Farnborough

1.000						
0.302	1.000					
- 0.021	0.393	1.000				
- 0.092	0.066	0.394	1.000			
- 0.164	- 0.151	- 0.048	0.272	1.000		
- 0.238	- 0.142	- 0.151	- 0.078	0.147	1.000	
- 0.228	- 0.091	- 0.061	- 0.185	- 0.186	0.065	1.000

Table 9.3 Lag-one correlations between summer and winter events of 4, 8, 12 and 16 hours duration

4	Sum	0.220	0.002						
	Win	0.160	0.193						
8	Sum	0.299	0.461	0.436	0.256	0.324	0.227		
	Win	0.302	0.393	0.394	0.272	0.147	0.065		
12	Sum	0.473	0.447	0.460	0.435	0.537	0.408	0.244	0.645
		0.211	0.281						
	Win	0.158	0.531	0.609	0.604	0.360	0.445	0.520	
		0.411	0.209	0.274					
16	Sum	0.620	0.633	- 0.087	0.743	0.599	0.749	0.236	0.567
		0.569	0.310	0.407	0.111	0.206	- 0.029		
	Win	- 0.176	0.576	0.438	0.662	0.265	0.621	0.357	0.216
		0.581	0.520	0.554	0.318	0.474	0.047		

Table 10.1 Definition of summer and winter seasons for dry period durations

	summer	winter
Hampstead	March-October	November-February
Abingdon	March-October	November-February
Farnborough	March-October	November-February
St. Mawgan	April-September	October-March
Rhoose	May-August	September-April

Table 10.2 Parameter values for the exponential, generlaised pareto and gamma distributions fitted to dry period durations by the method of maximum likelihood.

		exponential	gamma		generalised	pareto
		μ	ξ	γ	a	k
Hampstead	sum	0.040	0.478	52.78	6.35	-0.98
	win	0.05	0.54	37.19	7.01	-0.80
Abingdon	sum	0.04	0.48	52.13	6.55	-0.95
	win	0.05	0.56	32.27	7.61	-0.74
Farnborough	sum	0.04	0.47	54.04	6.27	-0.98
	win	0.05	0.56	25.14	7.51	-0.73
St. Mawgan	sum	0.05	0.50	39.52	5.66	-0.90
	win	0.07	0.50	28.15	4.22	-0.82
Rhoose	sum	0.04	0.49	46.97	6.80	-0.89
	win	0.05	0.41	39.04	4.98	-0.91

10. Modelling dry periods

Dry, or inter-event, periods within the rainfall record are defined only by their duration. In the rainfall model the duration of a dry period is treated as a random variable from a statistical distribution. Figure 10.1 and 10.2 show the variation in mean and standard deviation of dry period durations at Hampstead for each month in which the period starts. A broadly similar pattern is exhibited at each of the other four sites investigated with high values occurring in March, June and October. The only deviation is that at Farnborough the mid-summer peak occurs in July. Although a consistent pattern was evident, for model simplicity a two season restriction was imposed. The seasonal divisions for the five sites are given in Table 10.1. The seventeenth and eighteenth parameters of the model are the times of year for the start of summer and winter for dry period durations.

The same analyses were performed on the dry period durations as on the event durations. Because only positive integer values are acceptable for the number of hours duration, numbers generated from a continuous distribution are rounded up to the nearest whole number adding, on average, 0.5 to each observation. Consequently, as for event durations, 0.5 was subtracted from each observed dry period duration prior to analysis. The histogram of summer dry period durations at Rhoose is shown in Figure 10.3. As with events, long dry periods are less frequent than short ones with the probability of occurrence reducing in a systematic fashion with increasing duration, with a modal value of one hour. The tail of the observed distribution is much heavier than the corresponding distribution of event durations; observed values of skewness (Table 6.1) are in the range 35 - 55. Such high values suggest the need for the generalised pareto distribution. Probability density functions for each of these three distributions, fitted by the method of maximum likelihood, are shown in Figure 10.3 and the corresponding parameter values are given in Table 10.2. The observed data were divided into 20 classes and compared with the expected number given by integrating the density function between the class limits. These figures are shown in Table 10.3 for winter dry periods at Abingdon. The χ^2 goodness of fit statistic was used to test whether the observed data could be assumed to have been sampled from any of these distributions. χ^2 values for each location are given in Table 10.4. Given that $\chi^2_{(17,0.05)}$ is approximately 27.5, it is very unlikely that the sample data came from any of these distributions. However 30% of the contribution to the χ^2 statistics comes from the first class. Despite the poor χ^2 values the generalised pareto distribution was chosen to model the dry period durations. The two parameters of the generalised pareto distribution, a and k, differing between summer and winter, are the nineteenth, twentieth, twenty-first and twenty-second model parameters.

Table 10.3 Observed and predicted frequencies of winter dry period durations for Abingdon.

Class	Observed	Exponential	Gamma	Generalised Pareto
1	575	187.9	577.5	447.2
2	379	178.6	264.4	363.0
3	238	169.7	203.4	299.8
4	209	161.3	170.1	251.1
5- 6	350	299.1	279.2	393.5
7- 8	275	270.3	226.9	295.9
9- 10	214	244.1	191.6	228.5
11- 12	156	220.5	165.6	181.1
13- 14	146	1992.	145.3	146.5
15- 17	148	263.3	188.0	173.3
18- 20	155	226.1	159.7	135.6
21- 24	144	252.6	178.7	136.1
25- 28	105	206.2	148.1	103.3
29- 50	306	615.8	488.5	294.2
51- 70	149	191.0	199.5	111.2
71- 90	83	69.2	99.3	61.0
91-140	84	36.2	85.8	73.1
141-180	30	2.7	15.8	27.4
181-260	35	0.4	6.0	27.4
559-	13	0.0	0.6	28.6
	χ^2	29666.2	623.7	109.5

Table 10.4 χ^2 statistics for testing the goodness of fit of various distributions to dry period durations

		exponential	gamma	generalised pareto
Hampstead	sum	36933.3	1117.3	165.1
	win	21201.9	432.9	156.3
Abingdon	sum	39255.9	528.0	136.6
	win	29666.2	623.7	109.5
Farnborough	sum	41132.4	1358.0	126.9
	win	29365.4	633.5	110.9
St. Mawgan	sum	28679.5	432.8	60.8
	win	>99999.9	4154.8	44.9
Rhoose	sum	9642.7	263.8	65.9
	win	31439.3	1513.9	62.1

Fig. 10.1 Monthly variation in mean dry period durations at Hampstead.

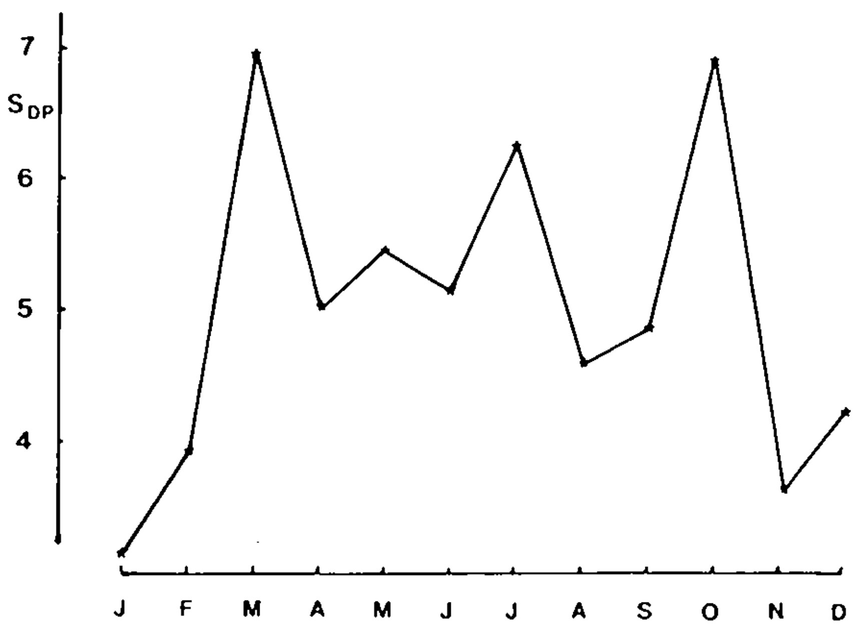

Fig. 10.2 Monthly variation in standard deviation of dry period durations at Hampstead.

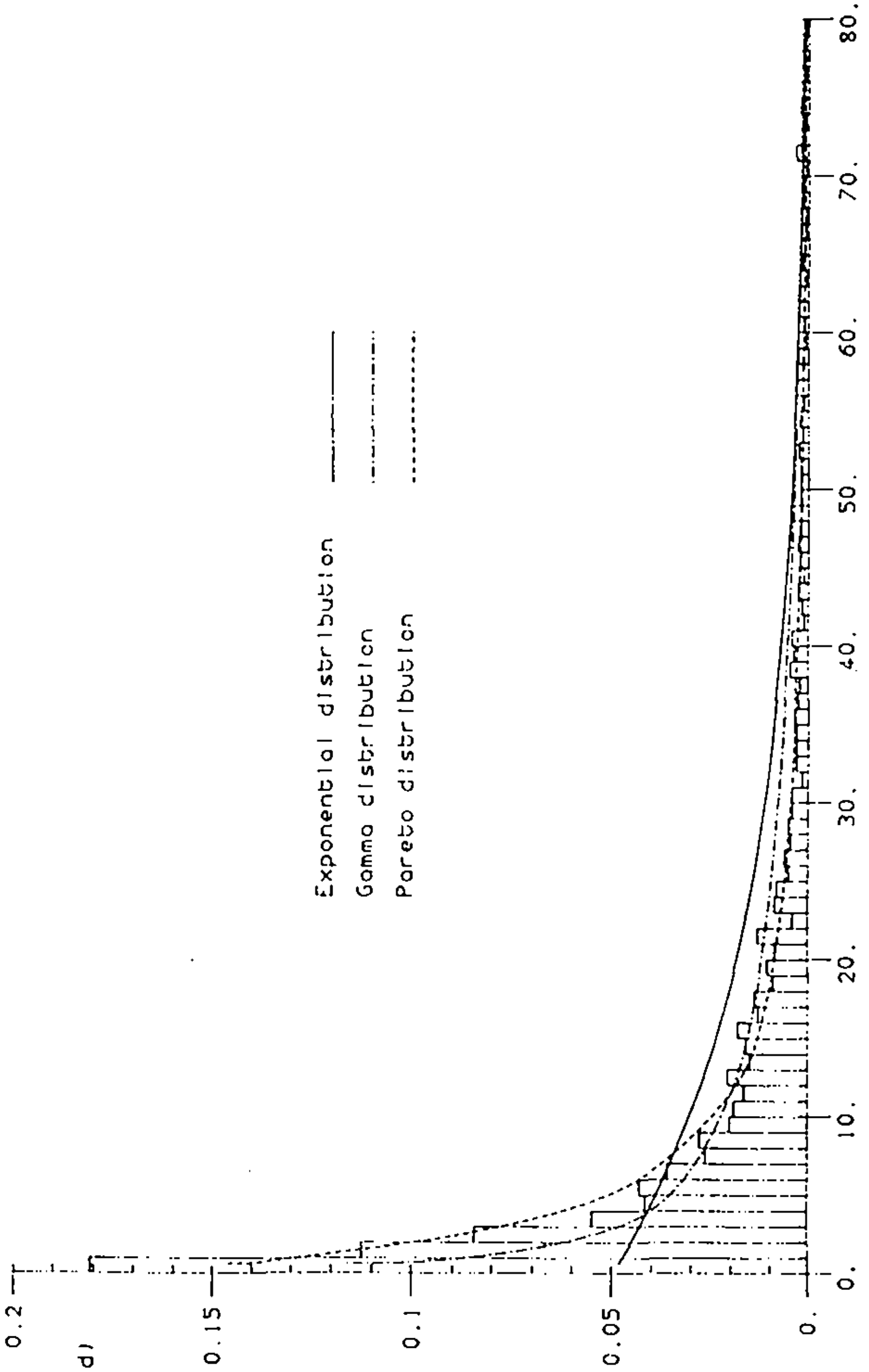

Fig. 10.3. Histogram of summer dry period durations at St. Mawgan together with fitted exponential, gamma and generalised pareto distributions.

11. Model implementation

The rainfall model is identified by the anachronym PHROG; Point Hourly Rainfall Ordinate Generator. PHROG has been implemented on a IBM computer at Wallingford using the FORTRAN 77 language. The model subroutine returns one years 'simulated rainfall in an array of size 16000 containing either 8760, or 8784, point hourly rainfall totals for non leap years and leap years respectively. A logical variable, set to true or false, is used as a leap year indicator. To obtain this output, the only input required is an integer year counter. Since the year rarely terminates exactly at the end of an event or dry period, if the input year is not 1, the first elements of the array contain hourly rainfalls generated in the previous year from an event or dry period which spanned the end of the year. In this way continuity between years is retained.

12. Comparsion of model results with observed data

The model was run to generate 2000 years of synthetic hourly rainfalls using the parameters calibrated for Farnborough. For each month the total rainfall volume was recorded. Table 12.1 shows the mean and standard deviation of each of these monthly rainfalls compared with those observed at Farnborough over the period of record used for calibration. Also shown in Table 12.1 are the means and standard deviations of the annual totals. The statistics of the simulated annual values are very close to those observed at Farnborough. However, given the nature of the model, average results are somewhat constrained to be similar to those observed. The average monthly simulated values show a smooth trend through the year, lowest in June highest in January. The trends are similar in the observed data but exhibit more variability. The standard deviation of the monthly totals show a similar behaviour with less variability in the model results. Nevertheless, since the model has only two seasons, the results are generally acceptable.

For each of the 2000 years of simulated data the maximum depth generated in any period of 1, 2, 4, 6, 12, 24 and 48 hours was recorded. These data are plotted using red symbols on Figure 12.1 against their respective return periods, T, calculated using the Gringorten plotting position

Table 12.1 Mean and standard deviation of monthly and annual rainfall totals observed at Farnborough (1941-1971) compared with those generalted by PHROG

Month	mean Observed	mean Simulated	standard deviation Observed	standard deviation Simulated
JAN	61.5	67.8	31.2	32.4
FEB	40.6	61.1	26.4	31.1
MAR	40.7	59.4	29.4	32.9
APR	45.0	54.7	21.4	32.4
MAY	52.0	49.2	23.4	30.1
JUN	46.4	47.7	31.2	30.1
JUL	53.6	48.2	26.5	29.5
AUG	64.5	48.6	27.0	30.3
SEP	54.6	48.4	36.2	30.4
OCT	62.3	54.5	40.6	33.3
NOV	71.8	60.9	41.9	33.0
DEC	60.4	67.0	23.2	31.7
ANNUAL	653.3	666.2	111.3	119.0
SAAR	669.0			

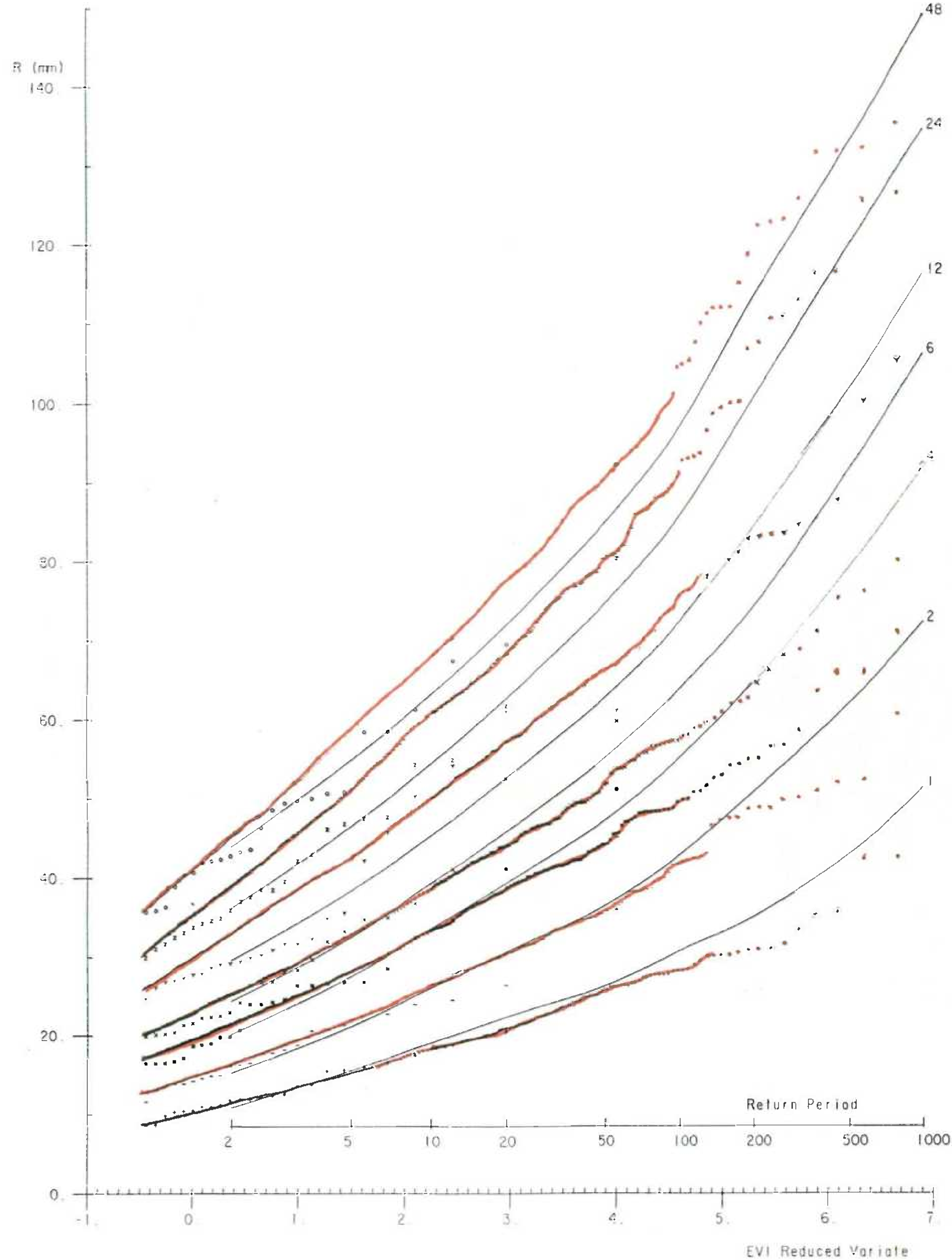

Fig. 12.1 Distribution of annual maximum simulated depths of 1, 2, 4, 6, 12, 24 and 48 hours duration compared with those observed at Farnborough.

Table. 12.2 Rainfall depths of specified duration and return period observed at Farnborough (1941-1971) compared with those generated by PHROG and those estimated by the methods given in the Flood Studies Report (NERC, 1975).

Duration	Return Period	Observed	Simulated	FSR
1	5	15.5	15.2	15.5
	10	18.9	18.1	19.0
	50	28.6	25.4	26.3
	100	33.9	29.0	30.8
	200		33.0	35.1
	500		38.7	43.0
	1000		43.5	51.4
2	5	21.1	21.9	15.2
	10	25.4	26.1	21.1
	50	30.1	36.7	36.5
	100	43.2	42.0	42.9
	200		47.9	50.9
	500		56.3	62.2
	1000		63.5	72.4
4	5	27.5	28.2	27.5
	10	33.0	33.1	33.6
	50	48.5	45.3	47.3
	100	56.8	51.2	55.3
	200		57.6	65.1
	500		66.6	82.4
	1000		74.2	92.7
6	5	32.5	32.9	32.5
	10	39.3	38.4	39.3
	50	58.4	52.2	45.8
	100	68.7	58.8	55.2
	200		66.0	74.4
	500		76.0	92.0
	1000		84.4	106.3
12	5	38.4	43.0	38.4
	10	45.8	50.0	45.3
	50	67.1	66.8	62.9
	100	78.8	74.5	72.5
	200		82.5	85.6
	500		93.6	102.0
	1000		102.6	116.3

Table 12.2 (cont'd)

Duration	Return Period	Observed	Simulated	FSR
24	5	46.5	51.5	46.5
	10	54.1	60.4	54.4
	50	73.6	81.7	74.8
	100	83.1	91.7	86.0
	200		102.2	100.8
	500		116.8	120.0
	1000		128.8	134.7
48	5	54.9	58.7	54.9
	10	63.4	68.3	63.2
	50	72.1	91.5	85.1
	100	94.1	102.4	97.2
	200		113.8	113.4
	500		129.7	133.0
	1000		142.7	148.6

$$T_i = \frac{i - 0.44}{n + 0.12} \quad (12.1)$$

where i is the ith ranked annual maximum depth in n years of record. Also shown, are the observed annual maxima of the same durations plotted in the same manner and solid lines depicting the rainfall frequency curves estimated by methods given in the Flood Studies Report (NERC, 1975 Vol. II). To derive these curves, the 5 year return period rainfall depths were estimated from the observed data at Farnborough and scaled up using the appropriate growth factors to give the magnitudes of the less frequent events. This procedure probably gives the best estimate of the true depths for the specified durations and frequencies. These data provide the basis for an independent test of the ability of the model to generate realistic extreme rainfalls. The envelope of data is coincident upto around the 50 year return period. This is the extent of the observed data. Above this point the Flood Studies Report curves for the 1, 2, 4 and 6 hour durations are slightly steeper than those which result from the model simulations. This suggests that the model is performing well over the range of observed data but is not so good at extrapolating to more extreme events. At the 12, 24 and 48 hour durations the model appears to slightly over estimate the depth of rainfall at all return periods. However, the shape of the frequency curves is closely matched. The dominant control over the frequency of the 1 and 2 hour durations is probably the form of the event profile, whereas the control over the 24 and 48 hour duration is more likely to be the distribution of the event depths for the longer durations.

In order to interpolate between the points and examine the depths estimated at specific return periods, a generalised extreme value distribution was fitted to these annual maxima, separately for each duration, by the method of probability weighted moments (Hosking et al, 1986). Rainfall depths for return periods 5, 10, 50, 100, 200, 500 and 1000 years were evaluated for each duration are given in Table 12.2. In general the model performs well over the range of return periods
at all durations.

13. Conclusions

A stochastic rainfall model has been developed to generate synthetic sequences of hourly rainfalls at a point. The model has been calibrated using up to 30 years of rainfall data for each of five sites in Southern Britain. These rainfall data series were divided into wet and dry spells; analysis of the durations of these spells suggests that they may be represented by exponential and pareto distributions respectively. The total volume of rainfall in wet spells is adequately fitted by a conditional gamma distribution. Random sampling from a beta distribution, defining the average shape of all rainfall profiles, is used in the model to obtain the rainfall profile for a given wet spell. The model has a total of 21 parameters, some of which are specific to winter or summer and vary at each site, whilst some are constant through the year and over all of southern Britain. Results obtained from the model compare favourably with observed monthly and annual rainfall totals and with annual maximum frequency curves of 1, 2, 6, 12, 24 and 48 hours duration.

In order to retain the goodness of fit to the Flood Studies Report curves as an independent test of the model it can not be used for model calibration. Furthermore, given that a large part of the model is empirically based, there is no justification for adjusting the parameter values on a theoretical basis. In general the generated data approximate those observed at Farnborough sufficiently well to make these adaptions unnecessary.

14. Future work

Work is continuing on calibrating and testing the model at the four other sites in Southern Britain. Data for other sites, in northern Britain, are to be aquired from the Meteorological Office to which the model will be fitted. Once the model has been calibrated at a wide range of locations the spatial distribution of the model parameters can be examined. Relationships can be sought between the model parameters and other mapped data. Maps showing the spatial distribution of of rainfall statistics, including average annual rainfall (SAAR) and two-day rainfall of five year return period (M52D) are available in the Flood Studies Report (NERC, 1975). The model will then be in a form suitable for generating data at any site within Brit ain, whether or not gauged rainfall data are available.

15. Acknowledgements

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