Institute of Hydrology

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Natural Environment Research Council

Impact of hydrological uncertainties on design flood estimation and the assessment of the benefits of flood alleviation

Report to MAFF

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November 1990

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Executive Summary

The objective of the study was to investigate the effect of a number of different types of hydrological uncertainty on the estimation of flood magnitudes and the evaluation of the benefits of flood alleviation.

The conventional approach to design flood estimation is to use a method which gives an unbiased estimate of the magnitude of the T-year flood. This estimate at a site, however, will, on average, be exceeded in the future more frequently than once in T years because of the non-linear relationship between flood magnitude and flood probability: a method which is unbiased in one direction is not therefore unbiased in the other. A method is presented which gives the increment which needs to be added to an estimate of the T-year flood in order to give a flood with an expected probability of being exceeded in the future of 1/T. An estimate of the 50-year flood made with the GEV-PWM procedure from a sample of just 10 years would need to be increased by over 18% to produce the magnitude with expected probability of 1/50 (assuming average flood characteristics). The adjustment, which varies with return period, record length and flood characteristics, can be seen as a safety factor to apply to an estimate of the T-year flood.

Uncertainties in the estimated flood frequency relationship feed through to bias and uncertainties in the estimation of average annual damage. A series of simulation experiments showed that average annual damage tended to be overestimated, with bias increasing as the return period at which damage commenced increased. The results emphasise the importance of estimating the return period of this damage threshold as accurately as possible.

Confidence intervals for estimates of both flood magnitudes and return periods were also studied using computer simulation experiments. The sampling distribution of magnitudes with a given return period is highly skewed, and methods to estimate confidence intervals based on the assumption that the distribution is Normal underestimate upper confidence limits. Until exact methods are developed, it is recommended that confidence intervals for flood magnitudes and return periods in practice are based on computer simulation experiments.

The conventional approach to scheme benefit assessment compares the present value of scheme costs with the present value of average annual benefits. In practice, the present value of the benefits that are actually realised over the project life will depend on the timing of flood events, and a method was developed which calculates, by computer simulation, the probability distribution of possible present values of future flood alleviation benefits. From the probability distribution it is possible to determine the probability that the present value of benefits will exceed particular target values, which may assist with scheme evaluation.

Contents

1. Introduction

1.1 PROJECT OBJECTIVES

Flood frequency analysis provides the basis for many design and investment decisions in and along a river course, but the relationship between flood magnitude and flood frequency at a site can never be known with complete certainty. The basic objective of this report is to investigate the impact of hydrological uncertainties on the estimation of design flood magnitudes and the assessment of scheme worthwhileness, and to develop procedures which allow for the effects of these uncertainties.

The project ran in parallel with a more general review of the statistical procedures used for flood frequency analysis in the United Kingdom as presented in the 1975 Flood Studies Report (NERC, 1975). Because this general review has not yet recommended a new technique, the procedures outlined in the current report should not be regarded as definitive methods for representing and coping with uncertainties. However, the studies do indicate the effects of hydrological uncertainties, and the general form of the suggested procedures is unlikely to change significantly as new estimation techniques are developed.

The investigations into risk and uncertainty and their impacts on scheme design and assessment provided the basis for a number of additional commissioned research projects into particular issues (including Beran, 1987). Some other avenues of research - in particular studies into confidence intervals - were followed after discussions with groups researching and applying methods for the assessment of the benefits of flood alleviation.

1.2 STRUCTURE OF REPORT

There are four main themes to the study, and each is summarised in a chapter. The themes are:

(i) estimating flood risk in the presence of uncertainty;

(ii) bias and uncertainty in the estimation of average annual flood damage;

(iii) confidence intervals for flood and return period estimates;

(iv) the effect of flood timing on the present value of the flood damages that are actually experienced.

Chapters 2 and 3 are based upon studies that have been reported in the international refereed literature: the papers are included as Appendices A and B, and contain much of the detail of the experimental design and conclusions.

2. Estimating flood risk

2.1 INTRODUCTION

The objective of this chapter is to outline several ways of expressing the risk of flood occurrence, and to introduce procedures which employ alternative definitions of risk. In particular, methods for estimating the magnitude of flood exceeded in the future with the desired degree of risk which is not the same as the more conventional approach of providing an unbiased estimate of the magnitude of that flood - are outlined.

It is appropriate to begin with some definitions:

p is the probability of experiencing an event greater than or equal to X in any one year;

T is the average return period between events greater than or equal to X. It is equal to 1/p. In annual maximum frequency analyses, the return period represents the average interval between years containing an event greater than or equal to X: the average interval between events may be rather shorter because some years will contain more than one event.

2.2 THE PROBABILITY OF M EVENTS IN N YEARS

It is well known that if an event has a probability p of occurring in any one year, then the probability of experiencing m events in N years can be derived using the binomial distribution, and is:

$$P(m \text{ events in } N \text{ years}) = \binom{N}{m} p^m (1-p)^{N-m} \qquad 2.1$$

The probability of experiencing at least one event in N years is

$$P(\text{at least one event}) = 1 - (1-p)^N \qquad 2.2$$

Table 2.1 shows the probability of experiencing at least one event during a range of different time horizons (N) for different event return periods (T) or probabilities (p). There is, for example, a 22% chance of experiencing the 100-year flood at least once in a 25 year period.

Equations 2.1 and 2.2 and Table 2.1 assume that each year is independent, and that the probability of experiencing an event is the same in each year. The probability of experiencing m events in N years when there is year-to-year dependence can be determined from the conditional probability of an event occurring given that the previous year did (or did not) contain an event. The probability of experiencing at least one event, for example, is

Table 2.1 The probability of experiencing at least one event over the next N years

Period of years N	Return period T: Probability P:	5 0.2	10 0.1	25 0.04	50 0.02	100 0.01	250 0.004
10		0.893	0.651	0.335	0.183	0.096	0.039
15		0.965	0.794	0.458	0.261	0.140	0.058
20		0.988	0.878	0.558	0.332	0.182	0.077
25		0.996	0.928	0.640	0.397	0.222	0.095
30		0.9988	0.958	0.706	0.455	0.260	0.113
50		0.99999	0.995	0.870	0.636	0.395	0.182

$$P(1 \text{ or more in } N \text{ years}) = 1 - \{P(\text{no event in first year}) \cdot P(\text{no event in year} | \text{no event in previous year})^{(N-1)}\} \qquad 2.3$$

and the probability of experiencing exactly one event in N years is

$$\begin{aligned} P(m=1) = {} & P(\text{event in first year}) \cdot P(\text{no event} | \text{one before}) \\ & \cdot P(\text{no event} | \text{none before})^{N-2} \\ & + P(\text{no event in first year}) \cdot P(\text{event} | \text{none before}) \\ & \cdot P(\text{no event} | \text{one before}) \\ & \cdot P(\text{no event} | \text{none before})^{N-3} \cdot (N-1) \end{aligned} \qquad 2.4$$

Similar - but longer - expressions can be derived for higher values of m. Table 2.2 shows the probability of experiencing none, one, two, three or four of more events over the next 20 years with different degrees of year-to-year correlation, assuming (i) that the last year before the 20 year period contained an event, and (ii) that it did not contain an event. The degree of clustering is represented by the ratio of the conditional to unconditional probability of experiencing an event: a ratio of three implies that the long-term 10-year flood has a 30% chance of occurring in a year following a 10-year flood, and represents a very high degree of clustering. In general, the probability of experiencing large numbers of events increases as the degree of clustering increases, as does the probability of no events occurring.

The probability of experiencing M events in N years can also be determined when peaks over a threshold (POT) flood data are used. If the mean number of peaks over the threshold q_0 is λ and the probability of a flood exceeding X, given that it is greater than q_0, is $p'(x) = (1-F'(x))$, then the mean number of peaks per year greater than X is

$$\lambda_x = \lambda\,(1 - F'(x)) \qquad 2.5$$

If it is assumed that the number of floods in a year follows a Poisson distribution (which is not unreasonable: NERC, 1975), then the probability of experiencing M floods above X in N years is:

Table 2.2 The probability of experiencing m events in 20 years, with different degrees of year-to-year correlation

Ratio of conditional to unconditional probability	0	1	2	3	4 or more
Record length is 20					
Long-term probability is 0.10					
Year zero did not contain an event					
3.00	0.198	0.254	0.216	0.147	0.185
2.50	0.175	0.261	0.233	0.157	0.173
2.00	0.155	0.266	0.249	0.168	0.161
1.50	0.137	0.269	0.266	0.179	0.148
1.00	0.122	0.270	0.284	0.190	0.134
Year zero did contain an event					
3.00	0.150	0.232	0.222	0.166	0.230
2.50	0.144	0.242	0.236	0.172	0.206
2.00	0.136	0.252	0.251	0.178	0.182
1.50	0.129	0.261	0.267	0.184	0.158
1.00	0.122	0.270	0.284	0.190	0.134
Record length is 20					
Long-term probability is 0.05					
Year zero did not contain an event					
3.00	0.400	0.334	0.168	0.064	0.034
2.50	0.389	0.345	0.173	0.064	0.029
2.00	0.379	0.356	0.178	0.062	0.025
1.50	0.369	0.367	0.183	0.061	0.021
1.00	0.358	0.377	0.188	0.060	0.017
Year zero did contain an event					
3.00	0.356	0.338	0.187	0.077	0.042
2.50	0.357	0.347	0.187	0.073	0.035
2.00	0.358	0.357	0.188	0.069	0.029
1.50	0.358	0.367	0.188	0.064	0.022
1.00	0.358	0.377	0.188	0.060	0.017

$$p(M \text{ events in } N \text{ years}) = \frac{e^{-\lambda_x N}(\lambda_x N)^M}{M!} = \frac{e^{-\lambda(1-F'(x))N}(\lambda(1-F'(x))^M}{M!} \qquad 2.6$$

F'(x) represents a conditional probability (i.e. the probability that X is exceeded, given that the flood exceeds the threshold q_o). The unconditional probability of a flood exceeding X is equal to

$$p(x) = \lambda\ (1-F'(x)) \qquad 2.7$$

and T=1/p(x) can be substituted into (2.6) to give

$$p(M \text{ events in } N \text{ years}) = \frac{e^{-N/T}(N/T)^M}{M!} \qquad 2.8$$

The probability of experiencing at least one event in M years is

$$p(\text{at least one}) = 1 - e^{-N/T} \qquad 2.9$$

As return period T increases and the period of interest N lengthens, equations 2.2 and 2.9 converge.

2.3 THE CONCEPT OF EXPECTED PROBABILITY

The conventional approach to flood frequency analysis is to provide the best estimate of the magnitude of the flood with exceedance probability p: a "good" flood frequency estimation procedure is one which gives unbiased estimates of the magnitude of the p-probability (or T-year) flood. However, the true probability of exceedance of this "best" estimate will be larger than the initially-specified probability p. This is illustrated in Figure 2.1.

Figure 2.1a shows the (hypothetical) sampling distribution of estimates of the magnitude of the flood with exceedance probability p=0.1 (and hence return period 10 years). The mean of the sampling distribution of estimated 10-year flood magnitudes is equal to the underlying true value, and the estimation procedure therefore gives an unbiased estimate of the magnitude of the 10-year flood. The sampling distribution of magnitudes is also approximately normally-distributed. Figure 2.1b shows the distribution of the true exceedance probabilities of each estimate of the 10-year magnitude, and it is clear that this distribution is highly skewed and with a mean different to - and greater than - a probability of 0.1. The mean of the true exceedance probabilities is not the same as the true exceedance probability of the mean of the magnitudes. Beard (1960) called the mean of the true probabilities of the estimates x_p the "expected probability", but Hardison and Jennings (1972) proposed the term "average exceedance probability". Appendix A contains a paper (Arnell, 1988) which provides more details of the concept of expected probability, including a number of alternative interpretations.

The practical implication of expected probability is that design floods, as estimated conventionally, will be exceeded in the future more frequently than desired. Stedinger (1983a) argued that flood managers did not need the "conventional" estimate of the magnitude of the T-year flood, but required instead the flood which would be exceeded in the future with the specified risk of occurrence. In other words, they needed the magnitude of flood which had an expected probability (or average exceedance probability) equal to p.

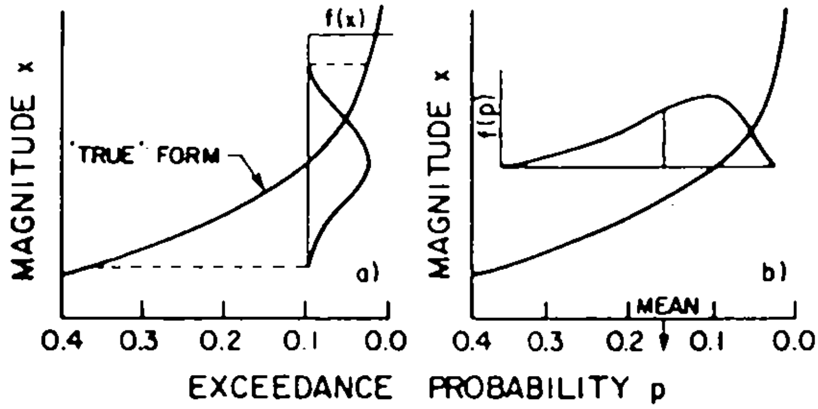

Figure 2.1 Sampling distribution of estimates of the 10-year flood (Arnell, 1988; Appendix A)

This magnitude will be higher than the conventional best unbiased estimate of the p-probability flood, with the difference depending on the form of probability distribution used, the parameter estimation procedure employed, the sample size available and the desired degree of risk. The difference between the two magnitude estimates can be seen as a "safety factor" representing parameter uncertainty (but not model uncertainty): as sample sizes increase, the two estimates converge.

2.4 ESTIMATING THE MAGNITUDE OF A FLOOD WITH THE DESIRED RISK

2.4.1 Introduction

The objective of the investigation was to produce a method for estimating the magnitude of a flood which would be exceeded in the future with a desired expected probability p. The method is based on the General Extreme Value (GEV) distribution (Jenkinson, 1955), with parameters estimated by the method of probability-weighted moments (PWM: Hosking et al, 1985). In general terms, the method determines the increment which needs to be added to a conventional GEV-PWM estimate of the magnitude of the p-probability flood, in order to produce a flood with an expected probability equal to p. Different increments (or "adjustment factors") would be necessary with different probability distributions or parameter estimation procedures.

Analytical expressions for estimating the magnitude of a flood with expected probability equal to p can unfortunately be derived for only a very few probability distributions. Stedinger (1983a) derived a method assuming floods (or their logarithms) were normally distributed, which is outlined in more detail in Appendix A, and Rasmussen and Rosbjerg (1989) presented a procedure for use with POT data. For other probability distributions it is necessary to derive empirically the average true exceedance probability of conventional estimates of the magnitude x_p. Hardison and Jennings (1972) used computer simulation experiments to develop corrections to be applied to "conventional" probabilities when using the log-Pearson type 3 distribution, and a similar approach was used in the current study with the General Extreme Value distribution.

2.4.2 Single-site analysis

Appendix A gives details of the derivation of the adjustment factors to be applied during the course of a single-site flood frequency analysis using the GEV-PWM procedure. To summarise, the construction of the adjustment factors involved the following stages:

(i) generate a sample of synthetic annual floods of size N from a specified GEV parent distribution;

(ii) estimate GEV parameters u, a and k using PWM;

(iii) compute the true probabilities of the estimated magnitude x_p and $x_p + AF.x_p$, where AF ranges from 0.01 to 0.5;

(iv) repeat the process many times and compute the mean true probability (the "expected probability") for x_p and each increment $x_p + AF.x_p$;

(v) interpolate to determine the value of AF which gives expected probability equal to p.

Table 2.3 (reproduced from Appendix A) gives the final adjustment factors AF for a range of sample sizes, parent distribution characteristics and return periods. The adjustment factor for the 100-year flood estimated from a 10 year sample from a GEV distribution with coefficient of variation 0.4 and k parameter -0.1, for example, is 0.2717. In other words, the GEV-PWM estimate of the 100-year flood magnitude must be increased by 27% to produce the magnitude exceeded with expected probability 0.01.

Table 2.3 indicates the magnitudes of the adjustment factors under a range of conditions, but is not easy to use in practice where interpolation is necessary. An interpolation model was therefore developed, as detailed in Appendix A. The model, by analogy with an analytical expression which can be derived for the lognormal distribution, has the form:

$$AF = [\ \exp\{\ (1+1/N)^{½}\ \ A\ -\ K_{GEV}\}]^{(\log_e(1+CV^2))^{½}} -1 \qquad 2.10$$

Table 2.3 Adjustment factors to convert a GEV-PWM estimate of X_T to a magnitude with expected probability 1/T. (Arnell, 1988; Appendix A)

GEV *k* is 0.10						GEV *k* is 0.00					
C.V. is 0.40						C.V. is 0.40					
	Return period						Return period				
N	10	25	50	75	100	*N*	10	25	50	75	100
10	0415	.0615	.0947	.1224	.1456	10	.0477	.0873	.1369	.1744	.2046
20	.0283	.0366	.0534	.0690	.0827	20	.0281	.0484	.0772	.1004	.1198
30	.0220	.0275	.0384	.0488	.0582	30	.0182	.0316	.0517	.0683	.0826
40	.0203	.0241	.0317	.0389	.0456	40	.0147	.0239	.0384	.0507	.0613
50	.0193	.0227	.0293	.0355	.0412	50	.0125	.0199	.0325	.0429	.0523
100	.0173	.0207	.0251	.0287	.0317	100	.0083	.0123	.0191	.0250	0302
C.V. is 0.60						C.V. is 0.60					
	Return period						Return period				
N	10	25	50	75	100	*N*	10	25	50	75	100
10	.0728	.0995	.1433	.1802	.2112	10	.0879	.1379	.1999	.2472	.2856
20	.0412	.0524	.0746	.0949	.1128	20	.0473	.0726	.1078	.1363	.1601
30	.0288	.0361	.0504	.0639	.0762	30	.0300	.0467	.0710	.0911	.1082
40	.0244	.0293	.0390	.0482	.0567	40	.0235	.0350	.0523	.0668	.0794
50	.0220	.0264	.0347	.0426	.0499	50	.0195	.0289	.0436	.0562	.0672
100	.0173	.0209	.0261	.0304	.0342	100	.0119	.0170	.0251	.0319	.0378
C.V. is 0.80						C.V. is 0.80					
	Return period						Return period				
N	10	25	50	75	100	*N*	10	25	50	75	100
10	.1001	.1303	.1814	.2250	.2617	10	.1216	.1776	.2481	.3025	.3469
20	.0523	.0650	.0907	.1143	.1351	20	.0625	.0906	.1299	.1619	.1888
30	.0348	.0430	.0594	.0748	.0888	30	.0392	.0577	.0846	.1069	.1259
40	.0285	.0339	.0448	0554	.0650	40	.0303	.0429	.0619	.0779	.0917
50	.0250	.0297	.0392	.0482	.0566	50	.0249	.0352	.0515	.0653	.0773
100	.0181	.0219	.0275	.0323	.0365	100	.0146	.0204	.0291	.0365	.0429

GEV *k* is -0.10						GEV *k* is -0.20					
C.V. is 0.40						C.V. is 0.40					
	Return period						Return period				
N	10	25	50	75	100	*N*	10	25	50	75	100
10	.0527	.1149	.1840	.2331	.2717	10	.0522	.1382	.2282	.2900	.3376
20	.0296	0652	.1088	.1413	.1677	20	.0296	.0829	.1432	.1864	.2207
30	.0171	.0419	.0737	.0982	.1183	30	.0159	.0547	.1005	.1341	.1611
40	.0124	.0306	.0546	.0734	.0890	40	.0106	.0405	.0765	.1033	1249
50	.0092	.0240	.0445	.0609	.0746	50	.0067	.0316	.0626	.0861	.1052
100	.0032	.0114	.0231	.0326	.0406	100	.0001	.0145	.0332	.0475	.0594
C.V. is 0.60						C.V. is 0.60					
	Return period						Return period				
N	10	25	50	75	100	*N*	10	25	50	75	100
10	.1005	.1776	.2611	.3209	.3681	10	.1061	.2117	.3186	.3920	.4487
20	.0543	.0975	.1487	.1870	.2181	20	.0592	.1230	.1927	.2424	2817
30	.0333	.0630	1000	.1283	.1515	30	.0361	.0820	.1341	.1721	.2025
40	.0249	.0467	.0743	.0956	.1133	40	.0267	.0617	.1022	.1319	.1559
50	.0195	.0373	.0608	.0794	.0949	50	0203	.0493	.0840	.1100	.1311
100	.0095	.0192	.0325	.0430	.0519	100	.0084	.0254	.0460	.0615	.0743
C.V. is 0.80						C.V. is 0.80					
	Return period						Return period				
N	10	25	50	75	100	*N*	10	25	50	75	100
10	1402	.2266	.3201	.3875	.4409	10	.1511	.2699	.3887	.4705	.5338
20	.0734	.1209	.1771	.2193	.2535	20	.0821	.1524	.2281	.2820	.3247
30	.0453	.0780	.1182	.1488	.1740	30	.0512	.1013	.1573	.1980	.2305
40	.0342	.0578	.0875	.1104	.1294	40	.0385	.0764	.1194	.1509	.1764
50	.0271	.0464	.0717	.0916	.1082	50	.0300	.0613	.0982	.1257	.1480
100	.0138	.0244	.0385	.0496	.0590	100	.0143	.0325	.0541	.0703	.0835

where

$$K_{GEV} = \frac{(-\log_e F)^k - (1+k)}{((1+2k) - (1+k))^{1/2}} \qquad k < 0$$

$$\frac{(-\log_e F)^k - (1+k)}{((1+2k) - (1+k))^{1/2}} \qquad k > 0 \qquad 2.11$$

$$= 0.45 + 0.779\ (-\log_e(-\log_e F)) \qquad k = 0$$

$$F = 1 - p$$

and

$$\log_e A = \text{const} + a\log_e N + b\log_e CV + c\log_e K_{GEV} \qquad 2.12$$

Table 2.4 (from Appendix A) gives the coefficients for Equation 2.11.

Table 2.4 Coefficients of model to estimate adjustment factors from n, cv and k. (Arnell, 1988; Appendix A)

Design return period (years)	const	*a*	*b*	*c*	R^2
10	0.1525	-0.02841	0.02177	0.9558	99.6
25	0.2013	-0.02839	-0.00142	0.9143	98.5
50	0.2007	-0.01997	-0.01316	0.9449	96.3
75	0.1456	-0.03101	-0.01866	1.0035	96.5
100	0.1046	-0.03167	-0.02211	1.0391	97.4

$\log_e A = \text{const} + a\log_e N + b\log_e CV + cK_{GEV}$; $AF = [\exp\{(1+1/n)^{1/2}A - K_{GEV}\}]^{(\log_e(1+CV^2))^{1/2}} - 1$.

2.4.3 Regional flood frequency analysis

Appendix A describes the calculation of adjustment factors when the flood frequency relationship is estimated from one sample. A similar approach could be adopted to estimate the adjustment factors required for a regional frequency analysis.

Table 2.5 shows, for illustrative purposes, the adjustment factors necessary for the 50 and 100 year flood estimates for several combinations of sample and region size, and for one homogeneous GEV parent distribution. The simulation experiments assumed that all sites in the region were from the same parent, and that there was no inter-site correlation. Different regional compositions would give different adjustment factors, which makes it very difficult to develop a generalised estimation method. However, it is clear from Table 2.5 that the adjustment factors for regional frequency analysis are small (indicating incidentally the benefits of règional flood frequency analysis over single site analysis).

Table 2.5 Adjustment factors for regional dimensionless flood $X_T/\bar{x}$, as estimate by regional GEV-PWM estimation procedure

GEV parent: k = -0.2, CV = 0.4

		Number of sites		
		10	20	40
T = 50				
	10	0.051	0.035	0.028
Number of years	20	0.025	0.018	0.015
at each site	40	0.012	0.010	0.009
T = 100				
	10	0.078	0.052	0.039
Number of years	20	0.038	0.026	0.021
at each site	40	0.019	0.014	0.012

2.5 CHAPTER SUMMARY

This chapter has presented some equations for estimating the probability of experiencing M events in N years, and has also outlined a procedure for determining the magnitude of event which will be exceeded in the future with the desired degree of risk. Conventional procedures, which aim to produce an unbiased estimate of the magnitude of the T-year event, tend to yield estimates that will, on average, be exceeded more frequently than once in T years in the future. This is due to sampling uncertainties in flood frequency analysis, and the difference between the two magnitude estimates can be seen as some form of "safety factor". Table 2.3 shows the increase in conventional estimate necessary to produce magnitudes exceeded in the future with the specified degree of risk, assuming the conventional estimates are made using a GEV distribution with parameters estimated by probability-weighted moments. The adjustment necessary increases as samples become shorter, as return periods increase, and as sample coefficient of variation and skewness increase: with a 10 year sample of flood data with average characteristics (CV of 0.4, GEV k parameter of -0.1), the conventional estimate of the 50-year flood magnitude would need to be increased by over 18% to produce an event with an expected probability of one in 50 years.

3. Estimating average annual damage

3.1 INTRODUCTION

Flood managers need a measure of the magnitude of a flood problem in order both to compare the extent of risk in different floodplains and to provide a basis for the economic evaluation of a flood management strategy. Such an index is provided by the average annual damage (or, in American usage, the expected annual damage), which is best understood as the average of flood damages computed over many years. One way to calculate this would be simply to add up a long time series of annual flood damage data and divide by the number of years, but this is not feasible in practice: long records are very rarely available, flood damages vary considerably from year to year making estimates from small samples very imprecise and, most importantly, exposure to flood loss will have changed over time.

Average annual damage is therefore estimated synthetically, by combining information on the damage incurred in a flood with the probability of experiencing that flood. A curve defining the relationship between flood damage and flood probability is usually constructed using relationships between flood discharge and frequency, flood discharge and depth, and flood depth and associated damages, as indicated in Figure 3.1. The flood magnitude-frequency relationship is of course based on hydrological analysis, the discharge-depth relationship is derived through hydraulic studies, and the floodplain depth-damage relationship is produced by combining depth-damage curves for each property (or property-type) on the floodplain. Such depth-damage curves were published for British floodplains by Penning-Rowsell and Chatterton (1977), with a major update in 1988 (Suleman et al, 1988). Although it is assumed that flood depth controls the bulk of damage incurred, procedures have been developed for estimating the differential effect of different flood durations, and the amount of indirect damages incurred (such as lost business: see Parker et al, 1987).

The average annual damage incurred in a floodplain can be derived from the probability distribution function of annual damages, but there is never enough information to define the form, let alone the parameters, of such a mathematical function: it is a combination of, for example, a General Extreme Value distribution characterising the flood discharge-frequency relationship, perhaps a logarithmic relationship defining the stage-discharge relationship, and an empirical, possibly stepped, function relating depth to flood damage. Average annual damage is therefore estimated by calculating the area under the graph of damage-probability function. The mean of a variable x distributed with probability density function f(x) is

$$E(x) = \int x \, f(x) \, dx \qquad 3.1$$

and since the cumulative distribution function F(x), defining the probability of experiencing an event less than or equal to X, is related to f(x) by

$$\frac{dF(x)}{dx} = f(x) \qquad 3.2$$

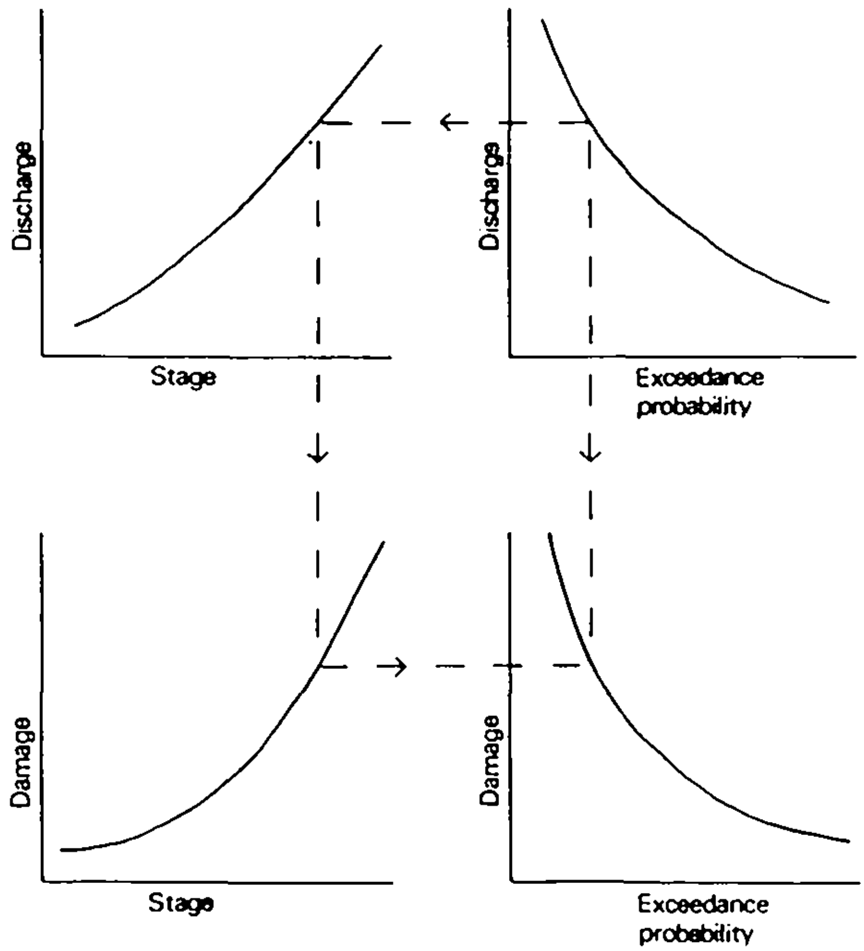

Figure 3.1 Estimation of damage probability curve

then

$$E(x) = \int x \, dF(x) \qquad 3.3$$

(or the area under the curve of x against F(x)).

A technique such as Simpson's rule could be used to determine this area, but in practice the relationship between flood probability and flood damage is based on only a very few points (such as the 25, 50 and 100 year flood profiles). Average annual damage is therefore estimated using the "mid-range probability" rule, as outlined in Figure 3.2.

$$\text{Av. Ann. Dam.} = \sum_{i=1}^{M-1} (P_i - p_{i+1}) \frac{(D_{i+1} + D_i)}{2} \qquad 3.4$$

where M is the number of pairs of data points, p_i is exceedance probability for point i and D_i is the associated damage. The precision of an estimate of average annual damage is influenced by the number of pairs of points considered, but the number of pairs required for a given degree of precision will depend on the smoothness of the damage-probability function: the greater the number of steps and abrupt changes, the larger the number of points required.

This chapter summarises some studies into the effect of hydrological uncertainties on bias and uncertainties in the estimation of average annual damage. More details are found in the paper reproduced as Appendix B (Arnell, 1989).

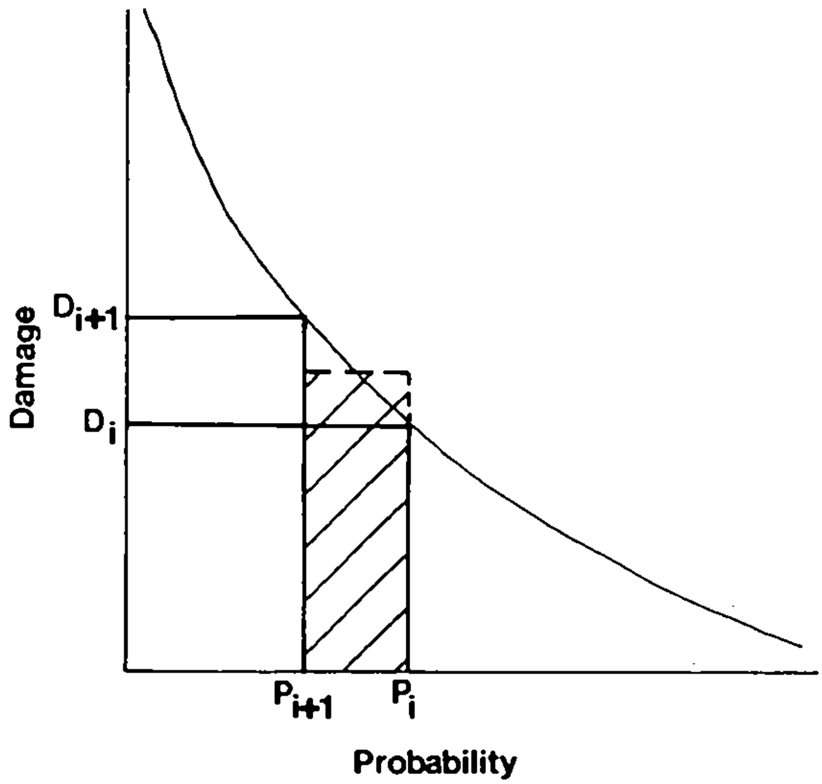

Figure 3.2 Calculation of average annual damage using the mid-range probability rule

3.2 BIAS AND UNCERTAINTY IN THE ESTIMATION OF AVERAGE ANNUAL DAMAGE

The average annual damage at a site is based on estimates of the various components which link together to produce average annual damage. The reasons for uncertainties in the flood frequency relationship at a site are well known frequency analyses are usually based on very limited samples but there are also uncertainties in the hydraulic analyses converting discharge to flood depths across and along a floodplain. In principle the depth-damage relationship (and all the other relationships predicting damage from flood characteristics) in a floodplain can be defined exactly, but in practice uncertainties arise because standard depth-damage relationships are applied in preference to property-by-property surveys and damages in a future event may be influenced by characteristics of the event assumed irrelevant (such as the time of day the flood peaks). The relative importance of each of these sources of uncertainty will vary between sites, but it is likely that hydrological uncertainties will frequently be the most important. This chapter therefore describes some investigations into the effects of hydrological uncertainties, as represented by the estimation of frequency distribution parameters from small samples, on bias and uncertainty in average annual damage. More details are provided in Appendix B.

The basic method adopted was to construct a series of simulation experiments to explore the effect of parameter uncertainty on the bias and precision of estimates of average annual damage. Three different approaches were compared:

(i) the conventional approach, with the parameters of the flood frequency distribution estimated using maximum likelihood;

(ii) an approach using expected probability (see Chapter 2): Beard (1978) argued that less biased estimates of average annual damage would be calculated when flood risk was expressed in terms of expected probability;

(iii) an approach taking explicit account of the sampling distribution of flood quantile estimates: the damage value for a given flood quantile is taken to be the mean of the sampling distribution of estimates for that quantile (James and Hall, 1986):

$$E(D) = \int_0^1 D\ h(D)\ \ dF \qquad 3.5$$

where h(D) is the probability density function of the estimate of damage D for a given frequency event. It is a combination of the probability density function of the flood discharge and the (determininstic) relationship relating discharge to flood damage.

A number of simplifying assumptions were made. Firstly, it was assumed that floods followed a log-normal distribution, and that the only uncertainties lay in the estimation of the parameters of such a distribution from a small sample. A log-normal distribution was used because all three estimation procedures could be readily applied (as outlined in Appendix B). Secondly, the damage functions were assumed to follow four simple mathematical functions (Figure 3 in Appendix B): this was to allow the investigation of the effects of the shape of the damage function on bias and uncertainties. Flood damage was assumed to begin at three different return period thresholds, namely the <u>true</u> 5, 25 and 100-year floods.

Table 3.1 (reproduced from Appendix B) shows the bias in estimates of average annual damage, for two of the four damage functions. It is clear that all the methods overestimate average annual damage, but that the conventional

Table 3.1 Bias in estimation of average annual damage, expressed as percentage of the true value (Arnell, 1989; Appendix B)

Probabilities (1)	DAMAGE FUNCTION					
	Quadratic			Logistic		
	10* (2)	20* (3)	40* (4)	10* (5)	20* (6)	40* (7)
Threshold probability = 0.2						
conventional method	7.6	0.4	1.7	33.8	12.0	8.7
expected probability method	38.5	16.0	9.6	120.9	52.6	28.5
expected damage method	41.4	17.5	10.5	143.5	64.4	35.5
Threshold probability = 0.04						
conventional method	53.3	20.9	14.1	169.3	68.6	41.8
expected probability method	86.5	80.0	42.6	622.9	232.0	108.5
expected damage method	200.9	84.6	44.5	846.4	340.0	168.0
Threshold probability = 0.01						
conventional method	179.3	72.7	43.3	568.4	194.7	105.3
expected probability method	644.0	246.7	118.0	2,647.3	742.1	278.9
expected damage method	714.7	273.3	129.3	4,163.6	1,400.0	589.5

*Sample size.
Note: Simulation results from 500 repetitions.

method produces the least biased estimates. Bias increases as the threshold at which damage begins rises: the greater the frequency of flooding at a site, the less the average annual damage is over-estimated. Bias also reduces as sample size increases.

The similarities between the expected probability method (method ii above) and the expected damage method (method iii) reflect similarities in their derivation: in the case of the lognormal distribution, both methods use the t-distribution, and the expected value of the p-probability flood is close to the value of the flood exceeded with expected probability p. With the linear damage function the two methods give very similar results, but differences increase as the damage function becomes less linear.

The magnitude of average annual damage at a site is closely dependent on the estimated probability at which flood damage begins. Although the expected probability method produces an estimate of the flood magnitude exceeded with the specified degree of risk, it produces a biased estimate of the probability associated with a particular, fixed, magnitude (because of the non-linear relationship between flood magnitude and probability: an estimator cannot produce estimates that are unbiased in all dimensions). The conventional approach, however, gives a much less biased estimate of the exceedance probability of a flood of a particular size (such as the size at which damage begins), and therefore produces a less biased estimate of average annual damage. Beard (1978) argued that conventional procedures underestimated average annual damage, and that the use of expected probability compensated: the results of this study imply that the use of expected probability leads to even greater overestimation of average annual damage.

Beard (1990), commenting on Arnell (1989), has reiterated his conclusion that the use of expected probabilities gives better estimates of average annual damage. However, his experiments and those reported in the current study start from different premises. If it is assumed that there is an underlying average annual damage that is waiting to be estimated, with the only uncertainty being in the probability of a given value of damage occurring - as in the current study - then the use of expected probabilities does not give the best estimate of the underlying average annual damage. Beard (1990), however, assumes that the observed sample could have come from a range of different populations, and that the most appropriate value of average annual damage is the average of all the possible parent values.

The effect of the shape of the damage function can be seen in Table 3.1. In each case damage is assumed to begin at the same level, but damage with the logistic function at first increases only slowly with flood magnitude: the point at which flood damages change significantly with magnitude is at a higher level than with the quadratic curve (where the greatest change in damage with magnitude is at the lowest levels).

Table 3.2 shows the standard deviation of the sample estimates of average annual damage, again for just two of the damage functions. Variability is high, and is highest for the expected probability and expected damage methods. With

Table 3.2 Standard deviation of estimation of average annual damage, divided by the true value (Arnell, 1989; Appendix B)

	DAMAGE FUNCTION					
	Quadratic			Logistic		
Probabilities (1)	10* (2)	20* (3)	40* (4)	10* (5)	20* (6)	40* (7)
Threshold probability = 0.2						
conventional method	0.80	0.53	0.40	1.48	0.89	0.66
expected probability method	0.86	0.56	0.41	1.86	1.05	0.72
expected damage method	0.86	0.56	0.42	1.84	1.05	0.72
Threshold probability = 0.04						
conventional method	2.03	1.17	0.86	5.07	2.35	1.57
expected probability method	2.69	1.44	0.96	8.58	3.60	2.04
expected damage method	2.71	1.44	0.96	8.71	3.69	2.10
Threshold probability = 0.01						
conventional method	5.35	2.54	1.75	18.11	6.26	3.58
expected probability method	440.11	3.79	2.22	40.11	12.68	5.63
expected damage method	8.92	3.80	2.20	43.11	14.21	6.47

*Sample size.
Note: Simulation results from 500 repetitions.

a sample size of 20 years, the standard deviation of the estimates of average annual damage when floods begin at the (true) 5-year flood is over 50% of the average annual damage, indicating that the precision of an individual estimate is low.

The standard error of the sampling distribution of average annual damage at a site, and hence some indication of the precision of the estimate, can be calculated from the the standard deviation of annual damages. The standard error is simply

$$\text{s.e. (D)} = \frac{\text{s.d. (D)}}{N^{1/2}} \qquad 3.6$$

and the standard deviation of annual damages can be estimated by calculating $E(D^2)$, the area under the 'damage-squared' curve:

$$\text{s.d. (D)} = [E(D^2) - E(D)^2]^{1/2} \qquad 3.7$$

The simulation experiments indicated that estimates of the standard error of average annual damage calculated using equations 3.6 and 3.7 were quite precise, but unfortunately the sampling distributions of average annual damage are highly (positively) skewed. It is not possible to estimate confidence intervals for average annual damage from the estimated standard error alone.

3.3 SUMMARY AND IMPLICATIONS

This chapter has described some studies into the effects of hydrological uncertainties on bias and uncertainty in the estimation of average annual damage. It has been found that conventional flood frequency estimation procedures tend to overestimate true underlying average annual damage, with the bias increasing as the threshold at which damage begins to occur rises. This bias is due to uncertainties in the estimation of the probability at which damage begins, and the non-linear relationship between damage and probability. Uncertainties in the estimation of flood frequency distribution parameters from small samples mean that estimates of average annual damage are very uncertain, with the standard error of their sampling distribution being considerably larger than the mean for high damage thresholds.

The expected probability procedure was found to produce more highly biased estimates of average annual damage than the conventional procedure. This is because the estimated average annual damage at a site is controlled by the estimated probability at which flood damage begins: whilst the expected probability procedure produces an estimate of the magnitude which will be exceeded with a given degree of risk, it produces a biased estimate of the probability of a fixed magnitude event.

The implications of these studies for the calculation of average annual damage are that estimates will be very uncertain and, on average, too high (by an unknown degree). The precision and bias of an estimate is largely determined by the accuracy with which the probability at which damage begins can be determined, and it is clear that the emphasis in a flood frequency analysis needs to be placed on the estimation of such floods.

4. Confidence intervals

4.1 INTRODUCTION

Flood frequency estimates are not precise, and one way of describing their uncertainty is to calculate confidence intervals around an estimate. This chapter summarises briefly the calculation of confidence intervals for an estimate of the T-year flood, and considers the derivation of confidence intervals for the return period associated with a particular magnitude flood. Finally, the chapter outlines the difficulties with estimating confidence intervals for average annual damage, and suggests a possible approach to use in practice.

4.2 CONFIDENCE INTERVALS FOR X_T

4.2.1 Introduction

A confidence interval for an estimate of X_T, the T-year flood, indicates the degree of precision of that estimate: the wider the interval, the less precise the estimate. There are two possible interpretations of a confidence interval, one deriving from classical statistics and the other following a Bayesian approach.

The classical approach assumes that there is a single true, but unknown, value of the T-year flood. The confidence intervals placed around an estimate from a sample define the probability that the true population value is within some defined limits: there is a 95% chance, for example, that the true T-year flood lies between 27 m^3/sec and 52 m^3/sec. The confidence interval is of course estimated from the sample, and different samples would give different confidence intervals. The classical interpretation is that, in the long run, 95% of all such estimated limits would contain the true parent value.

The Bayesian approach is rather different, and assumes that there is no single, fixed, true population value of the T-year flood. Instead, the approach asssumes that it is possible to estimate the probability of any particular value being the 'true' population value, given both sample and 'prior' information: what is the probability of the 'true' T-year flood being greater than 58 m^3/sec, for example, given the characteristics of the recorded sample? Under some limiting circumstances, the classical and Bayesian approaches yield numerically similar results.

Three main approaches have been used to estimate confidence limits for the T-year flood. The first two - using estimates of the variance of the T-year flood and attempting to derive the exact form of the sampling distribution of the T-year flood - are based on the classical approach, whilst the third follows a Bayesian interpretation. The "classical" methods consider only *parameter* uncertainty, and assume the form of the underlying model is known: different model assumption will therefore produce different confidence intervals. The

Bayesian approach can allow for the effect of model uncertainty by producing a composite frequency distribution from a range of candidates.

4.2.2 Methods based on the variance of the T-year flood

In the most general terms, these methods derive confidence intervals by estimating the variance of the T-year flood and assuming that the sampling distribution is normal (see Kite, 1975): confidence intervals are then determined as:

$$\hat{X}_T - z_p\ se(\hat{X}_T) < X_T < \hat{X}_T + z_p\ se(\hat{X}_T) \qquad 4.1$$

where $se(\hat{X}_T)$ is the standard error of estimate of X_T and z_p is the standard normal deviate with exceedance probability p.

Methods have been developed to estimate the variance of the T-year flood for practically all of the probability distributions and parameter estimation procedures used in practice. The quantile estimate $\hat{X}_T$ is a function of the estimated parameters $\hat{\theta}$:

$$\hat{X}_T = X_T(\hat{\theta}) \qquad 4.2$$

which can be expanded as a Taylor series around the true parameters θ:

$$\begin{aligned} X_T &= X_T(\theta + \hat{\theta}-\theta) \\ &= X_T(\theta) + (\hat{\theta}-\theta)\ X_T'(\theta) + \frac{(\hat{\theta}-\theta)^2}{2} X_T''(\theta) + \end{aligned} \qquad 4.3$$

where X_T' and X_T'' represent the first and second derivatives of X_T with respect to θ. Ignoring squared terms and above, the variance of X_T is (from the general form for the variance of aY+bZ)

$$var(X_T) = \sum_{r=1}^{n} \sum_{s=1}^{n} \frac{dX_T}{d\theta_r} \frac{dX_T}{d\theta_s} cov(\theta_r,\theta_s) \qquad 4.4$$

There are three approximations involved in the calculation of $var(X_T)$: first, the Taylor series expansion omits squared and higher terms; second, the derivatives are evaluated not at the true parameter values, but at their sample estimates; and third, the variance-covariance matrix of the parameter estimates must usually be approximated in practice. This third approximation is probably the most important. Approximations to the variance-covariance matrix for a particular probability distribution and parameter estimation procedure tend to be derived asymptotically.

Much less effort has been directed towards the derivation of the variance of an estimate X_T based on a regional flood frequency analysis, although an approximate procedure is given in the Flood Studies Report (NERC, 1975). If an estimate of X_T is derived by combining a regional estimate of the dimensionless quantile $X_T/\bar{X}$ with a local estimate of $\bar{X}$, the mean annual flood, the variance of the quantile estimate is

$$\mathrm{Var}(X_T) = \mathrm{Var}((X_T/\bar{X}).\bar{X})$$

$$= E(X_T/\bar{X})^2 \ \mathrm{var}(\bar{X}) + E(\bar{X})^2 \ \mathrm{var}(X_T/\bar{X}) \qquad 4.5$$

(assuming that the mean annual flood $\bar{X}$ and the dimensionless T-year flood $X_T/\bar{X}$ are independent). The variance of the mean annual flood $\bar{X}$ depends on how it is estimated, and Wiltshire (1987) presents a procedure for estimating the variance of an estimate derived from regression relationships of the form frequently used in regional analyses. In the Flood Studies Report (NERC, 1975) the variance of the dimensionless quantile $X_T/\bar{X}$ was based on an empirical assessment of the variability in estimated quantiles within a case study region, but Wiltshire (1987) calculated the variance for an estimate derived from a regional GEV-PWM fit using equation 4.4 and assuming that the region was homogeneous. In practice, the variance of a regional estimate of $X_T/\bar{X}$ is a function both of the within-region variability and the sampling error of an estimate based on m' *independent* catchments (which may be rather less than the actual number of catchments).

The most important assumption with regard to the estimation of confidence intervals, however, is the assumption that estimates of X_T from different samples are normally distributed. The derivation of exact confidence intervals - see below - and computer simulation experiments (eg Stedinger, 1983b) show this assumption to be false, particularly at high return periods, and sampling distributions are highly skewed. The assumption that they are normal means in practice that the upper confidence limit may be considerably underestimated. Figure 4.1 shows confidence intervals, based on computer simulation, for a sample of length 20 drawn from a GEV distribution with 'typical' UK parameters, together with those estimated from the variance of the T-year flood and the assumption that the sampling distributions were normal. The shorter the record length, the higher the return period and the higher the coefficients of variation and skewness, the further the sampling distribution of X_T departs from the normal distribution.

4.2.3 Exact confidence intervals for X_T

The exact form of the sampling distribution for estimates of the magnitude of the rth rank observation from a sample of size N is defined by

$$g(X_r) = \binom{N}{r} (N - r) \ [F(X)]^r \ [1\text{-}F(X)]^{N-r-1} \ f(x) \qquad 4.6$$

but this of course is only applicable to a subset of possible return periods.

Stedinger (1983b), however, derived exact confidence intervals for any return period for the normal and lognormal distributions. He showed that the variable

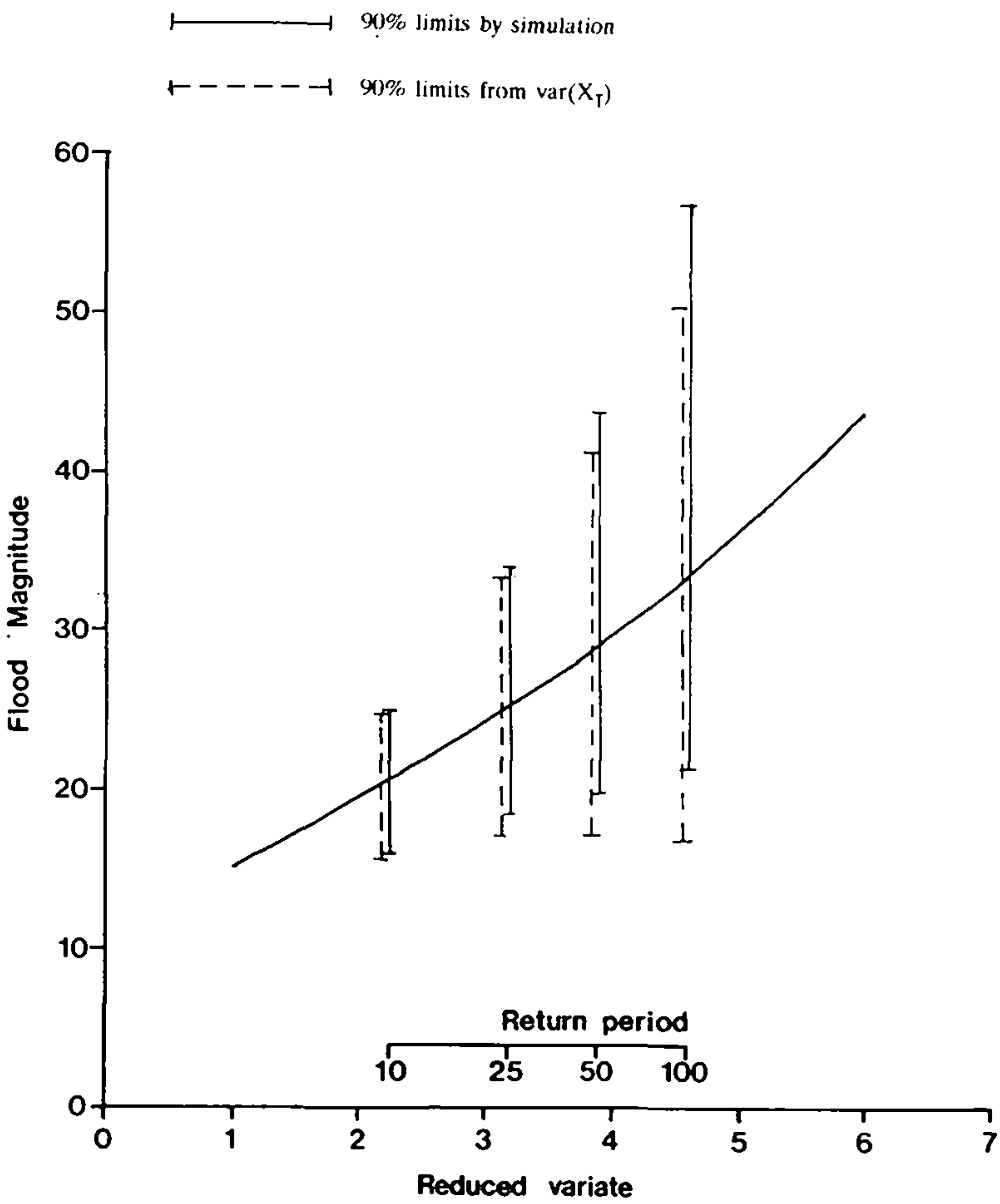

Figure 4.1 *Confidence intervals for X_T, with GEV parameters estimated by PWM: intervals based on var(X_T) and on simulation experiments. Simulation experiments with 1000 repetitions. GEV parent parameters u=10, a=4, k=-0.1, and sample size 20.*

$$S(T) = N^{½} \frac{X_T - \bar{X}}{s_x} \qquad 4.7$$

where $\bar{X}$ is the mean annual flood and s_x is the standard deviation of annual floods, was distributed following a non-central t-distribution with centrality parameter

$$c = z_p N^{½} \qquad 4.8$$

and N-1 degrees of freedom. Again, z_p is the normal reduced variate with exceedence probability p=1/T. Tables were produced showing the value of S(T) for a given sample size, return period and confidence level: the resulting confidence intervals are not normally distributed about the estimate X_T. A procedure was proposed for deriving 'approximate' exact confidence intervals for the Pearson type III distribution from those for the normal distribution. Ashkar *et al* (1987) followed a similar approach to derive exact confidence intervals for the Weibull and Gumbel (or EV1) distributions, and Ashkar and Bobee (1988) refined Stedinger's (1983b) procedure for deriving Pearson type III confidence intervals. No papers, however, have attempted to derive exact confidence intervals for the GEV distribution.

4.2.4 Confidence intervals from a Bayesian perspective

As indicated above, the Bayesian approach assumes that an assessment of the probability of a particular set of parent parameters being 'correct' can be based on a combination of sample information with prior expectations (see Wood and Rodriguez-Iturbe, 1975, for example). Each possible combination of parameters yields an estimate of X_T, and the probability of the 'true' quantile exceeding X_T can be derived by integrating across the joint probability distribution of parameter values (this probability distribution is the 'marginal posterior distribution' of X_T: it is a 'posterior' distribution because it combines sample information with prior knowledge). In practice, however, this can be extremely difficult, and some simplifying assumptions need to be made about the form of the underlying probability distributions of the parameters.

Cunnane and Nash (1971) used simulation experiments to derive the marginal posterior distribution of X_T, assuming that annual floods followed the Gumbel (or EV1) distribution and using two different forms for the assumed distributions of the Gumbel parameters. Stedinger (1983b) showed analytically for the normal (and lognormal) distribution that, under a particular set of assumptions about the form of the probability distribution of parameters, the variable

$$S(T) = N^{½} \frac{X_T - \bar{X}}{s_x} \qquad 4.9$$

has a non-central t-distribution. This is the same as equation 4.7, and the classical and Bayesian approaches to the calculation of confidence intervals for the normal (or lognormal) distribution therefore give exactly the same answers (if sample information only is used in the Bayesian approach). By combining several candidate probability distributions, a composite Bayesian distribution can incorporate the effects of model uncertainty as well as parameter uncertainty (Wood and Rodriguez-Iturbe, 1975): in principle, a marginal posterior distribution of X_T can be derived as in the single distribution case.

Whilst the Bayesian approach appears useful (both because of the ability to include non-sample information and the relatively simple interpretation of Bayesian confidence intervals), it is extremely difficult to apply in practice.

Analytical derivations are forced to make some restrictive assumptions about the probability distributions of model parameters, and numerical integration can be very difficult. There have been no published attempts to apply Bayesian analyses to the three-parameter GEV distribution.

4.2.5 Implications

The initial implication of this section is that it is easy to estimate incorrect confidence intervals, but considerably more difficult to estimate correct intervals (except for the lognormal distribution). More work is needed on the derivation of exact confidence intervals for the probability distributions and estimation procedures used in the UK (primarily the GEV distribution with parameters estimated by the method of probability-weighted moments), and in particular in regional frequency analyses.

In practice, it appears that the most appropriate method of estimating confidence intervals for the GEV distribution (or indeed many apart from the normal and lognormal) is through computer simulation experiments. The approach would be to estimate GEV parameters from the sample, repeatedly draw samples and build up the sampling distribution of X_T. The experiments reported in Stedinger (1983b) indicate that the confidence intervals derived from simulation compare very closely with exact confidence intervals (at least for the normal and lognormal distributions). It should be emphasised, however, that the confidence intervals assume that the data do follow a GEV distribution: different intervals would arise with different assumed distributions.

4.3 CONFIDENCE INTERVALS FOR T_X

Most effort tends to be placed on defining confidence intervals for an estimate of the magnitude of the T-year flood, but confidence limits for the estimated return period of a particular magnitude flood are often required in practice.

If the return period of an event X is estimated simply from

$$p = r/N \qquad 4.10$$

where r is the number of floods greater than or equal to X in a sample of size N, then confidence intervals for T=1/p can be defined using the binomial distribution. The probability of experiencing r events in N years with annual probability of occurrence p is

$$p(r|N,p) = \binom{N}{r} p^r(1-p)^{N-r} \qquad 4.11$$

but different values of p could produce the same number of occurrences r in N years. The probabilities of experiencing r or more, or r or fewer, events in N years are

$$p(r \text{ or more}) = \sum_{i=r}^{n} \binom{N}{i} p^i(1-p)^{N-i} \qquad 4.12a$$

$$p(r \text{ or fewer}) = \sum_{i=0}^{r} \binom{N}{i} p^i(1-p)^{N-i} \qquad 4.12b$$

One confidence limit can be calculated from equation 4.12a by solving to give the probability p which yields a probability of experiencing r or more events of, for example, 0.05, and the other confidence limit can be defined in a similar way from equation 4.12b. The equations can be solved by graphing the relationships between p and p(r or more) and p(r or less), or by using the incomplete beta function.

However, binomial confidence intervals are not of much use in flood frequency analysis in practice, because flood return periods are very rarely estimated from r/N: the return period of an event of magnitude X is more usually a function of the estimated parameter set $\hat{\theta}$:

$$T_X = f(\hat{\theta}) \qquad 4.13$$

The use of binomial confidence intervals when T_X is a function of distribution parameters (as was proposed by Oosterbaan, 1988) gives a misleading indication of the precision of an estimate. Simulation experiments indicate that binomial confidence intervals are too wide: they tend to underestimate the upper limit for T_X at a given level of confidence, but underestimate the lower limit even more.

The sampling distribution of p for the rth rank observation in a sample of size N is a beta distribution with parameters N and N-r-1 but, as is the case with equation 4.6, this is only useful for a small subset of cases. However, if it is assumed that the sampling distributions of p at all magnitudes follow the beta distribution, it is possible to determine confidence intervals by estimating beta distribution parameters from p and the standard error of p at a particular magnitude X: the method is analogous to the procedure for estimating confidence intervals on flood magnitudes outlined in 4.2.3. The standard error of p_X can in principle be determined using

$$\text{var}(p_X) = \sum_{r=1}^{n} \sum_{s=1}^{n} \frac{dp_X}{d\theta_r} \frac{dp_X}{d\theta_s} \text{cov}(\theta_r,\theta_s) \qquad 4.14$$

which is directly equivalent to equation 4.4. An alternative approach would be to estimate the variance of the linear reduced variate (such as the

Gumbel-reduced variate y=-ln(-ln(1-p))) at magnitude X (using an equation similar to 4.13) and assume that the sampling distribution of reduced variates was normal. Neither of these methods have been evaluated or attempted in practice.

One approach that is occasionally adopted is to derive confidence intervals for X_T, plot lines joining all the, for example, 95% confidence intervals, and define confidence intervals for a particular magnitude X by drawing horizontal lines to meet the confidence interval 'envelope' (as illustrated in Figure 4.2). However, there is no immediately apparent reason why this procedure should necessarily give the correct answer, and confidence intervals constructed in different directions - with different interpretations - need not coincide.

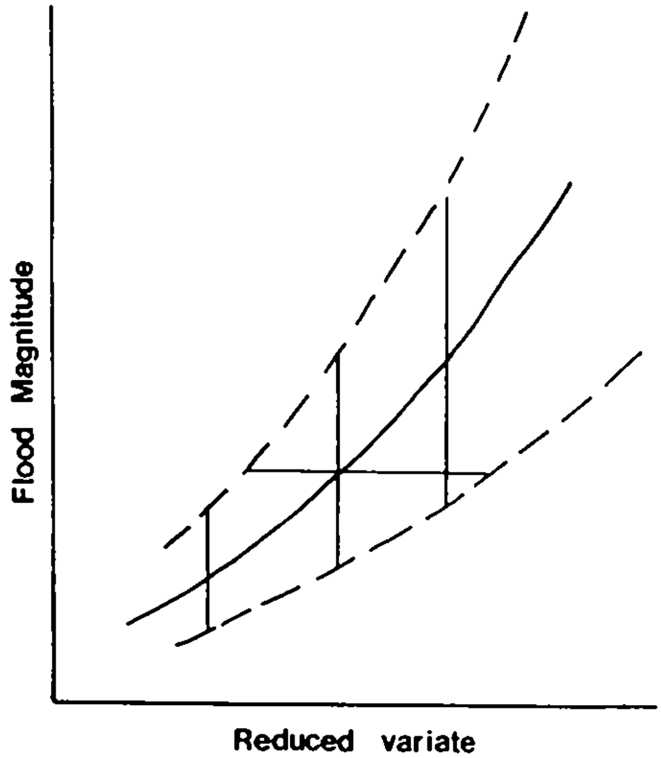

Figure 4.2 Estimating confidence intervals on T_X from intervals on X_T

The most appropriate way to construct confidence intervals on T_x (or indeed p_x) at present appears to be based on computer simulation experiments, as is the case for confidence intervals on X_T. Figure 4.3 shows confidence intervals for both X_T and T_X, assuming that annual maximum floods follow a GEV distribution with 'typical' parameters: the shorter the record, the more variable the data and the higher the return period the wider the confidence intervals. It is interesting to note that the confidence intervals on T_X derived by the simulation experiment are quite similar to those which could have been estimated from the confidence intervals on X_T.

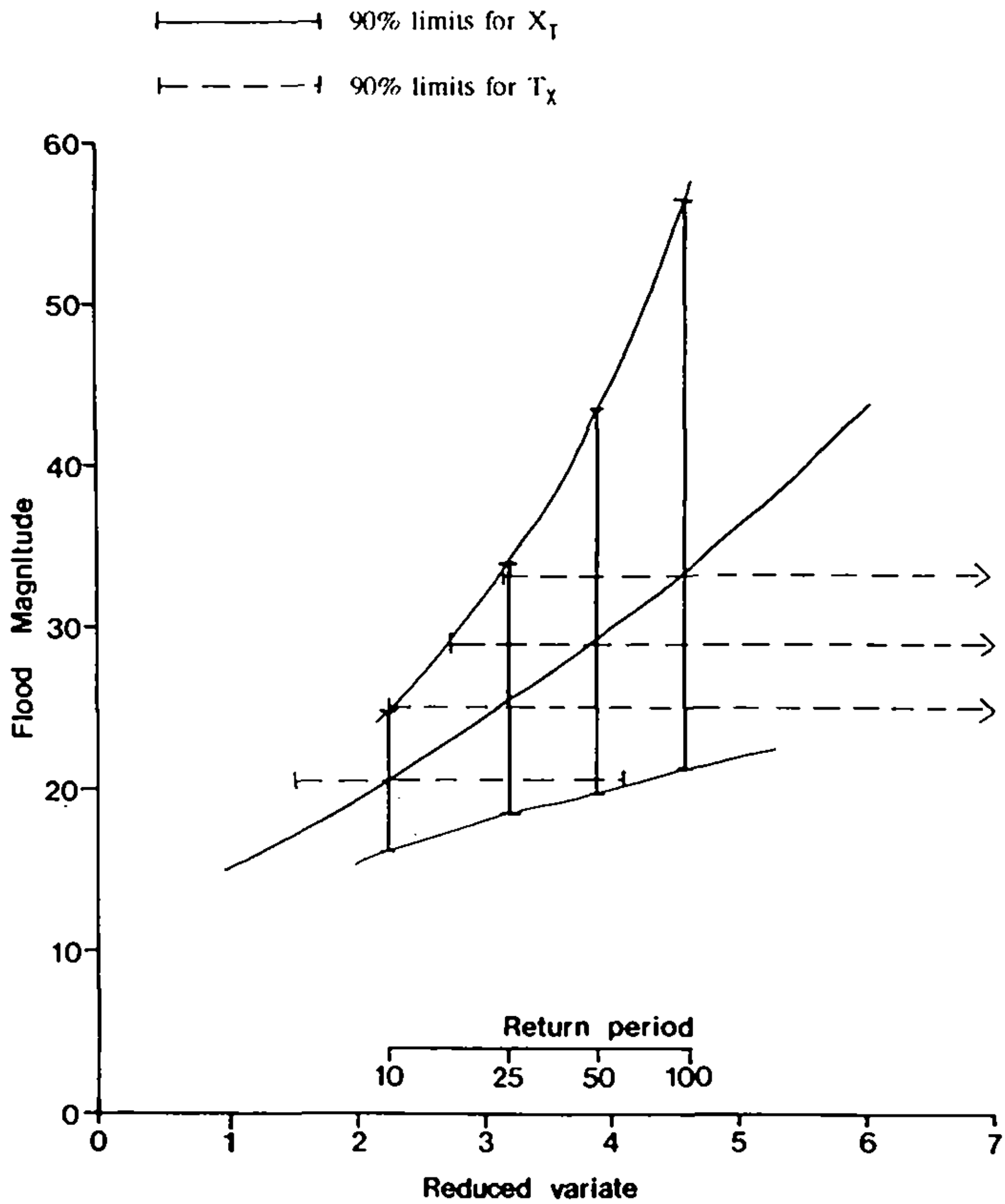

Figure 4.3 *Confidence intervals for X_T and T_X: GEV parameters estimated by PWM. Simulation experiments with 1000 repetitions, GEV parent parameters u=10, a=4, k=-0.1, and sample size 20*

4.4 CONFIDENCE INTERVALS ON AVERAGE ANNUAL DAMAGE

Formal confidence intervals are very rarely calculated for estimates of average damage. Analytical derivations are made difficult by the 'unfriendly' characteristics of depth-damage curves in practice, and although it is possible to estimate the standard error of estimate of average annual damage (as indicated in Chapter 3), the form of the sampling distribution is unknown: it is however clear that it is most unlikely to be normal.

Grigg (1978) developed a method to estimate confidence intervals on average annual damage, based on the confidence intervals on flood magnitudes. The method, illustrated schematically in Figures 4.4a to 4.4c (from Appendix B), is however based on a misinterpretation of confidence intervals. The locus of the, for example, 90% confidence intervals does not define the frequency curve which will be exceeded over all probabilities 90% of the time: one sample curve may yield an estimate of the 10-year flood outside the 90% confidence intervals for the 10-year flood, whilst yielding estimates of other magnitude floods closer to the mean value (as shown in Figure 4.4d). Such an approach will overestimate confidence intervals on average annual damage, and give an unduly pessimistic impression of precision.

As with estimates of flood magnitude and return period, it appears that the most practical method of estimating confidence intervals for average annual damage is with the aid of computer simulation experiments. No examples have been reported, but it is probable that confidence intervals in practice will be very wide.

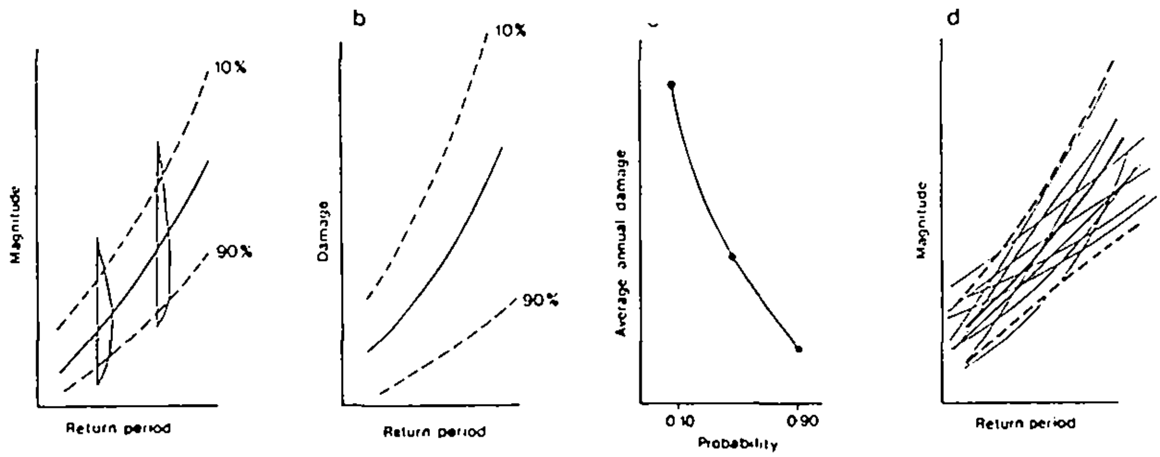

Figure 4.4 Calculation of sampling distribution of average annual damage from confidence intervals on flood magnitude: and why it is incorrect (Arnell, 1989; Appendix B)

4.5 SUMMARY

This chapter has summarised methods for estimating confidence intervals for (i) the flood magnitude with a particular return period; (ii) the return period associated with a particular flood magnitude and (iii) average annual damage. By far the greatest amount of attention has been placed on the derivation of confidence intervals for estimates of flood magnitudes, but accurate exact methods still need to be developed for flood frequency distributions and estimation procedures that are frequently used in practice. Very little attention has been directed towards the estimation of confidence intervals in regional flood frequency analysis.

For practical purposes, it appears that the most accurate way of deriving confidence intervals for flood magnitudes, flood return periods and average annual damage is through computer simulation experiments with the parent characteristics based on observed data.

5. Estimating flood alleviation benefits from time series of future flood losses

5.1 INTRODUCTION

The present value of the flood damages at a site (and hence the flood alleviation benefits) is conventionally determined by calculating the average annual damage and discounting this average value over the project life N_p. This assumes that the average damage occurs each year, which of course does not happen. In practice, the present value of the damages actually experienced during the next N_p years will depend on the distribution of events over time: the greater the number of large floods at the beginning of the period, the greater the present value of flood damages. The present value of flood damages over the next N_p years therefore has a probability distribution, with the probability of each different value equal to the probability of experiencing a different pattern of flood timing. From this distribution it would be possible to say, for example, that whilst the conventional estimate of the present value of floods damages over the next 30 years is £25million, there is a 10% chance that the present value could be less than £12million, or a 10% chance of it exceeding £50million: it would also be possible to say that a scheme costing £30million (at present values) would have a 45% chance of being cost effective over the next 30 years. A decision to implement a scheme could therefore be based on an assessment of the probability of experiencing different total benefits, rather than simply on the average potential benefit. Such a risk-based approach could be used to justify flood protection in areas where the conventional use of the present value of average annual damages indicates a scheme would not be economically effective.

This chapter describes a method for estimating the probability distribution of the present value of annual flood damage over a project design life. Much of the implementation of the method was done under contract to a Water Authority but the initial development work was done under the auspices of the MAFF project. The MAFF project had earlier provided the basis for Beran's (1987) time-dependent approach to costing the effects of floods. His procedure was based on the evolution of a flood relief fund which is increased by interest but depleted by withdrawals to pay for flood losses. The probability of the fund being exhausted depends on the initial sum placed in the fund, and the procedure allows the starting value with a given probability of exhaustion to be determined: this value can provide the basis, for example, for compensation against future flooding

5.2 THE METHOD

The number of possible future distributions of floods over the next N years is infinitely large, and remains large even when flood damages are grouped into discrete classes: there are 2^N different, equally likely, possible future time series if damages fall into just two categories. It is therefore impossible in practice to evaluate numerically the present value of future flood damages under all possible future outcomes, and it is necessary to resort to an approach based on computer simulation.

The approach developed has the following stages:

(i) define the input characteristics, namely the flood frequency relationship, the damage-magnitude/duration relationship (perhaps with and without the alleviation scheme), the discount rate, the project design life (or, more generally, the time horizon of interest N), the desired 'target' present value of damage (which may be equivalent to scheme costs, for example), and the number of repetitions in the experiment;

(ii) generate N years of floods, converting each annual value to flood damage and discounting to present values. Maintain a running sum of the present values;

(iii) store both the total present value of damages incurred and the year at which the accumulated present value exceeded the target value;

(iv) repeat stages (ii) and (iii)

(v) produce histograms and cumulative frequency distributions describing both the present value after N years and the time taken to achieve the target value.

The present value of damage incurred in year I is calculated from:

$$\text{Present value}_I = \frac{\text{Damage}_I}{(1+r)^I} \qquad 5.1$$

where r is the discount rate.

It is important to note that the mean of all possible estimates of the present value of damages after N years is exactly equal to the present value of average annual damages discounted over N years. In other words, the conventional estimate of the present value of flood damages at a site is equal to the expected present value.

Beran's (1987) procedure was also based on the simulation of time series of floods.

5.3 AN EXAMPLE APPLICATION

The method outlined in the previous section was implemented on a PC, and applied to the estimation of the present value of flood damages at a site in southern England. The following assumptions were made:

(i) the relationshp between flood magnitude and flood frequency remained constant into the future: this assumption could be relaxed to allow for,

for example, the effects of climate or land use change;

(ii) the exposure to flood loss remains constant over time: this too could be relaxed to allow for changes in floodplain land use;

(iii) only one flood can cause damage in any one year: this assumption could be relaxed by using a different flood generation process (such as one that generates a different number of floods each year);

(iv) the discount rate was fixed at 5%, and the target present value (in this case estimated scheme costs) was set at £26.68million;

(v) 1000 repetitions were made.

Figure 5.1 shows four different synthetic 50 year time series, each with a different present value of flood damages: the difference between Run 1 and Run 2 is particularly striking, and emphasises the greater 'value' of floods which occur towards the beginning of the time period.

The distributions of present values of flood damages after 25, 50 and 75 years are shown in Figure 5.2. There is clearly little difference between 50 and 75 years, because the present value of even large floods 50 years into the future is small (less than 9% of the monetary value, with a discount rate of 5%). Over the next 50 years, the present value of actual flood losses (and hence realised scheme benefits) ranges from £4million to nearly £100million. Although the present value of average annual benefits is less than the present value of scheme costs, there is a 34% chance that the actual benefit will be greater than the scheme costs. For comparative purposes, there would be only a 44% chance that a scheme that was just cost effective (with the present value of costs equal to the present value of benefits) would actually give benefits greater than costs over the next 50 years.

Figure 5.3 shows the distribution of times needed to accumulate the target present value. There is a 10% chance that the target will be reached within approximately 12 years, but a 45% probability that the benefits will never reach the target.

The sensitivity of the conclusions to changes in the target benefit value and flood frequency relationship were also explored in the original study.

5.4 CONCLUSIONS

This brief chapter has summarised a simple method for determining the probability that the present value of flood losses (and hence flood alleviation benefits) actually incurred over the next N years, which depends on the timing of flood events, exceeds particular specified target values. The method allows the adoption of a risk-based approach to scheme evaluation. The conventional approach is based on the expected value of future benefits, but an alternative would be to implement a scheme if there was, for example, at least a 30% chance of scheme costs being covered over the ensuing time period (note that

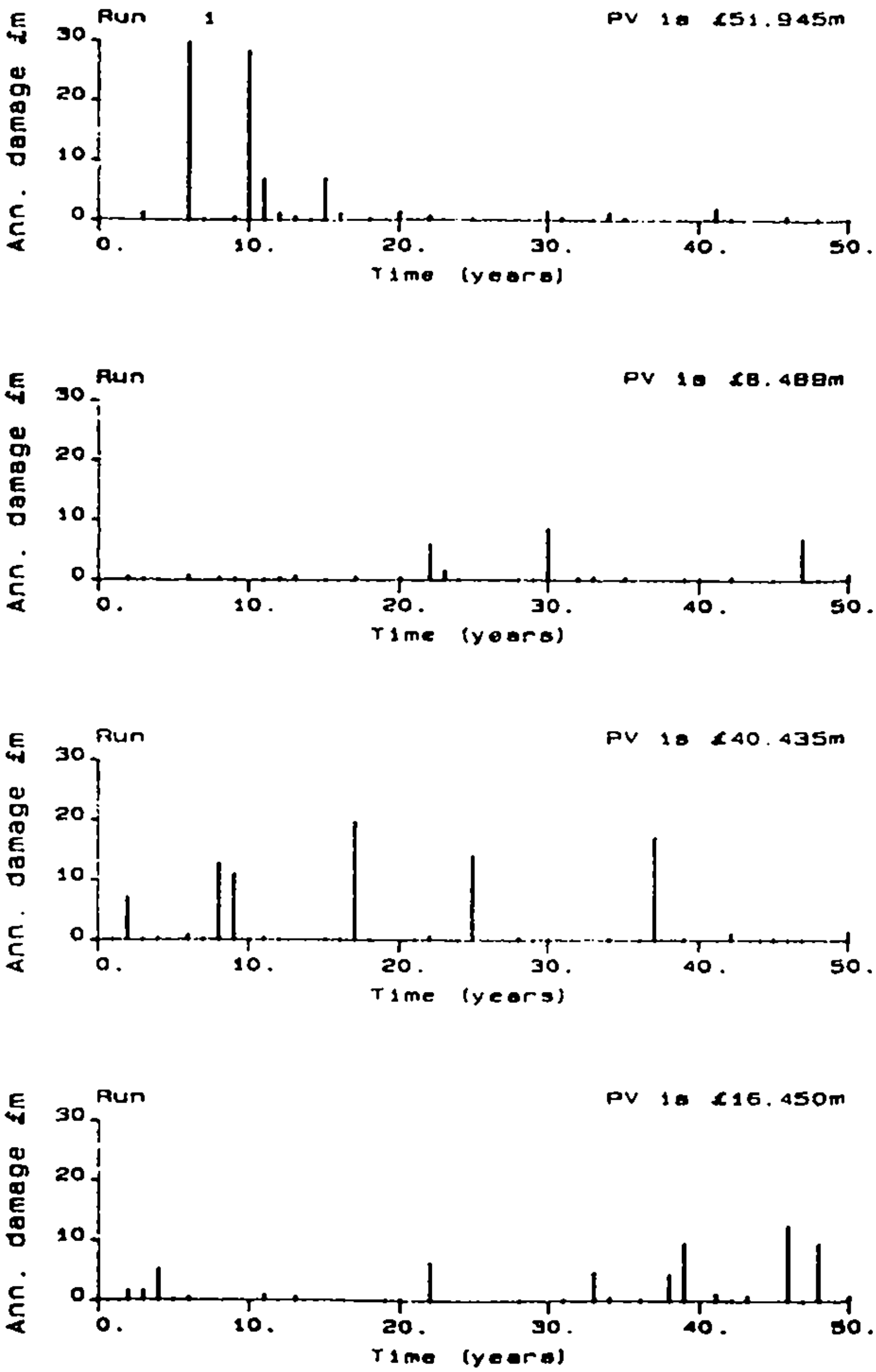

Figure 5.1 Four different synthetic 50 year flood damage time series

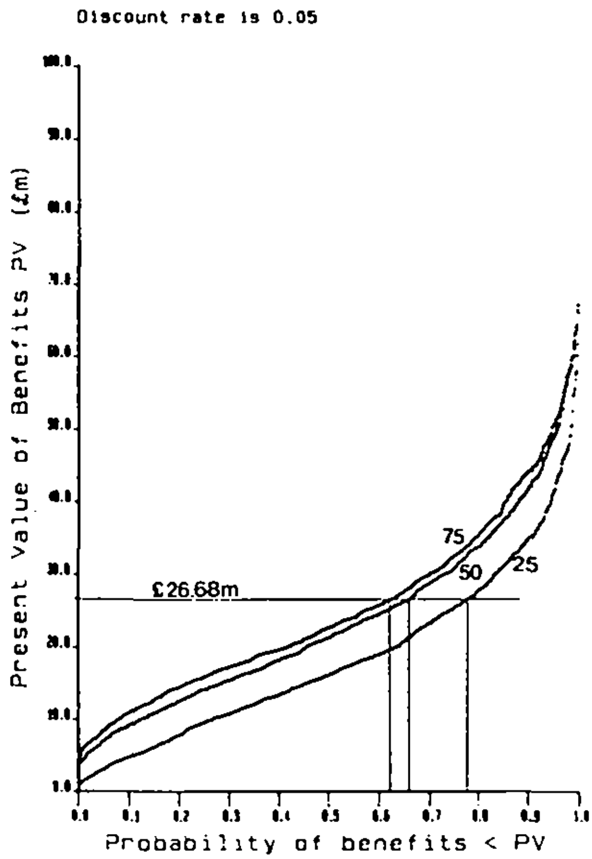

Figure 5.2 Distributions of the present value of flood damages after 25, 50 and 75 years

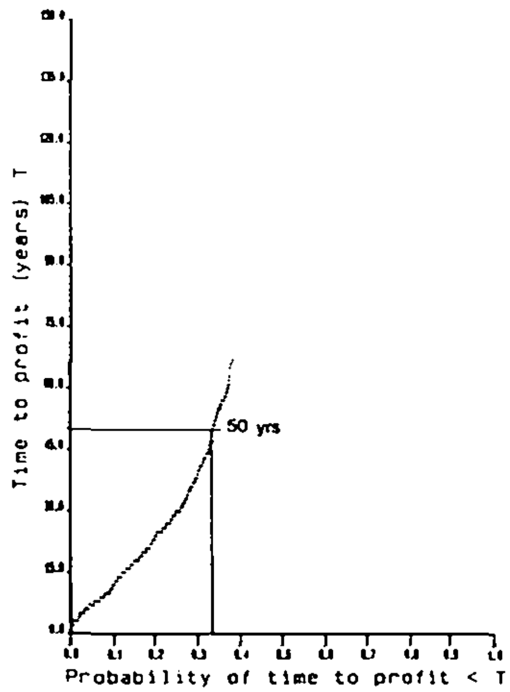

Figure 5.3 Distribution of times needed to accumulate the target present value of annual flood damages

in the example used in the chapter, the probability that a scheme which was just economically effective in conventional terms - would actually show benefits greater than costs was less than 50%).

Although the example makes some restrictive assumptions about changes in risk over time, it is possible to use the method to evaluate the frequency distribution of present values under changing climate, catchment and floodplain land use conditions.

6. Conclusions

6.1 A SUMMARY OF THE CONCLUSIONS

This report has examined the effect of a number of different types of uncertainty on the estimation of flood magnitudes and the evaluation of the benefits of flood alleviation.

Chapter 2 concentrated on the estimation of the rate of occurrence of floods during a N-year period and, in more detail, the estimation of the magnitude of the T-year flood exceeded with a given degree of risk. The conventional approach is to use a method which gives an unbiased estimate of the magnitude of the T-year flood, but it was shown that this estimate will probably be exceeded in the future more frequently than once in T years. A method was proposed, based on the concept of expected probability, to produce an estimate of flood magnitude that would be exceeded in the future, on average, once in T years. The method took the form of an increment to add to a conventional estimate of the T-year flood (produced by applying a GEV distribution with parameters estimated by probability-weighted moments): the size of the increment reflects sampling uncertainties, and increases as records become shorter, as the annual flood data become more variable, and as return period increases. A conventional estimate of the 50-year flood made from a sample of just 10 years would have to be increased by over 18% to produce a magnitude with an expected probability of 1/50 (assuming 'average' flood characteristics).

Chapter 3 examined the effect of uncertainties in the estimation of a flood frequency relationship at a site on the bias and precision of estimates of average annual damage. It was found that average annual damage tended to be overestimated, with bias increasing as the return period at which damage commenced increased: it is clearly important that this critical return period is estimated in practice as accurately as possible. Hydrological uncertainties lead to very large sampling uncertainties in the estimate of average annual damage. The use of expected probabilities in the calculation of average annual damage was found to results in even greater overestimation: although the expected probability approach gives an estimate of the magnitude exceeded with a given risk, it does not produce an unbiased estimate of the risk associated with a given magnitude, and it is the estimated probability at which damage commences which largely determines the magnitude of average annual damage.

Confidence intervals on estimates of both the estimated flood magnitude X_T and the return period of a given magnitude T_X were considered in Chapter 4. Computer simulation experiments showed that the assumption that the sampling distribution of estimates X_T was Normal, and therefore that confidence intervals could be estimated from the variance of X_T, was inappropriate: the true upper 95% confidence limit is larger than that based on a Normal distribution, particularly for high return periods, short sample sizes and highly variable flood data. Confidence intervals for estimates of the T-year flood are currently best approximated by computer simulation experiments, and the experiments can also give confidence intervals for the estimated return period for a given magnitude flood. Such confidence intervals are frequently required in practice.

The conventional approach to benefit assessment compares the present value of scheme costs with the present value of the average annual benefits of the scheme (i.e. flood damages averted). In practice, however, the actual benefits that will be realised will depend on the timing of flood events over the project life: the greater the number of floods early on, the larger the present value of the benefits realised. Chapter 5 outlined a method to simulate the probability distribution of the present value of flood benefits, from which the probability that the benefits will in practice exceed, for example, scheme costs can be determined. The method allows for uncertainty in the future state of the world (as represented by the actual pattern of future flooding), rather than uncertainty in the parameters of a particular model of the world, unlike chapters 2, 3 and 4.

6.2 SUMMARY OF RECOMMENDATIONS

The report makes the following recommendations:

Assessments of the precision of estimates of flood magnitudes, flood return periods and average annual damage be made using computer simulation experiments.

Expected probability should not be used in the calculation of average annual flood damages.

The possibility of basing scheme assessments on the likelihood of the present value of scheme benefits exceeding scheme costs be considered.

The safety factor to be added to an individual estimate of the design flood X_T should be such that the adjusted flood magnitude has an expected probability of occurrence in the future of 1/T. This would mean that the flood is exceeded, on average, with the desired risk.

6.3 FURTHER STUDIES

The report has shown how hydrological uncertainties may have very significant effects on design flood estimation and scheme assessment, and recommends that computer simulation experiments are used to indicate the precision of estimates of flood magnitudes, flood return periods and average annual damages. Further studies, however, could be undertaken to develop more analytical and hence faster methods, once the details of a new UK flood frequency estimation procedure are finalised.

In particular, studies will be needed in the following areas:

(i) exact confidence intervals around estimated flood magnitudes for the recommended single-site frequency analysis procedure;

(ii) methods for estimating the confidence intervals around regional dimensionless flood frequency curves. Such confidence intervals will need to reflect both the sampling uncertainties involved in estimating a regional curve from a number of possibly correlated catchments and the uncertainties due to within-region heterogeneity;

(iii) exact methods for determining the confidence interval around an estimate of the return period of a particular magnitude flood;

(iv) calculation of expected probability corrections to be applied when flood magnitudes are estimated from a regional analysis.

7. Acknowledgements

This report forms part of the Institute's River Flood Protection research commissioned by the Ministry of Agriculture, Fisheries and Food, for whom the liaison officer was Mr Roger Buckingham. The research summarised in this report has benefitted from discussions with IH staff, and in particular Max Beran, and with Colin Green and Professor Edmund Penning-Rowsell from Middlesex Polytechnic Flood Hazard Research Centre.

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Appendix A

Unbiased estimation of flood risk with the GEV distribution

Published in: Stochastic Hydrology and Hydraulics, 1988, vol. 2, pp 201-212

Stochastic Hydrol. Hydraul. 2 (1988) 201-212

Stochastic Hydrology
and Hydraulics

Unbiased estimation of flood risk with the GEV distribution

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Abstract: Conventional flood frequency analysis is concerned with providing an unbiased estimate of the magnitude of the design flow exceeded with the probability p, but sampling uncertainties imply that such estimates will, on average, be exceeded more frequently. An alternative approach is therefore, to derive an estimator which gives an unbiased *estimate of flow risk*: the difference between the two magnitudes reflects uncertainties in parameter estimation. An empirical procedure has been developed to estimate the mean true exceedance probabilities of conventional estimates made using a GEV distribution fitted by probability weighted moments, and adjustment factors have been determined to enable the estimation of flood magnitudes exceeded with, on average, the desired probability.

Key words: Flood risk, flood frequency analysis, generalised extreme value distributions.

1 Introduction

If the hydrologist had perfect knowledge of the flood frequency relationship at a site there would be no controversy over the estimation of flood magnitudes corresponding to specified frequencies. However, in practice the hydrologist knows neither the form of the most appropriate statistical model of flood frequencies nor the values of the parameters of this model and must therefore make assumptions and estimates. The conventional approach is to select a model and estimate its parameters in such a way that, because of sampling uncertainties, the best estimate of the magnitude of the flood with probability of exceedance p is obtained. Many probability distributions and parameter estimation procedures have been proposed and applied, but it has been noted that, because of sampling uncertainties, the best estimate of the magnitude $\hat{x}_p$ of the flood with probability p and return period $T = 1/p$ will probably be exceeded in the future more *frequently than once in T* years. In other words, if risk is defined as the probability that the design flood will be exceeded in any one year, the "expected" risk of having an event greater than the estimated magnitude $\hat{x}_p$ in the future is greater than p. Stedinger (1983) argued that flood managers did not need the best estimate of the magnitude of the p probability flood, but needed instead the flood with a specified risk of occurrence. It is therefore necessary to estimate the flood with an expected risk equal to p (Beard, 1960; Hardison and Jennings, 1972).

This magnitude will be higher than the conventional best unbiased estimate of the magnitude of the p probability flood, and the difference will depend on uncertainties in parameter estimation. The difference between the two magnitudes can be seen as an 'adjustment factor', which can be applied to a conventional estimate to determine the flood magnitude which, given available data, can be expected to be exceeded with the desired probability or risk p. It is the objective of this paper to develop a procedure for estimating this adjustment factor for the generalised extreme value (GEV) distribution. It is first necessary to examine more closely the reasons why the conventional best estimate of the flood with a probability of exceedance p gives a biased estimate of flood risk.

2 The risk of a flood greater than the estimated design flood

Figure 1a shows a hypothetical sampling distribution of estimates of the flood magnitude $\hat{x}_p$ exceeded with probability p. It can be seen that this particular method (the actual method is not important for this illustration) gives an unbiased estimate of the magnitude $\hat{x}_p$ (the mean of the sampling distribution is equal to the true value) and that estimates are approximately normally distributed. Figure 1b shows the distribution of true exceedance probabilities of each estimate $\hat{x}_p$, and it is immediately clear that this distribution is highly skewed with a mean different to - and greater than - p. In other words, the mean true probability of the estimates of design magnitude $\hat{x}_p$ is greater than p even though the mean magnitude is equal to the true magnitude, and this is due to sampling uncertainties (Beard 1960; Hardison and Jennings, 1972).

If there were many independent rivers in an area with the same underlying parent form, there would therefore be events greater than $\hat{x}_p$ on average more than once every $T = 1/p$ years. A similar effect would be observed, if many independent samples could be taken at one site of interest. Beard (1960) called the mean true probability of estimates $\hat{x}_p$ the expected probability, but Hardison and Jennings' (1972) term "average exceedance probability" is clearer and emphasises the idea of averaging across samples. The average exceedance probability of $\hat{x}_p$ is greater than the desired probability because of the shape of the relationship between magnitude and probability, although the precise difference varies between probability distributions.

Several approaches have been developed to produce an unbiased estimate of the probability of a flood of magnitude x. Moran (1957), for example, assumed that floods (or their logarithms) were normally distributed, and noted that a future value x and sample mean $\bar{x}$ are therefore both independent normal random variates with common mean μ and variances σ^2 and σ^2/N, where N is sample size. If the sample standard deviation s is substituted for σ, the statistic:

$$t = \frac{x - \bar{x}}{\{s^2 + s^2/N\}^{1/2}} \tag{1}$$

therefore follows Student's t distribution with $N - 1$ degrees of freedom. The probability p of magnitude x can be obtained from:

$$p = \text{prob}[\frac{x - \bar{x}}{s[1 + 1/N]^{1/2}} \leq t_{p,n-1}] \tag{2}$$

and the magnitude $\hat{x}_p$ exceeded with probability p is therefore simply obtained by rearranging Eq. (2) (Stedinger 1983):

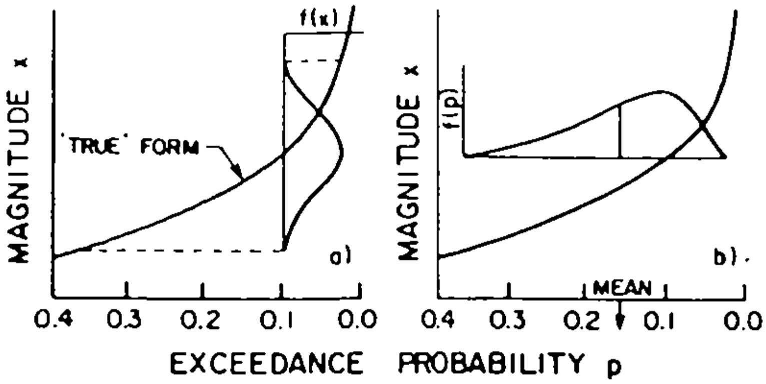

Figure 1. Sampling distribution of estimates of $x_{0.1}$ and distribution of true probabilities of estimates

$$x_p^0 = \bar{x} + s[1 + 1/N]^{1/2} t_{p,n-1}. \quad (3)$$

Here, $t_{p,v}$ is the quantile with exceedance probability p from a Student's t distribution with $v = N - 1$ degrees of freedom. Beard (1960) derived the same expression differently, following Proschan's (1953) proof that the expected probability (i.e. average true exceedance probability) of a statistic based on the mean and standard deviation of a sample from a normal distribution was a simple function of Student's t distribution. Although the two expressions are the same, there is a difference in interpretation: Moran's (1957) derivation makes no reference to the idea of averaging across many samples. Similar expressions have been derived for some other distributions (Lall 1987).

An alternative approach uses the idea that the expected value of a random variable y is equal to

$$E(y) = \int y\, f(y)\, dy \quad (4)$$

where $f(y)$ is the probability density function of y. The expected probability of flood magnitude greater than X can therefore be computed from

$$f(x) = \int f(x\,|\,\theta) f(\theta)\, d\theta \quad (5)$$

and

$$p(X) = 1 - F(X) = 1 - \int_0^X f(x)\, dx \quad (6)$$

Here, $f(x\,|\,\theta)$ gives the estimated probability of x given parameter set *the*, and $f(\theta)$ is the probability of parameter set θ being correct. If $f(\theta)$ is in fact a posterior distribution combining prior knowledge about the distribution of the parameters with sample information using Bayes theorem, Eq. (5) is the Bayesian distribution (also known as the marginal density function or the predictive density). Such Bayesian distributions have been derived from frequency distributions used in flood frequency analysis by, among others, Cunnane and Nash (1971), Wood and Rodriguez-Iturbe (1975) and Stedinger (1983), and are reviewed by Kuczera (1987). The form of the distribution depends to an extend on the assumed form of the prior distribution, and Stedinger (1983) showed how, under certain conditions, the Bayesian distribution derived from a lognormal distribution is numerically identical to Eq. (3).

The Bayesian approach, however, has the advantage that information from a

Table 1. Mean true exceedance probability of events greater than estimated design magnitudes. GEV parameters estimated by probability-weighted moments

	design return period (years)				
	10	25	50	75	100
Design probability	0.1	0.04	0.02	0.013	0.010
True magnitude	20.707	26.419	31.214	34.242	36.500
mean estimate	20.660	26.417	31.451	34.757	27.296
s.d. estimate	3.150	5.722	8.889	11.410	13.574
mean true exceedance probability	0.1154	0.0553	0.0340	0.0264	0.0224

GEV parent: $u = 10.0$, $a = 4.0$, $k = -0.15$; Sample size $n = 20$; Number of repetitions = 1000.

number of sources - site data, regional information and 'engineering judgement' - can be incorporated into the assessment of the probabilities of different parameter combinations.

The estimated magnitude with an average true exceedance probability p is larger than an unbiased estimate of flood magnitude, and there is a greater risk of overdesign than underdesign: although on average, sample estimates are exceeded with the desired risk, a higher proportion are exceeded less frequently. It is possible to derive an estimator which, instead of focusing on the mean true exceedance probability, aims at the median. Such an estimator would give estimates that are actually exceeded more or less frequently than desired with equal probability, but there is no reason why equal probabilities of under or overdesign should be sought (Stedinger 1983). Indeed, caution suggests that overdesign - as is more likely using the unbiased risk estimator - should be preferred.

3 Application to the GEV distribution

The generalised extreme value (GEV) distribution was recommended for use in flood frequency analysis in the U.K. Flood Studies Report (NERC 1975), and has been widely applied. It has the following form (Jenkinson 1955).

$$F(x) = \begin{cases} \exp[-\{1 - k(x - u)/a\}^{1/k}] & k \neq 0 \\ \exp[-\exp\{-(x - u)/a\}] & k = 0 \end{cases} \qquad (7)$$

When $k = 0$ the distribution reduces to the extreme value type 1 (EV1). Parameter estimation procedures include the method of sextiles (NERC 1975) and maximum likelihood (Prescott and Walden 1980), but Hosking et al. (1985) showed that parameters estimated using probability-weighted moments were less biased and more efficient for the short sample sizes encountered in hydrology.

Table 1 illustrates that estimates of flood quantiles, whilst nearly unbiased, are exceeded more frequently than desired: the table was constructed by repeatedly generating a synthetic sample from a GEV distribution, estimating the parameters by probability-weighted moments and determining the true probabilities of exceedance of estimated quantiles from parent parameters.

The form of the GEV distribution means it is very difficult to find analytically an exact expression to give unbiased estimates of the probability of flood magnitude similar to that derived by Moran (1957) for the lognormal distribution. It is therefore necessary to develop an empirical analogue to Beard's (1960) approach, and derive empirically the average true exceedance probability of conventional estimates of the magnitude $\hat{x}_p$. Hardison and Jennings (1972) employed such a procedure with the log-Pearson type III distribution to convert 'conventional' probabilities (as implied by parameters fitted by the method of moments) to average exceedance probabilities, and derived the relationship by computer simulation.

The approach adopted in this paper is to develop empirically a relationship between the probability of a magnitude x calculated using parameters estimated by the method of probability-weighted moments (GEV-PWM), and the average true exceedance probability of that magnitude, in a similar vein to Hardison and Jennings (1972). However, whereas with their method it is necessary to replot the flood frequency curve after correcting estimated probabilities and to find graphically the magnitude with average exceedance probability p, the method presented here allows the direct estimation of the magnitude of an event with a specified average exceedance probability. The method produces an increment to be added to the probability-weighted moments estimate of a flood quantile, and this increment can be seen as a risk-based 'adjustment factor' enabling the conversion from unbiased estimates of flood magnitudes to unbiased estimates of flood risk. As sample sizes increase, uncertainty and, thus, the difference between the two estimators, decrease; and the adjustment factor also diminishes. The Bayesian approach was not explored partly due to the difficulty of choosing appropriate forms for the prior distributions of parameters but mainly for practical reasons: the GEV-PWM estimation procedure is widely used and it is logical to conceive of a way that allows design magnitudes exceeded with a specified risk to be determined using an adjustment factor added to an estimate. It is appropriate at this juncture to note that different results would be obtained if an attempt was made to produce an unbiased estimate of the return period of a particular magnitude (because $1/\bar{p}$ is n ot equal to $(\overline{1/p})$). This approach was not followed as the distribution of true return periods of estimated flood magnitude is more highly skewed than the distribution of true exceedance probabilities (and is virtually unbounded). It is more difficult to obtain stable estimates of the average true return period by simulation.

4 Computation of adjustment factors

Adjustment factors can be determined using computer simulation according to the following stages:

1) generate a sample of synthetic annual floods of size N from a specified GEV parent distribution,
2) estimate parameters u, a and k using probability-weighted moments;
3) compute the true probabilities of the estimated magnitude $\hat{x}_p$ and $\hat{x}_p + AF \cdot \hat{x}_p$, where AF ranges from 0.01 to 0.5 in increments of 0.01;
4) repeat the process many times and compute the mean true probability (the 'expected probability') for $\hat{x}_p$ and each increment $\hat{x}_p + AF \cdot \hat{x}_p$.

A graph can be constructed from the results of stage 4, and an example is shown in Fig. 2. Values of $\hat{x}_p + AF \cdot \hat{x}_p$ with average exceedance probability p are then obtained by interpolation aided by log-linear regression of $\hat{x}_p + AF \cdot \hat{x}_p$ on p. Experiments were run with sample sizes of 10 (10) 50, 70, 100, parent coefficient of variation CV 0.4 (0.2) 1.0 and parent k parameter -0.3 (0.1) 0.1, and adjustment factors AF were computed for $T = 1/p$ of 10, 25, 50, 75 and 100 years. The experiments consisted of 10000 repetitions for all except the runs with sample sizes of 70 and 100, which used 1000 repetitions. Adjustment factors for other return periods can be approximated by drawing a curve through the points defined by the computed adjustment factors and interpolating. The shape of the relationship between probability and magnitude varies with parent distribution parameters.

Some of the results are tabulated in Table 2 by return period T, CV,k and sample size N, and the variations are illustrated in Fig. 3 which shows results for $k = -0.1$. It is clear that the adjustment factor AF reduces as record length

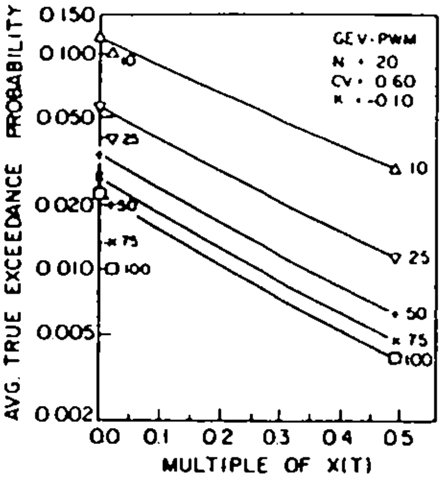

Figure 2. Variation of average exceedance probability with magnitude $\hat{x}_p + AF \cdot \hat{x}_p$ for various return periods $T = 1/p$

increases and as parent CV reduces, and from Table 2 it can be seen that for short records at least the adjustment factor reduces as k becomes larger (i.e., the skew of the parent reduces). The highest adjustment factor calculated for the 100 year flood, is 0.685 with $N = 10$, $CV = 1.0$ and $k = -0.3$: 0.685 $\hat{x}_{0.01}$ must be added to the GEV-PWM estimate of the 0.01 probability flood to yield the magnitude with an expected probability or risk of 0.01. For larger samples, however, the picture is less clear. In general, if T is less than sample size N, the lowest adjustment factor occurs with parents with k close to 0, reflecting the relative decline in GEV-PWM performance as k departs from zero: for shorter record lengths performance is influenced by the interaction between record length and parent k. It is interesting to compare adjustment factors for the GEV distribution with similar adjustment factors for the two-parameter lognormal distribution. These can be calculated directly from

$$AF = \frac{\exp[\bar{x} + s(1 + 1/n)^{1/2} t_{p,n-1}] - \exp[\bar{x} + z_p s]}{\exp[\bar{x} + z_p s]}$$

$$= [\exp\{(1 + 1/n)^{1/2} t_{p,n-1} - z_p\}]^{(\log_e(1+CV^2))^{1/2}} - 1 \qquad (8)$$

where z_p is the standard normal deviate with exceedance probability p. In general, the adjustment factors for the lognormal distribution are smaller for long record lengths, but for smaller samples are similar to those for the GEV with k between 0 and -0.2. Figure 4 shows the variations in adjustment factor with sample size for a parent CV of 0.6.

5 Application of the technique

Whilst Table 2 shows clearly the variation in adjustment factor across sample size, CV and k, it is not very easy to use in practice where interpolation is necessary. A simple model was therefore developed to predict adjustment factor AF from N, CV and k, and has the general form:

Table 2 Adjustment factors to convert a GEV-PWM estimate of X_T to a magnitude with expected probability $1/T$.

GEV k is 0.10

	C.V. is 0.40				
	Return period				
N	10	25	50	75	100
10	.0415	.0615	.0947	.1224	.1456
20	.0283	.0366	.0534	.0690	.0827
30	.0220	.0275	.0384	.0488	.0582
40	.0203	.0241	.0317	.0389	.0456
50	.0193	.0227	.0293	.0355	.0412
100	.0173	.0207	.0251	.0287	.0317
	C.V. is 0.60				
	Return period				
N	10	25	50	75	100
10	.0728	.0995	.1433	.1802	.2112
20	.0412	.0524	.0746	.0949	.1128
30	.0288	.0361	.0504	.0639	.0762
40	.0244	.0293	.0390	.0482	.0567
50	.0220	.0264	.0347	.0426	.0499
100	.0173	.0209	.0261	.0304	.0342
	C.V. is 0.80				
	Return period				
N	10	25	50	75	100
10	.1001	.1303	.1814	.2250	.2617
20	.0523	.0650	.0907	.1143	.1351
30	.0348	.0430	.0594	.0748	.0888
40	.0285	.0339	.0448	.0554	.0650
50	.0250	.0297	.0392	.0482	.0566
100	.0181	.0219	.0275	.0323	.0365

GEV k is 0.00

	C.V. is 0.40				
	Return period				
N	10	25	50	75	100
10	.0477	.0873	.1369	.1744	.2046
20	.0281	.0484	.0772	.1004	.1198
30	.0182	.0316	.0517	.0683	.0826
40	.0147	.0239	.0384	.0507	.0613
50	.0125	.0199	.0325	.0429	.0523
100	.0083	.0123	.0191	.0250	.0302
	C.V. is 0.60				
	Return period				
N	10	25	50	75	100
10	.0879	.1379	.1999	.2472	.2856
20	.0473	.0726	.1078	.1363	.1601
30	.0300	.0467	.0710	.0911	.1082
40	.0235	.0350	.0523	.0668	.0794
50	.0195	.0289	.0436	.0562	.0672
100	.0119	.0170	.0251	.0319	.0378
	C.V. is 0.80				
	Return period				
N	10	25	50	75	100
10	.1216	.1776	.2481	.3025	.3469
20	.0625	.0906	.1299	.1619	.1888
30	.0392	.0577	.0846	.1069	.1259
40	.0303	.0429	.0619	.0779	.0917
50	.0249	.0352	.0515	.0653	.0773
100	.0146	.0204	.0291	.0365	.0429

GEV k is -0.10

	C.V. is 0.40				
	Return period				
N	10	25	50	75	100
10	.0527	.1149	.1840	.2331	.2717
20	.0296	.0652	.1088	.1413	.1677
30	.0171	.0419	.0737	.0982	.1183
40	.0124	.0306	.0546	.0734	.0890
50	.0092	.0240	.0445	.0609	.0746
100	.0032	.0114	.0231	.0326	.0406
	C.V. is 0.60				
	Return period				
N	10	25	50	75	100
10	.1005	.1776	.2611	.3209	.3681
20	.0543	.0975	.1487	.1870	.2181
30	.0333	.0630	.1000	.1283	.1515
40	.0249	.0467	.0743	.0956	.1133
50	.0195	.0373	.0608	.0794	.0949
100	.0095	.0192	.0325	.0430	.0519
	C.V. is 0.80				
	Return period				
N	10	25	50	75	100
10	.1402	.2266	.3201	.3875	.4409
20	.0734	.1209	.1771	.2193	.2535
30	.0453	.0780	.1182	.1488	.1740
40	.0342	.0578	.0875	.1104	.1294
50	.0271	.0464	.0717	.0916	.1082
100	.0138	.0244	.0385	.0496	.0590

GEV k is -0.20

	C.V. is 0.40				
	Return period				
N	10	25	50	75	100
10	.0522	.1382	.2282	.2900	.3376
20	.0296	.0829	.1432	.1864	.2207
30	.0159	.0547	.1005	.1341	.1611
40	.0106	.0405	.0765	.1033	.1249
50	.0067	.0316	.0626	.0861	.1052
100	.0001	.0145	.0332	.0475	.0594
	C.V. is 0.60				
	Return period				
N	10	25	50	75	100
10	.1061	.2117	.3186	.3920	.4487
20	.0592	.1230	.1927	.2424	.2817
30	.0361	.0820	.1341	.1721	.2025
40	.0267	.0617	.1022	.1319	.1559
50	.0203	.0493	.0840	.1100	.1311
100	.0084	.0254	.0460	.0615	.0743
	C.V. is 0.80				
	Return period				
N	10	25	50	75	100
10	.1511	.2699	.3887	.4705	.5338
20	.0821	.1524	.2281	.2820	.3247
30	.0512	.1013	.1573	.1980	.2305
40	.0385	.0764	.1194	.1509	.1764
50	.0300	.0613	.0982	.1257	.1480
100	.0143	.0325	.0541	.0703	.0835

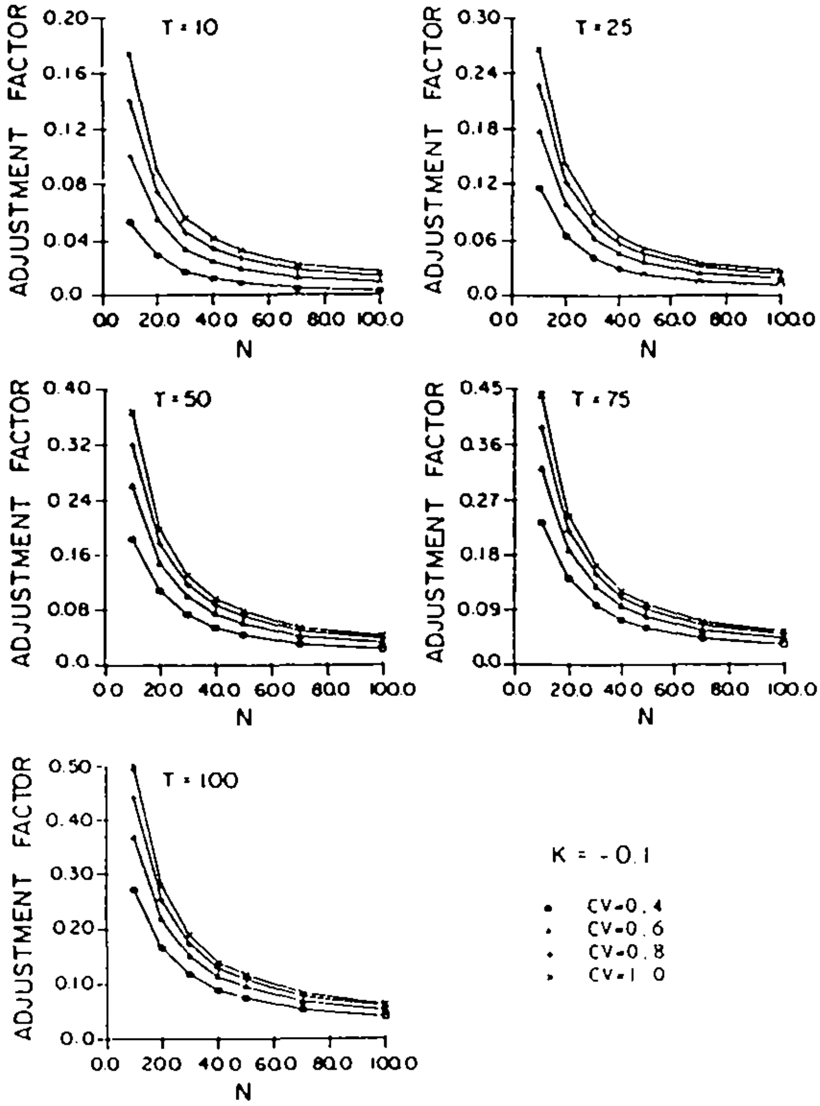

Figure 3. Variation of adjustment factor with N and CV, $k = -0.1$

$$\log_e AF = \text{const} + a\log_e N + b\log_e CV + ck. \tag{9}$$

Different equations were developed for each of the five return periods using multiple regression, and the coefficients are shown in Table 3a together with the coefficient of determination R^2. It is clear that all except the model for $p = 0.1$ provide a good fit to the data. The relatively poor fit with $p = 0.1$ is due to the highly non-linear relationship between adjustment factor and CV and k, particularly for long record lengths (as illustrated in Fig. 3). Another model was therefore developed from the equation to calculate adjustment factors for the lognormal distribution. The standard normal deviate z_p is equal to the frequency factor K in

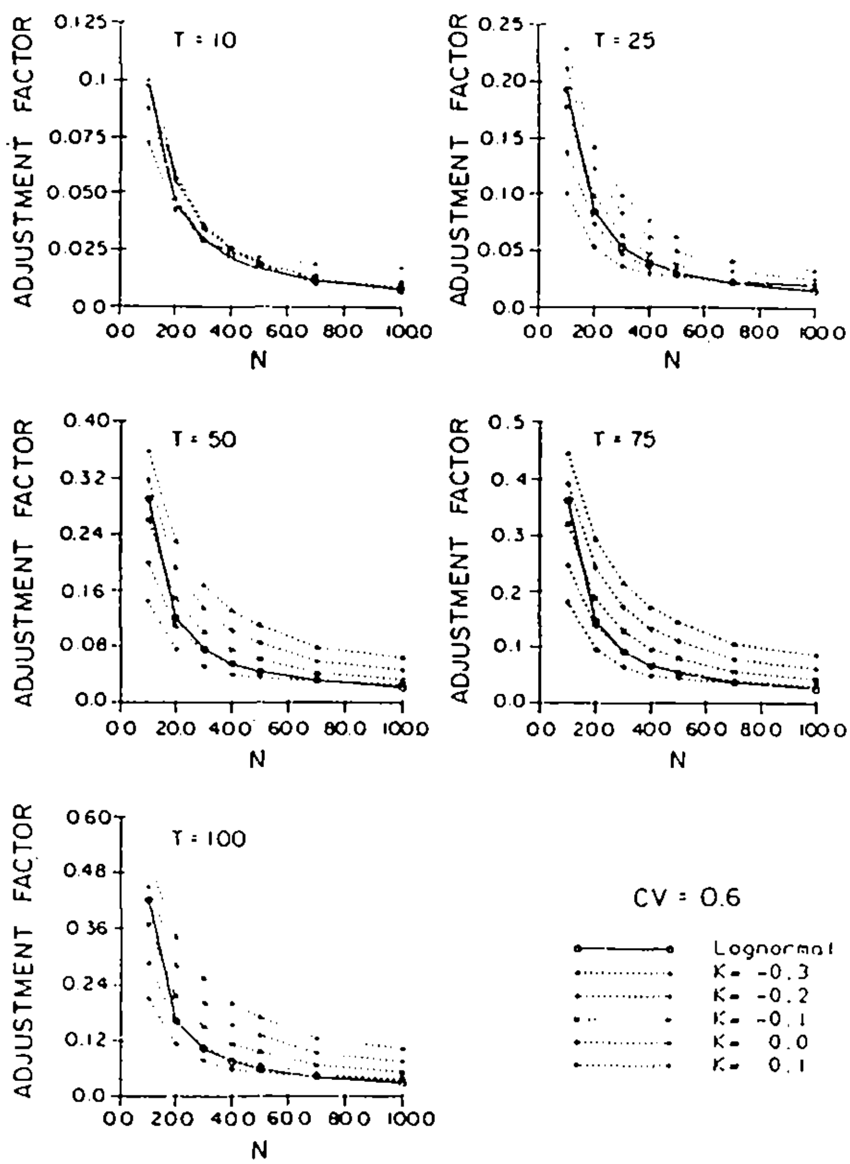

Figure 4. Variation of adjustment factor. Lognormal and GEV distributions, $CV = 0.6$

$$\hat{x}_p = \bar{x} + K_p s \tag{10}$$

and Eq. (8) can be modified by substituting the frequency factor for the GEV distribution for z_p and replacing $t_{p,n-1}$ with a parameter A:

$$AF = [\exp\{(1 + 1/n)^{1/2} A - K_{GEV}\}]^{(\log_e(1+CV^2))^{1/2}} - 1. \tag{11}$$

The frequency factor for the GEV distribution is, with $F = 1 - p$,

Table 3a. Coefficients of model to estimate adjustment factors from n, CV and k

Design return period (years)	const	a	b	c	R^2
10	1.166	-1.1188	1.713	0.965	71.2
25	0.4496	-0.9026	0.8093	-2.0970	97.1
50	0.6058	-0.8623	0.6193	-2.7255	98.5
75	0.7390	-0.8429	0.5525	-2.8459	98.8
100	0.8367	-0.8291	0.5172	-2.8670	98.9

$\log_e AF = \text{const} + a\log_e N + b\log_e CV + ck.$

Table 3b. Coefficients of model to estimate adjustment factors from n, CV and k

Design return period (years)	const	a	b	c	R^2
10	0.1525	-0.02841	0.02177	0.9558	99.6
25	0.2013	-0.02839	-0.00142	0.9143	98.5
50	0.2007	-0.01997	-0.01316	0.9449	96.3
75	0.1456	-0.03101	-0.01866	1.0035	96.5
100	0.1046	-0.03167	-0.02211	1.0391	97.4

$\log_e A = \text{const} + a\log_e N + b\log_e CV + cK_{GEV}$; $AF = [\exp\{(1+1/n)^{1/2}A - K_{GEV}\}]^{(\log_e(1+CV^2))^{1/2}} - 1$.
K_{GEV} as given in Eq (12).

$$K_{GEV} = \frac{(-\log_e F)^k - \Gamma(1+k)}{(\Gamma(1+2k) - \Gamma^2(1+k))^{1/2}} \quad k < 0$$
$$= -\left[\frac{(-\log_e F)^k - \Gamma(1+k)}{(\Gamma(1+2k) - \Gamma^2(1+k))^{1/2}}\right] \quad k > 0$$
$$= 0.45 + 0.779(-\log_e(-\log_e F)) \quad k = 0 \tag{12}$$

The parameter A in Eq. (11) varies with return period, sample size, CV and k, and a simple regression model of the form

$$\log_e A = \text{const} + a\log_e N + b\log_e CV + c\log_e K_{GEVp} \tag{13}$$

was constructed. The parameters are shown in Table 3b, and it can be seen that the fit of this model at lower return periods is much better than that of the more naive model in Table 3a. Whilst the exact form of this model derives of course from the lognormal distribution, it may be possible to develop a similar analytical expression for the GEV.

Application of the method therefore involves the following stages.

1) estimate GEV parameters from the sample using probability-weighted moments;

2) estimate the quantile $\hat{x}_p$ from parameters;

3) use the equations in Table 3a or 3b to predict AF from sample size N, sample CV and estimated parameter k;

4) compute new quantile estimate as $\hat{x}_p + AF \cdot \hat{x}_p$.

A further set of computer experiments was run to illustrate the effect of applying the risk-based factor to estimated quantiles. Adjusted quantiles were computed as previously described, and the mean true probability of events greater than the new adjusted $\hat{x}_p$ was calculated. The results are shown in Table 4a and 4b, and are directly comparable with those of Table 1: it can be seen that the use of the

Table 4a. Mean true exceedance frequency of events greater than estimated design magnitudes. GEV parameters estimated by probability-weighted moments and adjustment factor added to design estimates. Model from Table 3a

	design return period (years)				
	10	25	50	75	100
Design probability	0.1	0.04	0.02	0.013	0.010
True magnitude	20.707	26.419	31.214	34.242	36.500
mean estimate	21.260	28.857	36.697	42.513	47.326
s.d. estimate	3.422	7.957	15.327	17.084	28.344
mean true exceedance probability	0.1057	0.0445	0.0248	0.0180	0.0145

GEV parent: $u = 10.0$, $\alpha = 4.0$, $k = -0.15$; Sample size $n = 20$; Number of repetitions = 1000.

Table 4b. Mean true frequency of events greater than estimated design magnitudes. GEV parameters estimated by probability-weighted moments and adjustment factor added to design estimates. Model from Table 3b

	design return period (years)				
	10	25	50	75	100
Design probability	0.1	0.04	0.02	0.013	0.010
True magnitude	20.707	26.419	31.214	34.242	36.500
mean estimate	21.378	28.417	34.962	39.407	43.105
s.d. estimate	3.338	6.331	9.891	12.698	15.367
mean true exceedance probability	0.1037	0.0432	0.0235	0.0173	0.0142

GEV parent: $u = 10.0$, $\alpha = 4.0$, $k = -0.15$; Sample size $n = 20$; Number of repetitions = 1000.

adjustment factor produces estimates of flood quantiles which are exceeded with more nearly the specified probability or risk. The actual risk is not quite the same as the specified risk even after adding the adjustment, due to bias in estimating population CV from sample data and, more particularly, bias in estimating k (Hosking et al. 1985).

6 Conclusions

Flood frequency analysts conventionally use a method which gives an unbiased estimate of the magnitude of the p probability flood $\hat{x}_p$ to estimate design floods. However, the expected probability or risk of a future flood larger than the estimate $\hat{x}_p$ is greater than p due to sampling uncertainties, and Stedinger (1983) has argued that flood managers need a technique which provides an estimate of the flood $\hat{x}_p$ exceeded in the future with specified risk p. Such estimates may be obtained using Bayesian methods and several examples have been presented in the literature. Bayesian methods however, may be difficult to apply in practice, and results will depend on the form of the selected prior distribution.

An empirical technique has, therefore, been developed to estimate the magnitudes of floods exceeded with specified probabilities based on the generalised extreme value (GEV) distribution. The method takes the form of an adjustment factor to be added to an estimate of the magnitude of the p probability flood $\hat{x}_p$ obtained conventionally using the method of probability weighted moments, to yield an estimate exceeded with risk p. The adjustment varies with the uncertainty in parameter estimation, and hence with sample coefficient of variation, GEV shape parameter k and sample size. Adjustment factors were determined numerically using computer simulation experiments to calculate the average true exceedance probability of an an estimate and a simple model has been constructed to estimate the appropriate adjustment factor for the site of interest. Because the difference between an unbiased estimate of flood magnitude and an unbiased estimate of flood risk depends on parameter uncertainty, the adjustment factor can be seen as an index of this uncertainty.

Acknowledgements

The work presented in this paper was undertaken as part of a research project funded by the Ministry of Agriculture, Fisheries and Food. The author acknowledges the helpful comments of the anonymous reviewers.

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Accepted May 30, 1988.

Appendix B

Expected annual damages and uncertainties in flood frequency estimation

Published in: Journal of Water Resources Planning and Management, 1989.
vol. 115, pp. 94-107

Expected Annual Damages and Uncertainties in Flood Frequency Estimation

By Nigel W. Arnell[1]

Abstract: The expected annual damge is the most frequently used index of the impact of flooding at a site. However, estimates of expected annual damages are very uncertain as a result of uncertainties in both the estimation of the flood frequency relationship from limited data and the relationships between magnitude and damage. Computer simulation experiments using synthetic flood peak data and fixed magnitude-damage functions have shown that the sampling distribution of estimates of expected annual damages is highly skewed to a degree depending on the form of the damage function, and most importantly, that bias in the estimates is most closely related to error in the estimated probability at which damage begins. The use of expected probability leads to a very significant increase in bias in the estimation of expected annual damages.

Introduction

When designing a scheme to alleviate flooding, planners and engineers need an estimate of the costs of flood damage. The most commonly used measure is the expected annual damage, which is best understood as the average of flood damages computed over many years. One way of calculating this is simply to add up a long time series of annual damages and divide by the number of years. However, this is rarely possible in practice; a very long record would be necessary because damage would be zero in most years, and in any case exposure to damage would have changed considerably over time.

Expected annual damages are therefore calculated by first fitting a frequency distribution to flood magnitudes. A function relating flood magnitude to damage is then used to derive a relationship between flood damage and the probability of incurring that damage in any one year. All of these stages include unknowns and uncertainties—the relationship between flood discharge and depth may be poorly defined, as might the function relating depth to damage—but it is the objective of this paper to examine the effects of the uncertainties associated with the estimation of the flood frequency relationship. In particular, there is uncertainty about both the appropriate form of the statistical model of flood frequencies, and the value of model parameters. These uncertainties are primarily due to the problems caused by making inferences from small samples of flood peaks. In this paper, emphasis is placed on parameter uncertainty—the form of the model is assumed known—and three alternative procedures for estimating expected annual damages are compared. Practical implications of bias and variability are also considered.

Estimation of Expected Annual Damages

At its simplest, the mean of a random variable x such as annual flood damage is

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Note. Discussion open until June 1, 1989. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on April 18, 1988. This paper is part of the ***Journal of Water Resources Planning and Management***, Vol. 115, No. 1, January, 1989. ©ASCE, ISSN 0733-9496/89/0001-0094/$1.00 + $.15 per page. Paper No. 23129.

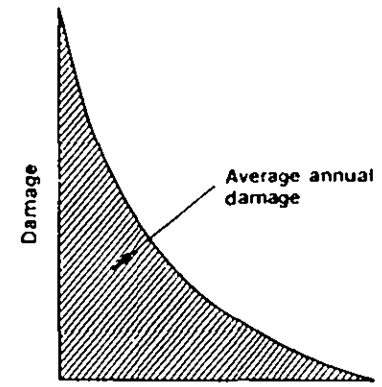

FIG. 1. Expected Annual Damages as Area under Damage-Probability Curve

$$E(x) = \int x f(x) dx \quad (1)$$

where $f(x)$ = the probability density function of that variable. Flood frequency analysts are more used to working with exceedance or non-exceedance probabilities, defined by the cumulative distribution

$$E(x) = \int_0^1 x \, dF$$

which shows that expected annual damages are equal to the area under the graph of damage against non-exceedance (or exceedance) probability (Fig. 1). This is well known to analysts, who routinely calculate expected annual damages by computing damage associated with several return period floods, drawing up a graph similar to Fig. 1 and measuring the area under the curve.

Several authors (Hardison and Jennings 1972; Beard 1978; Tai 1987) have maintained, however, that "conventional" flood frequency estimation procedures such as the methods of moments or maximum likelihood underestimate the true frequency of flooding and thus the value of expected annual damages (Arnell 1988). Beard (1960) illustrated the problem by considering a large number of independent but identical rivers, each with the same record length. If the flood with an exceedance probability of 0.01 was estimated from each sample and the true exceedance probabilities were determined for each estimate, it would be found that the average true exceedance probability would be greater than 0.01 even if the average magnitude was equal to the true magnitude (because the relationship between flood magnitude and frequency is not linear). Over all sites, events would therefore occur in the future with an average frequency greater than 0.01. Beard (1960) called the mean true exceedance probability of estimates of the magnitude of the p probability event the *expected probability* of that flood, and urged that the design flood be taken as the flood with an expected probability equal to p. If not, he argued, the risk of future flooding and hence expected annual damages would be underestimated. Hardison and Jennings (1972) and Tai (1987) showed that use of expected probability resulted in an increase in

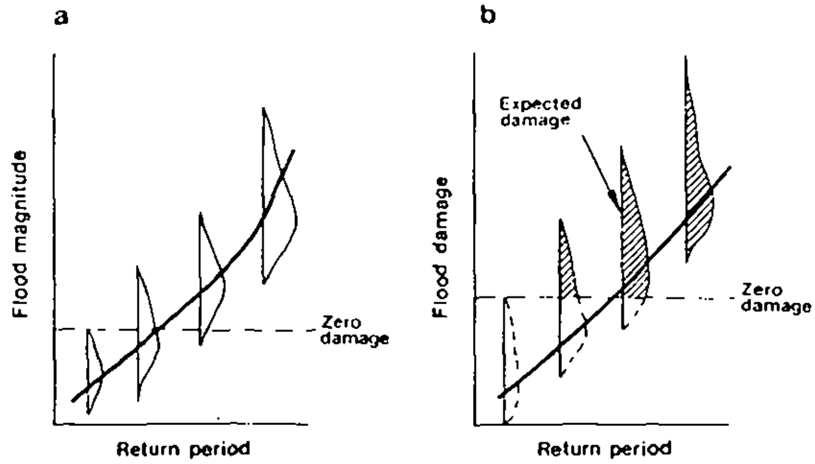

FIG. 2. Sampling Distribution of Flood Magnitudes and Damages

expected annual damages (because converting to expected probability increases the probability assigned to a particular magnitude event), and inferred that bias in the estimation of expected annual damages was therefore reduced. However, a method which gives an unbiased estimate of flood risk does not necessarily give an unbiased estimate of expected annual damages, and Gould (1973a, 1973b) argued that rather than eliminating bias, the use of expected probability increased it. He showed that bias in the estimation of expected annual damages using "conventional" methods was small and of the opposite direction to that implied by Hardison and Jennings (1972). Doran and Irish (1980) subsequently supported Gould's (1973a) conclusions using computer simulation experiments. The present investigations were designed to further clarify this issue.

A second refinement to the conventional procedure for estimating expected annual damages has been presented by James and Hall (1986), followed by Tung (1987) and Bao et al. (1987). The method is based on the recognition that uncertainty in the parameters of the flood frequency distribution can be expressed by sampling distributions for given flood quantile estimates, as shown in Fig. 2, from which confidence limits can be determined. The sampling distribution of the magnitudes of a given frequency flood can then be converted to a sampling distribution of flood damage using the magnitude-damage function [Fig. 2(*b*)], and the expected value of this sampling distribution can be taken as the appropriate estimate of damage for that frequency. This can be expressed as

$$E(D) = \int_0^1 [Dh(D)]dF$$

where $h(D)$ = the probability density function of the estimate of damage D for a given frequency event. James and Hall (1986), Tung (1987), and Bao et al. (1987) all found that the effect of this refinement was to increase the estimate of expected annual damages, although the magnitude of this effect depends of course on the sampling distribution of estimates of flood quantiles (which is strongly influenced by record length) and the shape of the function relating flood magnitude to damage. This procedure, too, was examined in the current study.

Experimental Design

The relative performances at the preceding conventional procedure for estimating expected annual damages and the two refinements were assessed using computer simulation experiments. A general analytical approach is not feasible. Gould (1973b) developed a theoretical expression for bias in expected annual damages, but was forced to assume a normal distribution of flood depths and a linear depth-damage function, and hence a normal distribution of damages. In essence, the simulation experiments involved: (1) Generating a synthetic sample of flood depths from a pre-defined parent distribution; (2) estimating the form of the depth-probability relationship from the sample; (3) converting depth to damage using a depth-damage function; and (4) computing the area under the depth-probability curve. By generating synthetic flood depths it is assumed that the relationship between flood discharge and depth is known with complete certainty; this will not of course be true in practice. Similarly, step (4) of the procedure neglects uncertainties in the relationship between flood depth and flood damage.

The two-parameter lognormal distribution was used as the parent distribution, with parameters selected such that the difference between the true 10- and 100-year flood depths was equal to 1 "synthetic" meter. This distribution was selected because it is possible to apply relatively easily the three alternative ways of estimating expected annual damages. The first, conventional approach involves the estimation of the lognormal parameters from the sample data. Using both the method of moments and the method of maximum likelihood, the parameters can be estimated from

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

$$s = \left[\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1} \right]^{1/2}$$

where x_i = the natural logarithm of flood magnitude; and N = sample size. The logged depth corresponding to a specified frequency can then be calculated from

$$\hat{x}_p = \bar{x} + s z_p \qquad (6)$$

where z_p = the standard normal deviate with exceedance probability p. An estimate of the logarithm of the flood which will be exceeded with an *expected probability* equal to p can be computed from (Beard 1960; Stedinger 1983a):

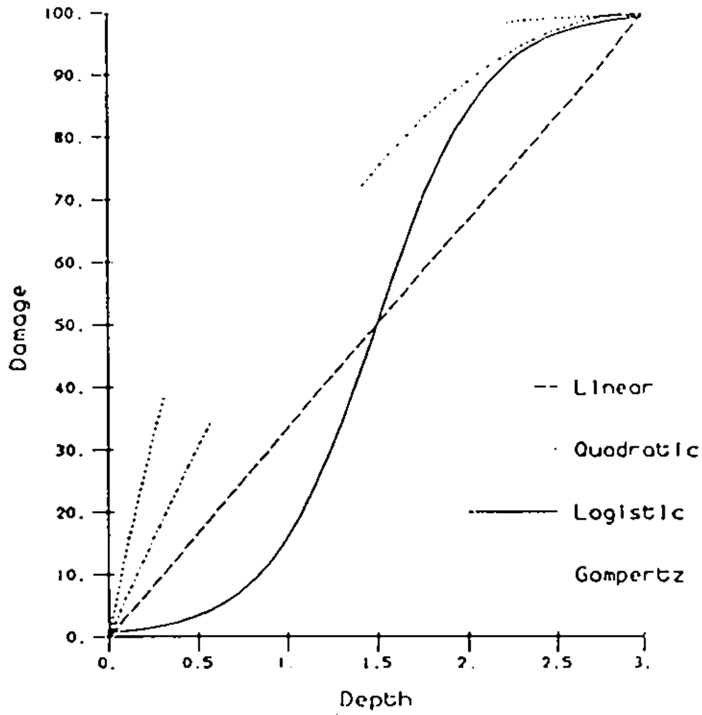

FIG. 3. Depth-Damage Functions

$$x_p = \bar{x} + s\left[1 + \frac{1}{N}\right]^{1/2} t_{p,N-1} \qquad (7)$$

t_p, υ = the quantile with exceedance probability p from a student's t distribution with υ degrees of freedom.

The third approach, involving sampling distributions of quantile estimates, is rather more complicated. Stedinger (1983b) showed that if floods (or their logarithms) were normally distributed, the sampling distribution of a quantile estimate could be derived using the non-central t distribution. The random variable $\sqrt{N}\xi(p)$, where

$$\xi(p) = \left(\frac{x_p - \bar{x}}{S}\right) \qquad (8)$$

has a non-central t distribution with non-centrality parameter $\delta = z_p\sqrt{N}$ and $\upsilon = N - 1$ degrees of freedom, and it is therefore possible to determine the value of $\xi(p)$ exceeded with probability α. Eq. 8 can then be rearranged to give the estimate of the p-probability flood $x_p(\alpha)$, which would be exceeded in samples of size N with probability α: in other words, of all samples of size N from a lognormal distribution with parameters μ and σ, a proportion α would yield estimates of x_p greater than $x_p(\alpha)$. The expected damage associated with exceedance probability p (Eq. 3) is computed by converting log depth $x_p(\alpha)$ to damage D_p using the depth-damage function, and calculating the area under the damage-α curve. This was done using Simpson's Rule, and the approximation to the non-central t distribution presented by Abramowitz and Stegun (1965) was used to compute $x_p(\alpha)$ for a given probability α. It is clear that this approach is much more consuming of computer simulation time than the other two.

Four depth-damage functions were defined as shown in Fig. 3. The quadratic has the form:

$$D = 100\left[\frac{1 - (3 - \text{depth})^2}{9}\right] \tag{9}$$

the Gompertz has the form (Ouellette et al. 1985):

$$D = \frac{100\{e^{(1-e^{-\gamma\,\text{depth}})\alpha^{-1}} - 1\}}{\{e^{\alpha^{-1}} - 1\}} \tag{10}$$

with $\gamma = 2.0$ and $\alpha = 0.5$, and the logistic function is defined as

$$D = 100\left[1 + \exp\left(\frac{-(\text{depth} - u)}{a}\right)\right] - 1. \tag{11}$$

where $u = 1.5$ and $a = 0.3$ All the damage functions give a damage of zero at zero depth and 100 at a depth of 3 "synthetic" meters.

The three procedures yield an array of pairs of damage and associated exceedance probabilities, which are used to construct a damage-probability curve. The area under this curve was calculated for all three methods using the "mid-range probability" method

$$EAD = \sum_{i=1}^{M-1} (p_i - p_{i+1}) \frac{[D_{i+1} + D_i]}{2} \tag{12}$$

(where M = the number of pairs; p = exceedance probability; and D = damage), rather than by the more accurate Simpson's Rule, for two reasons. First, it is more often used in practice, since there are rarely enough pairs of damage and probability available to justify the use of Simpson's Rule, and secondly, use of Simpson's Rule with the third method—which requires numerical integration for each exceedance probability—would be very costly in computer resources. Simulation experiments were undertaken with samples of size 10, 20 and 40; 500 repetitions were used for each experiment. Expected annual damages were calculated for situations where damage begins at the levels of the true 5, 10, 25, 50 and 100 year floods.

Results

Tables 1, 2, and 3 show, for the quadratic and logistic damage functions, the mean, standard deviation, and skewness of estimates of expected annual damages. Similar results were found with the other damage functions. In general, it is clear that all the methods overestimate expected annual damages, particularly when damage commences in infrequent events, but that the conventional method is least biased. This supports Gould's (1973a) and Doran and Irish's (1980) conclusions and conflicts with Hardison and Jennings' (1972) and Beard's (1978) inferences. Although the degree of dif-

TABLE 1. Bias in Estimates of Expected Annual Damage, Expressed as Percentage of True Value, for Different True Probabilities at which Damage Begins

Probabilities (1)	DAMAGE FUNCTION Quadratic 10* (2)	20* (3)	40* (4)	Logistic 10* (5)	20* (6)	40* (7)
Threshold probability 0.2						
conventional method	7.6	0.4	1.7	33.8	12.0	8.7
expected probability method	38.5	16.0	9.6	120.9	52.6	28.5
expected damage method	41.4	17.5	10.5	143.5	64.4	35.5
Threshold probability = 0.04						
conventional method	53.3	20.9	14.1	169.3	68.6	41.8
expected probability method	86.5	80.0	42.6	622.9	232.0	108.5
expected damage method	200.9	84.6	44.5	846.4	340.0	168.0
Threshold probability = 0.01						
conventional method	179.3	72.7	43.3	568.4	194.7	105.3
expected probability method	644.0	246.7	118.0	2,647.3	742.1	278.9
expected damage method	714.7	273.3	129.3	4,163.6	1,400.0	589.5

*Sample size.
Note: Simulation results from 500 repetitions.

ference varies with damage function, the results clearly show that use of either expected probabilities or the "expected damage" method would produce very biased estimates of expected annual damages. These two methods yield very similar results (when using a lognormal distribution), which re-

TABLE 2. Standard Deviation of Expected Annual Damage Estimates, Divided by True Value, for Different True Probabilities at which Damage Begins

Probabilities (1)	DAMAGE FUNCTION Quadratic 10* (2)	20* (3)	40* (4)	Logistic 10* (5)	20* (6)	40* (7)
Threshold probability = 0.2						
conventional method	0.80	0.53	0.40	1.48	0.89	0.66
expected probability method	0.86	0.56	0.41	1.86	1.05	0.72
expected damage method	0.86	0.56	0.42	1.84	1.05	0.72
Threshold probability = 0.04						
conventional method	2.03	1.17	0.86	5.07	2.35	1.57
expected probability method	2.69	1.44	0.96	8.58	3.60	2.04
expected damage method	2.71	1.44	0.96	8.71	3.69	2.10
Threshold probability = 0.01						
conventional method	5.35	2.54	1.75	18.11	6.26	3.58
expected probability method	8.80	3.79	2.22	40.11	12.68	5.63
expected damage method	8.92	3.80	2.20	43.11	14.21	6.47

*Sample size.
Note: Simulation results from 500 repetitions.

TABLE 3. Skewness of Expected Annual Damage Estimates, for Different True Probabilities at which Damage Begins

Probabilities	DAMAGE FUNCTION					
	Quadratic			Logistic		
	10*	20*	40*	10*	20*	40*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Threshold probability = 0.2						
conventional method	1.08	0.77	0.70	2.07	1.50	1.31
expected probability method	0.85	0.67	0.66	1.48	1.22	1.18
expected damage method	0.85	0.68	0.68	1.47	1.22	1.20
Threshold probability = 0.04						
conventional method	2.37	1.76	1.51	3.63	2.88	2.53
expected probability method	1.66	1.39	1.36	2.25	2.08	2.16
expected damage method	1.66	1.43	1.36	2.12	1.93	2.02
Threshold probability = 0.01						
conventional method	3.48	2.75	2.36	5.15	4.36	3.84
expected probability method	2.21	1.99	2.01	2.83	2.91	3.27
expected damage method	2.12	1.99	2.01	2.34	2.34	2.58

*Sample size.

Note: Simulation results from 500 repetitions.

flects similarities in their derivation. Both are based on the t-distribution (Stedinger 1983a, 1983b), and the expected value of the estimate of the p probability flood $\int x_p f(x) dx_p$ is very close to the estimate computed from Eq. 7. The actual difference between the two methods depends on the shape of the damage function (the expected damage is not equal to the damage associated with the expected magnitude, except with a linear damage function) and, to a lesser extent, the numerical approximation.

The contrasts in the degree of bias between the different damage functions depends on the rate of change of damage with magnitude, particularly at low magnitudes. With the logistic curve, damage is limited for floods just above the damage threshold (Fig. 3) but increases significantly at higher depths. The frequency with which floods reach this depth is estimated with greater bias and uncertainty than the frequency with which damage begins. For a given flood frequency relationship and threshold at which damage begins, therefore, the greater the proportion of damage which occurs in small floods, the less the bias and variability in estimate of expected annual damage.

As sample sizes increase, all the methods become less biased (bias falls from over 50% to just over 15% for the conventional method, with damage occurring with a true probability of 0.04, for example). The expected probability and expected damage methods improve the most and with very large samples all three methods would give the same results. Sample variability of estimates also falls as sample sites increase (Table 2) and, for high damage thresholds at least, there is less difference in variability than bias between the three methods. The coefficient of skew, given in Table 3, shows the high asymmetry in the sampling distribution of expected annual damages, due to the occasional very large estimates.

The magnitude of estimated expected annual damages depends partly on the estimated slope of the depth-frequency curve but much more closely on

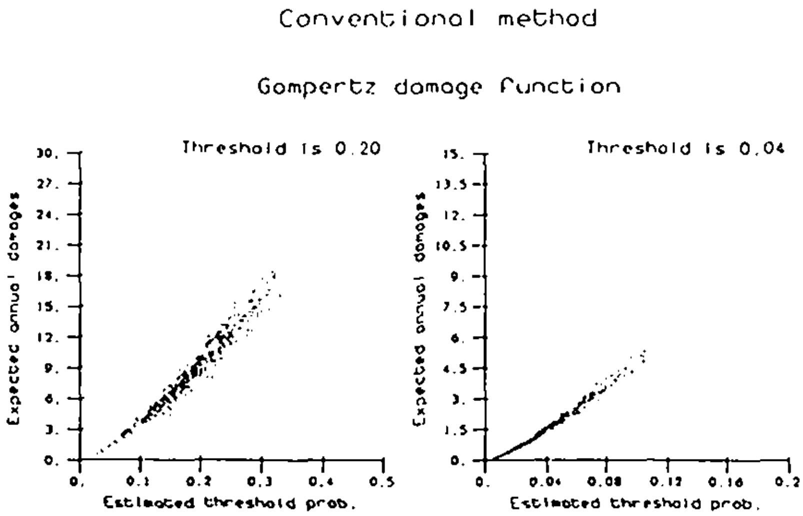

FIG. 4. Variation of Estimated Expected Annual Damages with Estimated Threshold Probability

the estimated probability at which damage begins. Fig. 4 shows the strong relationship between estimated threshold probability and computed expected annual damages (for the Gompertz damage function and a sample size of 20). The bias and variability in expected annual damages is clearly related to the bias and variability in the estimated threshold probability. The reason for the difference in bias between the conventional and expected probability methods can be seen in Table 4, which shows the mean estimated probability at which damage begins. The conventional estimator provides a good estimate of the threshold probability, but the expected probability method produces a very biased estimate of the risk of damage. This arises because a

TABLE 4. Mean Estimated Threshold Probability: Conventional and Expected Probability Estimators

Estimators (1)	True Threshold Probability 0.2 (2)	0.1 (3)	0.04 (4)	0.02 (5)	0.01 (6)
N = 10					
conventional method	0.199	0.103	0.047	0.027	0.016
expected probability	0.219	0.126	0.068	0.045	0.031
N = 20					
conventional method	0.194	0.098	0.042	0.023	0.013
expected probability	0.205	0.111	0.053	0.031	0.019
N = 40					
conventional method	0.199	0.101	0.042	0.022	0.012
expected probability	0.205	0.107	0.047	0.026	0.015

Note: Averaged over 500 repetitions.

method that gives an estimate of the flood exceeded on average with the desired risk p (i.e., unbiased); it does not produce an unbiased estimate of the risk of a specified magnitude (such as a floor level) being exceeded.

Implications of Uncertainty in Estimation of Expected Annual Damages

The results of the previous section have emphasized the potentially very large sampling variability in the estimation of expected annual damages due solely to sampling variability in the observed flood data. In current practice only a single "best" estimate of expected annual damages is used, derived from the "best" estimate of the flood frequency curve, but it may be useful to have information on the precision of this estimate. Some workers, for example Grigg (1978), have attempted to derive confidence limits for an estimate of expected annual damages directly from confidence intervals on flood magnitude estimates, as shown in Fig. 5(a)–5(c). This, however, is incorrect due to a misinterpretation of the meaning of confidence intervals for flood quantiles. These confidence limits should be interpreted solely as intervals for the range of magnitudes for a *specified* exceedance probability; the locus of 90% confidence interval values (i.e., 90% of estimates of magnitude for that probability are greater) does not define the frequency curve which will be exceeded over all probabilities 90% of the time. One sample curve may yield an estimate of the 10-year flood outside the 90% interval for that return period, for example, while yielding a 100-year flood estimate close to the mean value [Fig. 5(d)]. An approach such as this would overestimate confidence intervals and give an unduly pessimistic impression of precision.

It is well known that the standard deviation of the sampling distribution of the mean of a random variable is equal to the standard error, or the standard deviation of the variable divided by the square root of the sample size:

$$\text{standard error } (\bar{x}) = \frac{\text{s.d.}(x)}{\sqrt{N}} \tag{13}$$

It is therefore possible to estimate the standard error of the sampling distribution of expected annual damages by computing the standard deviation of annual damages using

$$\text{s.d.}(D) = [E(D^2) - E^2(D)]^{1/2} \tag{14}$$

where $E(D)$ = expected annual damages and $E(D^2)$ = the area under the "damage-squared"-probability curve. Table 5 compares the average standard error of expected annual damages (computed using Eq. 14), with the observed standard deviation of estimates of expected annual damages. It can be seen that, for cases where damage occurs in frequent events at least, the standard error provides a good estimate of sample standard deviation. However, the high skew of the sampling distribution (Table 3) means that confidence limits cannot be based on just expected annual damages and standard error, and, although it is possible to estimate the skew of annual damages using the area under the "damage-cubed"-probability curve, sample skewness estimates are notoriously unreliable.

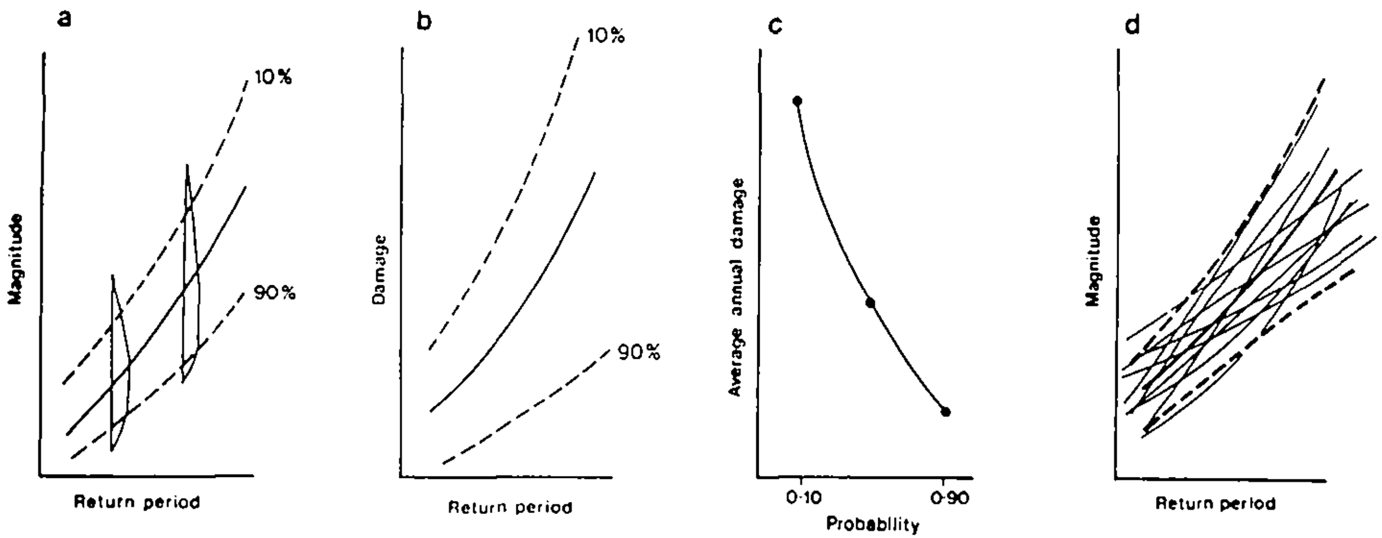

FIG. 5. Calculation of Sampling Distribution of Expected Annual Damages from Confidence Limits on Flood Magnitude—and Why It Is Incorrect

TABLE 5. Mean Standard Error of Expected Annual Damages and Observed Standard Deviation of Expected Annual Damages Estimates (Logistic Damage Function)

Expected annual damages (*EAD*) (1)	True Threshold Probability 0.2 (2)	0.1 (3)	0.04 (4)	0.02 (5)	0.01 (6)
N = 10					
average standard error of *EAD*[a]	2.301	1.410	0.778	0.510	0.343
standard deviation of estimated *EAD*[b]	2.332	1.410	0.776	0.511	0.344
N = 20					
average standard error of *EAD*[a]	1.553	0.883	0.423	0.244	0.144
standard deviation of estimated *EAD*[b]	1.398	0.765	0.359	0.206	0.119
N = 40					
average standard error of *EAD*[a]	1.114	0.612	0.271	0.143	0.077
standard deviation of estimated *EAD*[b]	1.036	0.548	0.240	0.128	0.068

[a]The "average standard error" is the average of 500 estimates of the standard error of *EAD*.

[b]The "standard deviation of estimated *EAD*" is the standard deviation of the 500 estimates of *EAD*.

To estimate the sampling distribution and confidence intervals for expected annual damages in practice, it would therefore be necessary to resort to the use of computer simulation. Such an approach would follow the form of the experiments reported here, with the parent distribution defined by the parameters as estimated at the site of interest. The computer experiments would allow the construction of a sampling distribution of expected annual damages and the identification of desired confidence intervals, but the mean of this distribution (the statistical "best" estimate of expected annual damages) would be different to—and greater than—the value of expected annual damages derived from the best estimate of the frequency curve. The analyst would have to insure that the parent used for the simulation experiments yielded "realistically variable" estimates of flood frequencies. This can be done by selecting a sample size which produces synthetic sampling distributions of flood quantiles consistent with previously defined confidence intervals calculated from the original site data. The synthetic sample size need not be the same as the observed sample size; additional (for example regional) information has an equivalent effect to providing extra years of data.

Conclusions

This paper has presented results of a series of computer simulation experiments into the effects of uncertainties in flood frequency estimation on the bias and variability of estimates of expected annual damages. It has been shown that the "conventional" approach (using a method such as moments or maximum likelihood to attempt to obtain unbiased estimates of flood magnitudes) slightly overestimates expected annual damages where damage begins in frequent events, with greater overestimation where damage begins in rare events. These results conflict with the assertion of Hardison and Jennings (1972) and Beard (1978) that conventional estimators underestimate expected annual damages, and it has been shown that their proposed approach—to use expected probabilities—gives even greater bias. This is because while the expected probability method gives an estimated magnitude exceeded on average with the specified risk, it does not give an unbiased estimate of the risk of a specified magnitude (such as the level at which damage begins) being exceeded. The third method considered, which computes the expected damage for each probability flood by averaging across the sampling distribution of that flood estimate, also gives higher estimates of damage for a given flood probability and hence also leads to very significant overestimation of expected annual damages. For all three methods, bias reduced rapidly as sample sizes increased.

The experiments have shown that estimates of expected annual damages are highly variable, particularly where damage begins in low-frequency events. The sampling distribution of expected annual damages is also very highly skewed. It has been shown that the bias and variability in the estimate of expected annual damages is closely linked to the bias and variability in the estimation of the probability at which damage begins, emphasizing again the importance of using as good an estimate of this threshold probability as possible.

The exact form of the flood magnitude-damage relationship determines the degree of bias in estimated expected annual damage. Bias is least if damages increase rapidly once the damage threshold is reached; conversely, it is higher the greater the magnitude that "significant" damage begins.

A simulation based method has been briefly described for deriving confidence intervals for an estimate of expected annual damages in practice.

Finally, it is important to note that the results show only the effect of uncertainties in flood magnitude-frequency estimation. In practice, the bias and variability that these produce are compounded by uncertainties in the relationships linking flood magnitude with damages.

Acknowledgments

The research presented in this paper was undertaken as part of a research project funded by the Ministry of Agriculture, Fisheries and Food for England and Wales.

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