Short pulse multi-frequency phase-based time delay estimation

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An approach for time delay estimation, based on phase difference detection, is presented. A multiple-frequency short continuous wave pulse is used to solve the well-known phase ambiguity problem when the maximum distance exceeds a full wavelength. Within an unambiguous range defined with the lowest frequency difference between components, the corresponding phase difference is unique and any distance within this range can be determined. Phase differences between higher frequency components are used to achieve a finer resolution. The concept will be presented and the effectiveness of the approach will be investigated through theoretical and practical examples. The method will be validated using underwater acoustic measurements, simulating noisy environments, demonstrating resolutions better than a 50th of a wavelength, even in the presence of high levels (−5 dB) of additive Gaussian noise. Furthermore, the algorithm is simple to use and can be easily implemented, being based on phase detection using the discrete Fourier transform.

I. INTRODUCTION

Time delay estimation is a fundamental step in source localization or beamforming applications and has attracted considerable research attention over the past few decades in many fields including radar, sonar, seismology, geophysics, ultrasonics, communication, and medical imaging. Various techniques are reported in the literature 1–5 and a state of the art review can be found in Ref. 6 which concerns critical techniques, limitations, and recent advances that have significantly improved the performance of time delay estimation in adverse environments. These techniques can be classified into two broad categories: correlator-based approaches and system-identification-based techniques. Both categories can be implemented using two or more sensors; in general, more sensors lead to increase robustness due to greater redundancy. When the time delay is not an integral multiple of the sampling rate, however, it is necessary to either increase the sampling rate or use interpolation. 6 In this paper, we present a new approach for time delay estimation, based on the received signal phase information. This avoids barriers encountered by alternative approaches based on phase information, which are limited by the need to use a reference signal, usually provided by a coherent local oscillator. 7 Ambiguities in such phase measurement, caused by the inability to count integer number of cycles (wavelengths), are resolved using the Chinese remainder theorem (CRT) taken from number theory, where wavelength selection is based on pair-wise relatively prime wavelengths. 7,8 However, the CRT is not entirely robust, in the sense that small errors in its remainders may induce a large error in the determined integer. 9,10

Another phase-based measurement approach, adopted to ensure accurate positioning of commercial robots, uses two or more frequencies in a decade scale in a transmitted signal. In this, the phase shift of the received signal with respect to the transmitted signal is exploited for ranging. 11–12 However, this approach is valid only when the maximum pathlength/displacement is less than one wavelength, otherwise a phase ambiguity will appear.

The time delay estimation approach proposed here is based on the use of local phase differences between specific frequency components of a short continuous wave (cw) received signal pulse. Within an unambiguous range defined by the lowest frequency difference between components, the corresponding phase difference is unique and any distance within this range can be determined. Phase differences between higher frequency components are used to achieve a finer resolution. Thus our approach overcomes the need to cross-correlate the received signal with either a reference signal or the transmitted signal. Unlike conventional methods, this approach is not limited by phase ambiguity; therefore most practical situations, where the range to be determined is beyond one wavelength, can be accommodated.

II. METHODOLOGY

A. Concept

The inspiration for development of the technique comes from the observation that bats have been shown to have exceptional resolution with regard to target detection when searching during flight. 13–15 Au and Simmons, 16 somewhat controversially, concluded that bats using pulses with a center frequency of about 80 kHz (40 kHz bandwidth) can achieve a distance resolution in air approaching 20 μm. At

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this frequency, and using $\lambda/2$ as the guide for resolution, we see that the resolution reported by Au and Simmons is about 200 times better than that predicted conventionally. We sought to develop a methodology which would allow us to approach a resolution comparable to that achieved by such creatures which is still better than any man-made system.

Consider an acoustic pulse containing a single frequency component $f_1$ with an initial zero phase offset. This pulse is emitted through the medium, impinges on a target, is reflected and returns. The signal is captured and its phase measured relative to the transmitted pulse. Given this situation, we cannot estimate the distance to and from an object greater than one wavelength away (hence, usually, we would estimate the time of arrival of the pulse and assume a value for the velocity of sound in the medium to estimate the distance to the target).

For simplicity, assume the pulse contains a single cycle of frequency $f_1$ of wavelength $\lambda_1$. The distance $D$ to the target can be expressed as

$$D = n_1\lambda_1 + r_1,$$

where $\lambda_1 = v/f_1$, $n_1$ is an integer, $r_1$ is a fraction of the wavelength $\lambda_1$, and $v$ is the speed of sound in the medium.

$r_1$ can be expressed as follows:

$$r_1 = \lambda_1 \times \frac{\phi_1}{360},$$

where $\phi_1$ is the residual phase angle in degrees. Combining Eqs. (1) and (2) and rearranging

$$D = n_1\lambda_1 + \lambda_1 \times \frac{\phi_1}{360},$$

$$D = \frac{v}{f_1} \left(n_1 + \frac{\phi_1}{360}\right).$$

If we transmit a second frequency component $f_2$ within the same pulse, then it will also have associated with it a wavelength $\lambda_2$ and a residual phase $\phi_2$; similarly,

$$D = \frac{v}{f_2} \left(n_2 + \frac{\phi_2}{360}\right).$$

Equations (3) and (4) can be solved by finding Eq. (4) $-\text{Eq. (3)} \times (\lambda_2/\lambda_1)$ and rearranged to give

$$D = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} \left( n_2 - n_1 \right) + \frac{\left(\phi_2 - \phi_1\right)}{360}.$$  

Using $v = f \times \lambda$, we obtain

$$D = \frac{v}{\Delta f} \left(\Delta n + \frac{\Delta \phi}{360}\right).$$  

where $\Delta f = f_2 - f_1$, $\Delta n = n_2 - n_1$, and $\Delta \phi = \phi_2 - \phi_1$.

Knowing $D = v \times t$, we deduce the time delay $t$ as

$$t = \frac{1}{\Delta f} \left(\Delta n + \frac{\Delta \phi}{360}\right).$$

If we impose the condition that $\Delta n \leq 1$, then Eq. (7) can be solved. This restriction on $\Delta n$ is imposed as follows.

- A distance $D$ is selected within which we require an unambiguous range measurement.
- Select a frequency $f_1$ within the bandwidth of the system, and its corresponding wavelength $\lambda_1$ [from Eq. (1)].
- Similarly, using Eq. (1), select frequency $f_2$ with its corresponding wavelength $\lambda_2$ such that the number of cycles is $n_2 = n_1 + 1$.

Considering Eq. (6), the maximum range is achieved by this approach when $\Delta n = 1$;

$$R = \frac{v}{\Delta f}.$$  

Therefore, $R$ is the maximum unambiguous range that can be achieved using the two frequencies $f_1$ and $f_2$ as described above. As the phase differences $\Delta \phi$ will be unique within this range $R$, any distance within this range can be determined unambiguously.

### B. Example

We demonstrate in this example how we could measure a range by using two frequencies and their related phase differences.

Consider an unambiguous range $R$; two frequencies $f_1$ and $f_2$ comprising integer number of cycles $n_1$, $n_2$ of wavelength $\lambda_1$, $\lambda_2$, respectively, as shown in Table I. Within this range $R$, consider a distance to target $d = 1000.1234$ mm we wish to estimate. Assume that a short cw pulse comprising these two frequencies $f_1$ and $f_2$ is sent toward the target at the distance $d$.

Using $f_1$, $f_2$ and Eqs. (1) and (6) for this distance $d$ gives an integer number $n_1 = 133$ and a residual fraction of cycle of $r_1 = 0.349786$ corresponding to a residual phase of $\phi_1 = 125.923^\circ$. Similarly for the frequency $f_2$, we find the residual phase $\phi_2 = 5.953^\circ$ corresponding to $n_2 = 134$ and $r_2 = 0.0165356$. Thus, $\Delta \phi = \phi_2 - \phi_1 = -119.970^\circ$.

We use this value in the formula given in Eq. (6), since $\Delta \phi$ is negative; this means $\Delta n = 1$. Using $v$ from Table I, $\Delta f = 1$ kHz, and substituting into Eq. (7) give a first estimate of the range $d_{f_1/f_2} = 1000.1233$ mm (caret is used here to mean an estimate). The unambiguous range $R$ [Eq. (8)] is independent of the frequencies used depending only on the difference in frequency $\Delta f$. 

<table>
<thead>
<tr>
<th>$R$ (mm)</th>
<th>$v$ (mm/µs)</th>
<th>$f_1$ (kHz)</th>
<th>$\lambda_1$ (mm)</th>
<th>$n_1$</th>
<th>$f_2$ (kHz)</th>
<th>$\lambda_2$ (mm)</th>
<th>$n_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>1.50</td>
<td>200</td>
<td>7.50</td>
<td>200</td>
<td>201</td>
<td>7.462</td>
<td>201</td>
</tr>
</tbody>
</table>

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Note that in practice such resolution may not be achievable and limitations must be considered. For example, if the accuracy of estimating the phase is within ±0.5°, then the phases in the example above become φ₁ = 126.0° and φ₂ = 6.0°, giving d = 1000.0 mm implying an error of 0.1234 mm. To get a finer resolution, we have to add other frequency components.

C. Using multiple frequencies through a “Vernier approach”

We demonstrate in this example how we could obtain a finer resolution in a range by using more than two frequencies and their related phase differences.

In Eq. (6), we imposed the condition that |Δn| = 1. The values of frequencies f₁ and f₂ were chosen to ensure this condition and to obtain a first estimate of the distance ******̂d₁f₂**,** and an estimate of the time delay ******̂t₁f₂** [Eq. (7)].

Introducing a third frequency f₃ as shown in Table II, such that f₃−f₁ = 10×(f₂−f₁); f₂ differs from f₁ by 1 kHz and f₃ differs from f₁ by 10 kHz.

Again from Eq. (1), for f₃ and d = 1000.1234 mm, φ₃ = 6.219°, which we would measure as 6.5°. Thus, Δφ₁₃ = φ₃−φ₁ = −119.5°. We add 360° to give 240.5°. However, Δn₁₃ between frequencies f₁ and f₃ is now 7 (in fact, 6 since we have already added in 360° to make the phase difference positive).

Using Eq. (6) with Δφ₁₃ and different values of Δn₁₃ (0–6) to get different distance estimate ******̂d₁f₃**,** where k = 0, ... , 6, in this case.

Applying Eq. (6) recursively for Δn₁₃ = 0, ... , 6 to calculate ******̂d₁f₃**,** selecting ******̂d₁f₃**,** closest in value to ******̂d₁f₂**,** as the optimum value ******̂d₁f₃** = 1000.2083 mm |k|=6. Hence, a new best time delay estimate ******̂t₁f₃**,** |k|=6. Note that the new best delay estimate is with an error of 0.0849 mm.

If a fourth frequency f₄ is introduced, as shown in Table II, such that f₄−f₁ = 100.0 kHz, using Eq. (1) again gives φ₄ = 8.8848° which we measure as 9°. Thus, Δφ₁₄ = φ₄−φ₁ = −117° which gives Δφ₁₄ = 249.5° after adding 360°. Note that Δn₁₄ = 66 in this case.

Similarly, select the estimate ******̂d₁f₄**,** (Δn=0,1,2,...,66) closest in value to ******̂d₁f₃**,**. This occurs at Δn₁₄ = 66 giving ******̂d₁f₄**,** = 1000.125 mm. Taking this as the best estimate, the final error is 0.0016 mm = 1.6 μm.

Note that the frequencies need not be in decadal scale variation; however, they should be selected within the bandwidth of the system. All the selected frequencies should be a multiple of the lowest difference ensuring an integer number of cycles; in this way, we ensure that the phase calculated by the discrete Fourier transform (DFT) will be correct (i.e., corresponding exactly to the frequency bins). Thus, this example is reminiscent of the operation of a Vernier gauge as follows:

- Δφ₁₂, related to the frequencies f₁ and f₂, gives the first estimate of the distance ******̂d₁f₂**,** hence ******̂t₁f₂**,**
- A higher frequency f₃ is then used (decade difference) to measure the same range but with a finer resolution. So a more accurate approximation to the measured range is obtained ******̂d₁f₃**,**
- Similarly, the measured range ******̂d₁f₄** corresponding to Δφ₁₄ within f₁ and f₄ will give the ultimate estimate of the measured range ******̂d**.
- Consequently, the maximum distance and the minimum resolution achieved are determined by the choice of the frequencies f₁, f₂, f₃, and f₄.

An example algorithm calculating the phase at each frequency and outlining the iterative phase difference measurements is given in the Appendix.

D. Phase offset measurement calibration

In the numerical example above, it is assumed that all phases are accurately transmitted and received, with no phase error on transmission or reception, and that all frequencies have zero phase offset with respect to each other. In practice this is almost certainly not the case and phase offsets between frequencies should be accounted for as discussed below.

Considering two frequencies f₁ = 200.0 kHz and f₂ = 201 kHz, and assuming the speed of sound in water (v = 1.5 mm/μs), from Eq. (8), the unambiguous range R = 1500 mm and Δn is 0 or 1. Considering the above

\[
\Delta = \frac{D}{v} = \frac{n + \phi/360}{f}.
\]

Consider two distances d₁, d₂ corresponding to two “times” t₁ and t₂ such that the number of cycles n is the same for both frequencies over these distances, and assuming the phase measured includes a phase offset for that frequency. As an example, suppose the unknown phase offset for f₁ is 10°, for f₂ is 30°, and assume d₁ = 100 mm.

From Eq. (9), the term (n₁+φ₁/360) would be calculated as 13.33 cycles, where φ₁ = 120°. The “measured” φ₁ = 120°+10° = 130° (φ₁_m = φ₁_distance + φ₁_offset).

Similarly, for f₂ we obtain 13.40 cycles, where φ₂ = 144°. The measured φ₂ = 144°+30° = 174°; from Eq. (7), t₁ = 12.22 μs. The actual time should be 6.66 μs.

Assume a second distance d₂ = 200 mm. Using Eq. (9), for f₁ we obtain 26.66 cycles, which gives φ₁ = 240°. The measured φ₁ = 240°+10° = 250°. Similarly, for f₂ we obtain 26.80 cycles, which gives φ₂ = 288°. The measured φ₂ = 288°+30° = 318°. Thus, using Eq. (7), t₂ = 18.88 μs. The actual time should be 13.33 μs.

A linear relationship could be deduced between t and d as follows:

\[
t = m \times d + c,
\]

TABLE II. Parameters used in example 2.

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>v (mm/μs)</th>
<th>f₃ (kHz)</th>
<th>λ₁ (mm)</th>
<th>n₁</th>
<th>f₄ (kHz)</th>
<th>λ₄ (mm)</th>
<th>n₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>1.5</td>
<td>210</td>
<td>7.142</td>
<td>210</td>
<td>300</td>
<td>5.0</td>
<td>300</td>
</tr>
</tbody>
</table>
where the slope \( m = 1 / 1.5 \approx 0.66 \) is the speed of sound measured as 1 mm per 0.66 \( \mu \)s or 1/0.66 \( \approx 1.5 \) \( \mu \)s/\( \mu \)s. The intercept \( c = 5.55 \) \( \mu \)s is a measure of the relative phase between \( f_1 \) and \( f_2 \). Since \( \Delta f_{12} = 10 \) kHz, 1 cycle is 100 \( \mu \) long; consequently, the offset of \( 5.55 \mu s = 360 \times (5.55/100) = 20^\circ \), which is equal to the relative phase (30–10) between the two frequencies. If we had known the phase offset between the two frequencies (20\( ^\circ \)), then in the calculation of times we would have obtained for \( t_1 \) a new phase difference of \((174 - 130 - 20)\approx 24^\circ \) giving a time for \( t_1 = 6.66 \mu \)s.

Similarly, for \( t_2 \) we obtain a new phase difference of \((318 - 250 - 20)\approx 48^\circ \) giving a time for \( t_2 = 13.33 \mu \)s. Both \( t_1 \) and \( t_2 \) are now correct. Note the following.

- If we assumed \( d_1 \) was 100 mm but it was actually, say, 120 mm and that \( d_2 \) was 200 mm but it was actually 220 mm, then we obtain the phase offset as 15.2\( ^\circ \). The slope of Eq. (10) above, however, is unaffected. For example, such uncertainty may arise if the distance traveled by the wave within the transducers is not taken into consideration.
- If the temperature changes, so \( v \) changes; this changes the slope of Eq. (10) but not the time at which the phase offset. For example, if \( v = 1.6 \) mm/\( \mu \)s, then Eq. (10) becomes \( t = (1/1.6) \times d + 5.55 = 0.625 \times d + 5.55 \).

III. APPLICATION

A. Experiment

To demonstrate this approach, a series of measurements was performed in a water tank measuring 1530 \( \times \) 1380 \( \times \) 1000 mm\(^3\). Two broadband ultrasonic transducers (Alba Ltd., Glasgow, UK), having a wide bandwidth with a center frequency between 100 and 130 kHz, operate as both transmitters (Tx) and receivers (Rx) of ultrasound with a beam width of around 10\( ^\circ \) at the center frequency, where \( -3 \) dB bandwidth is 99 kHz (72–171 kHz). These were mounted on a trolley, movable in the X-Y directions. Linear encoders (Newall Ltd., Leicester, UK) were used to measure displacement of the rails in the \( x \)-direction and a software provided readouts of transducer positions. The temperature in the tank was measured by thermocouples calibrated using a quartz thermometers which are traceable to national standard. They were positioned on the four sides panels of the tank and recorded \( 19.80 \pm 0.05 \) \( ^\circ \)C during the experiment. The experiment setup is shown in Fig. 1. The transmitter was driven by a 20 V peak-to-peak waveform consisting of short cw pulse comprising four frequencies (70, 71, 80, and 170 kHz). A modular system comprising a 16-bit arbitrary waveform generator (Ztec ZT530PXI) and a 16-bit digital storage oscilloscope (Ztec ZT410PXI) was used for transmit/receive process. Software was used to control signal transmission and acquisition. Distances between Tx and Rx of 305.800, 345.778, 481.128, 535.131, 535.252, 624.515, 759.182, and 862.887 mm were selected to be within an unambiguous range of \( R = 1500 \) mm [Eq. (8)], as set by the linear encoders. A set of ten signals, noise free signal and signals with different signal to Gaussian noise ratios (SNR=−20,−10,−5,−2,0,2,5,10,20 dB), was transmitted at each distance described above. Note that the added Gaussian noise was generated by software and added to the original signal in each case. Before transmitting, each signal was multiplied by a Tukey window (cosine-tapered window with a 0.2 taper ratio) to reduce the “turn on” and “turn off” transients of the transducers. At each distance, three repeat pulses were transmitted and received for each SNR and each distance. Furthermore, 60 repeat pulses were transmitted and received while keeping the distance constant at 862.887 mm to assess the repeatability of the system (see Fig. 3). The sampling frequency \( F_s \) was set to 10 MHz, giving a number of samples \( N = 20000 \) and a 2 ms pulse length. A DFT was then applied to the received pulses to obtain the magnitude and phase information for each of the four frequency components, using a window of \([N/2+1:N]\) for each received signal. This gave a resolution \( F_s/N/2=1 \) kHz which was consistent with the smallest step between the four frequencies comprised in the pulse.

Figure 2(a) shows the noise free transmitted (top) and received (bottom) signals when Tx and Rx were 305.800 mm apart, while Fig. 2(b) shows transmitted (top) and received (bottom) signal in the greatest noise case SNR (−20 dB). Note the DFT reports phase with respect to cosine, whereas sine waves were used in this experiment. Sine waves are returned with a phase of −90\( ^\circ \) relative to cosine waves by the DFT. This was not an issue, since relative phase differences were used. Using the phase for each distance obtained by the DFT, the phase-based time delay algorithm given in the Appendix was applied to obtain the corresponding estimated times for each phase difference \( \Delta \phi_{12}, \Delta \phi_{13}, \Delta \phi_{14}, \Delta \phi_{23}, \Delta \phi_{24}, \) and \( \Delta \phi_{34} \) for the pairs \( f_1f_2, f_1f_3, f_1f_4, f_2f_3, f_2f_4, \) and \( f_3f_4 \), respectively.

Using a simple calculation of the first estimate by \( t_{12} = \Delta \phi_{12}/(f_2-f_1) \) as a first estimate using Eq. (7) gave corresponding estimated times \( \hat{t}_{12}, \hat{t}_{13}, \hat{t}_{14}, \hat{t}_{23}, \) and \( \hat{t}_{34} \), respectively. For each distance, \( \hat{t}_{14} \) should be the best estimate (i.e., the greatest \( \Delta f \)). The sensitivity of the algorithm to noise was reduced by the following.

- Limitation of the noise beyond the sensitivity of the transducer. The transducer act with its bandwidth as a bandpass filter.
- Signals were averaged 32 times during acquisition. When acquiring the signal with averaging, this reduces the noise level.
- The DFT operates on sine waves preferentially, even in the
B. Results and discussion

The best estimated time delays $\hat{t}_{14}$ for the eight different distances and using different signal-to-noise ratios are shown in Table III. For each distance, ten estimations were given corresponding to different signal-to-noise ratios. The column 2 corresponding to $\hat{t}_{14,\text{Org}}$ represents the estimated times when no noise was added to the signal. Using $\hat{t}_{14,\text{Org}}$ as best estimate time reference related to noise free signal, we can see that the technique became more robust to noise when the signal-to-noise ratio becomes greater than 2 dB. The variation is within 50, 30, 20, and 10 μs when the SNRs are 2, 5, 10, and 20 dB, respectively.

For the two rows in Table III corresponding to the distances 535.131 and 535.252 mm, where the displacement is about 120 μm, the estimated time for this displacement is about 120 ± 20 ns when the SNR is greater than –5 dB. As the sampling frequency used in this experiment was 10 MHz, using the conventional method (e.g., cross-correlation) to estimate the time delay, we would not get a best time resolution of less than $\frac{1}{10^7}=100$ ns, which corresponds to a displacement of 150 μm, when assuming a speed of sound of 1500 m/s. Hence, such displacement could not be measured without interpolation or oversampling. Moreover, if the cross-correlation approach was used to estimate the time delay in this experiment, all the measurements in Table III would be an integer of the sampling rate; in this case, 100 ns, as the technique is sampling-frequency dependent.

Figure 3(a) shows the time-distance plots, with the least squares fitting (linear regression), corresponding to the most noisy case (SNR = –20 dB). The error plot (bottom) shows the residuals with a norm of about 170 μs. Figure 3(b) shows the case of the best SNR (20 dB) and the corresponding residual error norm equal to 0.268 μs. The estimated time shown in y-axis corresponds to the best estimate $\hat{t}_{14}$ which is based on the phase difference between the two components $f_1=70$ kHz and $f_2=170$ kHz. The time delay estimation values with and without SNR ratios can be seen to be almost identical for each distance except for the two lowest SNR –20 and –10 dB. The data plotted in Fig. 3(b) (top) corresponds to a linear equations of the form $t=m d+c$, where $c$ is a measure of the phase offset for the two frequencies equal to $c\times(f_2-f_1)\times360$ and $1/m=v$ (mm/μs) is the estimated sound velocity in water, where $m$ is the slope. Solving $t =md+c$ by least squares fitting gave the best estimate $m$, where $(1/m)$ is the speed of sound $v$ in water.

Table IV represents the results after a least squares fitting estimation, showing the estimated sound velocity and the related offset for each situation (original and –20–20 dB)

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>Estimated time (μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{t}_{14,\text{Org}}$</td>
<td>$\hat{t}_{14,-20}$</td>
</tr>
<tr>
<td>345.778</td>
<td>269.18036</td>
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<tr>
<td>481.128</td>
<td>360.43477</td>
</tr>
<tr>
<td>535.131</td>
<td>396.63307</td>
</tr>
<tr>
<td>535.252</td>
<td>396.76493</td>
</tr>
<tr>
<td>759.182</td>
<td>547.80206</td>
</tr>
<tr>
<td>862.887</td>
<td>617.79228</td>
</tr>
</tbody>
</table>
The speed of sound in pure water with similar temperature was reported in Ref. 16 to be 1481.727 m/s and in Ref. 17 to be 1482.36 m/s. Figure 4 shows the speed of sound variation versus different SNR ratios in a logarithmic scale; the approach can be seen to be less sensitive to noise when the SNR is greater than −10 dB.

Table V shows the measured distances and the corresponding least squares fitting showing the residual errors.

### IV. CONCLUSION

In this paper, a time delay estimation approach based on phase differences between components of the received short cw pulse signal is demonstrated. A validation underwater experiment showed that a resolution greater than 1/50 of the wavelength was achievable. Using local phase difference information, no ambiguity in phase measurement arises; hence there is no need to use the Chinese remainder theorem or a coherent local oscillator to overcome the well-known phase ambiguity problem. As phase information is usually regarded to be useless in most correlator-based techniques, the approach developed in this paper is distinctive. Compared to correlator-based approaches, which have an estimation accuracy on the order of the used bandwidth, the suggested approach is not limited by the bandwidth but only by the ability to measure phase differences accurately. This leads to much improved performance once the SNR is sufficiently high. This approach is tolerant to additive Gaussian noise when the SNR is acceptable. Consequently, the technique offers the potential to outperform animals in subwavelength measurements. Although a bat can achieve a resolution of 20 μm in air, potentially we would get a resolution of 4 mm/360 = 11 μm. As a new approach, we expect that the algorithm will be improved to the point where one degree of phase can be resolved, suggesting that performance similar to the bat will be achieved by such technology in the near future.

### ACKNOWLEDGMENTS

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TABLE V. The measured (using the encoders) and the estimated distances with different SNR in Rx.

<table>
<thead>
<tr>
<th>Measured (mm)</th>
<th>Estimated distances (mm)</th>
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</thead>
<tbody>
<tr>
<td>d_{ave}</td>
<td>\hat{d}_{ave}</td>
</tr>
<tr>
<td>305.800</td>
<td>305.785</td>
</tr>
<tr>
<td>345.778</td>
<td>346.017</td>
</tr>
<tr>
<td>481.128</td>
<td>481.372</td>
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<td>535.131</td>
<td>535.065</td>
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</tbody>
</table>

APPENDIX: TIME DELAY ESTIMATION ALGORITHM USING PHASE DIFFERENCES

Input: Set of phases (ϕ₁, ϕ₂, ϕ₃, ϕ₄) and frequencies (f₁, f₂, f₃, f₄)
Output: Set of estimated time delay (t₁₂, t₁₃, t₁₄, t₂₃, t₂₄, t₃₄)

foreach Received signal, k do
    \text{Calculate DFT} (f₁, f₂, f₃, f₄, k);
    \text{Get} ϕ₁, ϕ₂, ϕ₃, ϕ₄
end

foreach (i, j)={(1, 3);(1, 4);(2, 3);(2, 4);(3, 4)} do
    \Delta ϕᵢ = ϕᵢ - ϕᵣ;
    \text{if} (|\Delta ϕᵢ|<0) then
        \Delta ϕᵢ = \Delta ϕᵢ + 360.0;
    end
end

\Delta ϕ₁₂ \equiv \frac{\Delta \phi₁₂}{\Delta \phi₁₂}; \Delta f₁₂ \equiv f₂ - f₁; \hat{t}_₁₂ \equiv \frac{\Delta \phi₁₂}{\Delta \phi₁₂}; \hat{t}_₁₂ \equiv \hat{t}_₁₂

foreach (i, j)={(1, 3);(1, 4);(2, 3);(2, 4);(3, 4)} do
    \text{if} (|\hat{t}_₁₂| \times Δ \phiᵢ < 1.0) then
        \hat{t}_₁₂ = \frac{\Delta \phiᵢ}{Δ \phiᵢ};
        \text{nmin} = -1;
    end
end

\text{for each} n=0 \text{ to 1000 do}
\text{Calculate} \hat{t}_n = (n+\Delta \phiᵢ)/\Delta \phiᵢ;
\Delta \tau = |\hat{t}_n - \hat{t}_₁₂|;
\text{if} (\Delta \tau < \text{min}) then
    \text{min} = \Delta \tau;
    \hat{t}_\text{new} = \hat{t}_n;
    \text{nmin} = n;
end
end

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\text{W. L. Whitlows and J. A. Simmons, “Echolocation in dolphins and bats,” Phys. Today 60(9), 40–45 (2007).}