Three-Dimensional Aquifer Heterogeneity in Pumping Test Analysis
A Numerical Investigation Using Radial Flow Models.

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ABSTRACT

Pumping test results are often analysed on the assumption of radial symmetry. This may be because analytical solutions are used, which are limited to two-dimensional problems, or because of the widespread acceptance that this assumption is satisfactory when estimating average values of the aquifer hydraulic characteristics. A three-dimensional R-θ-Z model, built using an object-oriented approach, is used to investigate the effects of spatial heterogeneity on time-drawdown curves. Particular attention is paid to the variation of aquifer properties in the circumferential direction, a factor ignored in most pumping test analyses.

The mechanisms in the numerical model include a logarithmically increasing mesh spacing in the radial direction, features operating at the pumped borehole and a moving water table. The model is applied to two and three-dimensional idealised aquifers to establish the impact of aquifer heterogeneity. Three-dimensional aquifer heterogeneity is shown to produce time-drawdown curves that can be matched using two-dimensional numerical models, giving a misleading interpretation of the aquifer flow processes. For example, a delayed response to pumping at an observation borehole due to a combination of low radial permeability and a fracture in the circumferential direction can be mistaken for large aquifer storage when radial symmetry is assumed. A field example is presented where this mechanism is believed to operate.

INTRODUCTION

The shapes of the time-drawdown curves obtained from pumping tests reflect the physical characteristics of the aquifer. Analysis of these curves allows the characteristics to be determined. Analytical and numerical solutions have been developed to analyse observed time-drawdown data and to estimate the corresponding parameter values. Simplifications are needed in both cases but numerical solutions prevail over analytical solutions because they include more flow processes. Radial symmetry is one simplification that is commonly used when analysing pumping test results. This may be caused by the widespread acceptance that pumping test results yield average aquifer transmissivity values as demonstrated by Tumlinson et al. (2006).

This paper uses a multi-layer R-θ object-oriented finite difference model to investigate the role of circumferential flow components, which are zero when radial symmetry is assumed. Two theoretical investigations and one field application are used for this purpose.

MODEL DEVELOPMENT

The axi-symmetric flow equation describing flow through a porous medium to a well under confined condition is given, in terms of the drawdown s, by Equation 1. Where k_r, k_θ and k_z, are the hydraulic conductivities (m d^{-1}) in the radial, circumferential and vertical directions respectively. S_s (m^{-1}) and N (d^{-1}) are the specific storage and an external source term respectively. The derivation of the above equation uses the transformation function $a = \ln r$. The distance between nodes increases logarithmically in terms of r but they are equally spaced in the transformed coordinate system. The vertical dimension is divided into numerical layers with each representing one physical layer. The top and base elevations of a layer are defined at

$$\frac{k_r}{r^2}\left(\frac{\partial^2 s}{\partial a^2}\right)+\frac{k_\theta}{r^2}\left(\frac{\partial^2 s}{\partial \theta^2}\right)+k_z\left(\frac{\partial^2 s}{\partial z^2}\right)=-S_S\frac{\partial s}{\partial t}+N$$

Equation 1. Axi-symmetric flow equation

each of its constituent nodes to allow spatial variation of the layer thickness. The horizontal conductance between nodes is based on the average saturated thicknesses at the nodes and the hydraulic conductivity between them. The storage coefficient at each node is calculated by multiplying the specific storage value by the saturated thickness.

To simulate groundwater flow in an unconfined aquifer an additional mesh, which represents the moving water table, is added above the top layer. The equation describing free surface movement is non-linear and its complexity has to be reduced to obtain an implicit numerical solution. This is achieved by assuming that the hydraulic gradients are small so that their products yield terms with even smaller values that can be ignored. The moving water table equation becomes $\partial\varphi/\partial t = 1/S_y\left(k_z\,\partial s/\partial z\right)$, where φ is a function that represents the location of the water table and S_y is the specific yield. The term $k_z\,\partial s/\partial z$ is a vertical flow term equal to $S_y\partial\varphi/\partial t$. The water table is taken into account by adding this term to the basic flow equation and applying it at the water table nodes (Rushton and Redshaw, 1979).

Water table nodes provide the vertical flow component to the underlying aquifer and they also transfer water horizontally and circumferentially to adjacent water table nodes. Both the water table and the aquifer nodes can move vertically based on the head values calculated at the end of each time step. The positions of the aquifer nodes that are directly connected to the water table nodes are modified so that they always occupy the mid-point between the water table and the base of the corresponding aquifer layer (Figure 1). This collapsing numerical grid significantly reduces the groundwater head oscillations caused by the creation of internal boundaries. These boundaries are created when the free surface falls sufficiently and the horizontal conductance between two nodes is set to zero to disconnect an aquifer node from its adjacent free surface node. The process is explained below.

At time zero the water table is horizontal and there is only a vertical connection between the water table nodes and their underlying aquifer nodes (Figure 1a). In addition the conductance value between nodes A1 and A2 is based on the full layer thickness (the hashed area in Figure 1a). Later the water table and aquifer nodes move downward due to pumping as shown in Figure 1b. The conductance between the two-aquifer nodes is then based on the smallest saturated thickness between these nodes (elevation of W1 minus the elevation of the base of the layer as shown in Figure 1b). This depth is always smaller than the initial layer thickness and reduces towards zero with time. At the same time, horizontal conductances between the water table nodes (W1 and W2) are modified. The conductance between the free surface nodes is calculated as the mid-point saturated thickness (half the elevation of W2 minus the elevation of W1) multiplied by the hydraulic conductivity value of the layer where the water table nodes are located. If adjacent water table nodes occur in different layers, W1 in Layer 2 and W2 and Layer 1 for example, the conductance term uses the average hydraulic conductivity of the layers. When the distance between the water table node and the base of a layer becomes smaller than a user defined value, the water table disconnects from the underlying aquifer node which becomes inactive, and connects to the next lower aquifer node (W1 disconnects from A1 and connects to A3 in Figure 1c and A1 becomes inactive). Since the saturated thickness at the node which becomes inactive is small at this time (Figure 1c), the de-activation of the node does not lead to groundwater head oscillations.

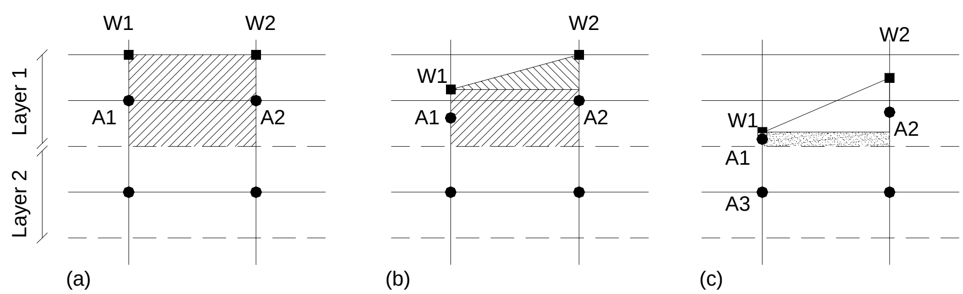

Figure 1. Movement of the water table and its effects on the connections between model nodes.

INVESTIGATION OF AQUIFER HETEROGENEITY

The ability of the model to simulate aquifers that are asymmetric about the pumped well is demonstrated by treating flow in a sloping bed aquifer. Analytical solutions developed by Hantush (1962a,b,c) describe flow in aquifers with a variable thickness. Investigating the drawdown at observation wells, Hayes (1998) notes that the results of pumping in sloping bed aquifer give a graphic illustration of the averaging of aquifer properties. He concludes, “Observation of pumping in the sloping bed aquifer gives aquifer transmissivity estimates that are an average of the properties present”. The aim of this exercise is to reproduce Hayes’ observation by simulating an identical aquifer. The aquifer is circular in plan and has a planar sloping bed. It is 10 m and 210 m thick at its thinnest and thickest points respectively. The hydraulic conductivity is 1 m d^{-1} and the specific storage is 10^{-5} m^{-1}. The aquifer is pumped at its centre at a rate of 1256 m^3 d^{-1} where the thickness is 110 m (Figure 2). Drawdowns are monitored for 100 days at two observation wells 10 m away from the pumped well. The model is divided in plan into four segments of which the first and the third contain the observation wells and represent the thickest and thinnest parts of the aquifer, respectively.

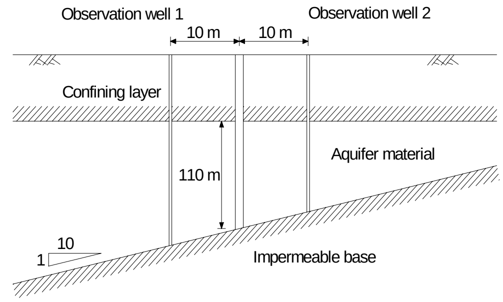

Figure 2. The sloping bed aquifer.

Figure 3a shows the time-drawdown curves at the two observation wells compared with the curve at an observation well at the same radius in a uniform, 110 m thick aquifer. The curves are indistinguishable and confirm Hayes’ conclusion. Hayes (1998) also demonstrates that the circumferential flow components that compensate for the lower transmissivity and storage on one side of the aquifer are the cause of this observation. However, noticeable differences are observed between the time drawdown curves that are produced at more distant observation wells. These differences become more pronounced as the distance between the observation and pumped wells increases (Figure 3b).

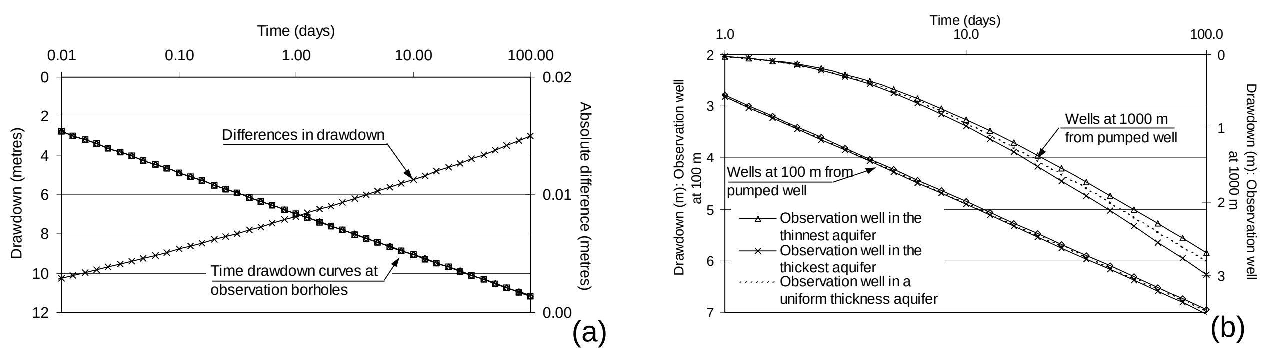

Figure 3. Time drawdown curves at observation wells on opposite sides of the pumped well. (a) 10 m from the abstraction well. (b) 100 m and 1000 m from the abstraction well.

The large differences between observation wells 1000 m from the pumped well (Figure 3a) also questions the concept of using the results obtained at these wells to estimate the regional average characteristics. The reduction in thickness on the one side of the aquifer far from the centre decreases the circumferential conductance and consequently causes difficulties in balancing the drawdown values. When the circumferential hydraulic conductivity is increased to 10 m d^{-1}, the differences in drawdown are greatly reduced as shown in Figure 4.

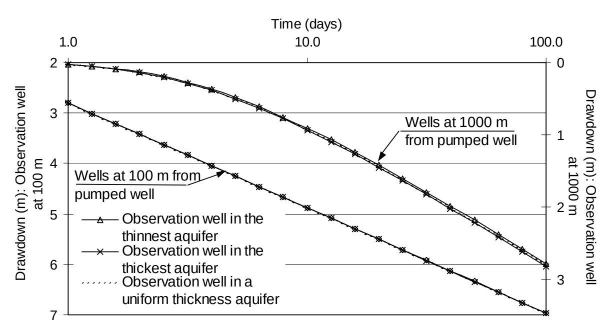

Figure 4. Response at boreholes 100 m and 1000 m on opposite sides of the pumped well.

The significance of circumferential flow components and their effects on masking aquifer variability can be demonstrated by considering lateral spatial variations. In this example the aquifer is of uniform thickness but is heterogeneous. It is divided into four segments as shown in Figure 5. The first quarter has a hydraulic conductivity of 0.01 m d^{-1} while the other three quarters are set to 10 m d^{-1}. Impermeable boundary conditions are imposed 10 km from the centre of the aquifer and the specific storage is 10^{-5} m^{-1} everywhere. Drawdowns are monitored at two observation wells; one located in the middle of the low permeability segment (Well 1), and one in the opposite high permeability zone (Well 2). Both wells are at 100 m from the abstraction well. It is anticipated that the drawdown will be observed in Well 2 before Well 1 because of the permeability differences. This is clear in the time-drawdown curves shown in Figure 6 where significant differences are observed at early times. However, the two curves converge at later times as observed by Toth (1966). An examination of the flow components reveals that the convergence is caused by significant flows moving circumferentially at later times.

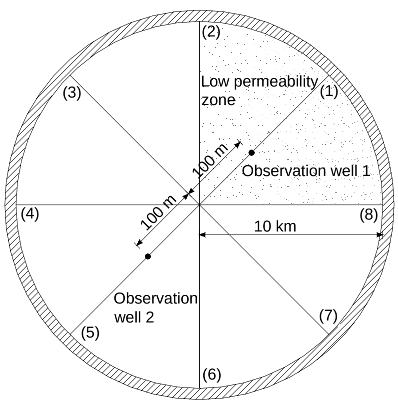

Figure 5. Plan of the two-permeability zone aquifer.

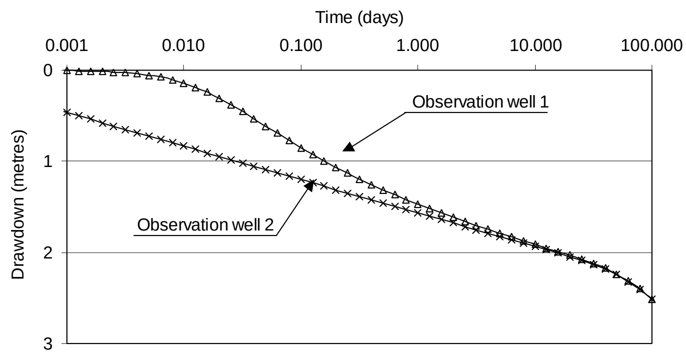

Figure 6. Response in the two observation wells.

FIELD APPLICATION

The implications of horizontal aquifer heterogeneity are investigated using the results of a pumping test conducted in the Chalk aquifer in the south-east of the London Basin. The test consists of three abstraction phases and one recovery phase. The pumped well is approximately 2 m in diameter and penetrates 12.5 m of saturated aquifer. The test includes many irregularities such as pump failures during the test and the close proximity of a large Chalk quarry. To simplify the discussion, the drawdown measured during the first part of the first abstraction phase at a selected observation borehole is studied.

The observation borehole is approximately 325 m away from the abstraction well. The abstraction rate increases from 3.6 to 5.4 Ml d^{-1} after eight days. The study that analysed the full data set (Spink and Mansour, 2003) suggests that a horizontal hydraulic conductivity of 140 m d^{-1}, a specific yield of 0.006 and a specific storage of 10^{-5} m^{-1} produce numerical results that match the field data. The response at the selected observation borehole is shown in Figure 7 where the curve produced using the above parameters is labelled Scenario 1. It should be noted that a high permeability zone exists around the pumped well. This provides storage that is readily available to the abstraction well and is essential to obtain a good fit to the field results at the abstraction well. This zone does not affect the response at the observation borehole. The specific storage is also increased to improve the fit at the observation borehole. Figure 7 shows a curve labelled Scenario 2 with a specific storage value of 10^{-4} m^{-1} that is in

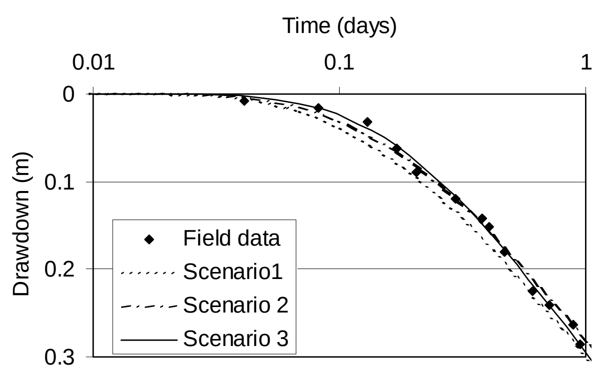

Figure 7. Response at the observation well in the Chalk aquifer.

better agreement with the field results. However, the sensitivity analysis carried out by Spink and Mansour (2003) shows that this storage coefficient reduces the agreement during the recovery at the pumped well and is discarded.

An alternative assumes that the observation borehole is located in a low permeability zone which delays the effects of abstraction as demonstrated above. A single layer r-θ model containing eight segments is used. The aquifer characteristics obtained from Spink and Mansour (2003) are used except in the first two segments where the values of the radial and circumferential hydraulic conductivities are reduced to 80 m d^{-1}. The observation borehole is 325 m from the abstraction well in the middle of the low permeability zone as shown in Figure 5. The model curve is labelled Scenario 3 and is shown in Figure 7. This curve is also in close agreement with the field data and suggests that radial symmetry is not a valid assumption here. This is reasonable given the highly variable nature of the Chalk.

SUMMARY

In this paper, a radial flow model with a moving free surface is introduced. This model is derived from that used by Mansour et al. (2003) but includes the capability to simulate sloping bed aquifers. In addition, the new model implements a collapsing grid to minimise groundwater head oscillations when aquifer nodes de-water. The radial flow model is used to study the impact of aquifer heterogeneity on the response to pumping. The main findings are:

- In sloping bed aquifers time-drawdown curves at observation wells that are close to the pumped well give transmissivity estimates that are close to the aquifer average. This is not true at observation wells that are located at significant distances from the pumped well.
- In heterogeneous aquifers time-drawdown curves at two observation wells that are equally spaced from a pumped well converge to the same value at the later times. This is consistent with Toth's (1966) observation of drawdown data for multiple observation wells converging on a single curve.
- Field investigation shows that radial symmetry is an assumption that has to be considered carefully. Here field data are matched with two sets of parameters. The first set is derived from a numerical model that assumes radial symmetry and the second set includes variations of horizontal hydraulic conductivity. While both fit reasonably well, the likelihood is that the aquifer is regionally variable.

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REFERENCES

Hantush M.S., 1962a. Flow of groundwater in sands of nonuniform thickness; Part 1, flow in a wedge-shaped aquifer. J. Geophys. Res., 67, 703-710.

Hantush M.S., 1962b. Flow of groundwater in sands of nonuniform thickness; Part 2, approximate theory. J. Geophys. Res., 67, 711-720.

Hantush M.S., 1962c. Flow of groundwater in sands of nonuniform thickness; Part 3, flow to wells. J. Geophys. Res., 67, 1527-1534.

Hayes P.J., 1998. An r-θ coordinate numerical model, incorporating mesh refinement, for the investigation of pumping tests in heterogeneous aquifers. Ph.D. Thesis, The University of Birmingham, England.

Mansour M.M., Spink A.E.F. and Riches J., 2003. Numerical investigation of flow to pumped wells in heterogeneous layered aquifers using radial flow models. MODFLOW and More 2003: Understanding through Modeling – Conference Proceeding, 1, 388-392.

Rushton K.R., Redshaw S.C., 1979. Seepage and groundwater flow. John Wiley and Sons, Chichester.

Spink A.E.F and Mansour M.M., 2003. Swanscombe Phase 2. Task 6: Preliminary Modelling. Report prepared for the British Geological Survey. Wallingford, England.

Toth J., 1966. Groundwater, geology, movement, chemistry and resources, near Olds, Alberta. Research council of Alberta Bulletin 17, Edmonton.

Tumlinson L.G., Osiensky J.L. and Fairley J.P., 2006. Numerical evaluation of pumping well transmissivity estimates in laterally heterogeneous formations. Hydrogeology Journal, 14, 1-2, 21-30.