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The influence of sampling frequency, non–linear interaction, and frictional effects upon the accuracy of the harmonic analysis of tidal simulations.

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Abstract

A two dimensional tidal model of the northwest European shelf is used to examine the influence of sampling rate, number of harmonic constituents analysed for, and length of data upon the accuracy of tidal constituents. Calculations show that in shallow water, where non–linear interactions give rise to higher harmonics, an accurate analysis can be obtained from a short span of data provided the higher harmonics are included in the analysis. In very shallow water where the tidal range is comparable to the water depth, asymmetry in the tidal signal due to substantial differences in friction at times of high and low water produces a number of semi–diurnal constituents in particular ν_2 and L_2 that must be included in the harmonic analysis. When these constituents together with the "classical" shallow water constituents are used in the harmonic analysis then an accurate analysis can be performed on a short span of data. The significant saving in computer time, particularly for a fine grid three dimensional model of using frequent sampling and analysing for a full set of constituents is stressed.

1. INTRODUCTION

In recent years there has been a significant increase in computing power and storage which has lead to the development of finer grid tidal models covering larger geographical areas. Although initially the emphasis was on refining the grids in two–dimensional (2D) vertically integrated models (for example the early shelf wide model of Flather (1976) compared with the recent finer grid models of Verboom et al. (1992); Gerritsen and Berentsen (1998); and regional models, Gjevik et al. (2006)) recently high resolution three–dimensional (3D) models have been developed (e.g. Davies and Aldridge (1993); Davies et al. (1997c,a,b); Davies and Hall (2000); Lee and Jung (1999); Young et al. (2000); Xing and Davies (2001)), requiring additional computational resources.

In early 3D models eddy viscosity was computed from the flow field, but recently e.g. Davies and Jones (1990); Luyten et al. (1996); Davies and Hall (2000) turbulence energy closure models have been used to compute this parameter. The additional calculations required with a turbulence closure model have also lead to an increasing demand in computational resources. Classically models were used to examine only the M_2 tidal constituent and its higher harmonics namely M_4 and M_6. However, recently other tidal constituents in particular S_2, N_2, K_1, and O_1 have been included in order to examine their spatial dependency. These additional constituents are important in shallow water regions where the system is highly nonlinear. Since the total level of turbulence depends upon these additional constituents and consequently if the benefits of using turbulence closure schemes are to be realised then these constituents need to be included. As finite difference grids become finer, or a finite element model is used with a refined grid in shallow water, then the importance of using a full set of tidal constituents increases.

The main problem of including a large number of constituents is that an harmonic analysis is required to separate out the various constituents, and also as we will show to take account of those produced by non–linear interaction in shallow water. Based upon classical tidal theory Godin (1972) used to analyse elevation and current observations, in which there is significant noise due particularly to meteorological effects, this can require a long time series (often in excess of a year) depending upon the constituents to be re-

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solved and the accuracy required. The computation of such a long time series with a 3D fine grid model, covering both shallow and deep water (which restricts the time step), and including a turbulence closure scheme is significant. Consequently any reduction in the time series required to perform the harmonic analysis and means of ensuring an accurate solution from this analysis is a significant bonus in numerical modelling both in terms of computer time, and memory required to store and process time series at every grid point for output.

Although we are primarily concerned with the accuracy of the harmonic analysis of time series of data derived from a fine grid three dimensional non–linear model (since this is a computationally expensive model) we will use a 2D non–linear model for illustrative purposes. In this paper a 2D model of the North West European Shelf together with synthetic time series produced by the addition of harmonics with a given amplitude and phase are used to examine how long a time series, and with what data interval, is required to perform an accurate harmonic analysis, particularly in shallow water regions. The synthetic time series is used in addition to the numerical model results since the number of harmonics contributing to the time series and in the subsequent harmonic analysis can be exactly controlled. By this means the influence of including or excluding various constituents upon the accuracy of other constituents derived by harmonic analysis can be exactly determined. In the numerical model the generation of constituents at frequencies other than the forcing frequency depends purely on location (deep or shallow water) and cannot be pre–determined as in the case of synthetic data. Also the analysis of the synthetic data is a check that conclusions based on model output are not a function of the model but depend only on analysis method. In the comparisons performed with the numerical model results the emphasis is on shallow water where bottom friction effects are particularly important and higher harmonics are produced. As we will show it is in these regions where there are problems in determining an accurate harmonic analysis, and hence if the benefits of using fine grid models are to be realised, accuracy is required.

In many tidal models, calculations are performed with only M_2 tidal forcing and conclusions as to the accuracy of the model are based on this single constituent calculation. The effect of including additional constituents in particular S_2, N_2, K_1 and O_1 upon the accuracy of the M_2 tide in shallow water is also examined. In this paper the emphasis is on the harmonic analysis of elevations, although similar conclusions apply to currents derived from 3D models (e.g. Davies and Aldridge (1993); Davies et al. (1997c); Davies and Jones (1992, 1996); Davies et al. (1997a,b); Kwong et al. (1997)). The form of the model is described in the next section, with subsequent sections dealing with the tidal calculations, and means of determining an accurate harmonic analysis.

2. The hydrodynamic model

The two–dimensional model used here covers the European shelf (Fig. 1), and hence a range of water depths from over 2000m to less than 10m. Since detailed plots of the depth distribution in the model are given in Davies et al. (1997c) and Kwong et al. (1997) they will not be presented here. Open boundary forcing, namely a radiation condition with input at the K_1, O_1, M_2, S_2, and N_2 frequencies is used to force the model. Any higher harmonics are generated by non–linear interaction in shallow water, in particular the non–linear momentum advection terms, and bottom frictional term.

Since extensive details of the two dimensional model formulation can be found in the literature, and means of solving the equations (e.g. Davies et al. (1997c,a,b)) only essential details will be presented here.

Boundary conditions took the form of a zero surface stress condition, while at the sea bed a quadratic friction law was applied. On closed boundaries, a no normal flow condition was applied, while along open boundaries a radiation condition namely,

$$q = q_T + \frac{c}{h}\zeta - \zeta_T \qquad (1)$$

was adopted, with q the normal component of depth mean current, ζ sea surface elevation, h water depth, $c = (gh)^{1/2}$ with g the acceleration due to gravity and

$$q_T = \sum_{i=1}^{m} Q_i cos(\omega_i t - \gamma_i) \qquad (2)$$

$$\zeta_T = \sum_{i=1}^{m} H_i cos(\omega_i t - G_i) \qquad (3)$$

In these equations, ω_i denotes the speed of constituent i with H_i and Q_i the amplitude of tidal elevation and current, and G_i and γ_i, the phase.

3. Tidal calculations

In a preliminary series of calculations the model was forced with only theM_2 tide (Table 1), and an harmonic analysis was performed using the M_2 tide and its higher harmonics (Group A, Table 2), although subsequently Groups B, C and D (Table 2) were used. In later calculations the model was forced with the O_1, K_1, N_2, M_2 and S_2 tides (Table 1) and harmonically analysed using various groups (namely E to G) of tidal constituents (Table 2). By this means the influence of the choice of constituents used in the harmonic analysis method and the frequency of storing data (summarised in Table 3) upon the accuracy of the solution could be assessed. In addition to analysing data at 10min and 60min intervals, in some cases data at 30min intervals was analysed and results are discussed in the text. Changes in the M_2 tide due to the presence of additional constituents could also be examined.

Fig. 1. Finite difference grid of the shelf tidal model.

3.1. *Harmonic analysis of numerical model tides generated with M_2 tidal forcing*

In an initial calculation the model was forced at the M_2 period and tidal elevations over the whole domain were saved at 10min intervals for a period of 25 days. The nature of the non–linear terms in the hydrodynamic equations is such that besides generating a tidal residual, higher harmonics of the tide at the M_4, and higher frequencies are produced particularly in shallow water. To examine the accuracy of theM_2 tide derived by harmonic analysis using various groups of constituents (Groups A to D in Table 2), the time series was analysed in various ways, for a number of shallow and deep water sites (see later).

3.1.1. *Determination of a "benchmark" solution*

In this section we initially determine that the model can produce an accurate M_2, M_4 and M_6 tide in the region. To this end output is saved at 10min intervals and the resulting time series is analysed using the full set of tidal constituents (Group A in Table 2). This accurate analysis approach (see later) yielded an accurate "benchmark" solution that was used to check that the model was determining a physically realistic tidal distribution. Initially we briefly examine the spatial distribution of the M_2, M_4, and M_6 tides and check their accuracy at a number of locations. Subsequently using time series from Lerwick and Avonmouth we briefly describe a method to check how long the model must be integrated for to remove the effect of the initial conditions and ensure that the harmonic analysis is independent of the in-

Tide	Period (°/h)	Boundary Forcing
O_1	13.943	Yes
K_1	15.041	Yes
μ_2	27.968	No
N_2	28.440	Yes
ν_2	28.512	No
M_2	28.984	Yes
L_2	29.528	No
T_2	29.959	No
S_2	30.000	Yes
MN_4	54.424	No
M_4	57.968	No
MS_4	58.984	No
M_6	86.952	No
$2MS_6$	87.968	No
Mm	0.544	No
MSf	1.0159	No
Mf	1.0980	No
$2MS_2$	27.968	(see μ_2)
$2SM_2$	31.016	No
MNS_2	27.424	No
MSN_2	30.544	No

Table 1
Symbol and frequency of the various tidal constituents used to force the model and in the analysis

(A) M_2, M_4, M_6, M_8, M_{10}, M_{12}
(B) M_2, M_4, M_6
(C) M_2, M_4
(D) M_2
(E) M_2, S_2, N_2, O_1, K_1, μ_2, MN_4, M_4, MS_4, M_6, ν_2, L_2, T_2
(F) M_2, S_2, N_2, O_1, K_1, μ_2, MN_4, M_4, MS_4, M_6
(G) M_2, S_2, N_2, O_1, K_1

Table 2
Groups of harmonics used in the analysis, with tidal constituents having frequencies shown in Table 1

Harmonic analysis method	Group of harmonics	Time interval in time series (min)
1	A	10
2	B	10
3	C	10
4	D	10
5	A	60
6	B	60
7	C	60
8	D	60

Table 3
Range of harmonic methods used to analyse the computed time series derived with M_2 forcing

tegration time period. In later sections by comparison to the "benchmark" solution we will examine how the M_2, M_4 and M_6 harmonics change when other analysis approaches (Table 3) are used and when other tidal constituents (i.e. O_1, K_1, N_2, S_2) are included.

Since detailed and accurate co–tidal charts are given elsewhere (Davies et al., 1997c; Kwong et al., 1997) and the co–tidal charts computed with the present model are comparable to these, only a brief discussion will be given here. The M_2 co–tidal chart (not presented) derived using method 1, (Table 3) shows amphidromic points in the Southern Bight of the North Sea and off the west coast of Denmark with degenerate amphidromic points at the southern tip of Norway and south–east of Ireland. The tidal distribution is in good agreement with other cotidal charts based on models and observations (Howarth and Pugh, 1983; Howarth, 1990). A detailed comparison with a limited number of A class gauges is shown in Table 4. (The A class gauges are highly accurate gauges which have been used in other model comparisons (Davies et al., 1997c,a,b)).

The computed M_4 and M_6 cotidal charts (not presented) show that tidal amplitude increases in shallow water regions where the non–linear terms are significant. These distributions are in good agreement with those derived from other models (Davies et al., 1997c) and based on observations (Howarth, 1990).

At shallow water ports such as Avonmouth and Heysham the model significantly overestimates the M_4 and M_6 amplitude, in part due to resolution problems. At other ports, given the model's limited resolution there is reasonable agreement. Since the main objective of the work here is to examine the influence of harmonic analysis method and the inclusion of other constituents, these results particularly for M_2 are of acceptable accuracy for this to be achieved.

In order to compare the effect of the various harmonic analysis approaches upon the accuracy of the tides in a range of regions from deep water to shallow, a number of harmonic analysis results from both deep and shallow water ports were examined and compared with the "benchmark" solution. Here for illustrative purposes we will concentrate upon the ports of Lerwick, and Avonmouth which were examined in detail. To determine the accuracy of the harmonic analysis method chosen (Table 3) an analysis was performed using a period (window) of 12h 30min starting at t = 0h and then moving the 12h 30min window through time in intervals of 4h. By this means it was possible to check that the results of the initial conditions had been removed, and also to see if a consistent harmonic analysis was obtained. Subsequently to investigate the effect of reducing the number of analysis constituents only a M_2, M_4 and M_6 analysis was performed, (method 2, Table 3), then M_2 and M_4 (method 3, Table 3) and finally only the M_2 tide was included (method 4, Table 3). The influence of using hourly data (Section 3.1.3) compared with 10min values was examined in a subsequent analyses, as was the influence of using 30min values.

3.1.2. *Harmonic analysis of 10min sampled data*

Time series of the amplitude of the M_2 tide at Avonmouth and Lerwick derived using methods 1 to 4 in Table 3 are shown in Fig. 2a and b (method 2 gave similar results to 1 and is not shown). From these figures it is clear that at both locations the amplitude of the M_2 harmonic derived using method 1 has reached a constant value after 7 days. This suggests that a model spin up time of this order is sufficient to remove the effects of the initial conditions after which the results from the analysis are independent of the period chosen. Namely a periodic steady state has been achieved.

At deep water ports (illustrated here with reference to Lerwick, Fig. 2b) a comparable level of accuracy to that

	Obs		Calc 1		Calc 2	
M_2	h_o	g_o	h_c	g_c	h_c	g_c
St Marys	176.5	130	172.2	151	169.6	166
Newlyn	169.7	135	168.9	151	165.4	166
Avonmouth	424.9	202	430.8	242	406.8	258
Hilbre	292.3	317	288.6	336	282.5	351
Heysham	315.6	293	304.9	2	292.6	17
Cromer	152.1	188	196.9	202	189.6	216
Sheerness	203.2	354	212.1	2	201.3	17
Dover	223.5	331	247.3	357	236.9	11
Portsmouth	141.7	326	137.9	354	132.6	8
Portland	60.7	191	70.9	189	71.3	207
St Helier	336.0	182	338.3	201	330.8	217
M_4						
St Marys	6.6	184	4.2	247	3.6	274
Newlyn	10.9	169	7.3	216	6.3	246
Avonmouth	28.5	347	107.0	66	87.7	103
Hilbre	19.8	203	17.3	237	15.4	263
Heysham	19.7	243	55.5	311	48.8	341
Cromer	7.4	277	11.1	331	9.8	360
Sheerness	11.0	16	17.1	312	14.6	346
Dover	26.6	220	19.2	277	16.8	307
Portsmouth	19.9	13	12.5	91	10.6	121
Portland	13.6	25	9.5	112	7.1	141
St Helier	19.9	299	19.3	355	15.8	27
M_6						
St Marys	1.5	350	0.6	84	0.3	135
Newlyn	0.6	335	0.6	71	0.4	145
Avonmouth	12.0	272	15.3	278	13.0	332
Hilbre	2.4	32	11.1	77	7.0	120
Heysham	2.0	10	12.7	326	6.6	8
Cromer	3.0	43	5.8	6	3.9	55
Sheerness	5.4	38	10.7	85	6.1	129
Dover	6.8	104	12.7	135	7.5	184
Portsmouth	11.5	148	17.2	255	10.2	302
Portland	6.0	60	10.5	114	6.5	161
St Helier	1.2	358	5.1	337	3.5	28

Table 4
Comparison of observed (h_o,g_o) and computed (h_c,g_c) amplitude (cm) and phase (degrees) of the M_2, M_4 and M_6 tide at a number of gauges, based on an accurate analysis

achieved using method 1 was found with method 2, although a slight variation in the M_2 tidal amplitude was found at the shallow water locations. Although an analysis based upon just the M_2 and M_4 tides (method 3) gave a comparable level of accuracy (with some small fluctuations) to that found with method 1, at deep water locations (e.g. Lerwick), a significant error was found at shallow water locations (e.g. Avonmouth), with changes in amplitude of the order of 1cm, depending upon the period chosen. When an analysis based on just the M_2 tide was performed (method 4), there was a significant variation at Avonmouth of the order of over 6cm, although at Lerwick the variation was not significant.

3.1.3. *Harmonic analysis of 60min sampled data*

When the harmonic analysis was based on hourly values (methods 5, 6, 7 and 8), the variation increased significantly at the shallow water locations (Fig. 2c). In this case a variation of over 10cm occurred at Avonmouth in an M_2 only analysis, which reduced to 4cm when M_4 was included. Results from an M_2, M_4, and M_6 solution were not significantly different from those found with an M_2 to M_10 analysis using hourly or ten minute values (Fig. 3a and c). At Lerwick, the difference between anM_2 only analysis using hourly data (method 8), and an accurate analysis was less than 0.1cm (Fig. 2d).

3.1.4. *Discussion of the importance of data interval and higher harmonics upon accuracy*

Results using 30min sampled data were slightly better than those with 60min but 10min values were really required for an accurate analysis. From these calculations it is evident that accuracy is improved by using 10min samples and analysing for constituents in group A, even when only the M_2 constituent is required. The use of group A constituents and 10min values is more important at shallow water than deep water ports. In order to understand this it is useful to compare the contribution of the higher harmonics to the solution at each location.

Results from the accurate harmonic analysis (method 1, Table 3) show (Table 5) that at a deep water location namely Lerwick the solution is dominated by the M_2 tide, while at shallow water ports namely Avonmouth higher harmonics are appreciable. A consequence of performing an harmonic analysis for the M_2 tide only is that these higher harmonics are not taken into account and hence contribute to noise in the record (Godin, 1972), and influence the accuracy of the M_2 tide. If they are included in the harmonic analysis, even though constituents such as M_8 and above cannot be resolved and hence may not be physically meaningful, their inclusion in the harmonic analysis does improve the accuracy of the M_2 tide. This is particularly important when only hourly output from the model is saved. In this

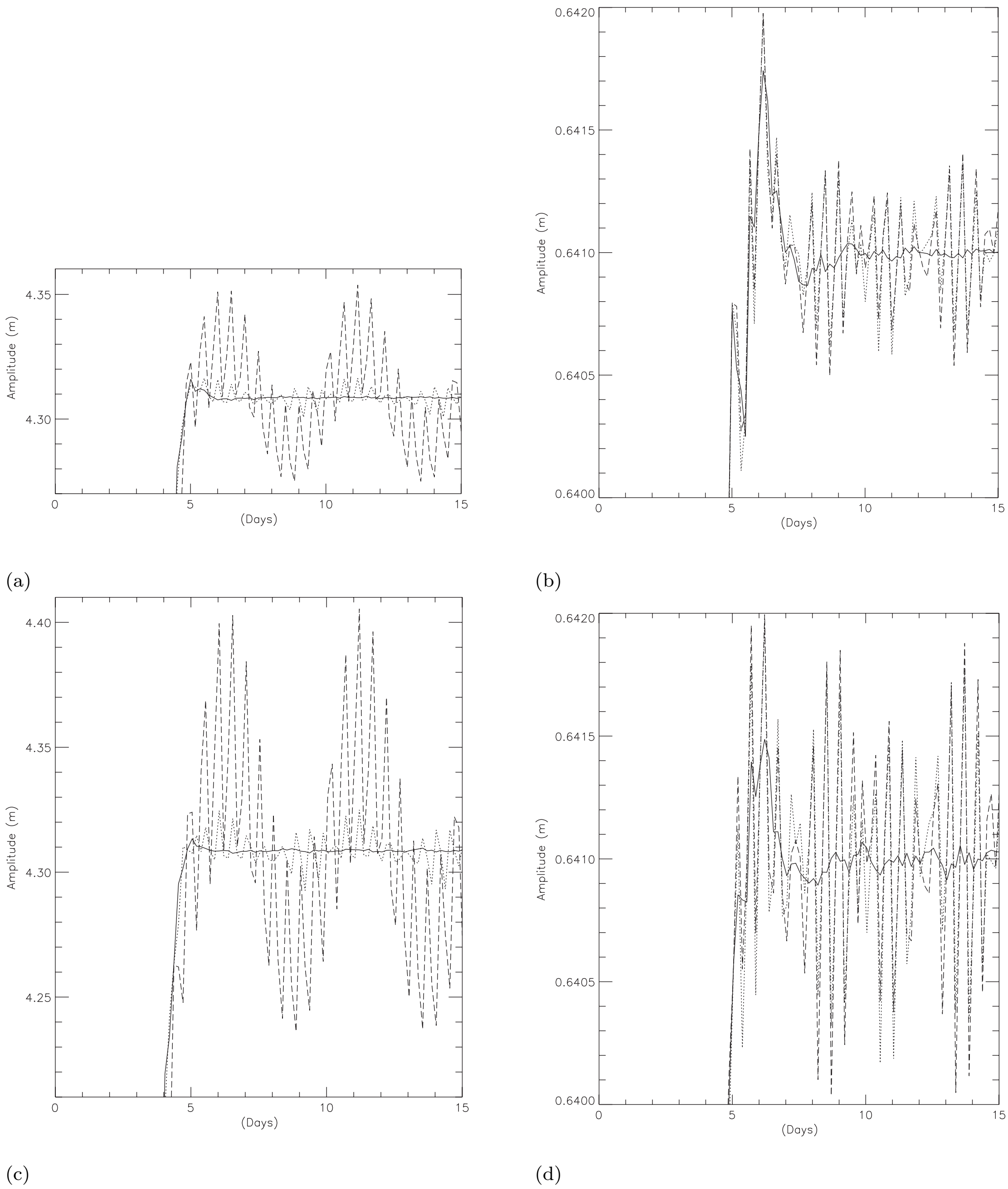


Fig. 2. (a) Time series at Avonmouth of the variation of the M_2 tidal amplitude with time based on 10min values, for constituent Set A (solid line), Set C (dotted line) and Set D (dashed line). (b) As (a), but for Lerwick. (c) As (a), but for hourly data. (d) As (b), but for hourly values. (e) As (a), but for synthetic data. (f) As (e), but for M_4 amplitude from synthetic data using Set A (solid line), Set B (dotted line) and Set C (dashed line). (g) As (e), but for M_6 amplitude from synthetic data using Set A (solid line), Set B (dotted line).

case the conditioning of the matrix in the harmonic analysis method is reduced from that computed using 10min values and hence any noise in the record (in this case higher harmonics not included in the analysis) reduced the accuracy of the M_2 tide.

This calculation suggests that provided an harmonic analysis is performed for the M_2, M_4 and M_6 constituents, then even when only hourly output is saved from the model, an accurate M_2 can be obtained. However, there may be some inaccuracies in the M_4 and M_6 harmonics. To obtain maximum accuracy (namely a "best practice" approach) it is essential to use 10min data, and a full set of harmonics (namely group A, Table 1).

3.2. *Harmonic analysis of synthetic data*

In the previous section numerical model results were used to examine factors influencing the accuracy of the M_2 tide in both deep and shallow water locations. In order to examine how these factors influence the accuracy of theM_4 andM_6 tide and complement the numerical model results,

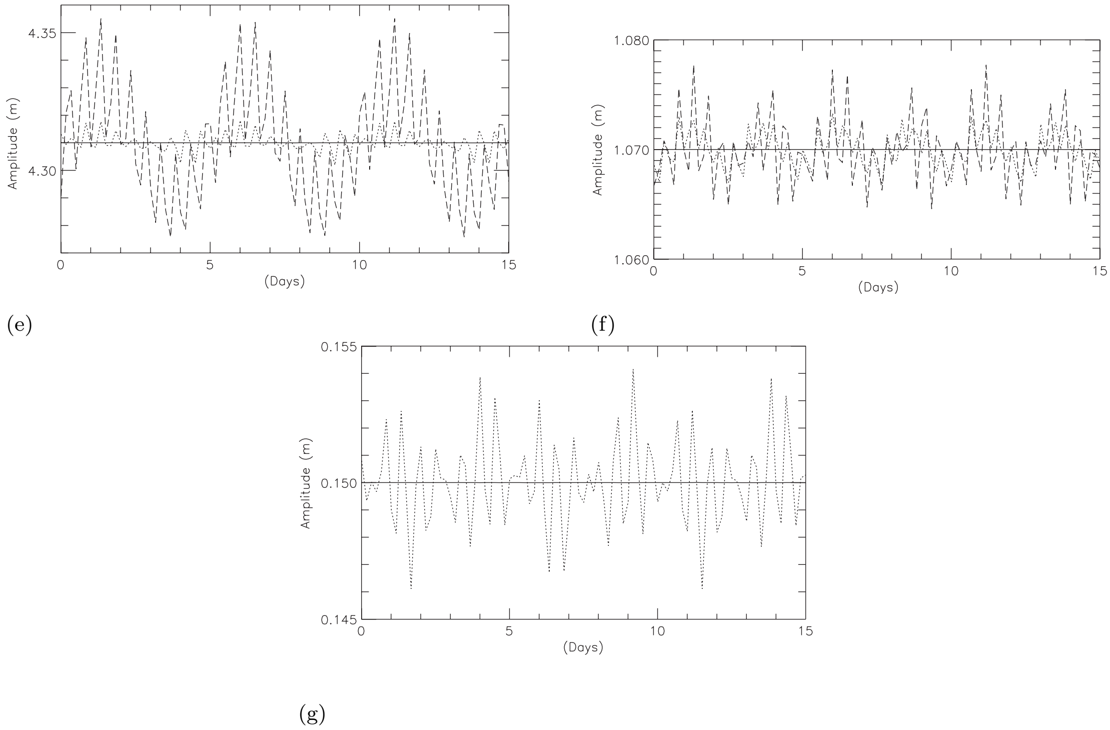


Fig. 2. (Contd.)

the analysis of a synthetic time series is considered here. By this means we can be sure that the time variation found initially in the M_2 harmonic analysis is not a function of the numerical model.

To examine in detail the influence of the harmonic analysis method (Table 3), upon the accuracy of the M_2, M_4, and M_6 tides at shallow water locations, a synthetic time series was generated using the amplitude and phase of the M_2 to M_{12} harmonics for Avonmouth as given in Table 5. The time variability of the M_2 amplitude based upon an analysis of 10min values for only this constituent is shown in Fig. 2e. As in the case with model output the harmonic analysis using only theM_2 tide shows appreciable time variability in its amplitude. This confirms that the variability is a function of the harmonic analysis approach and not due to the numerical model. However, the time variability in the analysis decreases giving an accurate value of M_2 amplitude when a full set (constituent set A) is used, as in the numerical model case.

At many shallow water locations, it is physically interesting to examine the amplitude of both the M_4 and M_6 tide which are appreciable in such regions. The amplitude of the M_4 tide derived from an M_2, M_4 analysis (Set C, Table 2), shows (Fig. 2f) an appreciable error (relative to its magnitude). This error diminishes whenM_6 is included (Set B), with an accurate analysis when the full set is used (Fig. 2f). Similarly for M_6 there is a significant error (Fig. 2g) using Set B, although this is reduced using a full set of constituents. When hourly data is used the accuracy is appreciably reduced.

Although in the case of the M_2 tide, performing an analysis in which the higher harmonics are included does not present a problem, as we will show later this does present some difficulties when a number of harmonics are involved.

Constituent	Avonmouth		Lerwick	
	h	g	h	g
M_2	431	224	64	305
M_4	107	42	1	36
M_6	15	248	1	282
M_8	9	141	0	94
M_{10}	1	338	0	24
M_{12}	3	282	0	232

Table 5
Amplitude h (cm) and phase g (degrees) of the M_2 tide and its higher harmonics at two locations

3.3. *Harmonic analysis of numerical model tides generated with M_2, S_2, N_2, K_1, and O_1 tidal forcing*

3.3.1. *Tidal distributions and influence of shallow water*

Although theM_2 tide has the largest magnitude, the S_2, N_2, K_1, and O_1 tides are significant and cannot be neglected in any calculation aimed at an accurate determination of the M_2 tide, as they contribute to the frictional effects in shallow water. The non–linear interaction of these tidal constituents in shallow regions gives rise to a range of shallow water constituents, which as we will show have to be taken into account when performing an harmonic analysis if an accurate description of the principal constituents, namely the M_2, S_2, N_2, K_1, and O_1 tides is to be derived.

In this calculation the 2D model with the same coefficient of bottom friction as used previously and M_2, S_2, N_2, K_1, and O_1 forcing applied along the open boundary was integrated in time for a period of 215 days. This length of data was used so that the variations due to analysis method (Table 3) of a range of constituents (Table 1) could be examined in detail. The M_2 cotidal chart (not shown) derived from the most accurate analysis (see later discussion) showed a similar distribution to that found previously, although tidal amplitudes were reduced in some shallow water areas (Table 4) due to the presence of increased friction arising from including the other constituents. This suggests that in shallow regions it is important to include the effects of other harmonics if an accurateM_2 tide is to be obtained. The spatial distributions of the S_2 and N_2 tides (not shown) were similar to those found for the M_2 tide, and other model studies (Davies et al., 1997c; Kwong et al., 1997), and a comparison with observations at some A class gauges is given in Table 4. The accuracy of the model was comparable to that of Kwong et al. (1997).

Cotidal charts of the higher harmonics M_4 and M_6 showed similar distributions to those presented previously, although their values had been influenced by the presence of the other constituents. The diurnal (K_1 and O_1) cotidal charts (not presented but comparable to Kwong et al. (1997)), show an amphidromic point off the south–west corner of Norway with tidal amplitudes within the North Sea reaching a maximum in the region of the Wash. At the shelf edge, there are regions of local intensification due to a shelf edge resonance in this area (Huthnance et al., 1986; Huthnance, 1989; Heaps et al., 1988). In the present model the grid is not sufficiently fine to resolve these features, which have been examined in more detail using localised shelf edge models (Proctor and Davies, 1995; Xing and Davies, 1996). A comparison of computed values based on an accurate analysis (method 1, Table 3b) with observations at a limited number of locations is given in Table 6. This comparison suggests that the model can accurately (given its limited resolution in shallow water regions) reproduce the main tidal constituents, and hence the time series from the model can be used to examine the sensitivity of tidal constituents to analysis method.

Before considering the accuracy of a range of harmonic analysis approaches it is useful to examine the time series of elevations at a deep water port (Lerwick) and shallow port (Avonmouth) (Fig. 3a and b). The first 50 days from the model are shown for illustrative purposes. The elevation time series at Lerwick (Fig. 3a(i)) shows a spring–neap cycle modulated by the N_2 tide, and a diurnal variation due to the K_1 and O_1 tide. At any given time the maximum high water and minimum low water tidal elevations are comparable. Similar time variations (Fig. 3b(i) and (ii)) are found at Avonmouth. However, at this port the magnitude of low water is significantly less than high water. This is due to the shallow water at this location and the nature of the bed stress term $k\rho U|U|/(h+\zeta)$, where k is the bottom drag coefficient, h mean water depth, ρ density, and U instantaneous current magnitude. As free surface elevation ζ decreases the bed stress term for a given U increases, with bed stress becoming infinite if $\zeta = -h$. For this reason a drying condition (see Jones and Davies (1995)) is used in the model which prevents ζ falling below h by effectively stopping the flow out of the region when $h + \zeta$ is small, usually of order 0.1m, and consequently keeping ζ constant with time until the region floods again. In a fine resolution model (e.g. the 1km grid model of the eastern Irish Sea of Davies and Lawrence (1995)) this can lead to a time series with an essentially fixed ζ at low waters and varying ζ only at high waters when the near coastal region has flooded. In the present coarse grid model drying has not occurred at Avonmouth but high frictional dissipation has reduced the amplitude of the tide at low water, to give the sinusoidal time series shown in Fig. 3b.

In shallow water non–linear interaction between the major tidal constituents in particular M_2, S_2, and N_2 can lead to both higher harmonics (e.g. M_4, MS_4, MN_4. . . see Kwong et al. (1997) for detail) and also additional semi–diurnal constituents in particular μ_2 due to interaction between M_2 and S_2 (Kwong et al., 1997). Consequently besides analysing for the tides at the forcing frequencies (M_2, S_2, N_2, K_1 and O_1, Group G, Table 2), these other constituents (Group F, Table 2) must be considered. (The consequences of not including them will be examined later.) As we will show, even group F is insufficient for an accurate analysis at Avonmouth.

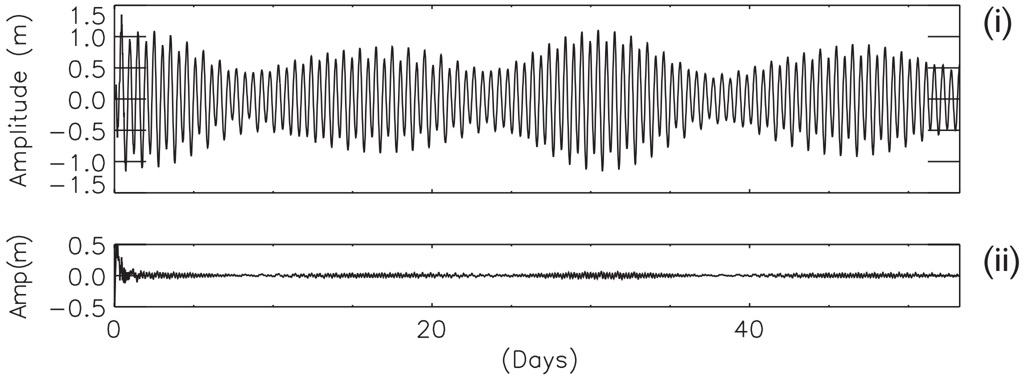
It is evident from Fig. 3a(ii) that at Lerwick, apart from an initial period of order a few days (when the model is "spinning up") the residual after removing the tide based on the Group G analysis was small. When this group was expanded (Group F) to take account of shallow water constituents which were small at this location, the computed residual (not shown) was negligible. However, at Avonmouth (Fig. 3a(ii)(iii)) even though the residual decreases going from Group G to F, it is evident that the residual time series, determined by subtracting the time series computed using the Group F constituents, clearly shows (Fig. 3b(iii)) that there is semi–diurnal energy still in the record. To reduce this and find the distribution of this energy over the semi–diurnal band, additional constituents namely ν_2, L_2 and T_2 were added giving Group E (Table 2). The addition of these constituents slightly reduced the residual. To understand the importance of including these constituents at some ports and not others it is necessary to examine their magnitude. The magnitude of these constituents, based on accurate analysis of the model data, and that of μ_2 for a number of shallow water and deep water ports is given in Table 7.

At deep water ports e.g. Lerwick and Stornoway where non–linear interaction is small, and the influence of bottom frictional effects upon higher harmonic generation is negligible and the amplitudes of μ_2, ν_2, L_2, and T_2 are below 1cm (Table 7). However, at shallow water ports e.g. Avonmouth, Heysham, St Helier, Immingham and Hilbre Island the amplitude of the μ_2, ν_2, L_2, and T_2 is significant (Table 7). Of these constituents μ_2, due to the non–linear interaction of M_2 and S_2 is clearly the most important. However, the other constituents particularly ν_2 and L_2 produced by

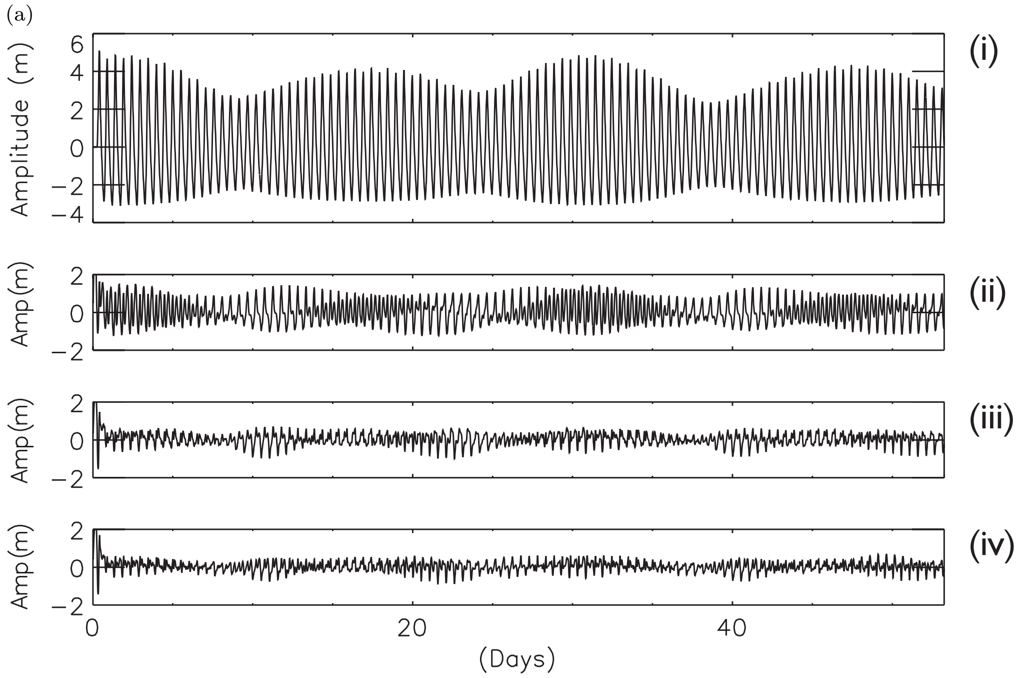
	Observed		90 or 215 days		30 days					
			E,F,G		E		F		G	
M$_2$										
St Marys	177	130	170	166	171	166	As 90 days		As 90 days	
Newlyn	169	135	165	166	167	165				
Avonmouth	425	201	406	258	409	255				
Hilbre	292	317	283	351	286	351				
Stornoway	139	197	133	227	133	227				
Lerwick	58	312	64	339	64	339				
Portland	61	191	71	207	70	205				
St Helier	336	182	331	217	329	216				
S$_2$										
St Marys	61	170	61	207	51	208	As 90 days		As 90 days	
Newlyn	57	178	59	209	49	227				
Avonmouth	150	261	130	319	248	357				
Hilbre	95	0	98	35	75	43				
Stornoway	55	230	56	263	57	264				
Lerwick	21	346	25	10	25	11				
Portland	32	242	35	263	59	270				
St Helier	132	231	133	267	165	280				
N$_2$										
St Marys	34	110	34	134	35	151	33	133	33	132
Newlyn	32	116	33	134	36	156	31	133	31	132
Avonmouth	75	187	69	236	170	251	56	242	53	240
Hilbre	56	294	56	318	53	336	55	316	56	316
Stornoway	28	174	30	199	32	199	30	199	31	199
Lerwick	12	291	15	305	16	303	15	305	15	306
Portland	14	184	15	188	32	176	15	196	14	197
St Helier	65	166	66	189	92	192	64	192	63	189
K$_1$										
St Marys	5.4	99	6.4	122	6.4	122	6.4	122	6.2	122
Newlyn	6.2	110	6.4	126	6.3	125	6.3	125	6.3	126
Avonmouth	7.2	143	8.1	163	8.0	162	8.0	162	7.6	162
Hilbre	11.2	189	12.5	187	12.1	187	12.1	187	12.1	187
Stornoway	13.2	134	14.2	146	14.2	146	14.2	146	14.2	146
Lerwick	7.6	164	11.3	165	11.3	165	11.3	165	11.3	165
Portland	8.4	111	7.2	130	6.8	130	6.8	130	6.9	130
St Helier	9.0	97	9.4	130	9.2	130	9.2	130	9.1	130

	Days (30-45)				Days (60-75)			
	G		F		G		F	
M$_2$								
St Marys	166	168	170	168	167	167	171	167
Newlyn	161	168	166	168	163	167	168	167
Avonmouth	409	266	427	262	409	263	429	225
Hilbre	275	353	284	353	277	352	286	353
Stornoway	134	228	134	228	134	228	134	228
Lerwick	65	339	65	339	65	339	65	339
Portland	81	210	80	206	78	209	78	204
St Helier	339	220	345	219	337	219	344	218
S$_2$								
St Marys	60	208	61	207	60	208	61	208
Newlyn	57	209	58	208	57	209	59	209
Avonmouth	126	324	124	319	130	325	131	320
Hilbre	96	35	99	34	96	35	99	35
Stornoway	56	264	56	264	56	263	56	263
Lerwick	25	11	25	11	25	10	25	10
Portland	35	267	34	264	36	266	34	263
St Helier	132	269	132	268	134	269	134	268
N$_2$								
St Marys	28	126	30	138	27	132	33	142
Newlyn	24	123	27	140	24	133	32	145
Avonmouth	18	355	52	265	47	306	76	255
Hilbre	46	306	47	325	45	313	53	327
Stornoway	31	201	31	200	31	200	31	199
Lerwick	14	310	15	308	15	309	15	307
Portland	18	234	21	206	21	218	23	189
St Helier	47	208	63	207	59	207	76	200
K$_1$								
St Marys	6.5	125	6.5	123	6.4	122	6.6	120
Newlyn	6.5	129	6.5	127	6.3	125	6.5	123
Avonmouth	8.7	173	8.5	163	7.8	168	8.7	158
Hilbre	12.5	187	12.8	189	12.5	183	12.5	185
Stornoway	14.3	146	14.2	146	14.3	146	14.3	146
Lerwick	11.5	164	11.5	164	11.3	164	11.3	164
Portland	7.4	132	7.3	132	6.7	125	6.7	123
St Helier	9.5	133	9.5	131	8.7	130	9.0	127

Table 6
Sensitivity of M_2, S_2, N_2 and K_1 tidal amplitude (cm) and phase (degrees) to time span and groups of harmonics (E, F, G) used in harmonic analysis method



(a)



(b)

Fig. 3. (a) Time series of (i) tidal elevation at Lerwick and (ii) residual after removing the tide due to Group G (Table 2) constituents. (b) Time series of (i) tidal elevation at Avonmouth, and residual after removing tide computed with (ii) Group G constituents, (iii) Group F constituents and (iv) Group E constituents.

Port	Amplitude of constituent (cm)			
	μ_2	ν_2	L_2	T_2
Stornoway	0.7	0.1	0.4	0.1
Lerwick	0.8	0.3	0.6	0.0
Avonmouth	53.2	15.1	19.8	2.4
Heysham	20.1	4.1	9.7	0.8
St Helier	16.2	3.7	7.9	0.0
Immingham	15.3	3.2	5.7	0.2
Hilbre	11.1	2.3	5.3	0.5

Table 7
Amplitude of shallow water semi-diurnal constituents at a number of ports from an accurate analysis

the drying condition and asymmetry in tidal elevation due to increased friction at low water are significant, especially at Avonmouth. In the calculation in which only the M_2 tide was present, the tidal range was much smaller and this asymmetry was less significant. When other tidal constituents were added the tidal range became comparable with the water depth, and significant fluctuations in bed stress arose during the tidal cycle.

Cotidal charts of the shallow water constituent (μ_2) and additional constituents ν_2 and L_2 (Group E, Table 2) exhibited significant spatial variability in their amplitude. The T_2 cotidal chart (not shown) revealed that its amplitude was the order of 2cm and confined to near coastal regions. The cotidal charts of μ_2, ν_2 and L_2 (not presented) show that they have a significant contribution in the near coastal regions where bottom friction and non–linear interaction can be substantial. In the present model, unlike in the calculations of Kwong et al. (1997), there is no forcing at the ν_2 or L_2 period through the open boundary or from the tide generating terms, and consequently these constituents are generated here by local effects and their amplitudes will be influenced by how bottom frictional effects and drying in shallow water regions are parameterised in the model. Obviously if the drying condition is set such that adjacent grid boxes in essence become land earlier or later in the tidal cycle, this will influence the tidal range and hence minimum water depth at the point under consideration. Significant changes in this minimum water depth through frictional effects will determine if constituents such as ν_2 or L_2 need to be included in the analysis. However, some care is required if these constituents are omitted, since as shown previously if shallow water constituents are omitted in the harmonic analysis they can influence the accuracy of the major constituents. The influence of these constituents in particular μ_2 will be considered in the next section.

3.3.2. *Influence of harmonic analysis method upon accuracy*

The accuracy of the harmonics derived from analysing different lengths of data with a range of tidal constituents, are given in Table 6, for a representative number of deep and shallow water ports, using groups of constituents E, F and G (Table 2). In the case of an analysis of a 215 or 90 day time series all groups of constituents yielded an identical analysis for the M_2, S_2, N_2, and K_1 tide. (Similar results were found for O_1 and K_1 in all the analysis methods used, and hence only K_1 is presented here.) A comparable level of accuracy was found with a 60 day analysis for the major constituents, although the accuracy of the smaller constituents in shallow water was reduced (not presented).

The M_2 and S_2 harmonics derived from a 30 day analysis with groups G and F were not significantly different from those derived with a 90 day analysis, although when group E was used the amplitude of harmonics such as μ_2, ν_2, L_2, and T_2 became artificially large and an error appeared in the amplitude of the M_2 and S_2 tide, particularly at shallow water ports such as Avonmouth and St Helier (Table 6). The diurnal tide was not appreciably influenced by the group of constituents used, and an analysis of 30 days yielded an accurate solution. Similarly an accurate M_2 and S_2 solution could be derived from a 30 day analysis with groups G or F. However, for N_2 at shallow water ports the 30 day analysis with groups G or F was less accurate than for M_2 and S_2. In the case of N_2 and M_2 the synodic period (the time taken for two constituents to come into phase) is twice as long as for M_2 and S_2, and this together with the smaller amplitude of N_2 will influence its accuracy.

At deep water ports such as Stornoway or Lerwick the generation of the μ_2 tide, and other constituents namely ν_2, L_2, and T_2 is negligible and hence, an accurate N_2 analysis is possible from a 30 day time series. Similarly at ports such as St Marys and Newlyn where the model computes an appreciable (of order 6cm) amplitude for μ_2, the N_2 analysis has an acceptable accuracy. However, in shallow water regions (e.g. Avonmouth and St Helier) where μ_2 and L_2 are appreciable (Table 7), the N_2 harmonic has a significant error.

In the case of a 15 day analysis, besides using groups G and F, two time periods were chosen, namely 30 days and 60 days after the start of the calculation, in order to examine the accuracy and variability in the analysis. For deep water ports such as Stornoway and Lerwick, accurate (identical with those from a 90 or 215 day analysis) M_2, S_2, N_2, O_1, and K_1 constituents could be obtained from either 15 day period using group G or F. For shallow water ports (e.g. Avonmouth) there were some errors in the M_2 and S_2 harmonics, with a larger error in the smaller semi–diurnal namely N_2, although the N_2 accuracy could be improved by including μ_2 within the harmonic analysis. This is because the frequency of μ_2 is much closer to N_2 than M_2 and S_2 (Table 1) and hence energy from μ_2 can 'leak' into N_2 and also because N_2 is a smaller constituent and hence more affected by noise (i.e. constituents not included in the analysis) than either M_2 or S_2. Although the diurnal constituents are small, they are well removed in frequency from both the semi–diurnal tides (e.g. μ_2, ν_2, L_2, T_2) and higher harmonics (e.g. M_4, MS_4, MN_4, M_6) which are significant in shallow water and hence an accurate analysis could be obtained at both deep and shallow water ports (although a variation of 1cm in amplitude was found at Avonmouth) using a 15 day time series.

Obviously the amount of energy which can leak from μ_2 to other semi–diurnal tidal constituents depends upon their relative amplitude and the time period considered. To examine this in detail it is valuable to use a synthetic time series containing the M_2, S_2, N_2, and μ_2 tides.

3.4. *Harmonic analysis of M_2, S_2, N_2 and μ_2 using synthetic data*

To complement the analysis of model output and further examine the extent to which energy can leak from μ_2 to other semi–diurnal tides it is valuable to generate a synthetic time series with constituent's amplitude and phase as shown in Table 8. The amplitudes of M_2, S_2, and N_2 were comparable to those found in the model at Avonmouth, although the μ_2 amplitude was increased slightly to illustrate what would occur in a very shallow region with an enhanced μ_2 due to increased non–linear interaction. The phase of the constituents is arbitrary.

Tidal amplitude and phase from an analysis of a 15 day time series for the M_2, S_2, N_2, and μ_2 harmonic (Analysis A, Table 8) were identical with those used to compute the synthetic time series. Although with such a short time span the matrix to be inverted is very badly conditioned, it can be accurately inverted, and since there is no noise (namely constituents that are not included in the analysis) in the time series, an accurate analysis is possible. When the same time series is analysed but only for the M_2, S_2 and N_2 constituents, (Analysis B, Table 8) then the contribution of the μ_2 tide to the time series is effectively noise. Because of the ill–conditioned nature of the analysis matrix this leads

Constituent (frequency)		Synthetic		Analysis A (15 days)		Analysis B (15 days)		Analysis C (15 days)	
		h	g	h	g	h	g	h	g
S_2	30.0000	133	60	133	60	151	57	134	60
M_2	28.9841	450	50	450	50	503	48	451	50
N_2	28.4397	90	40	90	40	145	78	86	41
μ_2	27.9682	86	30	86	30	–	–	–	–

Table 8
Sensitivity of tidal amplitude h (cm) and phase g (degrees) for a number of tidal constituents at Avonmouth to inclusion/exclusion of μ_2 in the analysis and length of data

to a 61% error in N_2, 12% in M_2 and 14% in S_2. However, when the analysis period is extended to 30 days (Analysis C, Table 8) the matrix is better conditioned and only the N_2 tide is significantly in error. This error is larger than would occur in the model, since the amplitude of μ_2 relative to N_2 has been slightly increased in this example. The analysis of the synthetic data time series confirms the main conclusions from the model time series. It clearly shows that the μ_2 tide should be included in the analysis in order to maximise the accuracy of the computed M_2 and S_2 constituents.

4. Discussion and conclusions

4.1. *Model forced with M_2 only*

A two dimensional model of the European Continental shelf has been used to examine the accuracy of computed tidal elevations to harmonic analysis method. Initial calculations were performed with only an M_2 tidal forcing, and output was saved at coastal locations at 10min intervals. In shallow water regions higher harmonics of the M_2 tide were generated, and calculations showed that it was essential to include these in the harmonic analysis, if an accurate M_2 tide was to be obtained. At deeper water ports where the higher harmonics are less important, then an accurate solution could be obtained from an M_2 only analysis. If the analysis was based on 30min or hourly values then it was particularly important to include the higher harmonics at the shallow water locations. In the case of a shallow water port where the M_4 tide is important, since its amplitude is significantly less than that of M_2, then it is essential to include all the higher harmonics in the time series analysis, if an accurate M_4 is to be derived.

4.2. *Model forced with multiple constituents*

Subsequent calculations showed that the presence of other tidal constituents in particular S_2, N_2, K_1 and O_1 could influence the amplitude of the M_2 tide by increasing bottom friction. These other constituents produced an increase in tidal range, which could lead to significant time variations in bed stresses in shallow regions with an associated asymmetry producing energy at the ν_2, L_2 and T_2 periods. Also, non–linear interaction between the M_2 and S_2 tides, give rise to the μ_2 tide. The analysis of a 90 day time series was identical to that from a 215 day period and was used as the "benchmark" in determining the accuracy of results from shorter period analysis.

Harmonic analysis at shallow water ports where μ_2 was appreciable showed that it was particularly important to include this constituent in order to avoid appreciable errors in the M_2, S_2 and N_2 constituents if a short (of order 15 days) model run was analysed.

4.3. *Conclusions and "best practice"*

In essence results from the detailed analysis of a range of methods used to harmonically analyse data from numerical models showed that 10min sampling was preferential to using 30min or hourly output. In a numerical model unlike in field measurements the only source of noise is that due to tidal constituents which are omitted from the harmonic analysis. Consequently provided all forcing constituents and those arising through non–linear interaction in shallow water are included, and an accurate matrix solver is used, then an accurate solution from a short span of data is possible. In very shallow regions where the tidal range is comparable to the water depth, it is essential to include a large number of constituents.

As the geographical extent of models increases and grids become finer, computational requirements rapidly increase particularly in three dimensional models. Consequently any means of achieving an accurate harmonic analysis from a short model run becomes more important.

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